Quarkonia as Tools (Aussois, Jan 2024)

Theory of (Quarkonia production) in AA Collisions

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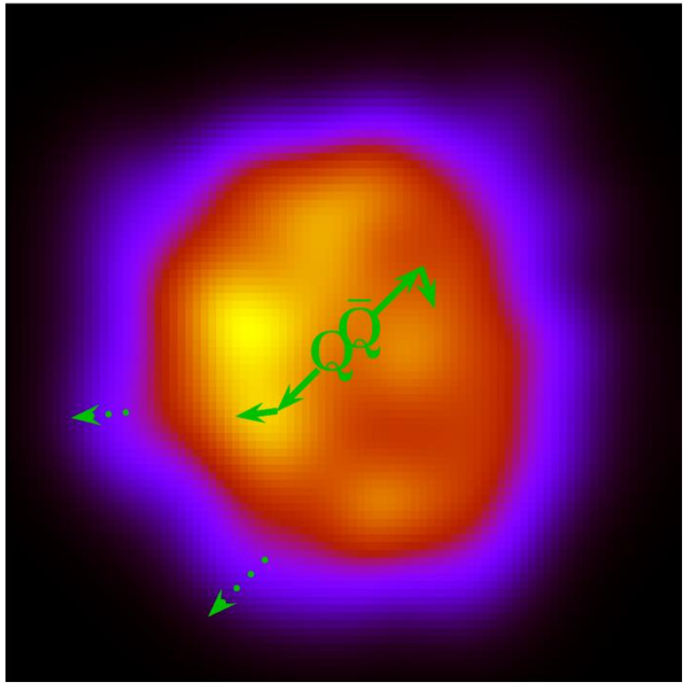
and Pays de la Loire

What is a quarkonia… in a hot QGP medium ?

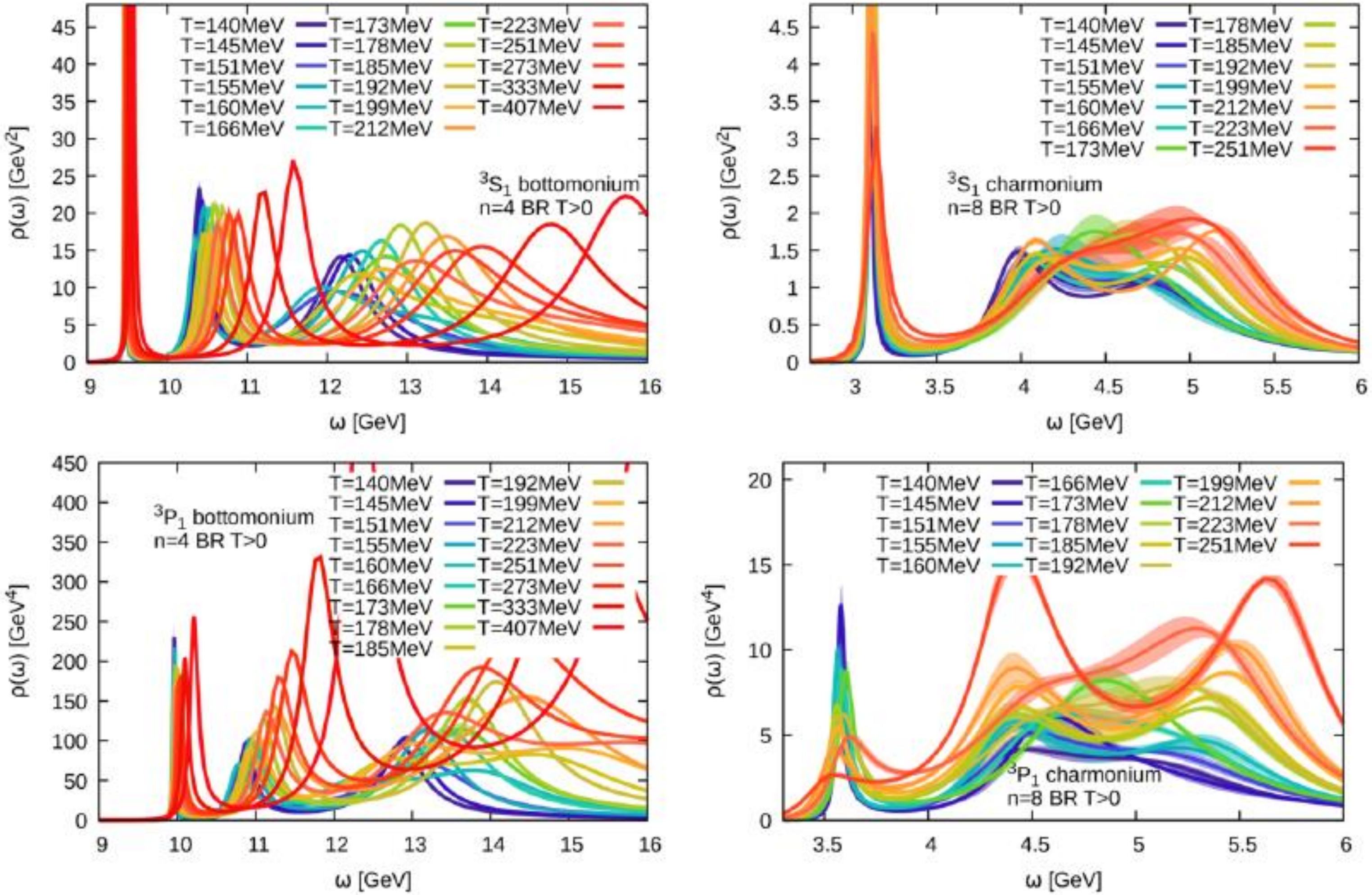


Answer may vary depending on how hot is the QGP, and how long you observe

- Not to high T, not too long : Same as in vacuum (see Maxim's talk) + some external perturbation
- If not : probably better to speak a $Q\bar{Q}$ pair

lQCD perspective : spectral function

Kim et al, JHEP11(2018)088



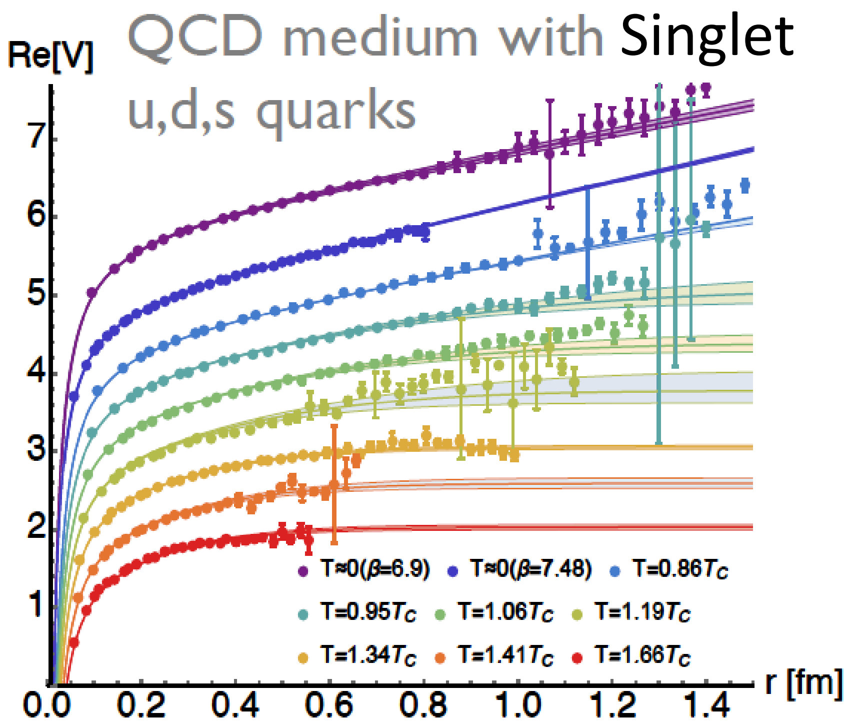
Rich structure : broadening and mass shift. What are the underlying “ingredients’” ?

The 3 pillars of quarkonia production in AA

Implicitly in the pNRQD EFT.

Screening of the real potential

Protential (recent lQCD calculations)



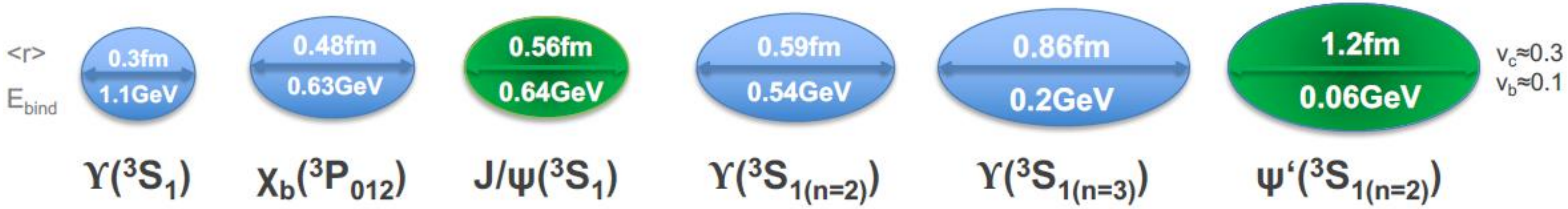
At T=0, well described by the Cornell shape:

$$V(r) = -\frac{\alpha}{r} + Kr$$

<u>Quarkonia scales</u>

- m_Q
- In vacuum: Binding energy / separation energy btwn levels: $\Delta E \alpha\ m_Q$ g^4 (Coulomb part) => v α g^2
- Radius : $(m_Q\ g^2)^{-1}$
- For a linear potental $\hbar\omega_0 = \left(\frac{\hbar^2 K_l^2}{m_b/2}\right)^{\frac{1}{3}} \approx 0.504\ \text{GeV}$

$$v \propto \left(\frac{K_l}{m_b^2}\right)^{\frac{1}{3}}$$



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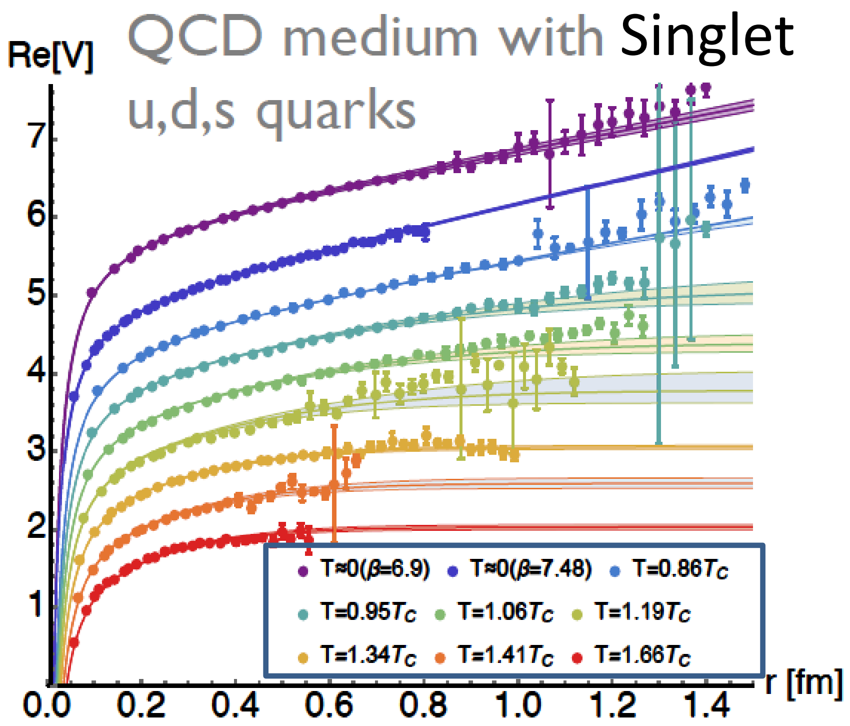

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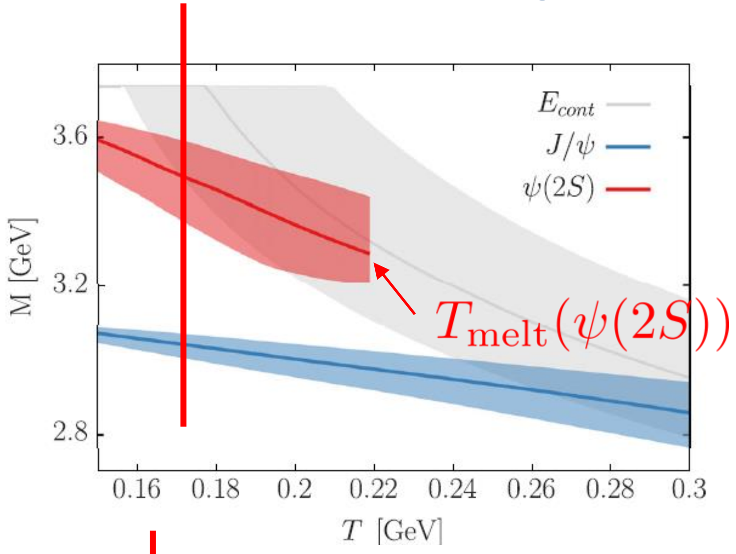
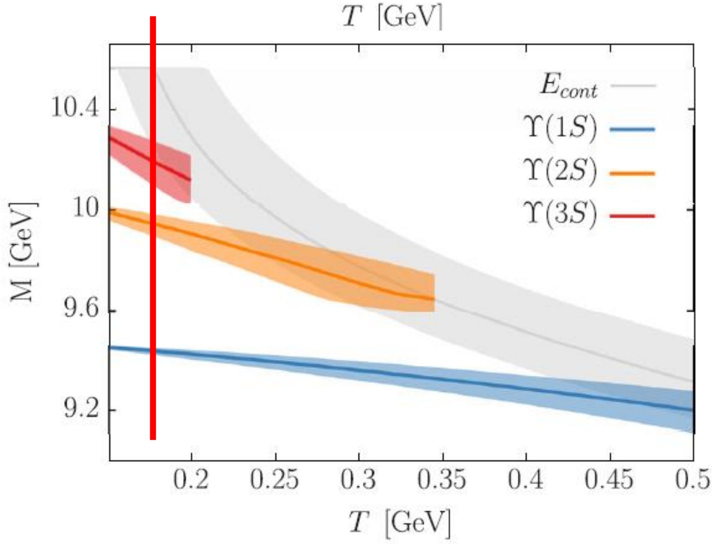
Compact and tightly bound states (at least for the lowest ones) => could survive QGP at low/mid T as well as to interactions with hadronic matter.

