Review of rare H,Z,W, top decays involving quarkonia

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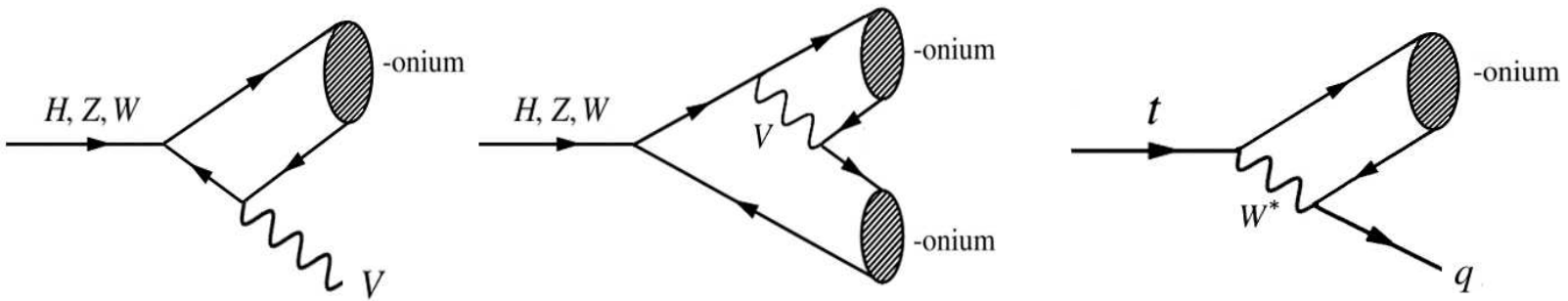
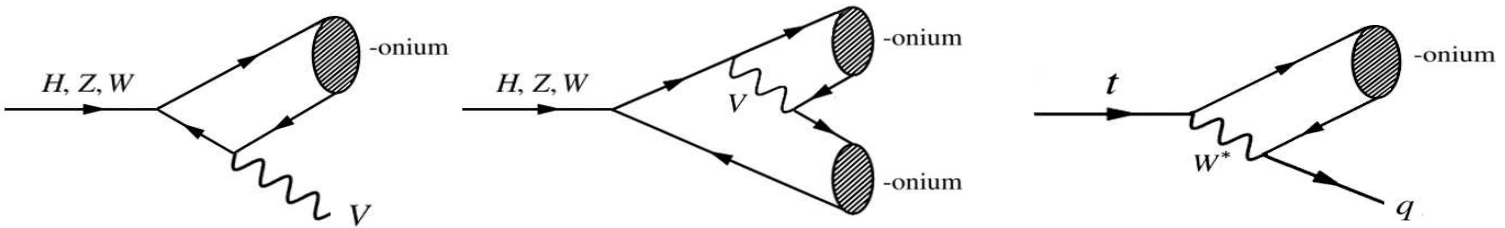


Introduction

- The work:
  - Comprehensive review of all exclusive decays to quarkonium (BR<10^{-5}) of the 4 heaviest particles:

    ~100 unobserved channels (~50 upper limits today).
  - Explicitly compute a few new decays (Z,H → leptonium+gamma, Higgs FCNC exclusive decays,...).
  - Make projections for HL-LHC/FCC-ee searches: Help guide and prioritize future experimental searches.
- Physics motivations:
  - Searches for BSM physics that might enhance very rare partial decay widths.
  - Probe lighter quark Yukawa couplings: Hcc, Hqq, via exclusive final states with mesons.
  - Stringent tests of the QCD factorization formalism, constraint poorly known nonperturbative hadronic bound-state parameters



Theoretical predictions

- For all rare decays collected, we indicate the BR of each channel and the theoretical framework used to compute them
  - Exclusive hadronic channels are all based on pQCD factorization: cross-section = perturbative ⊗ non-perturbative. Models of QCD factorization:
    - Light cone (LC): nonperturbative objects described by LCDAs. Applied for light-quark mesons (uds), and double quarkonia decays
    - Soft-Collinear Effective Theory (SCET): Resums multiple scales. Nonperturbative LCDAs. Mostly light mesons.
    - Heavy-Quark Effective Theory (HQET): LCDA describes mixed formation of light-heavy-quark mesons
    - Non-Relativistic QCD (NRQCD): For charmonium & bottomonium objects described by LDMEs
  - Leptonium channels: similar to hadronic ones, with much smaller BR. Never computed before. We have applied similar methods and derived the BR predictions.

Example:

				Exp. limits		Producible at	
$H \to \gamma$ +	X	Branching fraction	Framework	2023	HL-LHC	FCC-ee	FCC-hh
	ρ^0	$(1.68 \pm 0.18) \times 10^{-5}$	SCET+LCDA [13]	$< 8.8\times10^{-4}$ [74]	$\lesssim 6.8 \times 10^{-5}$	✓	✓
	ω	$(1.48 \pm 0.17) \times 10^{-6}$	SCET+LCDA [13]	$< 1.5\times10^{-4}$ [76]	$\lesssim 2.2 \times 10^{-5}$	✓	✓
	ϕ	$(2.31 \pm 0.26) \times 10^{-6}$	SCET+LCDA [13]	$< 4.8\times10^{-4}$ [74]	$\lesssim 3.7 \times 10^{-5}$	✓	✓
	J/ψ	$(2.95 \pm 0.38) \times 10^{-6}$	SCET+LCDA [13]	$< 3.5\times10^{-4}$ [77]	$\lesssim 5.5 \times 10^{-5}$ [54]	✓	✓
		$(3.01 \pm 0.15) \times 10^{-6}$	NRQCD (NLL)+LDME [78]				
		$\left(3.0^{+0.2}_{-0.1}\right) \times 10^{-6}$	NRQCD+LCDA [79]				

Experimental limits: Present & projections

- For all rare decays collected, we:
  - Indicate all current limits (LEP, Tevatron, LHC), including most recent ones (not yet on PDG).
  - Provide extrapolation of limits for the HL-LHC either from
    - Existing dedicated CMS/ATLAS studies.
    - Our statistical projection from previous results
      - For LHC limits: scale the 13-TeV bounds down by $\sqrt{2 \times 3\ \mathrm{ab}^{-1}/\mathcal{L}_{\mathrm{int}}(13\,\mathrm{TeV})}$ ~ Improvement by ~6.5 factor

Example:

$H \to \gamma$ +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
	ρ^0	$(1.68 \pm 0.18) \times 10^{-5}$	SCET+LCDA [13]	$< 8.8\times10^{-4}$ [74]	$\lesssim 6.8 \times 10^{-5}$	✓	✓
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    - Our statistical projection from previous results
      - For LHC limits: scale the 13-TeV bounds down by $\sqrt{2 \times 3\ \mathrm{ab}^{-1}/\mathcal{L}_{\mathrm{int}}(13\,\mathrm{TeV})}$ ~ Improvement by ~6.5 factor
      - For CDF limits, scale bounds down by $\sqrt{N_{\mathrm{X}}(\text{HL-LHC})/N_{\mathrm{X}}(\text{Tevatron})}$ ~ Improvement (W,Z) by ~70 factor

Collider	$W^{\pm}$ bosons		Z bosons		H bosons		top quarks	
	σ(W)	N(W)	σ(Z)	N(Z)	σ(H)	N(H)	$\sigma(t\bar{t})$	N(top)
LEP	4.0 pb	0.8×10^5	59 nb	2×10^7	~2, 1 fb	~5	–	–
FCC-ee	4.0 pb	5×10^8	59 nb	6×10^{12}	200, 30 fb	1.9×10^6	0.5 pb	3.8×10^6
Increase factor LEP ↦ FCC-ee	1	6250	1	300, 000	70, 30	400, 000	–	–
Tevatron (1.96 TeV, 10 fb^{-1})	25.3 nb	2.5×10^8	7.6 nb	7.6×10^7	1.1 pb	1.1×10^4	7.1 pb	1.4×10^5
HL-LHC (14 TeV, $2 \times 3\ \mathrm{ab}^{-1}$)	200 nb	1.2×10^{12}	62.5 nb	3.8×10^{11}	58 pb	3.5×10^8	1 nb	1.2×10^{10}
FCC-hh (100 TeV, 30 ab^{-1})	1300 nb	4.1×10^{13}	415 nb	1.2×10^{13}	0.93 nb	2.8×10^{10}	35 nb	2.1×10^{12}
Increase factor Tevatron ↦ HL-LHC	8	4800	8.2	5000	52.7	31 800	141	86 000
Increase factor HL-LHC ↦ FCC-hh	6.5	34	6.7	32	16	80	35	175

Square root of these numbers

Future limits: FCC-ee and FCC-hh reaches

- For all rare decays collected, we:
  - Indicate whether the decay will be producible at FCC-ee/FCC-hh by simply checking the relation $[BR(X)\times N(X)]>1\,?$

Number of H,W,Z,top produced

Collider	W± bosons		Z bosons		H bosons		top quarks	
	σ(W)	N(W)	σ(Z)	N(Z)	σ(H)	N(H)	$\sigma(t\bar{t})$	N(top)
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Future limits: FCC-ee and FCC-hh reaches

- For all rare decays collected, we:
  - Indicate whether the decay will be producible at FCC-ee/FCC-hh by simply checking the relation $[BR(X)\times N(X)]>1\,?$
  - Graphically presented by Vertical (dashed) line for FCC-ee (FCC-hh) in bar-limits plots:

Example:

$H\to\gamma$ +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
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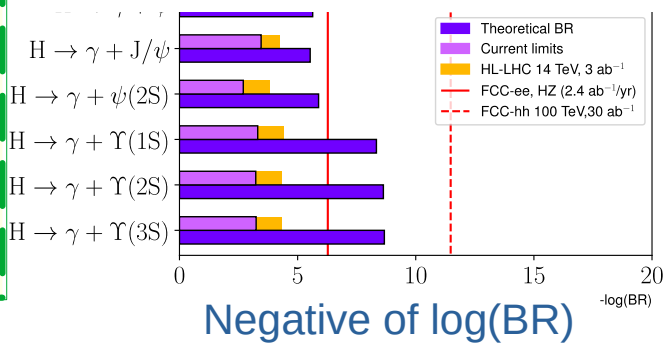
		$\left(3.0^{+0.2}_{-0.1}\right)\times10^{-6}$	NRQCD+LCDA [79]				
	$\psi(2S)$	$(1.3\pm0.1)\times10^{-6}$	SCET+LCDA [13]	$<2.0\times10^{-3}$ [80]	$\lesssim 1.6\times10^{-4}$	✓	✓
		$\left(4.6^{+3.9}_{-2.8}\right)\times10^{-9}$	SCET+LCDA [13]				
$H\to\gamma$ +							



Negative of log(BR)

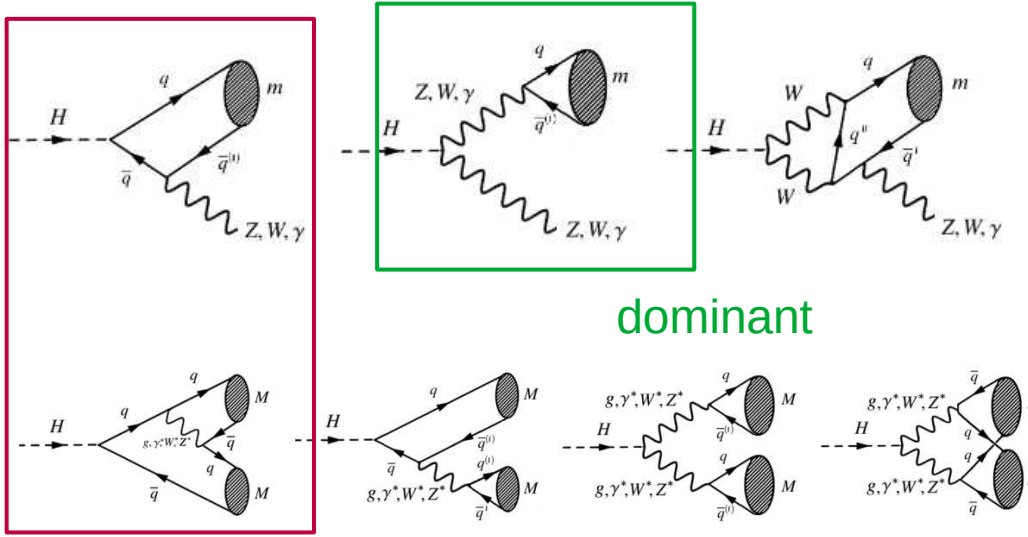
Purple bar below the red line means the process is producible at the FCC-ee
Yellow bar reaching the purple bar mean the process is close to be measured at the HL-LHC

