Heavy quarkonium production ("in vacuum"): theory overview

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Motivations (I): understanding hadronisation

Description of production of any high-p_T($\gg \Lambda_{\rm QCD}$) hadrons in QCD = (perturbative) production of quarks/gluons + *hadronisation.*

1. For light and heavy-light hadrons, hadronisation is studied phenomenologically:
   - Fragmantation Functions: based on factorisation theorems, fitted to describe data (first attempts to compute on the lattice)
   - Monte-Carlo models: hard to derive from QCD Lagrangian (string-based in Pythia, cluster hadronisation in Herwig,...)
2. Quarkonia – "Hydrogen atoms of QCD" ⇒ corrections to the "naive" quark model should be suppressed by powers of relative velocity (v) of heavy quarks in the bound state:

$$
\begin{aligned}
|J/\psi\rangle &= O(1)\left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle + O(v)\left|c\bar{c}\left[{}^3P_J^{(8)}\right]+g\right\rangle \\
&+ O(v^{3/2})\left|c\bar{c}\left[{}^1S_0^{(8)}\right]+g\right\rangle + O(v^2)\left|c\bar{c}\left[{}^3S_1^{(8)}\right]+gg\right\rangle + \ldots,
\end{aligned}
$$

3. ⇒ let's try to use understand production of quarkonia. **This understanding will be a small-v limit for any future theory of hadronisation!**

Motivations (II): quarkonia as tools

If hadronisation mechanism **was well understood,** *then quarkonium production would be:*

1. An excellent tool to study gluon content of a proton/nucleus:
   - Small (or negligible) "valence" c and b content – production predominantly through coupling to gluons at high energies
   - Clean experimental signatures for J/ψ, $\Upsilon(nS)$, ...
   - relatively small $M_{J/\psi} \simeq 3GeV$ – *access to very small* $x \sim Me^{-y}/\sqrt{s} \sim 10^{-4} - 10^{-6}$ at the LHC.
2. A tool to study double/multiple parton scattering: due to significant cross sections of multiple/associated production and lower p_T/scales in comparison to vector bosons/jets
3. A probe for QGP: melting/recombination/parton energy loss could be studied
4. A tool to study of c-Higgs and b-Higgs couplings through associated production and Higgs decays
5. ...

Quarkonium production models

Unfortunately no existing model can describe all data on inclusive quarkonium hadro/photo/electro/e^+e^- production and polarisation observables.

Old ideas:

1. Colour Singlet Model: only **colour-singlet** $Q\bar{Q}$ pairs with the same orbital momentum/spin as corresponding potential-model state hadronise to the quarkonium.
2. NRQCD factorisation: based on the hierarchy of different colour/orbital momentum/spin states of the $Q\bar{Q}$-pair in the v-expansion for the quarkonium state
3. (Improved) Colour Evaporation Model assumes "democracy" of colour/orbital momentum/spin states of the $Q\bar{Q}$-pair

New ideas: Potential NRQCD, Soft-gluon factorisation, Shape-functions, ...

Motivation for new ideas:

- reduction of the numeber of free parameters
- improvement of perturbative convergence
- phenomenological problems

Quarkonium in the potential model

Cornell potential:

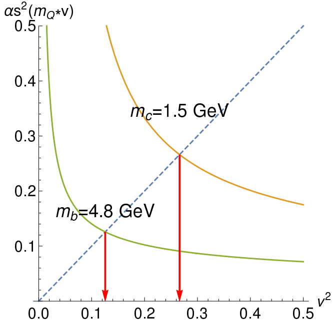
$$V(r) = -C_F \frac{\alpha_s(1/r)}{r} + \sigma r,$$

neglect linear part, because quarkonium is "small" (~ 0.3 fm) $\rightarrow$ Coulomb wavefunction (for effective mass $\frac{m_1 m_2}{m_1+m_2} = \frac{m_Q}{2}$):

$$R(r) = \frac{\sqrt{m_Q^3 \alpha_s^3 C_F^3}}{2} e^{-\frac{\alpha_s C_F}{2} m_Q r}$$

$$\langle v^2 \rangle = \frac{C_F^2 \alpha_s^2}{2}, \langle r \rangle = \frac{3}{2C_F} \frac{1}{m_Q v}$$