Screening of the real potential

Recent In-medium spectrum (Lafferty and Rothkopf 2020)

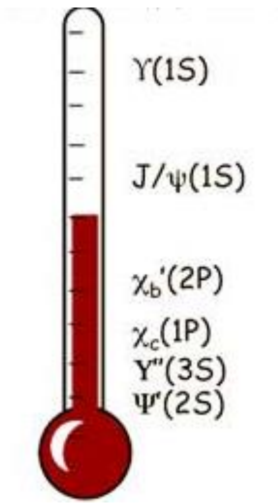




« all or nothing scenario»:

- If $T_{\text{early QGP}} > T_{\text{melt}}$ => the state is not produced
- If $T_{\text{early QGP}} < T_{\text{melt}}$ => the state is produced like in pp

=> *SEQUENTIAL SUPPRESSION; Quarkonia as early QGP thermometer*



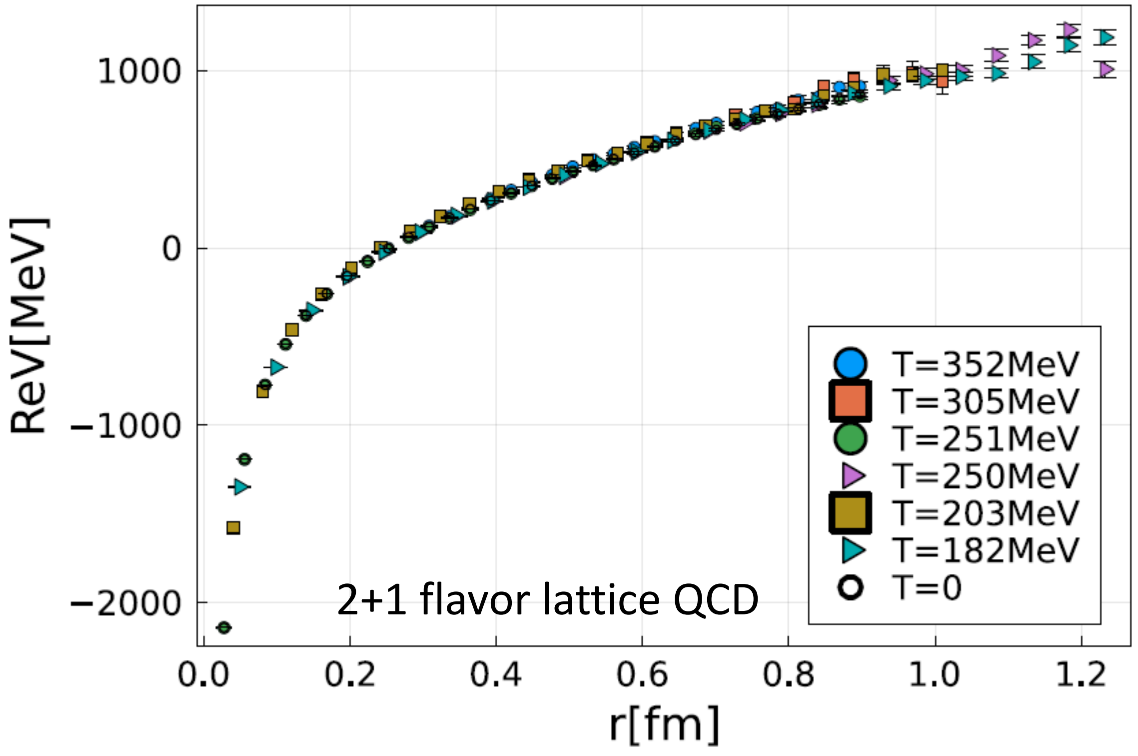
Most prominently : probing new state of matter in AA collision: Original idea by Matsui and Satz (86)…

… and advertized as a motivation in hundreds of talks (and papers) since then

David Lafferty, Alexander Rothkopf , Phys. Rev. D 101, 056010 (2020)

Screening of the real potential

Recent news : the real potential is not screened at temperatures reached in AA collisions !!!



Bazazov et al 2023 (Hot QCD collaboration)

How to define properly a “potential” on the lattice ?

<u>Historically</u> : thermodynamical potential like the free energy (in presence of a static dipole) or the total internal energy.

Modern approach : evaluate the Wilson loop and connect it to the r-dependent spectral density

$$W(\tau, r, T) = \int_{-\infty}^{+\infty} d\omega e^{-\omega\tau} \rho_r(\omega, T)$$

A “peak” contribution in the spectral density modelled as

$$\rho_r^{\mathrm{peak}}(\omega, T) = \frac{1}{\pi} \mathrm{Im} \frac{A_r(T)}{\omega - \mathrm{Re}V(r,T) - i\Gamma(\omega, r, T)}$$

=> Lattice data then unfolded with this Ansatz.

Does not seems quite intuitive, may not be the end of the story

Screening of the real potential

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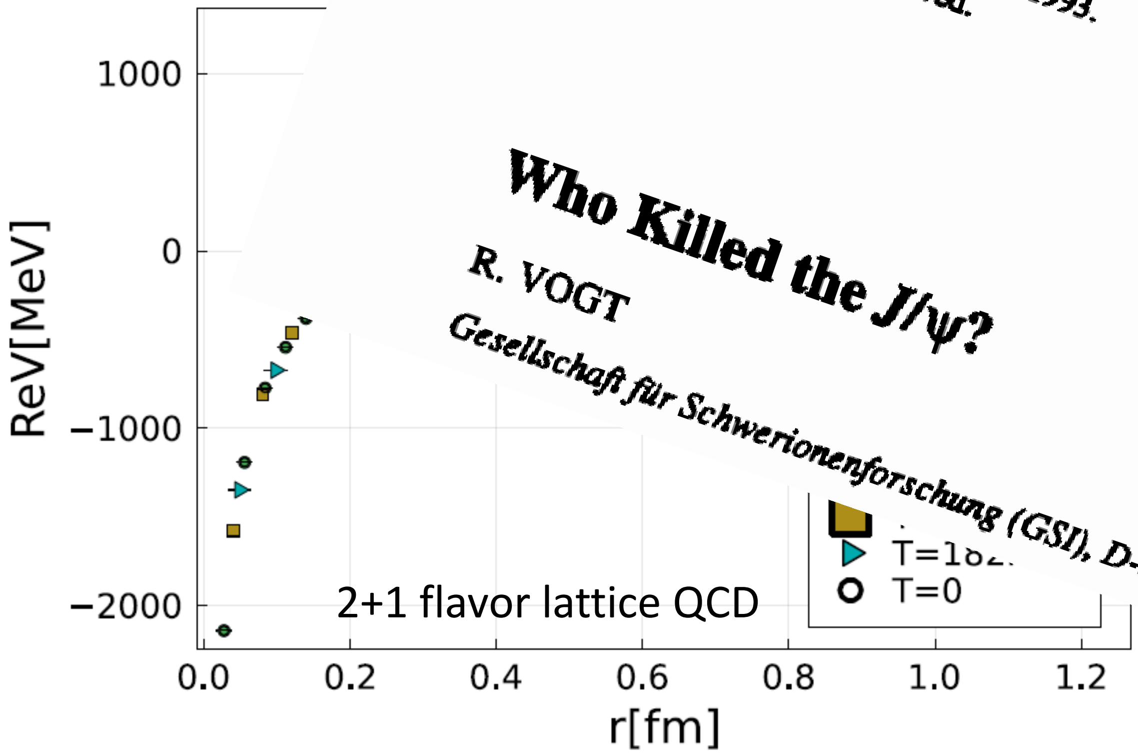

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…elled as

$\rho_r^{\text{peak}}(\omega, T) = \ldots \pi \ldots ; T)$

=> Lattice data then deconvoluted wi… …satz.

Prog. Part. Nucl. Phys., Vol. 30, pp. 405–406, 1993.

0146-6410/93 $24.00

Who Killed the J/ψ?

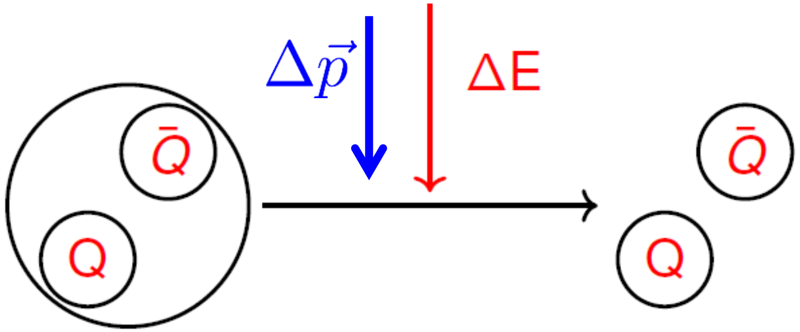
R. VOGT
Gesellschaft für Schwerionenforschung (GSI), D-6100 Darmstadt 11, Germany

Does not seems quite intuitive, may not be the end of the story

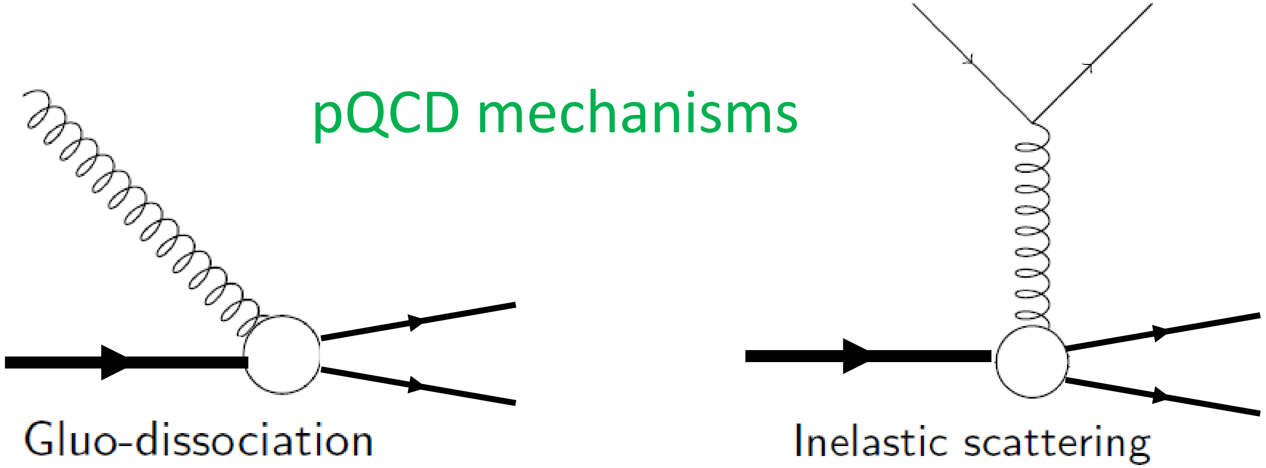
Collisions with the QGP

- Besides arguments based on the Debye mass / screening, it was pointed out already in the 90's that interactions with partons in the QGP could lead to dissociation of bound states (whose spectral function thus acquire some width Γ corresponding to the dissociation rate)

- Energy-momentum exchange with the QGP (gluo-dissociation, q – quarkonia quasi elastic scattering)



pQCD mechanisms



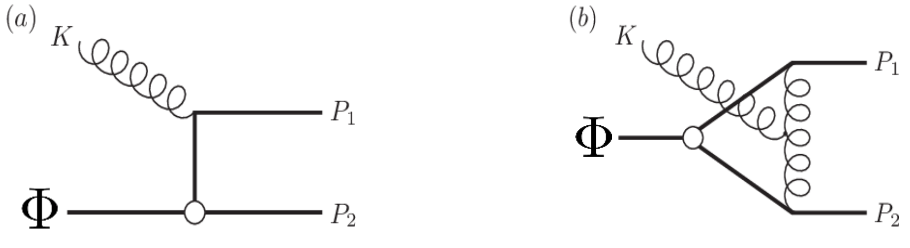
- => pair dissociation => **Suppression**

- ⇔ loss of probability of the quarkonia … Often described by some imaginary potential W in modern approaches

A central quantity: the decay rate Γ

Many approaches

pQCD view (Bhanot & Peskin), later on consolidated by NRQCD (Brambilla & Vairo)



$$\Phi + g \to Q + \bar{Q}$$

Dissociation cross section σ

$$\Gamma_\Phi(T) = \langle \sigma n_g \rangle_T$$

Other mechanisms : $x + \Phi \to x + Q + \bar{Q}$

QFT/Lattice QCD

Time correlator

$$\mathcal{C}_>(t,\vec{r}) \approx \langle \psi(t,\tfrac{\vec{r}}{2})\bar{\psi}(t,-\tfrac{\vec{r}}{2})\psi(0,0)\bar{\psi}(0,0)\rangle$$

Satisfies Schroedinger equation with complex potential V+iW . Breakthrough by Laine et al. (2006)

$$\Gamma_\Phi(T) = -2\langle\Phi|W|\Phi\rangle$$

Concept better suited at it genuinely encodes the "in medium" propagation

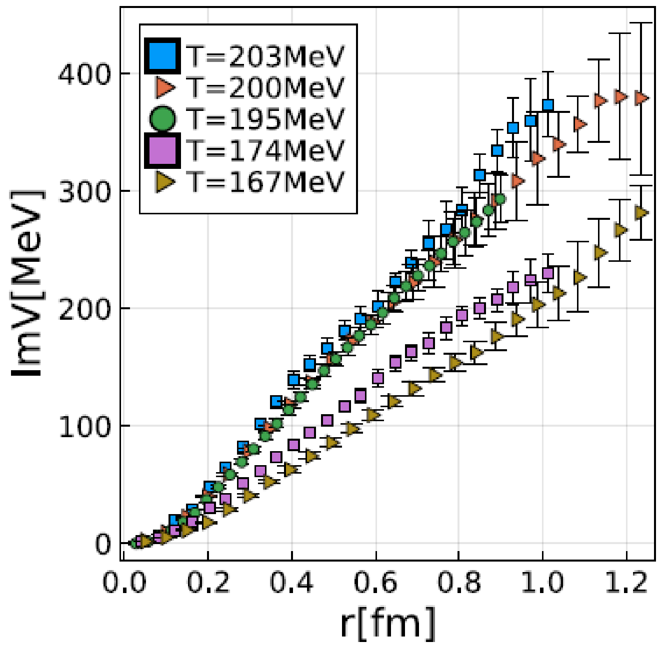
=> Simple decay law : Prob survival $= \exp\left(-\int_{t_0}^{t_{\rm fin}} \Gamma(T(t))dt\right)$

