Review on quarkonium production and CGC

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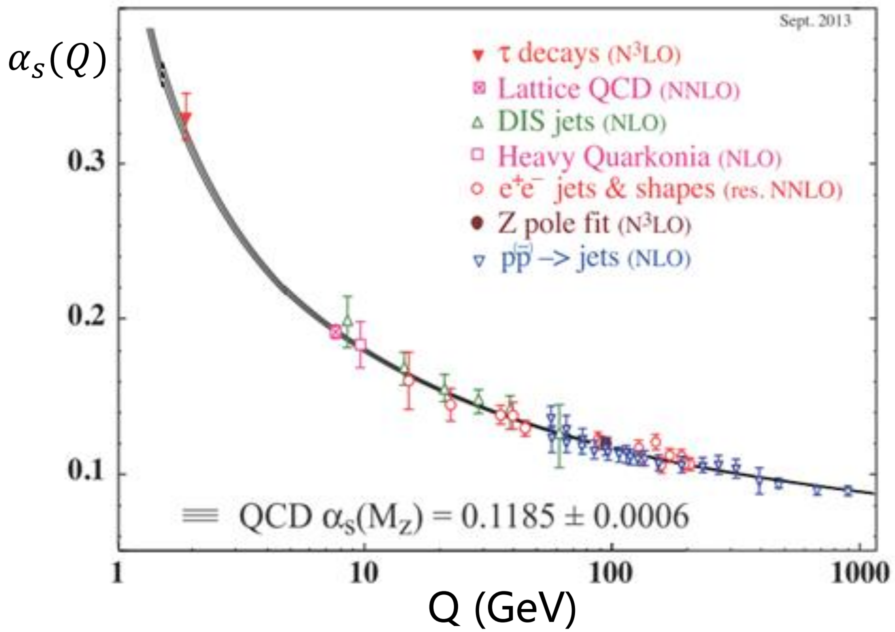
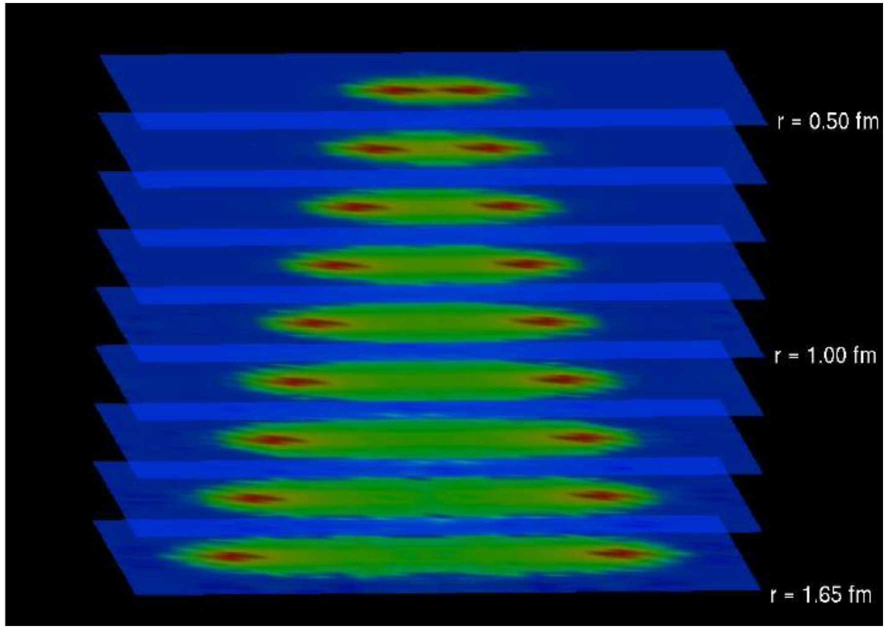
Outline

QCD

➢ Extremely hard

- Asymptotic freedom: perturbative at short distance
- Confinement: nonperturbative at long distance

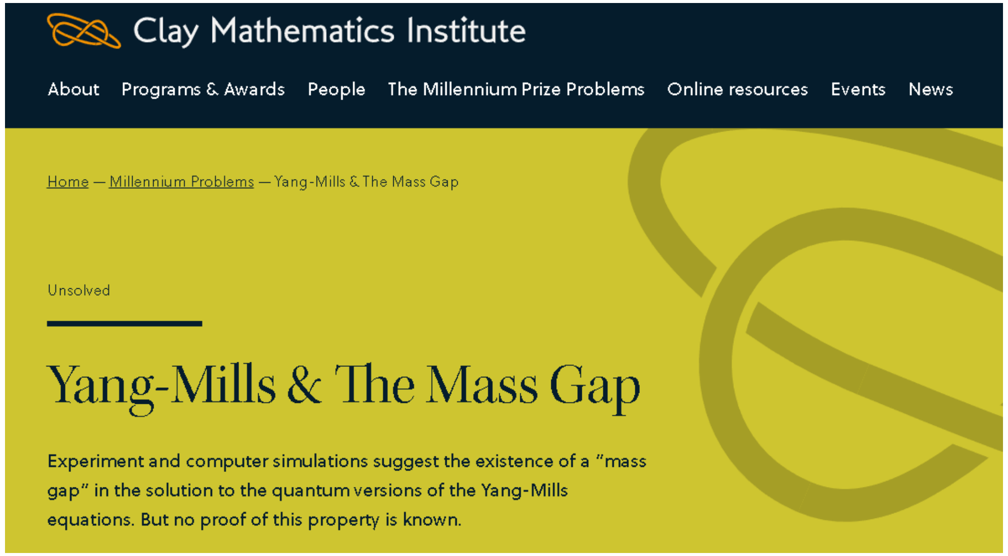




Confinement and hadronization

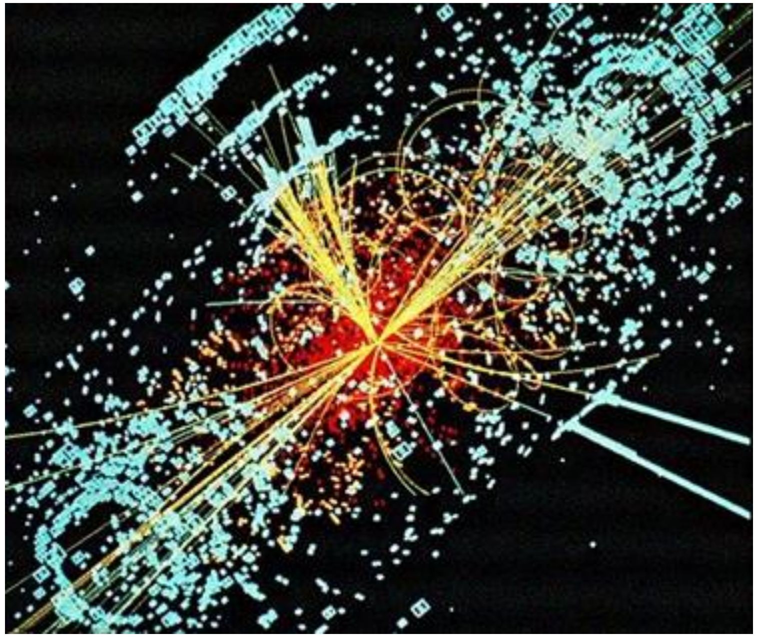
➢ Confinement

- 1/7 millennium prize problems in 21st century
- Not yet understood
- Equivalent: why and how produced quarks and gluons become hadrons?



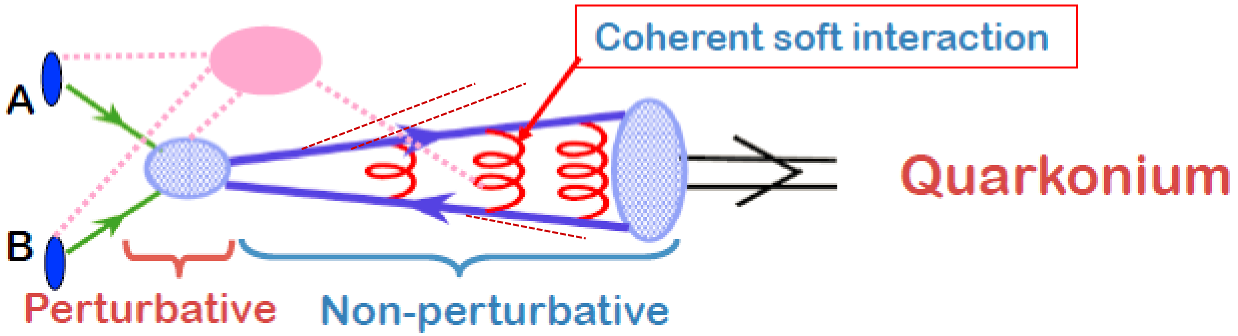
➢ Hadronization-QaT

- Light hadrons: factorization → fragmentations functions, do not know how to compute
- Heavy quarkonium: localized color charge, perturbative QCD can help, the simplest system
- HQ production: 50 years after the discovery, still not well understood



Space-time picture for production

➢ Production of an off-shell heavy-quark pair and hadronization



$$\Delta r \sim \frac{1}{2m}, \quad \frac{1}{m\,v}, \quad \frac{1}{m\,v^2}$$

- Time scale for producing heavy quark pair: $1/(2m)$
- Time scale for expansion: $1/(mv)$
- Time scale for forming bound state: $1/(mv^2)$

➢ Expand intermediate heavy-quark pair around on-shell limit

- Off-shellness is small, comparing with m
- If lucky enough: factorization → disentangle nonperturbative interactions

NRQCD factorization

➢ Factorization formula (assumption)

Bodwin, Braaten, Lepage, 9407339

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} = \sum_n d\hat{\sigma}_n(P_H)\langle\mathcal{O}_n^H\rangle$$

Production of a heavy quark pair
Expansion in: α_s

Hadronization (LDMEs)
Expansion in: v

- n: quantum numbers of the pair: color, spin, orbital angular momentum, total angular momentum, spectroscopic notation $^{2S+1}L_J^{[c]}$

➢ A glory history-thanks to color-octet mechanism

- Solved IR divergences in P-wave quarkonium decay
- Explained ψ' surplus, χ_{c2}/χ_{c1} production ratio, ….