$$\Rightarrow \boxed{\alpha_s^2(m_Q v) \simeq v^2}$$



Non-relativistic QCD

The velocity-expansion for quarkonium eigenstate is a copy of corresponding arguments from atomic physics:

$$\begin{aligned} |J/\psi\rangle &= O(1)\left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle + O(v)\left|c\bar{c}\left[{}^3P_J^{(8)}\right]+g\right\rangle \\ &+ O(v^{3/2})\left|c\bar{c}\left[{}^1S_0^{(8)}\right]+g\right\rangle + O(v^2)\left|c\bar{c}\left[{}^3S_1^{(8)}\right]+gg\right\rangle + \ldots, \end{aligned}$$

for validity of this arguments, we should work in *non-relativistic EFT*, dynamics of which conserves number of heavy quarks. In such EFT, $Q\bar{Q}$-pair is produced in a point, by local operator:

$$\mathcal{A}_{\rm NRQCD} = \langle J/\psi + X|\,\chi^\dagger(0)\kappa_n\psi(0)\,|0\rangle\,,$$

Different operators “couple” to different Fock states:

$$\chi^\dagger(0)\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^1S_0^{(1)}\right]\right\rangle,\ \chi^\dagger(0)\sigma_i\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle,$$
$$\chi^\dagger(0)\sigma_i T^a\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^3S_1^{(8)}\right]\right\rangle,\ \chi^\dagger(0)D_i\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^1P_1^{(8)}\right]\right\rangle, \ldots$$

squared NRQCD amplitude (=LDME):

$$\sum_X |\mathcal{A}|^2 = \langle 0|\underbrace{\psi^\dagger\kappa_n^\dagger\chi a^\dagger_{J/\psi}a_{J/\psi}\chi^\dagger\kappa_n\psi}_{\mathcal{O}_n^{J/\psi}}|0\rangle = \left\langle \mathcal{O}_n^{J/\psi}\right\rangle,$$

Non-relativistic QCD

Velocity-scaling of LDMEs follows from velocity-scaling of corresponding Fock states and of operators $\chi^\dagger \kappa_n \psi$:

	${}^1S_0^{(1)}$	${}^3S_1^{(1)}$	${}^1S_0^{(8)}$	${}^3S_1^{(8)}$	${}^1P_1^{(1)}$	${}^3P_0^{(1)}$	${}^3P_1^{(1)}$	${}^3P_2^{(1)}$	${}^1P_1^{(8)}$	${}^3P_0^{(8)}$	${}^3P_1^{(8)}$	${}^3P_2^{(8)}$
η_c	1		v^4	v^3					v^4			
J/ψ		1	v^3	v^4						v^4	v^4	v^4
h_c			v^2		v^2							
χ_{c0}				v^2		v^2						
χ_{c1}				v^2			v^2					
χ_{c2}				v^2				v^2				

Note that:

- Colour-singlet LDMEs are LO in v for S-wave states $\Rightarrow$ *Colour-Singlet Model*
- For P-wave states the CS and CO LDMEs are of the same order $\Rightarrow$ *mixing*
- Connection between LDMEs for η_c and J/ψ through *Heavy-Quark Spin Symmetry*

Matching procedure between QCD and NRQCD:

$$v \ll 1 : \mathcal{A}_{\rm QCD}(gg \to Y_{Q\bar{Q}(v)}) = \sum_n f_n \left\langle Y_{Q\bar{Q}(v)} \right| \chi^\dagger(0)\kappa_n\psi(0) \left|0\right\rangle + O(v^{\#}),$$

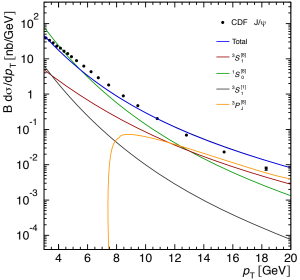
$\Rightarrow$ NRQCD factorization formula ("theorem") [Bodwin, Braaten, Lepage 95'] :

$$\sigma(gg \to \mathcal{H} + X) = \sum_n \sigma(gg \to Q\bar{Q}[n] + X) \left\langle \mathcal{O}_n^{\mathcal{H}} \right\rangle .$$