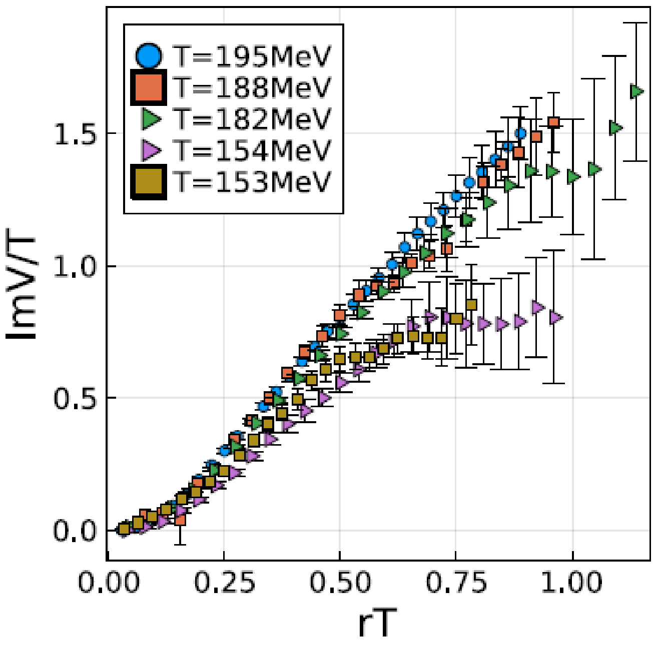
A central quantity: the decay rate Γ

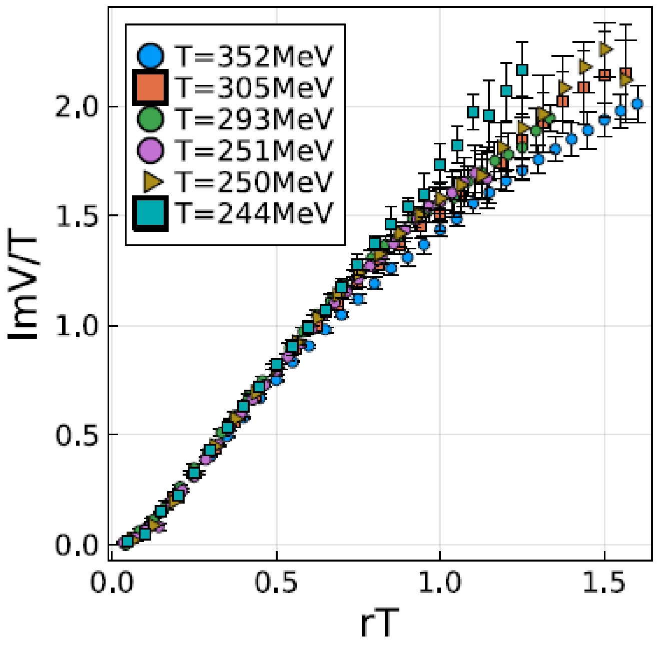
Recent lQCD calculations of W(r) = Im(V(r)) (at ω=0)

$$\rho_r^{\rm peak}(\omega,T) = \frac{1}{\pi}{\rm Im}\frac{A_r(T)}{\omega - {\rm Re}V(r,T) - i\Gamma(\omega,r,T)}$$

Bazazov et al 2023 (Hot QCD collaboration)





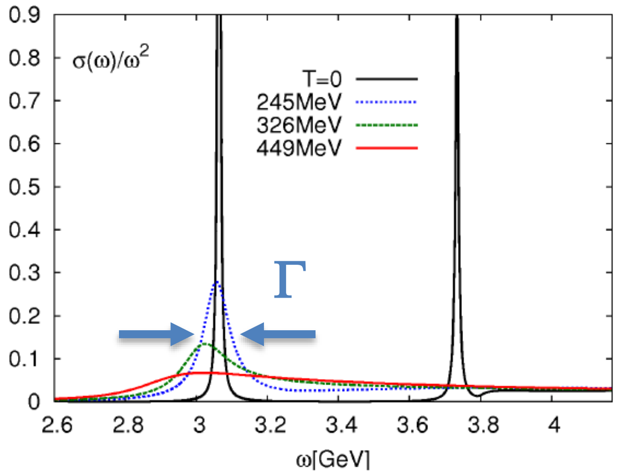


- Nice r T scaling
- Dipole structure at small r, no saturation seen at “large” r

Quarkonia at finite T

- Pheno: Yet, these pictures might still be compatible with the notion of sequential « suppression »…

- However, this notion has to be made more precise : (LQCD) spectral function lQCD



$$\rho(\omega, p, T) = \frac{1}{2\pi}\mathrm{Im}\int_{-\infty}^{\infty} dt e^{i\omega t}\int d^3x e^{ipx}\langle[J(x,t), J(0,0)]\rangle_T$$

At T=245 MeV, ψ' has disappeared but J/ψ still surviving for ≈ 1/Γ ≈ a couple of fm/c … which needs to be compared with the local QGP cooling time τ_{cool} : Γ x τ_{cool} > 1 ⇔ suppressed

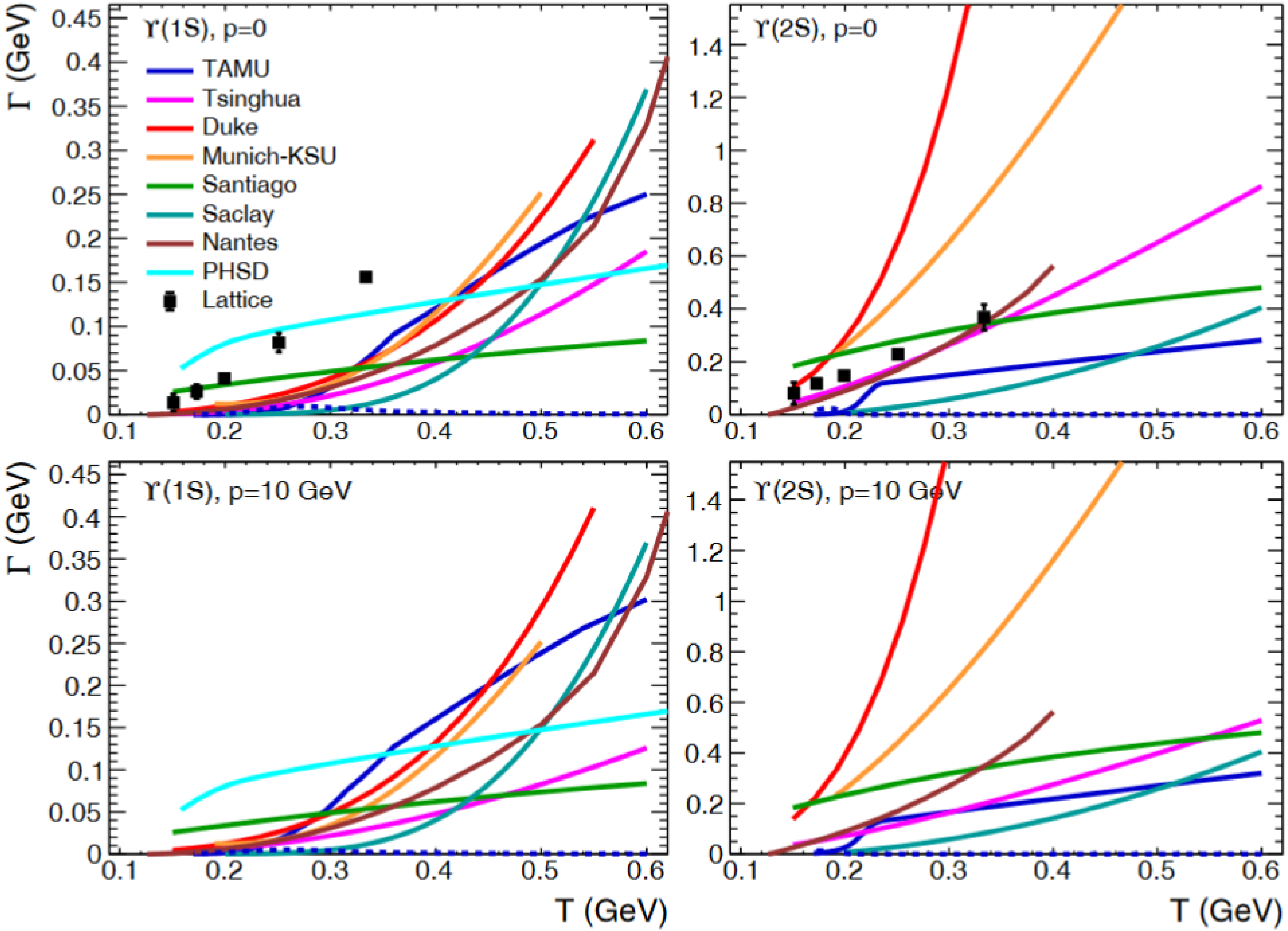
- N.B.: The opposite phenomenom might also be relevant: some state above the « melting » temperature can survive (for a short while < 1/Γ) before getting lost definitively.

Will it melt (even party) ?

- Key question : do the quarkonia states (chemically) equilibrate with the QGP ?

Modern era

Diversity in the approaches



EMMI RRTF on QUARKONIA (Dec 2019 & Dec 2022)

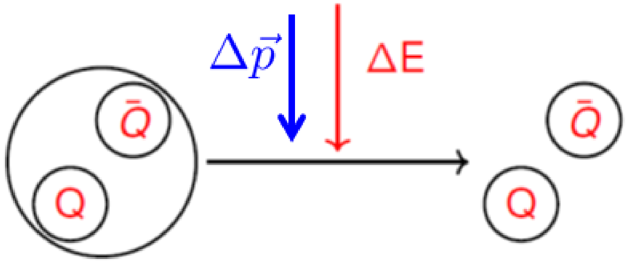
significant subtle variations in each model:

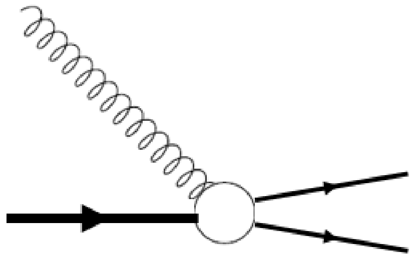
- Underlying binding force between Q & Qbar
- Binding energy
- Whether, on the top of dissociation, some « melting » is allowed
- ….

See https://indico.gsi.de/event/9314/overview (manuscript in preparation)

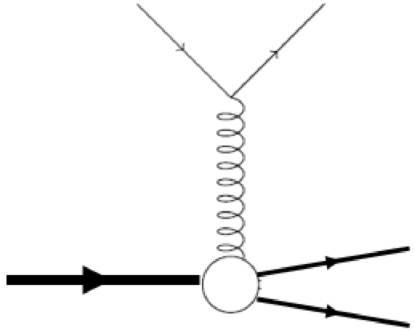
Regeneration

Detailed balance :



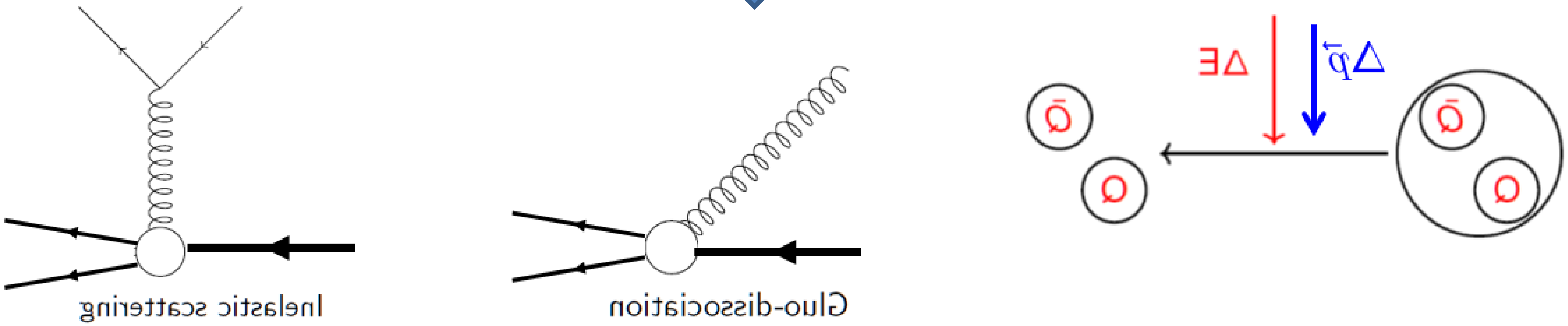


Gluo-dissociation



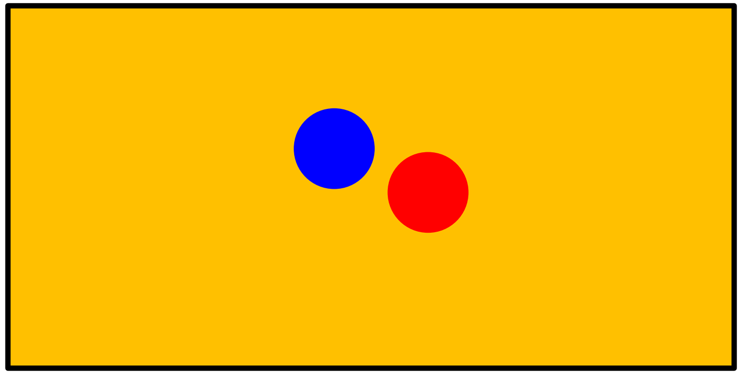
Inelastic scattering

Reverse mechanisms



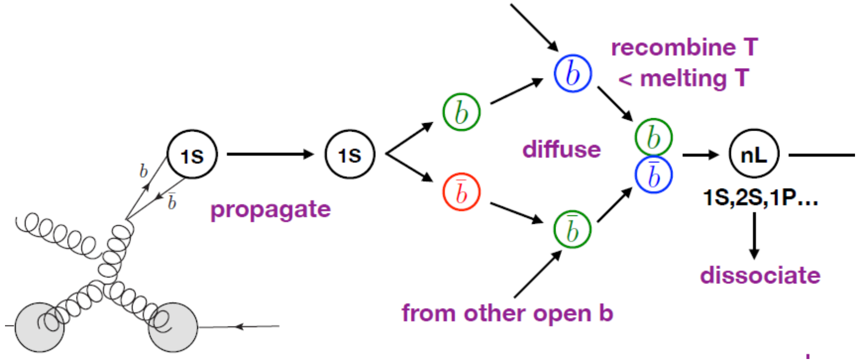
Regeneration: Dilute vs Dense

Bottomia



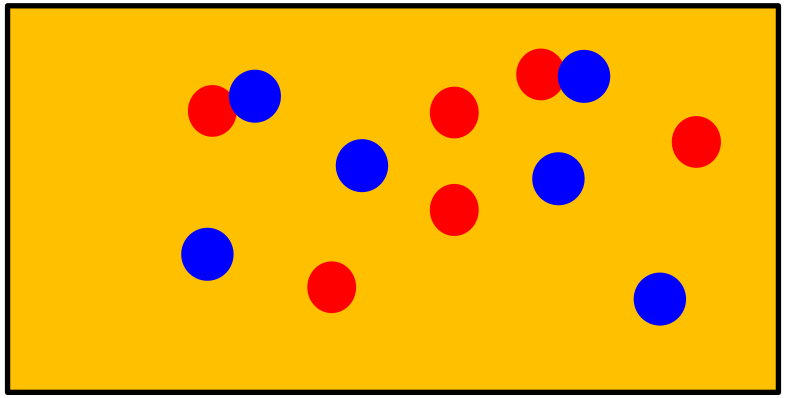
No exogenous recombination : only the b-bbar pairs which are initially close together will emerge as bottomia states

In some SC formalisms : intermediate regeneration



Yao, Mehen, Müller

Charmonia



Exogenous recombination : c & cbar initially far from each other may recombine and emerge as charmonia states

No full quantum treatment possible => need semi-classical approximation(s)

Key question : when does the recombination (dominantly) happen ? Crucial role of the binding force.

