Complete 1-loop study of exclusive J/ψ photoproduction at small x

Quarkonia as Tools 2024

Aussois

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IJCLab

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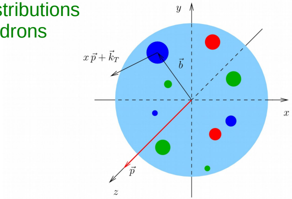
Based on work in progress with Chris Flett, Jean-Philippe Lansberg, Maxim Nefedov, Pawel Sznajder, Jakub Wagner

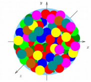
6D/5D

Wigner distributions for hadrons



$W(x,\vec{b},k_T)$

Experimentally inaccessible directly



perturbative Regge limit

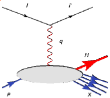
uPDFs (gluons)

Unintegrated parton distributions

$\int d^3\vec{b}$

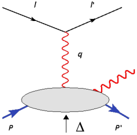
||

3D



$\int d^2k_T \int d\,b_z$

$\int d^2k_T \int Fourier(\vec{b})$



TMDs

$f(x,k_T)$

$b_T \Leftrightarrow \Delta$

$f(x,b_T) \longleftrightarrow H(x,0,t)$

$t=-\Delta^2$

$\xi=0$

GPDs

$H(x,\xi,t)$

Semi-inclusive processes

Transverse momentum dependent distributions

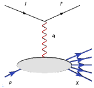
Impact parameter distributions

generalised parton distributions

exclusive processes

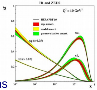
$\int d^2k_T$ $\quad$ $\int d^2b_T$ $\quad$ $t=0$ $\quad$ $\int dx$ $\quad$ $\int dx\, x^{n-1}$

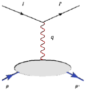
1D



PDFs

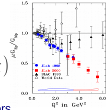
$f(x)$





FFs

$G_{E,M}(t)$



GFFs

generalized form factors

inclusive and semi-inclusive processes

parton distributions

elastic processes

form factors

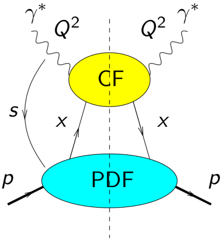
lattices

Deep Inelastic Scattering DIS: inclusive process

$\Rightarrow$ 1-dimensional structure

$\Rightarrow$ Collinear factorisation at the *cross section* level

Coefficient Function (hard) $\otimes$ Parton Distribution Function (soft)



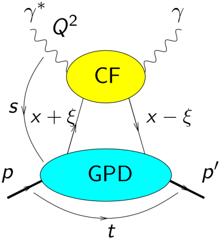
DVCS: exclusive process (non forward amplitude)

Fourier transf.: $t \leftrightarrow$ impact parameter

$\Rightarrow$ 3-dimensional structure

Collinear factorisation implies

Coefficient Function $\otimes$ Generalized Parton Distribution
(hard) (soft)



[X. Ji: hep-ph/9609381]

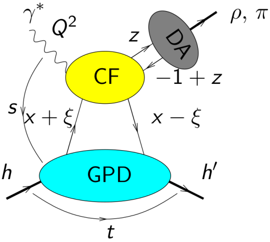
[A. Radyushkin: hep-ph/9604317, hep-ph/9704207]

[J. Collins, A. Freund: hep-ph/9801262]

[D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Horejsi: hep-ph/9812448]

DVMP: γ replaced by $\rho, \ \pi, \cdots$

GPD (soft) $\otimes$ CF (hard) $\otimes$ Distribution Amplitude (soft)



[J. Collins, L. Frankfurt, M. Strikman: hep-ph/9611433]

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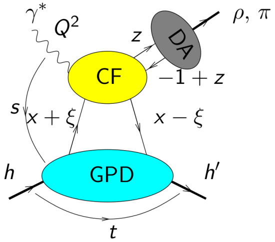

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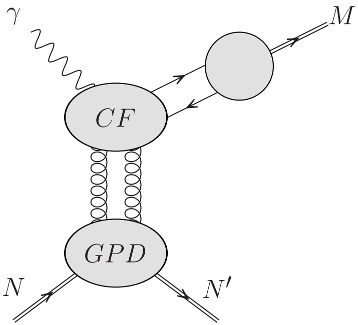
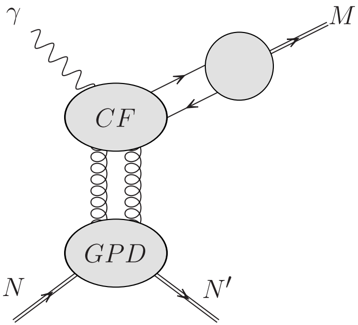
proofs valid only for some restricted cases



For $Q^2 \gg M^2$, proof of factorisation same as before.