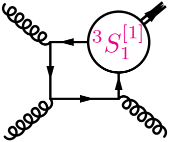
NRQCD factorisation: p_T-behaviour in pp

$$\frac{d\sigma}{dp_T^2}(pp \to \mathcal{H} + X) = \sum_n \frac{d\sigma}{dp_T^2}(pp \to Q\bar{Q}[n] + X)\,\langle \mathcal{O}_n^{\mathcal{H}} \rangle\,.$$

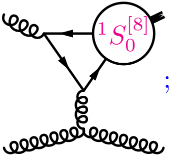
NLO, plot from hep-ph/1403.3970 :



At **LO**:



$\sim p_T^{-8}$



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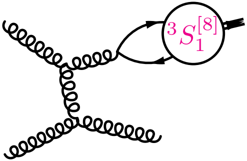
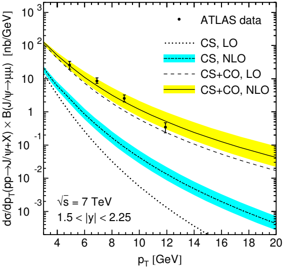


$\sim p_T^{-4}$

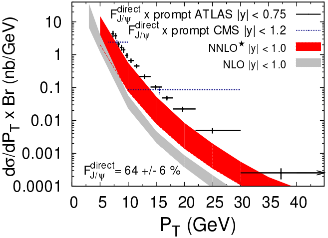
NRQCD factorisation: what does work?

- *Un-polarized* p_T distributions of J/ψ, χ_{cJ} in hadro- and photoproduction, as well as e^+e^- data can be described. The same is true for $\Upsilon(nS)$, $\chi_{bJ}(nS)$.
- Solves the problem of non-cancelling IR divergence at NLO in CSM for P-wave states production and decay through mixing with ${}^3S_1^{(8)}$ or ${}^1S_0^{(8)}$ states at $O(v^2)$.
- Covers the gap between CSM (@LO and NLO) and data at high-p_T in hadroproduction, due to contribution of CO states. If NNLO corrections in CS are as large as needed to close this gap, then perturbative expansion is just useless and we should stop doing quarkonia.

NLO NRQCD, [Butenschön, Kniehl, '11]



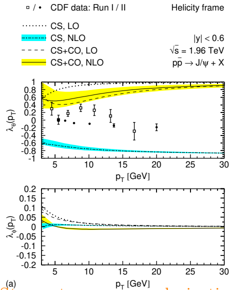
NLO and NNLO* CSM [Lansberg '11]



Problems: Polarisation

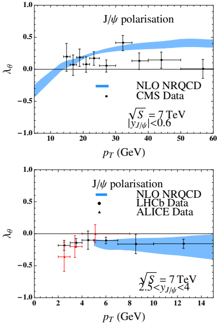
LDME fit	J/ψ hadropr.	J/ψ photopr.	J/ψ polar.	η_c hadropr.
Butenschön et al.	✓($p_T > 3$ GeV)	✓	✗	✗
Chao et al. + η_c	✓($p_T > 6.5$ GeV)	✗	✓	✓
Zhang et al.	✓($p_T > 6.5$ GeV)	✗	✓	✓
Gong et al.	✓($p_T > 7$ GeV)	✗	✓	✗
Chao et al.	✓($p_T > 7$ GeV)	✗	✓	✗
Bodwin et al.	✓($p_T > 10$ GeV)	✗	✓	✗

Global fit [Butenschön, Kniehl, '12]



Strong transverse polarisation due to $^3S_1^{[8]}$ and $^3P_J^{[8]}$ states at high p_T

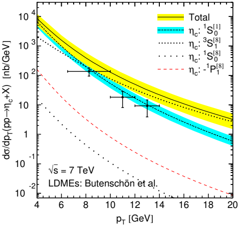
Example hadroproduction dominated fit [Chao et.al., '14]

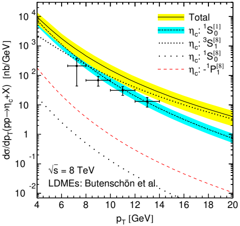


Problems: HQSS and photoproduction

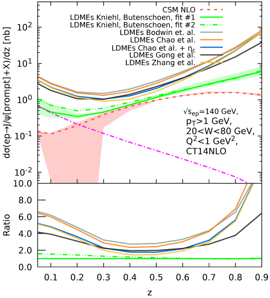
LDME fit	J/ψ hadropr.	J/ψ photopr.	J/ψ polar.	η_c hadropr.
Butenschön et al.	✓($p_T > 3$ GeV)	✓	✗	✗
Chao et al. + η_c	✓($p_T > 6.5$ GeV)	✗	✓	✓
Zhang et al.	✓($p_T > 6.5$ GeV)	✗	✓	✓
Gong et al.	✓($p_T > 7$ GeV)	✗	✓	✗
Chao et al.	✓($p_T > 7$ GeV)	✗	✓	✗
Bodwin et al.	✓($p_T > 10$ GeV)	✗	✓	✗