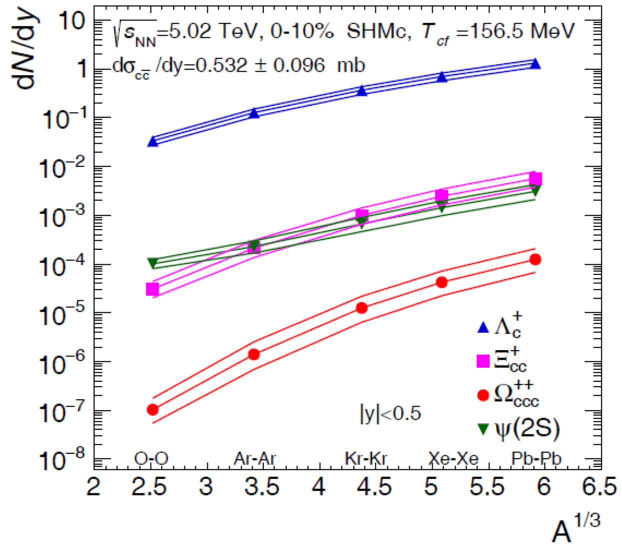
One extreme viewpoint : regeneration happens at the end of the QGP (Statistical Hadronization Model)

Statistical Hadronization Model

Alternate hypothesis :

- All heavy quarks are formed during the very early stage of the collision according to pQCD

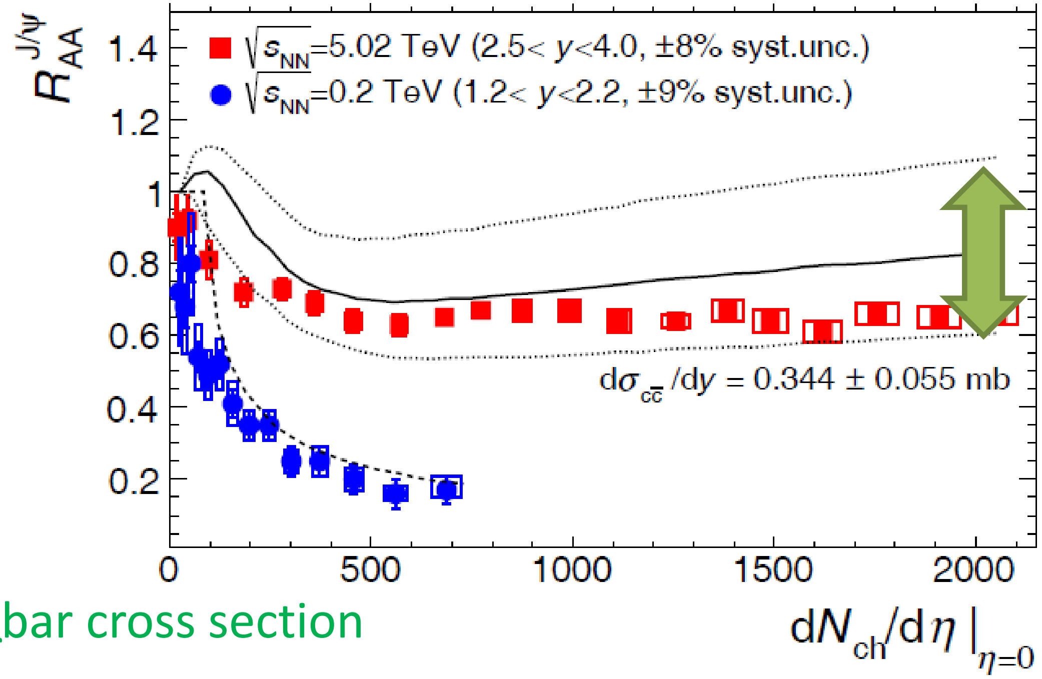
$$N_{c\bar{c}}^{\text{direct}} \propto N_{\text{coll}}\sigma_{c\bar{c}} \propto N_{\text{part}}^{4/3}\sigma_{c\bar{c}}$$



- These quarks survive through QGP and **are statistically distributed** at the time of hadronization (freeze out) => All quarkonia bound states are dissociated / melted during evolution

- All Q and Qbar recombine « instantaneously » around T_c.

  Predictions for:
  - dN_{Φ}/dy (large uncertainties stemming from $\sigma_{c\text{-}cbar}$)
  - dN_{Φ}/dp_T at low p_T.
  - Higher states
  - $v_{2\Phi}$
  - Multi-charmed hadrons



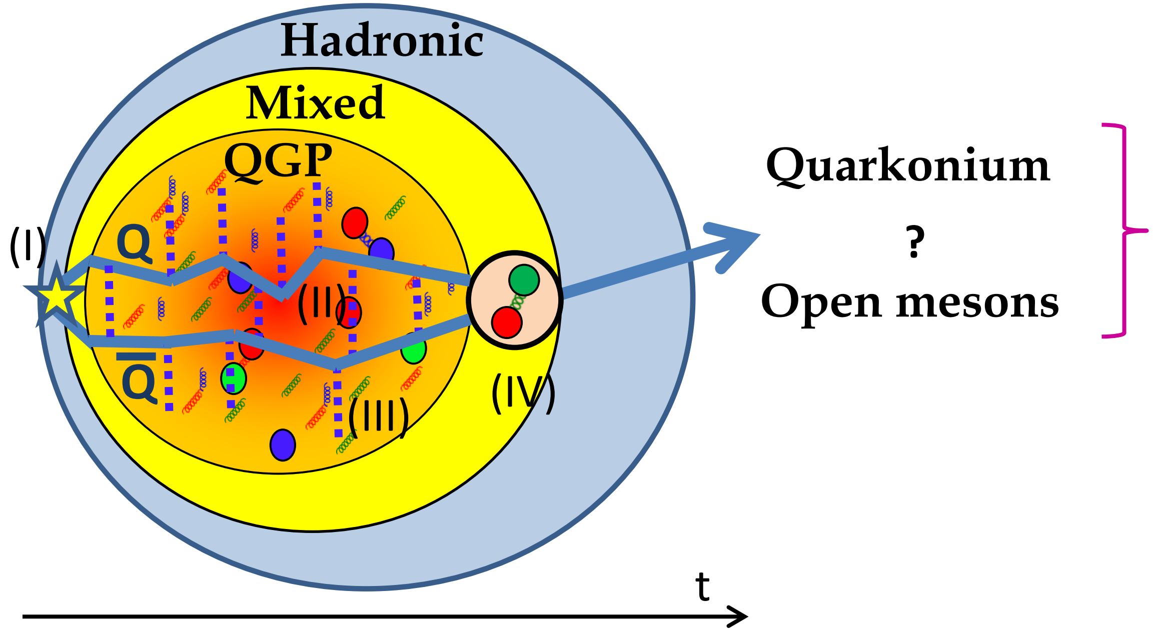
In this approach, uncertainties mainly stem from the total Q-Qbar cross section

The present challenge

Unravel the Q-Qbar interactions under the influence of the surrounding QGP and with the QGP

Develop a scheme able to deal with the evolution of one (or many) $Q\bar{Q}$ pair(s) in a QGP, fulfilling all fundamental principles (quantum features, gauge invariance, equilibration,…)

The full scheme



Strictly speaking, only resolved at the end of the evolution

Beware of quantum coherences during the whole evolution !

Especially at early time…

In practice, what counts is the so-called decoherence time, not the "Heisenberg time"

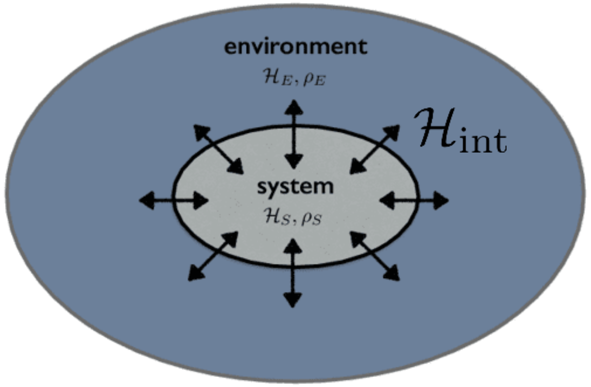
Complicated QFT problem (also due to the evolving nature of the QGP that mixes several scales)… only started to be addressed at face value recently

1) Initial state
2) (Screened) interaction between both HQ
3) Interactions with surrounding QGP partons
4) Projection on the final quarkonia

How to proceed ?

First incomplete QM treatments dating back to Blaizot & Ollitrault, Thews, Cugnon and Gossiaux; early 90's

Open Quantum Systems & Quantum Master Equations



Quite generally, system (Q-Qbar pair) builds correlation with the environment thanks to the Hamiltonian $\hat{H} = \hat{H}^{(0)}_{Q\bar{Q}} + \hat{H}_E + \hat{H}_{\text{int}}$ with $\hat{H}_E = \hat{H}_{\text{QGP}}$

Von Neumann equation for the total density operator ρ

$\frac{\mathrm{d}}{\mathrm{d}t}\rho = -i[H, \rho]$

System + environment (QGP)

$\rho(t=0) = \rho_{Q\bar{Q}} \otimes \rho_{QGP}$

Evolution of the total system

$\rho(t) = U(t,0)\left[\rho_{Q\bar{Q}} \otimes \rho_{QGP}\right] U(t,0)^\dagger$

Trace out QGP degrees of freedom =>
Reduced density operator $\rho_{Q\bar{Q}}$

System ($Q\bar{Q}$ pair)

$\rho_{Q\bar{Q}}(t=0)$

Evolution of the system

$\rho_{Q\bar{Q}}(t) = Tr_{QGP}\left[U(t,0)\rho(t=0)U(t,0)^\dagger\right]$

Can be formulated differentially ./. time :

$$\frac{\mathrm{d}\rho_{Q\bar{Q}}}{\mathrm{d}t} = \mathcal{L}[\rho_{Q\bar{Q}}]$$

Definition of $\mathcal{L}[\cdot]$

Open Quantum Systems & Quantum Master Equations

$$\hat{\rho}_{Q\bar{Q}} = \sum_{\alpha,\beta} d_{\alpha,\beta} |\alpha\rangle\langle\beta|$$

QME deal with the (coupled) evolution of probabilities ($d_{\alpha,\alpha}$) and coherences ($d_{\alpha,\beta\neq\alpha}$)

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Evolution of the system

$\rho_{Q\bar{Q}}(t) = Tr_{QGP}\left[U(t,0)\rho(t=0)U(t,0)^\dagger\right]$

However, $\mathcal{L}[\cdot]$ is generically a non local super-operator in time (linear map)

A special QME: The Lindblad Equation

There are many different QME… a special one :

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)\right] + \sum_i \boxed{\gamma_i}\left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \tfrac{1}{2}\left\{L_i L_i^\dagger, \rho_{Q\bar{Q}}(t)\right\}\right]$$

γ_i Characterize the coupling of the system (Q-Qbar) with the environment

$H_{Q\bar{Q}}$: $\{Q, \bar{Q}\}$ $\underbrace{\text{kinetics + Vacuum potential V}}_{\hat{H}^{(0)}_{Q\bar{Q}}}$ + Lamb shift / screening (every unitary term that is generated by tracing out the environment)

L_i : Collapse (or Lindblad) operators, depend on the properties of the medium

3 important conservation properties :

$\rho^\dagger_{Q\bar{Q}} = \rho_{Q\bar{Q}}$	$\mathrm{Tr}[\rho_{Q\bar{Q}}] = 1$	$\langle\varphi\|\rho_{Q\bar{Q}}\|\varphi\rangle > 0, \forall\|\varphi\rangle$
(Hermiticity)	(Norm)	(Positivity)

… but in general, non unitary !!! (relaxation)

<u>Nice feature</u> : Can be brought to the form of a stochastic Schroedinger equation (quantum jump method : QTRAJ)

A special QME: The Lindblad Equation

Non unitary / dissipative evolution ≡ decoherence

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)\right] + \sum_i \gamma_i \left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2}\left\{L_i L_i^\dagger, \rho_{Q\bar{Q}}(t)\right\}\right]$$

Genuine transitions :

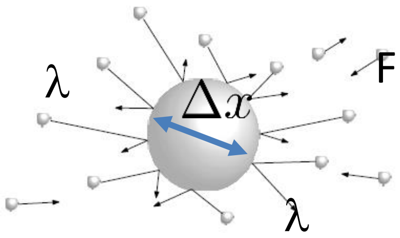
- ✓ Singlet <-> octet
- ✓ Octet <-> octet

Can be reshuffled into non Hermitic effective hamiltonian

$$\hat{H}_{Q\bar{Q},\mathrm{eff}} = \hat{H}_{Q\bar{Q}} - i\sum_j \gamma_j \frac{L_j L_j^\dagger}{2}$$

≡ Dissociation width

For **infinitely massive single Q** and environment wave length λ >> wave packet size Δx:



Fluctuations from env. ⟺ $$\frac{\partial \rho_Q(x_Q, x'_Q)}{\partial t} = -F(x_Q - x'_Q)\rho_Q(x_Q, x'_Q)$$

Decoherence factor: $F \approx \kappa\,(x_Q - x'_Q)^2$

HQ momentum diffusion coefficient (adjoint)

In Q world: smaller objects live longer !

At 1rst order in $1/m_Q$: recoil corrections ⟺ friction / dissipation

Pictorial summary

Inspired from Yao Int. J. of Mod. Phys. A, Vol. 36, No. 20, 2130010 (2021)

τ_E: environment autocorrelation time τ_S: system intrinsic time scale τ_R: system relaxation time

Subsystem + environment: von Neumann equation

↓ Trace out environment

Subsystem: non-unitary, time-irreversible evolution

↓ Weak syst-environment coupling + Markovian limit $\tau_E \ll \tau_R$

Redfield equation

$$\frac{\partial}{\partial t}\rho_{\mathrm{I}}(t) = -\frac{1}{\hbar^2}\sum_{m,n}\int_0^\infty d\tau\Big(C_{mn}(\tau)\Big[S_{m,\mathrm{I}}(t), S_{n,\mathrm{I}}(t-\tau)\rho_{\mathrm{I}}(t)\Big] - C^*_{mn}(\tau)\Big[S_{m,\mathrm{I}}(t), \rho_{\mathrm{I}}(t)S_{n,\mathrm{I}}(t-\tau)\Big]\Big)$$

Similar structure to the Linblad equation but with time delay effects

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Redfield equation

Smallest time scales wins it all !