BUT: also many phenomenological difficulties

Resummations: improving NRQCD

➢ Low $p_T \ll m$: k_T-dependent factorization

YQM, Venugopalan, 1408.4075
Watanabe, Xiao, 1507.06564
Lansberg, Nefedov, Ozcelik, 2112.06789

- Color Glass Condensate, or high energy factorization
- Resum Sudakov log $\ln(p_T/m)$, small-x log $\ln(x)$

➢ High $p_T \gg m$: Collinear factorization

Kang, Qiu, Sterman, 1109.1520
Fleming, Leibovich, Mehen, Rothstein 1207.2578
Kang, YQM, Qiu, Sterman, 1401.0923
Kang, YQM, Qiu, Sterman, 1411.2456

- Power expansion, double parton fragmentation
- Resum large log $\ln(p_T/m)$

➢ All p_T: Soft gluon factorization

YQM, Chao, 1703.08402
Chen, YQM, 2005.08786

- Kinematic effect in NRQCD can be very important
- Resum a series of v^2 corrections

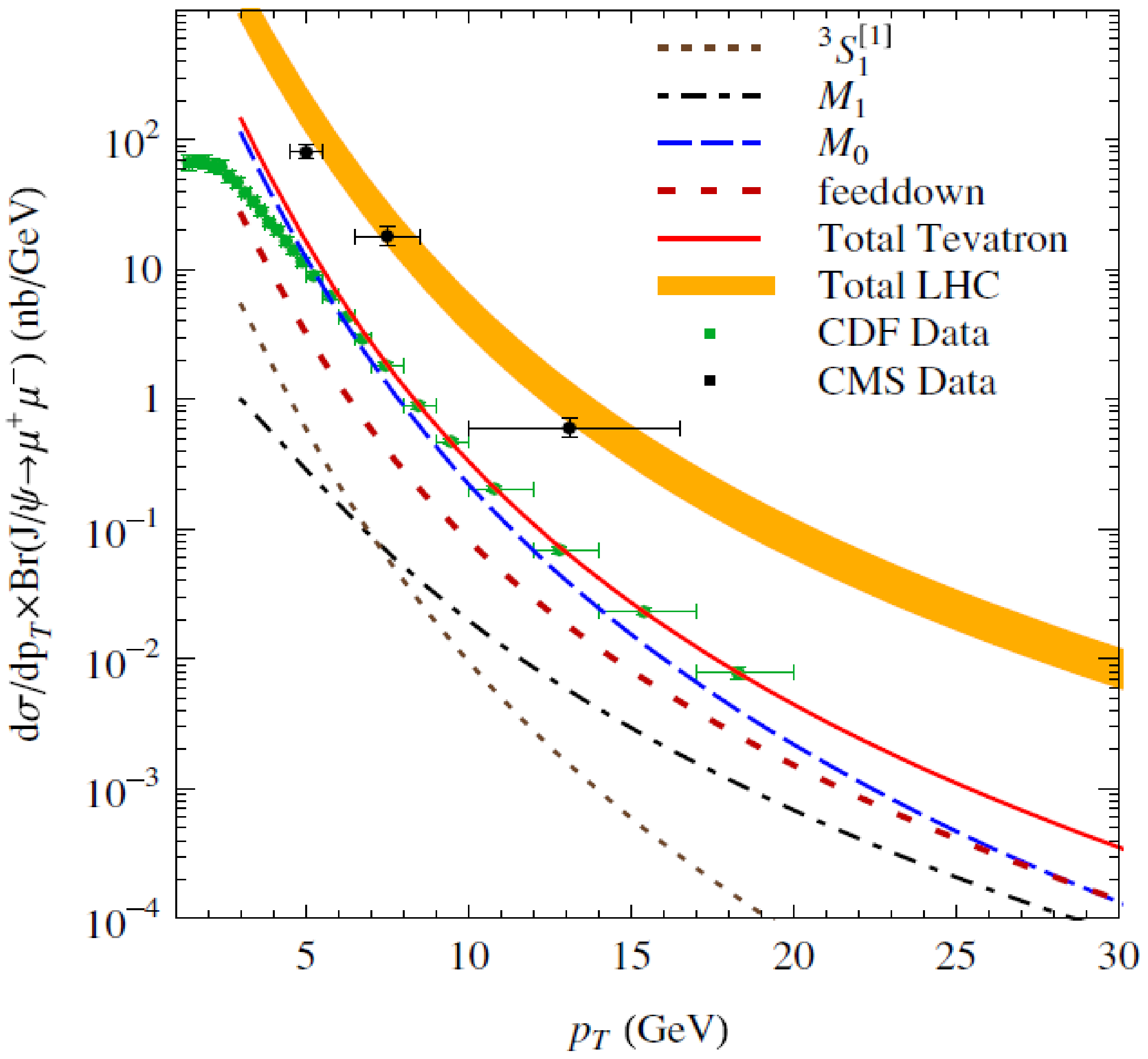
Outline

Low p_T quarkonium production

- **Moderate p_T region: fine**

- **Small p_T region**

  - When $p_T \ll m_H$, fixed order gives $\frac{d\sigma}{dp_T} \propto \frac{1}{p_T}$, data goes to zero
  - Dominate the total cross section



Small p_T and small x

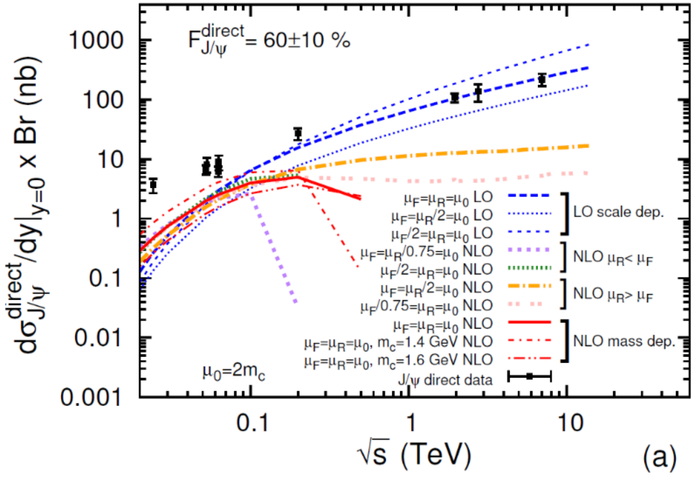
➢ Sudakov double logarithm

Berger, Qiu, Wang, 0404158
Sun, Yuan, Yuan, 1210.3432

- Sudakov resummation: $\ln^2(p_T/m_H)$ important at small p_T regime
- Sudakov resummation can be dominant for Υ production (large mass scale)
- But, itself still hard to explain the J/ψ data

➢ Why $\ln^2(p_T/m_H)$ resummation is not enough?

- Total cross section is free of $\ln(p_T/m_H)$
- Total cross section can be negative
- Fixed order NRQCD fails to explain data



Feng, Lansberg, Wang, 1504.00317

➢ Small-x effect can be important

- The only large logarithm is $\ln(x^2) \sim \ln(m_H^2/S)$

CGC effective field theory

➢ Color Glass Condensate

McLerran, Venugopalan, 9309289

- An effective field theory of QCD
- A tool to deal with small-x physics : separate $x < x_0$ configuration from $x > x_0$ configuration
- Small-x configuration: large saturation scale, perturbatively calculable
- Large-x configuration: $\Delta t^+ \sim \frac{1}{k^-} = \frac{2k^+}{k_\perp^2} \sim x$, life time of parton is long, determined before the collision, randomly distributed, CGC average
- JIMWLK evolution: guarantees the independence of separation point x_0

CGC+NRQCD

➢ CGC: production of $Q\bar{Q}$-pair

Kang, YQM, Venugopalan, 1309.7337

- Using CGC to calculate gluon distribution
- Small x resummation is accounted by solving JIMWLK or BK evolution equations

➢ NRQCD factorization:

- Control the formation of quarkonium from $Q\bar{Q}$-pair
- Via many channels, both CS and CO

$$d\sigma_H = \sum_\kappa d\hat{\sigma}^\kappa \langle \mathcal{O}_\kappa^H \rangle$$

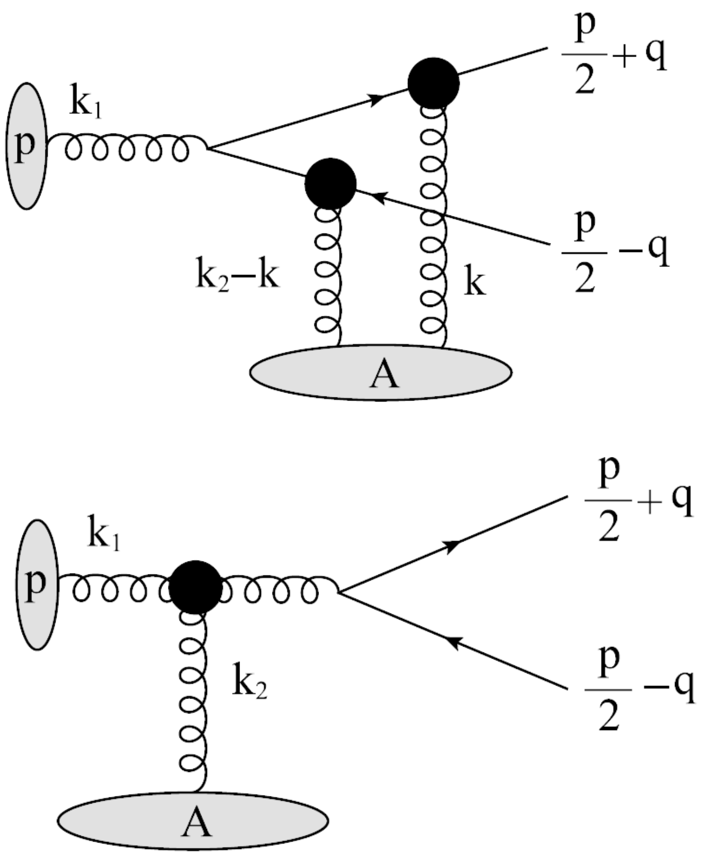
➢ Scope of application:

- Assume a dilute-dense formula, factorization is possible
- High energy p+A or p+p collision
- Quarkonium produced in forward rapidity region

High p_T and NLO

➢ **With LO calculation: can only describe small p_T region data!**

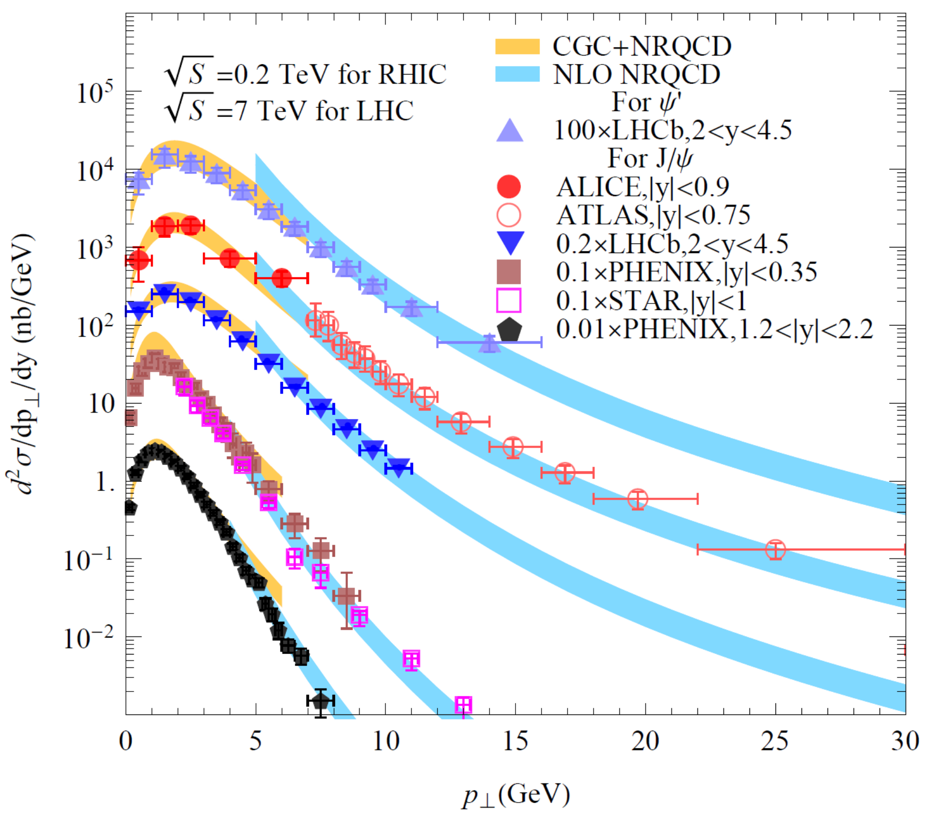
- No final state radiation
- Correct only if initial state radiation dominate (p_T can not be much larger than the saturation scale)
- To describe higher- p_T data in CGC+NRQCD, NLO is needed