Exclusive Higgs decays: radiative + meson, meson + meson

- Due to the smallness of the H → cc,qq partial widths, it has been proposed to constrain quark Yukawa couplings via exclusive decays of Higgs into:
  - EW boson + 1 meson:
    - Contributions from 2 main mechanisms (direct, indirect) which interfere destructively.
    - Can be used to probe hZγ effective couplings

direct

indirect

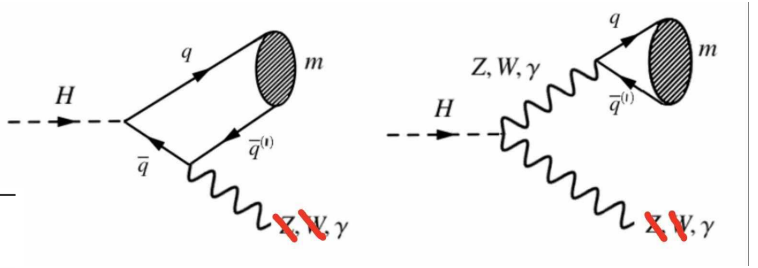


dominant

  - double meson:
    - Doubly suppressed → very small BR
      → Can't be produced until FCC-hh rates
    - Theoretical predictions have included more of these diagrams with time...

sub-dominant

Exclusive Higgs decays
H → γ + meson



H → γ +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
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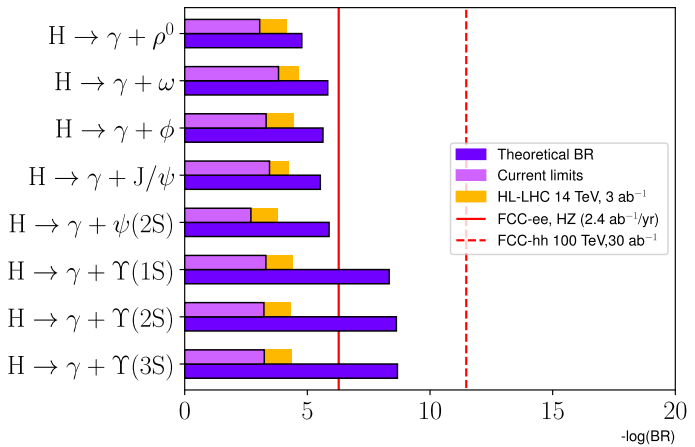
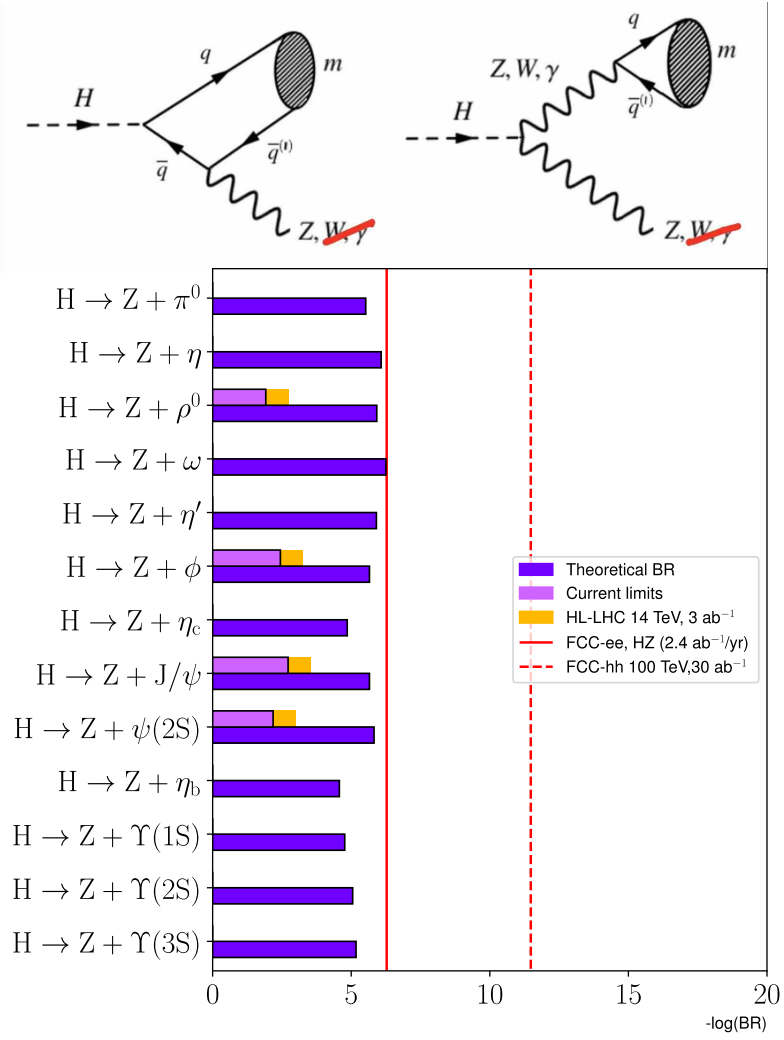
	$\Upsilon(1S)$	$\left(4.6^{+3.9}_{-2.8}\right) \times 10^{-9}$	SCET+LCDA [13]	$< 4.9\times10^{-4}$ [80]	$\lesssim 3.8 \times 10^{-5}$	✘	✓
		$\left(10.0^{+4.0}_{-3.0}\right) \times 10^{-9}$	NRQCD (NLL)+LDME [78]				
		$\left(5.2^{+2.0}_{-1.7}\right) \times 10^{-9}$	NRQCD+LCDA [79]				
	$\Upsilon(2S)$	$\left(2.3^{+1.7}_{-2.2}\right) \times 10^{-9}$	SCET+LCDA [13]	$< 5.9\times10^{-4}$ [80]	$\lesssim 4.6 \times 10^{-5}$	✘	✓
		$\left(2.6^{+1.4}_{-0.9}\right) \times 10^{-9}$	NRQCD (NLL)+LDME [78]				
		$\left(1.4^{+0.7}_{-0.6}\right) \times 10^{-9}$	NRQCD+LCDA [79]				
	$\Upsilon(3S)$	$\left(2.1^{+1.7}_{-2.5}\right) \times 10^{-9}$	SCET+LCDA [13]	$< 5.7\times10^{-4}$ [80]	$\lesssim 4.4 \times 10^{-5}$	✘	✓
		$\left(1.9^{+1.1}_{-0.7}\right) \times 10^{-9}$	NRQCD (NLL)+LDME [78]				
		$\left(9.1^{+4.8}_{-3.8}\right) \times 10^{-10}$	NRQCD+LCDA [79]				



- Theory BRs: O(10^{-5} – 10^{-10}). Exp. limits: O(10^{-3} – 10^{-4})
- 9 channels studied . 5 (8) producible channels at FCC-ee (FCC-hh).
- H → γ + J/ψ maybe observed at HL-LHC, with the ratios BR_theo/B_limit ~ 1/10

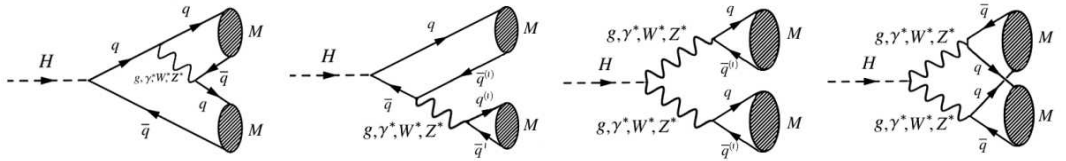
Exclusive Higgs decays
H → Z + meson

H → Z +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
H → Z +	π^0	$(2.3 \pm 0.1) \times 10^{-6}$	EFT+LCDA [86]	–	–	✓	✓
		3.0×10^{-6}	EFT+NRQM [11]				
	η	$(8.3 \pm 0.9) \times 10^{-7}$	EFT+LCDA [86]	–	–	✓	✓
	ρ^0	$(7.19 \pm 0.29) \times 10^{-6}$	EFT+LCDA [86]	$< 1.2 \times 10^{-2}$ [87]	$\lesssim 1.8 \times 10^{-3}$	✓	✓
		1.2×10^{-6}	EFT+NRQM [11]				
	ω	$(5.6 \pm 0.2) \times 10^{-7}$	EFT+LCDA [86]	–	–	✓	✓
	η'	$(1.24 \pm 0.13) \times 10^{-6}$	EFT+LCDA [86]	–	–	✓	✓
	ϕ	$(2.42 \pm 0.10) \times 10^{-6}$	EFT+LCDA [86]	$< 3.6 \times 10^{-3}$ [87]	$\lesssim 5.4 \times 10^{-4}$	✓	✓
		2.2×10^{-6}	EFT+NRQM [11]				
	η_c	$(1.00 \pm 0.01) \times 10^{-5}$	EFT+LCDA [88]	–	–	✓	✓
		1.4×10^{-5}	EFT+NRQM [11]				
	J/ψ	3.4×10^{-6}	NRQCD (NLO)+LMDE [89]	$< 1.9 \times 10^{-3}$ [90]	$\lesssim 2.9 \times 10^{-4}$ [56]	✓	✓
		$(2.3 \pm 0.1) \times 10^{-6}$	EFT+LCDA [86]				
		2.2×10^{-6}	EFT+NRQM [11]				
		3.2×10^{-6}	EFT+NRQM [91]				
	$\psi(2S)$	1.5×10^{-6}	EFT+NRQM [91]	$< 6.6 \times 10^{-3}$ [90]	$\lesssim 1.0 \times 10^{-3}$	✓	✓
	η_b	$(2.69 \pm 0.05) \times 10^{-5}$	EFT+LCDA [88]	–	–	✓	✓
		$\left(4.7^{+0.3}_{-0.2}\right) \times 10^{-5}$	EFT (NLO)+LCDA [92]				
	$\Upsilon(1S)$	1.7×10^{-5}	NRQCD (NLO)+LMDE [89]	–	–	✓	✓
		$(1.54 \pm 0.06) \times 10^{-5}$	EFT+LCDA [86]				
		1.7×10^{-5}	EFT+NRQM [91]				
	$\Upsilon(2S)$	$(7.5 \pm 0.3) \times 10^{-6}$	EFT+LCDA [86]	–	–	✓	✓
		8.9×10^{-6}	EFT+NRQM [91]				
	$\Upsilon(3S)$	$(5.63 \pm 0.24) \times 10^{-6}$	EFT+LCDA [86]	–	–	✓	✓
		6.7×10^{-6}	EFT+NRQM [91]				



- Theory BRs: O(10^{-5} – 10^{-7}). Exp. limits: O(10^{-2} – 10^{-3})
- 4 channels searched for . All channels are producible at FCC-ee
- Botomonia have largest BRs, but no bound set so far
- No observable channel at HL-LHC