$\tau_S \ll \tau_R$ Quantum Optical Regime → **Lindblad equation** — Eigenstates of the HQ Hamiltonian

$\tau_E \ll \tau_S$ Quantum Brownian Motion → **Lindblad equation** — Phase space densities

← Not the same basis ! →

↓ Wigner transform + gradient expansion

Boltzmann equation — Rate equations: ⇔ transport models

Fokker-Planck equation — Good method for many $c\bar{c}$ pairs

Semi-classical approx : density matrix ≈ diagonal

QCD time scales

τ_E: environment autocorrelation time

$$\tau_E \approx \frac{1}{m_D} \approx \frac{1}{CT} \approx \frac{1}{T}$$ (C taken as close to unity)

τ_S: system intrinsic time scale

$$\tau_S \approx \underbrace{\frac{1}{\Delta E}} \approx \frac{1}{m_Q v^2}$$ with $v \approx \alpha_S$ … at the beginning of the evolution

Difference btwn energy levels

τ_R: system relaxation time

$$\Gamma = \tau_R^{-1} \sim 2\langle\psi|W\psi\rangle \approx \alpha_s T \times \Phi(m_D r) \approx \alpha_s T \times \Phi(\frac{CT}{m_Q \alpha_s})$$

At "small" T $\left(T \lesssim \frac{m_Q \alpha_S}{C}\right)$: dipole approximation : $\Gamma = \tau_R^{-1} \approx \frac{C^2 T^3}{\alpha_s m_Q^2}$

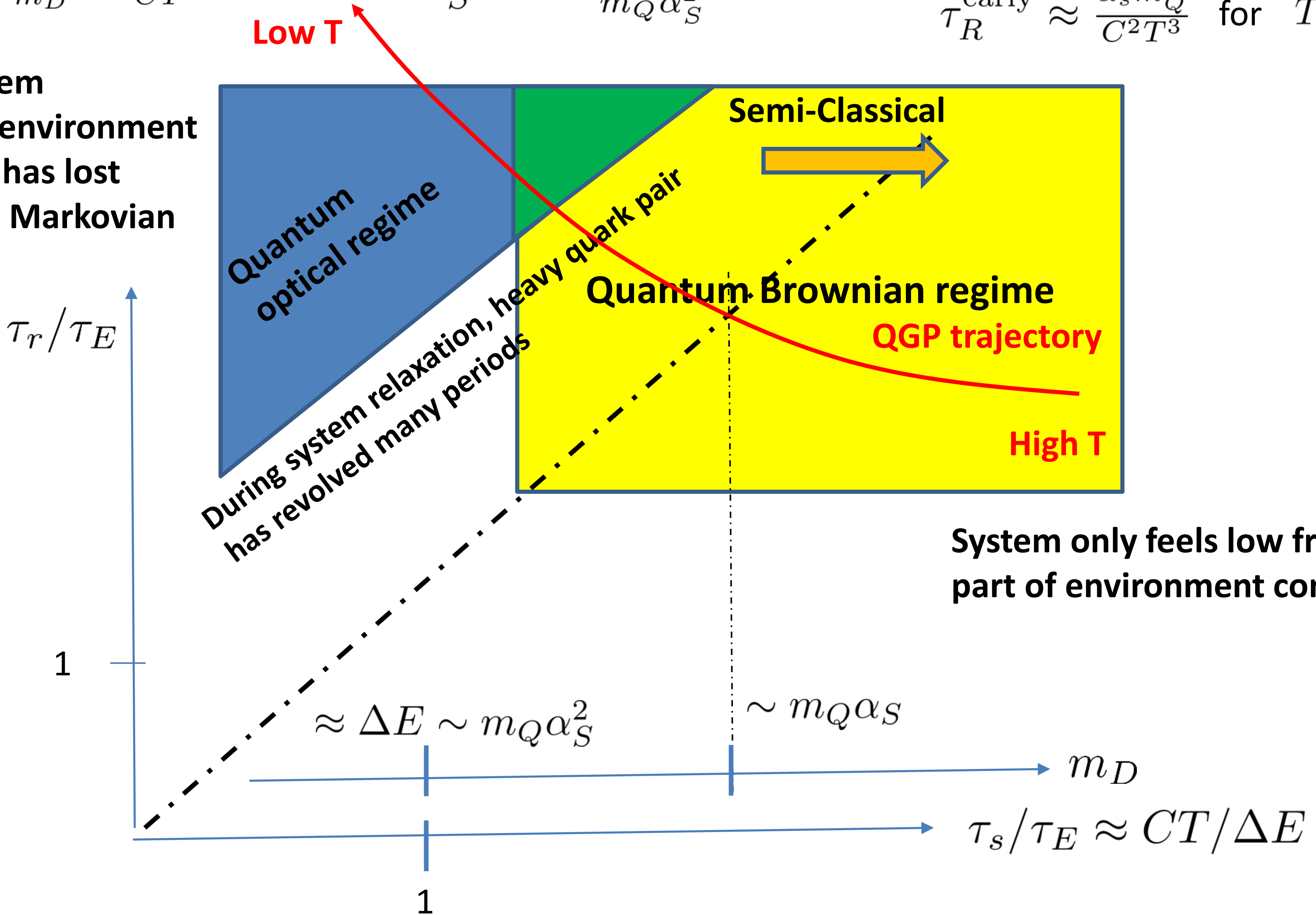
$$\frac{\tau_R}{\tau_E} = \frac{\alpha_s m_Q^2}{CT^2} \gg 1$$ And $$\frac{\tau_R}{\tau_S} = \frac{\alpha_s^3 m_Q^3}{C^2 T^3} \gg 1$$ for $T \lesssim m_Q \frac{\alpha_S}{C^{2/3}}$

Fine with the Markovian assumption

QCD time scales

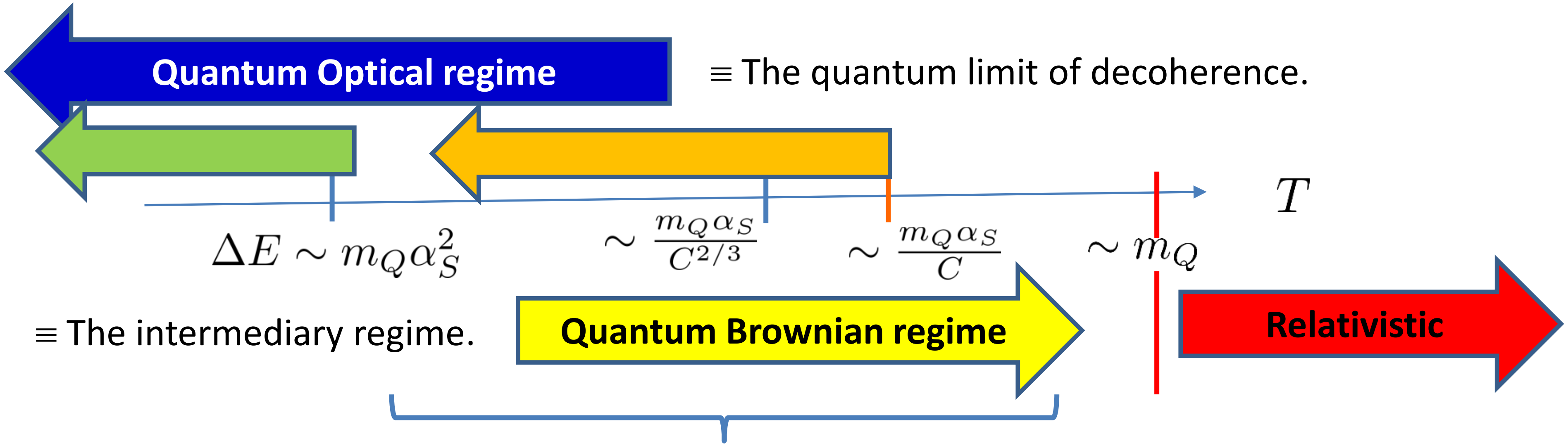
$\tau_E \approx \frac{1}{m_D} = \frac{1}{CT}$ $\quad \tau_S^{\text{early}} \approx \frac{1}{m_Q \alpha_S^2}$ $\quad \tau_R^{\text{early}} \approx \frac{\alpha_s m_Q^2}{C^2 T^3}$ for $T \lesssim \frac{m_Q \alpha_S}{C}$

During system relaxation, environment correlation has lost memory => Markovian process



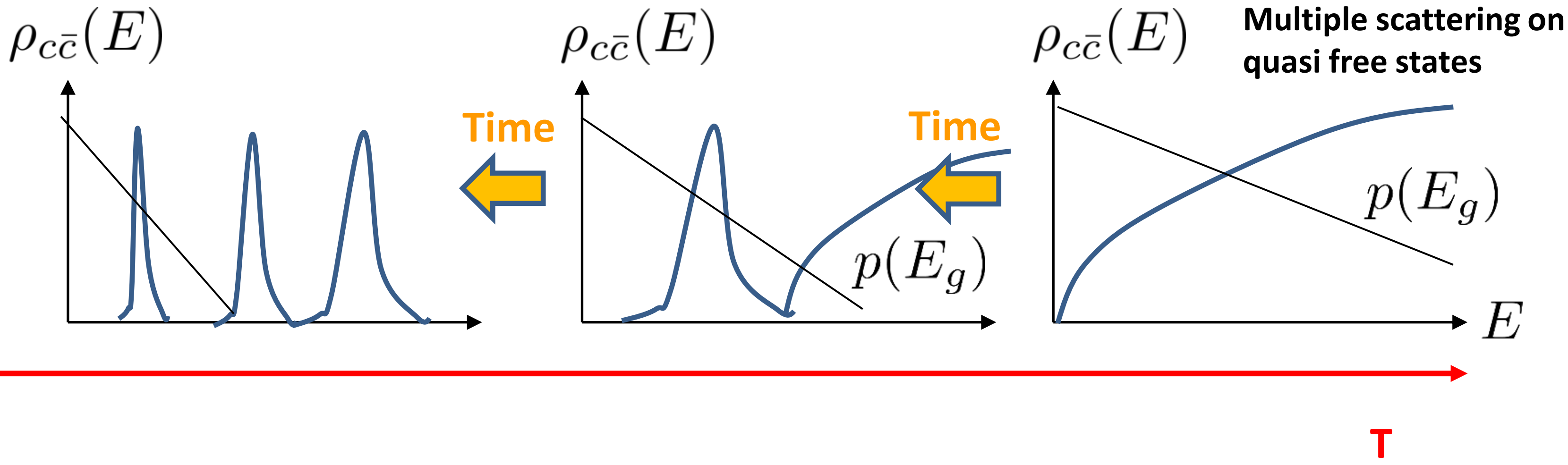
System only feels low frequency part of environment correlation

QCD Temperature scales



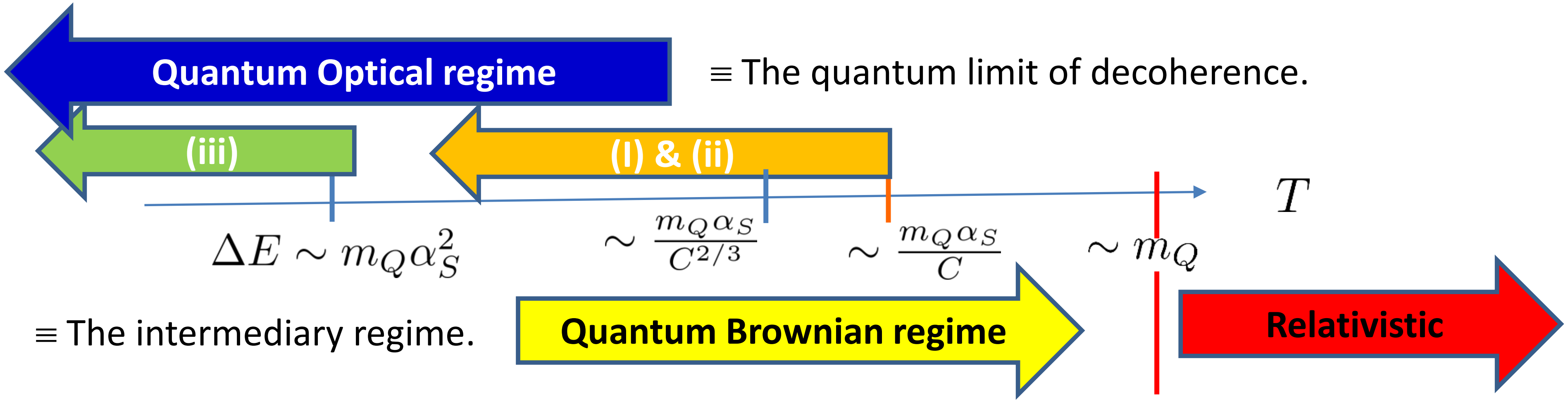
For these « large » temperatures, the Q-Qbar gain enough energy to overwhelm the real binding potential => larger distance => larger decoherence ….

dissociation of well identified levels by scarce "high-energy" modes (dilute medium => cross section ok)



In // : continuous evolution of the $Q\bar{Q}$ spectral function

QCD Temperature scales



Refined subregimes when playing with the scales of NRQCD / pNRQCD (series of recent papers by N. Brambilla, M.A. Escobdo, A. Vairo, M Strickland et al, Yao and Mehen,…)