Global fit [Butenschön, Kniehl, '12]





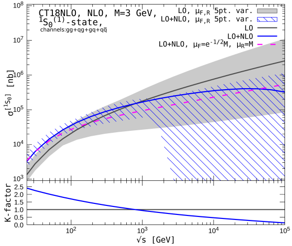
J/ψ-photoproduction at the EIC vs $z = (p_{J/\psi}P)/(qP)$, using NLO calculation data by M. Butenschön



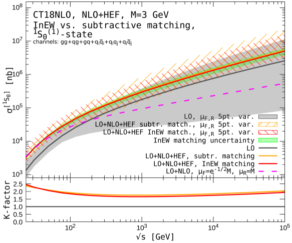
p_T-integrated cross sections, another "puzzle"?

Inclusive η_c-hadroproduction (CSM):[Lansberg, Ozcelik '20; Lansberg, M.N., Ozcelik '22]

NLO:

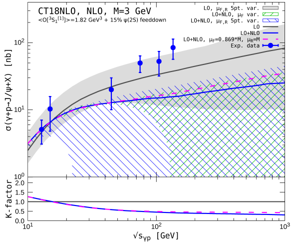


NLO+HEF matching:

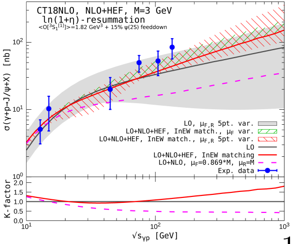


Inclusive J/ψ-photoproduction (CSM): [Lansberg et al. '21; Lansberg, M.N., Ozcelik, '23]

NLO:



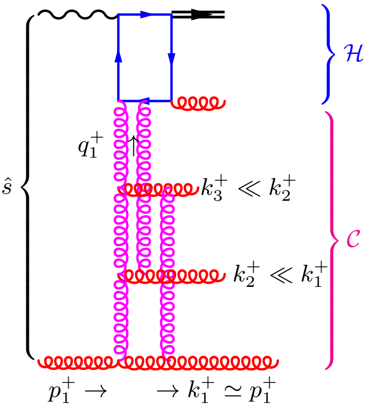
NLO+HEF matching:



High-Energy Factorisation

The **LLA** ($\sum_n \alpha_s^n \ln^{n-1}(\hat{s}/M^2)$) formalism is due to [Collins, Ellis, 91'; Catani, Ciafaloni, Hautmann, 91',94']

Physical picture in the **LLA** for photoproduction:



Glauber exchanges($k_+ k_- \ll \mathbf{k}_T^2$) form the **Reggeized gluon** in the t-channel.

$$\hat{\sigma}_{\rm HEF}(\hat{s}) \propto \int\limits_0^{\hat{s}/M^2} \frac{dy}{y} \int\limits_0^{\infty} d\mathbf{q}_{T1}^2 \mathcal{C}\left(\frac{yM^2}{\hat{s}}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right) \times \mathcal{H}(y, \mathbf{q}_{T1}^2) + \text{NLLA} + O(M^2/\hat{s}).$$

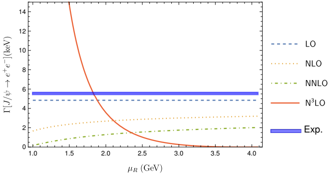
- The resummation factor $\mathcal{C}$ is the solution of the LL **BFKL** equation with collinear divergences subtracted,
- The coefficient function $\mathcal{H}$ can be calculated at LO and NLO (needed for **NLLA**),
- For consistency with fixed-order **DGLAP** evolution the anomalous dimension γ_{gg} in $\mathcal{C}$ should be taken at LO: $\gamma_{gg} = \hat{\alpha}_s C_A/(\pi N)$.
- Expansion of $\hat{\sigma}_{\rm HEF}(\hat{s})$ in α_s **correctly reproduces** $\hat{\sigma}_{\rm NLO}(\hat{s} \gg M^2)$ and predicts the LLA term in $\hat{\sigma}_{\rm NNLO}(\hat{s} \gg M^2)$.