Small p_T region

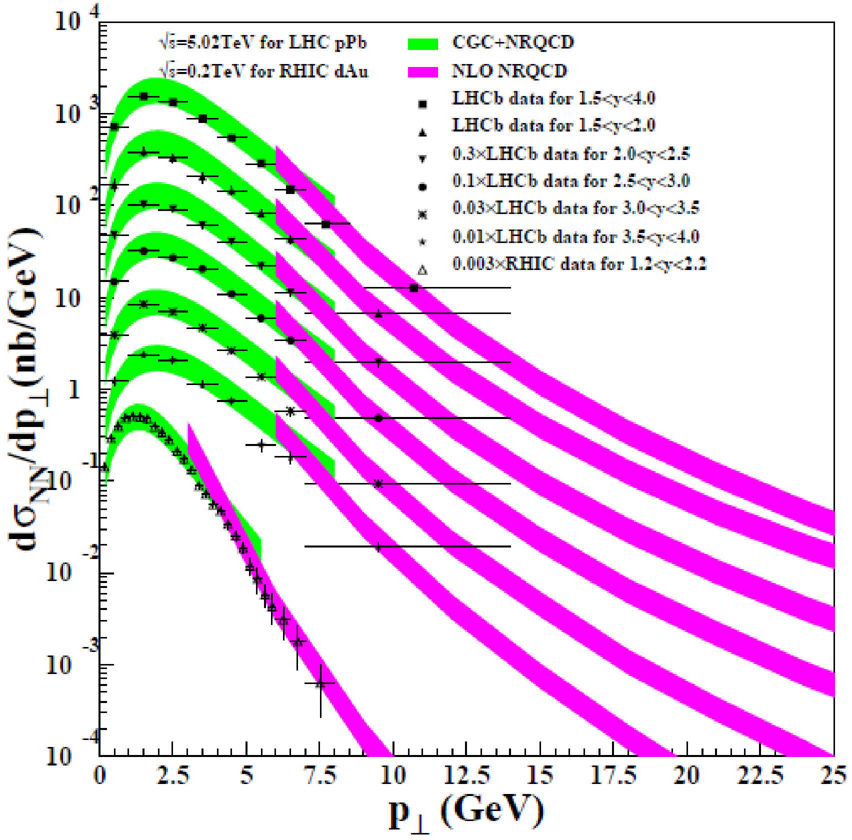
- CGC+NRQCD : comprehensive description of $\psi(nS)$ production

YQM, Venugopalan, 1408.4075



ψ(nS) in p+p collisions

YQM, Venugopalan, Zhang, 1503.07772



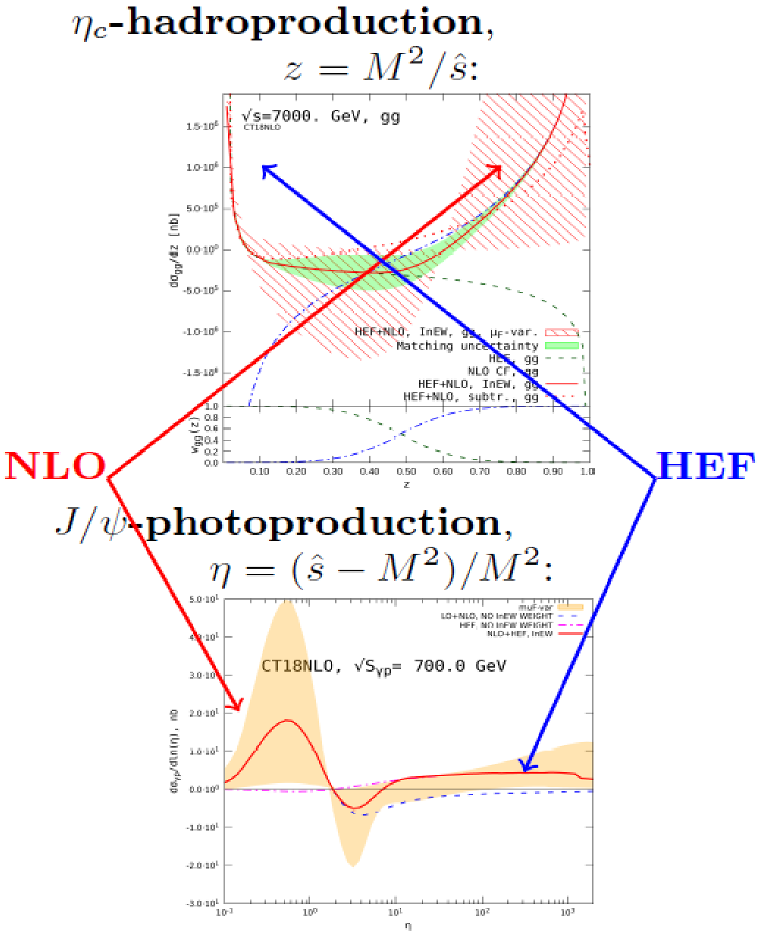
J/ψ in p+A collisions

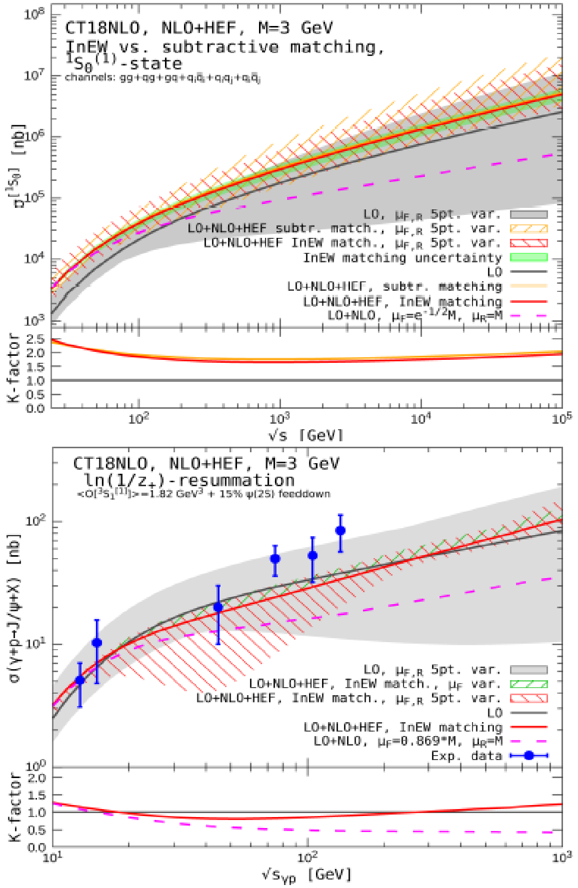
High energy factorization

- Resum large log $\ln(m_H^2/S) \sim \ln(x^2)$
- Resolve the problem of negative total cross section

Lansberg, Nefedov, Ozcelik, 2112.06789

See Maxim Nefedov's talk





Outline

Collinear factorization for high p_T production

- When $p_T \gg m_H$, power expansion m_H^2/p_T^2 first, then α_s

- LP: collinear factorization, single parton fragmentation

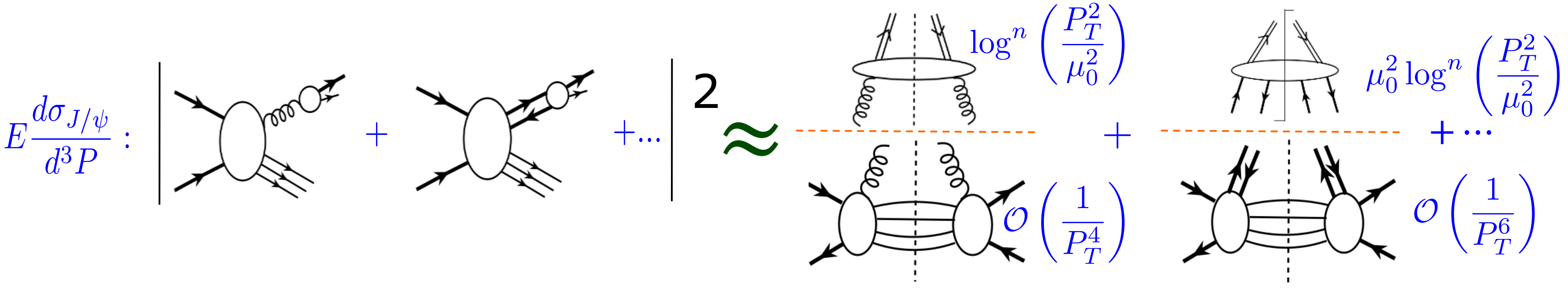
Collins, Soper (1982)
Braaten, Yuan, 9303205
Nayak, Qiu, Sterman, 0509021

- NLP: important for $p_T < 10m_H$, double parton fragmentation

Kang, Qiu, Sterman, 1109.1520
Fleming, Leibovich, Mehen, Rothstein 1207.2578
Kang, YQM, Qiu, Sterman, 1401.0923
Kang, YQM, Qiu, Sterman, 1411.2456

Collinear factorization approach

- **Ideas**



- **Factorization correct to all orders**

Qiu, Sterman (1991)
Kang, YQM, Qiu, Sterman, 1401.0923

Predictive power

- Calculation of short-distance hard parts in pQCD:

Kang, YQM, Qiu, Sterman, 1411.2456

  - Power series in α_s , without large logarithms

- Calculation of evolution kernels in pQCD:

  - Power series in α_s, without large logarithms

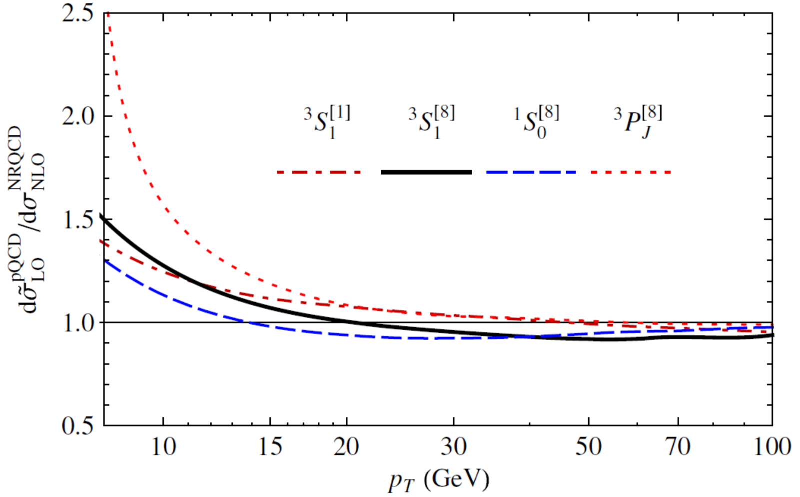
Kang, YQM, Qiu, Sterman, 1401.0923

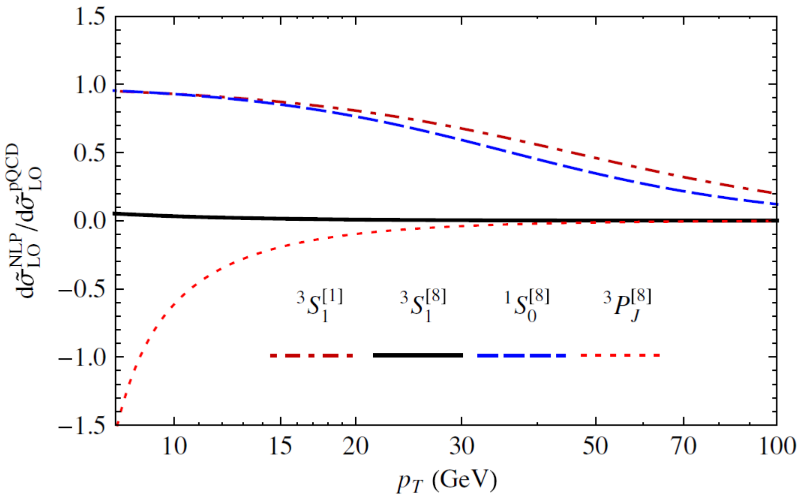
- Universality of input fragmentation functions at the initial scale μ_0