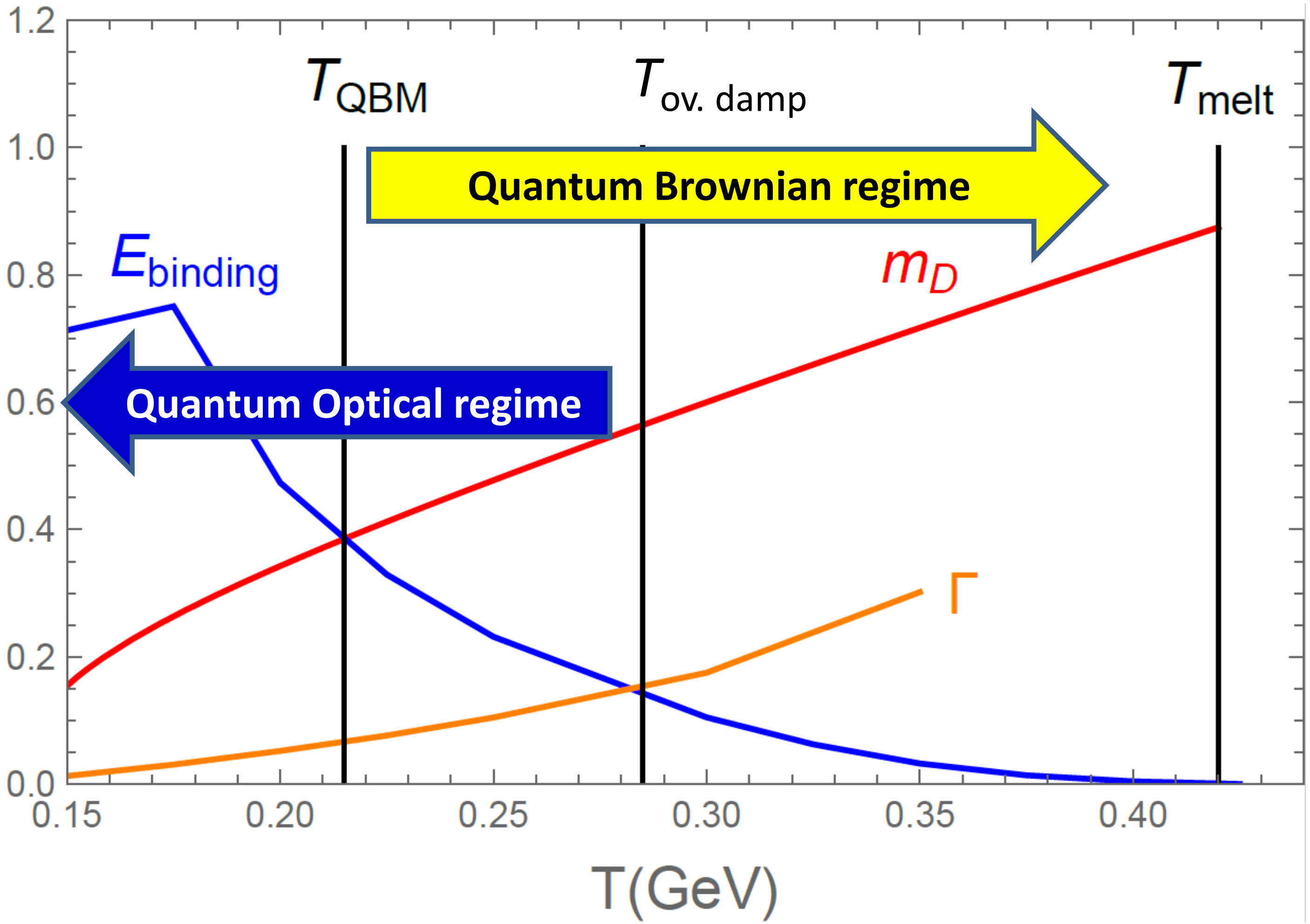
NRQCD: $Mv, \Lambda_{\mathrm{QCD}}, T \ll \mu_{\mathrm{NR}} \ll M$: most general scheme for markovian OQS !

pNRQCD:

(Singlet and octet quarkonium fields)

- (i) $1/r \gg T \sim m_D \gg E$: « strongly coupled » QME same as small dipole limit of NRQCD (applies for small time evolution)
- (ii) $1/r \gg T \gg E \gg m_D$: « weakly coupled » : g T << T : essential contribution is gluo – dissociation from hard mode T : does not apply in QCD
- (iii) $1/r \gg T \sim E \gg m_D$: Quantum optical regime

Two types of dynamical modelling



$c\bar{c}$ pair

Numbers extracted from a specific potential model : Katz et al, Phys. Rev. D 101, 056010 (2020)

Two types of dynamical modelling

$m_D \ll E_{\text{bind}}$

Quantum Optical Regime

$m_D \sim E_{\text{bind}}$

$m_D \gg E_{\text{bind}}$

Quantum Brownian Motion

- **Well identified resonances**
- Time long enough wrt quantum decoherence time

Good description with transport models (TAMU, Tsinghua, Duke)

Central quantities :
2->2 and 2->3 Cross sections, decay rates

Equilibrium : $\exp(-E_n/T)$ (theorem)

SC Approx: rate equations

?

- Correlations growing with cooling QGP
- **Best described in position-momentum space**
- Time short wrt quantum decoherence time ?

Quantum Master Equations for **microscopic dof (QS and Qbars)**

Equilibrium / asympt* : some limiting cases

SC Approx: Fokker-Planck equations in position-momentum space

* Since one is facing both dissociation and recombination, obtaining a correct equilibrium limit of these model is an important prerequisite !!!

Recent OQS implementations (single $Q\bar{Q}$ pair)

regime	SU3 ?	Dissipation ?	3D / 1D	Num method	year	remark	ref
NRQCD ⇔ QBM	No	No	1D	Stoch potential	2018		Kajimotoet al. , Phys. Rev. D 97, 014003 (2018), 1705.03365
	Yes	No	3D	Stoch potential	2020	Small dipole	R. Sharma et al Phys. Rev. D 101, 074004 (2020), 1912.07036
	Yes	No	3D	Stoch potential	2021		Y. Akamatsu, M. Asakawa, S. Kajimoto (2021), 2108.06921
	No	Yes	1D	Quantum state diffusion	2020		T. Miura, Y. Akamatsu et al, Phys. Rev. D 101, 034011 (2020), 1908.06293
	Yes ✓	Yes ✓	1D	Quantum state diffusion	**2021**		Akamatsu & Miura, EPJ Web Conf. 258 (2022) 01006, 2111.15402
	No	Yes	1D	Direct resolution	**2021**		O. Ålund, Y. Akamatsu et al, Comput. Phys. 425, 109917 (2021), 2004.04406
	Yes ✓	Yes ✓	1D	Direct resolution	**2022**		S Delorme et al, https://inspirehep.net /literature/ 2026925
pNRQCD (i)	Yes	No	1D+	Direct resolution	2017	S and P waves	N. Brambilla et al, Phys. Rev. D96, 034021 (2017), 1612.07248
(i) Et (ii)	Yes	No	1D+	Direct resolution	2017	S and P waves	N. Brambilla et al, Phys. Rev. D 97, 074009 (2018), 1711.04515
(i)	Yes	No	Yes	Quantum jump	**2021**	See SQM 2021	N. Brambilla et al. , JHEP 05, 136 (2021), 2012.01240 & *Phys.Rev.D* 104 (2021) 9, 094049, 2107.06222
(i)	Yes ✓	Yes ✓	Yes ✓	Quantum jump	**2022**		N. Brambilla et al. 2205.10289
(iii)	Yes ✓	Yes ✓	Yes ✓	Boltzmann (?)	2019		Yao & Mehen, Phys.Rev.D 99 (2019) 9, 096028, 1811.07027
NRQCD & « pNRQCD »	Yes	Yes	1D	Quantum state diffusion	**2022**		Miura et al. http://arxiv.org/abs/2205.15551v1
Other	No	Yes	1D	Stochastic Langevin Eq.	2016	Quadratic W	Katz and Gossiaux

(Year > 2015)

Not exhaustive

See as well table in 2111.15402v1

…

Bottomonia… the 50 shades of pNRQCD

Yao, Mehen, Muller: low T pNRQCD hierarchy (weak coupled small dipole): QOR

$1/r \gg E \gg T \gg m_D > \Lambda_{QCD}$ regime (treats late T> E as « instantaneous dissociation)

2018 — 2020 — 2021 — time

Yao & Mehen, Phys.Rev.D 99 (2019) 9, 096028, 1811.07027

Equivalence between QME and Boltzmann equations:

- For the evolution of the singlet BS
- After the Wigner transform is performed
- Need/justify smooth octet distribution f_8 along relative distance

Yao et al, JHEP 01 (2021) 046, 2004.06746

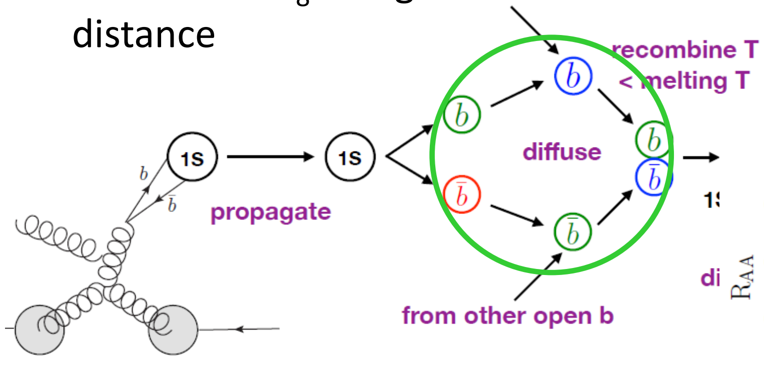
Coupled transp. Eq. in LIDO:

- Initial $b\bar{b}$ state : uncorrelated bottomonia (no open contribution)
- Good agreement with exp. results for α_s**=0.3**
- Crucial role of the correlated regeneration. Could be tested by $R_{AA}(\chi_b(1P))/R_{AA}(Y(2S))$

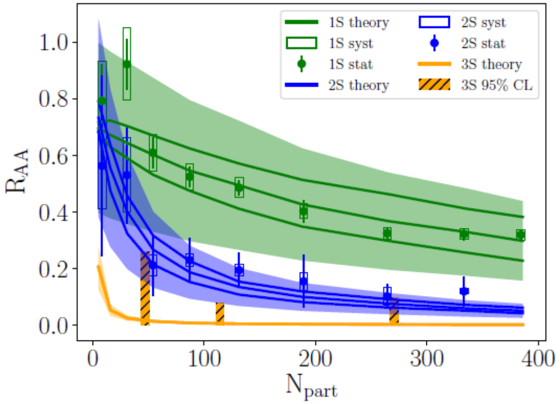
Yao & Mehen al, JHEP 02 (2021), 2009.02408

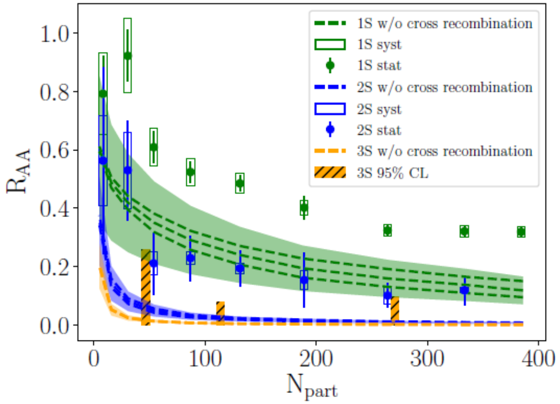
Revisit & extend previous work:

- Boltzmann ⇔ semi-classical gradient expansion of the octet distribution f_8 along relative distance
- Quantum correction derived formally… **Awaits for quantitative estimate**



Correlated regeneration : same $b\bar{b}$ pair





Bottomonia… the 50 shades of pNRQCD

Yao, Mehen, Muller: low T pNRQCD   hierarchy (weak coupled small dipole):

Boltzmann-like equation:

$$\frac{\partial}{\partial t} f_{nl}(\boldsymbol{x},\boldsymbol{k},t) + \frac{\boldsymbol{k}}{2M}\cdot\nabla_{\boldsymbol{x}} f_{nl}(\boldsymbol{x},\boldsymbol{k},t) = \mathcal{C}^{+}_{nl}(\boldsymbol{x},\boldsymbol{k},t) - \mathcal{C}^{-}_{nl}(\boldsymbol{x},\boldsymbol{k},t)$$

gain   Loss (dissociation)

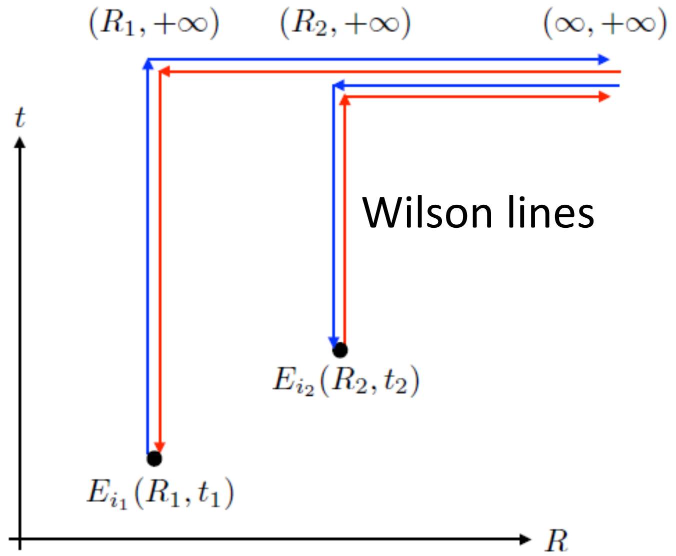
Dissociation:

$$\mathcal{C}^{-}_{nl}(\boldsymbol{x},\boldsymbol{k},-t/2) = \sum_{i_1,i_2}\int \frac{\mathrm{d}^3 p_{\mathrm{cm}}}{(2\pi)^3}\frac{\mathrm{d}^3 p_{\mathrm{rel}}}{(2\pi)^3}\frac{\mathrm{d}^4 q}{(2\pi)^4}(2\pi)^4\delta^3(\boldsymbol{k}-\boldsymbol{p}_{\mathrm{cm}}+\boldsymbol{q})\delta(E_{nl}-E_p+q^0)$$
$$\times\, d^{nl}_{i_1 i_2}(\boldsymbol{p}_{\mathrm{rel}})\, g^{E++}_{i_1 i_2}(q^0,\boldsymbol{q})\, f_{nl}(\boldsymbol{x},\boldsymbol{k},-t/2)$$

Factorization (OPE)

Dipole function   Fourier transform of the electric-electric correlator (in the adjoint representation)

$$g^{E++}_{i_1 i_2}(t_1,t_2,\boldsymbol{R}_1,\boldsymbol{R}_2) = \left\langle \mathrm{Tr}_{\mathrm{color}}\left(E_{i_1}(\boldsymbol{R}_1,t_1)\mathcal{W}_{[(\boldsymbol{R}_1,t_1),(\boldsymbol{R}_1,+\infty)]}\mathcal{W}_{[(\boldsymbol{R}_2,+\infty),(\boldsymbol{R}_2,t_2)]}E_{i_2}(\boldsymbol{R}_2,t_2)\right)\right\rangle_T$$



Bottomonia… the 50 shades of pNRQCD

Yao, Mehen, Muller: low T pNRQCD hierarchy (weak coupled small dipole):

