

# Di- $J/\psi$ photoproduction at the EIC (...and DPS off light-nuclei)

Matteo Rinaldi


INFN sezione di Perugia






# Outline

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 Introduction to double parton scattering (DPS) and hadronic Physics

 Data and interpretation

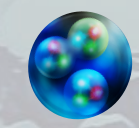
 DPS at the EIC?

 Nuclear DPS at the EIC?

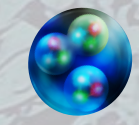


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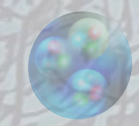
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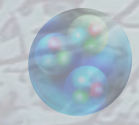
Introduction to double parton scattering (DPS) and hadronic Physics



Data and interpretation



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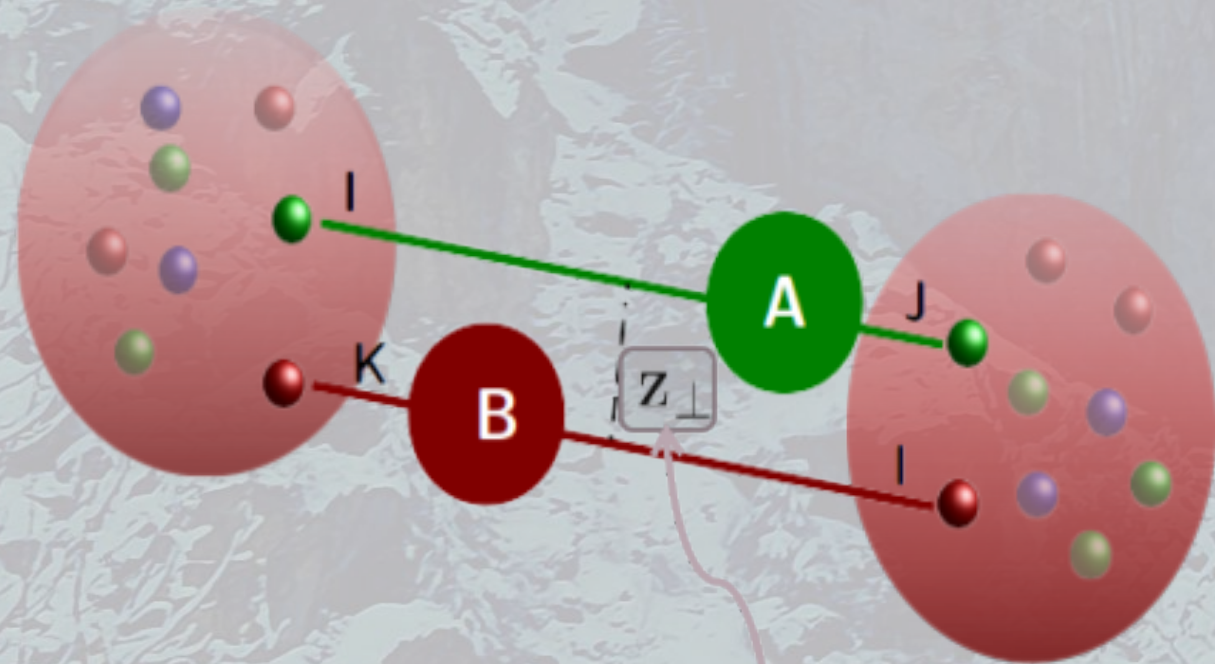
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# Double Parton Scattering

Multiparton interaction (MPI) can contribute to the, pp, pA and AA cross section @ the LHC:



Transverse distance between two partons

$$d\sigma \propto \int d^2z_{\perp} \underbrace{F_{ij}(x_1, x_2, z_{\perp}, \mu_A, \mu_B) F_{kl}(x_3, x_4, z_{\perp}, \mu_A, \mu_B)}$$

Double Parton Distribution (DPD)

N. Paver and D. Treleani, *Nuovo Cimento* 70A, 215 (1982)

Mekhfi, *PRD* 32 (1985) 2371

M. Diehl et al, *JHEP* 03 (2012) 089

$$F_{ij}^{\lambda_1, \lambda_2}(x_1, x_2, \vec{k}_{\perp}) = (-8\pi P^+) \frac{1}{2} \sum_{\lambda} \int d\vec{z}_{\perp} e^{i\vec{z}_{\perp} \cdot \vec{k}_{\perp}} \\ \times \int \left[ \prod_l^3 \frac{dz_l^-}{4\pi} \right] e^{ix_1 P^+ z_1^- / 2} e^{ix_2 P^+ z_2^- / 2} e^{-ix_1 P^+ z_3^- / 2} \\ \times \langle \lambda, \vec{P} = \vec{0} | \hat{O}_i^1 \left( z_1^- \frac{\vec{n}}{2}, z_3^- \frac{\vec{n}}{2} + \vec{z}_{\perp} \right) \hat{O}_j^2 \left( z_2^- \frac{\vec{n}}{2} + \vec{z}_{\perp}, 0 \right) | \vec{P} = \vec{0}, \lambda \rangle$$

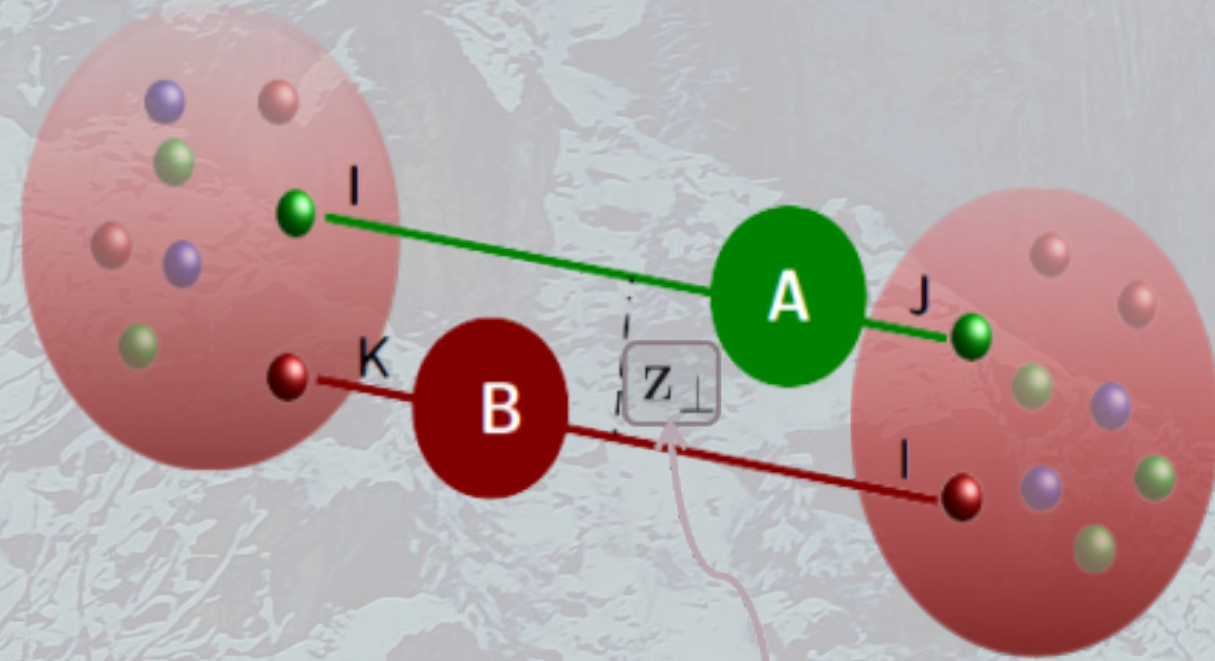
$$\hat{O}_i^k(z, z') = \bar{q}_i(z) \hat{O}(\lambda_k) q_i(z')$$

$$\hat{O}(\lambda_k) = \frac{\not{n}}{2} \frac{1 + \lambda_k \gamma_5}{2} .$$



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Mekhfi, *PRD* 32 (1985) 2371

M. Diehl et al, *JHEP* 03 (2012) 089

A **formal all-order proof** of the factorization formulae in perturbative QCD **has been achieved for DPS** in the case of a **colorless final state**, both for the TMD and the collinear case. Current status is at the **same level as for the SPS** counterpart.

Diehl et al. [JHEP 03 \(2012\) 089](#), [JHEP 01 \(2016\) 076](#)

Vladimirov [JHEP 04 \(2018\) 045](#)

Buffing et al. [JHEP 01 \(2018\) 044](#)

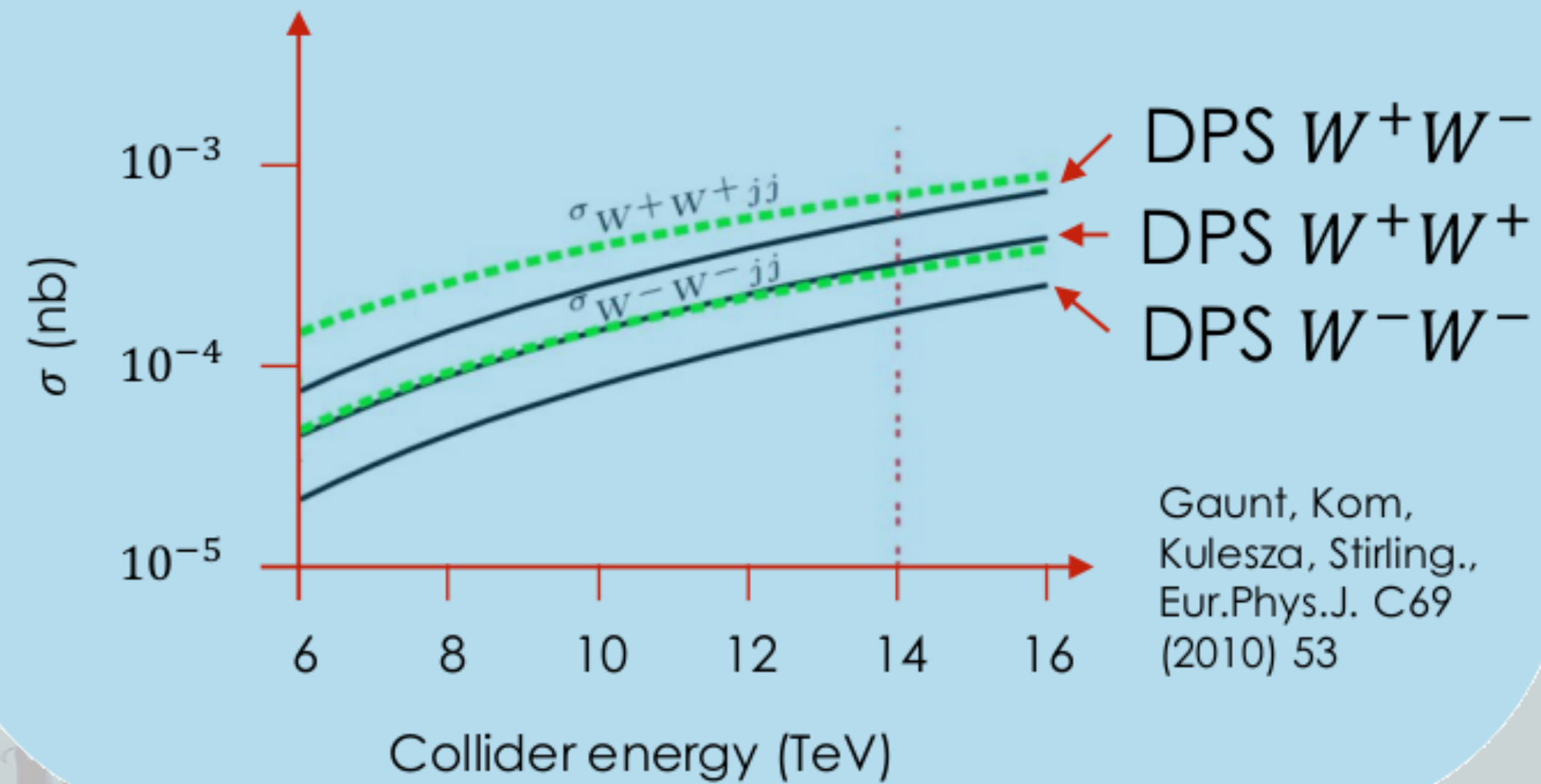
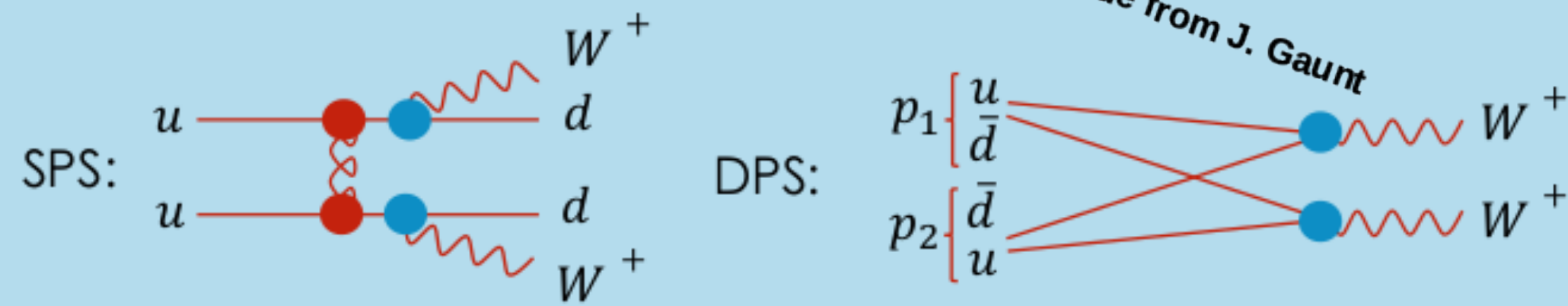
Diehl, RN [JHEP 04 \(2019\) 124](#)

R. Nagar's talk MPI 2021

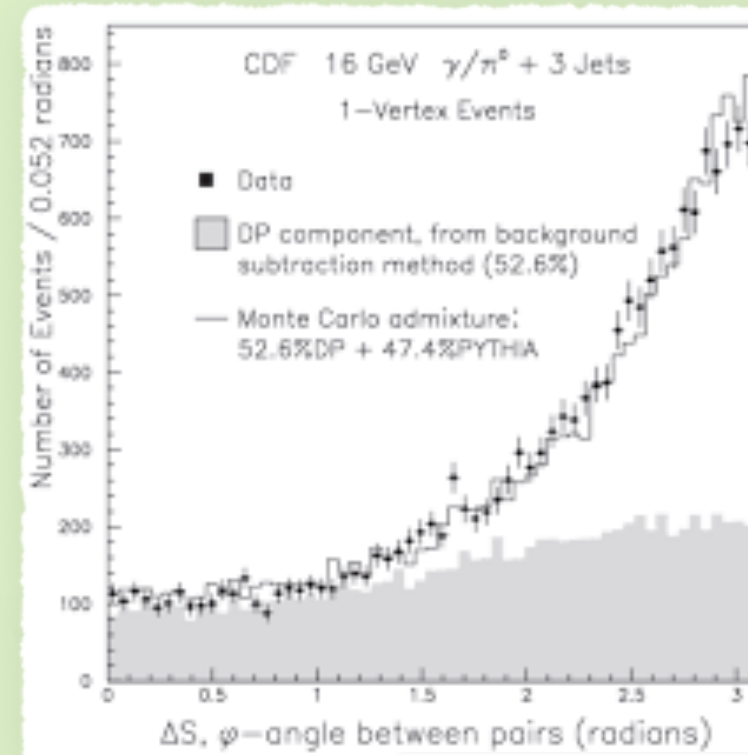


# Where and Why DPS?

DPS can give a significant contribution to processes where SPS is suppressed by small/multiple coupling constants:

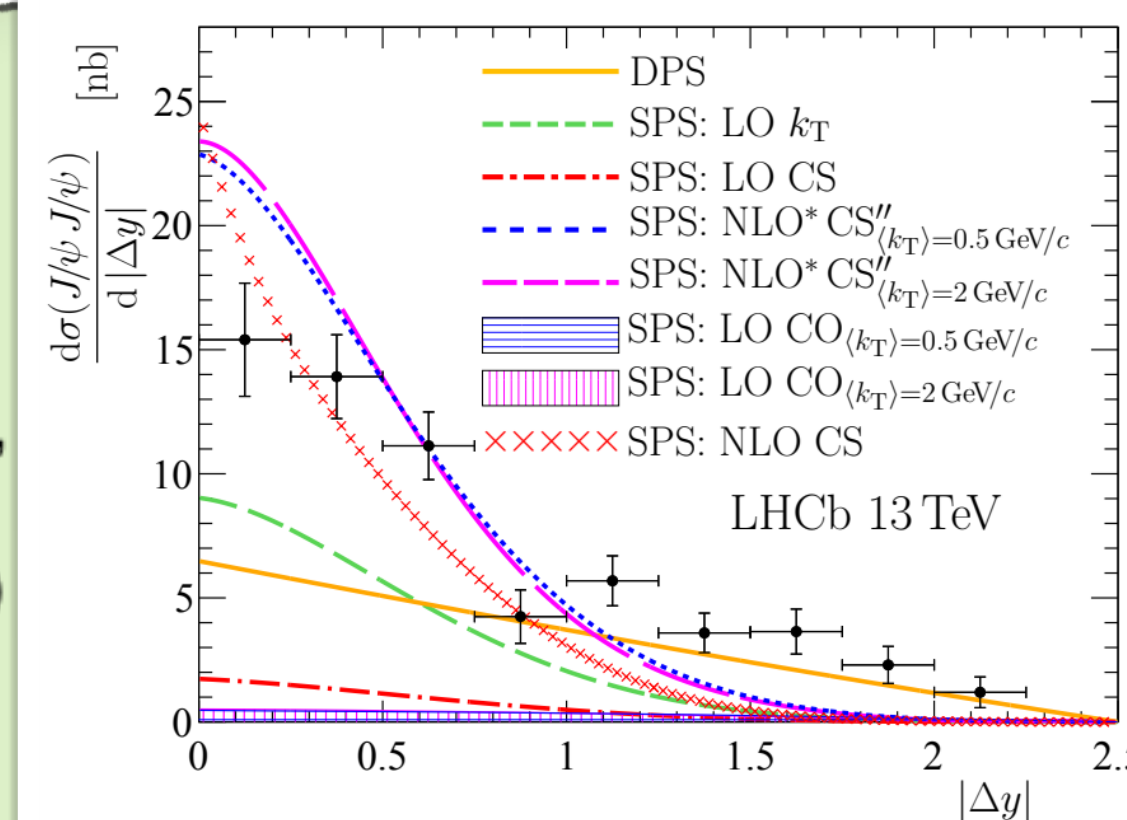


...or in certain phase space regions

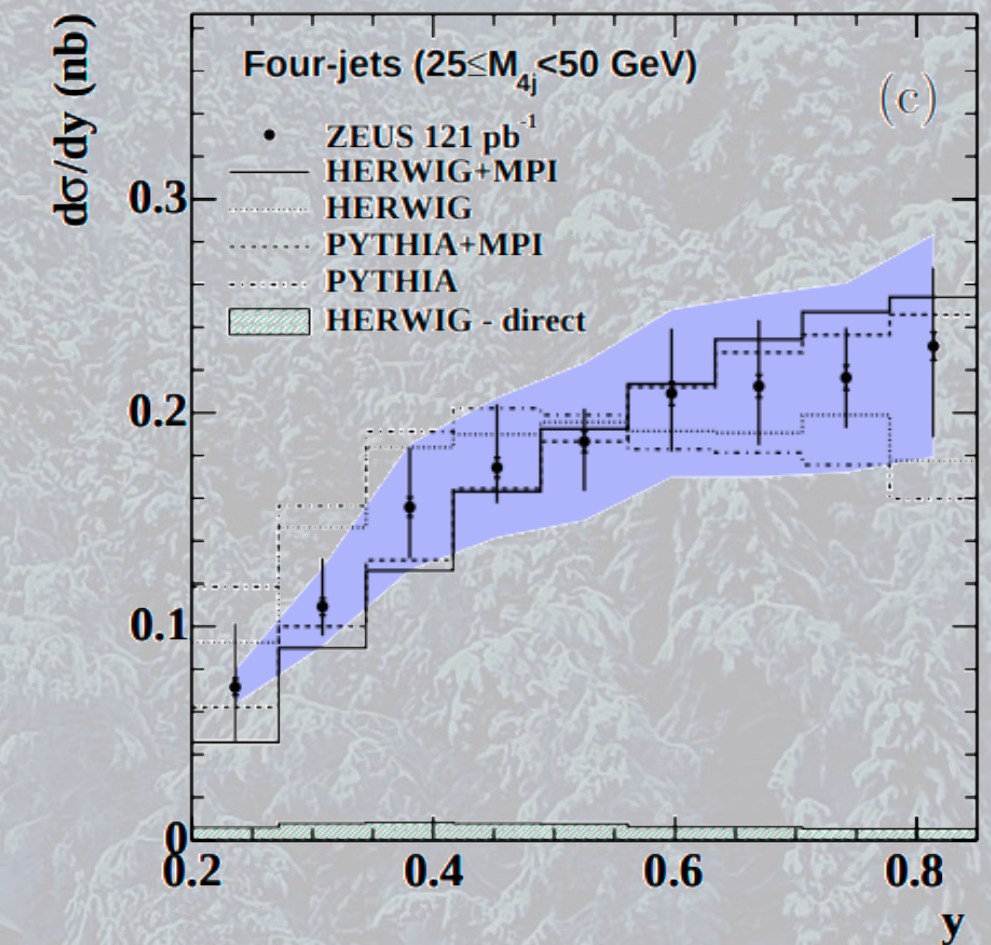


CDF,  $\gamma + 3j$ , Phys.Rev. D56 (1997) 3811-3832

LHCb, double  $J/\psi$ , JHEP 06, 047, (2017)



in ep Colliders?



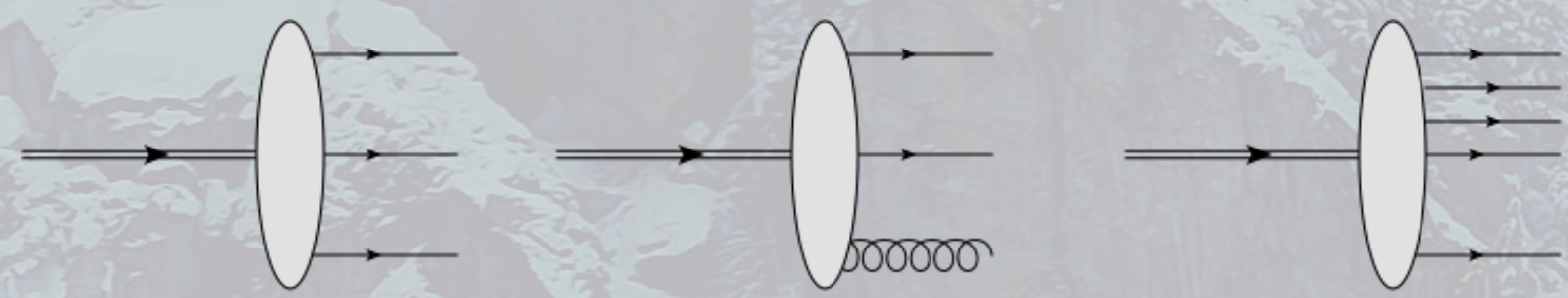
HERA data, ZEUS coll, Nucl. Phys. B 729, 1 (2008)

Access to:  
 - double parton correlations  
 - the transverse distance distribution of partons!!

all UNKNOWN

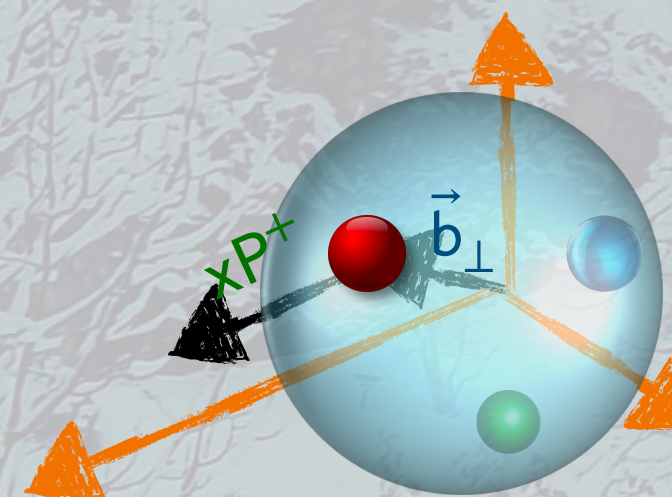


# Multidimensional picture of hadrons

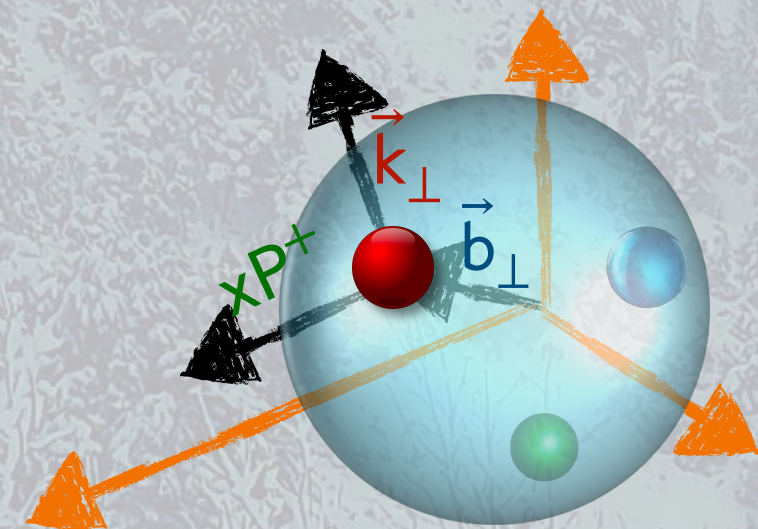


Light-Front wave-function

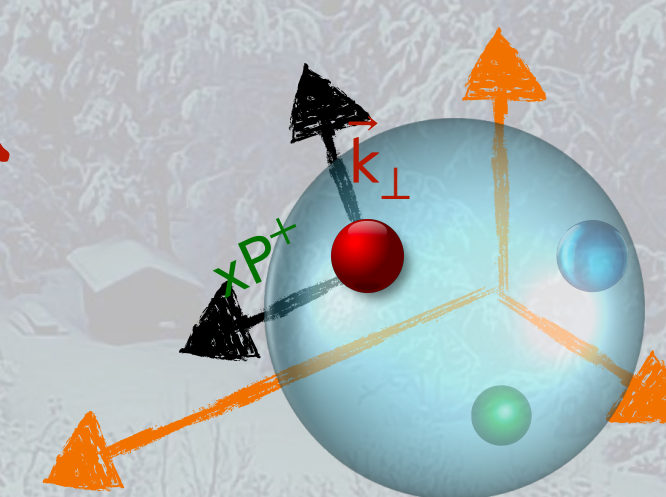
GPD in impact parameter space



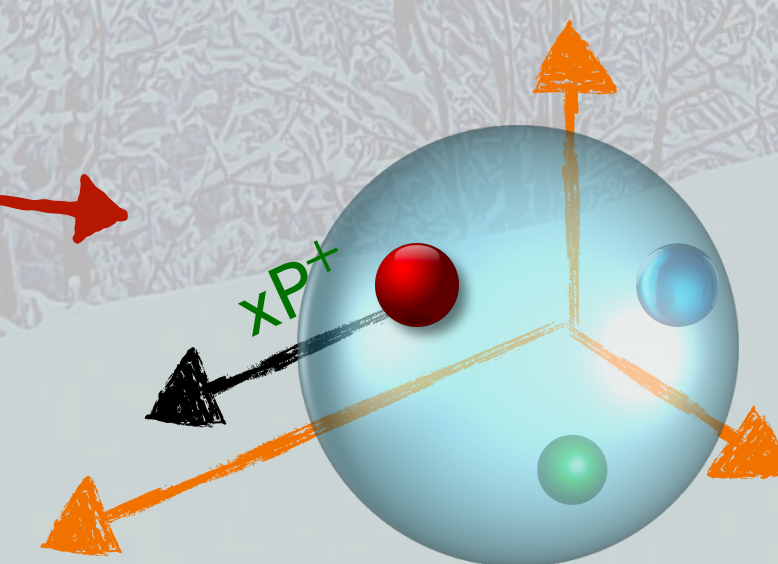
GTMD



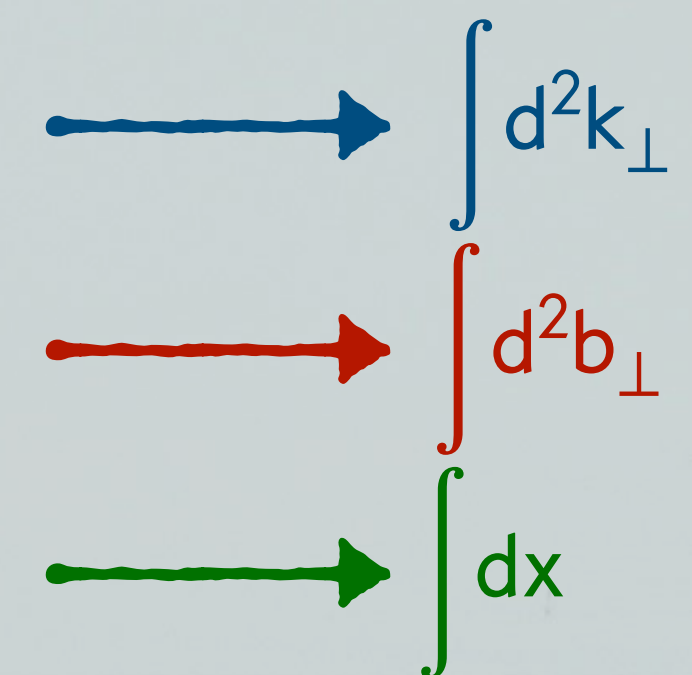
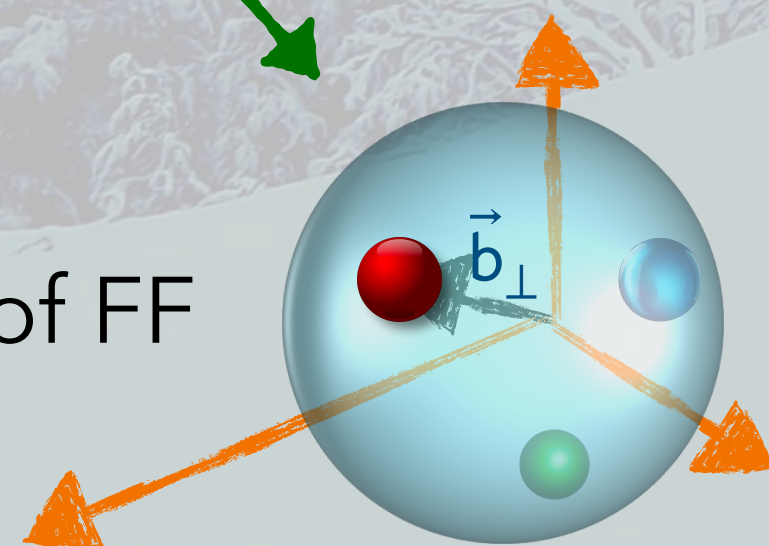
TMD