Exclusive Higgs decays
H → meson + meson



H →	X	+	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
H→	ϕ	+	J/ψ	1.0 $\times 10^{-9}$	LC+LCDA [96]	–	–	✘	✓
	J/ψ	+	J/ψ	(5.8 – 6.0) $\times 10^{-9}$	NRQCD+LDME [101]	$< 3.8\times10^{-4}$ [90]	$\lesssim 5.8\times10^{-5}$	✘	✓
				1.7 $\times 10^{-10}$	RQM [98]				
				2.1 $\times 10^{-10}$	RQM [100]				
				(5.9 ± 2.3) $\times 10^{-10}$	NRQCD/NRCSM [99]				
				1.5 $\times 10^{-10}$	LC+LCDA [96]				
	$\psi(2S)$	+	J/ψ	5.0 $\times 10^{-11}$	–	$< 2.1\times10^{-3}$ [90]	$\lesssim 3.2\times10^{-4}$	✘	✓
			$\psi(2S)$	(5.1 ± 2.0) $\times 10^{-11}$	NRQCD/NRCSM [99]	$< 3.0\times10^{-3}$ [90]	$\lesssim 4.5\times10^{-4}$	✘	✓
	$B_c^{*\mp}$	+	$B_c^{*\pm}$	(1.4 – 1.7) $\times 10^{-10}$	RQM [97]	–	–	✘	✓
	$B_c^{\mp}$	+	$B_c^{\pm}$	(2.0 – 3.0) $\times 10^{-10}$	RQM [97]	–	–	✘	✓
	$\Upsilon(1S)$	+	J/ψ	(2.7 – 3.6) $\times 10^{-10}$	NRQCD+LDME [101]	–	–	✘	✓
				1.6 $\times 10^{-11}$	LC+LCDA [96]				
			$\Upsilon(1S)$	(8.5 – 9.2) $\times 10^{-10}$	NRQCD+LDME [101]	$< 1.7\times10^{-3}$ [90]	$\lesssim 2.6\times10^{-4}$	✘	✓
				1.8 $\times 10^{-10}$	RQM [98]				
				2.3 $\times 10^{-9}$	RQM [100]				
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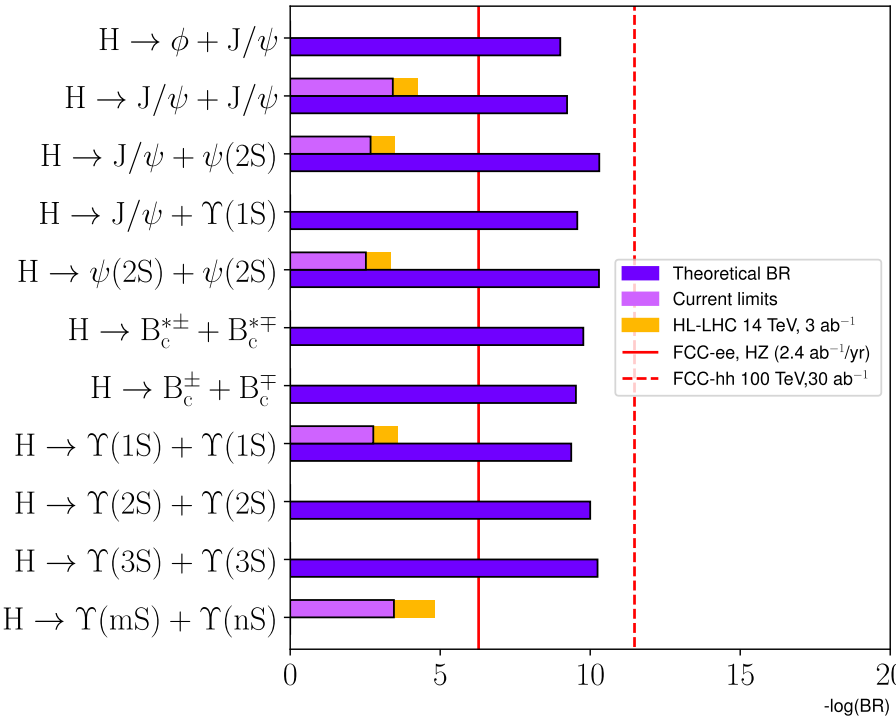
	$\Upsilon(2S)$	+	$\Upsilon(2S)$	(1.0 ± 0.2) $\times 10^{-10}$	NRQCD/NRCSM [99]	–	–	✘	✓
	$\Upsilon(3S)$	+	$\Upsilon(3S)$	(5.7 ± 1.2) $\times 10^{-11}$	NRQCD/NRCSM [99]	–	–	✘	✓
	$\Upsilon(mS)$	+	$\Upsilon(nS)$		–	$< 3.5\times10^{-4}$ [90]	$\lesssim 1.5\times10^{-5}$ [56]	✘	✘



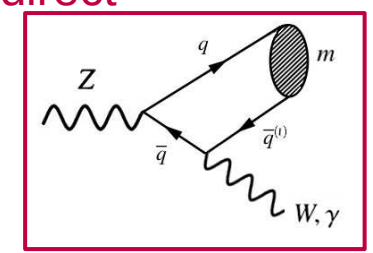
- Theory BRs: O(10^{-9}–10^{-11}). Exp. limits: O(10^{-3} – 10^{-4}).
- 5 channels searched-for. No (all) producible channels at FCC-ee (FCC-hh)
- Many predictions for double-QQbar from adding more contributing diagrams.

Exclusive Z decays

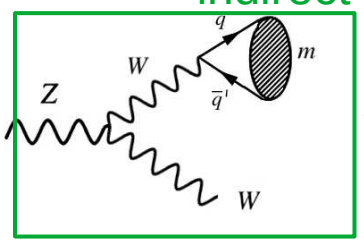
- Exclusive Z decays:
  - Similar mechanism to the exclusive Higgs radiative decays.
  - Large Z boson yields at colliders.

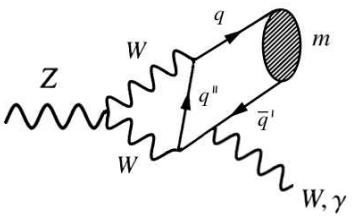
    Provides valuable information both theoretical (SCET & NRQCD validation, and LCDAs/LDMEs' params) and experimental (optimization of search techniques to study exclusive Higgs boson decays).
  - Exclusive Z decays into:
    - γ,W + 1 meson:
    - Double mesons:

direct

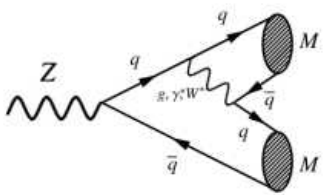


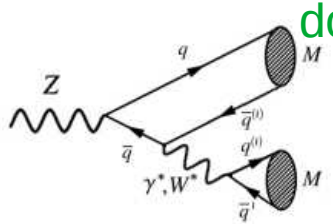
indirect

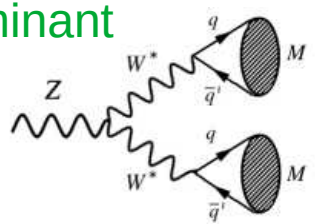




dominant

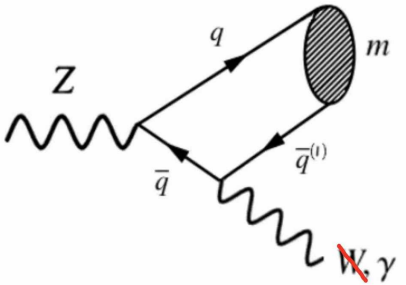




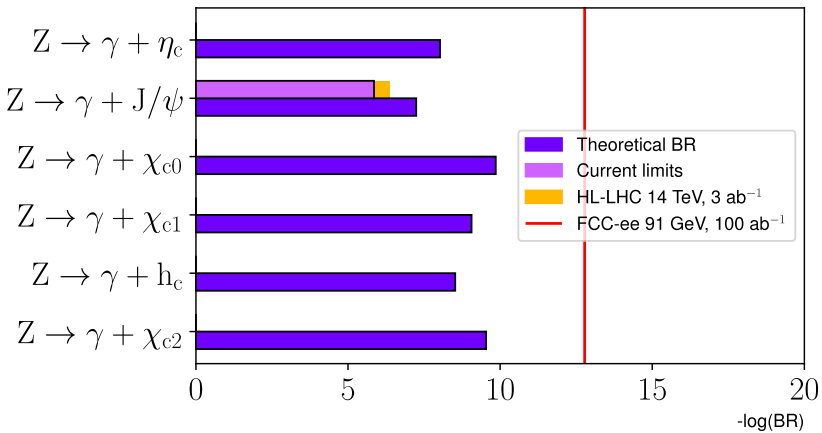


Exclusive Z decays:
Z → γ + charm meson

Z → γ +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee
Z → γ +	η_c	$(9.5 \pm 0.2) \times 10^{-9}$	NRQCD (NNLO+NLL) [120]	–	–	✓
		$(7.42 \pm 0.61) \times 10^{-9}$	NRQCD (NLO+NLL) [121]			
		6.6×10^{-9}	NRQCD+LDME [122]			
		$(9.4 \pm 1.0) \times 10^{-9}$	LC+LCDA [122]			
	J/ψ	$\left(5.75^{+0.08}_{-0.09}\right) \times 10^{-8}$	NRQCD (NNLO+NLL) [120]	$< 1.4\times10^{-6}$ [114]	$\lesssim 4.4 \times 10^{-7}$ [54]	✓
		$\left(9.0^{+1.5}_{-1.4}\right) \times 10^{-8}$	LC+LCDA [123]			
		4.5×10^{-8}	NRQCD+LDME [122]			
		$(8.8 \pm 0.9) \times 10^{-8}$	LC+LCDA [122]			
		$(9.96 \pm 1.86) \times 10^{-8}$	NRQCD+LDME [119]			
		$(8.02 \pm 0.45) \times 10^{-8}$	SCET+LCDA [58]			
	χ_{c0}	$(3.74 \pm 0.05) \times 10^{-10}$	NRQCD+LDME [124]	–	–	✓
		1.4×10^{-10}	NRQCD+LDME [122]			
		$(5.0 \pm 2.0) \times 10^{-10}$	LC+LCDA [122]			
	χ_{c1}	$\left(2.38^{+0.01}_{-0.02}\right) \times 10^{-9}$	NRQCD+LDME [124]	–	–	✓
		8.7×10^{-10}	NRQCD+LDME [122]			
		$(5.6 \pm 2.0) \times 10^{-9}$	LC+LCDA [122]			
	h_c	$\left(3.49^{+0.21}_{-0.23}\right) \times 10^{-9}$	NRQCD+LDME [124]	–	–	✓
		3.0×10^{-9}	NRQCD+LDME [122]			
		$(1.0 \pm 0.4) \times 10^{-8}$	LC+LCDA [122]			
	χ_{c2}	$\left(3.38^{+0.19}_{-0.22}\right) \times 10^{-10}$	NRQCD+LDME [124]	–	–	✓
		2.9×10^{-10}	NRQCD+LDME [122]			
		$(1.0 \pm 0.4) \times 10^{-9}$	LC+LCDA [122]			