Introduction
GPDs: Exclusive Heavy quarkonium photoproduction



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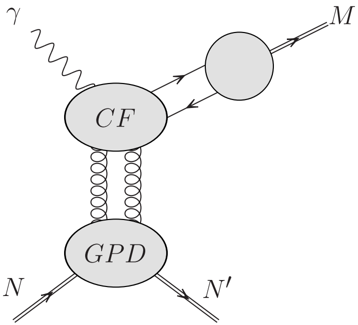
In *photoproduction*, situation not clear: [J. Qiu, Z. Yu: 2210.07995]



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But *fixed order NLO* result indicates that it works [D. Ivanov, A. Schafer, L. Szymanowski, G. Krasnikov: hep-ph/0401131]

Quark GPDs at twist 2 [M. Diehl: hep-ph/0307382]

without helicity flip (chiral-even Γ matrices): 4 chiral-even GPDs:
(Note: $\Delta = p' - p$)

$$\begin{aligned} F^q &= \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, \gamma^+ q(\tfrac{1}{2}z)\, |p\rangle \Big|_{z^+=0,\, z_\perp=0} \\ &= \frac{1}{2P^+}\left[H^q(x,\xi,t)\, \bar{u}(p')\gamma^+ u(p) + E^q(x,\xi,t)\, \bar{u}(p')\frac{i\,\sigma^{+\alpha}\Delta_\alpha}{2m} u(p)\right], \end{aligned}$$

$$\begin{aligned} \tilde{F}^q &= \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, \gamma^+\gamma_5\, q(\tfrac{1}{2}z)\, |p\rangle \Big|_{z^+=0,\, z_\perp=0} \\ &= \frac{1}{2P^+}\left[\tilde{H}^q(x,\xi,t)\, \bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q(x,\xi,t)\, \bar{u}(p')\frac{\gamma_5\,\Delta^+}{2m} u(p)\right]. \end{aligned}$$

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$H^q \xrightarrow{\xi=0,t=0}$ PDF q $\qquad$ $\tilde{H}^q \xrightarrow{\xi=0,t=0}$ polarised PDF Δq

Gluon GPDs at twist 2 [M. Diehl: hep-ph/0307382]

$$F^g = \frac{1}{P^+}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p'|G^{+\mu}(-\frac{z}{2})G_\mu^+(\frac{z}{2})|p\rangle\Big|_{z^+=0, z_\perp=0}$$
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$H^g \xrightarrow{\xi=0,t=0}$ PDF $xg(x)$

$\tilde{H}^g \xrightarrow{\xi=0,t=0}$ polarised PDF $x\Delta g(x)$

Exclusive J/ψ photoproduction

$$\mathcal{A} = \int_{-1}^{1} dx \int_{0}^{1} dz \, H(x)\phi(z) T(x,z)$$

- Factorise further using *NRQCD factorisation*:
  $\Longrightarrow \phi(z) \sim \delta(z - 1/2)$.

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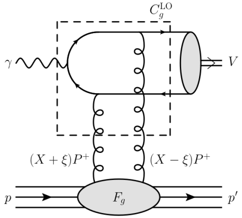
- Amplitude calculated up to NLO: D. Ivanov, A. Schafer, L. Szymanowski, G. Krasnikov [hep-ph/0401131]
  $\Longrightarrow$ *Collinear factorisation works*
- Also extended to *electroproduction* by C. Flett, J. Gracey, S. Jones, T. Teubner [2105.07657]