  - Fit data, or compute them using other methods, like NRQCD

Reproducing plain NRQCD

YQM, Qiu, Sterman, Zhang, 1407.0383





- **LO LP+NLP comparing with NLO NRQCD**
  - Compute input functions using NRQCD
  - LO analytical results reproduce NLO NRQCD calculations (numerical) !
- **For $p_T = 5\,m$**
  - LP dominates: $^3S_1^{[8]}$ and $^3P_J^{[8]}$ channels
  - NLP dominates: $^1S_0^{[8]}$ and $^3S_1^{[1]}$ channels

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- I. Introduction
- II. Resummation at low p_T region
- III. Resummation at high p_T region
- IV. Resummation at all p_T region
- V. Current difficulties
- VI. Outlook

Relativistic corrections in NRQCD

➢ Relativistic (power) corrections

- Equations of motion of NRQCD EFT: $\left(iD_0 - \frac{\boldsymbol{D}^2}{2m} + \cdots\right)\psi = 0$
- NRQCD factorization: use EOM to remove ∇_0, leaving operators like:

(Warning: here D replaced by ∇, needs proper gluon fields to make them gauge invariant)

$$\chi^\dagger\psi, \chi^\dagger\sigma^i\psi, \chi^\dagger T^a\psi, \chi^\dagger\sigma^i T^a\psi$$

Type 0: Different colors, spins

$$\chi^\dagger g E^i \psi$$

Type 1: Intrinsic gluons (E, B fields)

$$\chi^\dagger \overleftrightarrow{\nabla}^i \psi$$

Type 2: Orbital angular momentum

$$\chi^\dagger \overleftrightarrow{\nabla}^2 \psi$$

Type 3: Relative momentum

$$\nabla^i \left(\chi^\dagger \psi\right)$$

Type 4: Total momentum

Corrections in type 3 widely studied, for charmonium production in pp collision, about 30%-50% corrections

CS-channel:
Fan, YQM, Chao, 0904.4025

CO-channel:
Xu, Li, Liu, Zhang,1203.0207

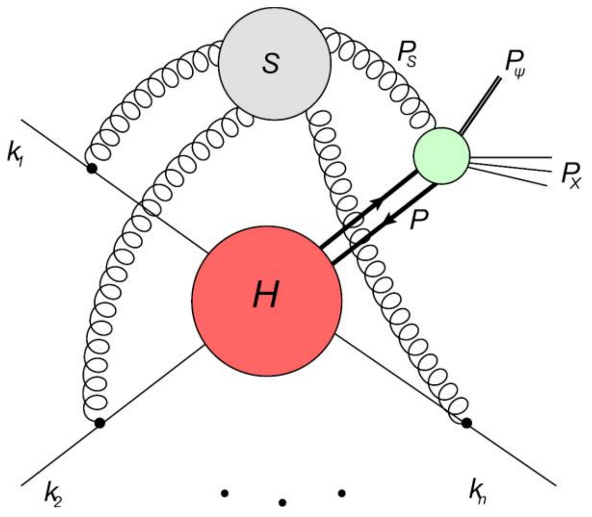
S-D mixing-channel:
He, Kniehl, 1507.03882

LP in p_T, all order in v:
Li, Chen, Huang, YQM, 1909.03554

However, more relativistic-correction terms may be needed!

Soft gluon emission

- **Soft gluon emission in color-bleaching process**
  - P_ψ **is different from** P, $P = P_\psi[1+O(\lambda)]$
  - **NRQCD expand** P **around** P_ψ



YQM, Vogt, 1609.06042

- **Bad convergence of NRQCD expansion**
  - **Cross section approximately** $\propto P^{-4} = P_\psi^{-4}[1+O(\lambda)]^{-4}$

Mangano, Petrelli, 9610364

$$\int_{-1}^{1} \frac{d\cos\theta}{2(1+\lambda+\lambda\cos\theta)^4} = 0.42 = 1 - 4\lambda + \frac{40}{3\lambda^2} - 40\lambda^3 + \cdots$$

$$= 1 - 1.2 + 1.2 - 1.08 + 0.91 - 0.73 + \cdots$$

With $\lambda \approx v^2 \approx 0.3$

  - **Soft gluon momentum should be kept but not expanded, which means to resum relativistic corrections (due to kinematic effects) to all powers in** v**!**

Soft gluon factorization

➢ Different way to use EoM in NRQCD EFT

Ma, Chao, 1703.08402
Chen, Ma, 2005.08786

- NRQCD factorization: use EOM to remove ∇_0
- SGF: remove relative derivatives $\overleftrightarrow{\nabla}_0$, $\overleftrightarrow{\nabla}^2$, leaving only total derivatives

➢ Factorization formula

$P = P_H + P_X$: momentum of $Q\bar{Q}$

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3 P_H} \approx \sum_n \int \frac{d^4 P}{(2\pi)^4} \mathcal{H}_n(P) F_{n\to H}(P, P_H)$$

- $\mathcal{H}_n$: perturbatively calculable hard parts
- $F_{n\to H}$: nonperturbative soft gluon distributions (SGDs)
- UV renormalization scale is suppressed

$$F_{n\to H}(P, P_H) = \int d^4 b e^{-iP\cdot b} \langle 0|[\overline{\Psi}\mathcal{K}_n\Psi]^\dagger(0)\left(a_H^\dagger a_H\right)[\overline{\Psi}\mathcal{K}_n\Psi](b)|0\rangle_{\rm S}$$

- Subscript "S": evaluate the matrix element in the region where off-shellness of all particles is much smaller than heavy quark mass

Fragmentation functions in SGF

Chen, YQM, Meng, 2304.04552

- **Gluon FFs $g \to Q\overline{Q}\,({}^3\mathrm{P}_J^{[1]}) + X$**
  - In NRQCD: plus-function result in negative results

$$\hat{d}^{(2)}_{g\to {}^3P_J^{[1]}} = \frac{4}{9N_c}\left\{\left[\frac{Q_J}{2J+1} - \frac{1}{2}\ln\left(\frac{\mu_\Lambda^2}{4m_Q^2}\right)\right]\delta(1-z) + \frac{z}{(1-z)_+} + \frac{P_J(z)}{2J+1}\right\}$$

  - In SGF: plus-functions are factorized into nonperturbative functions, can be positive

$$\hat{D}^{LO,(0)}_{g\to Q\bar{Q}[{}^3P_0^{[1]}]} = \frac{32\alpha_s^2}{M_H^5 N_c}\frac{2}{9}\left[\frac{1}{36}z(837-162z+72z^2+40z^3+8z^4) + \frac{9}{2}(5-3z)\ln(1-z)\right]$$

Fragmentation functions in SGF

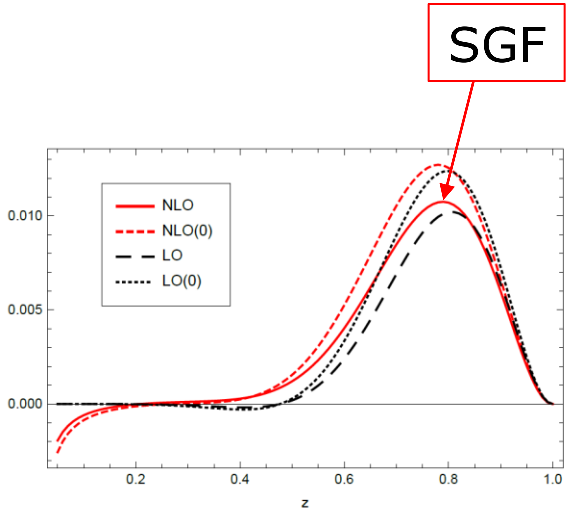
- **Gluon FFs** $g \to Q\overline{Q}\,({}^3S_1^{[8]}) + X$

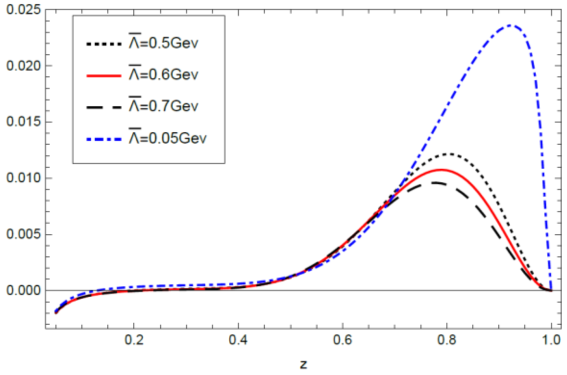
Chen, Jin, YQM, Meng, 2103.15121

$$D_{g\to H}(z, M_H, m_Q, M_H) = \int_z^1 \frac{dx}{x} \hat{D}_{[SS]}(\hat{z}, M_H/x, m_Q, M_H, M_H) \times F_{[SS]\to H}(x, M_H, m_Q, M_H),$$

$$D^{(0)}_{g\to H}(z, M_H, m_Q, M_H) = \int_z^1 \frac{dx}{x} \hat{D}^{(0)}_{[SS]}(\hat{z}, M_H/x, M_H, M_H) \times F_{[SS]\to H}(x, M_H, m_Q, M_H).$$

$\bar{\Lambda}$: average momentum emitted





NRQCD

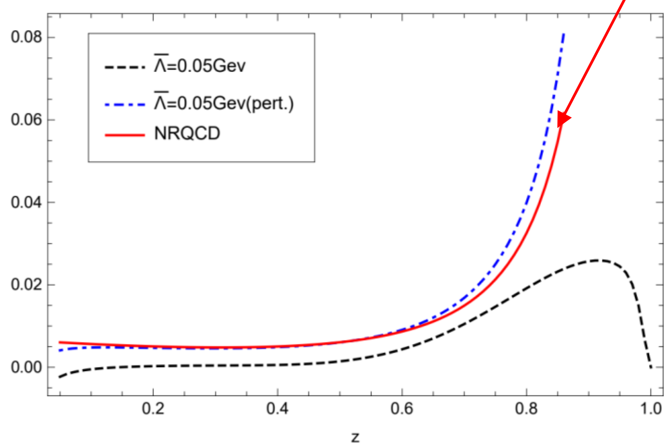


Figure 7. Left figure: Comparison of the gluon FF obtained in different approximations. Rig[ht] figure: $\bar{\Lambda}$ dependence of gluon FF at NLO.