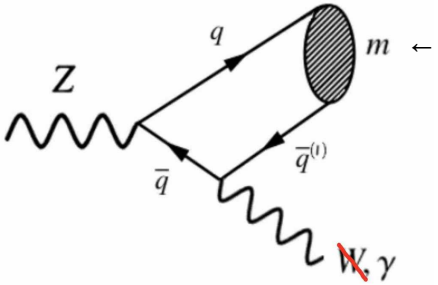


← c quarks

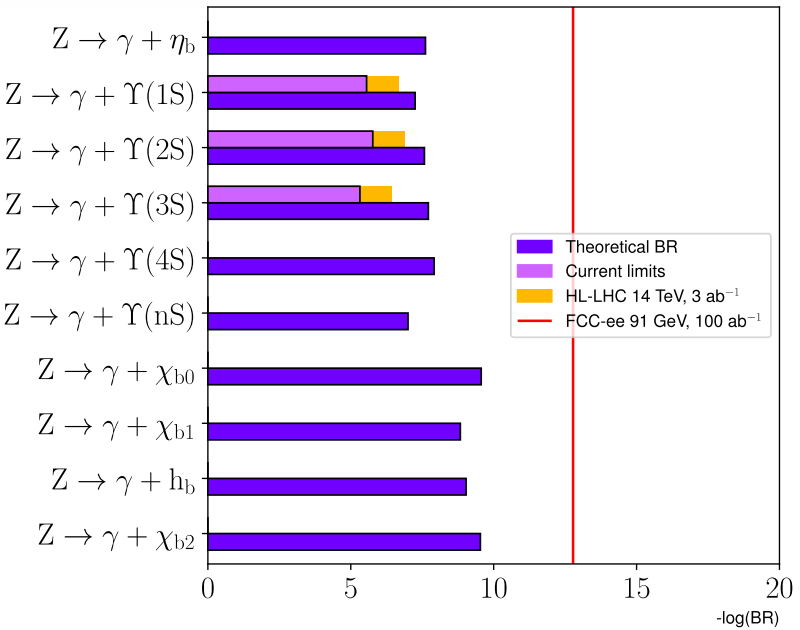


- Theory BRs: O(10^{-8} – 10^{-10}). Multiple calculations (LC, SCET, NRQCD). Exp. limits: O(10^{-6}).
- 1 channel searched for. 6 producible channel at FCC-ee
- Z → γ+ J/ψ maybe visible at HL-LHC

Exclusive Z decays:
Z→ γ + bottom meson



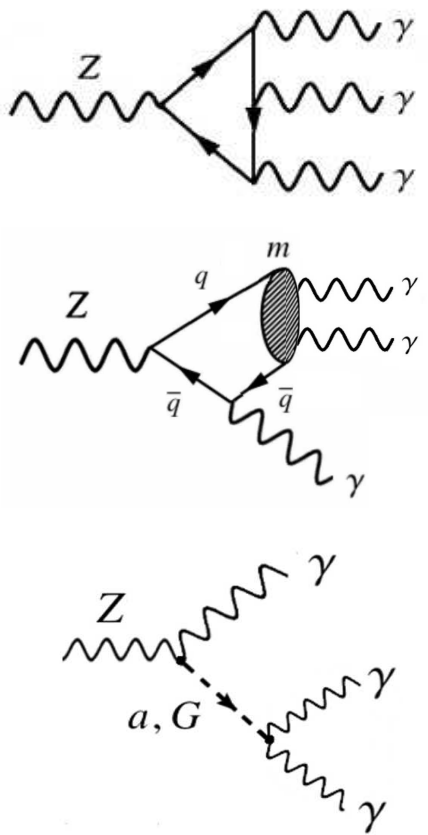
Z → γ +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee
Z → γ +	η_b	$(2.43 \pm 0.01) \times 10^{-8}$	NRQCD (NNLO+NLL) [120]	–	–	✓
		$(2.8 \pm 0.5) \times 10^{-8}$	NRQCD (NLO+NLL) [121]			
	$\Upsilon(1S)$	$(4.63 \pm 0.02) \times 10^{-8}$	NRQCD (NNLO+NLL) [120]	$< 2.8\times10^{-6}$ [80]	$\lesssim 2.2 \times 10^{-7}$	✓
		$(5.61 \pm 0.29) \times 10^{-8}$	NRQCD+LDME [125]			
		$\left(4.8^{+0.3}_{-0.2}\right) \times 10^{-8}$	LC+LCDA [123]			
		$(4.93 \pm 0.51) \times 10^{-8}$	NRQCD+LDME [119]			
		$(5.39 \pm 0.16) \times 10^{-8}$	SCET+LCDA [58]			
	$\Upsilon(2S)$	$(2.66 \pm 0.31) \times 10^{-8}$	NRQCD+LDME [125]	$< 1.7\times10^{-6}$ [80]	$\lesssim 1.3 \times 10^{-7}$	✓
		$\left(2.44^{+0.14}_{-0.13}\right) \times 10^{-8}$	LC+LCDA [123]			
	$\Upsilon(3S)$	$(1.93 \pm 0.25) \times 10^{-8}$	NRQCD+LDME [125]	$< 4.8\times10^{-6}$ [80]	$\lesssim 3.7 \times 10^{-7}$	✓
		$\left(1.88^{+0.11}_{-0.10}\right) \times 10^{-8}$	LC+LCDA [123]			
	$\Upsilon(4S)$	$(1.22 \pm 0.13) \times 10^{-8}$	SCET+LCDA [58]	–	–	✓
	$\Upsilon(nS)$	$\left(9.96^{+0.28}_{-0.26}\right) \times 10^{-8}$	SCET+LCDA [58]	–	–	✓
	χ_{b0}	$\left(2.7^{+0.1}_{-0.0}\right) \times 10^{-10}$	NRQCD+LDME [124]	–	–	✓
	χ_{b1}	$\left(1.473^{+0.010}_{-0.011}\right) \times 10^{-9}$	NRQCD+LDME [124]	–	–	✓
	h_b	$\left(9.27^{+0.36}_{-0.41}\right) \times 10^{-10}$	NRQCD+LDME [124]	–	–	✓
	χ_{b2}	$\left(2.92^{+0.12}_{-0.14}\right) \times 10^{-10}$	NRQCD+LDME [124]	–	–	✓



- Theory BRs: O(10^{-8} – 10^{-10}). Multiple calculations (LC, SCET, NRQCD). Exp. limits: O(10^{-6}).
- 3 channels searched-for (ATLAS). 10 producible channel at FCC-ee
- Z→ γ+Y(1S), might be visible at HL-LHC, (BR = 1/4 projected limit)

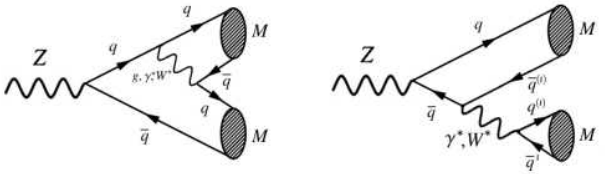
Radiative Z → onium decays as a background for BSM Z decays

Z → γ	+	$X(\gamma\gamma)$	Branching fraction
		$\pi^0(\gamma\gamma)$	9.7×10^{-12}
		$\eta(\gamma\gamma)$	6.3×10^{-11}
		$\eta'(\gamma\gamma)$	1.1×10^{-10}
		$\eta_c(\gamma\gamma)$	2.1×10^{-12}
Z → γ	+	$\chi_{c0}(\gamma\gamma)$	7.6×10^{-14}
		$\chi_{c1}(\gamma\gamma)$	1.5×10^{-14}
		$\chi_{c2}(\gamma\gamma)$	9.6×10^{-14}
		$\chi_{b0}(\gamma\gamma)$	1.6×10^{-14}
		$\chi_{b2}(\gamma\gamma)$	1.6×10^{-14}
Sum			1.8×10^{-10}
Z	⟶	γγγ	6.4×10^{-10}
Total			8.2×10^{-10}

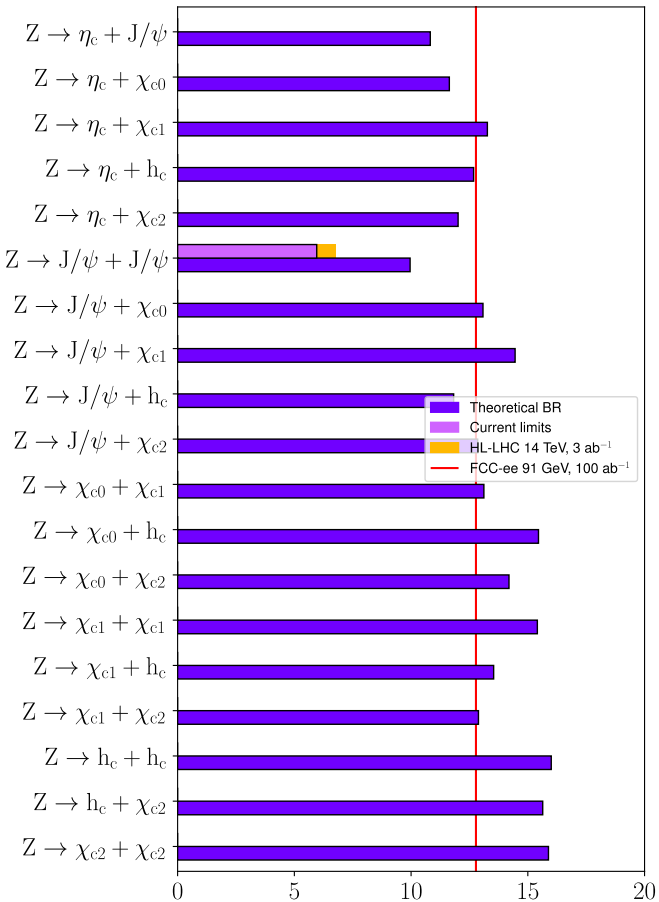


- Z → 3γ decay is very suppressed in the SM (6.4 10^{-10}).
- Z → γ + a(γγ) is a typical ALP/graviton search channel. 10 mesonic channels share same final state
- Z → γ + meson(γγ) provides about 30% extra contributions to the SM BR[Z→3γ]

Exclusive Z decays
Z → c-meson + c-meson

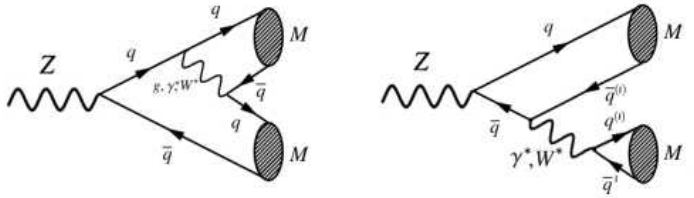


Z →	X	+	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee
Z →	η_c	+	J/ψ	$(1.5 \pm 0.4) \times 10^{-11}$	NRQCD/NRCSM [130]	–	–	✓
				$(1.8 - 2.7) \times 10^{-11}$	NRQCD+LDME [128]			
				2.7×10^{-14}	NRQCD+LDME [131]			
			χ_{c0}	2.3×10^{-12}	NRQCD+LDME [131]	–	–	✓
				$(2.3 \pm 1.0) \times 10^{-12}$	LC+LCDA [131]			
			χ_{c1}	5.4×10^{-14}	NRQCD+LDME [131]	–	–	✘
			h_c	2.1×10^{-13}	NRQCD+LDME [131]			
				$(1.0 \pm 0.5) \times 10^{-12}$				
			χ_{c2}	9.7×10^{-13}				
				$(4.6 \pm 2.0) \times 10^{-12}$				
	J/ψ	+	J/ψ	$(1.1 \pm 0.3) \times 10^{-10}$				
				$(1.1^{+0.3}_{-0.2}) \times 10^{-10}$				
				$(1.1 - 1.3) \times 10^{-10}$				
				2.3×10^{-14}				
				2.7×10^{-11}				
			χ_{c0}	$(1.1 - 4.1) \times 10^{-12}$				
				8.3×10^{-14}				
			χ_{c1}	$(4.7 \pm 2.0) \times 10^{-13}$				
				$(3.5 - 4.4) \times 10^{-12}$				
				3.5×10^{-15}				
	χ_{c0}	+	h_c	1.5×10^{-12}	NRQCD+LDME [131]	–	–	✓
				$(9.5 \pm 5.0) \times 10^{-12}$	LC+LCDA [131]			
			χ_{c2}	$(9.6 - 24.8) \times 10^{-13}$	NRQCD+LDME [128]			
				1.4×10^{-13}	NRQCD+LDME [131]	–	–	✓
				$(9.3 \pm 4.0) \times 10^{-13}$	LC+LCDA [131]			
	χ_{c1}	+	χ_{c1}	7.6×10^{-14}	NRQCD+LDME [131]	–	–	✘
				$(1.4 \pm 1.0) \times 10^{-12}$	LC+LCDA [131]			
			h_c	3.5×10^{-16}	NRQCD+LDME [131]	–	–	✘
			χ_{c2}	6.4×10^{-15}	NRQCD+LDME [131]	–	–	✘
	χ_{c2}	+	χ_{c1}	3.9×10^{-16}	NRQCD+LDME [131]	–	–	✘
			h_c	2.9×10^{-14}	NRQCD+LDME [131]	–	–	✘
				$(6.1 \pm 5.0) \times 10^{-13}$	LC+LCDA [131]			
			χ_{c2}	1.3×10^{-13}	NRQCD+LDME [131]	–	–	✘
				$(2.8 \pm 2.0) \times 10^{-12}$	LC+LCDA [131]			
	h_c	+	h_c	9.9×10^{-17}	NRQCD+LDME [131]	–	–	✘
			χ_{c2}	2.3×10^{-16}	NRQCD+LDME [131]	–	–	✘
	χ_{c2}	+	χ_{c2}	1.3×10^{-16}	NRQCD+LDME [131]	–	–	✘