Boltzmann-like equation:
$$\frac{\partial}{\partial t} f_{nl}(\boldsymbol{x},\boldsymbol{k},t) + \frac{\boldsymbol{k}}{2M}\cdot\nabla_{\boldsymbol{x}} f_{nl}(\boldsymbol{x},\boldsymbol{k},t) = \mathcal{C}^{+}_{nl}(\boldsymbol{x},\boldsymbol{k},t) - \mathcal{C}^{-}_{nl}(\boldsymbol{x},\boldsymbol{k},t)$$

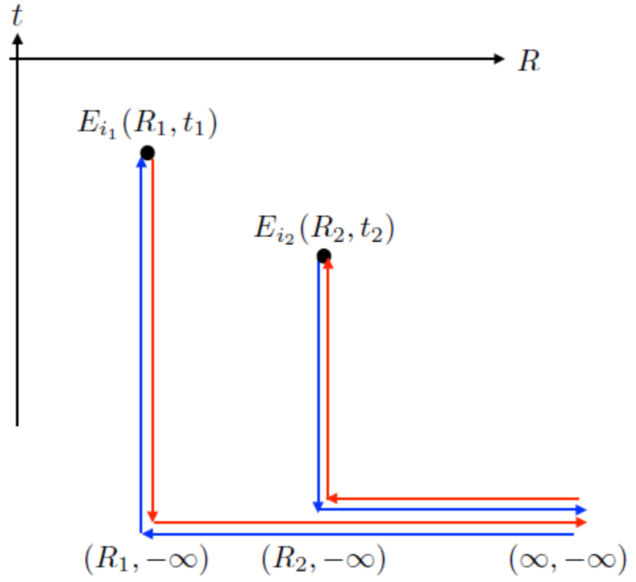
gain Loss (dissociation)

Dissociation:
$$\mathcal{C}^{-}_{nl}(\boldsymbol{x},\boldsymbol{k},-t/2) = \sum_{i_1,i_2}\int \frac{\mathrm{d}^3p_{\mathrm{cm}}}{(2\pi)^3}\frac{\mathrm{d}^3p_{\mathrm{rel}}}{(2\pi)^3}\frac{\mathrm{d}^4q}{(2\pi)^4}(2\pi)^4\delta^3(\boldsymbol{k}-\boldsymbol{p}_{\mathrm{cm}}+\boldsymbol{q})\delta(E_{nl}-E_p+q^0)$$
$$\times\, d^{nl}_{i_1 i_2}(\boldsymbol{p}_{\mathrm{rel}})\, g^{E++}_{i_1 i_2}(q^0,\boldsymbol{q}) f_{nl}(\boldsymbol{x},\boldsymbol{k},-t/2)$$

Dipole function Fourier transform of the electric-electric correlator (in the adjoint representation)

Recombination :
$$[g^{E--}_{i_2 i_1}(t_2,t_1,\boldsymbol{R}_2,\boldsymbol{R}_1)]^{A_2A_1} =$$
$$T_F\Big\langle\Big(\mathcal{W}_{[(\boldsymbol{R}_2,-\infty),(\boldsymbol{R}_2,t_2)]}E_{i_2}(\boldsymbol{R}_2,t_2)\Big)^{A_2}\Big(E_{i_1}(\boldsymbol{R}_1,t_1)\mathcal{W}_{[(\boldsymbol{R}_1,t_1),(\boldsymbol{R}_1,-\infty)]}\Big)^{A_1}\Big\rangle_T$$

These correlators , as well as the zero frequency limit of the Fourier transform κ_{adj} have received growing interest from the community (NLO calculation, paving the way to the lQCD evaluation, AdS/CFT, KMS relation,…)



Bottomonia… the 50 shades of pNRQCD

TUM + Kent State + … : most of the studies performed in the « strongly coupled » pNRQCD hierarchy :

$1/r \gg T \sim m_D \gg E \gg \Lambda_{QCD}$ regime (Strongly coupled ⬄ Quantum Brownian Regime)

Quarkonium fields : S & O + dipole approximation for g-quarkonium coupling (best for bottomonia, but can be questionable for higher states as b and bbar diffuse away)

Bonus from this simplification : only 2 parameters describing the QGP-quarkonium coupling:

Linblad operators

$$\Sigma_s(t) = \frac{r^2}{2}\left[\kappa(t) + i\gamma(t)\right] ,$$
$$\Sigma_o(t) = \frac{N_c^2-2}{2(N_c^2-1)}\frac{r^2}{2}\left[\kappa(t) + i\gamma(t)\right] ,$$

Dipole self energy (complex => relaxation)

$$\Xi_{so}(\rho_o, t) = \frac{1}{N_c^2-1} r^i \rho_o r^i \kappa(t) ,$$
$$\Xi_{os}(\rho_s, t) = r^i \rho_s r^i \kappa(t) ,$$
$$\Xi_{oo}(\rho_o, t) = \frac{N_c^2-4}{2(N_c^2-1)} r^i \rho_o r^i \kappa(t) .$$

Transition / « jump » operators in the Lindblad equations

κ: HQ momentum diffusion coefficient & γ : ≈ dipole Lamb-shift energy

$$\hat{\kappa} = \frac{1}{T^3}\frac{g^2}{6N_c}\int_0^\infty \mathrm{d}s \left\langle \left\{ \tilde{E}_i^a(s,\vec{0}), \tilde{E}_i^a(0,\vec{0}) \right\} \right\rangle ,$$
$$\hat{\gamma} = -\frac{i}{T^3}\frac{g^2}{6N_c}\int_0^\infty \mathrm{d}s \left\langle \left[\tilde{E}_i^a(s,\vec{0}), \tilde{E}_i^a(0,\vec{0}) \right] \right\rangle ,$$

Lattice QCD estimates: $1.8 \lesssim \frac{\kappa}{T^3} \lesssim 3.4$; presently no direct estimate for γ.

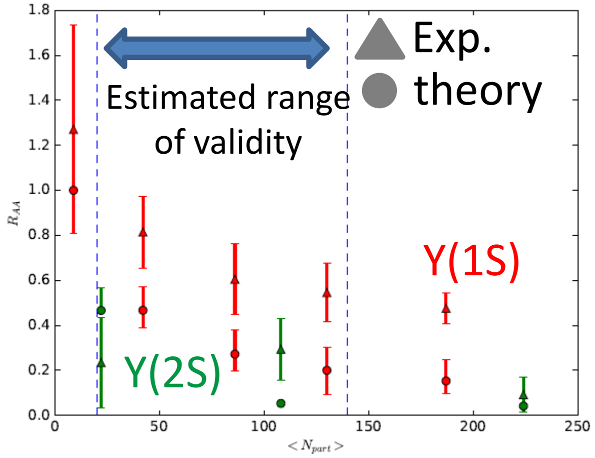
TUM + KSU: Strongly coupled pNRQCD

2017 — 2020 — 2021 **(SQM)** — 2022 — 2023 → time

N. Brambilla et al, Phys. Rev. D 97, 074009 (2018), 1711.04515

Direct solution, restrained to l=0 & l=1 sph. harmonics

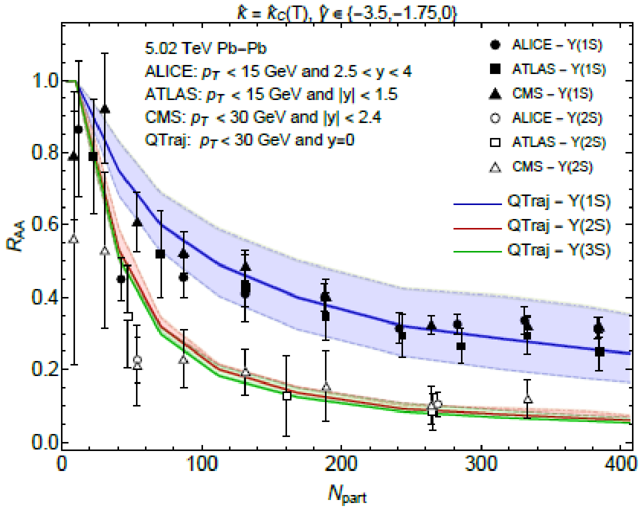
$\rho_{l,l'}$ assumed diagonal



N. Brambilla et al. , JHEP 05, 136 (2021), 2012.01240 & *Phys.Rev.D* 104 (2021) 9, 094049, 2107.06222

Numerical improvement due to the use of Quantum Jump algorithm (QTRAJ)

- No limit on ang. harmonics
- More realistic T(t) sampling QGP hydro-evolution
- => constrains on the DLS γ

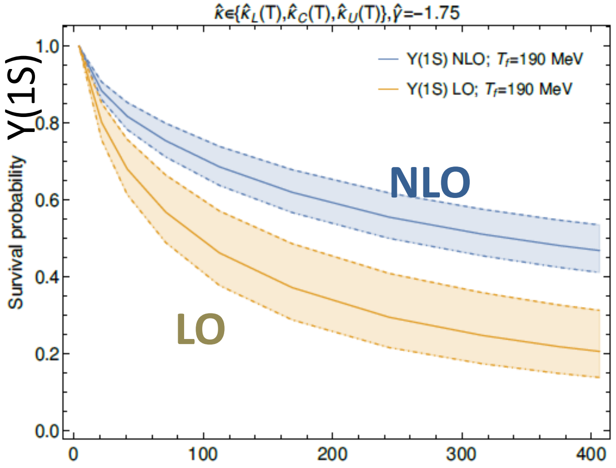


- $R_{AA}(p_T)$: flat

N. Brambilla et al. 2205.10289

Next To Leading order in T/E:

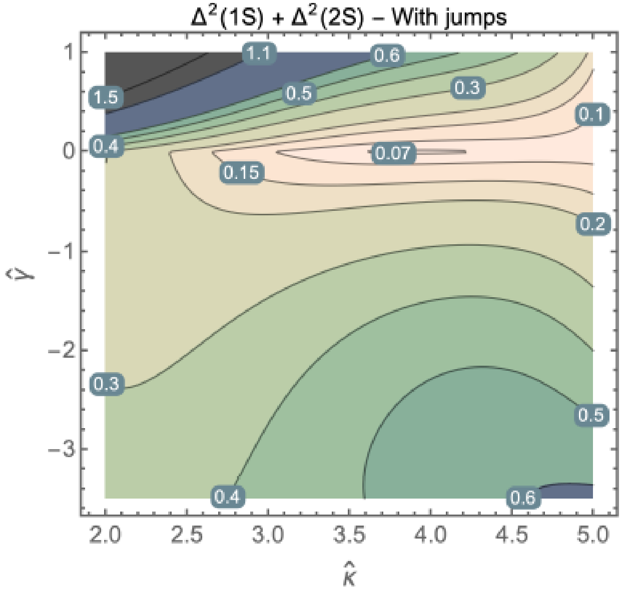
- ≈ recoil terms / friction
- Allows to go to lower T



- Sizable effect on the survival (friction prevents the $b\bar{b}$ from overheating => less suppr.)
- Complex H_{eff} & parameter uncertainties dominate over Q-jumps => not syst. included

N. Brambilla et al. 2302.11826v2

- First study including the jump operators (quite demanding), motivated by more precise data on the Y excited states



- Comparison with LHC data favors 0 mass-shift (⇔no « screening »)

A consistent picture emerging in the bottomia sector

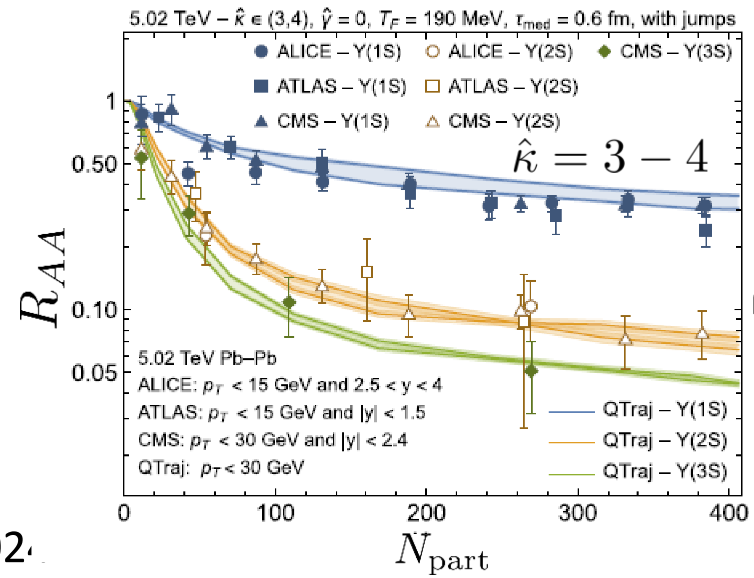
Beauty sector: good overall consistency of the following facts:

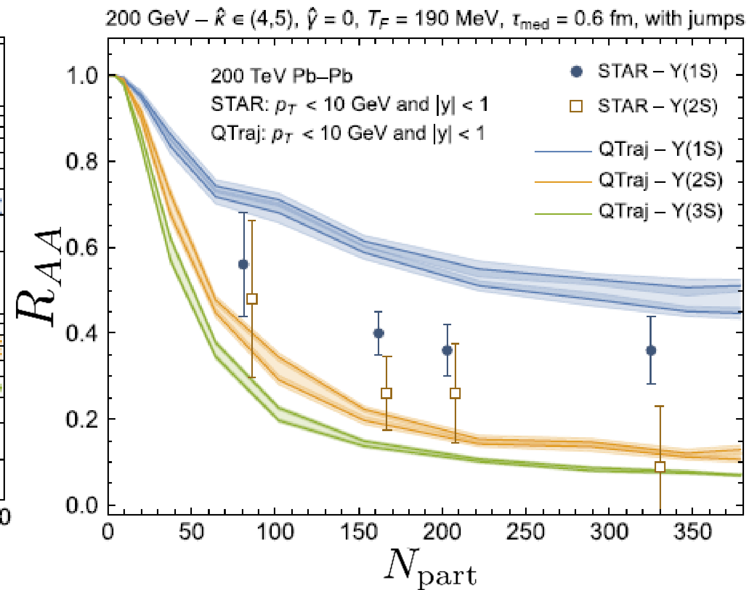
- Similar production of Y(1S) from RHIC -> LHC
- Higher states strongly suppressed
- Washing out of the spectral function (but the Y(1S) which survive up to T = 0.45 GeV)

Not paying too much attention at CNM effects:

With the interpretation that higher states (which contribute to the prompt Y(1S)) are suppressed both at RHIC and LHC in the QGP, while the ground state Y(1S) survives and is thus a genuine hard QGP probe; higher states could be produced (partly) through recombination

N.B.: No precise v_2(Y) measured up to now. One would expect very small v_2(Y(1S)) and slightly larger v_2(Y(2S))… but will be hard to measure.