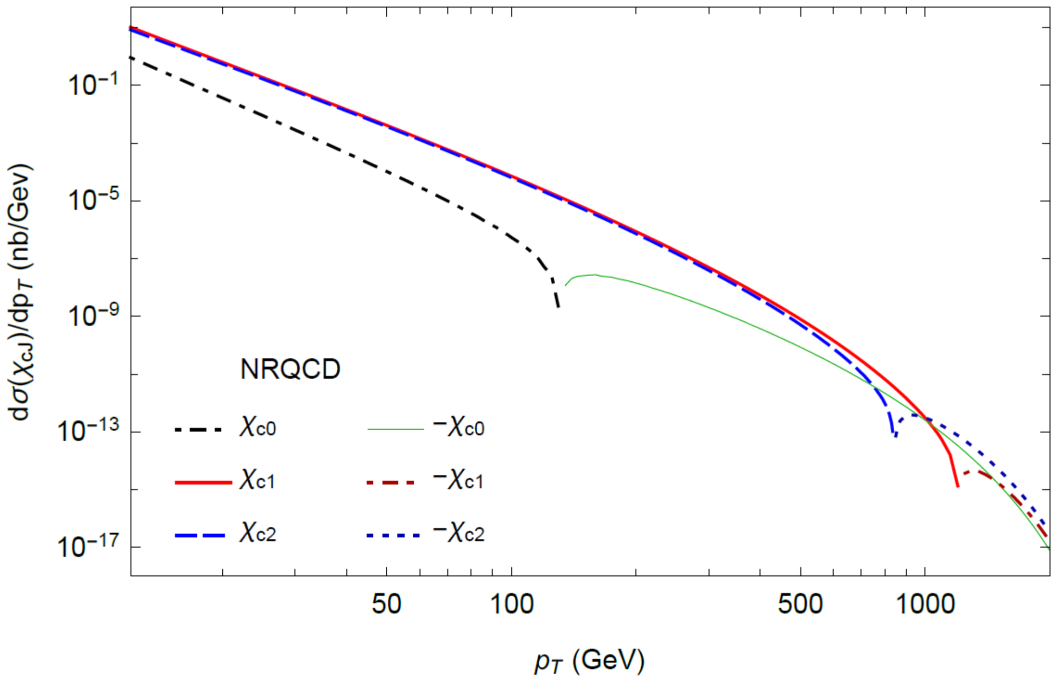
$$R^X(n) \equiv \frac{\int_0^1 dz z^n D^X_{g\to H}(z, M_H, m_Q, \mu)}{\int_0^1 dz z^n D_{g\to H}(z, M_H, m_Q, \mu)},$$

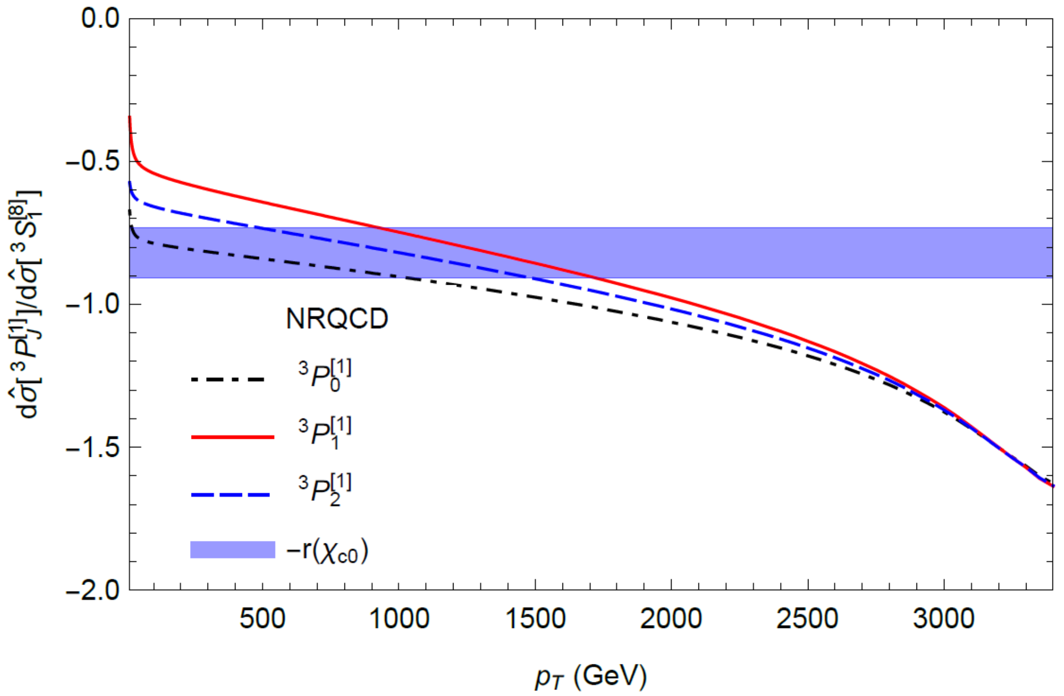
$R^{NRQCD} \approx 6$

Negative differential cross sections in NRQCD

- Cross sections become negative at exceptionally high p_T

$$d\sigma(\chi_{cJ}) = (2J+1)d\hat{\sigma}[^3S_1^{[8]}]\frac{\langle\mathcal{O}^{\chi_{c0}}(^3P_0^{[1]})\rangle}{m_c^2}\left[r(\chi_{c0}) + \frac{d\hat{\sigma}[^3P_J^{[1]}]}{d\hat{\sigma}[^3S_1^{[8]}]}\right] \qquad r(\chi_{c0}) \equiv \frac{\langle\mathcal{O}^{\chi_{c0}}(^3S_1^{[8]})\rangle}{\langle\mathcal{O}^{\chi_{c0}}(^3P_0^{[1]})\rangle/m_c^2}$$

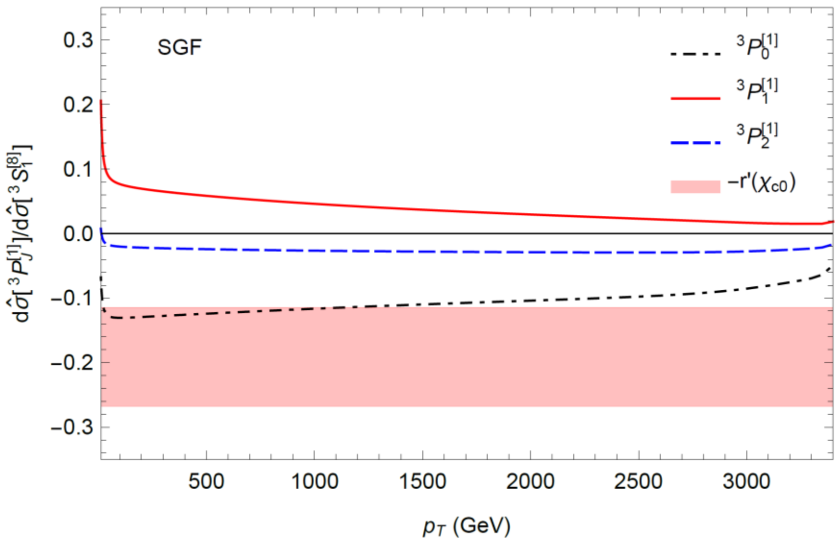


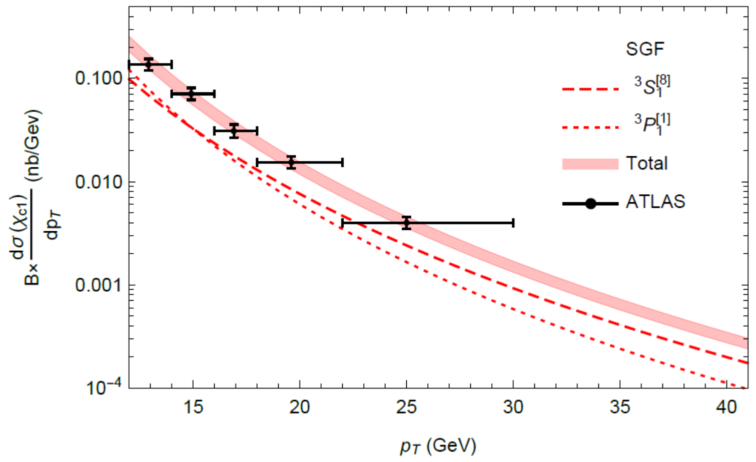


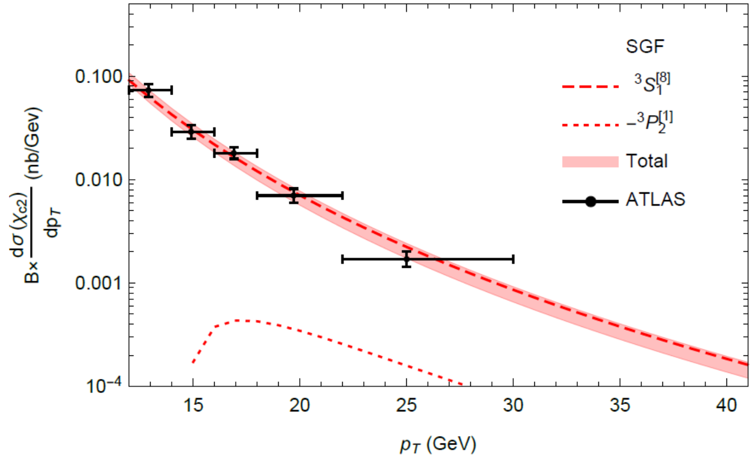
Resolve negative differential cross section in SGF

Chen, YQM, Meng, 2304.04552

- χ_{cJ} **production**
  - $\chi^2/d.o.f = 0.63/8$, as good as NRQCD
  - No substantial cancellations
  - Cross sections are positive at high p_T







- See also resummation within NRQCD framework

Chung, 2303.17240

Outline

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II. Resummation at low p_T region

III. Resummation at high p_T region

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V. Current difficulties

VI. Outlook

1. Hierarchy and universality problems

- **Fit J/ψ yield data at Tevatron with $p_T > 7$ GeV**

YQM, Wang, Chao, 1009.3655

- Due to p_T^{-4} and p_T^{-6} behaviors, constrain two combinations
- $M_0 = \langle O({}^1S_0^{[8]})\rangle + 3.9\langle O({}^3P_0^{[8]})\rangle/m_c^2 \approx (7.4 \pm 1.9)\times 10^{-2}\text{GeV}^3$
- $M_1 = \langle O({}^3S_1^{[8]})\rangle - 0.56\langle O({}^3P_0^{[8]})\rangle/m_c^2 \approx (0.05 \pm 0.02)\times 10^{-2}\text{ GeV}^3$

See also:
Butenschoen, Kniehl, 1105.0820
Gong, Wan, Wang, Zhang, 1205.6682

- **Two orders difference: hierarchy problem**
  - Velocity scaling rule of NRQCD

$$\langle O({}^1S_0^{[8]})\rangle \sim \langle O({}^3S_1^{[8]})\rangle \sim \langle O({}^3P_0^{[8]})\rangle/m_c^2$$

  - Thus natural expectation: $M_0 \sim M_1$

- **Upper bound from Belle total cross section**

$$M_0 < 2\times 10^{-2}\text{GeV}^3$$

Zhang, YQM, Wang, Chao, 0911.2166

- No universality of NRQCD LDMEs!