Leading order amplitude

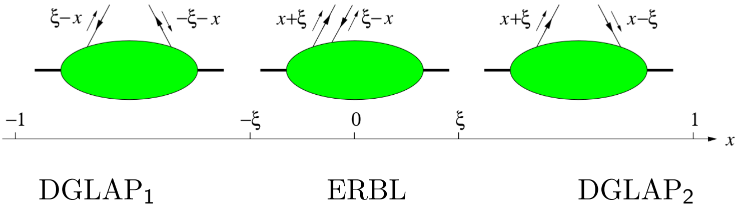
Due to the heavy quark in the coefficient function, the *LO amplitude* is sensitive to *gluon GPDs only*:



$$\mathcal{T}^{\mu\nu}_{\mathrm{LO}} = -g_{\perp}^{\mu\nu} \int_{-1}^{1} \frac{dx}{x} \left[C_g^{\mathrm{LO}} \left(\frac{\xi}{x} \right) \frac{F_g(x, \xi, \mu_F)}{x} \right]$$

$$C_g^{\mathrm{LO}} \left(\frac{\xi}{x} \right) = \frac{4\pi\alpha_s e e_q}{\left[1 + \frac{\xi}{x} - i\delta\,\mathrm{sgn}(x)\right]\left[1 - \frac{\xi}{x} + i\delta\,\mathrm{sgn}(x)\right]} \frac{2T_F}{N_c} \left(\frac{\left\langle \mathcal{O}\left[{}^3S_1^{[1]}\right]\right\rangle}{3m_c^3} \right)^{\frac{1}{2}}$$

DGLAP and ERBL regions in LO amplitude



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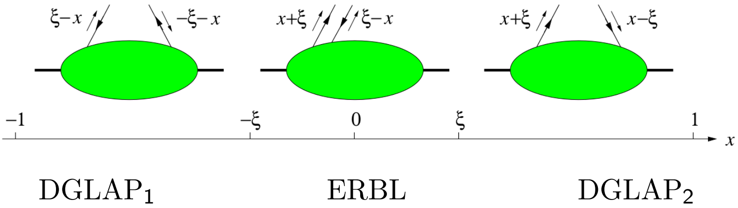


For LO amplitude:

- Picks up *imaginary part* at $x = \pm\xi$.

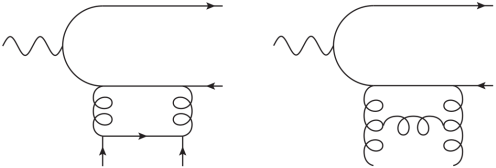
$$C_g^{\text{LO}}\left(\frac{\xi}{x}\right) = F_{LO}\frac{-i\pi}{2}\left[\delta\left(\frac{\xi}{x}-1\right)+\delta\left(\frac{\xi}{x}+1\right)\right]$$

$$\mathcal{T}^{\mu\nu}_{\text{LO}} = -i\pi\frac{g_\perp^{\mu\nu}F_{LO}}{\xi}F_g(\xi,\xi)$$

- Otherwise, amplitude fully real (PV contribution).
- Can fold on 0 by symmetry of the amplitude (at any order).

NLO calculation in collinear factorisation

NLO amplitude has contributions from *both* quark and gluon GPDs:



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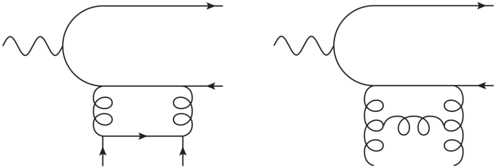

Imaginary part comes fully from the *DGLAP region* ($\xi \leq x \leq 1$)

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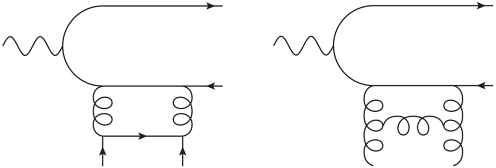

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$$\xi = \frac{M^2}{2S - M^2} \sim \frac{M^2}{2S}$$

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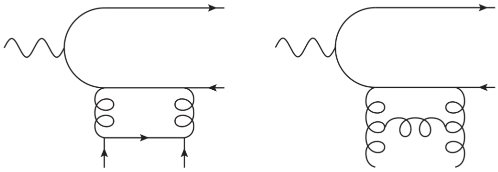

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Large S (small x_B in inclusive physics) $\leftrightarrow$ *small ξ*

High-energy limit of NLO calculation

At high energies, it is possible to relate the real part of the amplitude to the imaginary part through the *high-energy Regge dispersion relation*:

$$\frac{\mathrm{ReA}}{\mathrm{ImA}} = \frac{\pi}{2}\left(\frac{\partial \ln \mathrm{ImA}}{\partial \ln(1/\xi)}\right)$$