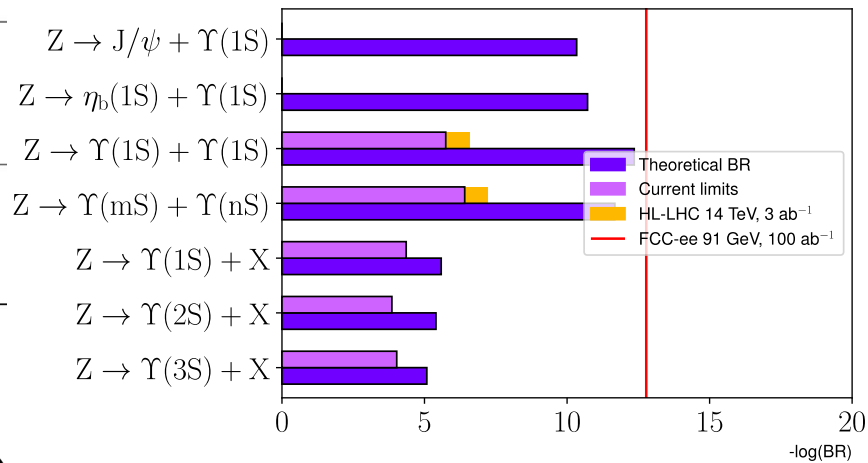


- Theory BRs: O(10^{-10} – 10^{-17}). Exp. limits: O(10^{-6})
- 1 channel searched for. 10 producible channels at FCC-ee
- Large uncertainty, variation between theoretical predictions. LC approach is more preferable
- Most promising place to study double-charmonia decay.

Exclusive Z decays
Z → b-meson + b-meson



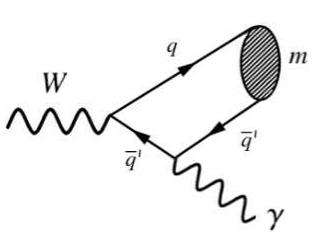
Z →	M	+	M	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at at FCC-ee
Z →	J/ψ	+	$\Upsilon(1S)$	4.6×10^{-11}	NRQCD [142]	–	–	✓
	$\eta_b(1S)$	+	$\Upsilon(1S)$	$(1.9 \pm 0.2) \times 10^{-11}$	NRQCD/NRCSM [143]	–	–	✓
	$\Upsilon(1S)$	+	$\Upsilon(1S)$	$\left(4.4^{+0.6}_{-0.3}\right) \times 10^{-13}$	NRQCD/NRCSM [143]	$< 1.8\times 10^{-6}$ [98]	$\lesssim 2.7 \times 10^{-7}$	✓
	$\Upsilon(mS)$	+	$\Upsilon(nS)$	2.1×10^{-12}	NRQCD [142]	$< 3.9\times 10^{-7}$ [98]	$\lesssim 5.9 \times 10^{-8}$	✓
Z →	$\Upsilon(1S)$	+	X	$(2.6 - 2.9) \times 10^{-6}$	NRQCD+LDME (NLO) [147]	$< 4.4\times 10^{-5}$ [148]	–	✓
	$\Upsilon(2S)$	+	X	$(3.7 - 4.3) \times 10^{-6}$	NRQCD+LDME (NLO) [147]	$< 1.4\times 10^{-4}$ [149]	–	✓
	$\Upsilon(3S)$	+	X	$(7.6 - 8.3) \times 10^{-6}$	NRQCD+LDME (NLO) [147]	$< 9.4\times 10^{-5}$ [149]	–	✓
	$\Upsilon(nS)$	+	X	$(1.4 - 1.5) \times 10^{-5}$	NRQCD+LDME (NLO) [147]	$(1.0 \pm 0.5)\times 10^{-4}$ [150]	–	✓

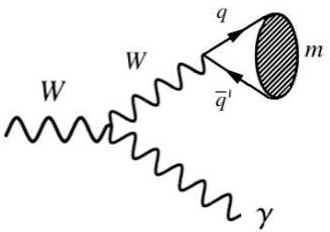


- Theory BRs: O(10^{-10} – 10^{-17}). Exp. limits: O(10^{-4} – 10^{-7})
- 5 channels searched for. 7 producible channels at FCC-ee
- Most promising place to study double-bottomonium decays.

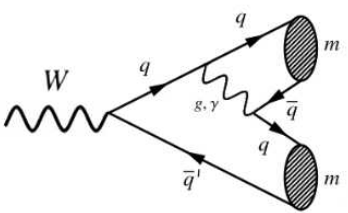
Exclusive W decays

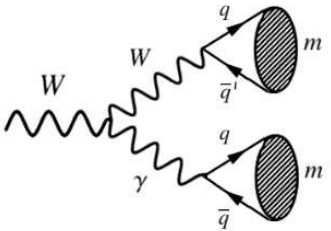
- Exclusive W decays:
  - Similar mechanism to exclusive Z and H decays.
  - Provides cross-check of pQCD factorization models and info on open-flavour meson form factors.
  - Exclusive Z decays into:
    - γ + 1 charged meson

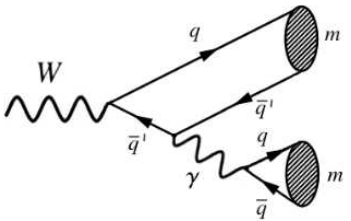




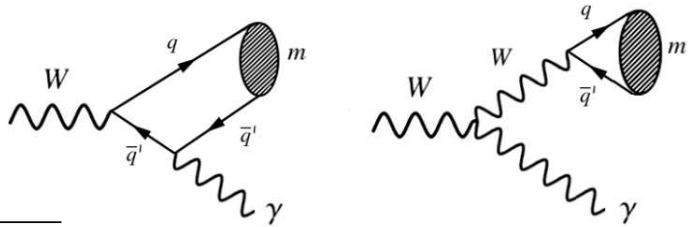
    - Meson + charged meson



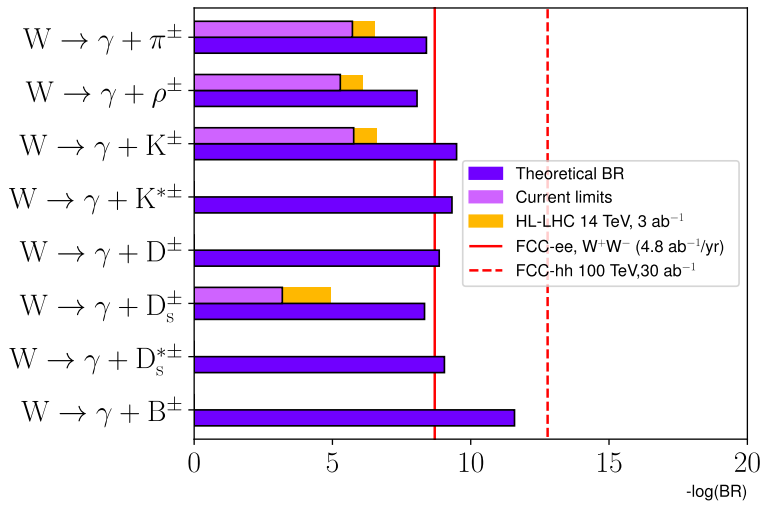




Exclusive W decays
W → γ + meson

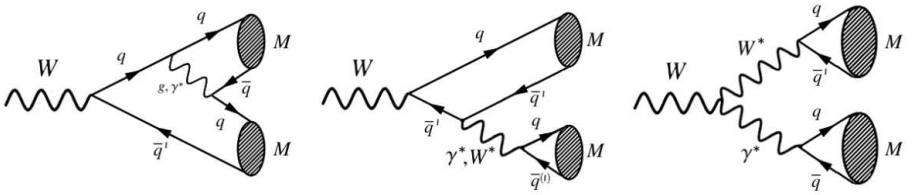


$W^{\mp} \to \gamma$ +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
	$\pi^{\pm}$	$(4.0 \pm 0.8) \times 10^{-9}$	SCET+LCDA [58]	$< 1.9\times10^{-6}$ [136, 137]	$\lesssim 2.9 \times 10^{-7}$	✓	✓
	$\rho^{\pm}$	$(8.74 \pm 1.91) \times 10^{-9}$	SCET+LCDA [58]	$< 5.2\times10^{-6}$ [136]	$\lesssim 7.9 \times 10^{-7}$	✓	✓
	$K^{\pm}$	$(3.25 \pm 0.69) \times 10^{-10}$	SCET+LCDA [58]	$< 1.7\times10^{-6}$ [136]	$\lesssim 2.6 \times 10^{-7}$	✘	✓
	$K^{*\pm}$	$(4.78 \pm 1.15) \times 10^{-10}$	SCET+LCDA [58]	–	–	✘	✓
	$D^{\pm}$	$\left(1.4^{+0.5}_{-0.3}\right) \times 10^{-9}$	SCET+LCDA [58]	–	–	✘	✓
	$D_s^{\pm}$	$\left(3.7^{+1.5}_{-0.8}\right) \times 10^{-8}$	SCET+LCDA [58]	$< 6.5\times10^{-4}$ [115]	$\lesssim 1.2 \times 10^{-5}$	✓	✓
		4.7×10^{-9}	NRQCD+LDME [138]				
		3.4×10^{-9}	LC+LCDA [138]				
	$D_s^{*\pm}$	8.9×10^{-10}	NRQCD+LDME [138]	–	–	✘	✓
		3.4×10^{-9}	LC+LCDA [138]				
	$B^{\pm}$	$\left(1.6^{+0.8}_{-0.6}\right) \times 10^{-12}$	SCET+LCDA [58]	–	–	✘	✓
		$\left(2.6^{+3.1}_{-1.3}\right) \times 10^{-12}$	HQET+LCDA [139]				
		$\left(2.0^{+2.5}_{-0.8}\right) \times 10^{-12}$	SCET+LCDA[a] [139]				

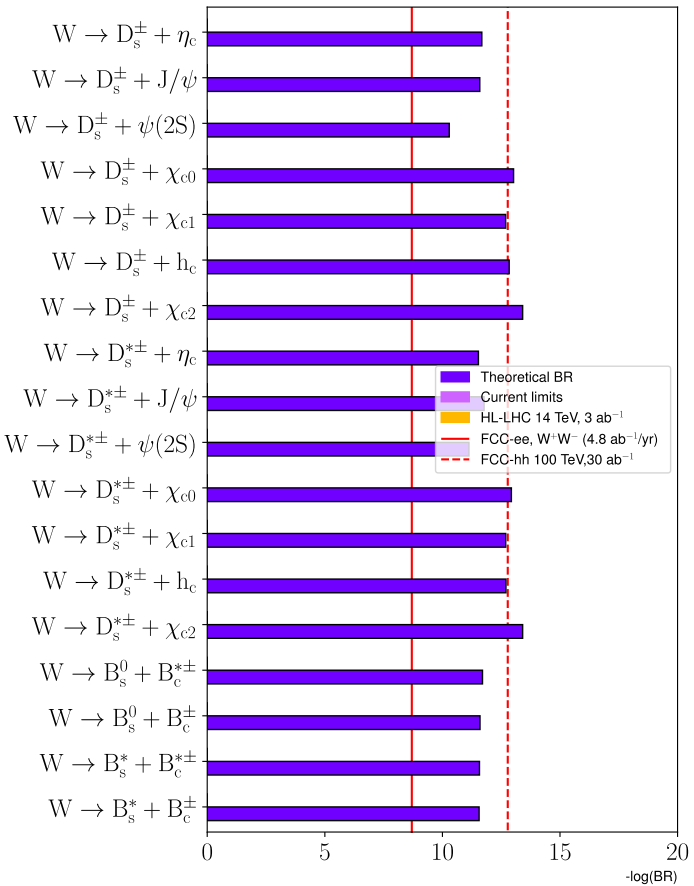


- Theory BRs: O(10^{-8} – 10^{-12}). Exp. limits: O(10^{-4} – 10^{-6})
- 4 channels searched-for.
- 3 (5) channels producible at FCC-ee (FCC-hh)