PDF

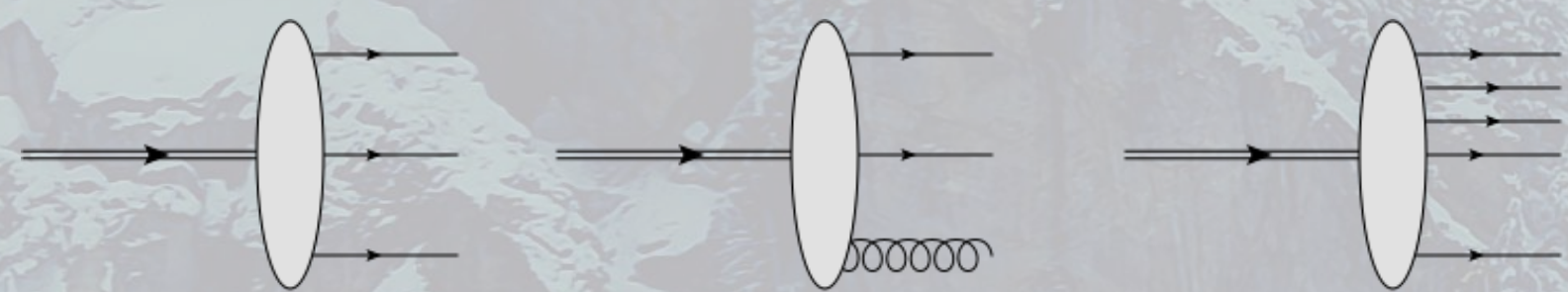


FT of FF



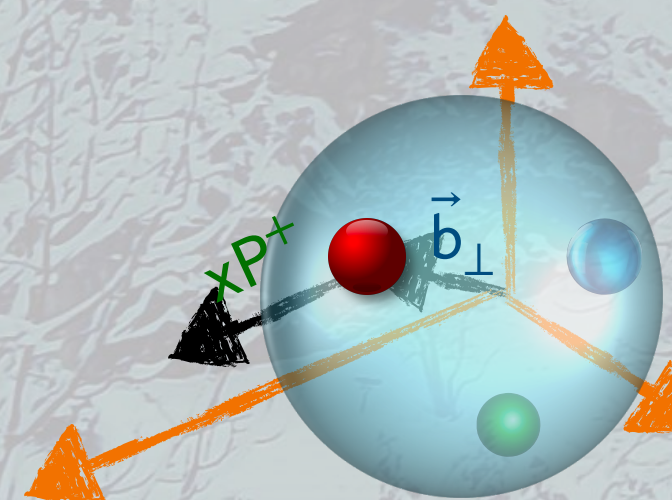


# Multidimensional picture of hadrons

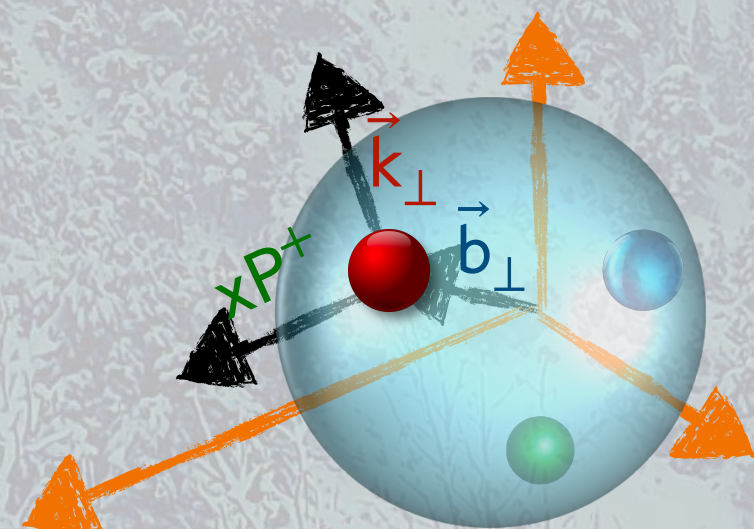


Light-Front wave-function

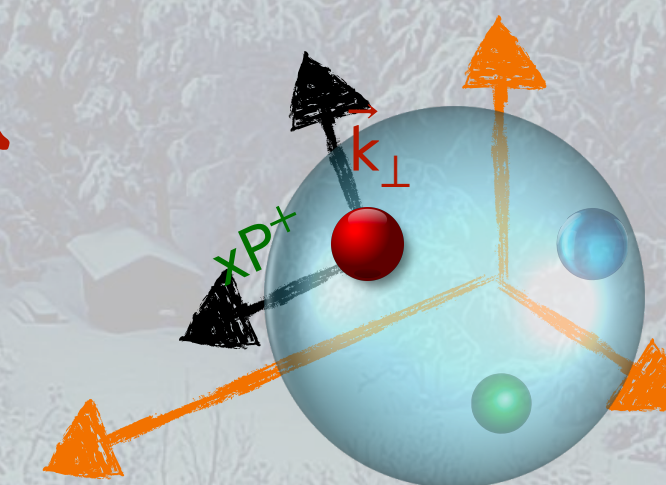
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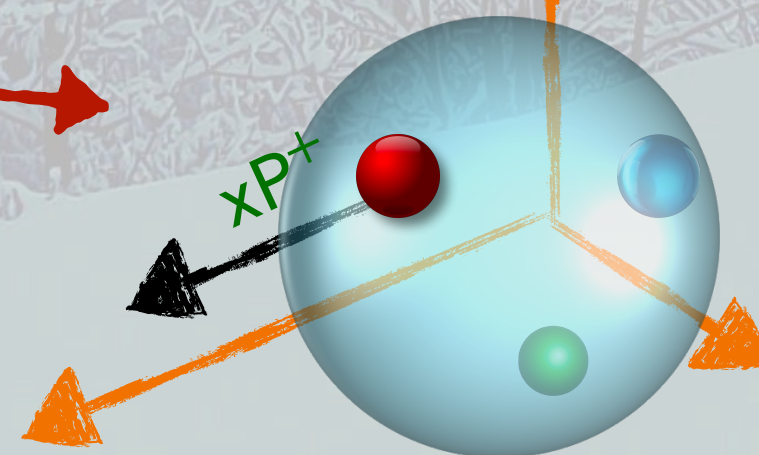
GTMD



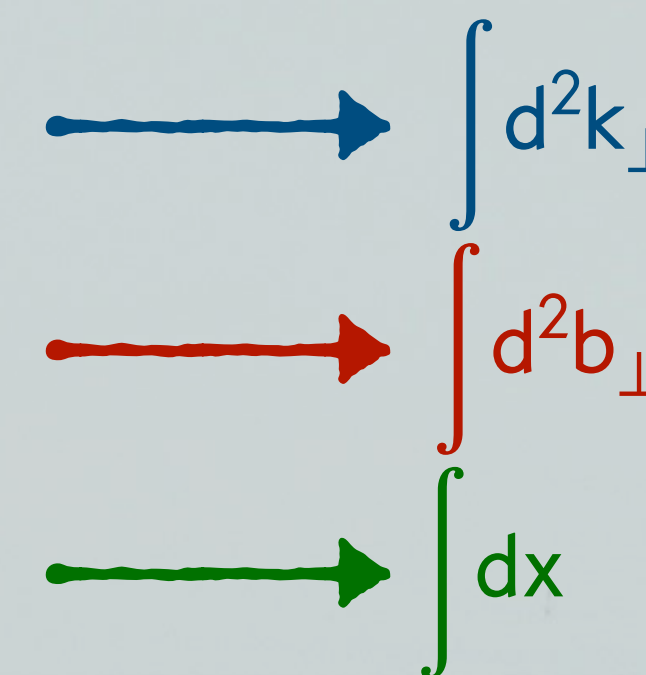
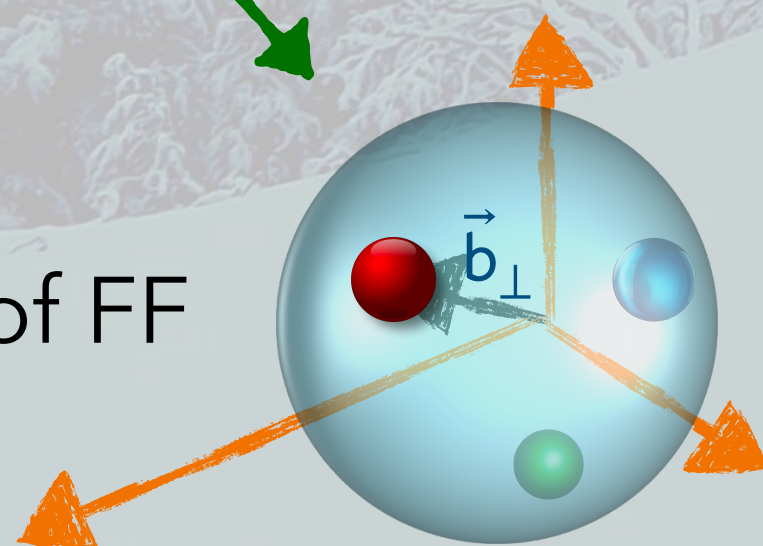
TMD



**1-body Functions!**

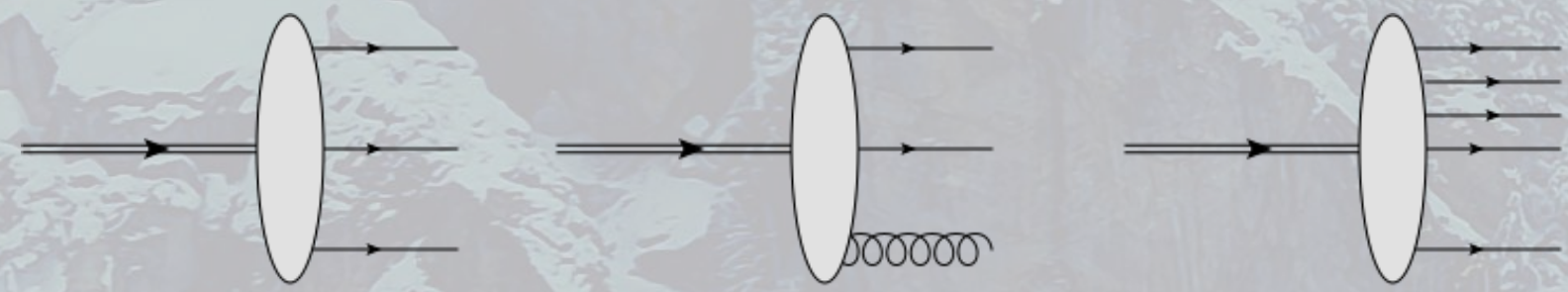


FT of FF





# Multidimensional picture of hadrons



Light-Front wave-function

$\int d^2z_{\perp}$

**2-body Function!**

GPD in impact parameter space

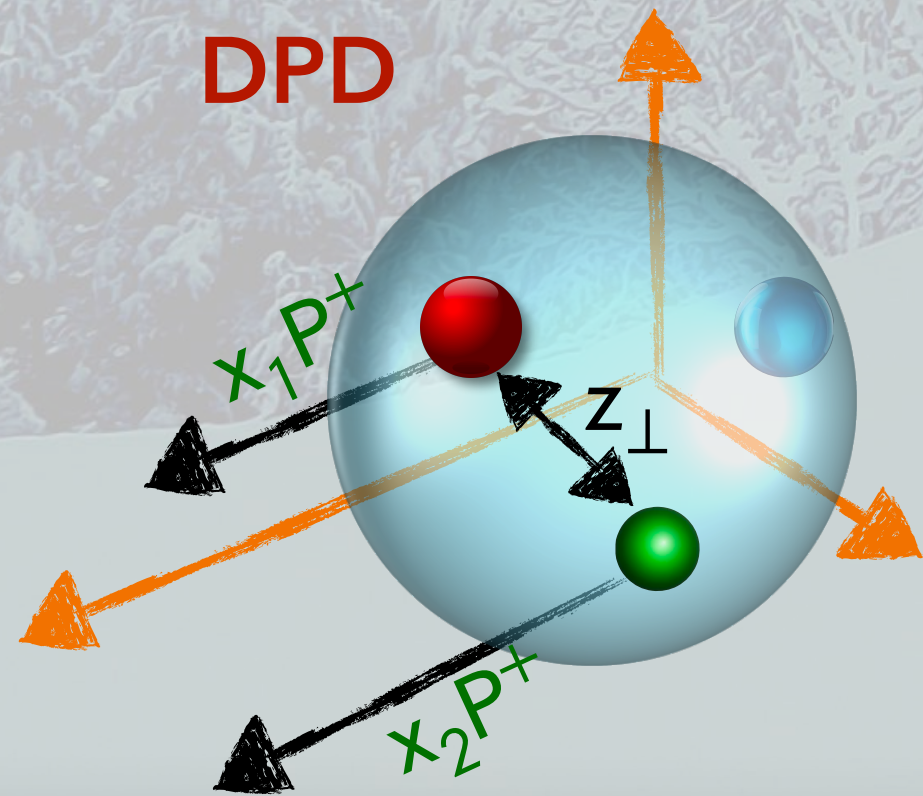
GTMD

TMD

FT of FF

PDF

DPD



Sum Rules  
Gaunt et al, JHEP (2010) 03, 005

$\int d^2k_{\perp}$   
 $\int d^2b_{\perp}$   
 $\int dx$



# How to build up a DPD

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$F_{ik}(x_1, x_2, \vec{z}_\perp)$  is unknown. For phenomenology @LHC kinematics (small  $x$  and many partons produced)



# How to build up a DPD

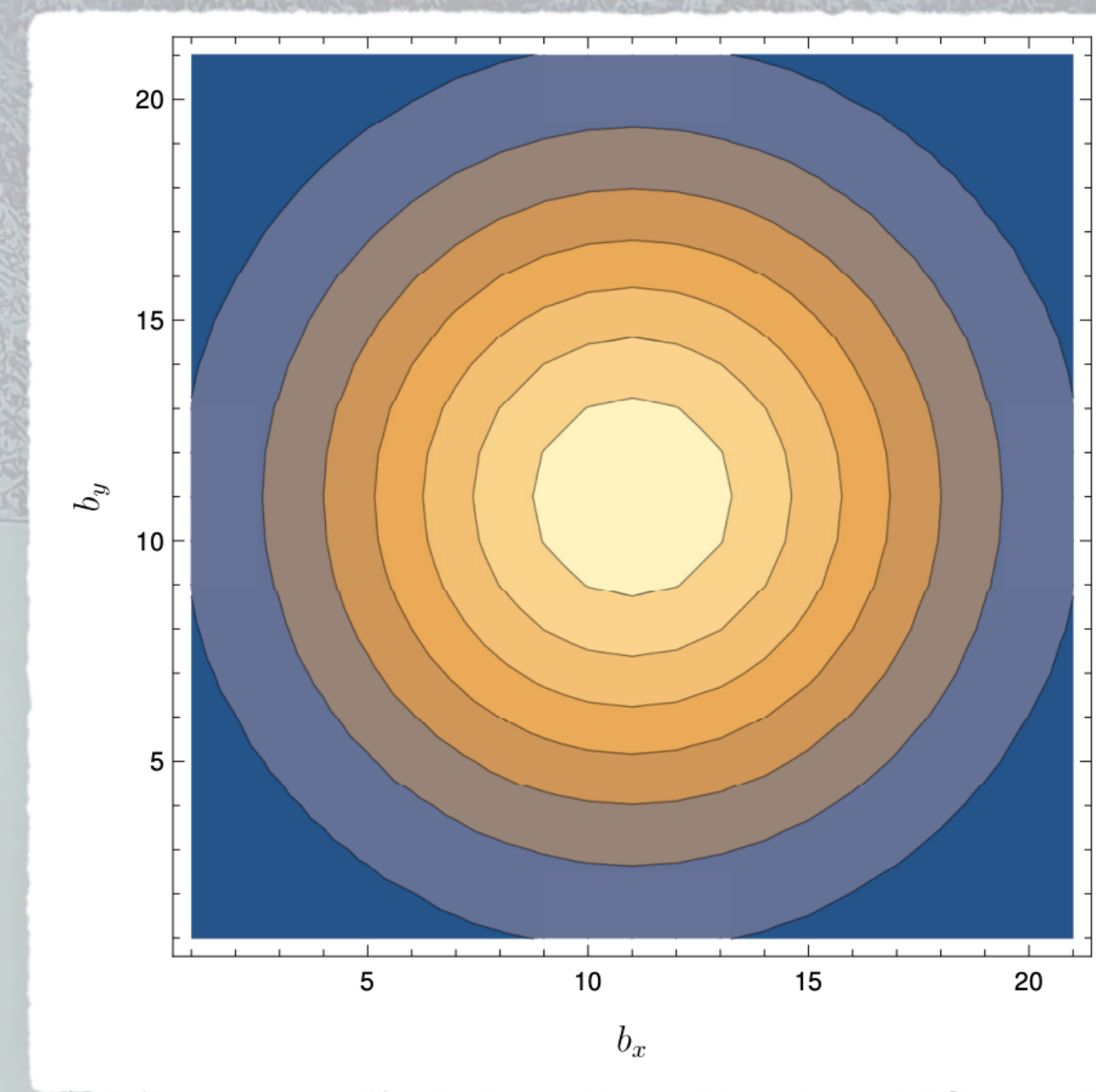
$F_{ik}(x_1, x_2, \vec{z}_\perp)$  is unknown. For phenomenology @LHC kinematics (small  $x$  and many partons produced)

$$F_{ik}(x_1, x_2, \vec{z}_\perp) \sim g(x_1, x_2) \tilde{T}(\vec{z}_\perp)$$

Models can help to grasp general features

M.R., S. Scopetta et al, PRD 87 (2013) 114021  
 M.R., S. Scopetta et al, JHEP 12 (2014) 028  
 A. V. Manohar et al, PRD 87 (2013) 3, 034009

$$\langle b_\perp^2 \rangle_{x_1, x_2}^{ij} = \frac{\int d^2 b_\perp b_\perp^2 \tilde{F}_{ij}(x_1, x_2, b_\perp, Q^2)}{\int d^2 b_\perp \tilde{F}_{ij}(x_1, x_2, b_\perp, Q^2)}$$



M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097



# How to build up a DPD

$F_{ik}(x_1, x_2, \vec{z}_\perp)$  is unknown. For phenomenology @LHC kinematics (small  $x$  and many partons produced)

uncorrelated scenario:  $F_{ik}(x_1, x_2, \vec{z}_\perp) \sim \underbrace{g(x_1, x_2)}_{\text{double PDF}} \tilde{T}(\vec{z}_\perp)$

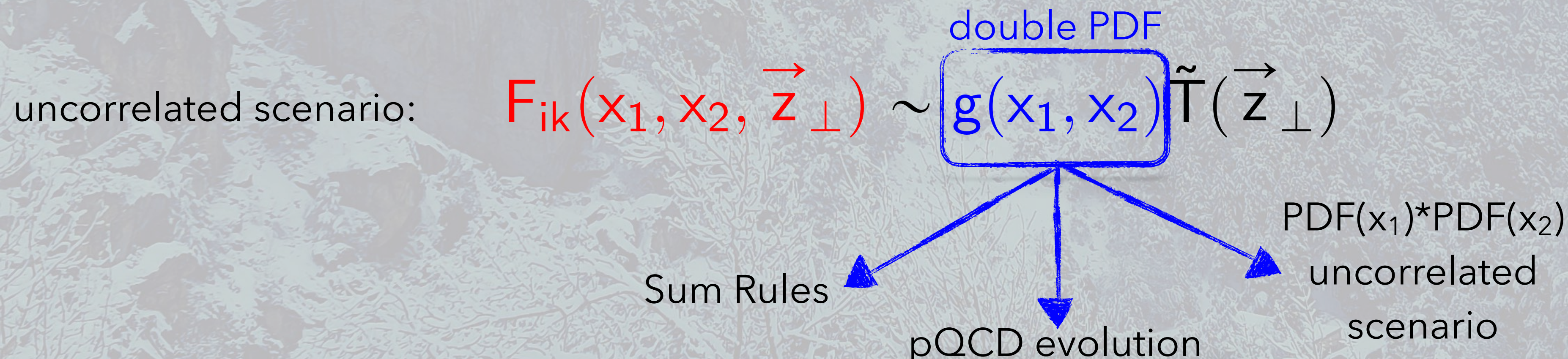
Sum Rules  $\swarrow$   $\searrow$  PDF( $x_1$ )\*PDF( $x_2$ )  
 pQCD evolution  $\downarrow$  uncorrelated scenario

$$\begin{aligned}
 & \frac{\alpha_s(t)\Delta t}{2\pi} P_{j' \rightarrow j_1 j_2} \left( \frac{x_1}{x_1+x_2}, \frac{\delta x_1}{x_1+x_2} \right) \\
 & + \sum_{j'} \int_0^1 dx_2 D_h^{j'}(x_1+x_2; t) \delta x_2
 \end{aligned}$$

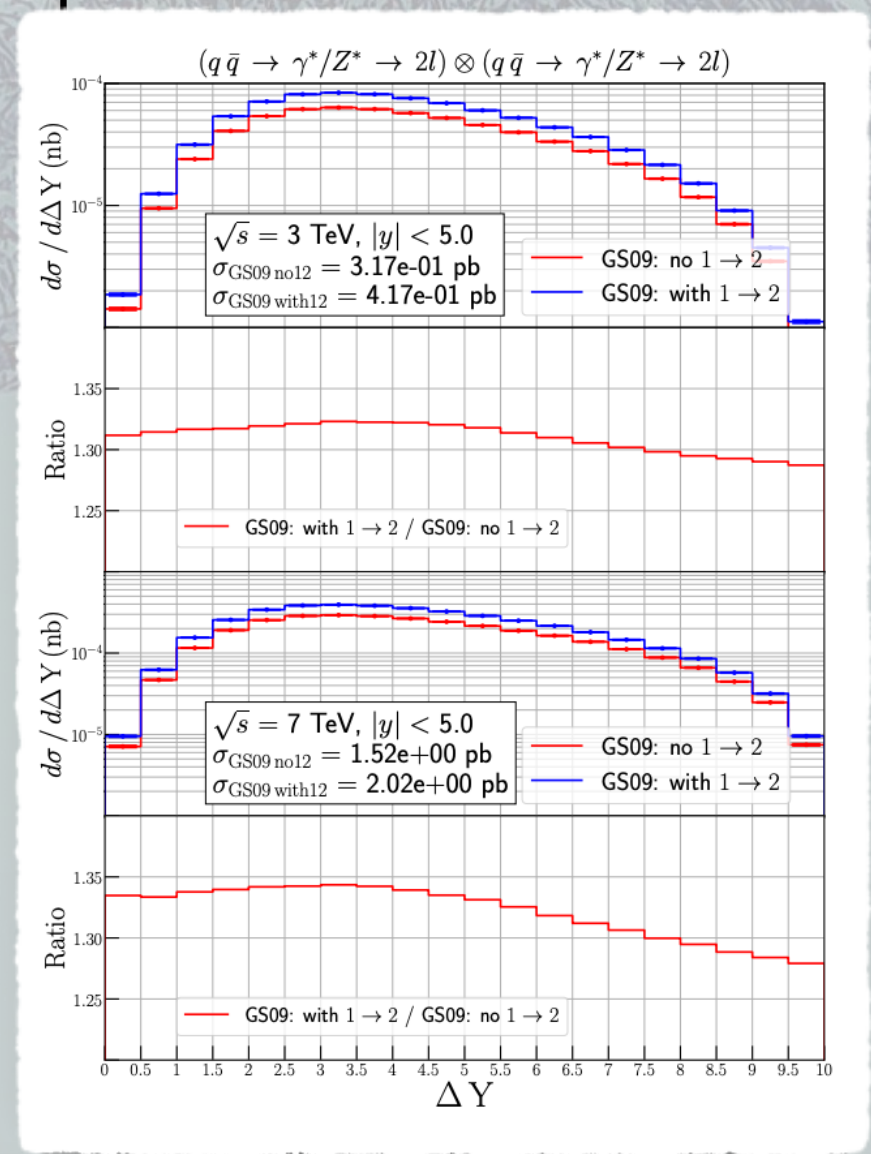
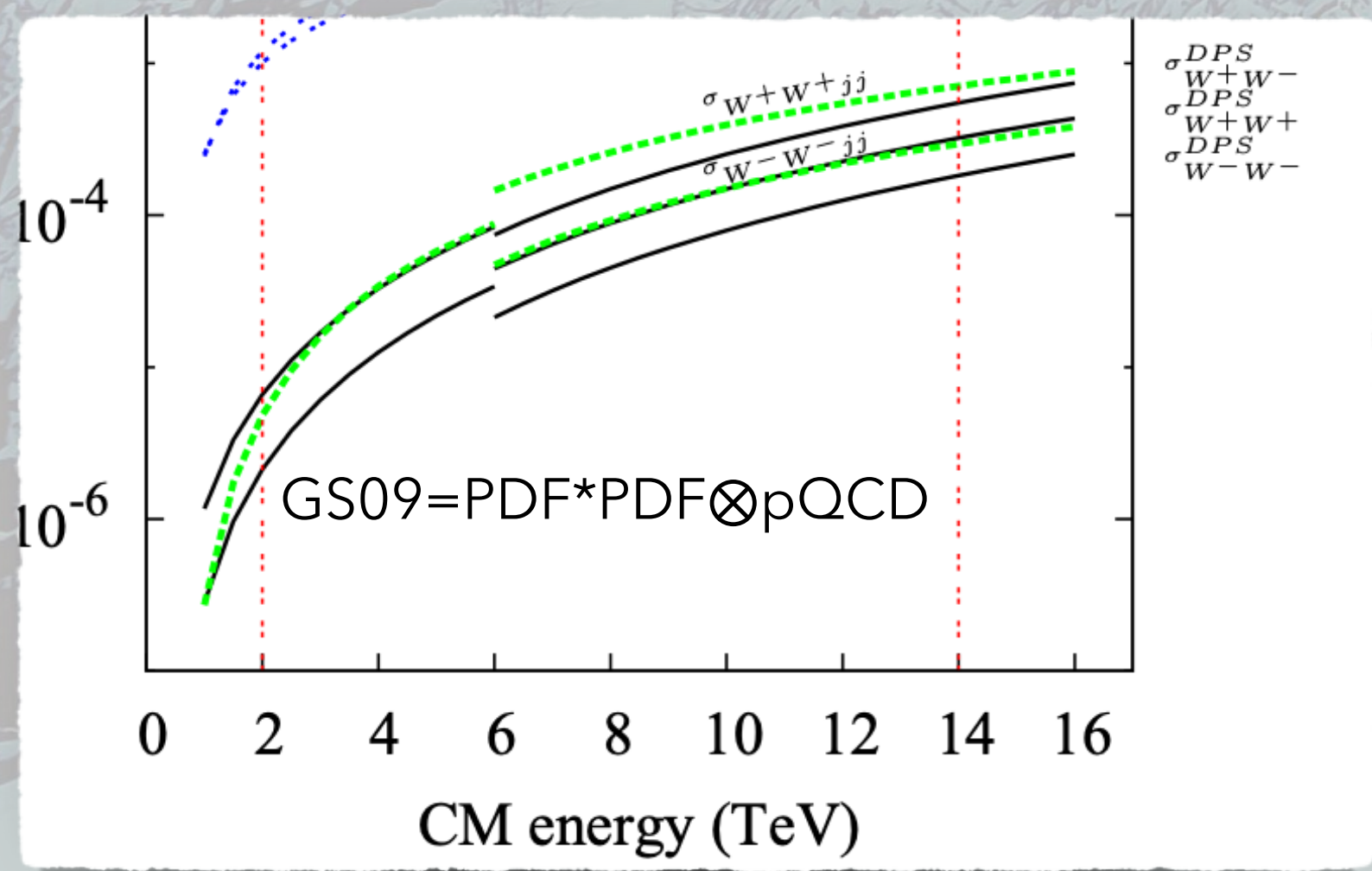


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$F_{ik}(x_1, x_2, \vec{z}_\perp)$  is unknown. For phenomenology @LHC kinematics (small x and many partons produced)



J. R. Gaunt et al, EPJC 69 (2010) 54-65

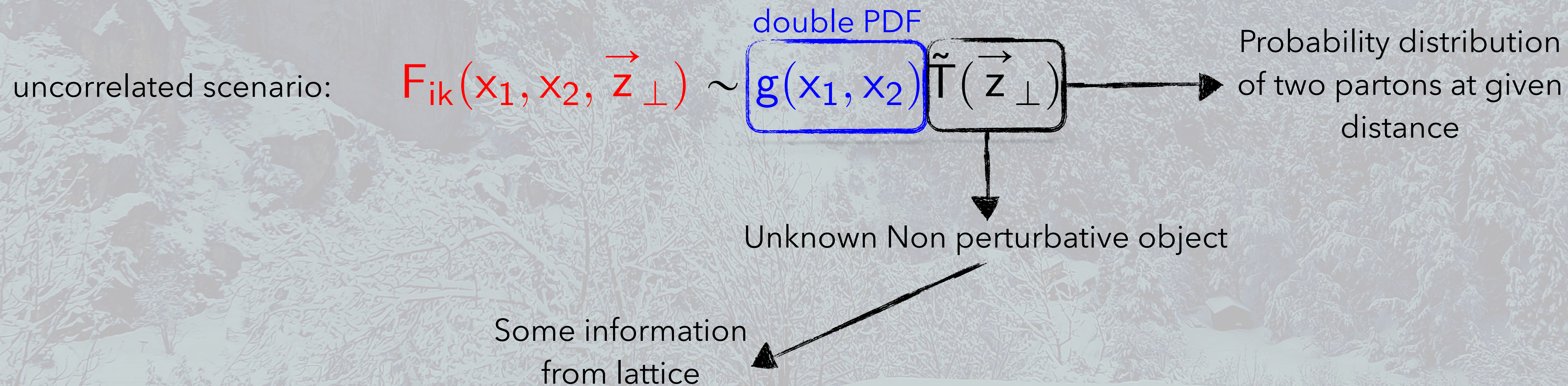


O. Fedkevych, J. R. Gaunt, JHEP 02 (2023) 090

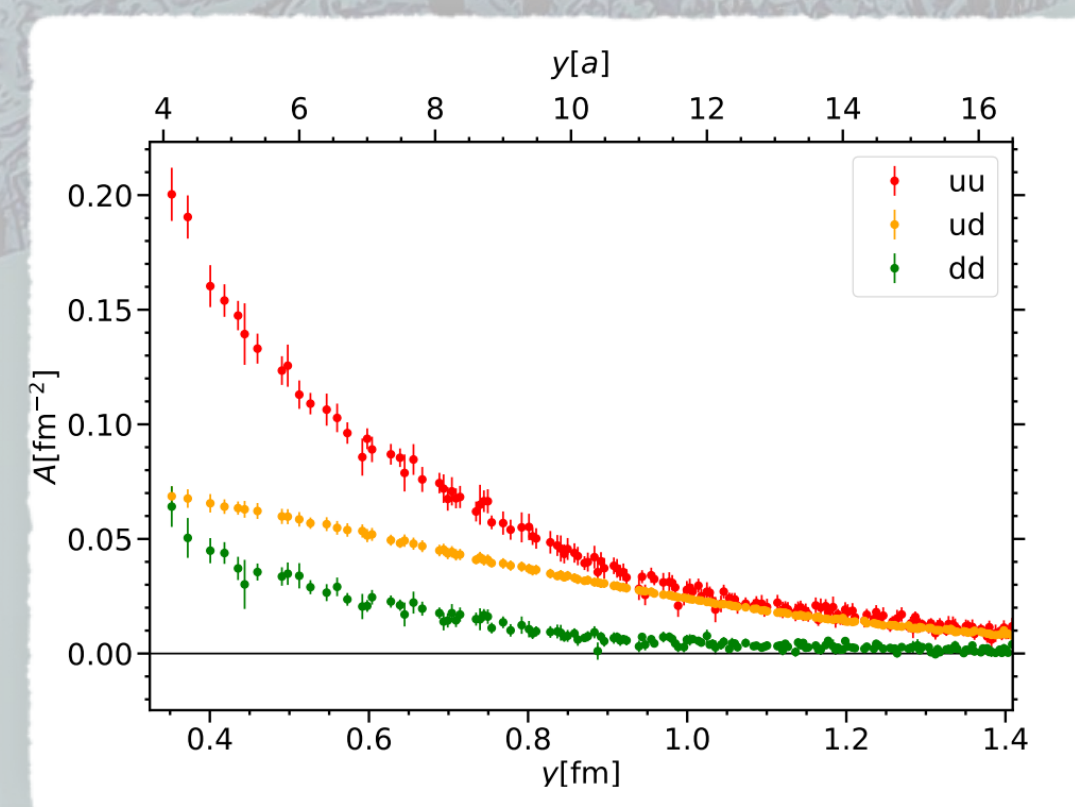


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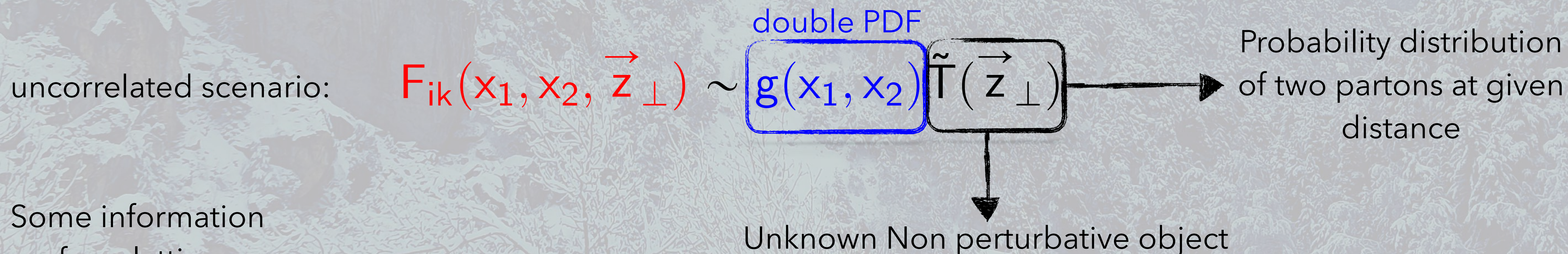
G. S. Bali et al, JHEP 09 (2021) 121



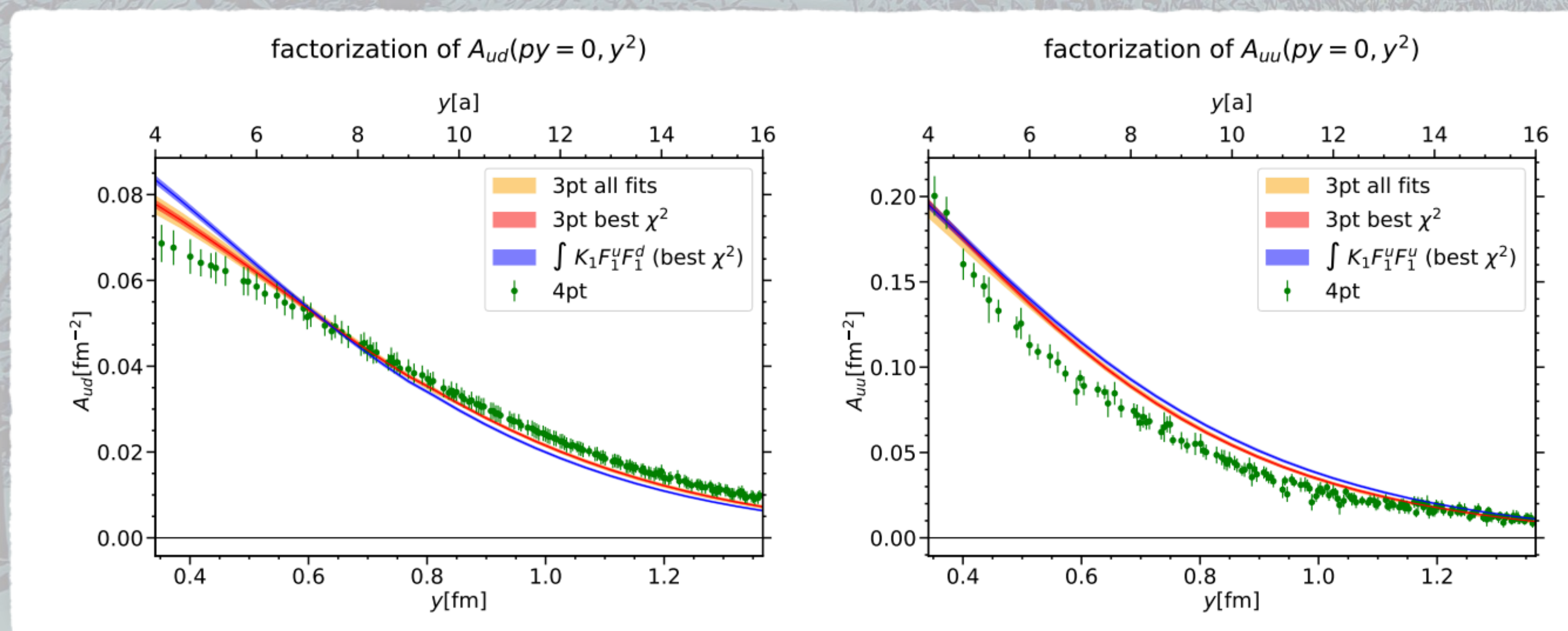


# How to build up a DPD

$F_{ik}(x_1, x_2, \vec{z}_\perp)$  is unknown. For phenomenology @LHC kinematics (small x and many partons produced)



Some information from lattice



Comparison from  $\tilde{T}(y)$  and the convolution of 2 form factors (FT)..this is a test for models in

which the

$$\text{DPD} = \text{GPD} \times \text{GPD}$$

NOT WELL REPRODUCED!