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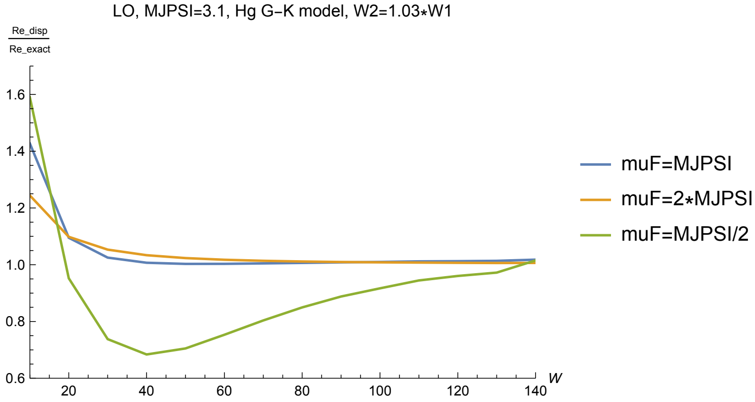
To test the validity of the above, we calculate the cross-section in two ways:

1. using the above.
2. computing the real part directly.

High-energy limit of NLO calculation: Checks



Comparison of Re disp/Re exact for scales $\mu_F = 0.5,\ 1,\ 2.0\ M_{J/\psi}$ using H_g G-K model at LO.

Approaches differ noticeably only in first few bins.

High-energy limit of NLO calculation: Pheno

Plot from K. Eskola, C. Flett, V. Guzey, T. Loytainen, H. Paukkunen [2203.11613]

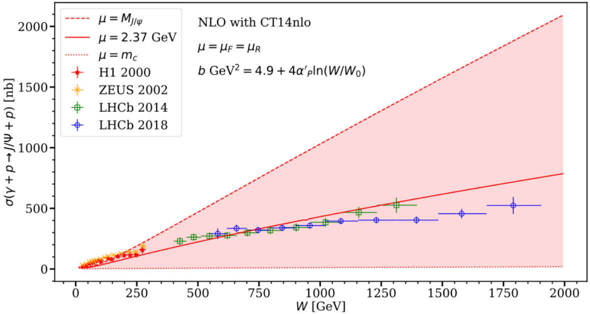


FIG. 2. The scale-choice uncertainty-envelope of exclusive J/ψ photoproduction NLO cross sections in *ep* and *pp* collisions as a function of the photon-proton c.m.s. energy W, computed to NLO pQCD with the CT14NLO [62] PDFs and compared against the experimental HERA data from H1 [26] and ZEUS [27], and LHC data from LHCb [28, 29]. The solid (red) line corresponds to the "optimal" scale explained in the text.

Origin of problem for NLO calculation

$$\mathcal{T}^{\mu\nu}_{\text{NLO}} \supset -i\pi \frac{g_\perp^{\mu\nu} F_{LO}}{\xi} \left[\hat{\alpha}_s \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 \frac{dx}{x} F_g(x,\xi) \right.$$
$$\left. + \hat{\alpha}_s \frac{C_F}{C_A} \ln\left(\frac{M^2}{4\mu_F^2}\right) \int_\xi^1 dx \left(F_q(x,\xi) - F_q(-x,\xi)\right) \right]$$

$\hat{\alpha}_s = \alpha_s(\mu_R) C_A / \pi$.

$F_g(x,\xi) \sim \mathrm{const}$, as $x \to \xi$
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Large $\ln \xi$ contributions are purely imaginary and come from the DGLAP region

Why large scale uncertainties present?

In the DGLAP evolution of low ξ GPDs, the probability of emitting a new gluon is strongly enhanced by the large value of $\ln \xi$

In contrast, the NLO coefficient function allows for the emission of *only one gluon*.

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$\Longrightarrow$ we cannot expect compensation between the contributions coming from the GPD and the coefficient function as we vary the scale μ_F .

$\Longrightarrow$ Hints towards a solution through *resummation* of these logarithms...

Scale fixing?

$$\mathcal{T}^{\mu\nu}_{\rm NLO} \supset -i\pi \frac{g_\perp^{\mu\nu} F_{LO}}{\xi} \left[\hat{\alpha}_s \ln\left(\frac{M^2}{4\mu_F^2}\right) \int\limits_{\xi}^{1} \frac{dx}{x} F_g(x,\xi) \right.$$
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Choose $\mu_F = m_c$. $\implies$ Large $\ln \xi$ terms cancel [S. Jones, A. Martin, M. Ryskin, T. Teubner: 1507.06942].