Exclusive W decays
W→ meson + meson



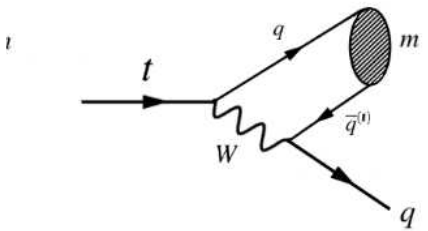
$W^{\pm}\to$	X	+	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
	η_c	+	$D_s^{\pm}$	2.1×10^{-12}	NRCSM+LCDA [140]	–	–	✘	✓
				$(1.3^{+0.3}_{-0.2})\times10^{-11}$	LC+LCDA [140]				
			$D_s^{*\pm}$	3.0×10^{-12}	NRCSM+LCDA [140]	–	–	✘	✓
				$(1.5^{+0.4}_{-0.2})\times10^{-11}$	LC+LCDA [140]				
	J/ψ	+	$D_s^{\pm}$	2.6×10^{-12}	NRQCD+LDME [138]	–	–	✘	✓
				2.1×10^{-12}	NRCSM+LCDA [140]				
				$(1.8^{+0.4}_{-0.2})\times10^{-11}$	LC+LCDA [140]				
			$D_s^{*\pm}$	1.7×10^{-12}	NRQCD+LDME [138]	–	–	✘	✓
				3.0×10^{-12}	NRCSM+LCDA [140]				
				$(2.0^{+0.5}_{-0.2})\times10^{-11}$	LC+LCDA [140]				
	$\psi(2S)$	+	$D_s^{\pm}$	5.1×10^{-11}	NRQCD+LDME [138]	–	–	✘	✓
			$D_s^{*\pm}$	7.4×10^{-12}	NRQCD+LDME [138]	–	–	✘	✓
	χ_{c0}	+	$D_s^{\pm}$	9.4×10^{-14}	NRQCD+LDME [138]	–	–	✘	✘
				4.7×10^{-14}	NRCS				
				$(7.1^{+3.5}_{-3.1})\times10^{-13}$	LC+L				
			$D_s^{*\pm}$	1.2×10^{-13}	NRQ	–	–	✘	✓
				8.1×10^{-14}	NRCS				
				$(8.0^{+3.7}_{-3.1})\times10^{-13}$	LC+L				
	χ_{c1}	+	$D_s^{\pm}$	2.0×10^{-13}	NRQ	–	–	✘	✘
				2.9×10^{-13}	NRCS				
				$(7.8^{+3.4}_{-3.0})\times10^{-12}$	LC+L				
			$D_s^{*\pm}$	2.0×10^{-13}	NRQ	–	–	✘	✓
				4.0×10^{-13}	NRCS				
				$(8.8^{+3.5}_{-3.1})\times10^{-12}$	LC+L				
			$D^{\pm}$	1.4×10^{-13}	NRCS	–	–	✘	✘
	h_c	+	$D_s^{\pm}$	1.4×10^{-13}	NRCSM+LCDA [140]	–	–	✘	✘
				$(2.1^{+1.0}_{-0.8})\times10^{-12}$	LC+LCDA [140]				
			$D_s^{*\pm}$	2.0×10^{-13}	NRCSM+LCDA [140]	–	–	✘	✓
				$(2.4^{+1.1}_{-0.9})\times10^{-12}$	LC+LCDA [140]				
	χ_{c2}	+	$D_s^{\pm}$	3.9×10^{-14}	NRQCD+LDME [138]	–	–	✘	✘
				9.6×10^{-14}	NRCSM+LCDA [140]				
				$(1.4^{+0.6}_{-0.5})\times10^{-12}$	LC+LCDA [140]				
			$D_s^{*\pm}$	3.9×10^{-14}	NRQCD+LDME [138]	–	–	✘	✘
				1.4×10^{-13}	NRCSM+LCDA [140]				
				$(1.6^{+0.7}_{-0.6})\times10^{-12}$	LC+LCDA [140]				
	B_s^0	+	$B_c^{*\pm}$	2.0×10^{-12}	NRQCD+LDME [138]	–	–	✘	✓
			$B_c^{\pm}$	2.5×10^{-12}	NRQCD+LDME [138]	–	–	✘	✓
	B_s^*	+	$B_c^{*\pm}$	2.7×10^{-12}	NRQCD+LDME [138]	–	–	✘	✓
			$B_c^{\pm}$	2.7×10^{-12}	NRQCD+LDME [138]	–	–	✘	✓



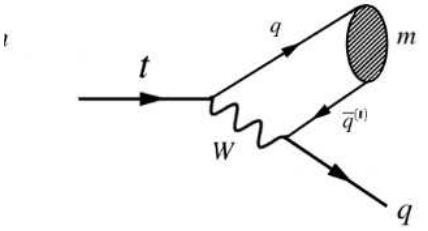
- Theory BRs: O(10^{-11} – 10^{-14}). No existing exp. Limit
- 0 channels searched-for. 0 (13) channels producible at FCC-ee (FCC-hh)
- Tiny rates. No searches so far anywhere

Semi-exclusive top decays

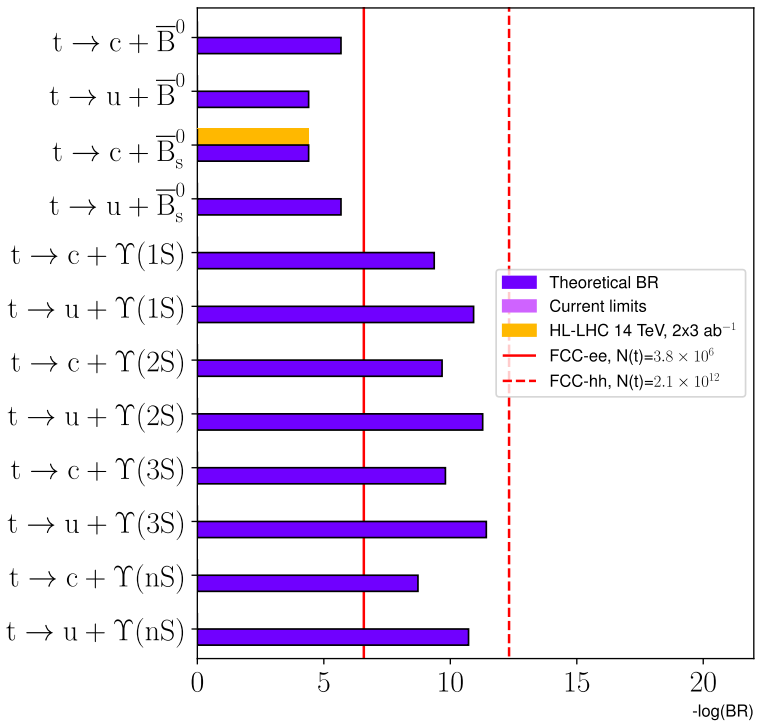
- two-body t → (qq) + q’ decay can provide an alternative top mass extraction



Semi-exclusive decays
t → meson + c/u quark



t →	M	+	q	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
t →	$\overline{B}^0$	+	c	$(2.1^{+2.1}_{-1.1}) \times 10^{-6}$	NRQCD+LDME [26]	–	–	✓	✓
			u	$(4.0^{+4.0}_{-2.0}) \times 10^{-5}$	NRQCD+LDME [26]	–	–	✓	✓
	$\overline{B}_s^0$	+	c	$(4.0^{+4.0}_{-2.0}) \times 10^{-5}$	NRQCD+LDME [26]		$\lesssim 4.0 \times 10^{-5}$ [26]	✓	✓
			u	$(2.1^{+2.1}_{-1.1}) \times 10^{-6}$	NRQCD+LDME [26]	–	–	✓	✓
	$\Upsilon(1S)$	+	c	4.3×10^{-10}	NRQCD+CSM [174]				
				$(1.0 – 1.5) \times 10^{-9}$	NRQCD+LDME [26]	–	–	✘	✓
				$(6.4 \pm 1.3) \times 10^{-10}$	NRQCD+COM [173]				
			u	$(1.0 – 1.5) \times 10^{-11}$	NRQCD+LDME [26]	–	–	✘	✓
	$\Upsilon(2S)$	+	c	2.1×10^{-10}	NRQCD+CSM [174]	–	–	✘	✓
				$(1.7 – 5.3) \times 10^{-10}$	NRQCD+LDME [26]				
			u	$(1.7 – 5.3) \times 10^{-12}$	NRQCD+LDME [26]	–	–	✘	✓
	$\Upsilon(3S)$	+	c	1.6×10^{-10}	NRQCD+CSM [174]	–	–	✘	✓
				$(2.7 – 3.8) \times 10^{-10}$	NRQCD+LDME [26]				
			u	$(2.7 – 3.8) \times 10^{-12}$	NRQCD+LDME [26]	–	–	✘	✓
	$\Upsilon(nS)$	+	c	$(1.9^{+0.2}_{-0.1}) \times 10^{-9}$	NRQCD+LDME [26]	–	–	✘	✓
			u	$(1.9^{+0.2}_{-0.1}) \times 10^{-11}$	NRQCD+LDME [26]	–	–	✘	✓



- Theory BRs: O(10^{-5} – 10^{-11}). No existing exp. limits
- No channel searched for. 4 (9) visible channels at FCC-ee (FCC-hh)
- Note: t→ B+c/u-quark have “large” BR: $4 \cdot 10^{-5}$

Extension to leptonium

- S state quarkonium and leptonium share the same relation between the leptonic decay constant

$$f_{\mathrm{X}}^2 = \frac{N_c m_{\mathrm{X}}}{4\pi Q_{\mathrm{X}}^2 \alpha^2(m_{\mathrm{X}})}\Gamma(\mathrm{X} \to l^+ l^-)$$

  and the wave function at the origin at leading order

$$f_{\mathrm{X}}^2 = 4N_{\mathrm{c}}\frac{|\phi_{\mathrm{X}}(0)|^2}{m_{\mathrm{X}}}.$$