**G. S. Bali et al, JHEP 09, 106 (2021)**



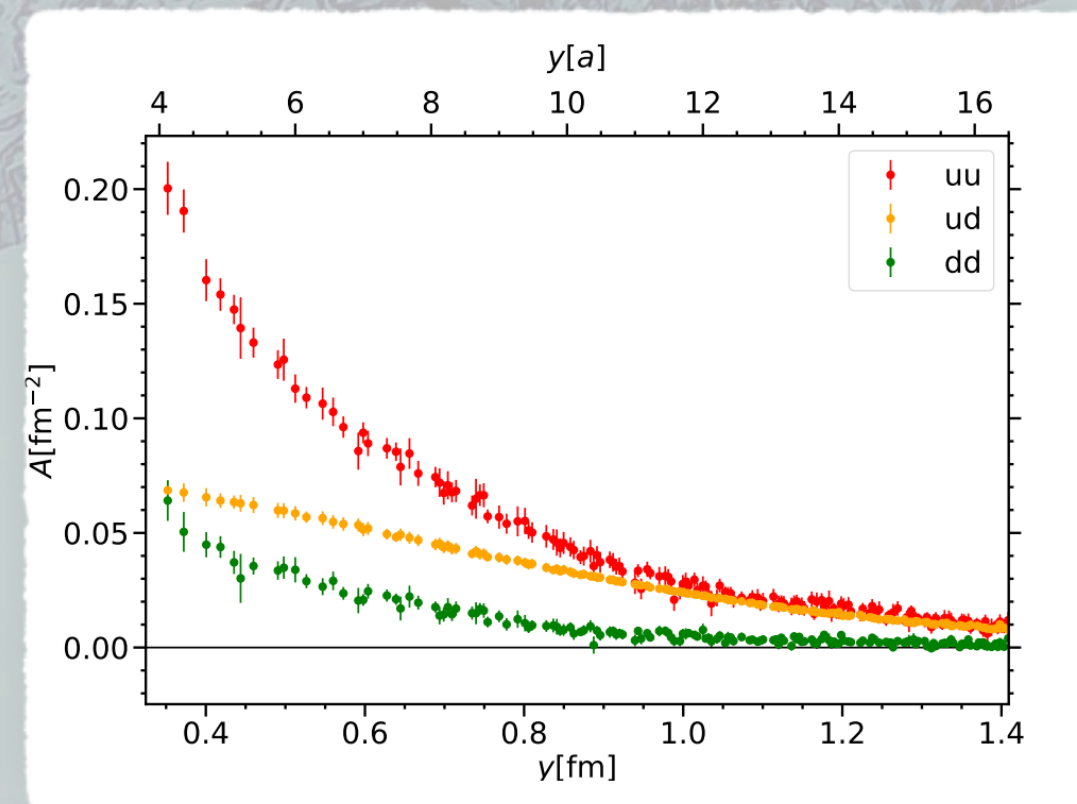
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$F_{ik}(x_1, x_2, \vec{z}_\perp)$  is unknown. For phenomenology @LHC kinematics (small x and many partons produced)

uncorrelated scenario:  $F_{ik}(x_1, x_2, \vec{z}_\perp) \sim \underbrace{g(x_1, x_2)}_{\text{double PDF}} \tilde{T}(\vec{z}_\perp)$  → Probability distribution of two partons at given distance

Unknown Non perturbative object

Some information from lattice



G. S. Bali et al, JHEP 09 (2021) 121

Some constraints from data

$$\sigma_{\text{eff}}^{-1} = \int d^2z_\perp \tilde{T}(z_\perp)^2$$

Some Constraints from general properties



# Some Data and Effective Cross Section

$$\sigma_{\text{eff}}^{pp} = \frac{m}{2} \frac{\sigma_A^{pp} \sigma_B^{pp}}{\sigma_{\text{DPS}}^{pp}}$$

POCKET FORMULA

Differential X-section single parton scattering for the process:  $pp \rightarrow A(B) + X$

Differential X-section double parton scattering for the process:  $pp \rightarrow A + B + X$



# Some Data and Effective Cross Section

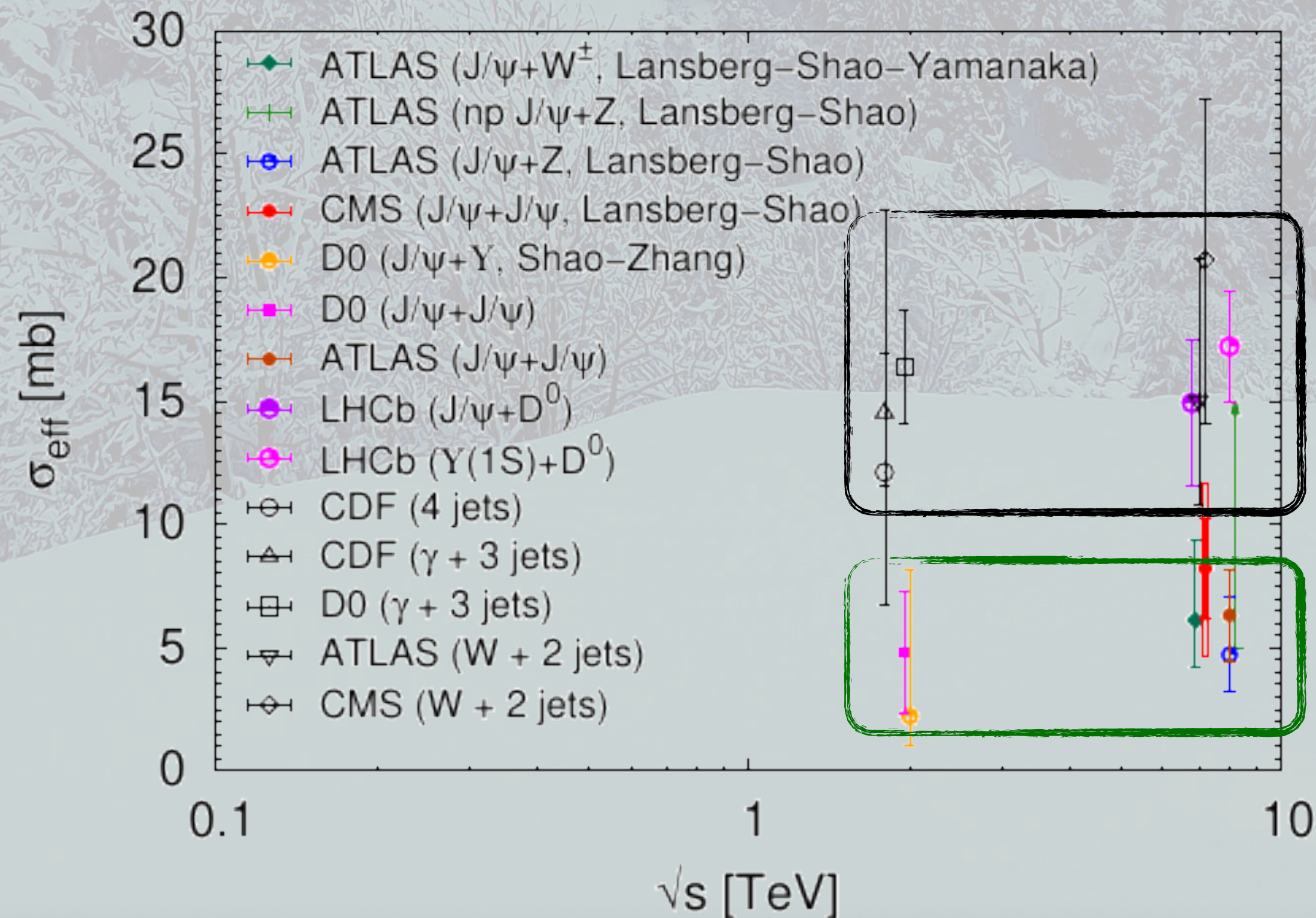
$$\sigma_{\text{eff}}^{pp} = \frac{m}{2} \frac{\sigma_A^{pp} \sigma_B^{pp}}{\sigma_{\text{DPS}}^{pp}}$$

→ Differential X-section single parton scattering for the process:  $pp \rightarrow A(B) + X$   
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**POCKET FORMULA**

— Results for W, Jet productions...

— Results for quarkonium productions



First observation of same sign WW via DPS:

$$\sigma_{\text{eff}} = 12.2^{+2.9}_{-2.2} \text{ mb}$$

[CMS coll.], PRL 131 (2023) 091803

$$\sigma^{\text{DPS}} \sim 6.28 \text{ fb}$$



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POCKET FORMULA

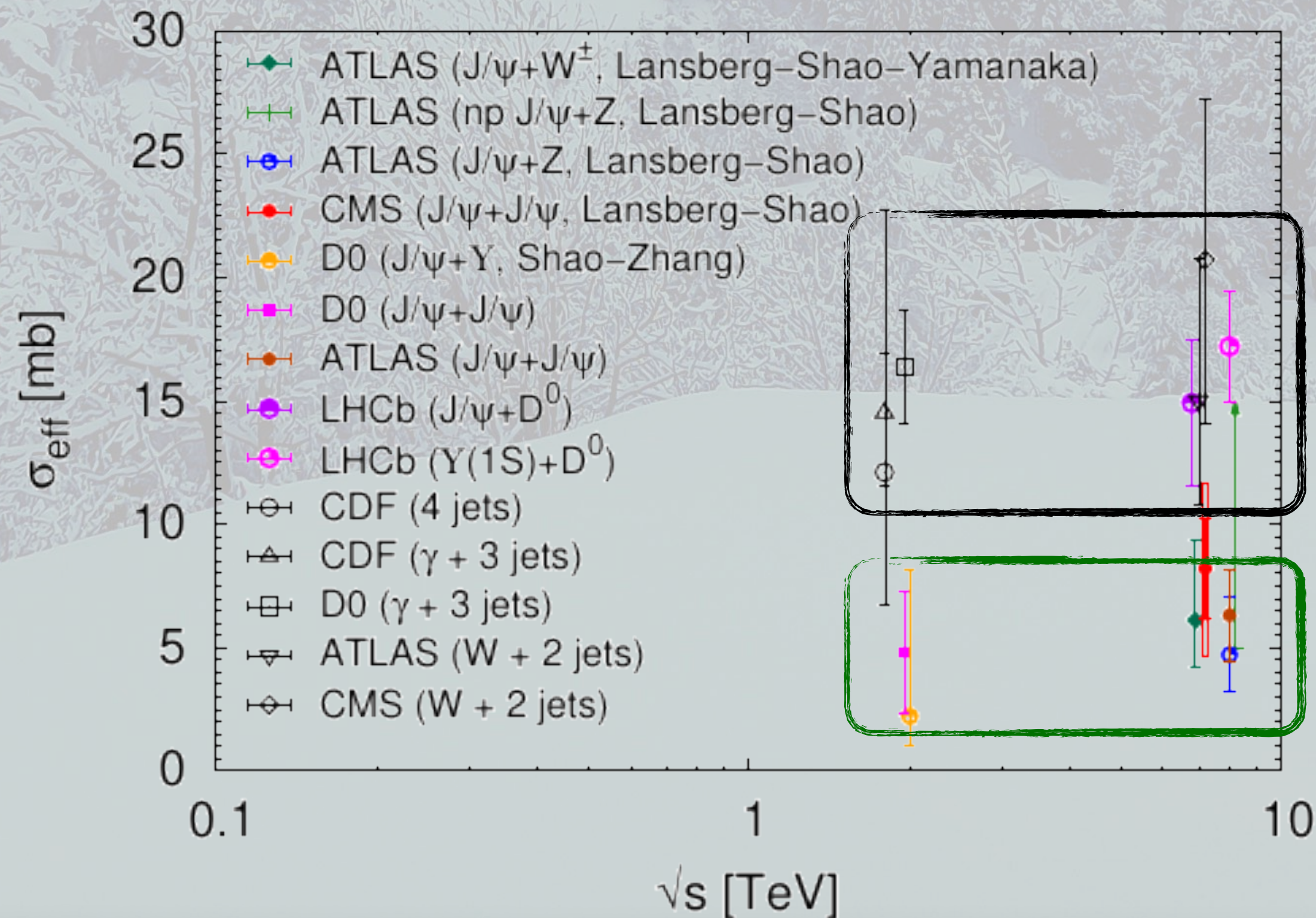
- 1) Process dependent?
- 2) Sensitive to correlations
- 3) Sensitive to the inner structure?

predicted by all models!

M.R. et al PLB 752,40 (2016)

M. Traini, M. R. et al, PLB 768, 270 (2017)

M. R. et al, Phys.Rev. D95 (2017) no.11, 114030



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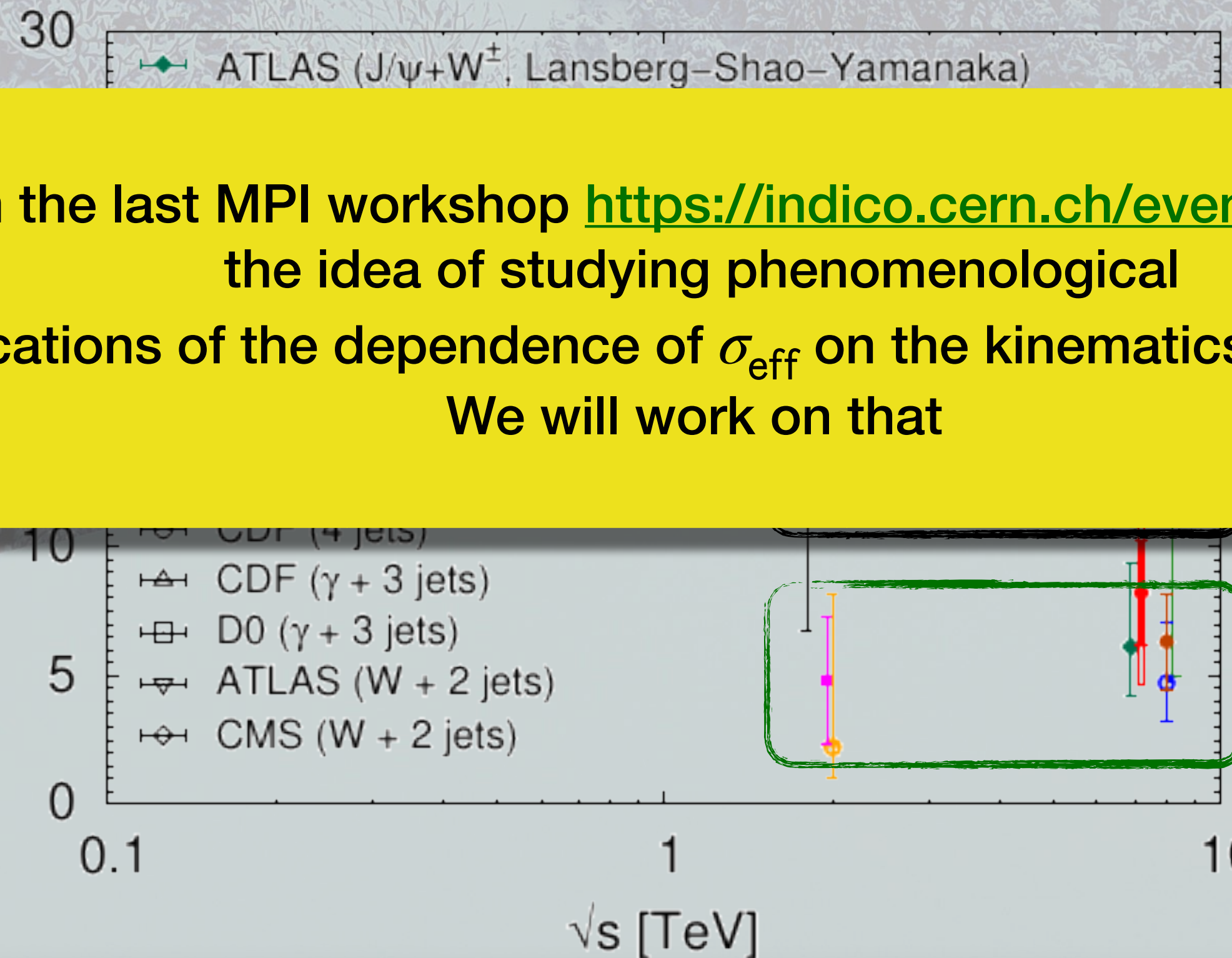
POCKET FORMULA

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From the last MPI workshop <https://indico.cern.ch/event/1281679/> the idea of studying phenomenological implications of the dependence of  $\sigma_{\text{eff}}$  on the kinematics came out!! We will work on that

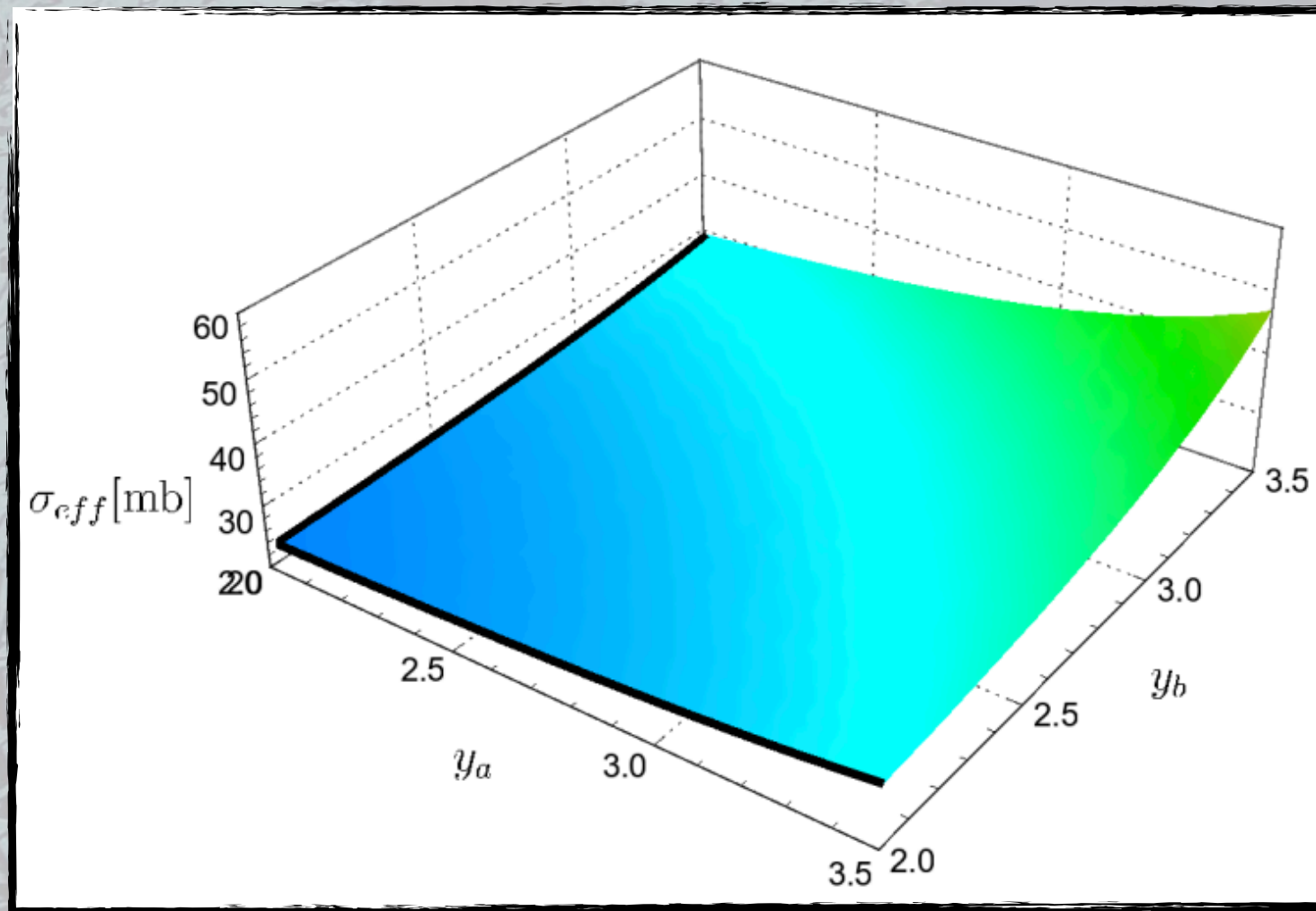




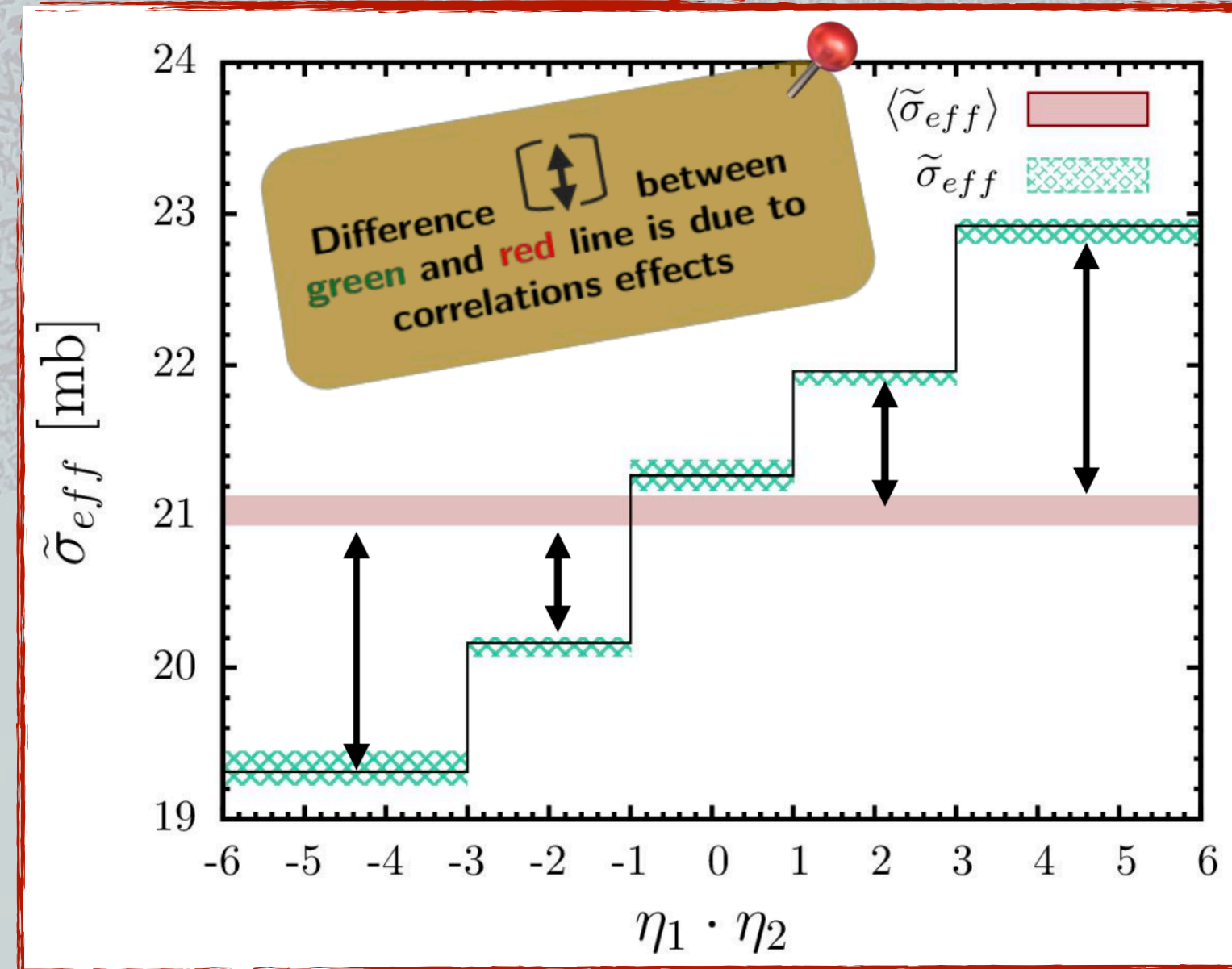
# Some Ideas on "nonconstant" $\sigma_{\text{eff}}$

$$\sigma_{\text{eff}} = \frac{m}{2} \frac{\sigma_A \sigma_B}{\sigma_{A+B}^{\text{DPS}}}$$

$$x_{1,3} = \frac{\sqrt{m_A^2 + k_{T,A}^2}}{\sqrt{s}} e^{\pm y_A}, \quad x_{2,4} = \frac{\sqrt{m_B^2 + k_{T,B}^2}}{\sqrt{s}} e^{\pm y_B}$$



An example for the calculation of  $\sigma_{\text{eff}}$  from gluon DPDs at high energy scales  
**M. R. and F. A. Ceccopieri, JHEP 09 (2019) 12, 125003**



Predictions from the calculation of same sign W's production at the LHC  
**F. A. Ceccopieri, M. R. and S. Scopetta, PRD 95 (2017), no.11, 114030**



# Effective Cross Section and proton structure

If DPDs factorize in terms of PDFs then

$$\sigma_{\text{eff}}^{-1} = \int d^2z_{\perp} \tilde{T}(z_{\perp})^2 = \int \frac{d^2k_{\perp}}{(2\pi)^2} \boxed{T(k_{\perp})^2}$$

Effective Form Factor (EFF) =  
FT of the probability distribution  $T$   
i.e. the probability of finding two partons  
at transverse distance  $z_{\perp}$



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$$T(k_{\perp}) \propto \int dx_1 dx_2 \tilde{F}(x_1, x_2, k_{\perp})$$

First moment of DPD



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As for the standard FF:

$$\langle z_{\perp}^2 \rangle \propto \left. \frac{d}{dk_{\perp}} T(k_{\perp}) \right|_{k_{\perp}=0}$$

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First moment of DPD

From the asymptotic behavior we got the following relation:

$$\frac{\sigma_{\text{eff}}}{3\pi} \leq \langle z_{\perp}^2 \rangle \leq \frac{\sigma_{\text{eff}}}{\pi}$$

M. R. and F. A. Ceccopieri, PRD 97, no. 7, 071501 (2018)



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$$\langle z_{\perp}^2 \rangle \propto \left. \frac{d}{dk_{\perp}} T(k_{\perp}) \right|_{k_{\perp}=0}$$

From the asymptotic behavior we got the following relation:

$$\frac{\sigma_{\text{eff}}}{3\pi} \leq \langle z_{\perp}^2 \rangle \leq \frac{\sigma_{\text{eff}}}{\pi} \quad \text{Verified in all model calculations:}$$

$$\text{DPD} = \text{GPD} \otimes \text{GPD}$$

Constituent quark models for:  
proton

M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097

Pion

M.R. EPJC 80 (2020) 7, 678

W. Broniovski and E. R. Arriola PRD 101 (2020), 1, 014019

$\rho$

M.R. EPJC 80 (2020) 7, 678

M. R. and F. A. Ceccopieri, PRD 97, no. 7, 071501 (2018)



# Effective Cross Section and proton structure

If DPDs factorize in terms of PDFs then

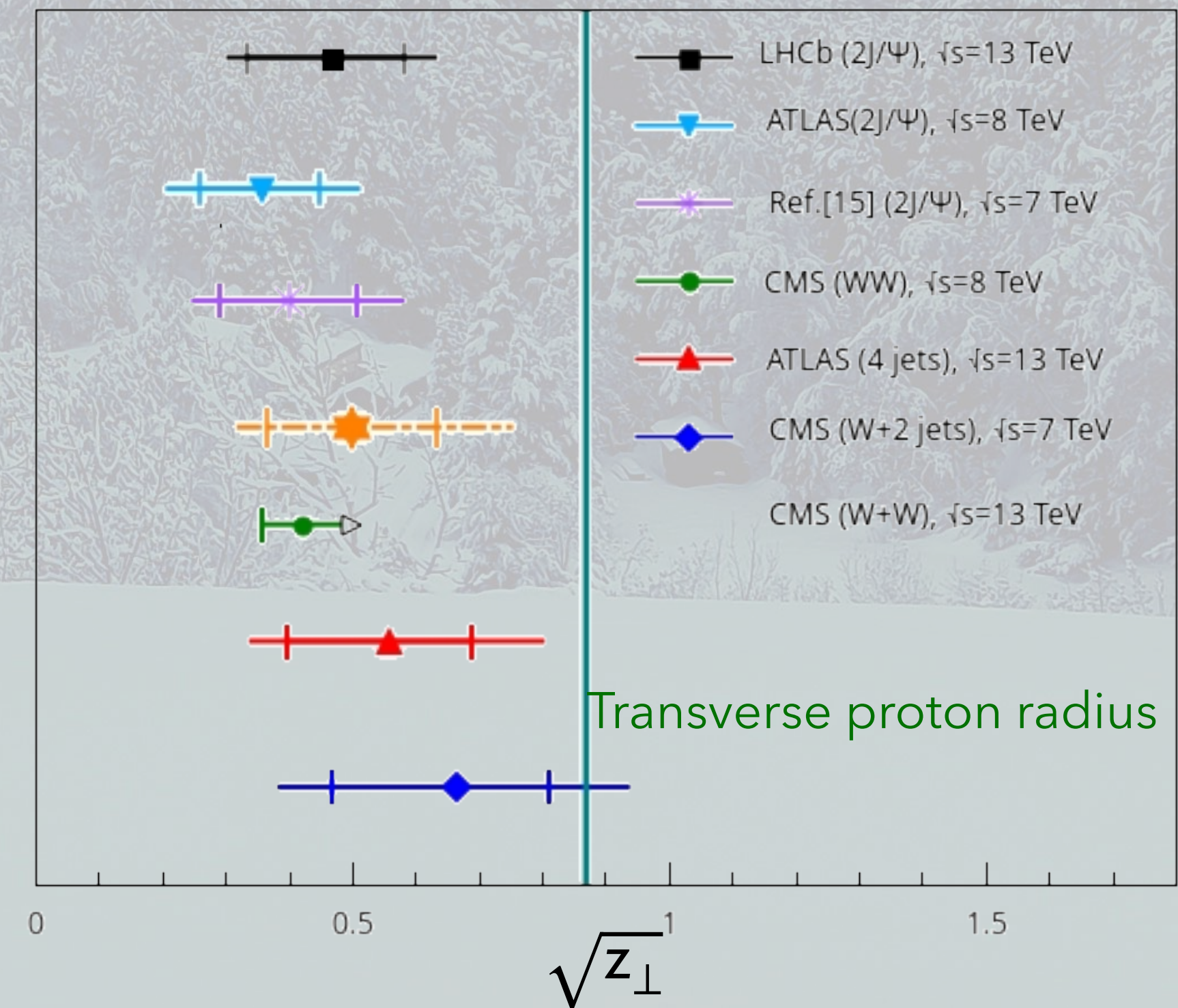
$$\sigma_{\text{eff}}^{-1} = \int d^2z_{\perp} \tilde{T}(z_{\perp})^2 = \int \frac{d^2k_{\perp}}{(2\pi)^2} \boxed{T(k_{\perp})^2}$$