M. Strickland & S. Thapa, Phys. Rev. D 108, 014031 (2023)

Good agreement with suppression at LHC but not at RHIC

Other implementations : Duke, Osaka, Saclay, Nantes, …

Fresh News from NRQCD - like

Akamatsu:
(recent)

2020 — 2021 — 2022 → time

T. Miura, Y. Akamatsu et al, Phys. Rev. D 101, 034011 (2020), 1908.06293

Akamatsu & Miura, EPJ Web Conf. 258 (2022) 01006, 2111.15402

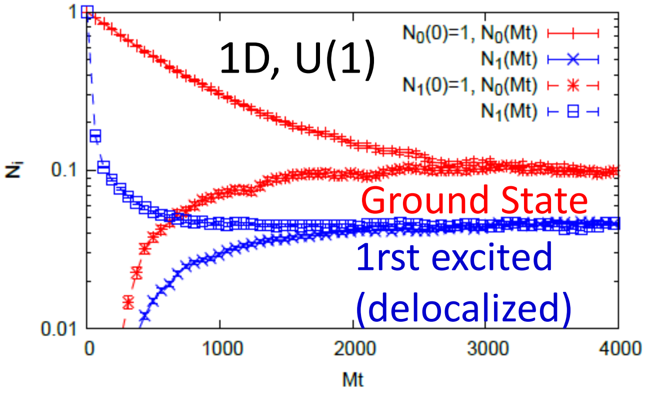
T. Miura et al, http://arxiv.org/abs/2205.15551v1

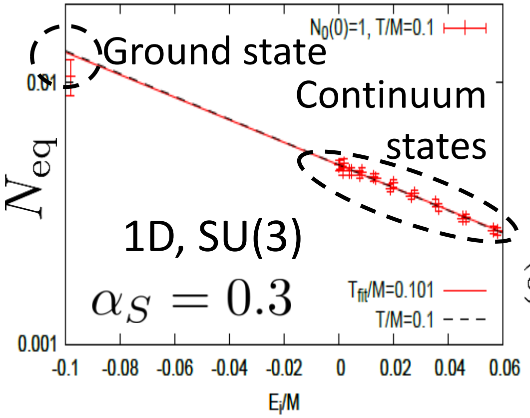
Comparison btwn full scheme & dipole limit (+nice scale analysis)

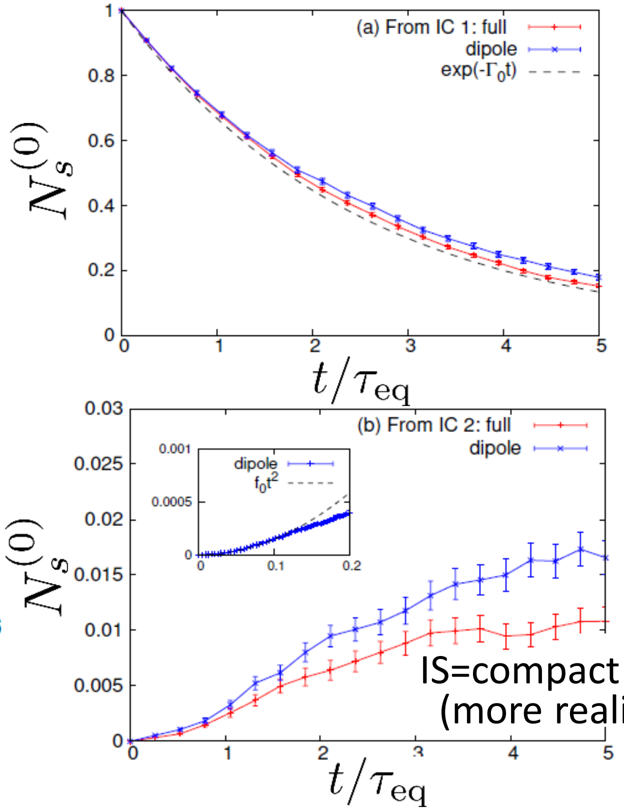
Quantum jump : numerical solution through Quantum State Diffusion Method => friction could be included

Extension -> SU(3) (still 1D)

Approach to equil can be studied







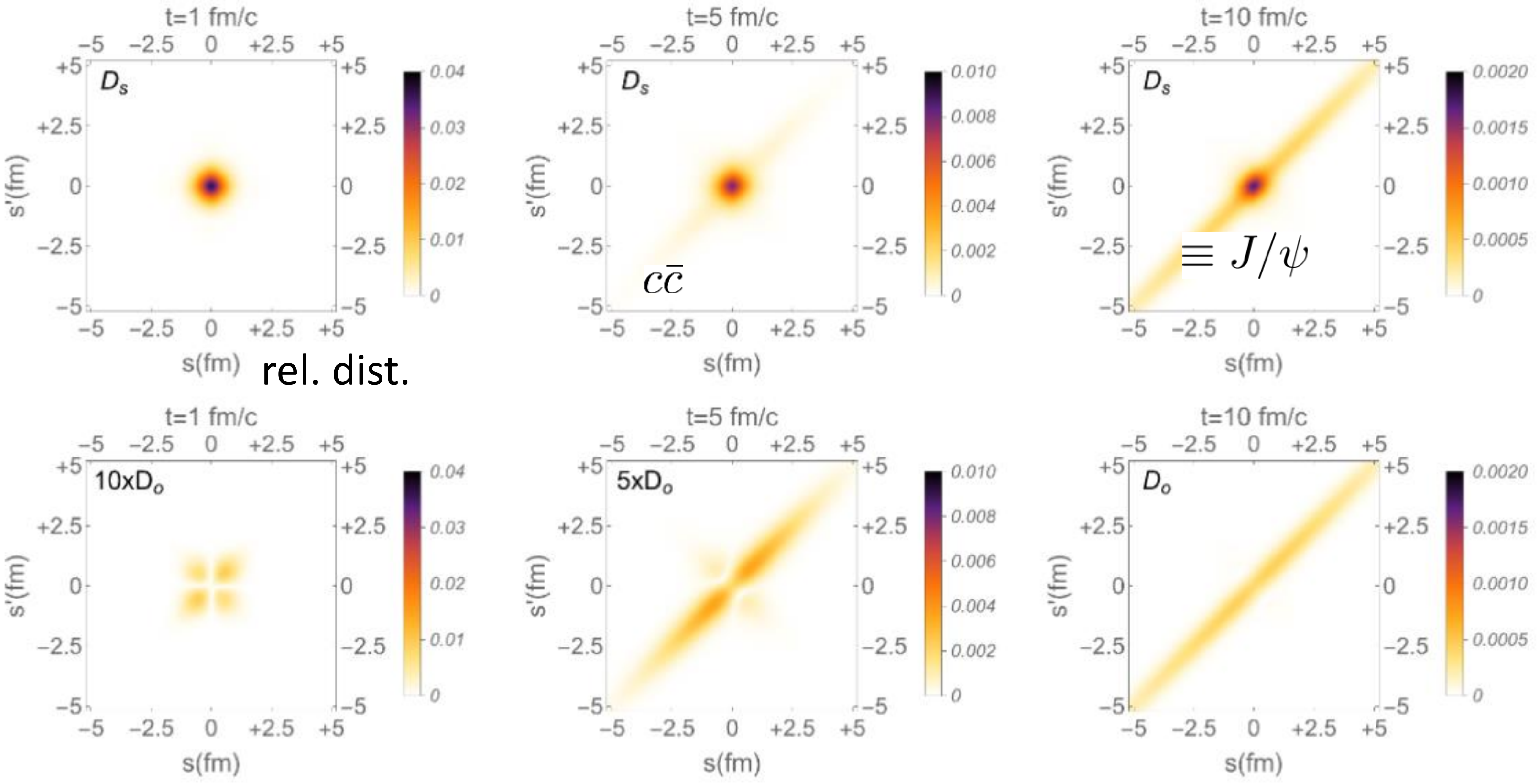
Quantum State -> equilibrium irrelevant of the starting condition -> Boltzmann distribution
See as well discussion in Katz & Gossiaux, Annals Phys. 368 (2016) 267-295, 1504.08087

b and $\bar{b}$ are unbound in 8-channel -> density decreases and feeds less the singlet channel.

Fresh News from NRQCD - like

- S. Delorme et al (https://inspirehep.net/literature/2026925 and Ph.D. thesis; manuscript coming soon on arxiv)
- solving BE equations in the QBM regime JP Blaizot & MA Escobedo JHEP 06 (2018) 034,1711.10812
- 1D potential tuned to 3D : Katz et al. , arxiv2205.05154

No dipole approximation => able to model pairs at finite distance

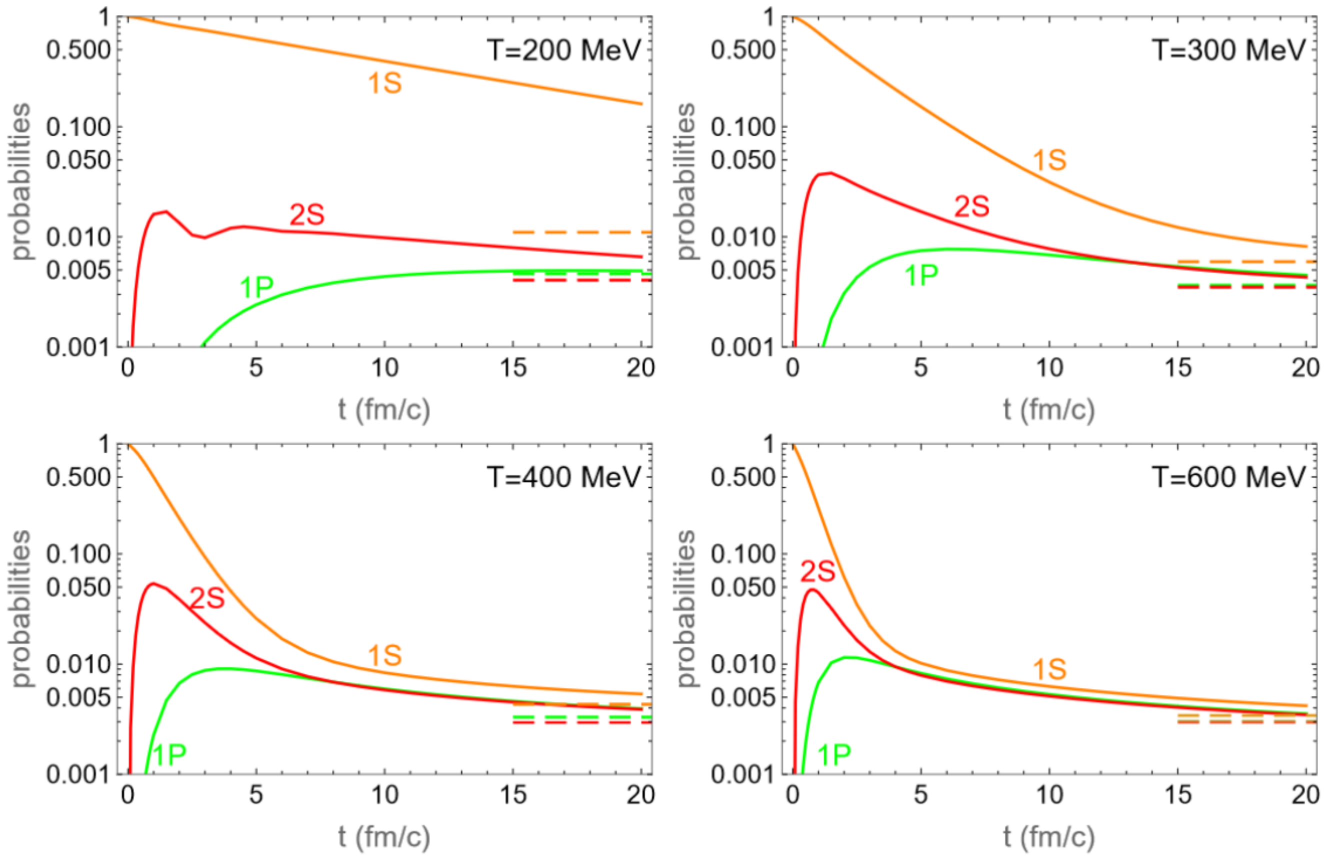


Evolution -> peak in S-like singlet channel, surviving at the end of the evolution

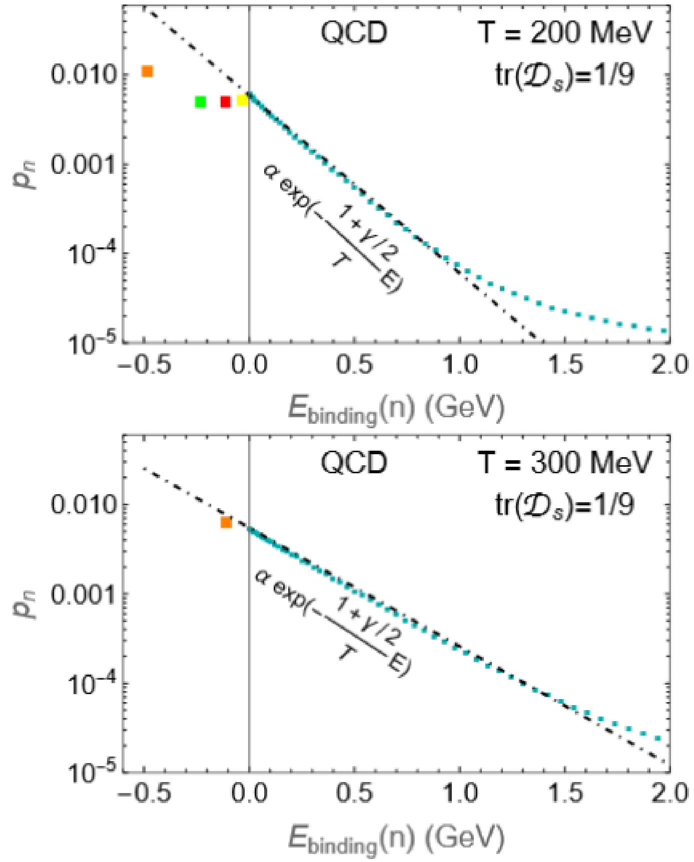
Fresh News from NRQCD - like

- S. Delorme et al (https://inspirehep.net/literature/2026925 and Ph.D. thesis; manuscript coming soon on arxiv)
- solving BE equations in the QBM regime JP Blaizot & MA Escobedo JHEP 06 (2018) 034,1711.10812
- 1D potential tuned to 3D : Katz et al. , arxiv2205.05154

… => able to model pairs at finite distance => able to study thermalization



2S and 1P are generated during the evolution (⇔ jump operators)



Deviations from Boltzmann law for bound states in the asymptotic distribution…

Quarkonia coupled to the QGP !!!

Summary

- Field that has recently benefitted from the impetus of talented young physicists and the maturity of older ones…
- keep going on !

Global picture (E. Ferreiro; QM 2018)

Caveat I: we need firm theoretical understanding of quarkonium production in pp collisions