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However, impossible to move all enhanced by powers of $\ln\xi$ contributions from the coefficient function into the GPD (through μ_F evolution)

Big part of NLO correction from the hard coefficient eliminated, but *not* from higher order contributions.

Result after scale-fixing procedure

Plot from S. Jones, A. Martin, M. Ryskin, T. Teubner [1610.02272]

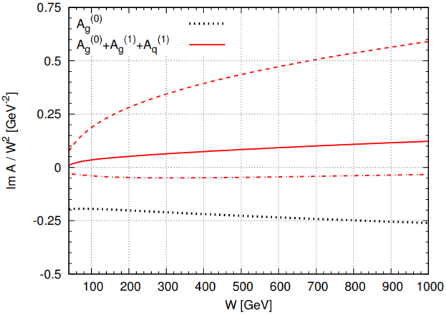
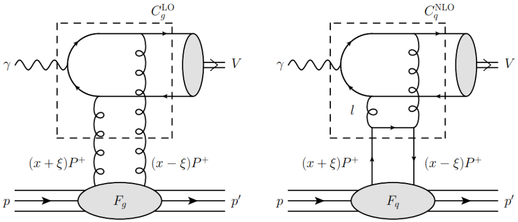


Figure 2: The dotted and continuous curves are the LO and NLO predictions, respectively, of $\mathrm{Im}A/W^2$ for the $\gamma p \to J/\psi + p$ amplitude, A, as a function of the γp centre-of-mass energy W, obtained using CTEQ6.6 partons [4] (with input $Q_0 = 1.3$ GeV) for the optimal scale choice $\mu_F = \mu_R = m_c$. The top three curves correspond to the NLO prediction for various values of the residual factorization scale μ_f, namely: $\mu_f^2 = 2m_c^2,\ m_c^2,\ Q_0^2$ respectively where $m_c^2 \equiv M_\psi^2/4 = 2.4$ GeV^2.

Q_0 subtraction procedure

S. Jones, A. Martin, M. Ryskin, T. Teubner [1610.02272]



To avoid double counting, exclude the $|l^2| < Q_0^2$ domain whose contribution is already included in the LO term using the input gluon GPD.

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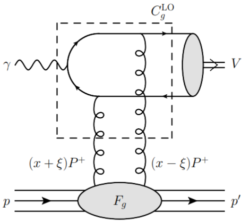

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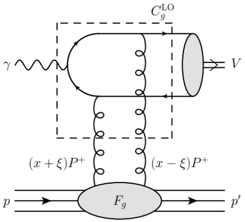

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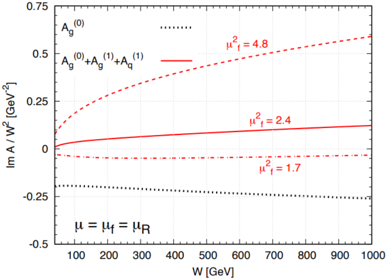
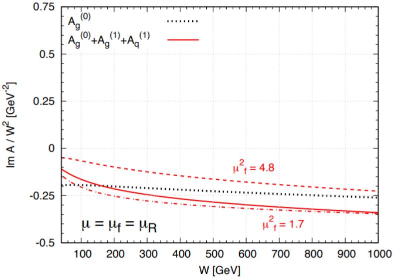
Typically power suppressed, but sizeable here: $\mathcal{O}(\frac{Q_0^2}{M^2})$

Result after Q_0 subtraction





Left: Scale-fixing procedure only

Right: Scale-fixing and Q_0 subtraction

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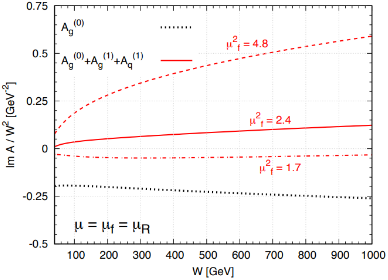

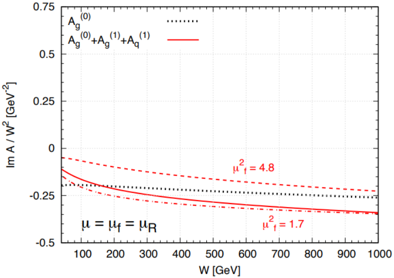

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Right: Scale-fixing and Q_0 subtraction

Process-dependent procedure!!