As for the standard FF:

$$\langle z_{\perp}^2 \rangle \propto \left. \frac{d}{dk_{\perp}} T(k_{\perp}) \right|_{k_{\perp}=0}$$

From the asymptotic behavior we got the following relation:

$$\frac{\sigma_{\text{eff}}}{3\pi} \leq \langle z_{\perp}^2 \rangle \leq \frac{\sigma_{\text{eff}}}{\pi}$$



M. R. and F. A. Ceccopieri, PRD 97, no. 7, 071501 (2018)



# Effective Cross Section and proton structure

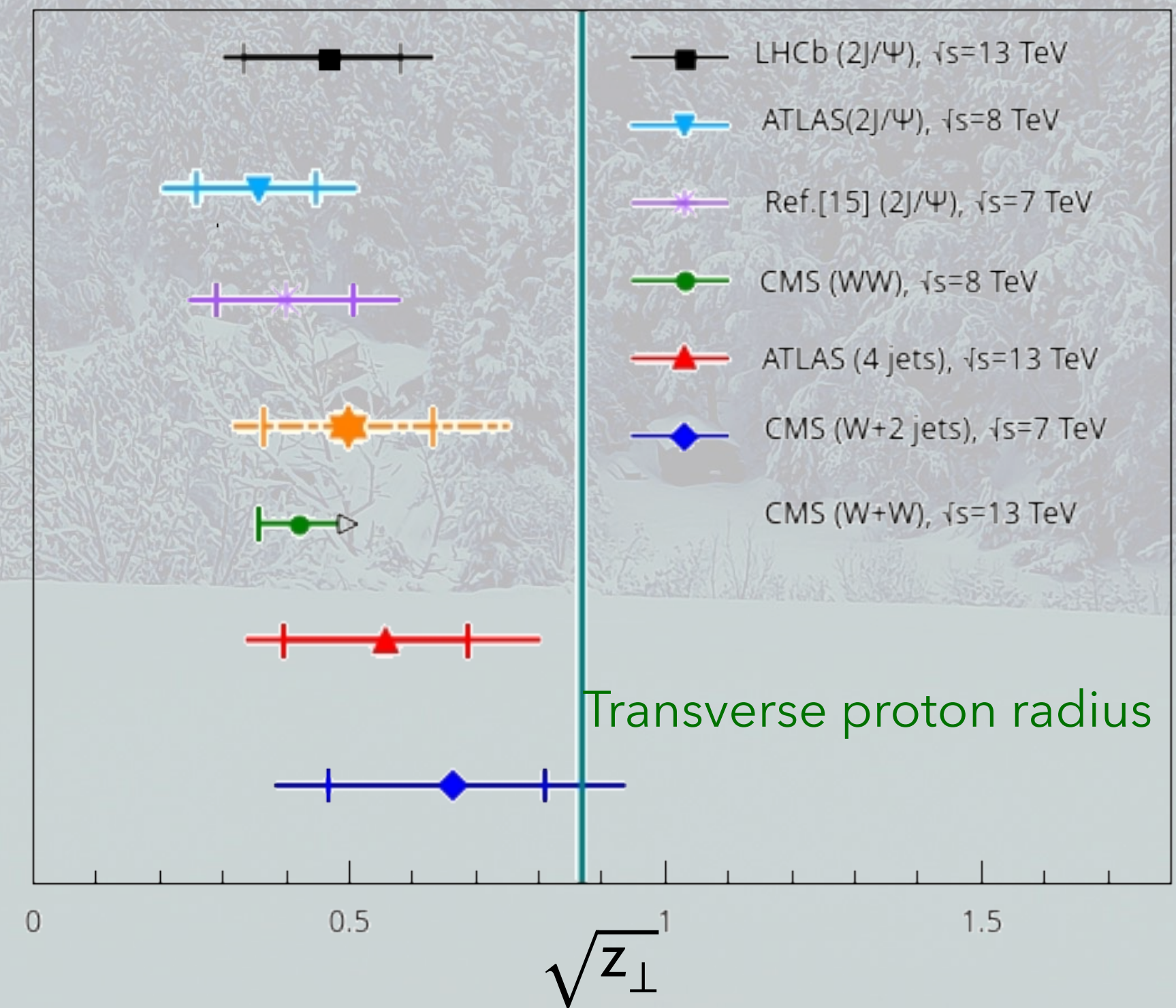
If DPDs factorize in terms of  $\tilde{T}(z_\perp)$  then  $\sigma_{\text{eff}}^{-1} = \int d^2z_\perp \tilde{T}(z_\perp)^2 = \int \frac{d^2k_\perp}{(2\pi)^2} T(k_\perp)^2$

- 1) THE MEAN DISTANCE IS LOWER THEN THE PROTON RADIUS!
  - 2) in hadron-hadron collisions we do not access directly the distance!
- M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097

From this behavior we got the following relation:

$$\frac{\sigma_{\text{eff}}}{3\pi} \leq \langle z_\perp^2 \rangle \leq \frac{\sigma_{\text{eff}}}{\pi}$$

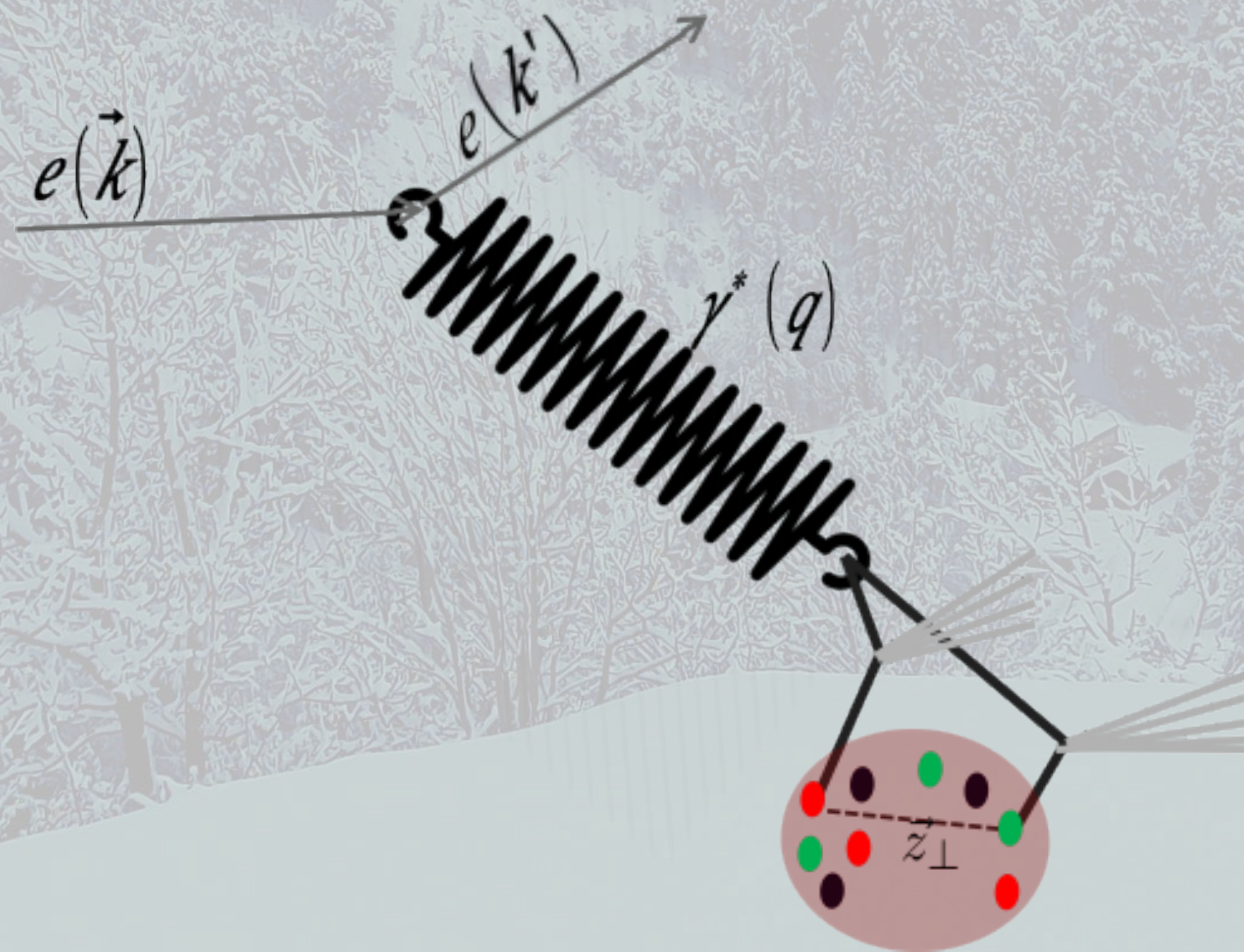
M. R. and F. A. Ceccopieri, PRD 97, no. 7, 071501 (2018)





# DPS in $\gamma - p$ interactions

We consider the possibility offered by a DPS process involving a photon FLUCTUATING in a quark-antiquark pair interacting with a proton:

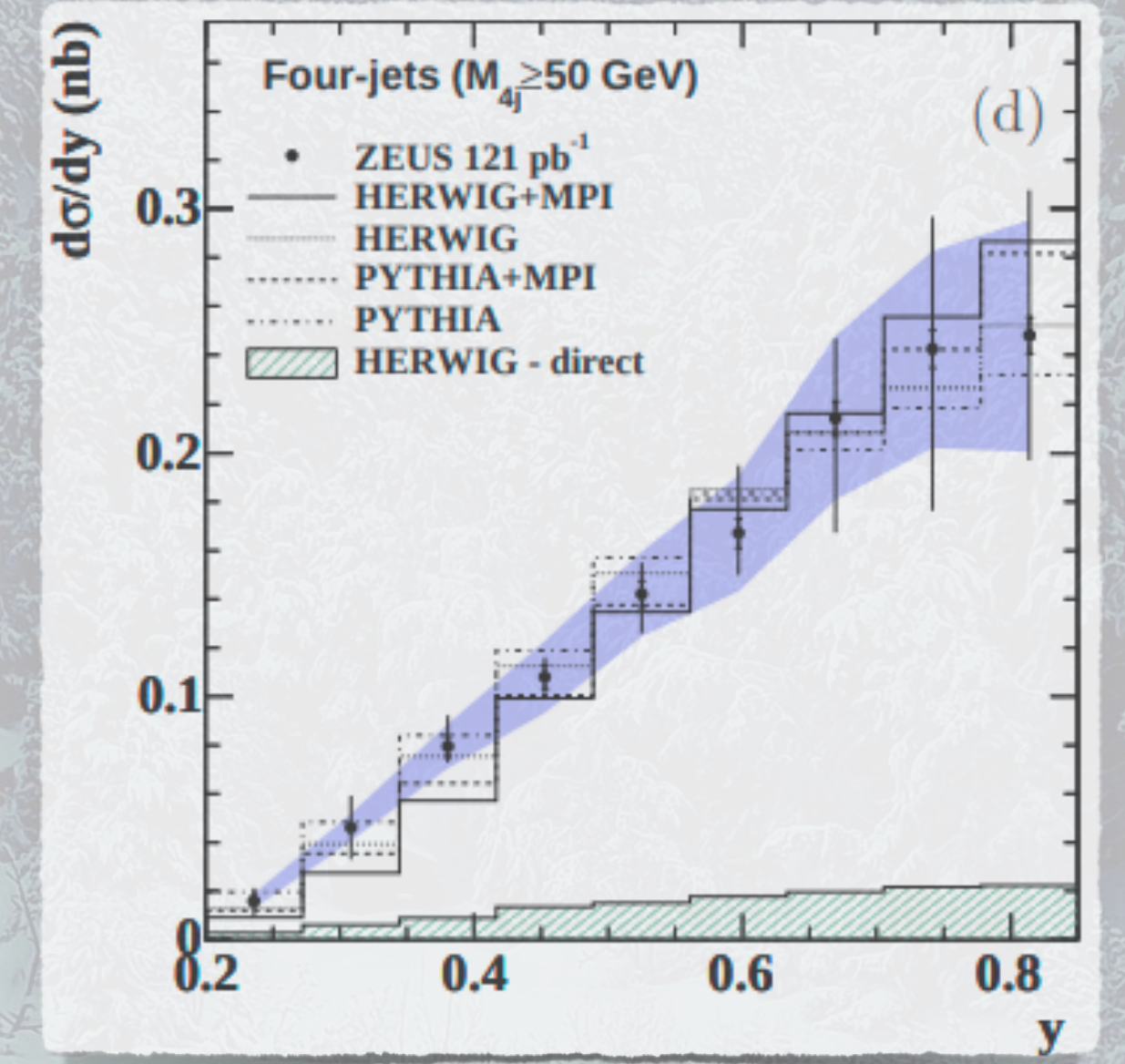
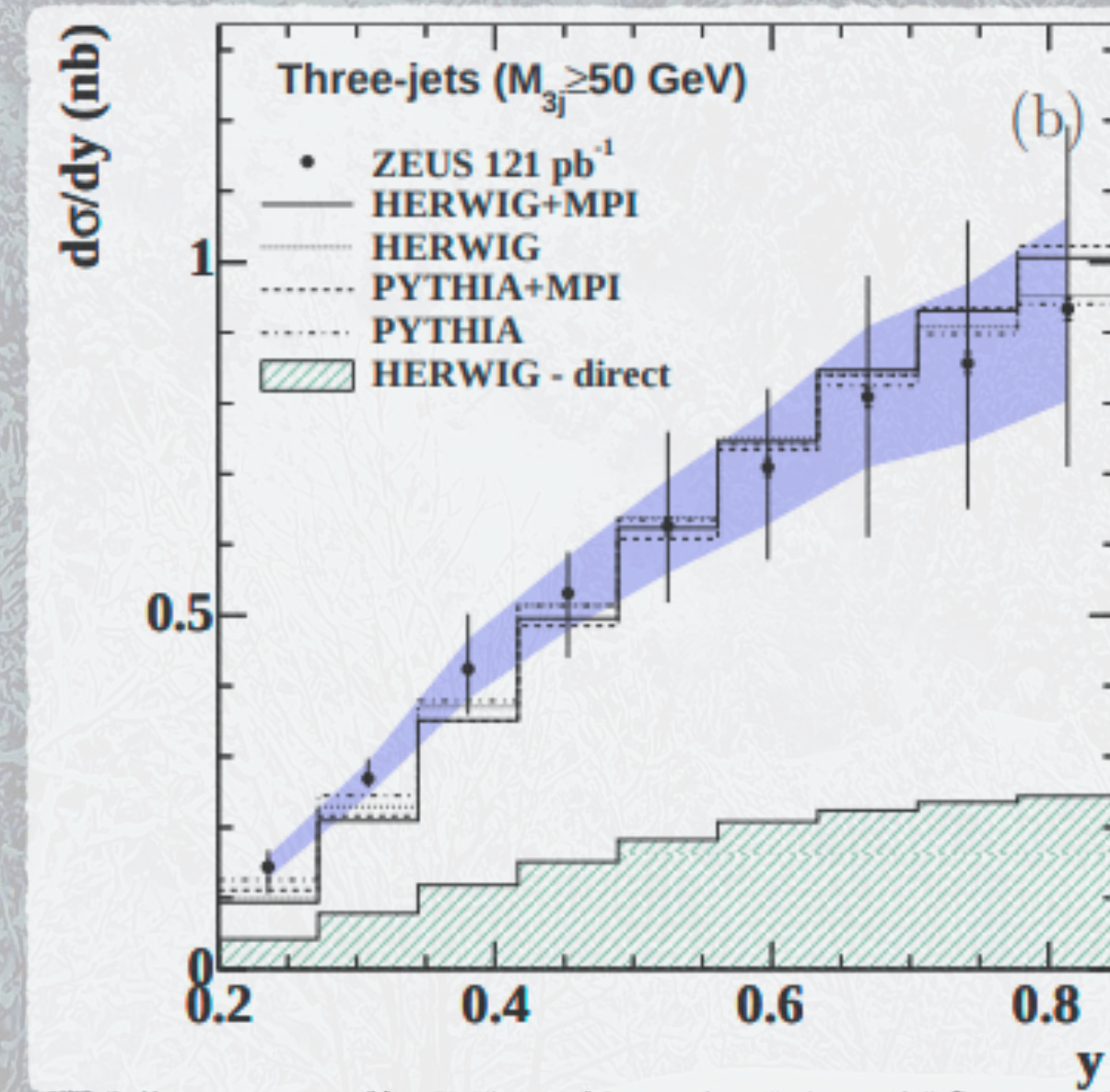
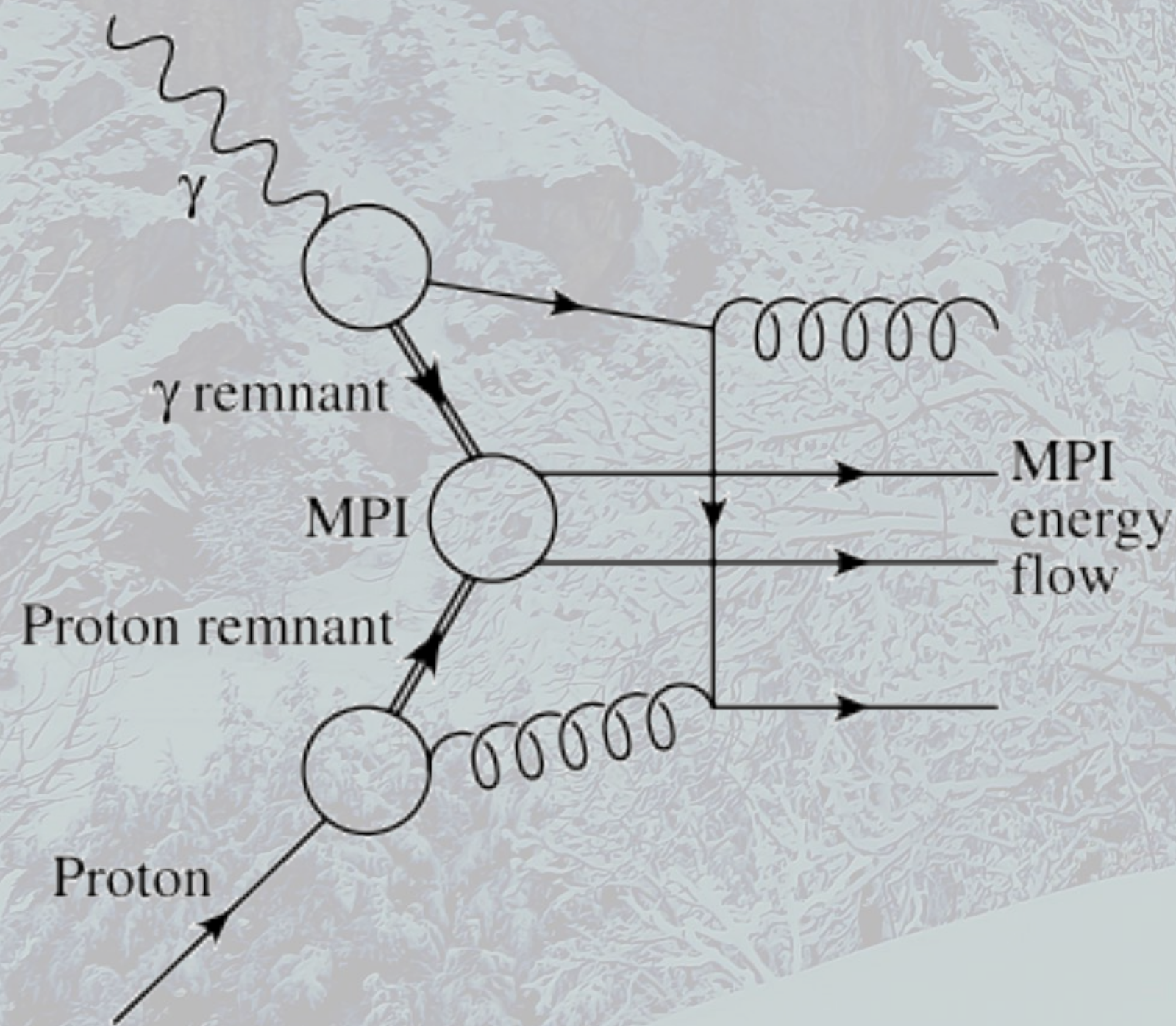


M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501



# DPS in $\gamma - p$ interactions

Already at HERA the importance of MPI for the **3,4 jets photo-production** has been addressed:



J. R. Forshaw et al, Z phys. C 72, 637

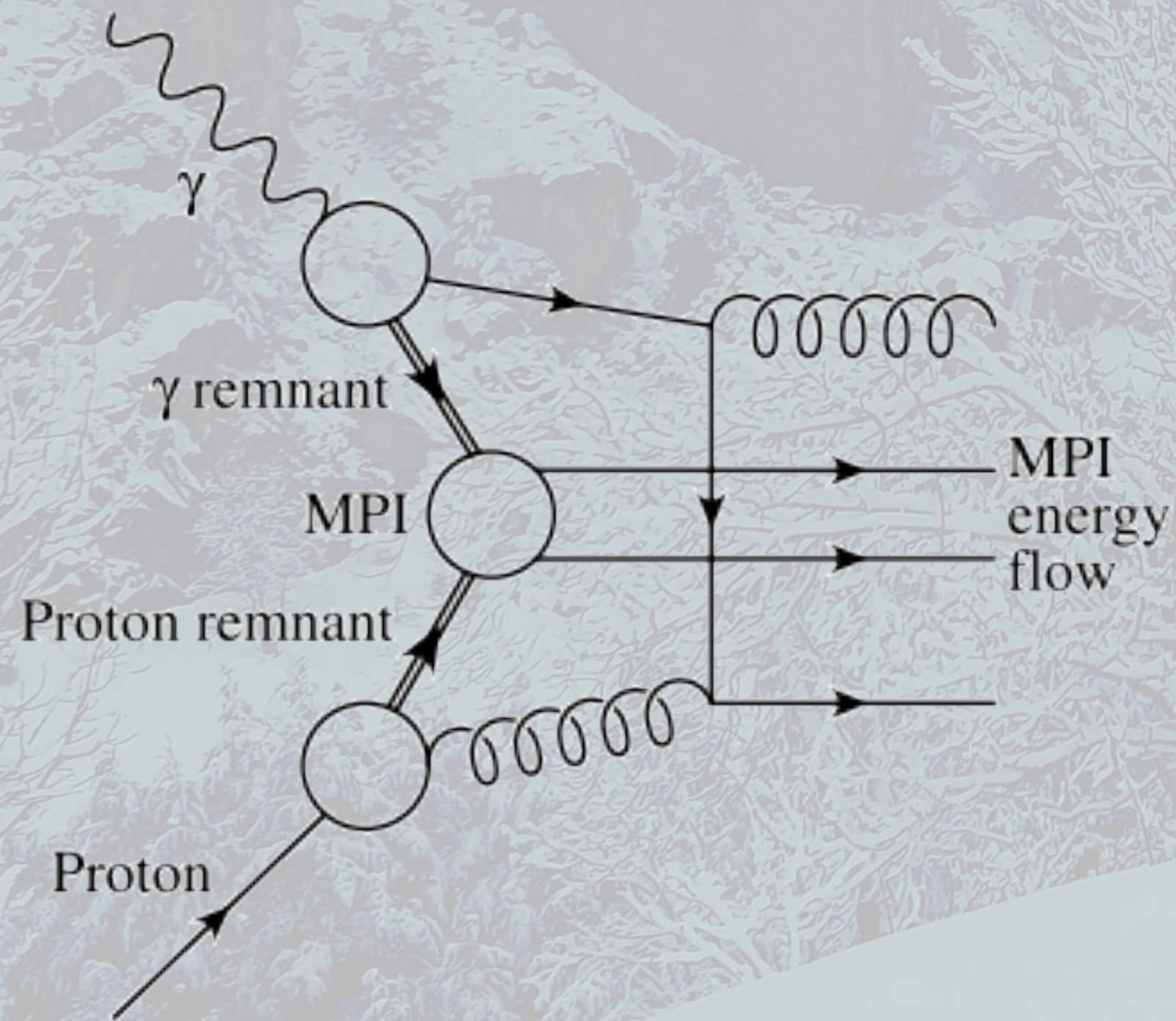
S. Chekanov et al [ZEUS coll.], Nucl. Phys B 792,1 (2008)



# DPS in $\gamma - p$ interactions

In order to study the impact of the DPS contribution to a process initiated via photon-proton interactions we evaluated the 4-JET photo-production at HERA (**S. Chekanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)**)

For this first investigation, we make use of the **POCKET FORMULA**:



Flux Factor  
P. Nason et al, PLB319

$$d\sigma_{\text{DPS}}^{4j} = \frac{1}{2} \sum_{ab,cd} \int dy dQ^2 f_{\gamma/e}(y, Q^2) \times \sigma_{\text{eff}}^{\gamma p}(Q^2)$$

$$\times \int dx_{p_a} dx_{\gamma_b} f_{a/p}(x_{p_a}) f_{b/\gamma}(x_{\gamma_b}) d\hat{\sigma}_{ab}^{2j}(x_{p_a}, x_{\gamma_b}) \left. \begin{array}{l} \text{SPS} \\ * \end{array} \right\}$$

$$\times \int dx_{p_c} dx_{\gamma_d} f_{c/p}(x_{p_c}) f_{d/\gamma}(x_{\gamma_d}) d\hat{\sigma}_{cd}^{2j}(x_{p_c}, x_{\gamma_d}) \left. \begin{array}{l} \text{SPS} \end{array} \right\}$$

Proton PDF

(J. Pumplin et al. JHEP 07, 012 (2002))

Photon PDF

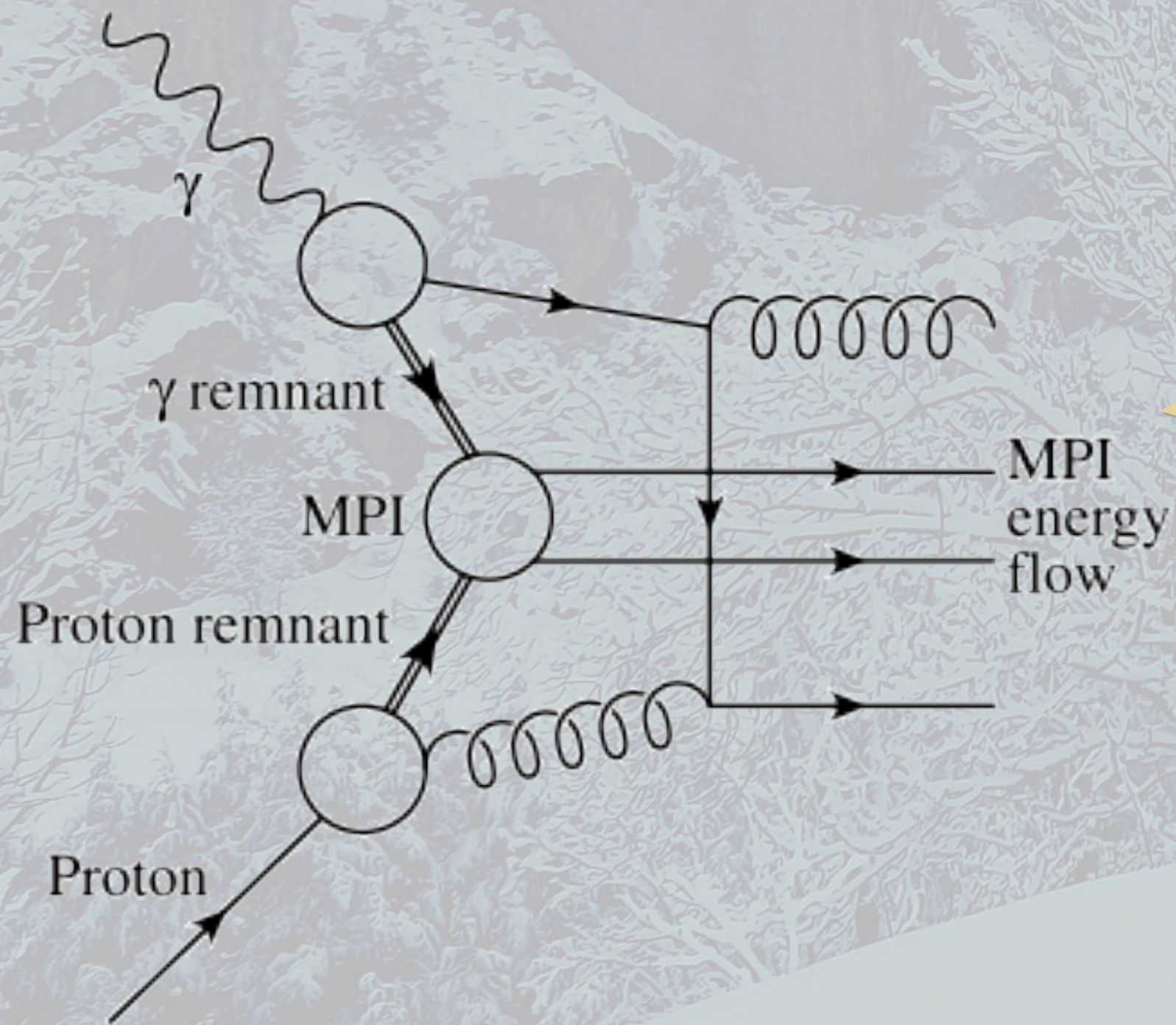
(M. Gluck et al. PRD46, 1973 (1992))



# DPS in $\gamma - p$ interactions

In order to study the impact of the DPS contribution to a process initiated via photon-proton interactions we evaluated the 4-JET photo-production at HERA (S. Chekanov et al. (ZEUS), Nucl. Phys B792, 1 (2008))

For this first investigation, we make use of the POCKET FORMULA:



Flux Factor  
P. Nason et al, PLB319

The main quantity we have to evaluate is:  
 $\sigma_{eff}^{\gamma p}(Q^2)$

$$f_{\gamma/e}(y, Q^2) \times \sigma_{eff}^{\gamma p}(Q^2)$$

$$\left. \begin{aligned} & (x_{\gamma b}) d\hat{\sigma}_{ab}^{2j}(x_{p_a}, x_{\gamma_b}) \\ & (x_{\gamma d}) d\hat{\sigma}_{cd}^{2j}(x_{p_c}, x_{\gamma_d}) \end{aligned} \right\} \begin{array}{l} \text{SPS} \\ * \\ \text{SPS} \end{array}$$

Photon PDF  
(M. Gluck et al. PRD46, 1973 (1992))



# The $\gamma - p$ effective cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The expression of this quantity is very similar to the proton-proton collision case and can be formally derived by comparing the product of SPS cross sections and the DPS one obtained in **Gaunt, JHEP 01, 042 (2013)** and describing a DPS from a vector bosons splitting with given  $Q^2$  virtuality

$$\left[ \sigma_{\text{eff}}^{\gamma p}(Q^2) \right]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} \overset{\text{Proton EFF}}{\boxed{T_p(k_{\perp})}} \overset{\text{Photon EFF}}{\boxed{T_{\gamma}(k_{\perp}; Q^2)}}$$

The full DPS cross section depends on the amplitude of the splitting photon in a  $q - \bar{q}$  pair. The latter can be formally described within a Light-Front (LF) approach in terms of LF wave functions



# The $\gamma - p$ effective cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The main ingredients of the calculations:

A hand-drawn diagram of a notepad with three numbered items. A vertical red line is on the left side. The items are:

- 1  $[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$
- 2  $T_p(k_{\perp})$  proton EFF
- 3  $\psi/\gamma$  Photon WF

For the proton EFF use has been made of three choices:

1) G1  $e^{-\alpha_1 k_{\perp}^2}$ ,  $\alpha_1 = 1.53 \text{ GeV}^{-2} \implies \sigma_{\text{eff}}^{\text{pp}} = 15 \text{ mb}$

2) G2  $e^{-\alpha_2 k_{\perp}^2}$ ,  $\alpha_2 = 2.56 \text{ GeV}^{-2} \implies \sigma_{\text{eff}}^{\text{pp}} = 25 \text{ mb}$

3) S  $\left(1 + \frac{k_{\perp}^2}{m_g^2}\right)^{-4}$ ,  $m_g^2 = 1.1 \text{ GeV}^2 \implies \sigma_{\text{eff}}^{\text{pp}} = 30 \text{ mb}$



# The $\gamma - p$ effective cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The main ingredients of the calculations:

1  $[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$

2  $T_p(k_{\perp})$  proton EFF

3  $\psi_{\gamma}$  Photon WF

For the photon W.F. use has been made of two choices representing two extreme cases:

1) QED at LO (S.J. Brodsky et al. PRD50, 3134 (1994)):

$$\psi_{q,\bar{q}}^{\lambda=\pm}(x, k_{1\perp}; Q^2) = -e_f \frac{\bar{u}_q(k) \gamma \cdot \varepsilon^{\lambda} v_{\bar{q}}(q - k)}{\sqrt{x(1-x)} \left[ Q^2 + \frac{k_{1\perp}^2 + m^2}{x(1-x)} \right]}$$

2) Non-Perturbative (NP) effects (E.R.Arriola et al, PRD74,054023 (2006))

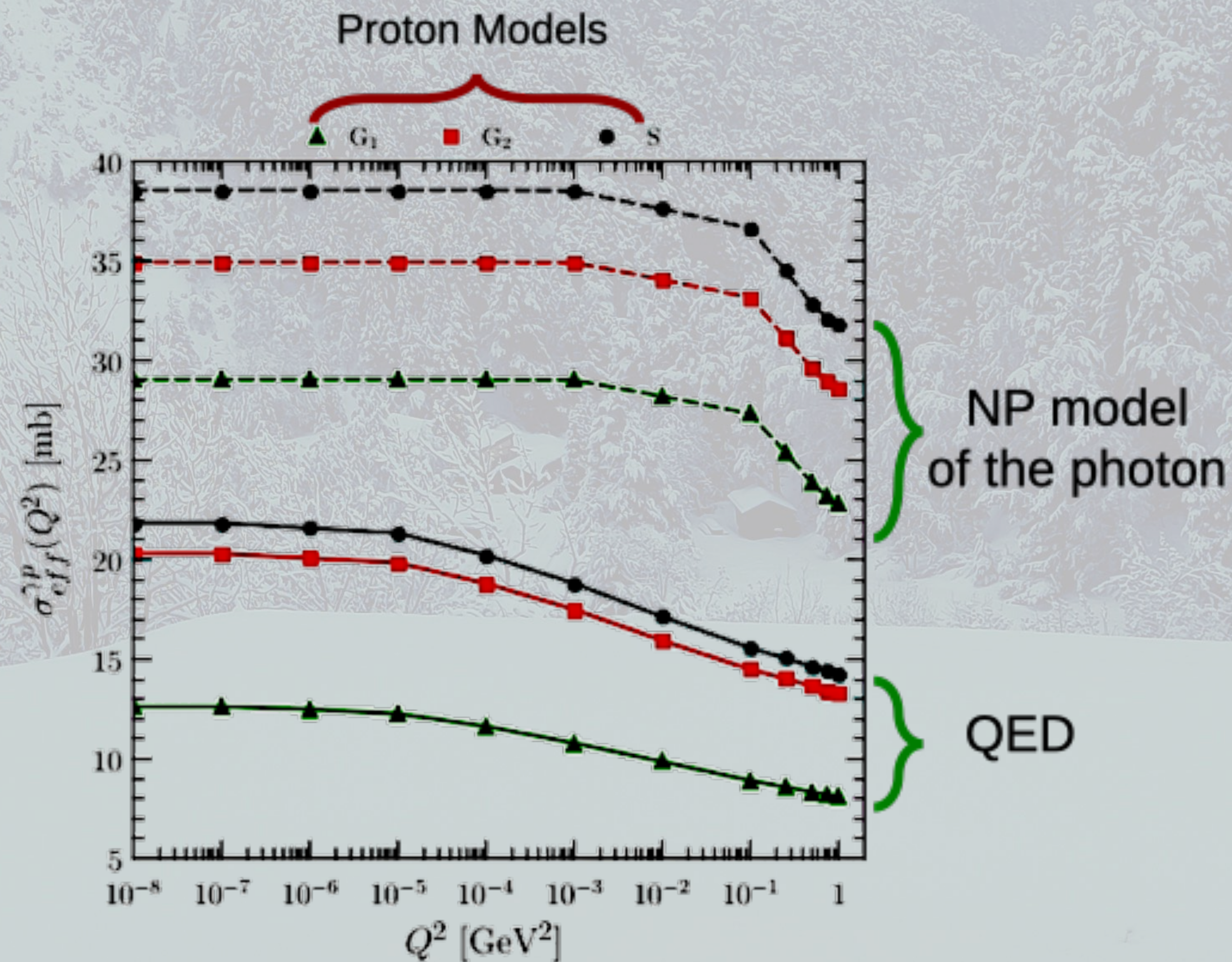
$$\psi_A^{\gamma}(x, k_{\perp 1}; Q^2) = \frac{6(1 + Q^2/m_{\rho}^2)}{m_{\rho}^2 \left( 1 + 4 \frac{k_{\perp 1}^2 + Q^2 x(1-x)}{m_{\rho}^2} \right)^{5/2}}$$



# The $\gamma - p$ effective cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

- 1  $[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$
- 2  $T_p(k_{\perp})$  proton EFF
- 3  $\psi/\gamma$  Photon WF



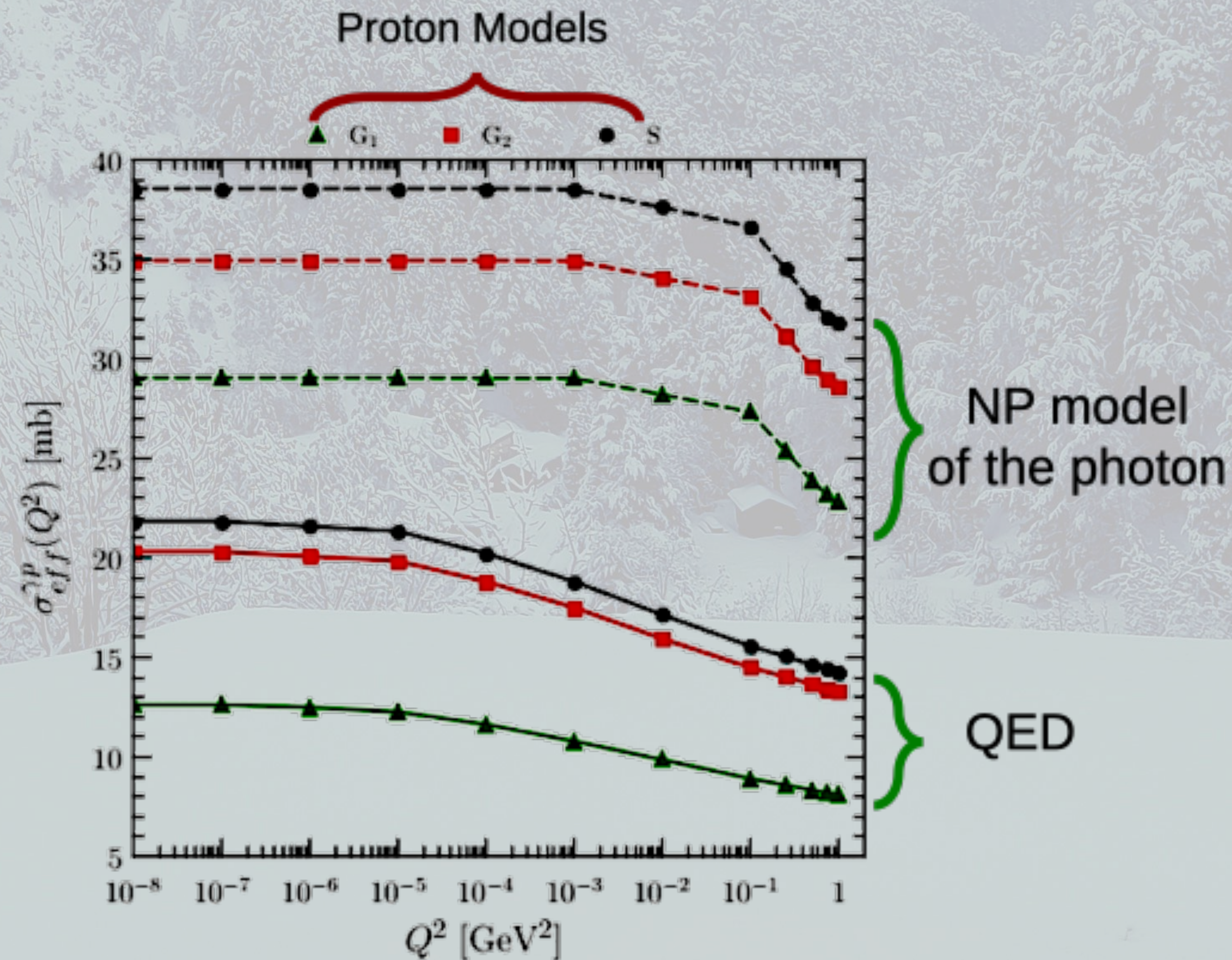


# The $\gamma - p$ effective cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

- 1  $[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$
- 2  $T_p(k_{\perp})$  proton EFF
- 3  $\psi/\gamma$  Photon WF

The effective cross-section depends on the photon virtuality! (NEW)





# The 4-jets DPS cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

$$d\sigma_{\text{DPS}}^{4j} = \frac{1}{2} \sum_{ab,cd} \int dy dQ^2 \frac{f_{\gamma/e}(y, Q^2)}{\sigma_{\text{eff}}^{\gamma p}(Q^2)} \times$$

$$\times \int dx_{p_a} dx_{\gamma_b} f_{a/p}(x_{p_a}) f_{b/\gamma}(x_{\gamma_b}) d\hat{\sigma}_{ab}^{2j}(x_{p_a}, x_{\gamma_b})$$

$$\times \int dx_{p_c} dx_{\gamma_d} f_{c/p}(x_{p_c}) f_{d/\gamma}(x_{\gamma_d}) d\hat{\sigma}_{cd}^{2j}(x_{p_c}, x_{\gamma_d})$$

KINEMATICS:

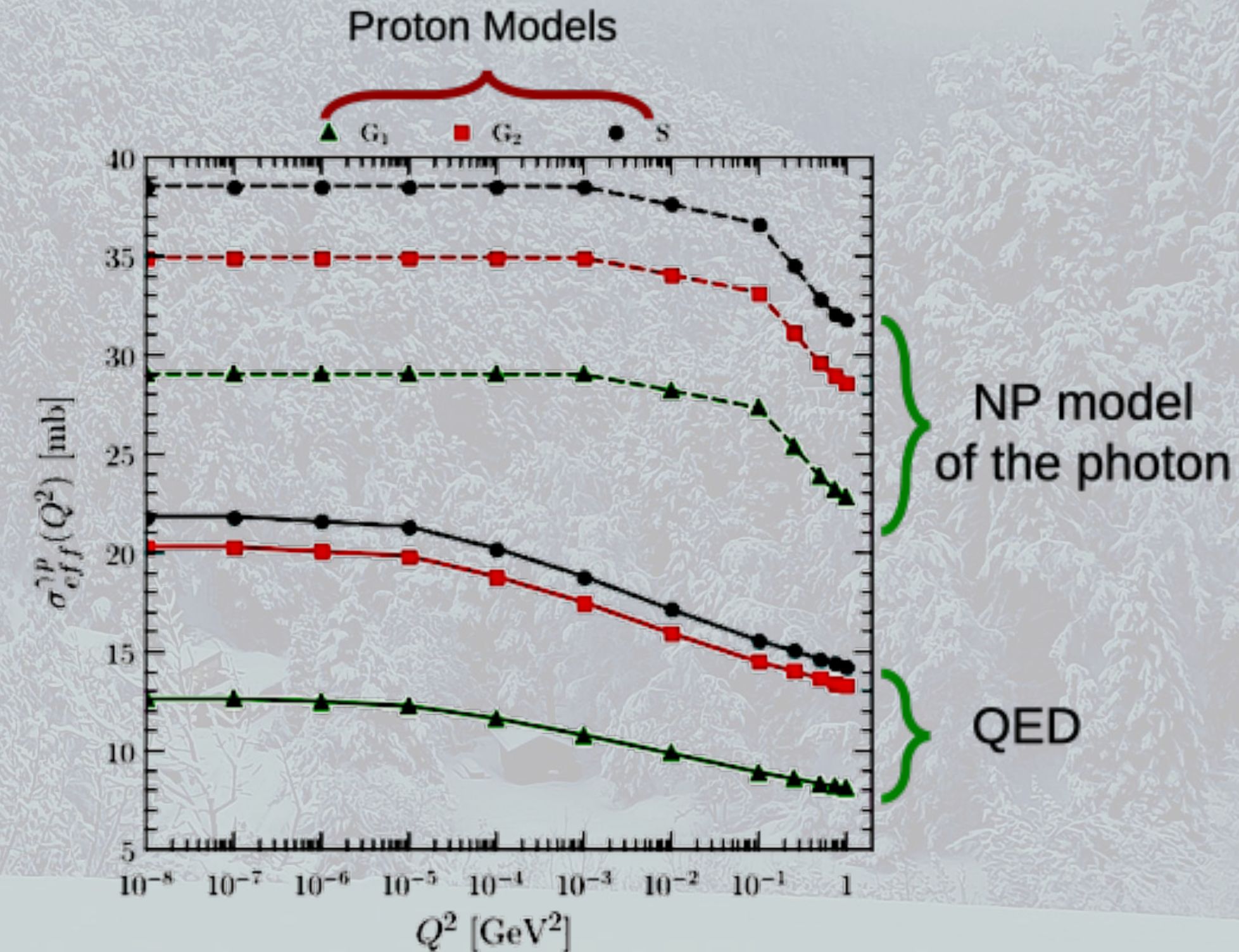
$$E_T^{\text{jet}} > 6 \text{ GeV}$$

$$|\eta_{\text{jet}}| < 2.4$$

$$Q^2 < 1 \text{ GeV}^2$$

$$0.2 \leq y \leq 0.85$$

The ZEUS collaboration quoted an integrated total 4-jet cross section of 136 pb  
**S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)**





# The 4-jets DPS cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

$$d\sigma_{DPS}^{4j} = \frac{1}{2} \sum_{ab,cd} \int dx_{pa} dx_{\gamma b} f_{a/p}(x_{pa}, Q^2) \int dx_{pc} dx_{\gamma d} f_{c/p}(x_{pc}, Q^2) \times \dots$$

KINEMATICS:

$$E_T^{\text{jet}} > 6 \text{ GeV}$$

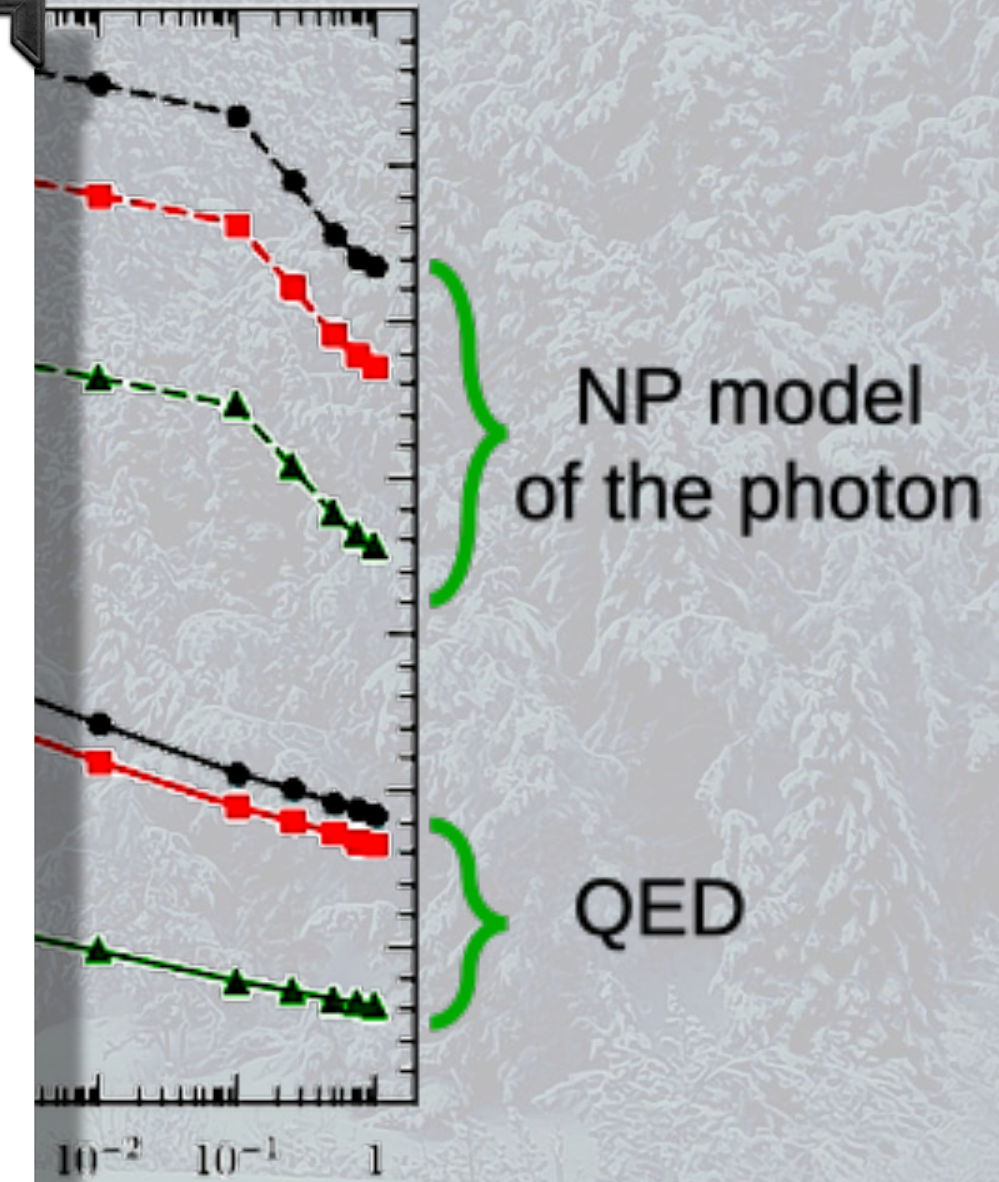
$$|\eta_{\text{jet}}| < 2.4$$

$$Q^2 < 1 \text{ GeV}^2$$

$$0.2 \leq y \leq 0.85$$

		$\sigma_{DPS}$ [pb]			
		$Q^2 \leq 10^{-2}$	$10^{-2} \leq Q^2 \leq 1$	$Q^2 \leq 1$	$\frac{\sigma_{DPS}}{\sigma_{tot}}$
		[GeV <sup>2</sup> ]	[GeV <sup>2</sup> ]	[GeV <sup>2</sup> ]	[%]
Proton	G <sub>1</sub>	35.1	18.6	53.7	40
	G <sub>2</sub>	29.1	15.2	44.3	33
	S	26.4	13.7	40.1	30
Photon	G <sub>1</sub>	87.8	54.3	142.1	101
	G <sub>2</sub>	54.3	33.4	87.7	65
	S	50.5	31.1	81.6	60

Proton Models



The ZEUS collaboration quoted an integrated total 4-jet cross section of 136 pb  
**S. Chekanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)**



# The 4-jets DPS cross-section

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

$$d\sigma_{DPS}^{4j} = \frac{1}{2} \sum_{ab,cd} \int dx_{pa} dx_{\gamma b} f_{a/p}(x_{pa}, Q^2) \times \int dx_{pc} dx_{\gamma d} f_{c/p}(x_{pc}, Q^2) \times \dots$$

KINEMATICS:

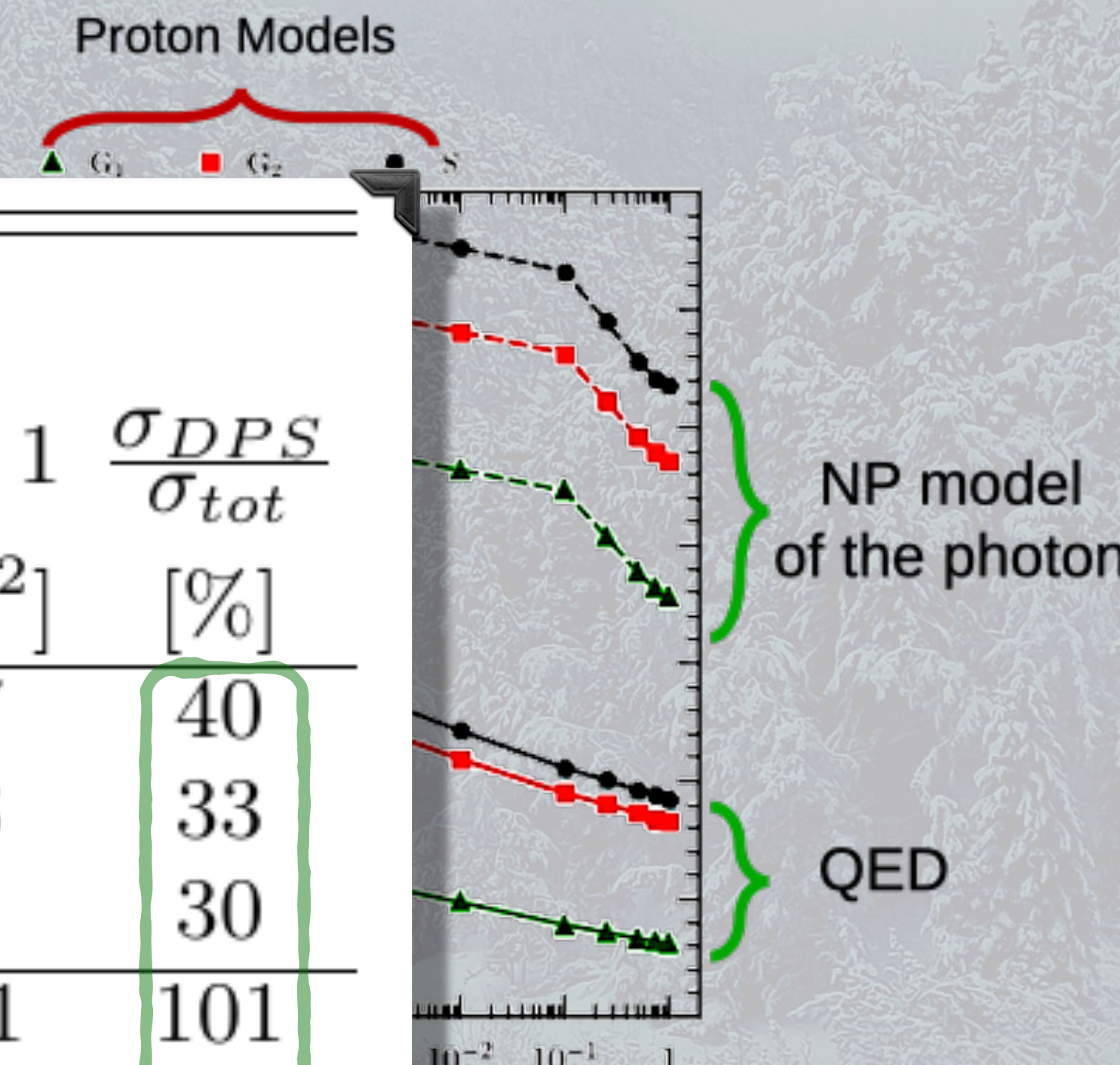
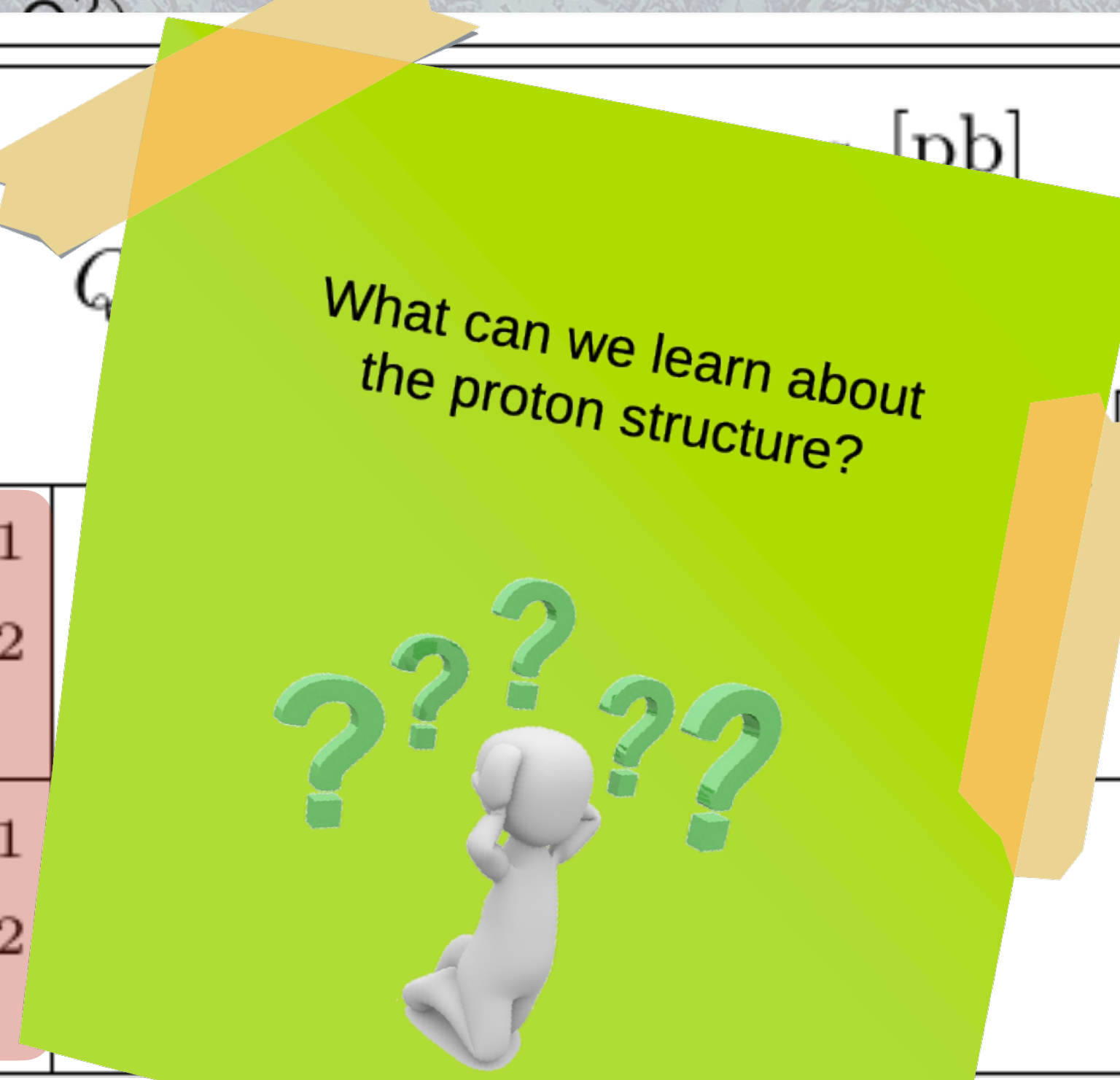
$$E_T^{\text{jet}} > 6 \text{ GeV}$$

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$$Q^2 < 1 \text{ GeV}^2$$

$$0.2 \leq y \leq 0.85$$

Proton		Photon		$Q^2 \leq 1 \text{ GeV}^2$	$\frac{\sigma_{DPS}}{\sigma_{tot}}$
				[nb]	[%]
NP Model	G <sub>1</sub>			53.7	40
	G <sub>2</sub>			44.3	33
	S			40.1	30
QED	G <sub>1</sub>			142.1	101
	G <sub>2</sub>			87.7	65
	S			81.6	60



The ZEUS collaboration quoted a total 4-jet cross section of 136 pb  
 S. Chekanov et al. (ZEUS), Nucl. Phys B772, 1 (2008)



# A key to the proton structure

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The effective cross section can be also written in terms of probability distribution:

$$\left[ \sigma_{\text{eff}}^{\gamma p}(Q^2) \right]^{-1} = \int d^2 z_{\perp} \tilde{F}_2^p(z_{\perp}) \tilde{F}_2^{\gamma}(z_{\perp}; Q^2)$$



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We can expand the distribution related to the photon:

$$\tilde{F}_2^{\gamma}(z_{\perp}; Q^2) = \sum_n C_n(Q^2) z_{\perp}^n$$

Coefficients determined in a given approach describing the photon structure



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Coefficients determined in a given approach describing the photon structure

$$\left[ \sigma_{\text{eff}}^{\gamma p}(Q^2) \right]^{-1} = \sum_n C_n(Q^2) \langle z_{\perp}^n \rangle_p$$

Mean value of the transverse distance between two partons in the PROTON



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M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

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Coefficients determined in a given approach describing the photon structure

$$\left[ \sigma_{\text{eff}}^{\gamma p}(Q^2) \right]^{-1} = \sum_n C_n(Q^2) \langle z_{\perp}^n \rangle_p$$

Mean value of the transverse distance between two partons in the PROTON

If we could measure  $\sigma_{\text{eff}}^{\gamma p}(Q^2)$  we could access NEW INFORMATION ON THE PROTON STRUCTURE



# A key to the proton structure

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

The effective cross section can be also written in terms of probability distribution:

$$\left[ \sigma_{\text{eff}}^{\gamma p}(Q^2) \right]^{-1} = \int d^2z_{\perp} \tilde{F}_2^p(z_{\perp}) \tilde{F}_2^{\gamma}(z_{\perp}; Q^2)$$

We can experimentally measure  $\sigma_{\text{eff}}^{\gamma p}(Q^2)$  and  $\tilde{F}_2^p(z_{\perp})$  (proton structure) and  $\tilde{F}_2^{\gamma}(z_{\perp}; Q^2)$  (photon structure):

We estimated that with an integrated luminosity of 200 pb<sup>-1</sup> Q<sup>2</sup> effects can be observed

Coefficients determined in a given approach describing the photon structure

$$\left[ \sigma_{\text{eff}}^{\gamma p}(Q^2) \right]^{-1} = \int d^2z_{\perp} \langle z_{\perp}^n \rangle_p \tilde{F}_2^p(z_{\perp}) \tilde{F}_2^{\gamma}(z_{\perp}; Q^2)$$

Mean value of the transverse distance between two partons in the PROTON

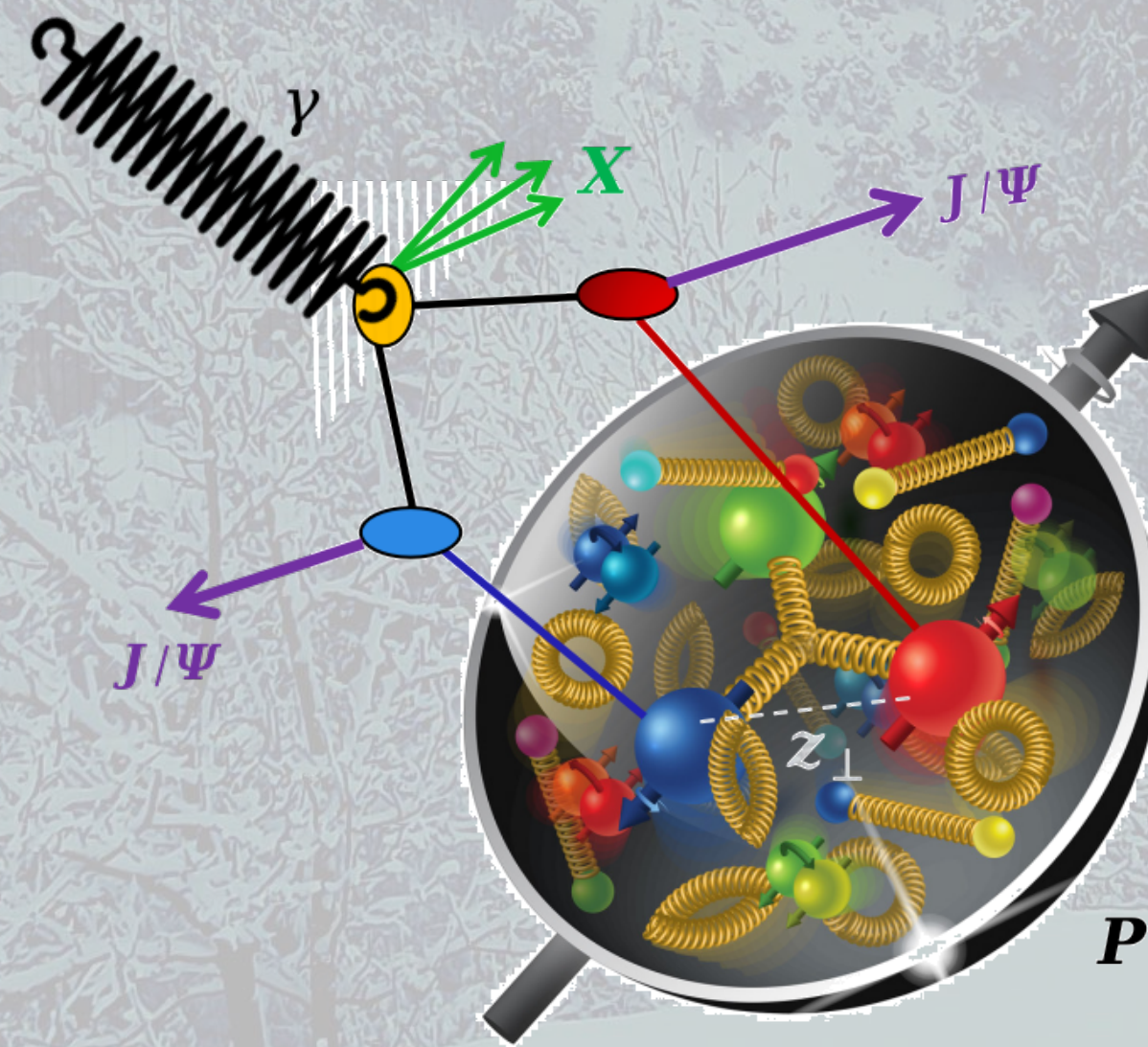
If we could measure  $\sigma_{\text{eff}}^{\gamma p}(Q^2)$  we could access NEW INFORMATION ON THE PROTON STRUCTURE