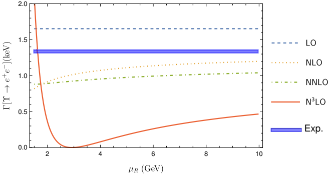
[MN, Lansberg, Ozcelik, '23]

Prospects for NNLO

Tere is no NNLO calculations for heavy qarkonium **production** yet, but the calculations for **decay rates** are rather advanced:

1. For $J/\psi \to \mu^+\mu^-$ the NNLO[Beneke, Smirnov] and (very recent!) N^{3}LO results are known [Feng et.al., 2207.14259] :





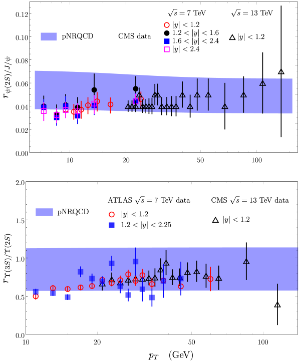
2. For $\eta_c \to \gamma\gamma$ the NNLO result was obtained in [Abreu et.al., 2211.08838] and similar behaviour of radiative corrections was found.

- Should we expect strong perturbative instability for production cross section at $p_T < M$ due to bound-state effects? Maybe separation of corrections between LDMEs and hard part is not optimal in NRQCD?
- The NLO corrections to parton$\to Q\bar{Q}[n]$ fragmentation functions tend to be moderate $\Rightarrow$ NNLO will stabilize at $p_T > M$ and $p_T \gg M$.

Potential NRQCD

The NRQCD logic can be pushed even further by assuming that $mv^2 \ll mv$ and dynamics at the scale mv^2 is strongly-coupled [Brambilla et.al., '22] . **At LO in** v:

Prompt cross section ratios:



$$\boxed{\langle \mathcal{O}^{\mathcal{H}}({}^3S_1^{[1]})\rangle = \frac{3N_c}{2\pi}|R_{\mathcal{H}}(0)|^2,}$$

$$\langle \mathcal{O}^{\mathcal{H}}({}^3P_J^{[8]})\rangle = \frac{2J+1}{18N_c}\frac{3|R_{\mathcal{H}}(0)|^2}{4\pi}\mathcal{E}_{00},$$

$$\langle \mathcal{O}^{\mathcal{H}}({}^1S_0^{[8]})\rangle = \frac{1}{6N_c m^2}\frac{3|R_{\mathcal{H}}(0)|^2}{4\pi}c_F^2\mathcal{B}_{00},$$

$$\langle \mathcal{O}^{\mathcal{H}}({}^3S_1^{[8]})\rangle = \frac{1}{2N_c m^2}\frac{3|R_{\mathcal{H}}(0)|^2}{4\pi}\mathcal{E}_{10;10},$$

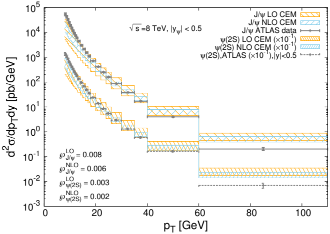
where $|R_{\mathcal{H}}(0)|^2$ – radial wave function at the origin from **potential model** for the quarkonium $\mathcal{H}$, and $\mathcal{E}_{00}$, $\mathcal{B}_{00}$, $\mathcal{E}_{10;10}$ – chromo electric/magnetic field correlators over QCD vacuum (i.e. **independent on** $\mathcal{H}$ up to RG running $m_c \to m_b$).