How to stabilise NLO result?

Our approach: High-energy resummation

Our appproach is to implement a *resummation* of these BFKL-type logs, compatible with the fixed order computation at NLO, which avoids double subtraction.

$\Longrightarrow$ *Doubly-logarithmic approximation (DLA)*

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Used in J-P. Lansberg, M. Nefedov, M. Ozcelik [2112.06789, 2306.02425] to cure instabilities in the total-inclusive-photoproduction cross sections of pseudoscalar quarkonia and vector S-wave quarkonia.

Implementation of high-energy resummation

HEF resummation of LLA contributions $\sim \alpha_s^n \ln^{n-1}(1/x)$ *at integrand level* to the imaginary part of the $C_g(x)$:

$$
\begin{aligned}
C_g^{\text{HEF}}(x) &= \frac{-i\pi}{2}\frac{F_{\text{LO}}}{x}\int_0^\infty d\mathbf{q}_T^2\;\mathcal{C}_{gi}(x,\mathbf{q}_T^2,\mu_F,\mu_R)h(\mathbf{q}_T^2), \\
h(\mathbf{q}_T^2) &= \frac{M^2}{M^2+4\mathbf{q}_T^2}.
\end{aligned}
$$

Implementation of high-energy resummation

Resummation factor, $\mathcal{C}_{gi}(x, \mathbf{q}_T^2, \mu_F, \mu_R)$ in the *Doubly-Logarithmic Approximation* (DLA) (in order to be consistent with fixed-order evolution of GPD) is given by the Blümlein-Collins-Ellis formula [hep-ph/9506403]

$$\mathcal{C}_{gg}^{(\mathrm{DL})}(x, \mathbf{q}_T^2, \mu_F^2, \mu_R^2) = \frac{\hat{\alpha}_s}{\mathbf{q}_T^2} \begin{cases} J_0\left(2\sqrt{\hat{\alpha}_s \ln\left(\frac{1}{x}\right) \ln\left(\frac{\mu_F^2}{\mathbf{q}_T^2}\right)}\right) & \text{if } \mathbf{q}_T^2 < \mu_F^2, \\ I_0\left(2\sqrt{\hat{\alpha}_s \ln\left(\frac{1}{x}\right) \ln\left(\frac{\mathbf{q}_T^2}{\mu_F^2}\right)}\right) & \text{if } \mathbf{q}_T^2 > \mu_F^2. \end{cases}$$

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For the quark channel, the resummation factor is given in the DLA by:

$$\mathcal{C}_{gq}(x, \mathbf{q}_T^2, \mu_F^2, \mu_R^2) = \frac{C_F}{C_A}\left[\mathcal{C}_{gg}(x, \mathbf{q}_T^2, \mu_F^2, \mu_R^2) - \delta(1-x)\delta(\mathbf{q}_T^2)\right].$$

Implementation of high-energy resummation

Useful representation in Mellin space:

$$\mathcal{C}_{gg}^{(\mathrm{DL})}(N, \mathbf{q}_T^2, \mu_F^2, \mu_R^2) \quad = \quad R(\gamma_{gg})\, \frac{\gamma_{gg}}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2}\right)^{\gamma_{gg}} .$$

γ_{gg} is the solution to the equation

$$\frac{\hat{\alpha}_s}{N}\chi(\gamma_{gg}) = 1, \quad \chi(\gamma) = 2\varphi(1) - \varphi(\gamma) - \varphi(1-\gamma), \quad \varphi(\gamma) = \frac{d \ln \Gamma(\gamma)}{d\gamma}$$