# Di $J/\psi$ photo-production@EIC

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

Illustration of DPS for  $\gamma + p \rightarrow J/\psi + J/\psi + X$



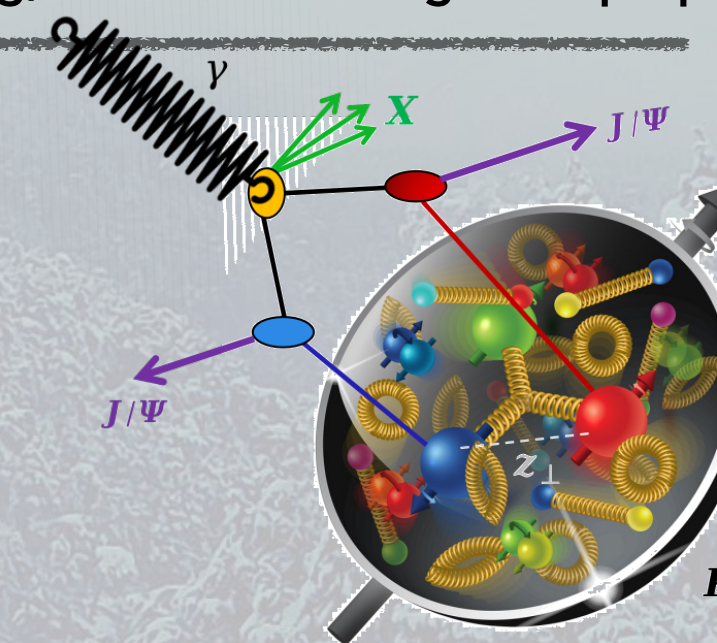
We consider the possibility of **resolved** photon to estimate the DPS cross section in quarkonium-pair photoproduction at the EIC



# Di $J/\psi$ photo-production@EIC

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

\*Slide from R. Sangem



$$\sigma_{SPS}^{(J/\psi, J/\psi)} \propto \sum_{a=g,q} \int dx_{p_a} f_{a/p}(x_{p_a}, \mu) d\hat{\sigma}^{\gamma a \rightarrow J/\psi + J/\psi + a} \quad \text{unresolved/direct}$$

$$\sigma_{SPS}^{(J/\psi, J/\psi)} \propto \sum_{a,b=g,q} \int dx_{\gamma_a} dx_{p_b} f_{a/\gamma}(x_{\gamma_a}, \mu) f_{b/p}(x_{p_b}, \mu) d\hat{\sigma}^{ab \rightarrow J/\psi + J/\psi} \quad \text{resolved}$$

$$\sigma_{DPS}^{(J/\psi, J/\psi)} \propto \frac{1}{2} \frac{1}{\sigma_{eff}^{\gamma p}} \sum_{a,b,c,d} \int dx_{\gamma_a} dx_{p_b} f_{a/\gamma}(x_{\gamma_a}, \mu) f_{b/p}(x_{p_b}, \mu) d\hat{\sigma}_{SPS}^{ab \rightarrow J/\psi}(x_{\gamma_a}, x_{p_b})$$

$$\times dx_{\gamma_c} dx_{p_d} f_{c/\gamma}(x_{\gamma_c}, \mu) f_{d/p}(x_{p_d}, \mu) d\hat{\sigma}_{SPS}^{cd \rightarrow J/\psi}(x_{\gamma_c}, x_{p_d})$$

Proton PDF

Photon PDF

Partonic x-sections

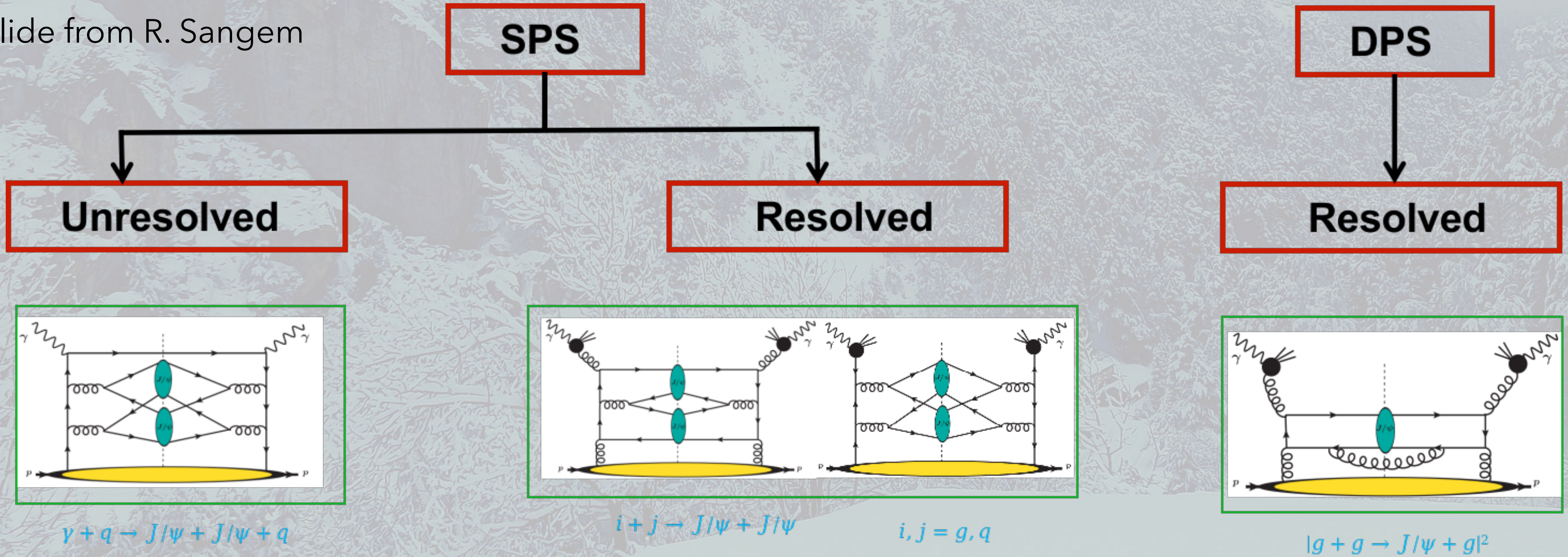
Single SPS resolved (namely same partonic cross section as hadroproduction)



# Di $J/\psi$ photo-production@EIC

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

\*Slide from R. Sangem



- GRV photon PDF is used [PRD 46, 1973 \(1992\)](#) , while CT18NLO PDF for proton [T.J. Hou et al., PRD 103, 014013 \(2021\)](#)
- HELAC-Onia latest version is used for generating matrix elements [HS Shao, CPC 184, 2562 \(2013\), 198, 238 \(2016\)](#)
- CO LDMEs are taken from [M. Butenschoen and B. A. Kniehl, PRD 84, 051501 \(2011\)](#)
- We expect at least 600 four-muon events with  $100 \text{ fb}^{-1}$  luminosity



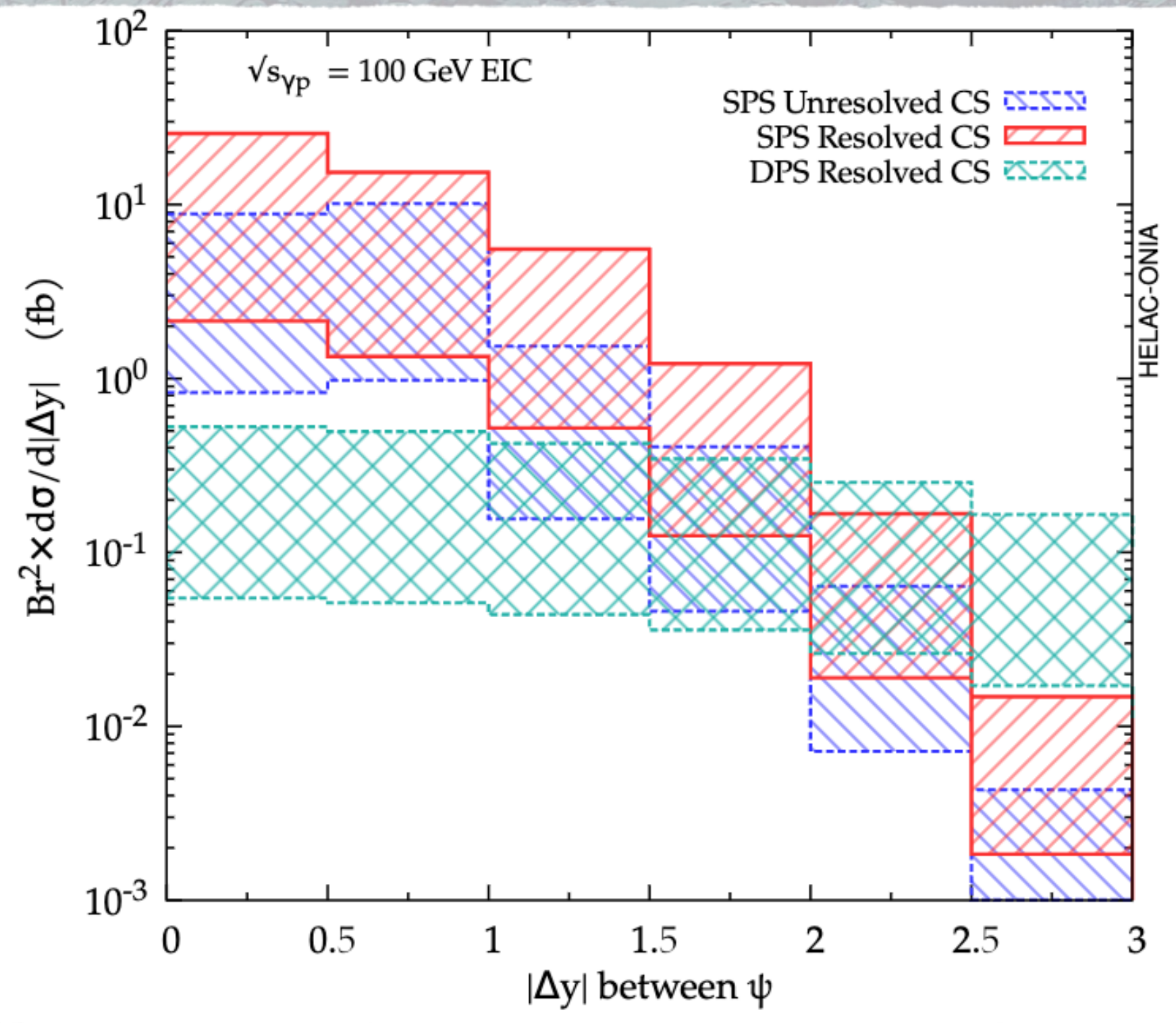
# Numerical Results

PRELIMINARY

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

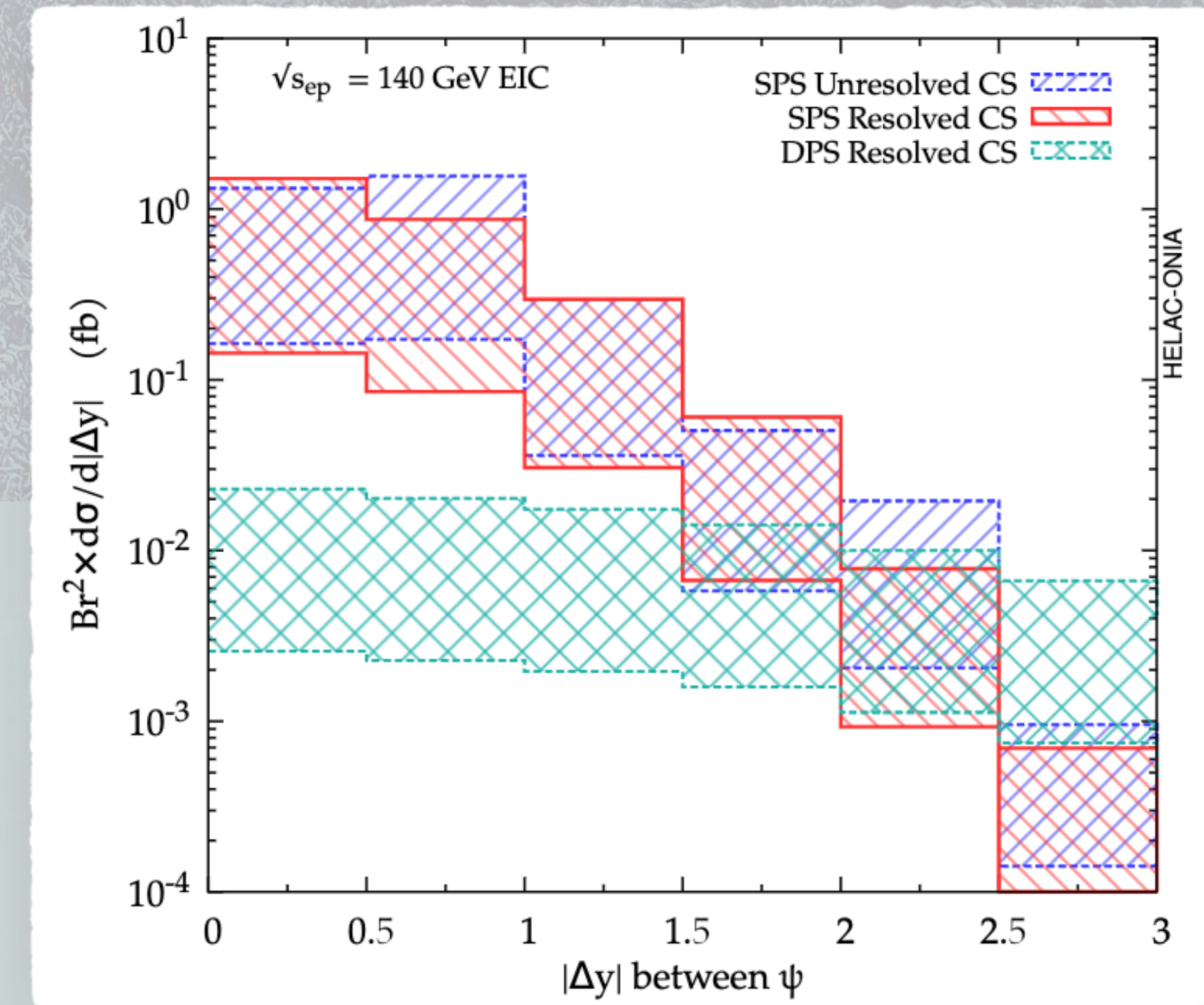
Absolute rapidity difference between the two  $J/\psi$

$$\sqrt{s_{\gamma p}} = 100 \text{ GeV}$$



- DPS dominates at high  $|\Delta y|$
- DPS is suppressed at low  $|\Delta y|$

$$\sqrt{s_{\gamma p}} = 140 \text{ GeV}$$





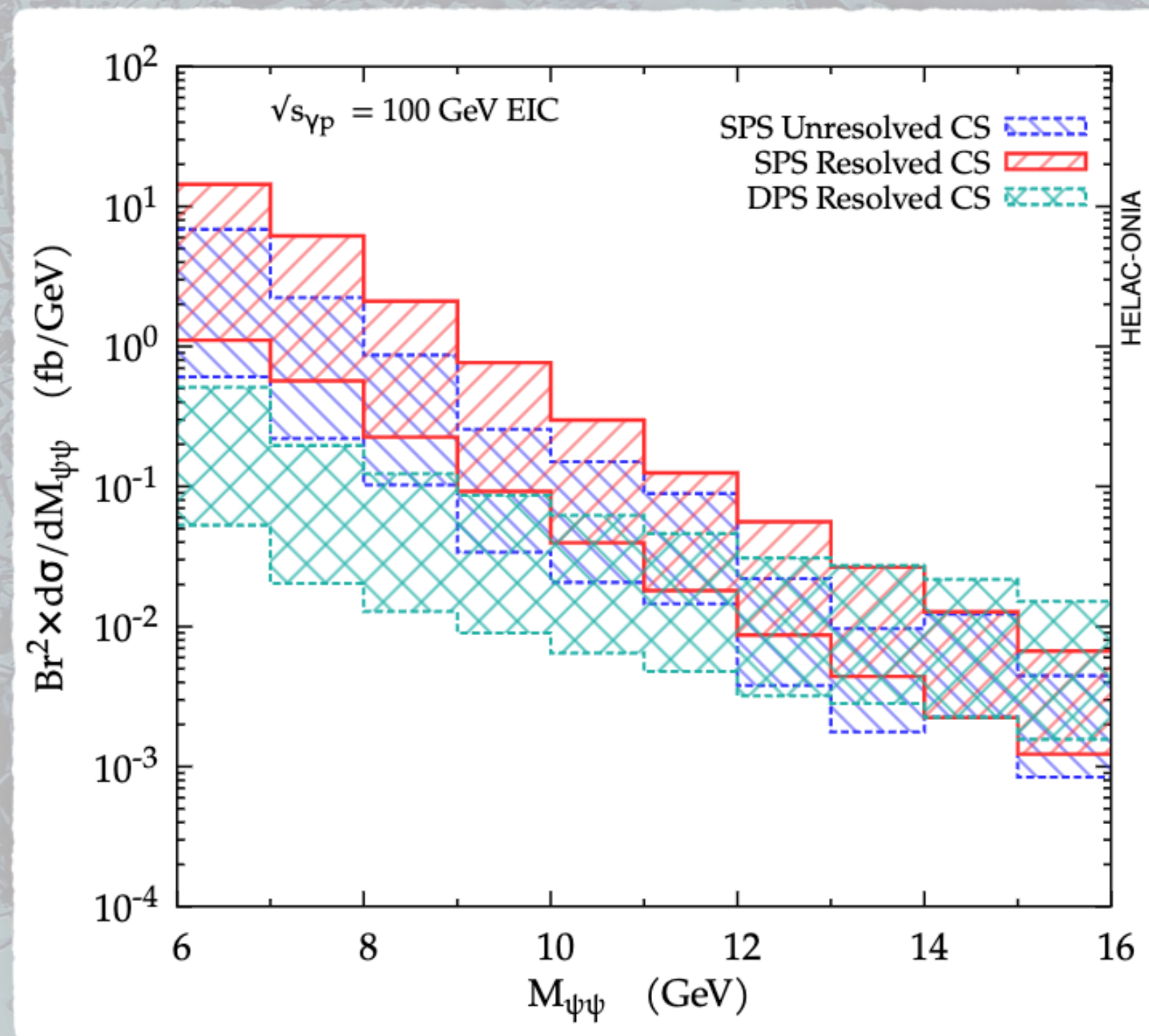
# Numerical Results

PRELIMINARY

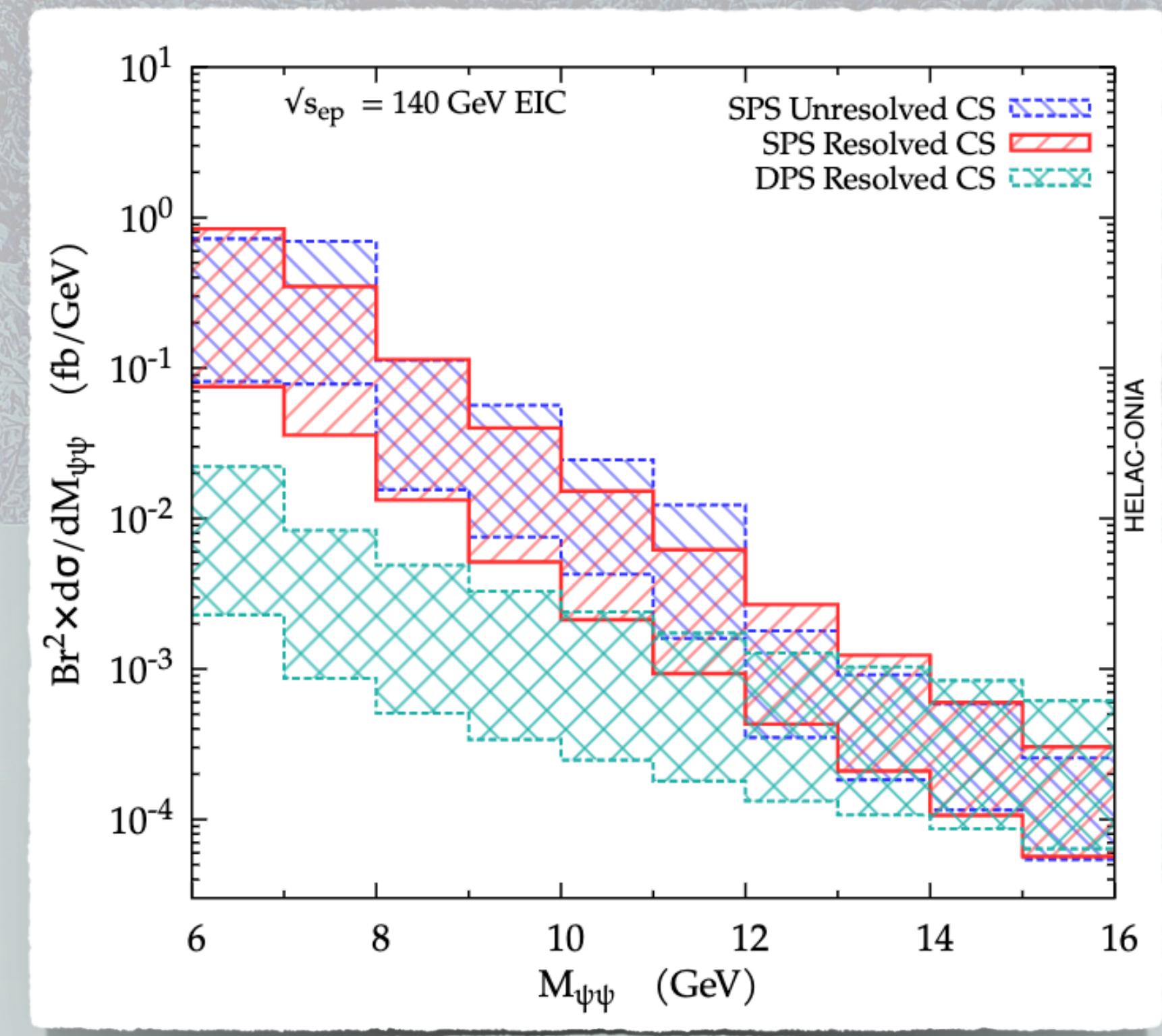
F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

Invariant mass of the  $J/\psi$  pair

$$\sqrt{s_{\gamma p}} = 100 \text{ GeV}$$



$$\sqrt{s_{\gamma p}} = 140 \text{ GeV}$$





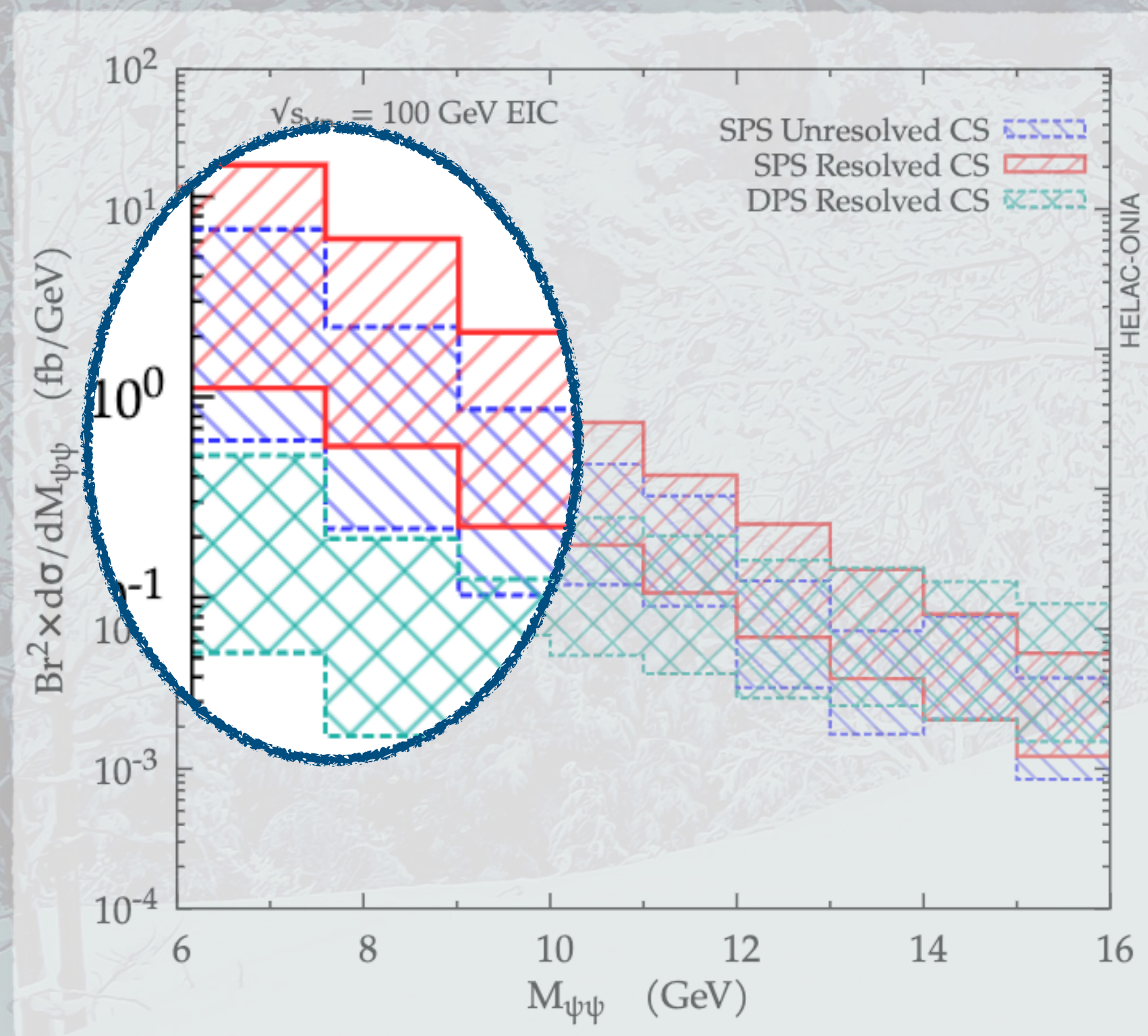
# Numerical Results

PRELIMINARY

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

Invariant mass of the  $J/\psi$  pair

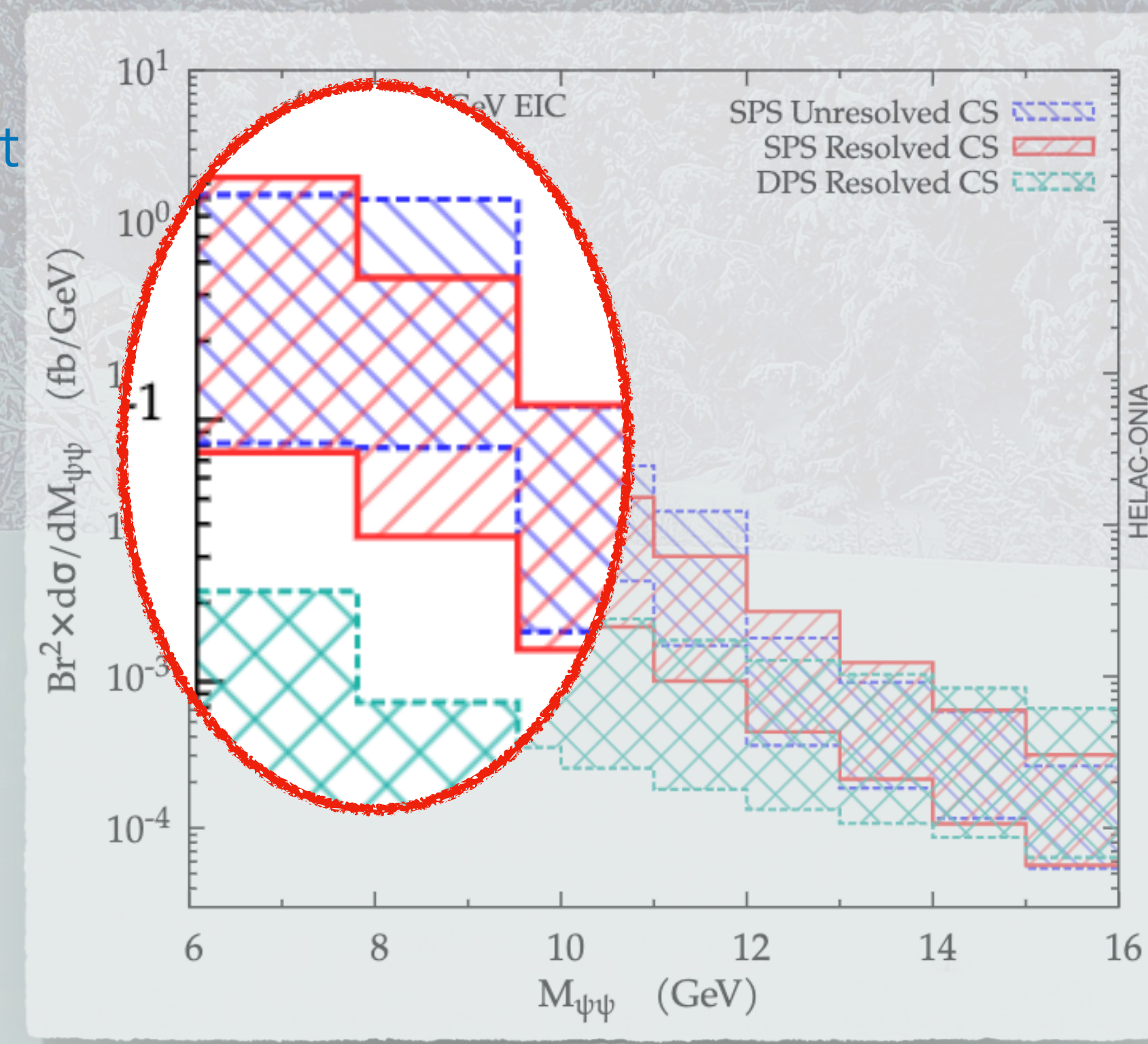
$$\sqrt{s_{\gamma p}} = 100 \text{ GeV}$$



a) at low invariant mass:

- DPS smaller than SPS, but not negligible
- DPS negligible

$$\sqrt{s_{\gamma p}} = 140 \text{ GeV}$$





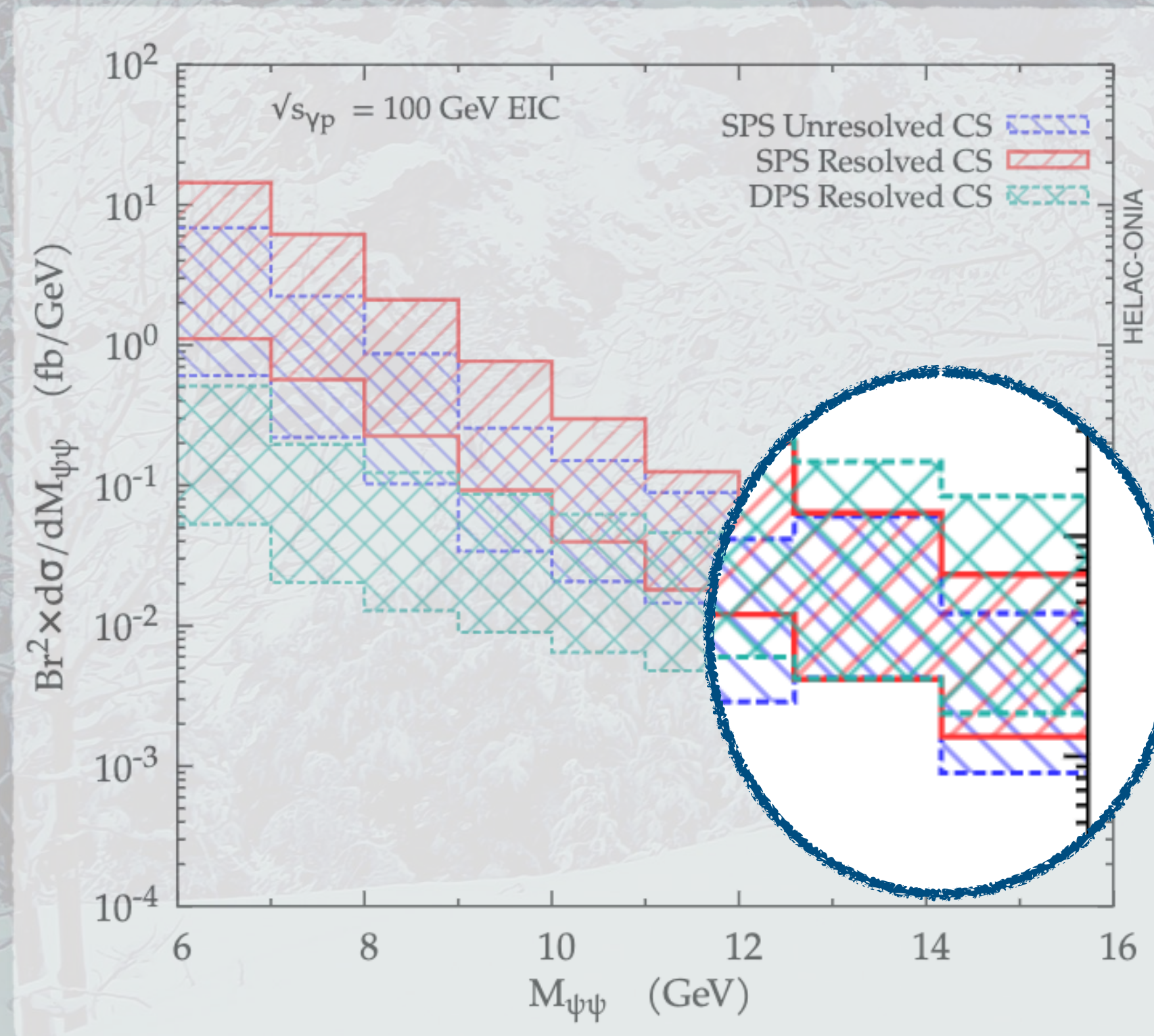
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PRELIMINARY

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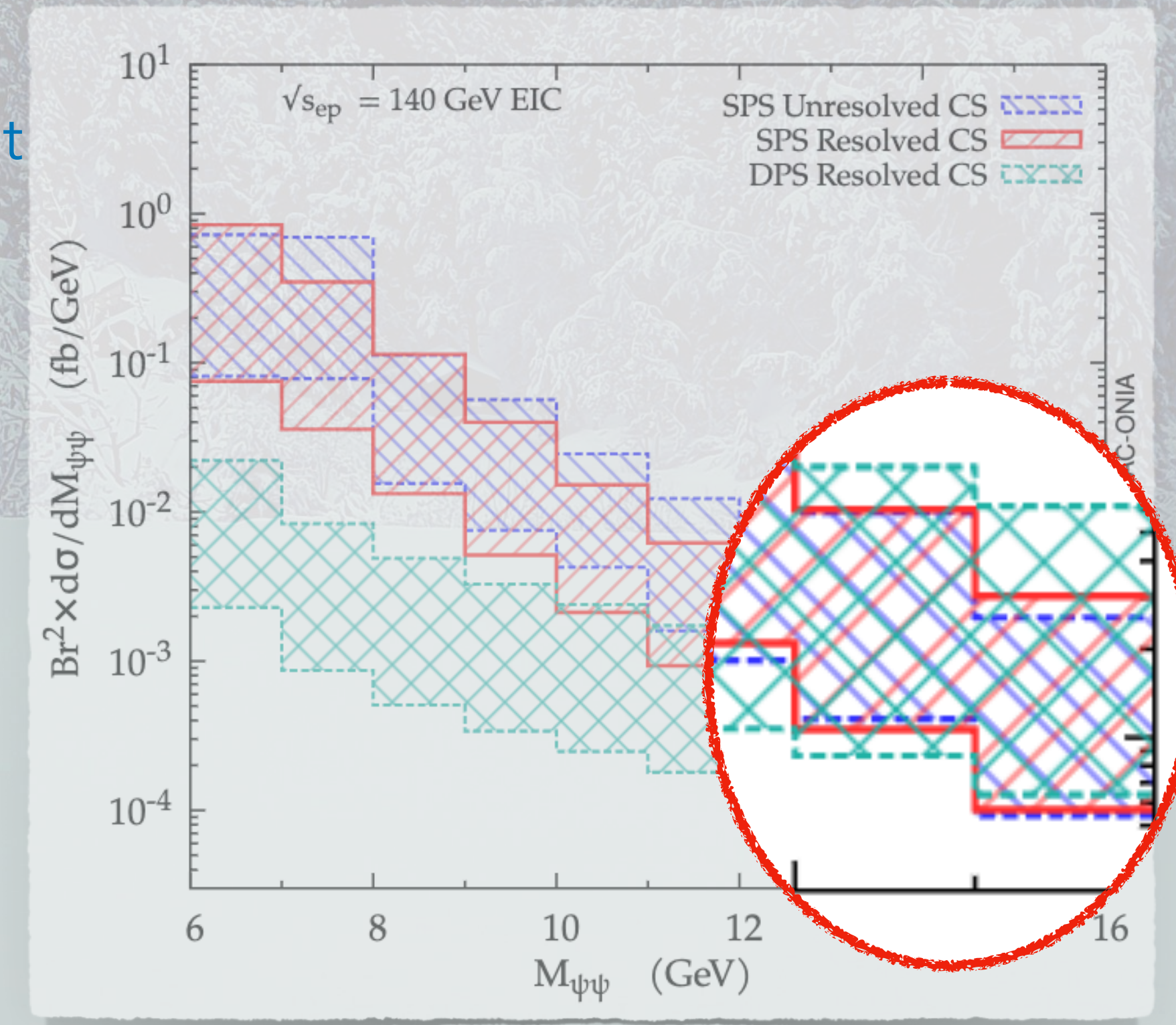
a) at low invariant mass:

- DPS smaller than SPS, but not negligible
- DPS negligible

b) at low invariant mass:

- DPS bigger than SPS
- DPS similar to SPS

$$\sqrt{s_{\gamma p}} = 140 \text{ GeV}$$

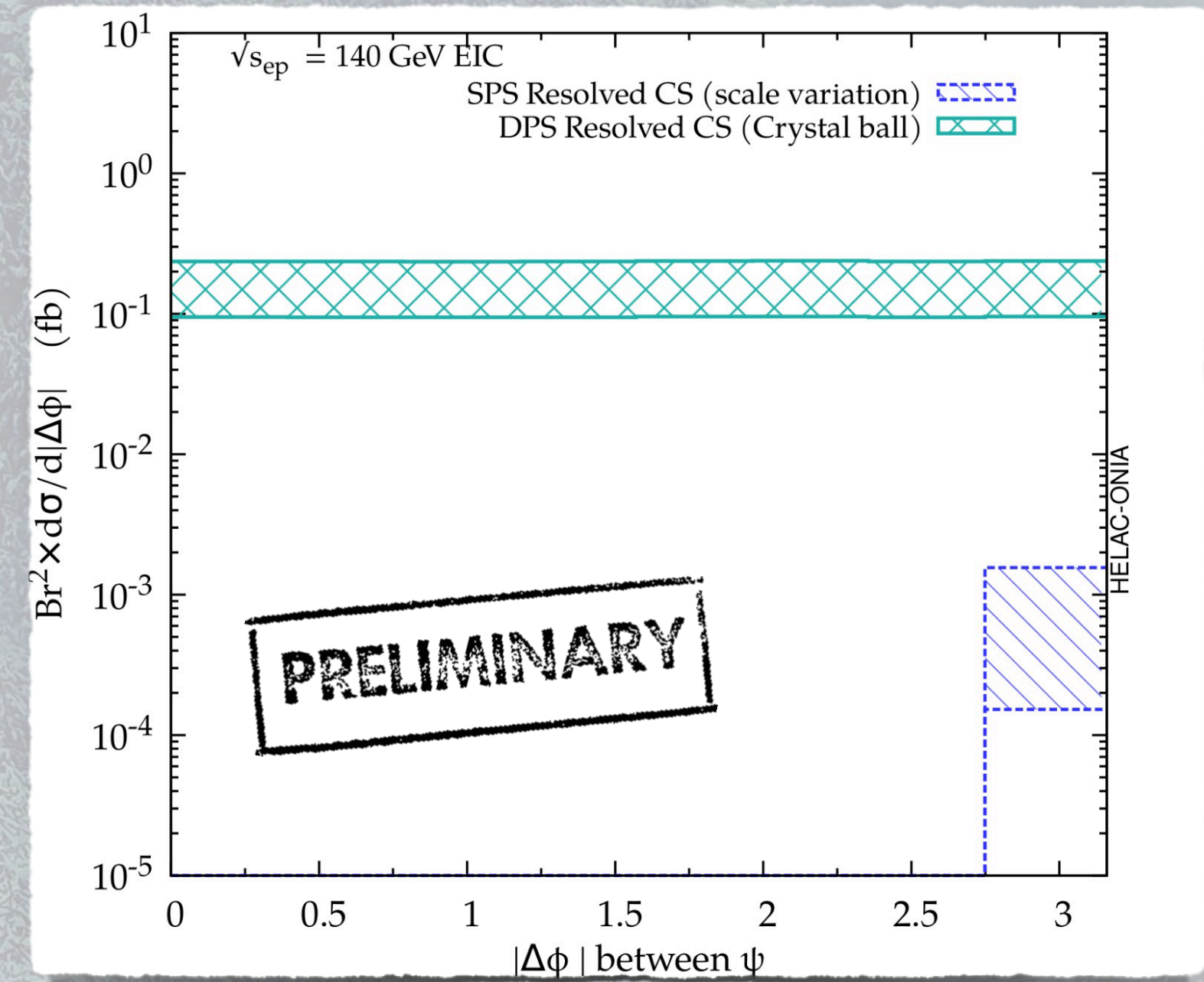
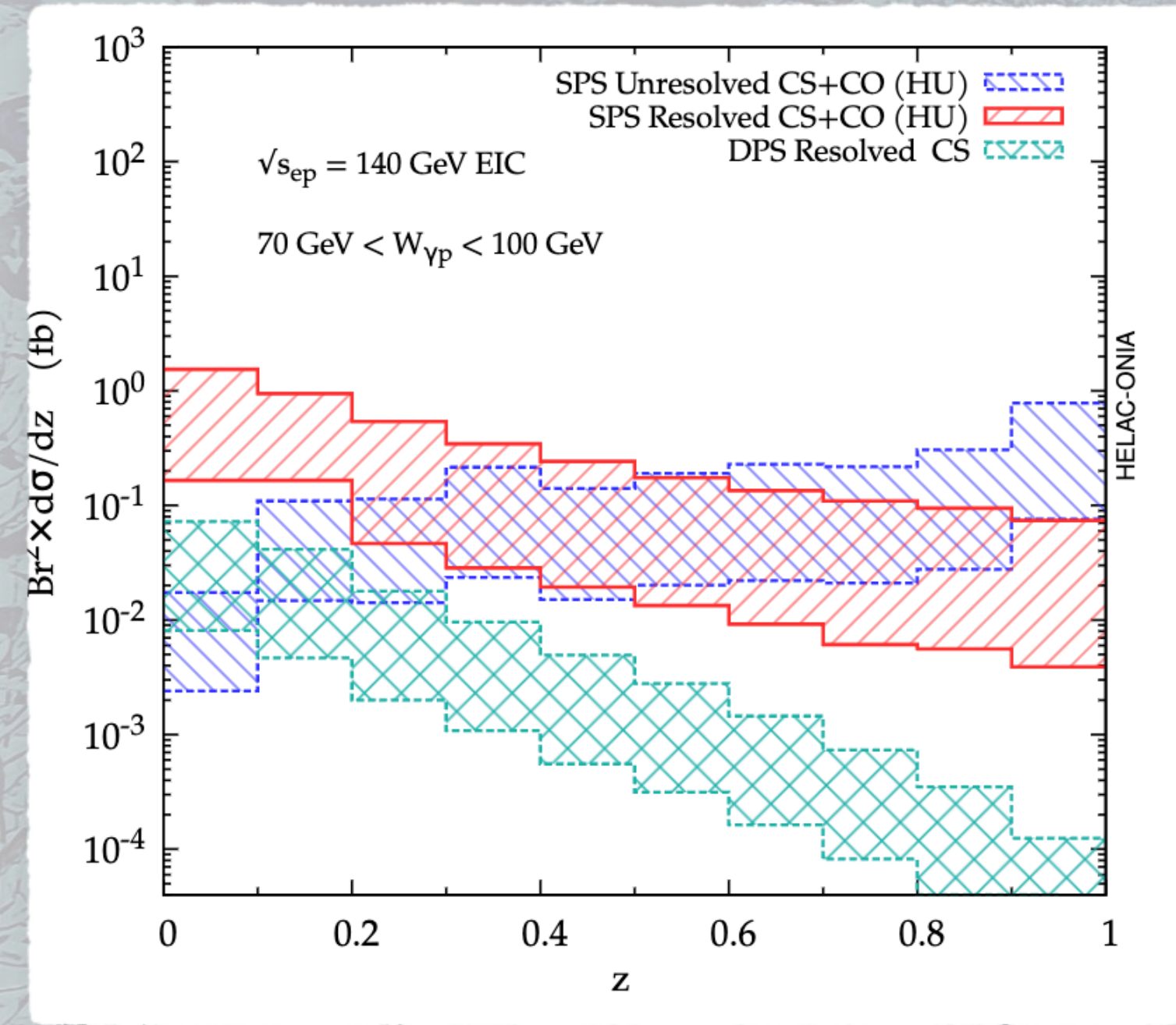




# Numerical Results

PRELIMINARY

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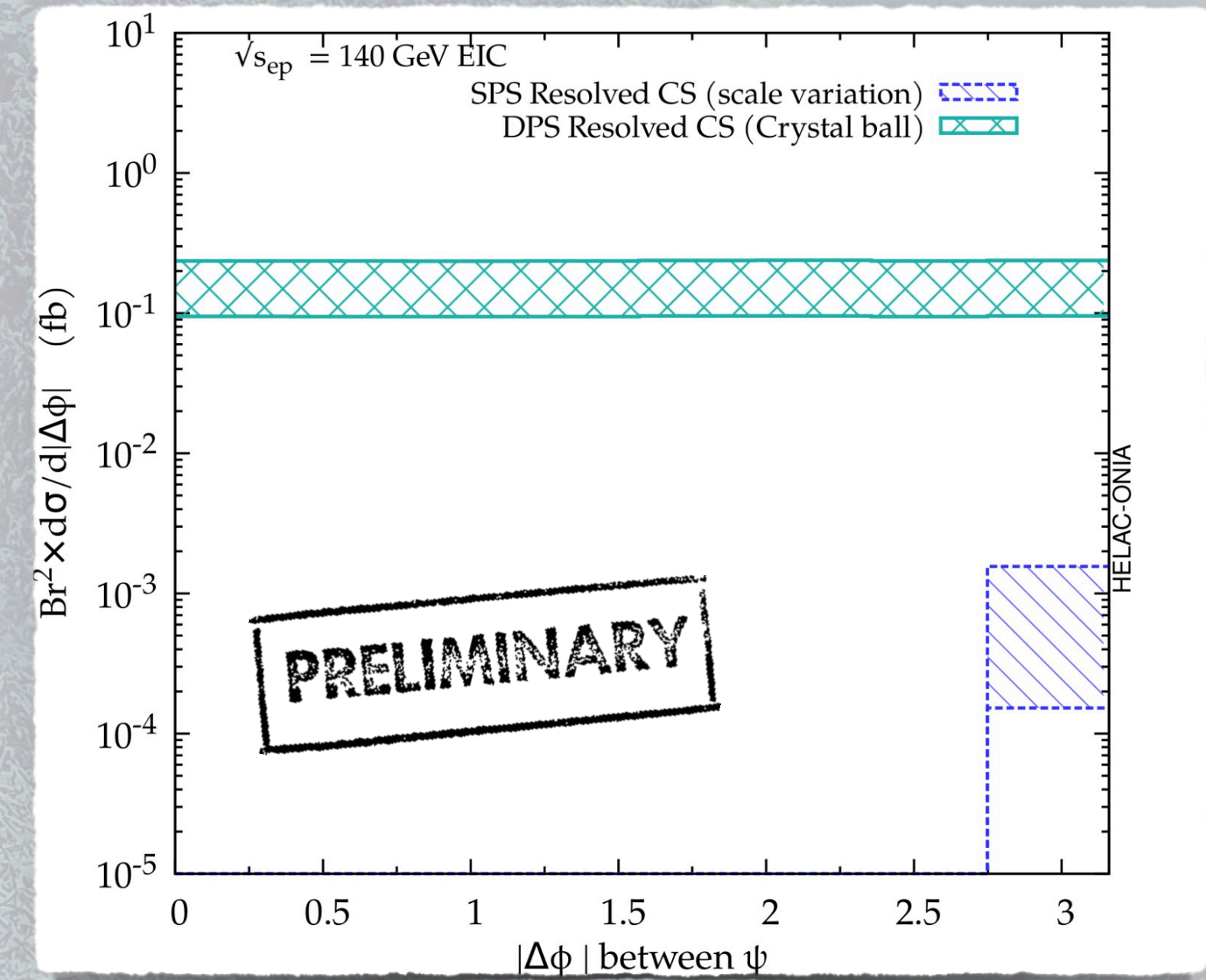
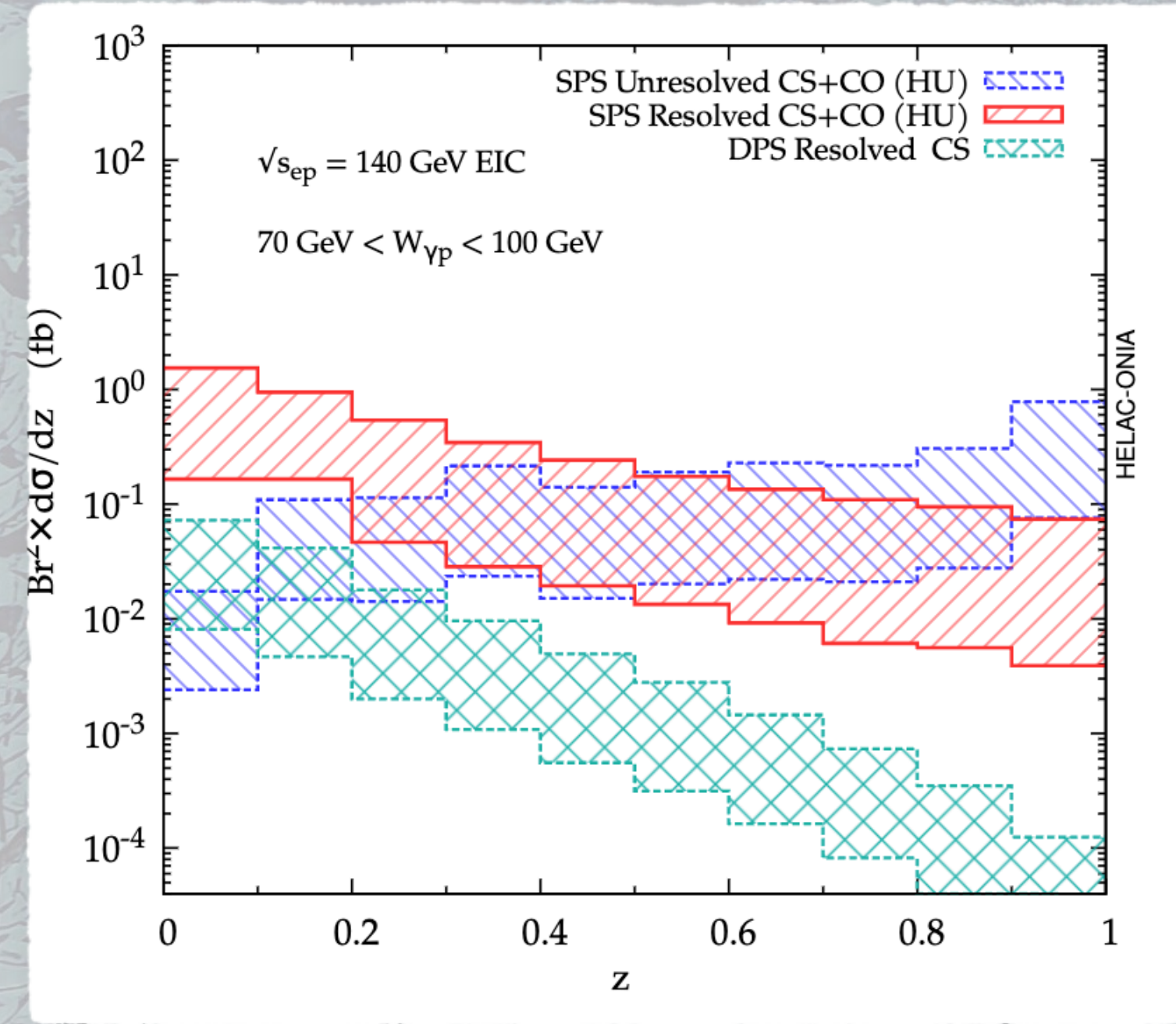
\* for  $z < 0.1$ , SPS resolved dominates → unique opportunity to investigate the PHOTON structure



# Numerical Results

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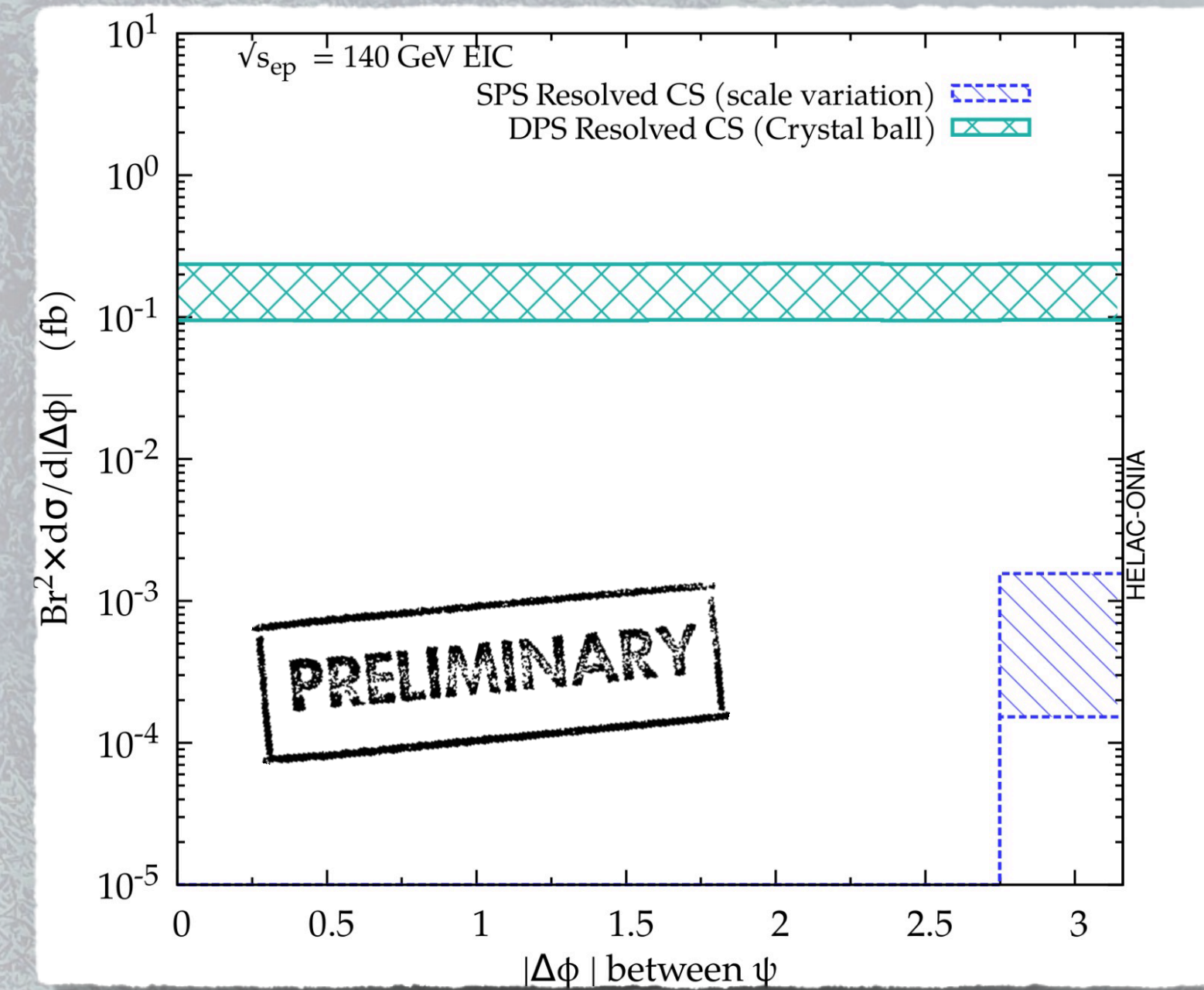
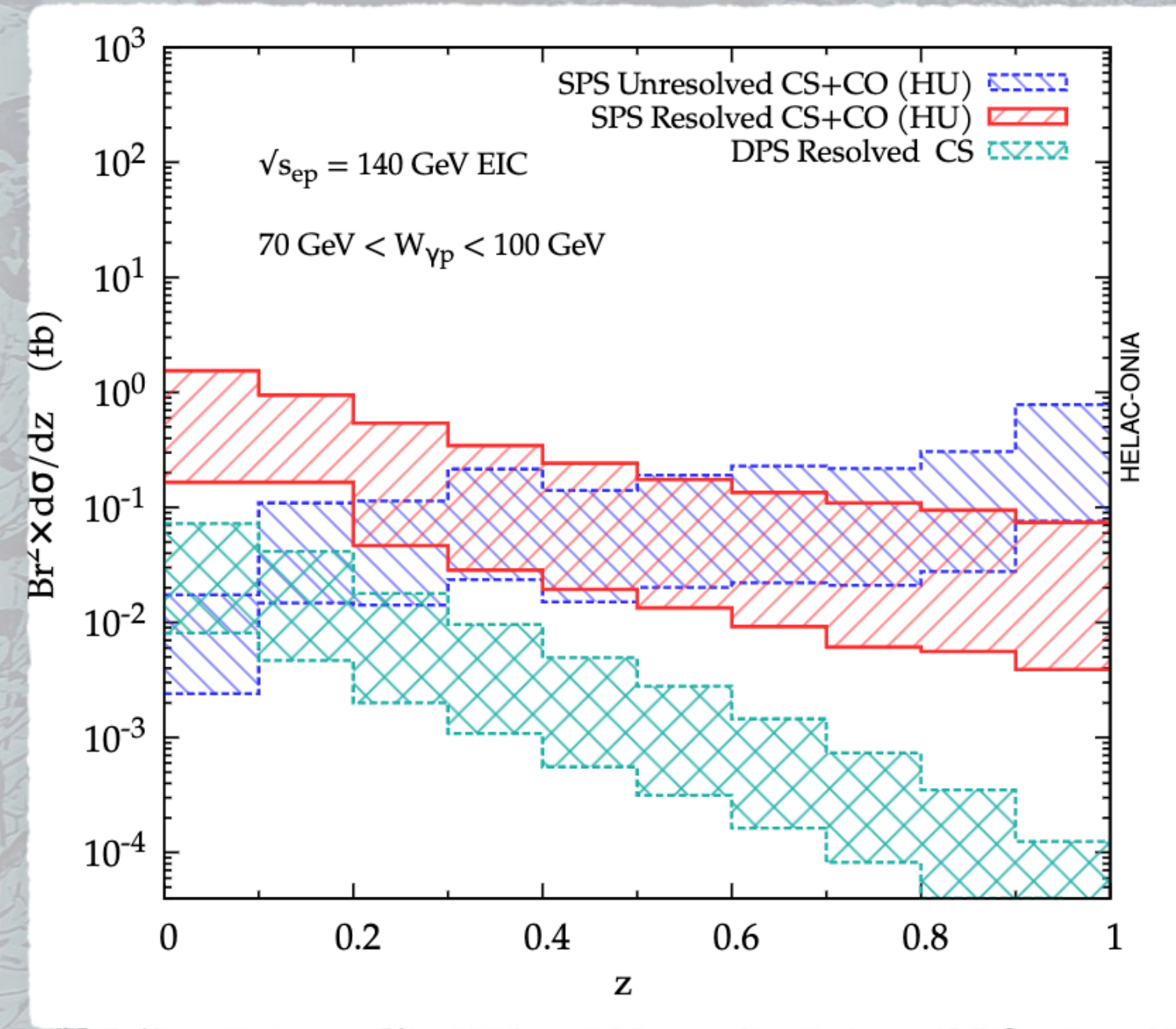
\* for high  $z$ , the direct SPS contribution dominates  $\longrightarrow$  we test the quarkonia production via direct photoproduction



# Numerical Results

PRELIMINARY

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.