2. Polarization puzzle

➢ LO NRQCD

- **Dominated by $^3S_1^{[8]}$, LO NRQCD predicts transversely polarized $\psi(\mathrm{nS})$, contradicts with CDF data**

CDF, 0704.0638

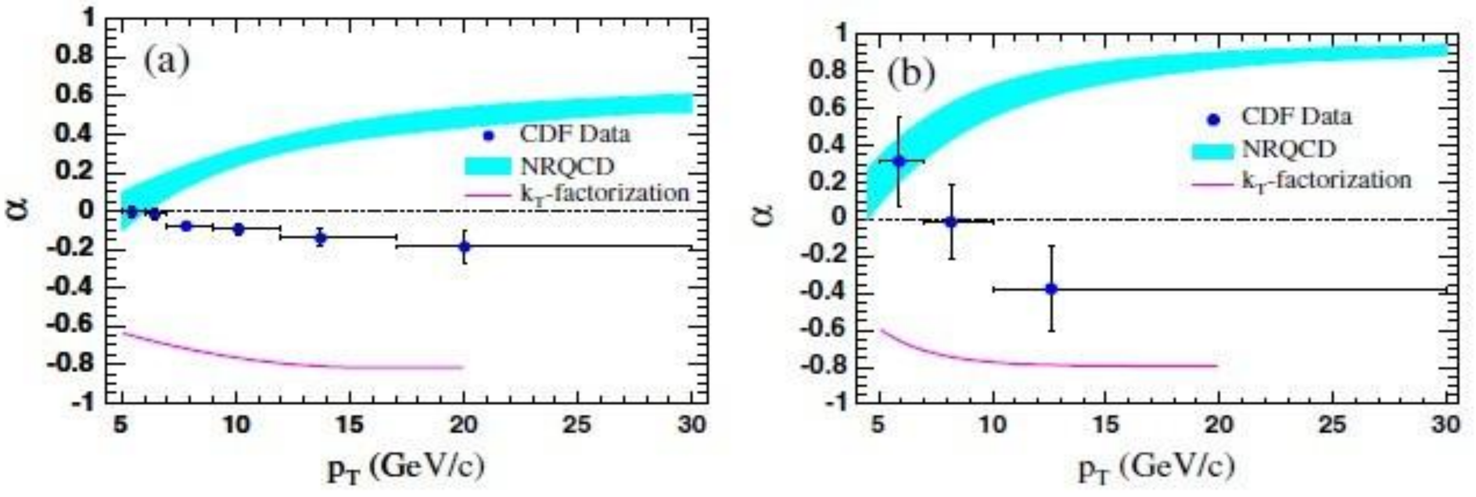
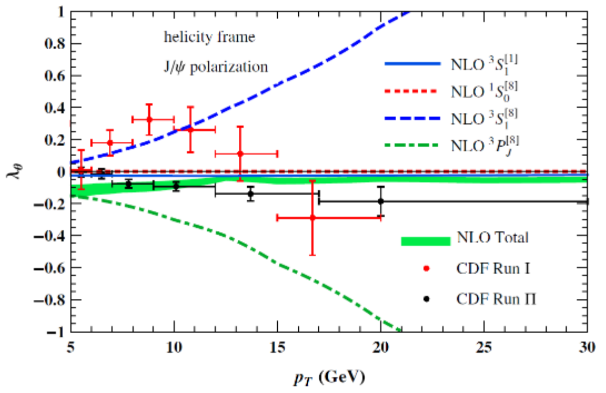


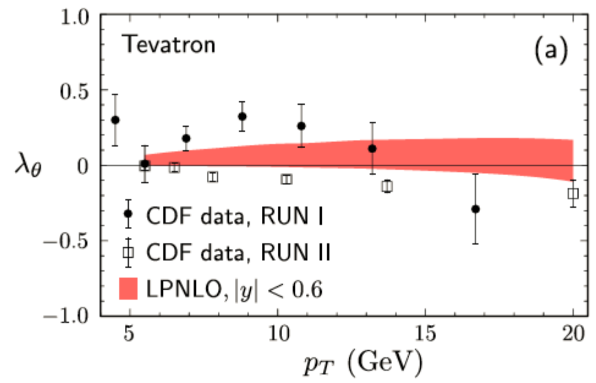
FIG. 4 (color online). Prompt polarizations as functions of p_T: (a) J/ψ and (b) $\psi(2S)$. The band (line) is the prediction from NRQCD [4] (the k_T-factorization model [9]).

Polarization puzzle at NLO

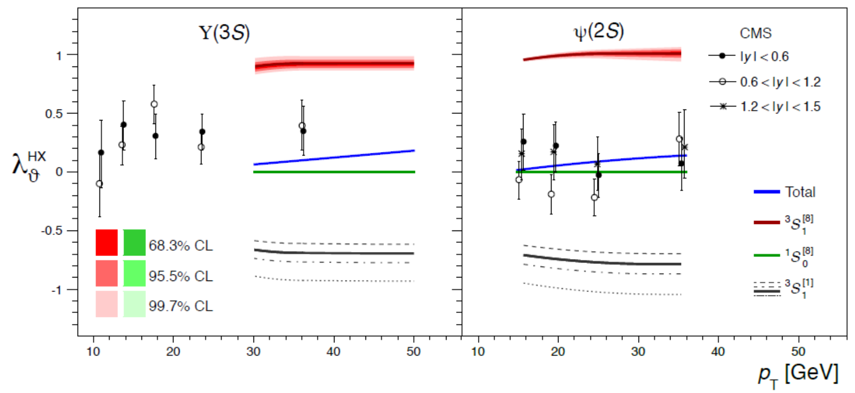
➢ J/ψ: transverse canceled (results in hierarchy) in ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$



Chao,YQM,Shao,Wang, Zhang,1201.2675

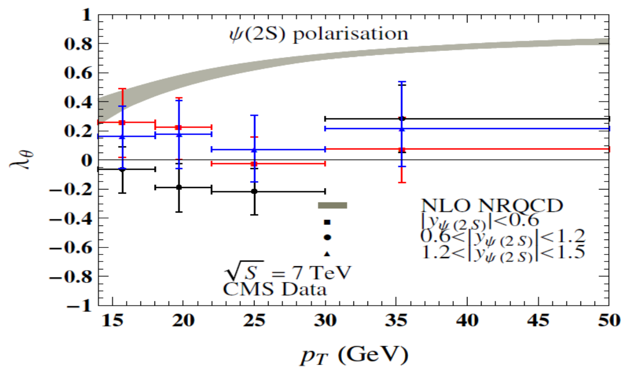


Bodwin, Chung, Kim, Lee, 1403.3612

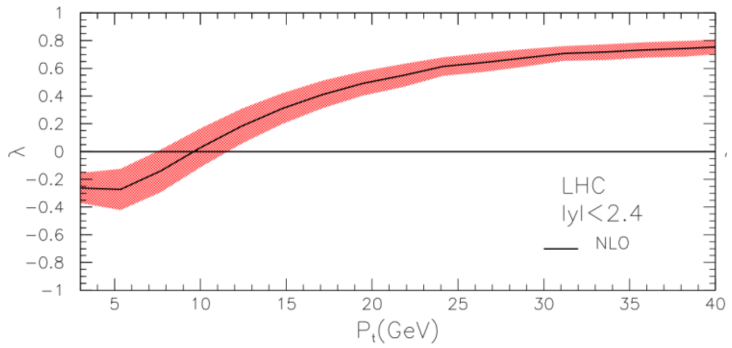


Faccioli,Knunz,Lourenco, Seixas,Wohri,1403.3970

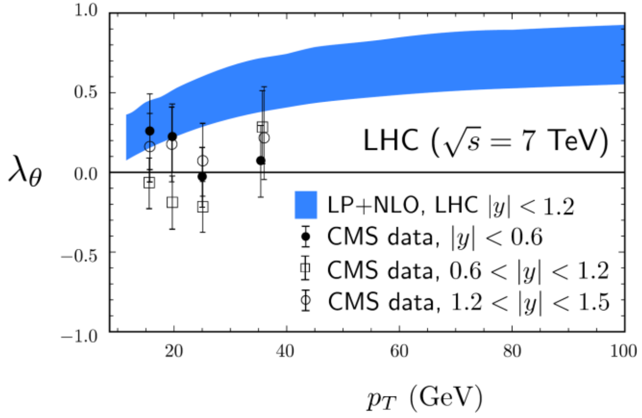
➢ $\psi(2S)$: cancelation weak, **hard to understand data**



Shao, Han, YQM, Meng, Zhang, Chao, 1411.3300



Gong, Wan, Wang, Zhang, 1205.6682



Bodwin et al., 1509.07904

3. η_c production

➢ **Heavy quark spin symmetry (HQSS)**

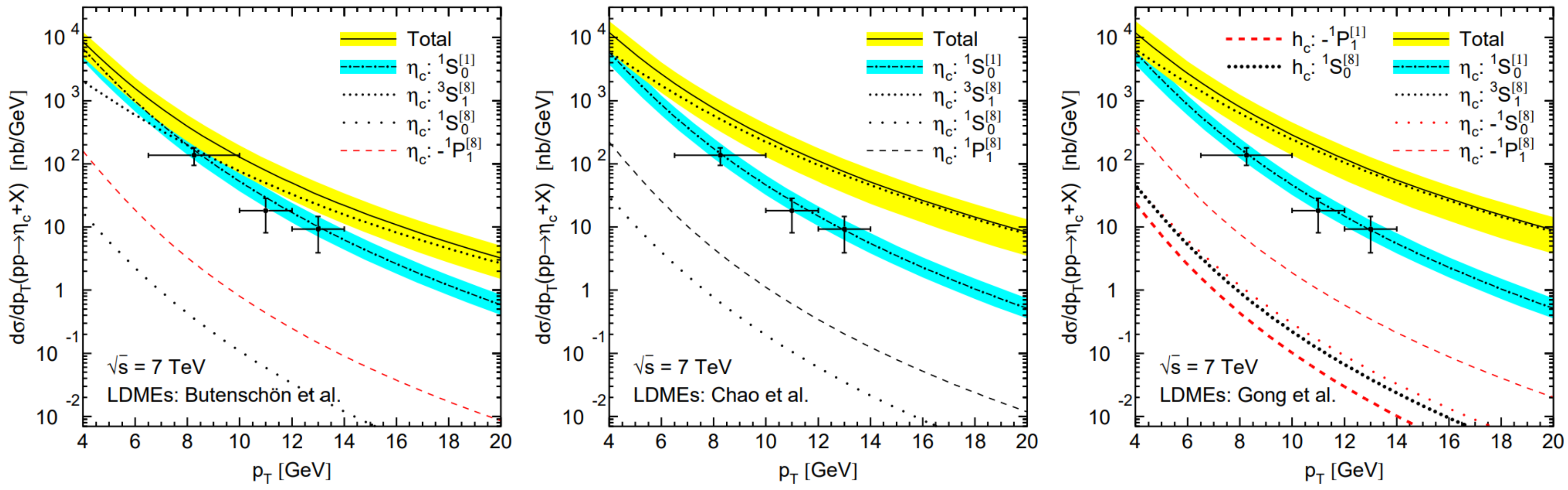
$$\left\langle \mathcal{O}^{\eta_c}\left({}^3S_1^{[8]}\right)\right\rangle = \left\langle \mathcal{O}^{J/\psi}\left({}^1S_0^{[8]}\right)\right\rangle,$$

$$\left\langle \mathcal{O}^{\eta_c}\left({}^1S_0^{[8]}\right)\right\rangle = \frac{1}{3}\left\langle \mathcal{O}^{J/\psi}\left({}^3S_1^{[8]}\right)\right\rangle,$$

$$\left\langle \mathcal{O}^{\eta_c}\left({}^1P_1^{[8]}\right)\right\rangle = \frac{3}{2J+1}\left\langle \mathcal{O}^{J/\psi}\left({}^3P_J^{[8]}\right)\right\rangle.$$

- Using the J/ψ LDMEs extracted by various groups, NLO NRQCD predictions greatly overshoot the LHCb data