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Mellin transform maps logarithms $\ln(1/x)$ to the poles at $N = 0$:

$$\frac{1}{x} \ln^{k-1} \frac{1}{x} \leftrightarrow \frac{(k-1)!}{N^k}.$$

Implementation of high-energy resummation

In Mellin space,

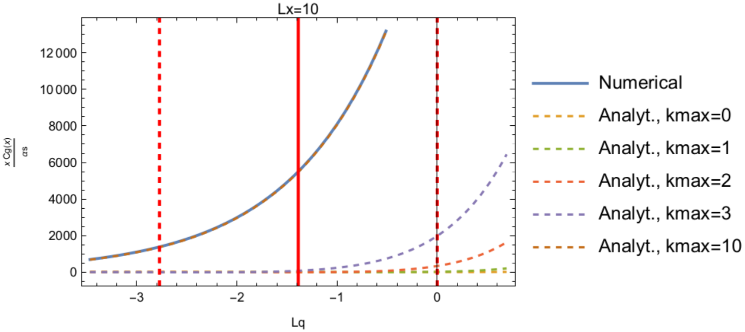
$$C_g^{\mathrm{HEF}}(N) \quad = \quad \frac{-i\pi}{2} F_{\mathrm{LO}} \left(\frac{M^2}{4\mu_F^2}\right)^{\gamma_N} \frac{\pi\gamma_N}{\sin(\pi\gamma_N)}.$$

In x-space, it becomes

$$\frac{2C_{\perp g}^{\mathrm{HEF}}(x)}{-i\pi\hat{\alpha}_s F_{\mathrm{LO}}} = \frac{1}{|x|}\sqrt{\frac{L_\mu}{L_x}} \left\{ I_1\left(2\sqrt{L_x L_\mu}\right) - 2\sum_{k=1}^{\infty} \mathrm{Li}_{2k}(-1) \left(\frac{L_x}{L_\mu}\right)^k I_{2k-1}\left(2\sqrt{L_x L_\mu}\right) \right\},$$

where $L_\mu = \ln[M^2/(4\mu_F^2)]$ and $L_x = \hat{\alpha}_s \ln 1/|x|$.

Convergence of HEF coefficient function



Solid: $\mu_F = M$, Dashed: $\mu_F = 2M, \frac{M}{2}$

Relatively *fast convergence* of HEF coefficient function

$\Longrightarrow$ *Convenient for numerical implementation*

Implementation of high-energy resummation

Expand in α_s (ie. in γ_N), and go back to x-space:

$$\frac{2C_g^{\mathrm{HEF}}(x)}{-i\pi F_{\mathrm{LO}}} = \delta(|x|-1) + \frac{\hat{\alpha}_s}{|x|}\ln\left(\frac{M^2}{4\mu_F^2}\right) + \frac{\hat{\alpha}_s^2}{|x|}\ln\frac{1}{|x|}\left[\frac{\pi^2}{6} + \frac{1}{2}\ln^2\left(\frac{M^2}{4\mu_F^2}\right)\right] + \ldots$$

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Quark coefficient function:

$$C_q^{\text{HEF}}(x) = \frac{2C_F}{C_A} C_g^{\text{HEF}}(x),$$

Matching

We use *additive matching*:

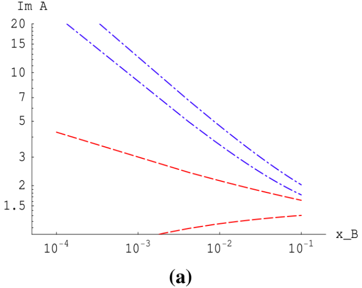
$$C^{\text{match.}}_{g,q}(x) = C^{\text{NLO CF}}_{g,q}(x) - C^{\text{asy.}}_{g,q}(x) + C^{\text{HEF}}_{g,q}(x),$$

$$C^{\text{asy.}}_{g}(x) = \frac{C_A}{2C_F} C^{\text{asy.}}_{q}(x)$$

$$= \frac{-i\pi F_{\mathrm{LO}}}{2} \frac{C_A \alpha_s}{\pi |x|} \ln\left(\frac{M^2}{4\mu_F^2}\right).$$

Attempt in Ivanov proceedings [0712.3193]

- He uses a very simple GPD model: $H_g(x,\xi) \sim xg(x) \sim x^{-0.2}$
- $C_g^{\text{HEF}}(x)$ is expanded in α_s.
- Vary μ_F^2 between $Q^2/2$ and $Q^2/4$



$Q^2 = 10\,\text{GeV}^2$ $\quad$ $Q^2 = 20\,\text{GeV}^2$

Red: Born + 1 term in $C_g^{\text{HEF}}(x)$ expansion
Blue: Born + 6 terms in $C_g^{\text{HEF}}(x)$ expansion

Conclusion

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- Perform *matching*.