Colour Evaporation Model

The $c\bar{c}$ pairs with $M_{c\bar{c}} < 2m_D$ can not hadronise to the pair of D-mesons. Where do they go?
CEM assumes that all of them hadronise to quarkonia with the same probability $F_{J/\psi}$, $F_{\psi(2S)}$, ... :

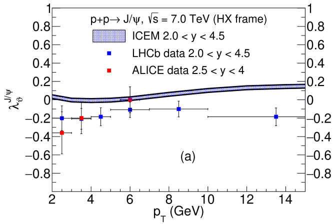
$$\sigma_{J/\psi} = F_{J/\psi} \times \int_{m_{J/\psi}}^{2m_D} dM_{c\bar{c}} \frac{d\sigma_{c\bar{c}}}{dM_{c\bar{c}}}.$$

CEM@NLO [Lansberg et al. '20]



Unpolarised production at high-p_T

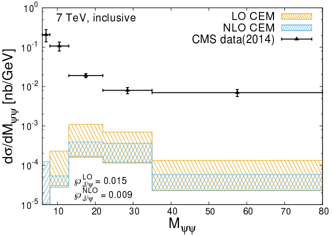
[Vogt, Chung '21]

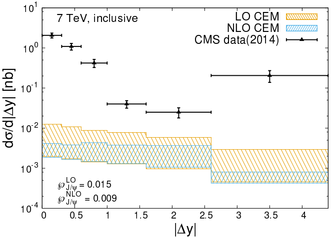


Problems of CEM

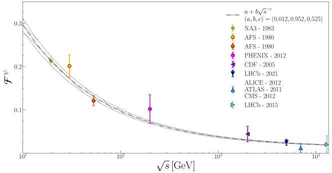
Pair production ($F_{2J/\psi} = (F_{J/\psi})^2$)

[Lansberg et al. '20] :

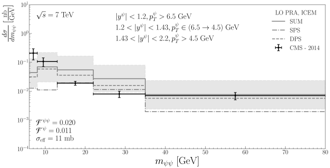




Energy-dependence (=non-universality) of $F_{J/\psi}$ [Saleev, Chernyshev '22]



Pair production ($F_{2J/\psi} \neq (F_{J/\psi})^2$ + DPS) [Saleev, Chernyshev '22] :



The IR problem of P-wave

It is well known that for the P**-wave** production, unexpected divergences appear.

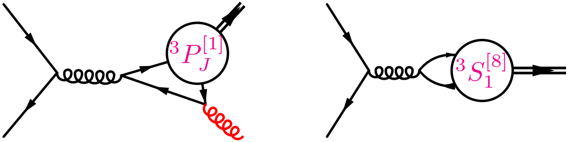
Simple example: The process

$$q + \bar{q} \to Q\bar{Q}[{}^3P_J^{[1]}] + g,$$

naively should not contain IR-divergences, because $g \to Q\bar{Q}[{}^3P_J^{[1]}]$ transition is forbidden. However for $\hat{s} \to M^2$:

$$\begin{aligned} |\mathcal{M}(q+\bar{q} \to Q\bar{Q}[{}^3P_J^{[1]}]+g)|^2 &\sim \alpha_s(2J+1)\frac{(M^4-\hat{t}^2)\hat{t}^2}{M^4(\hat{s}-M^2)^4} \\ &\times |\mathcal{M}(q+\bar{q} \to Q\bar{Q}[{}^3S_1^{[8]}])|^2. \end{aligned}$$

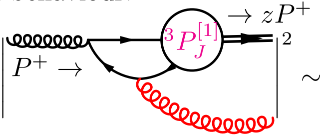
This new IR divergence can be absorbed through mixing between ${}^3P_J^{[1]}$ and ${}^3S_1^{[8]}$ LDMEs.



Soft-gluon factorisation

Usually in NRQCD, the fragmentation function $g^* \to Q\bar{Q}[^3P_J^{[1]}]$ has nasty IR behaviour:

$$D(z) = $$



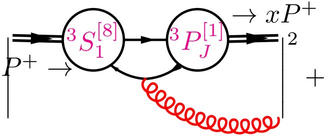
$$\sim \alpha_s^2 \left(\frac{\delta(z-1)}{\epsilon} + \frac{1}{(1-z)_+} + \dots \right).$$

Recently it has been proposed to re-factorise it using SGF [Ma, *et. al.*, '23]:

$$D_{g\to {}^3P_J^{[1]}}(z,\mu_0) = \sum_{n,n'={}^3P_J^{[1]},{}^3S_1^{[8]}} \int_0^1 dx \hat{D}_{n,n'}(z/x,\mu_0,M) F_{n,n'}(x,M),$$

where all "bad" terms at $x \to 1$ are absorbed into SGF $F_{n,n'}(x,M)$. *The SGF can be understood as the analog of LDME with x-dependence:*