Butenschoen, He, Kniehl,1411.5287



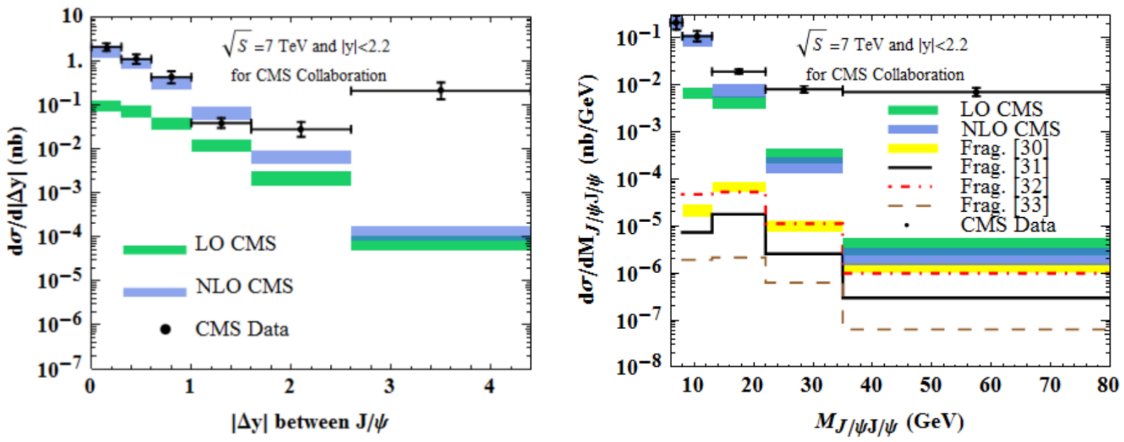
- Possible solutions: large cancelation between S-wave and P-wave (results in hierarchy)

Han, YQM, Meng, Shao, Chao, 1411.7350
Zhang, Sun, Sang, Li, 1412.0508

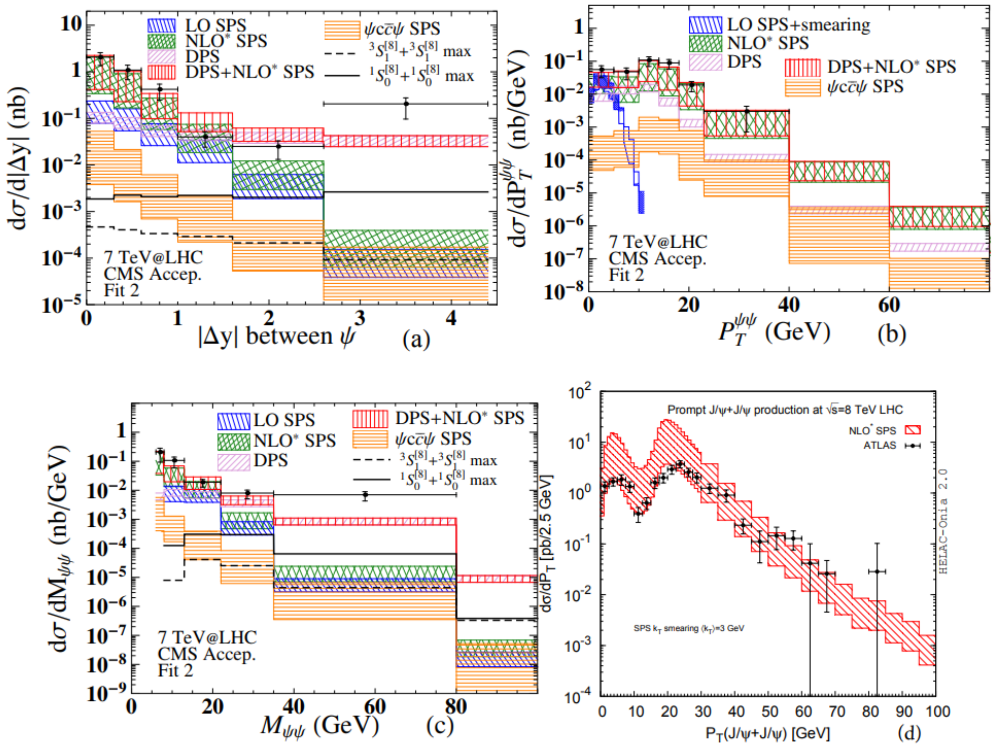
4. Double J/ψ production

➢ Cannot explain data

- 3 orders of discrepancy between data and single-parton scattering
- 1 order discrepancy still exist after including double-parton scattering
- What is missing?



Sun, Han, Chao, 1404.4042



Lansberg, Shao, 1410.8822

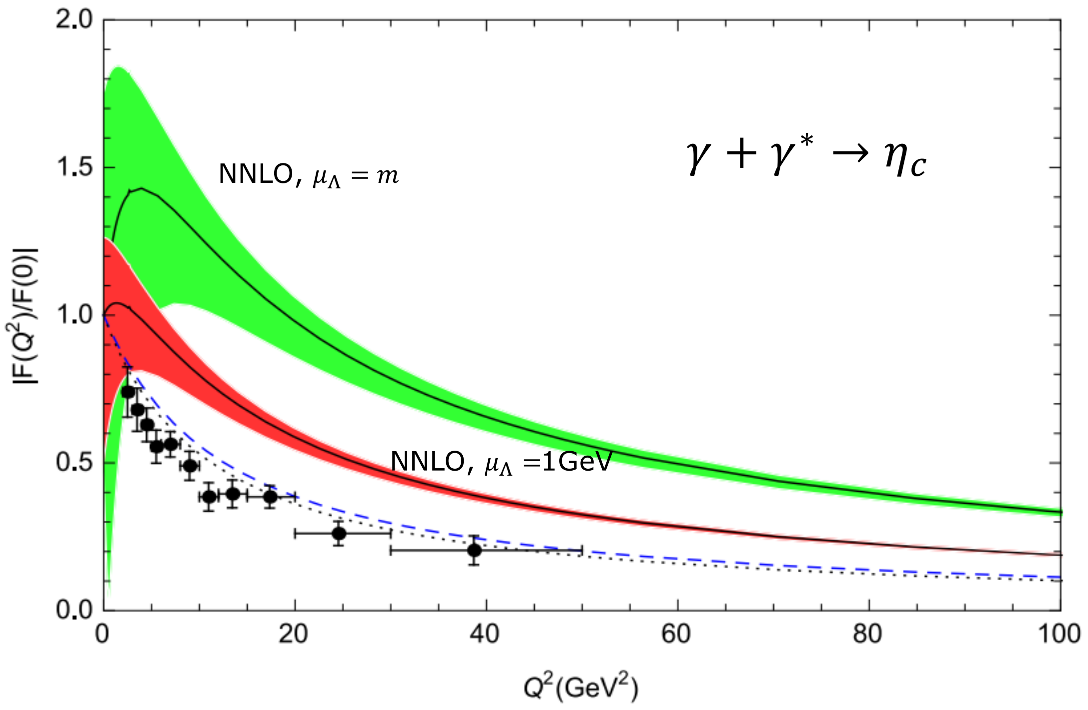
5. Beyond NLO

- **Very big high order correction!**

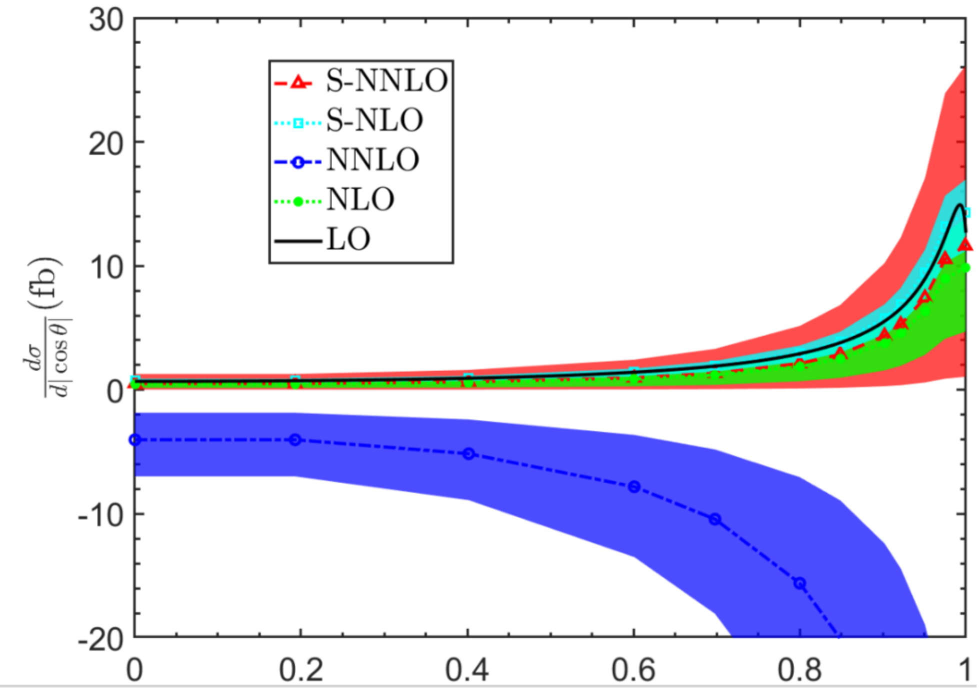
See also Jian-Xiong Wang's talk

- **Higher orders can fail to describe exclusive data**
- **Breaking down of perturbation theory? Or other mechanism?**

Feng, Jia, Sang, 1505.02665



Huang, Gong, Niu, Yu, Wang, 2311.04751



Outline

1. Heavy quark spin symmetry broken?

- **With finite quark mass, the symmetry is broken**
  - But how large of the breaking effect? $O(v^2)$? Other?
  - Production involves more scales, broken effect may be large
  - Relevant to understand polarization puzzle and η_c production data

- **Experimental input**
  - $J/\psi \Leftrightarrow \eta_c$
  - $\psi(2S) \Leftrightarrow \eta_c(2S)$
  - $\chi_{cJ} \Leftrightarrow h_c$

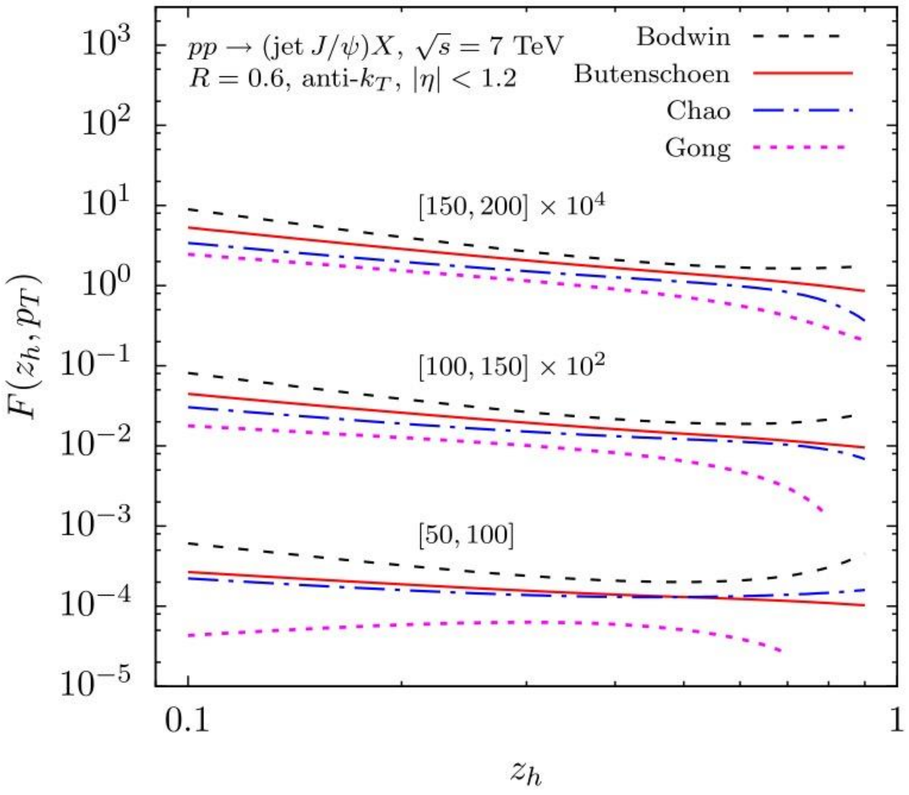
χ_{cJ}: 3 particles with 1 unknown LDME, very well described theoretically