- With the difference coming from strong and QED interaction

$$|\phi_{n,(qq)}(r=0)|^2 = \frac{(mC_F\alpha_s)^3}{8\pi n^3}$$

S wave quarkonium

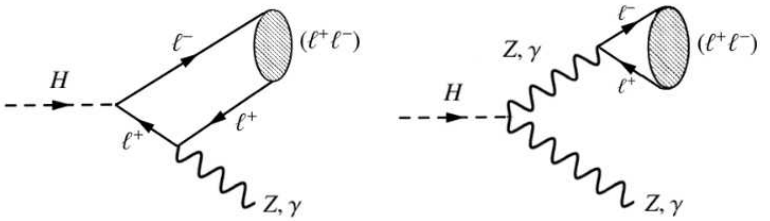
>>>

$$|\phi_{n,(\ell\ell)}(r=0)|^2 = \frac{(m_\ell \alpha(0))^3}{8\pi n^3}.$$

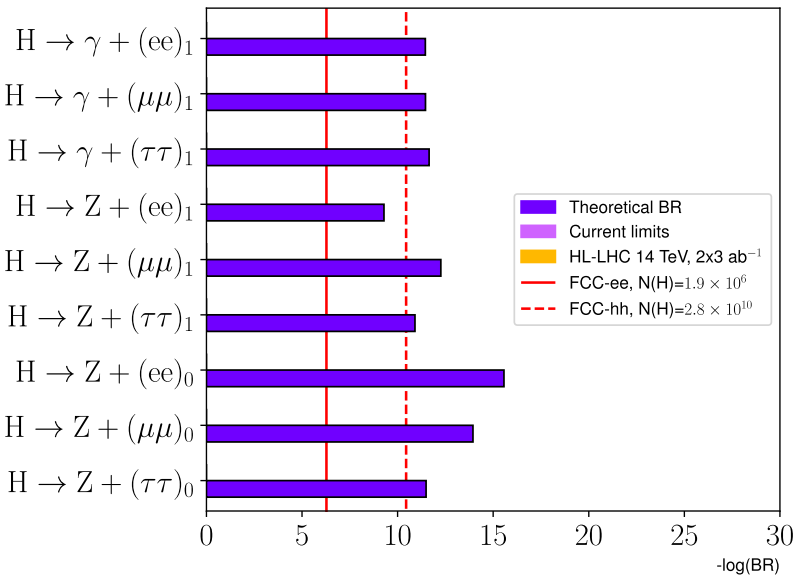
leptonium

- We have therefore reused the Z,H>QQbar+γ results to compute the Z,H>(ll)+γ decays

Exclusive Higgs decays
H → γ/Z + leptonium

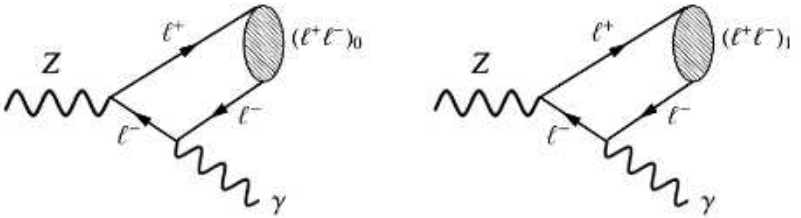


H → V +	$(\ell\ell)$	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
H → γ +	$(ee)_1$	3.5×10^{-12}	(this work)	–	–	✘	✘
	$(\mu\mu)_1$	3.5×10^{-12}	(this work)	–	–	✘	✘
	$(\tau\tau)_1$	2.2×10^{-12}	(this work)	–	–	✘	✘
H → Z +	$(ee)_1$	5.1×10^{-10}	(this work)	–	–	✘	✓
	$(\mu\mu)_1$	5.5×10^{-13}	(this work)	–	–	✘	✘
	$(\tau\tau)_1$	1.2×10^{-11}	(this work)	–	–	✘	✘
	$(ee)_0$	2.7×10^{-16}	(this work)	–	–	✘	✘
	$(\mu\mu)_0$	1.1×10^{-14}	(this work)	–	–	✘	✘
	$(\tau\tau)_0$	3.2×10^{-12}	(this work)	–	–	✘	✘



- Tiny BRs O(10^{-11} – 10^{-16}). First time computed here.
- No channel searched for. No (4) producible channels at FCC-ee (FCC-hh)
- Note: Leptonia are long-lived = LLP signature (displaced γ, e, μ vertices)

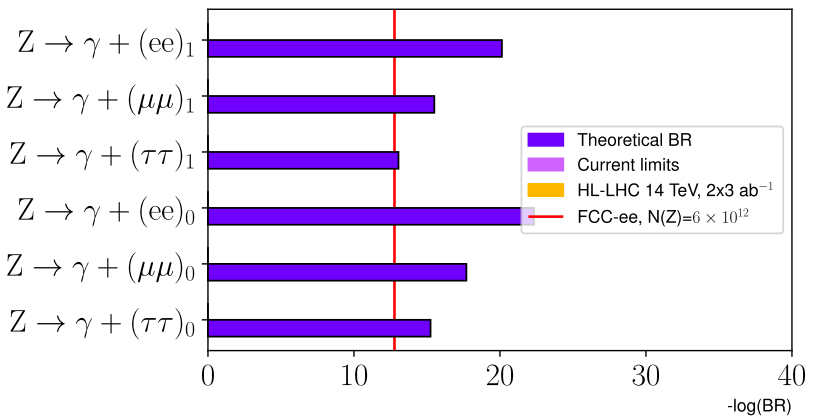
Exclusive Z decays
Z → γ + leptonium



$$\mathcal{B}(Z \to (\ell^+\ell^-)_0 + \gamma) = \frac{\alpha(0)^4\alpha(m_Z)^2}{9\cdot 256\, n^3}\frac{m^2_{\ell^+\ell^-}}{m^2_Z}\frac{\left(1-4s_w^2\right)^2\left(8s_w^4-4s_w^2+1\right)\left(m_Z^2-m^2_{\ell^+\ell^-}\right)}{\Gamma_{ee}\Gamma_Z s_w^4 c_w^4}$$

$$\mathcal{B}(Z \to (\ell^+\ell^-)_1 + \gamma) = \frac{\alpha(0)^4\alpha(m_Z)^2}{9\cdot 256\, n^3}\frac{m^2_{\ell^+\ell^-}}{m^4_Z}\frac{\left(8s_w^4-4s_w^2+1\right)\left(m_Z^4-m^4_{\ell^+\ell^-}\right)}{\Gamma_{ee}\Gamma_Z s_w^4 c_w^4}$$

				Exp. limits		Visible at
$Z \to \gamma$ +	X	Branching fraction	Framework	2023	HL-LHC	FCC-ee
$Z \to \gamma$ +	$(ee)_0$	4.7×10^{-23}	This work	–	–	✘
	$(\mu\mu)_0$	2.0×10^{-18}	This work	–	–	✘
	$(\tau\tau)_0$	5.7×10^{-16}	This work	–	–	✘
	$(ee)_1$	7.3×10^{-21}	This work	–	–	✘
	$(\mu\mu)_1$	3.1×10^{-16}	This work	–	–	✘
	$(\tau\tau)_1$	8.9×10^{-14}	This work	–	–	✘



- Tiny BRs: O(10^{-14} – 10^{-23}). First time computed here.
- No channel searched-for. No producible channel at FCC-ee.
- Note: Leptonia are long-lived = LLP signature (displaced γ, e, μ vertices)

Summary

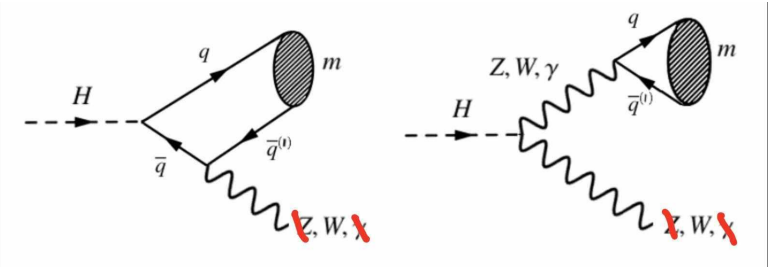
- Comprehensive survey of the theoretical & experimental status of more than 100 exclusive few-body decays into quarkonium of the 4 heaviest SM particles (H,Z,W,t): BR≈10^{-5} – 10^{-20}
  - Backgrounds to many BSM decays (H,Z → ALPs, gravitons, dark γ, ...), and allow study of QQbar production approaches and the ingredients: LDMEs,LCDAs,.
- Up-to-date collection of TH BRs and EXP limits from the literature.
- Calculation of new rare decay channels: radiative leptonium.
  - Tiny. Very hard to measure.
- Estimation of reachabilities of HL-LHC, FCC-ee/FCC-hh observations (if not BSM-enhanced):
  - HL-LHC can potentially observe a few of them: H→γ+ρ, Z→γ+ω, Z→γ+J/ψ,... all channel listed in the table below
  - FCC-ee can discover about 50% of such experimentally unobserved decays
  - FCC-hh can produce most of those decays channels

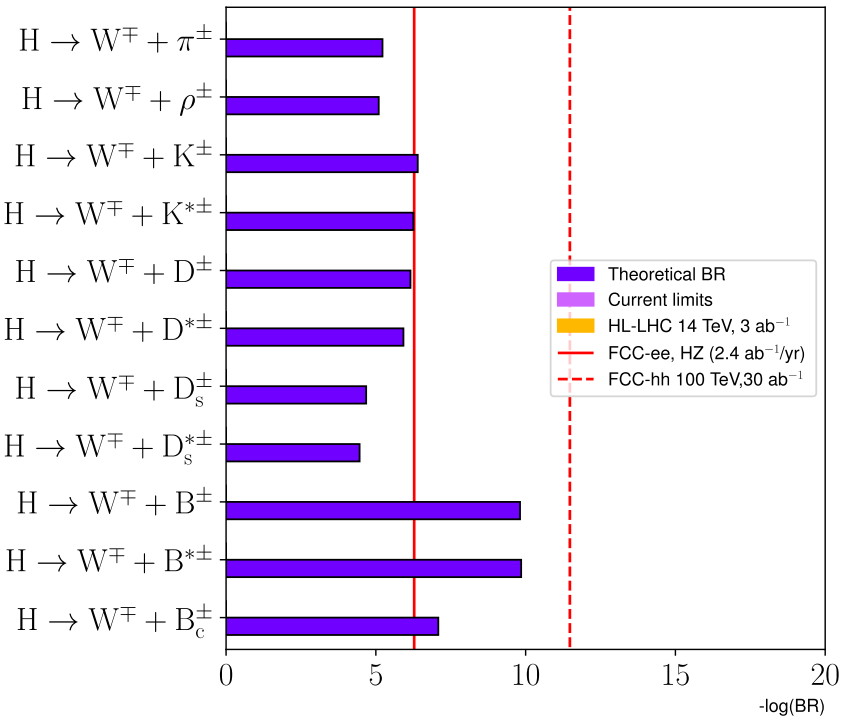
			Exp. limits		
		Branching fraction	2023	HL-LHC	$\mathcal{B}$(th)/$\mathcal{B}$(exp)
H →	$\gamma + J/\psi$	$(2.95 \pm 0.17) \times 10^{-6}$	$< 2.6\times 10^{-4}$ [84, 86]	$\lesssim 3.9 \times 10^{-5}$ [55]	~ 1/10
	$Z + \Upsilon(1S)$	1.7×10^{-5}	–	–	–
Z →	$\gamma + J/\psi$	$(9.96 \pm 1.86) \times 10^{-8}$	$< 6.0\times 10^{-7}$ [86, 127]	$\lesssim 3.1 \times 10^{-7}$ [55]	~ 1/3
	$\gamma + \Upsilon(1S)$	$(4.93 \pm 0.51) \times 10^{-8}$	$< 2.8\times 10^{-6}$ [83]	$\lesssim 2.2 \times 10^{-7}$	~ 1/4
t →	$c + \overline{B}_s^0$	$\left(4.0^{+4.0}_{-2.0}\right) \times 10^{-5}$		$\lesssim 4.0 \times 10^{-5}$ [26]	~ 1/1