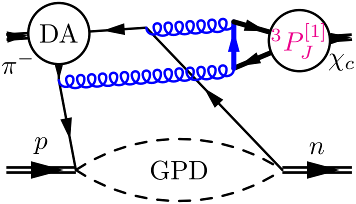
$$F_{{}^3S_1^{[8]},{}^3S_1^{[8]}}(x) = $$



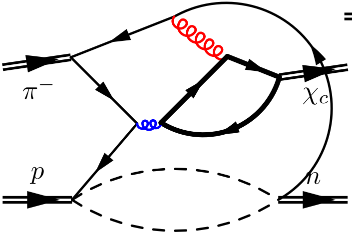
$$+ O(1-x).$$

Colour-octet for exclusive processes?

Recently, together with L. Szymanowski we have found a manifestation of the same problem in the hard exclusive reaction (see the talk by Saad Nabebacus about GPD formalism):



$$\pi^- + p \to \chi_{cJ} + n,$$



$$\longrightarrow \langle \chi_{cJ}, n | \mathcal{O}_8(0) | \pi^-, p \rangle$$

Similar problem arises in exclusive decays of χ_{cJ} [N. Kivel, '18]

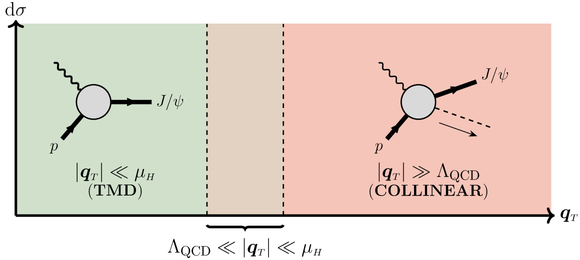
TMD factorization for heavy quarkonia?

Due to ${}^1S_0^{(1)}$-dominance of the η_c-production at small p_T, the factorization violation arguments for hadron-pair production are not valid for this state ?! But TMD factorization for the $p_T \ll M$ regime of η_c production is not simple. It includes new object – **TMD shape function** [Echevarria, 19'; Fleming, Makris, Mehen, 19'; J.Bor *et al.* '23] :

$$S(\mathbf{b}_T) = \langle 0| \psi^\dagger(\mathbf{b}_T)\kappa_n^\dagger \chi(\mathbf{b}_T) a^\dagger_{J/\psi} a_{J/\psi} \chi^\dagger(0)\kappa_n \psi(0) |0\rangle ,$$

where transverse coordinate $\mathbf{b}_T$ is Fourier-conjugate to the transverse momentum ($\mathbf{k}_T$) of the $Q\bar{Q}$-pair in the quarkonium, relative to the "light cloud".

The case of SIDIS: [J.Bor *et al.* '23]



Conclusions and outlook

- NRQCD factorisation can consistently describe hadroproduction data but the global description of *hadro, photoproduction and polarization* data has not been achieved at NLO
- This is attributed either to violation of universality of LDMEs across collision systems or problems of factorisation at $p_T \lesssim M$
- An important piece of the puzzle are high-energy enchanced corrections
- Theory effort is concentrated on consolidating the description of hadroproduction data at $p_T \gg M$ either by reducing the number of free parameters (pNRQCD) or improving the perturbative stability (SGF)
- SGF $\simeq$ Shape-functions ?
- There exist CO contributions not only in inclusive but also in exclusive physics

Thank you for your attention!

LDME fits

LDME fit	J/ψ hadropr.	J/ψ photopr.	J/ψ polar.	η_c hadropr.	$J/\psi + Z$
Butenschön et al.	✓($p_T > 3$ GeV)	✓	✗	✗	✗
Chao et al. + η_c	✓($p_T > 6.5$ GeV)	✗	✓	✓	✗
Zhang et al.	✓($p_T > 6.5$ GeV)	✗	✓	✓	✗
Gong et al.	✓($p_T > 7$ GeV)	✗	✓	✗	✗
Chao et al.	✓($p_T > 7$ GeV)	✗	✓	✗	✗
Bodwin et al.	✓($p_T > 10$ GeV)	✗	✓	✗	✗
Brambilla et al.	✓($p_T > 9$ GeV)	✗	✓	(✗✓)	✓