Measure the h_c production

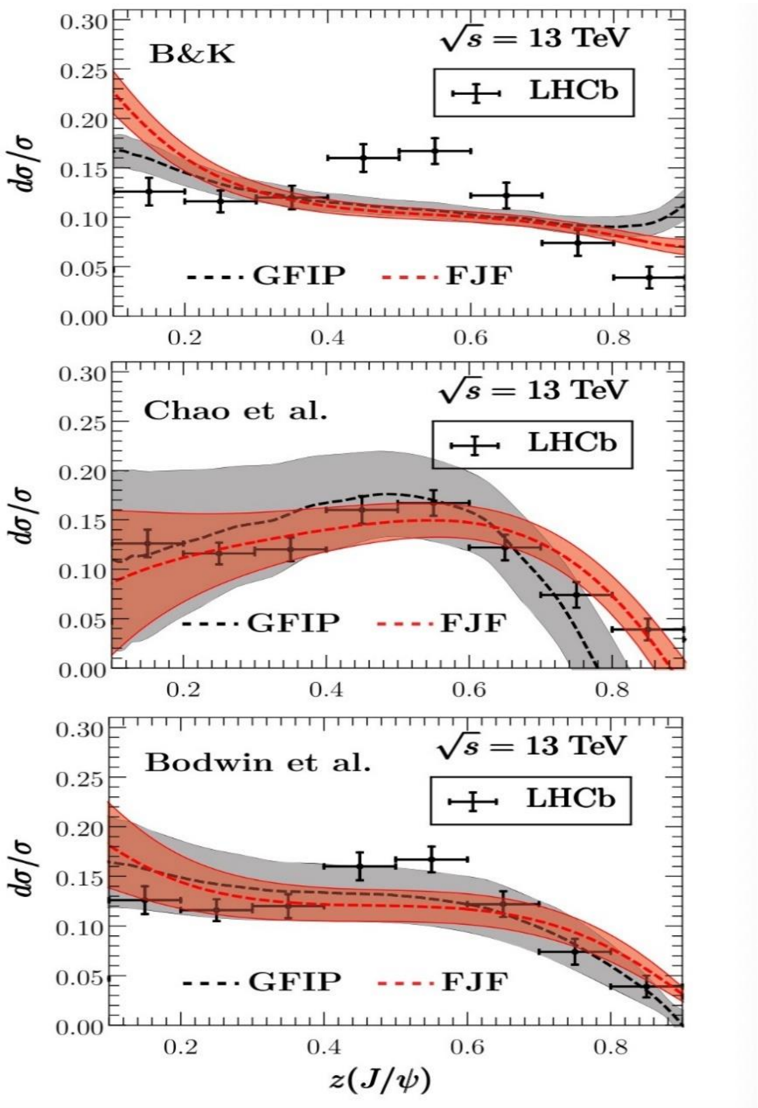
2. Observables more sensitive to production channels

➢ **Quarkonium produced in a jet**

- Sensitive to different production mechanisms



Kang, Qiu, Ringer, Xing, Zhang, 1702.03287



Bain, Dai, Leibovich, Makris, Mehen, 1702.05525

2. Observables more sensitive to production channels

- **Measure the energy emitted during hadronization**
  - Distinguish different production mechanism
  - E.g., to produce J/ψ, ${}^3S_1^{[8]}$ channel emits two gluons, while ${}^1S_0^{[8]}$ channel emits one gluon

- **Quarkonium-energy correlators**

$$\langle\Psi|\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\cdots\mathcal{E}(\vec{n}_k)|\Psi\rangle$$

  - Tag a J/ψ and measure the energy for each pixel
  - An observable under study …

Chen, Liu, YQM, to appear soon

New!

Summary

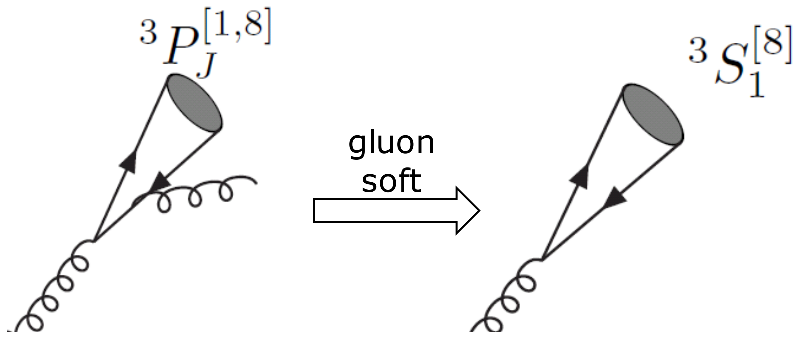
- Current difficulties: polarization puzzle, hierarchy problem, universality problem, J/ψ-pair puzzle, high-order puzzle,…
  - Very hard to understand
- Quarkonium production mechanism: a very important topic
  - New theoretical ideas needed
  - New data needed: confirm previous data; measure the spin symmetry broken effects ; measure the energy emitted during hadronization; …

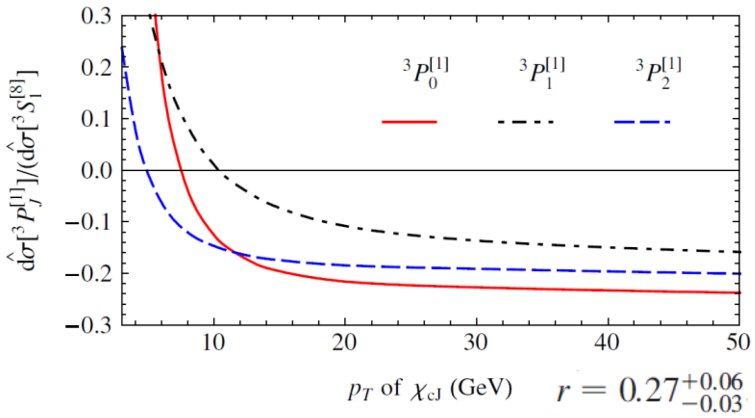
Thank you!

Over subtraction

➢ **Eg. χ_{cJ} production:** $d\sigma_{\chi_{cJ}}/(2J+1) \approx d\hat{\sigma}_{^3P_J^{[1]}}\langle O\left(^3P_0^{[1]}\right)\rangle + d\hat{\sigma}_{^3S_1^{[8]}}\langle O\left(^3S_1^{[8]}\right)\rangle$

Braaten, Chen, 9610401
YQM, Wang, Chao, 1002.3987





- Soft gluon in P-wave: factorized to S-wave matrix element
- Subtraction scheme: at ***zero momentum***, which contributes the largest production rate. Over subtracted! P-wave negative!
- Big cancellation between S-wave and P-wave! Perturbation unstable
- Solution: soft gluon momentum should be kept during subtraction process, or resum kinematic effects to all powers in v.

Threshold region

➢ At threshold region

- Large logarithms appear: can be resummed by introducing shape functions

Beneke, Rothstein, Wise, 9705286
Fleming, Leibovich, Mehen, 0306139
Leibovich, Liu, 0705.3230

- Soft gluon momentum: has leading contribution for quarkonium momentum distribution, cannot be ignored

➢ Combination of logs and powers resummation needed

- Keep soft gluon momentum unexpanded is the first step.

Comments

➢ Relativistic corrections with fixed power in v

- Bad convergence, too many terms are needed
- Involves too many LDMEs, very hard to fix them
- Solution: resum all LDMEs to obtain a function!

  (Like resum twist-2 local operators to obtain PDFs)

➢ What do we need to resum?

- Type 0 ($\chi^\dagger\psi, \chi^\dagger\sigma^i\psi, \chi^\dagger T^a\psi, \chi^\dagger\sigma^i T^a\psi$): finite number, can be studied exclusively
- Type 1-2 insertion ($\chi^\dagger gE^i\psi$, $\chi^\dagger \overleftrightarrow{\nabla}^i\psi$): usually not enhanced, less important
- Type 3 and 4 insertion ($\chi^\dagger \overleftrightarrow{\nabla}^2\psi$, $\nabla^i(\chi^\dagger\psi)$) : kinematic effects, enhanced if the observable has a steep distribution. E.g., p_T distribution in pp collision, momentum distribution in endpoint region.

Preliminary applications

➢ Application to $e^+e^- \to J/\psi({}^3P_J^{[8]}, {}^1S_0^{[8]}) + X$

- **Partonic differential cross sections**

Chen, Jin, Ma, Meng, 2201.04492

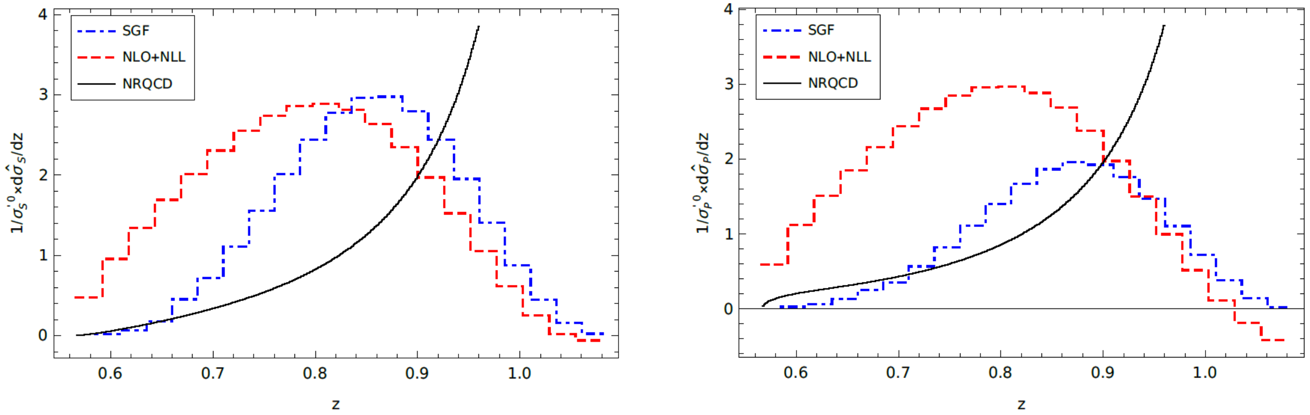


Figure 4. The differential cross sections in SGF and NRQCD factorization approaches.

- **Smaller partonic cross section, larger LDMEs allowed**

$$M_k^X = \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle + k\frac{\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]})\rangle}{m_c^2}$$

$$M_{3.9}^{\rm NRQCD} < (2.4 \pm 0.7) \times 10^{-2}\,{\rm GeV}^3,$$
$$M_{3.9}^{\rm NLO+NLL} < (5.8 \pm 1.8) \times 10^{-2}\,{\rm GeV}^3,$$
$$M_{2.5}^{\rm SGF} < (7.2 \pm 2.2) \times 10^{-2}\,{\rm GeV}^3.$$

- **LDMEs in e^+e^- can be consistent with that extracted in pp**

$$pp:\ M_0 = \langle O({}^1S_0^{[8]})\rangle + 3.9\langle O({}^3P_0^{[8]})\rangle/m_c^2 \approx (7.4 \pm 1.9)\times 10^{-2}\,\mathbf{GeV}^3$$