backup

Exclusive Higgs decays
H → W + meson

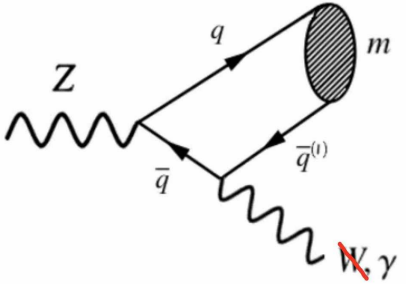
H → W∓ +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
H → W∓ +	$\pi^{\pm}$	$(4.3 \pm 0.2) \times 10^{-6}$	EFT+LCDA [86]	–	–	✓	✓
		6.0×10^{-6}	EFT+NRQM [11]				
	$\rho^{\pm}$	$(1.09 \pm 0.05) \times 10^{-5}$	EFT+LCDA [86]	–	–	✓	✓
		8.0×10^{-6}	EFT+NRQM [11]				
	$K^{\pm}$	$(3.3 \pm 0.1) \times 10^{-7}$	EFT+LCDA [86]	–	–	✘	✓
		4.0×10^{-7}	EFT+NRQM [11]				
	$K^{*\pm}$	$(5.6 \pm 0.4) \times 10^{-7}$	EFT+LCDA [86]	–	–	✓	✓
	$D^{\pm}$	$(5.6 \pm 0.5) \times 10^{-7}$	EFT+LCDA [86]	–	–	✓	✓
		7.0×10^{-7}	EFT+NRQM [11]				
	$D^{*\pm}$	$(1.04 \pm 0.14) \times 10^{-6}$	EFT+LCDA [86]	–	–	✓	✓
		1.2×10^{-6}	EFT+NRQM [11]				
	$D_s^{\pm}$	$(1.71 \pm 0.11) \times 10^{-5}$	EFT+LCDA [86]	–	–	✓	✓
		2.1×10^{-5}	EFT+NRQM [11]				
	$D_s^{*\pm}$	$(2.51 \pm 0.19) \times 10^{-5}$	EFT+LCDA [86]	–	–	✓	✓
		3.5×10^{-5}	EFT+NRQM [11]				
	$B^{\pm}$	$(1.54 \pm 0.40) \times 10^{-10}$	EFT+LCDA [86]	–	–	✘	✓
	$B^{*\pm}$	$(1.41 \pm 0.36) \times 10^{-10}$	EFT+LCDA [86]	–	–	✘	✓
	$B_c^{\pm}$	$(8.21 \pm 0.83) \times 10^{-8}$	EFT+LCDA [86]	–	–	✘	✓





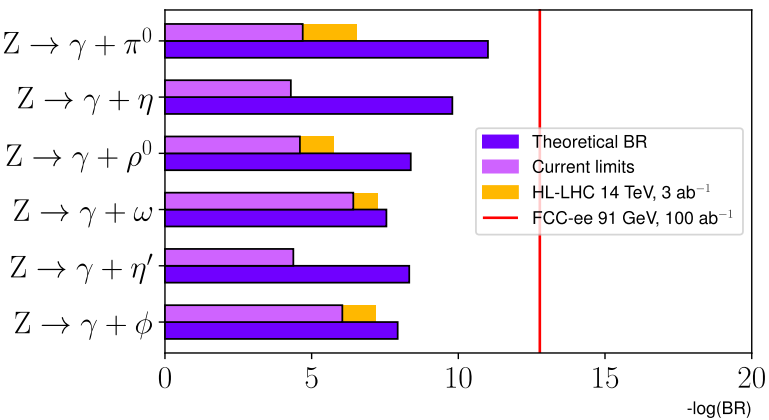
- Theory BRs: O(10^{-5} – 10^{-10}). No Exp. Limits.
- No search-performed so far.7 (4) producible channels at FCC-ee (FCC-hh)

Exclusive decays:
Z → γ + light meson



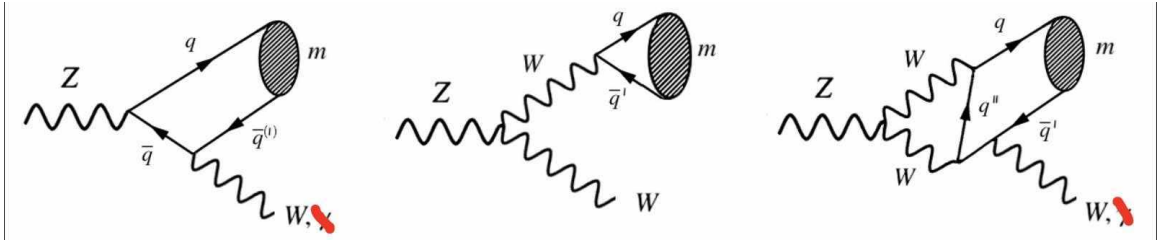
←light quarks (u, d ,s)

Z → γ +	M	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee
Z → γ +	π^0	$(9.8 \pm 1.0) \times 10^{-12}$	SCET+LCDA [62]	$< 2.0\times 10^{-5}$ [122]	$\lesssim 2.8 \times 10^{-7}$	✓
	η	$(1.0 - 17.0) \times 10^{-10}$	SCET+LCDA [131]	$< 5.1\times 10^{-5}$ [130]	–	✓
	ρ^0	$(4.19 \pm 0.47) \times 10^{-9}$	SCET+LCDA [62]	$< 2.5\times 10^{-5}$ [82]	$\lesssim 1.8 \times 10^{-6}$	✓
	ω	$(2.82 \pm 0.41) \times 10^{-8}$	SCET+LCDA [62]	$< 3.8\times 10^{-7}$ [85]	$\lesssim 5.7 \times 10^{-8}$	✓
	η'	$(3.1 - 4.8) \times 10^{-9}$	SCET+LCDA [131]	$< 4.2\times 10^{-5}$ [130]	–	✓
	ϕ	$(1.17 \pm 0.08) \times 10^{-8}$	LC+LCDA [132]	$< 9.0\times 10^{-7}$ [82]	$\lesssim 6.6 \times 10^{-8}$	✓
		$(1.04 \pm 0.12) \times 10^{-8}$	SCET+LCDA [62]			
	D^0	1.0×10^{-15}	SCET+LCDA [62, 129]	$< 2.1\times 10^{-3}$ [129]	$\lesssim 3.8 \times 10^{-5}$	✘

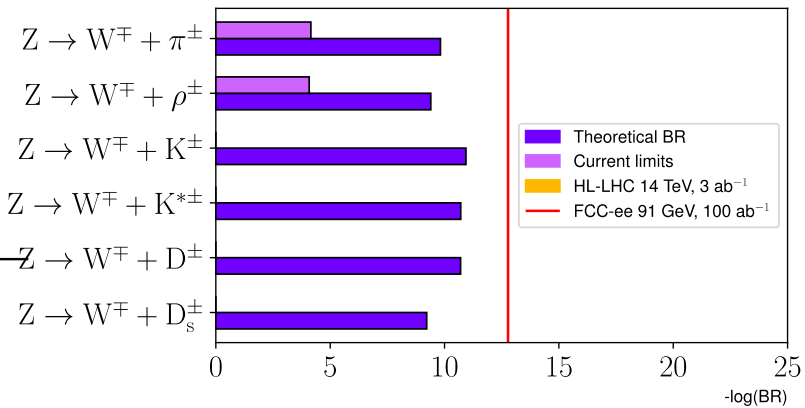


- Theory BRs: O(10^{-8} – 10^{-11}). Exp. Limits: O(10^{-5} – 10^{-7})
- 6 channels searched for. 6 producible channels at FCC-ee.
- Z → γ+ω is very close to be detected at HL-LHC (BR = ½ of projected limit)
- All channels will be visible at FCC-ee

Exclusive Z decays
Z → W + meson

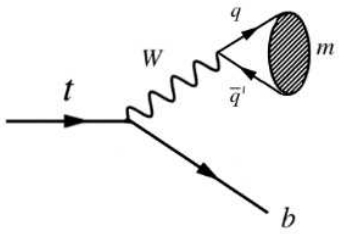


$Z \to W^{\mp}$ +	X	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee
$Z \to W^{\mp}$ +	$\pi^{\pm}$	$(1.51 \pm 0.01) \times 10^{-10}$	SCET+LCDA [58]	$< 7.0 \times 10^{-5}$ [118]	?	✓
	$\rho^{\pm}$	$(4.0 \pm 0.1) \times 10^{-10}$	SCET+LCDA [58]	$< 8.3 \times 10^{-5}$ [118]	?	✓
	$K^{\pm}$	$(1.16 \pm 0.01) \times 10^{-11}$	SCET+LCDA [58]	–	–	✓
	$K^{*\pm}$	$(1.96 \pm 0.12) \times 10^{-11}$	SCET+LCDA [58]	–	–	✓
	$D^{\pm}$	$(1.99 \pm 0.17) \times 10^{-11}$	SCET+LCDA [58]	–	–	✓
	$D_s^{\pm}$	$(6.04 \pm 0.30) \times 10^{-10}$	SCET+LCDA [58]	–	–	✓

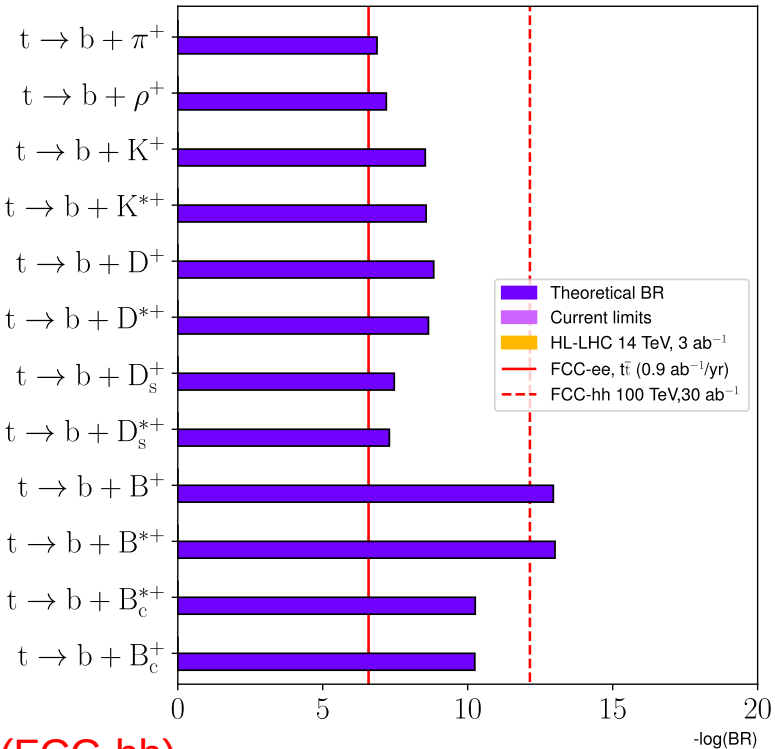


- Theory BRs: O(10^{-10} – 10^{-11}). Exp. limits: O(10^{-5})
- 2 channels searched for. 6 producible channel at FCC-ee

Semi-exclusive decays
t → meson + b quark



t →	X	+	Y	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
t →	π^+	+	b	1.3×10^{-7}	EFT+LCDA This work	–	–	✘	✓
	ρ^+	+	b	6.4×10^{-8}	EFT+LCDA This work	–	–	✘	✓
	K^+	+	b	2.9×10^{-9}	EFT+LCDA This work	–	–	✘	✓
	K^{*+}	+	b	2.7×10^{-9}	EFT+LCDA This work	–	–	✘	✓
	D^+	+	b	1.5×10^{-9}	EFT+LCDA This work	–	–	✘	✓
	D^{*+}	+	b	2.3×10^{-9}	EFT+LCDA This work	–	–	✘	✓
	D_s^+	+	b	3.4×10^{-8}	EFT+LCDA This work	–	–	✘	✓
	D_s^{*+}	+	b	5.1×10^{-8}	EFT+LCDA This work	–	–	✘	✓
	B^+	+	b	1.1×10^{-13}	EFT+LCDA This work	–	–	✘	✘
	B^{*+}	+	b	9.8×10^{-14}	EFT+LCDA This work	–	–	✘	✘
	B_c^{*+}	+	b	5.5×10^{-11}	EFT+LCDA This work	–	–	✘	✓
	B_c^+	+	b	5.7×10^{-11}	EFT+LCDA This work	–	–	✘	✓



- Theory BRs: O(10^{-7} – 10^{-14}). No existing exp. Limits
- 0 channel searched for. 0 (10) visible channels at FCC-ee(FCC-hh)
- First time those decays have been computed.