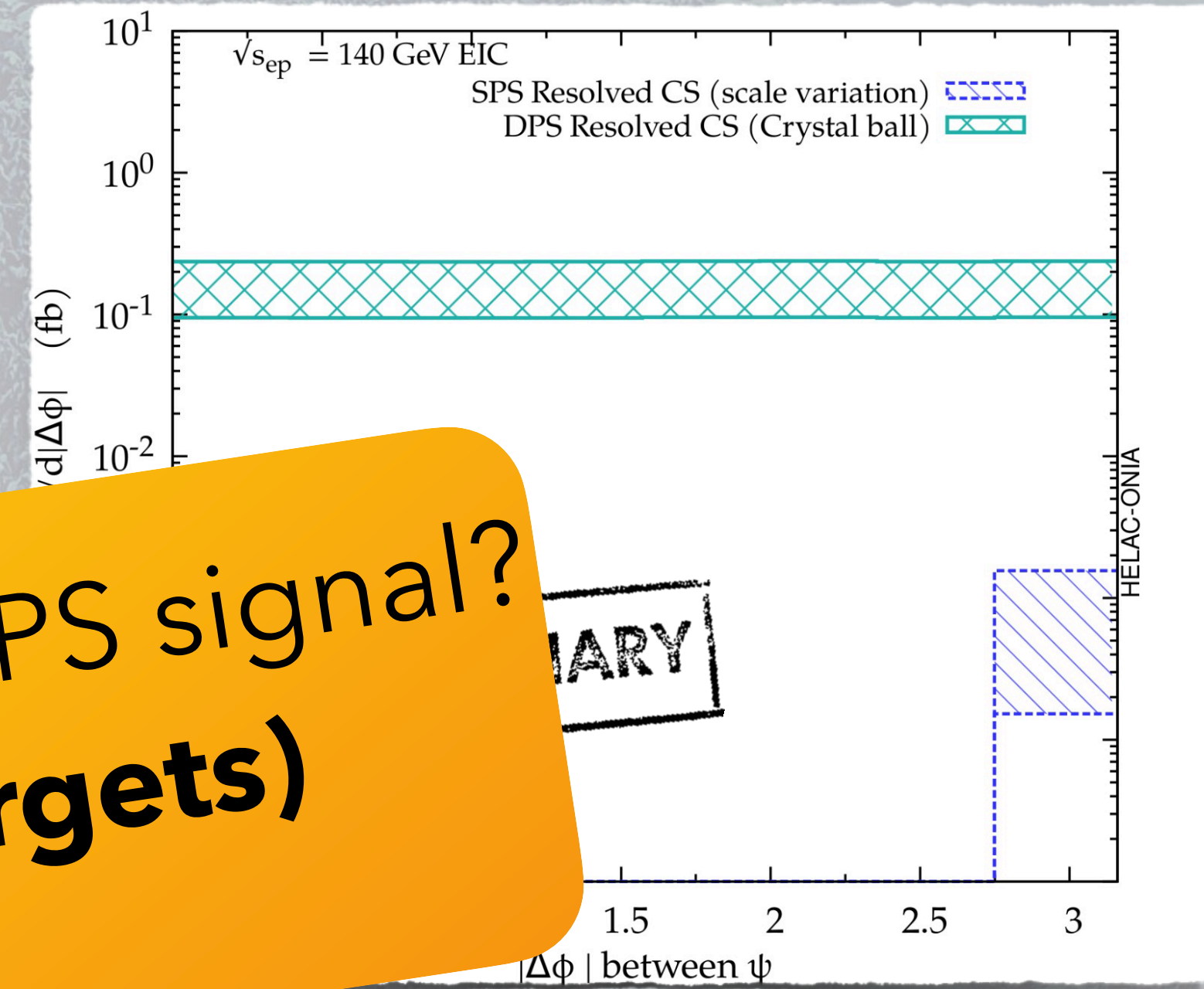
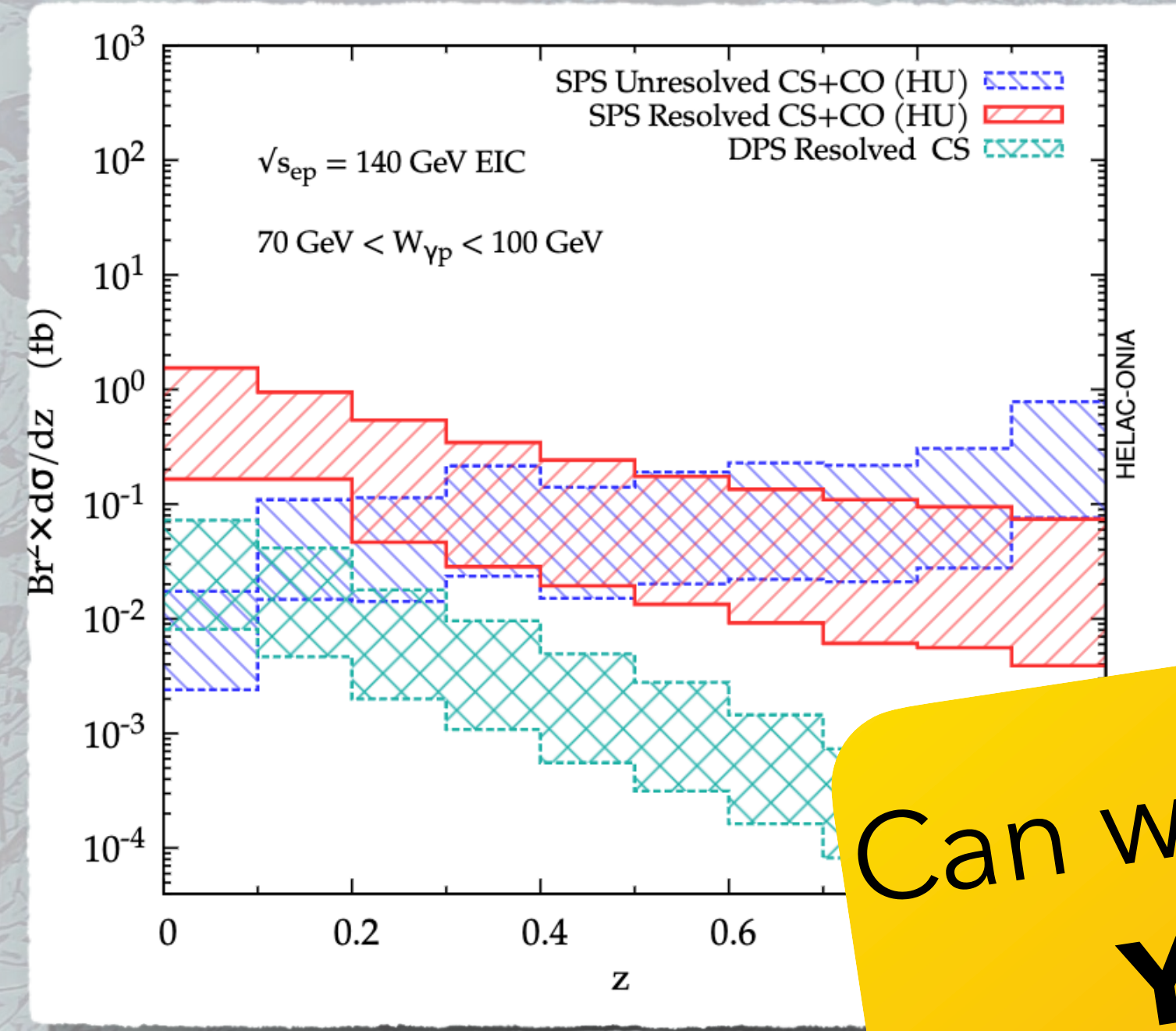
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- \* as for DPS studies @LHC, the cross-section dependence on the relative azimuthal angle is relevant to access the DPS contribution!



# Numerical Results

PRELIMINARY

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.



Can we increase the DPS signal?  
**YES! (Nuclear targets)**

- \* for  $z < 0.1$ , SPS resolved dominates → unique opportunity to investigate the PHOTON structure
- \* for high  $z$ , the direct SPS contribution dominates → we test the quarkonia production via direct photoproduction
- \* as for DPS studies @LHC, the cross-section dependence on the relative azimuthal angle is relevant to access the DPS contribution!



# DPS in pA collisions

For DPS in pA and AA collisions the following references were missing:

- 1) Same-sign WW production in proton-nucleus collisions at the LHC as a signal for double parton scattering  
**D. d'E. & A. Snigirev, PLB 718 (2013) 1395-1400**
- 2) Enhanced  $J/\psi$ / $\Psi$ -pair production from double parton scatterings in nucleus-nucleus collisions at the Large Hadron Collider  
**D. d'E. & A. Snigirev, PLB 727 (2013) 157-162**
- 3) Pair production of quarkonia and electroweak bosons from double-parton scatterings in nuclear collisions at the LHC  
**D. d'E. & A. Snigirev, Nucl. Phys. A 931 (2014) 303-308**

and for TPS:

Triple-parton scatterings in proton-nucleus collisions at high energies  
**D. d'E. & A. Snigirev, EPJC 78 (2018) 5, 359**



# DPS in pA collisions

---

$$F_{a_1 a_2}(x_1, x_2, \mathbf{y}_\perp) = 2p^+ \int \frac{dz_1^-}{2\pi} \frac{dz_2^-}{2\pi} dy^- e^{i(x_1 z_1^- + x_2 z_2^-)p^+} \\ \times \langle A | \mathcal{O}_{a_2}(0, z_2) \mathcal{O}_{a_1}(y, z_1) | A \rangle$$

In this case we have two mechanisms that contribute:



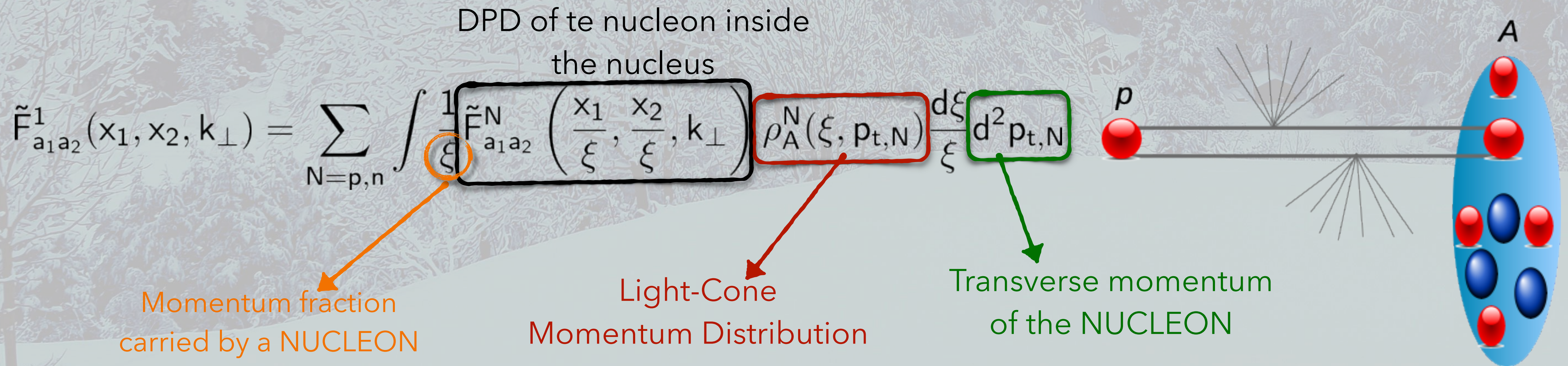
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B. Blok et al, EPJC (2013) 73:2422

**DPS 1:** The two partons belong to the SAME nucleon in the nucleus!





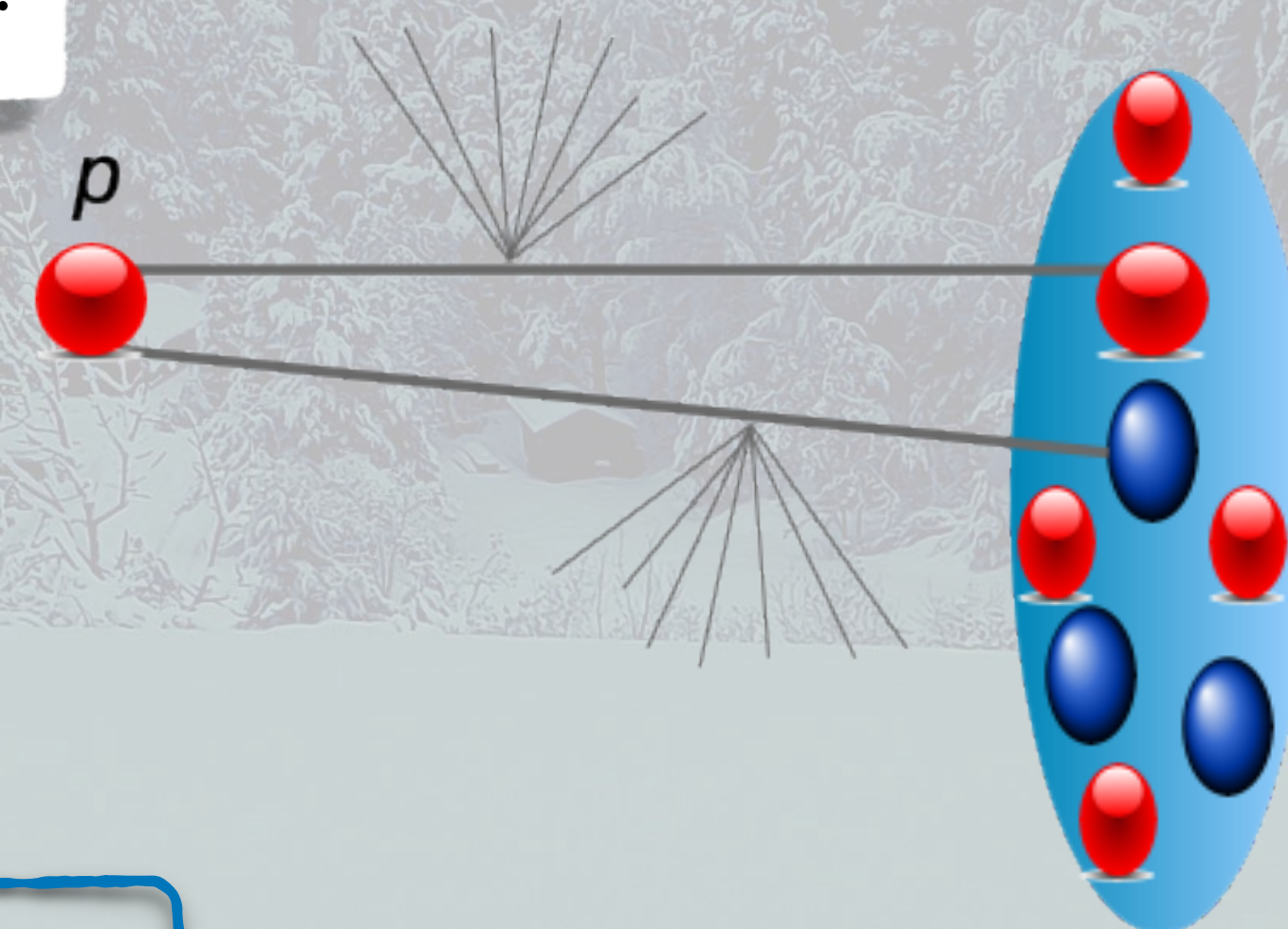
# DPS in pA collisions

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B. Blok et al, EPJC (2013) 73:2422

**DPS 2:** The two partons belong to the DIFFERENT nucleons in the nucleus!



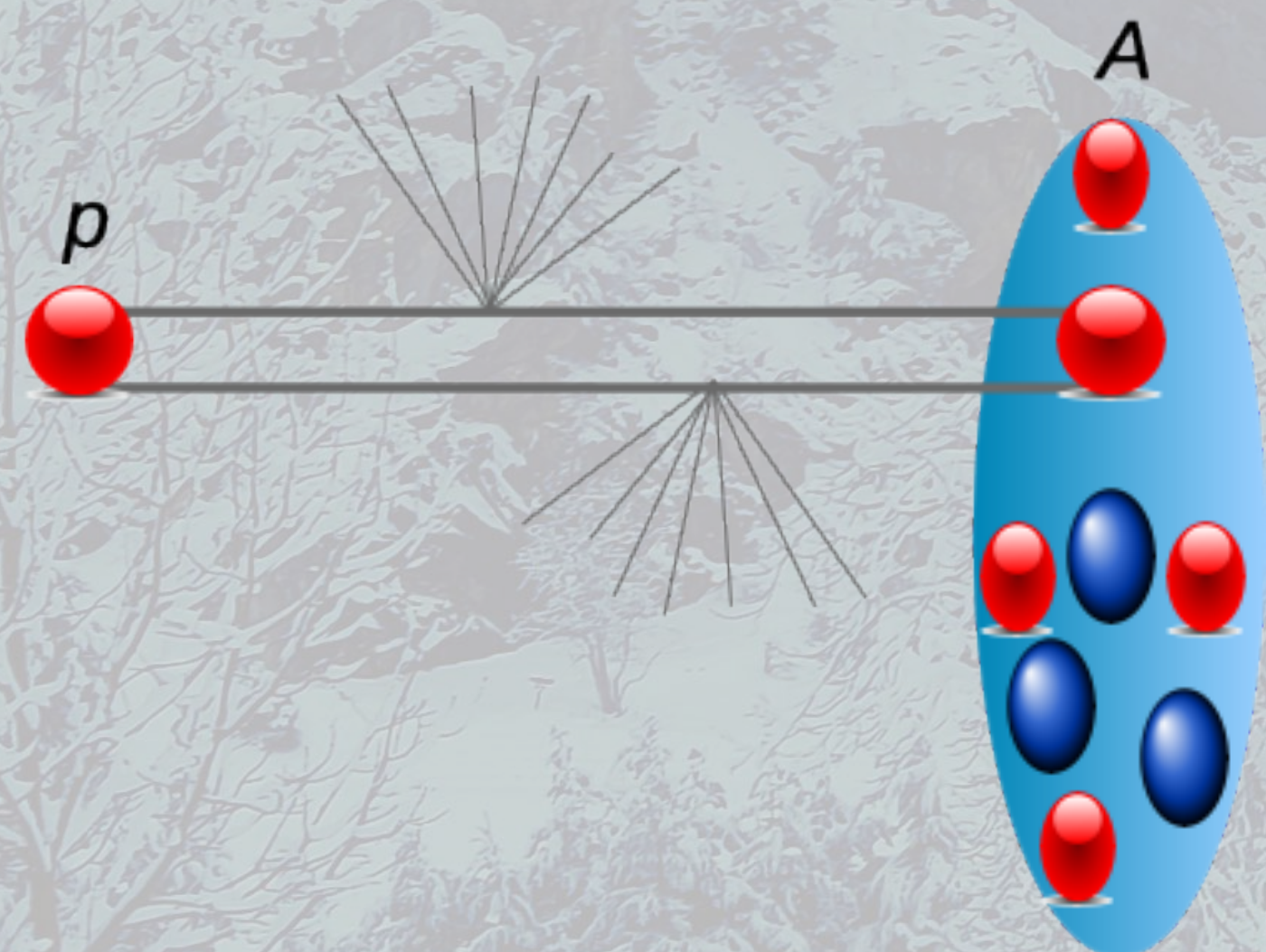
$$\tilde{F}_{a_1 a_2}^2(x_1, x_2, \vec{k}_\perp) \propto \int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 p_{ti}}{\xi_i} \delta\left(\sum_i \xi_i - A\right) \delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right) \psi_A^*(\xi_1, \xi_2, \mathbf{p}_{t1}, \mathbf{p}_{t2}, \dots) \times \psi_A(\xi_1, \xi_2, \mathbf{p}_{t1} + \vec{k}_\perp, \mathbf{p}_{t2} - \vec{k}_\perp, \dots) G_{a_1}^{N_1}(x_1/\xi_1, |\vec{k}_\perp|) G_{a_2}^{N_2}(x_2/\xi_2, |\vec{k}_\perp|)$$

Nucleus wf

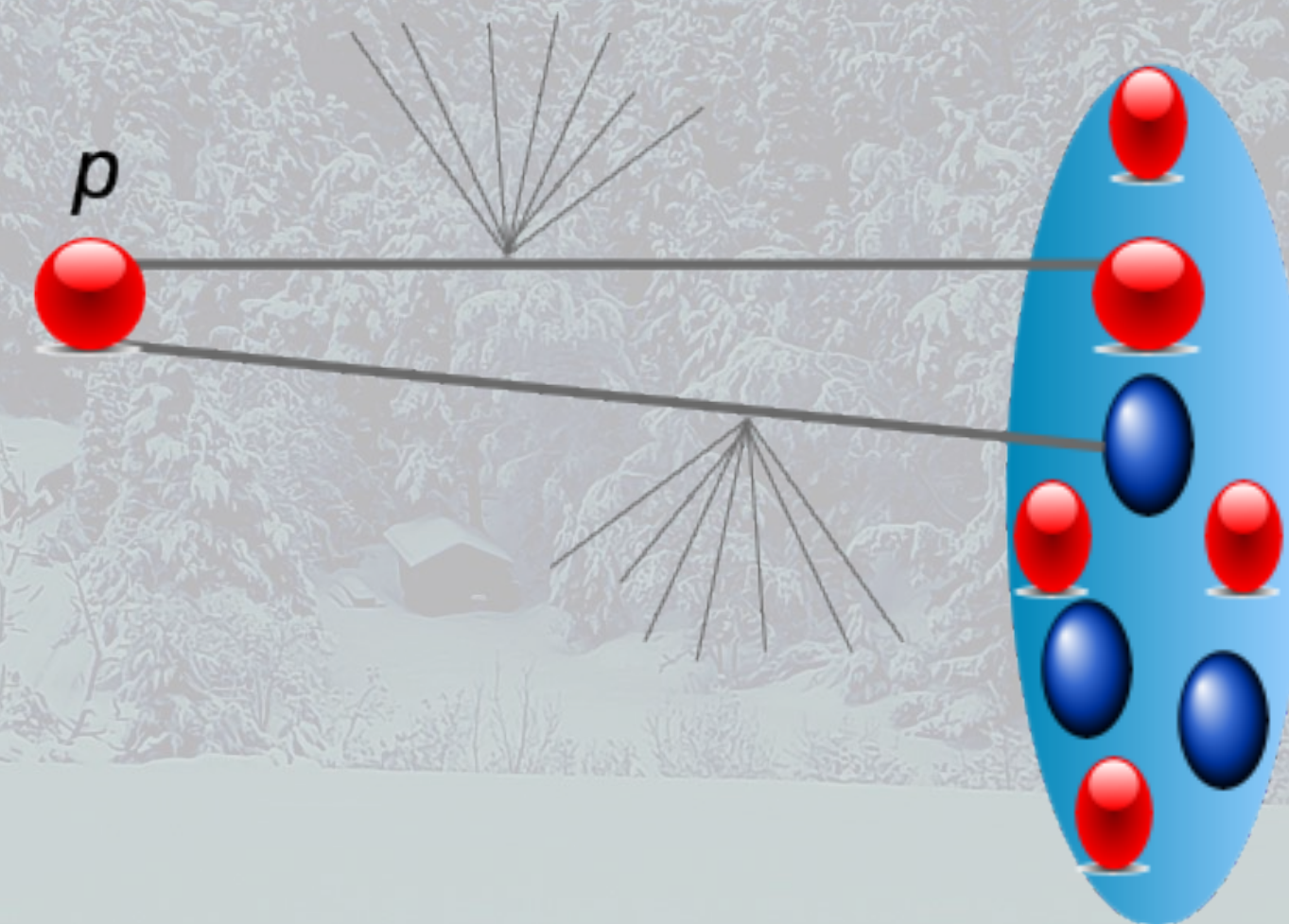
Nucleon GPD



# DPS in pA collisions



$$\sigma_{\text{DPS}2} \sim A^{1/3} \sigma_{\text{DPS}1}$$
$$\sigma_{\text{DPS}1} \sim A \sigma_{\text{DPS}}^{\text{pp}}$$






# DPS in $\gamma A$ collisions with light nuclei?

M.R. in progress

In p-Pb collisions there are some difficulties (personal view):


- 1) both cross-sections (DPS1 and DPS2) depends on proton DPD (still almost unknown) therefore both mechanisms are very important  could be difficult to extract some information on the proton DPD
- 2) for heavy nuclei is difficult to perform calculation with wave-function obtained from realistic potentials



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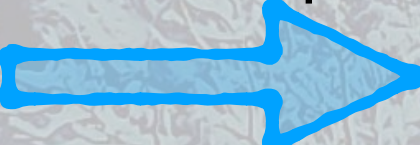
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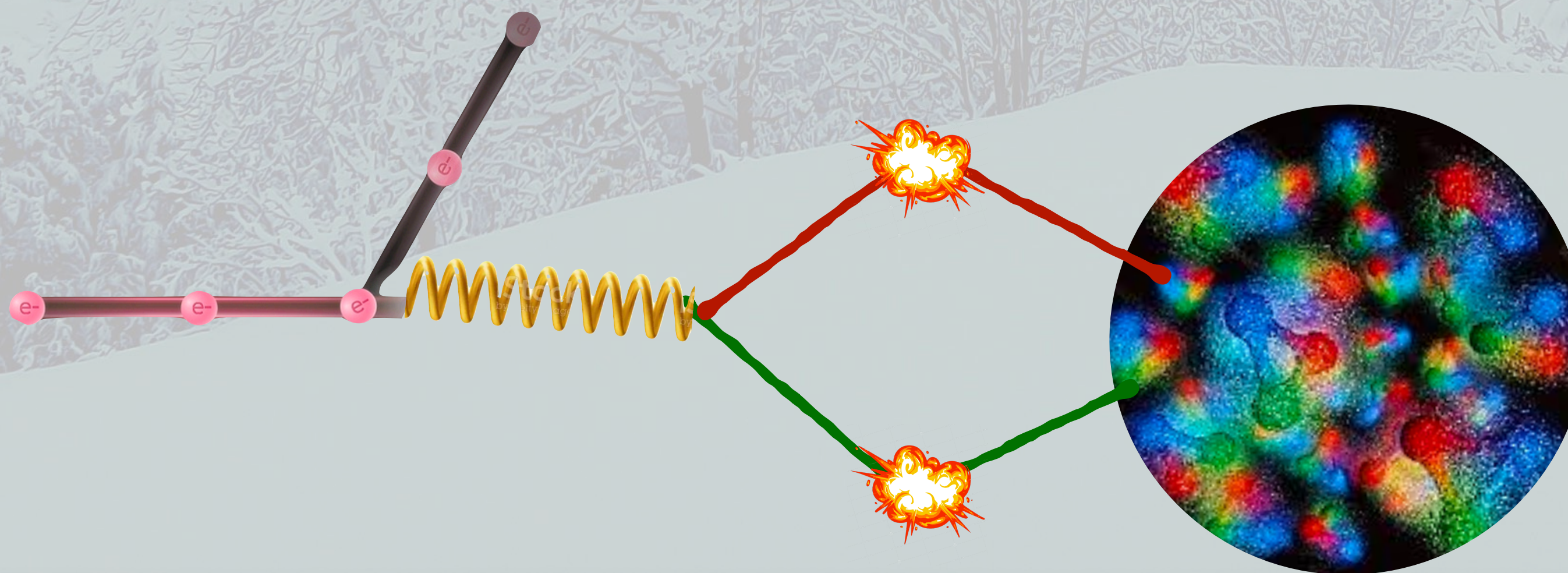
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## POSSIBLE SOLUTION?







# DPS in $\gamma A$ collisions with light nuclei?

M.R. in progress

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- 2) for heavy nuclei is difficult to perform calculation with wave-function obtained from realistic potentials

## POSSIBLE SOLUTION?

- 1) In  $\gamma A$  the DPS2 will not contain any DPD of the proton  this mechanism can now be viewed as a background that can be evaluated if we properly treat the photon (as previously discussed) and the Nuclear geometry
- 2) For light nuclei these calculations can be done starting from realistic wave-function (Av18 or chiral potential)!

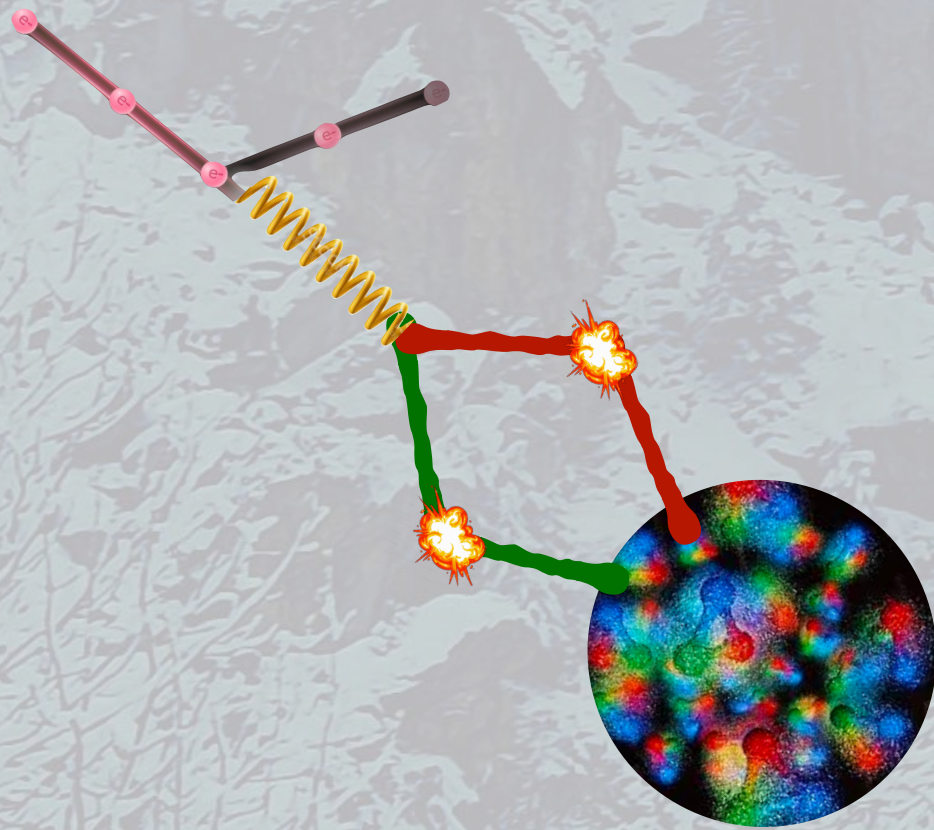
**Could we access the DPD of bound nucleons? Double EMC effect?**



# DPS in $\gamma A$ collisions with light nuclei?

M.R. in progress

For example in DPS1:



$$\tilde{F}_{a_1 a_2}^1(x_1, x_2, k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}_{a_1 a_2}^N \left( \frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp \right) \rho_A^N(\xi, p_{t,N}) \frac{d\xi}{\xi} d^2 p_{t,N}$$

The nuclear light-cone distribution can be evaluated with realistic wave-function (from Av18 +UIV potential) in a fully relativistic and Poincaré covariant approach for:

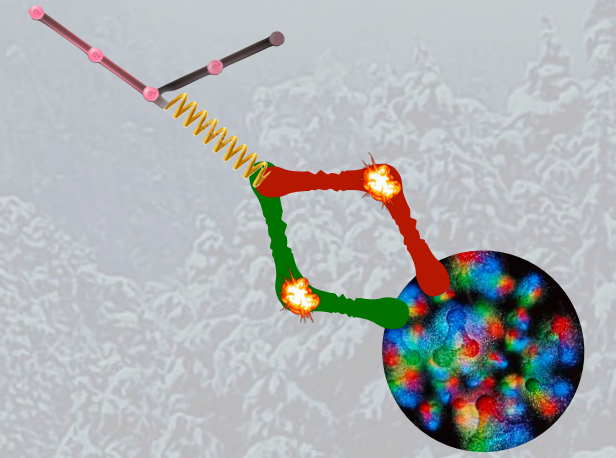
- 1)  $H^2$  in **E. Pace and G. Salmé, TNPI2000 (2001), arXiv:nucl-th/0106004**
- 2)  $He^3$  in e.g. **A. Del Dotto et al, PRC 95, 014001 (2017), M.R. et al, PLB 839 (2023), 137810**
- 3)  $He^4$  from **F. Fornetti, E. Pace, M. R., G. Salmé, S. Scopetta and M. Viviani, PLB submitted**



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M.R. in progress

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Let us check sum rules:

$$\int_0^1 dx_1 \int_0^{1-x_1} dx_2 F_{i_1 i_2}(x_1, x_2, k_\perp = 0) = \begin{cases} N_{i_1} N_{i_2} & \text{for } i_1 \neq i_2 \\ (N_{i_1} - 1) N_{i_2} & \text{for } i_1 = i_2 \end{cases}$$

Gaunt's sum rules

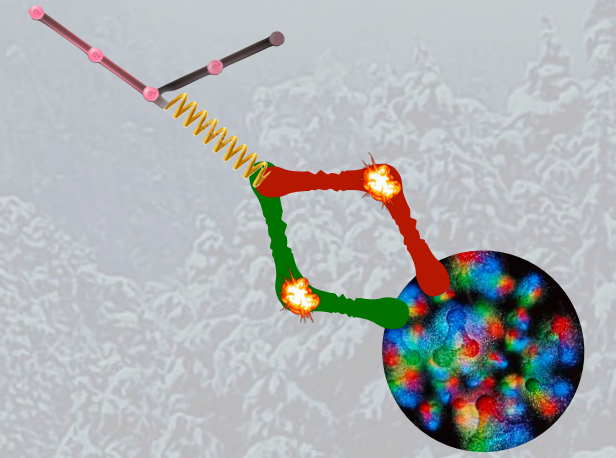
**J. R. Gaunt and W. J. Stirling,**  
**JHEP 03, 005 (2010)**



# DPS in $\gamma A$ collisions with light nuclei?

M.R. in progress

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$$\tilde{F}_{a_1 a_2}^1(x_1, x_2, k_\perp) = \sum_{N=p,n} \int \frac{1}{\xi} \tilde{F}_{a_1 a_2}^N \left( \frac{x_1}{\xi}, \frac{x_2}{\xi}, k_\perp \right) \rho_A^N(\xi, p_{t,N}) \frac{d\xi}{\xi} d^2 p_{t,N}$$



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Gaunt's sum rules

**J. R. Gaunt and W. J. Stirling,  
JHEP 03, 005 (2010)**

**However for the nuclear case one needs also the DPS2**



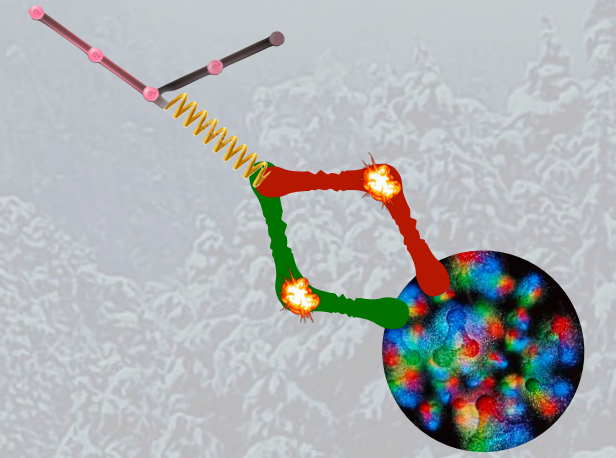
**Thus we can introduce approximated partial sum rules (APSR)**



# DPS in $\gamma A$ collisions with light nuclei?

M.R. in progress

For example in DPS1: 
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$$\int_0^1 dx_1 \int_0^{1-x_1} dx_2 F_{i_1 i_2}(x_1, x_2, k_\perp = 0) = \begin{cases} N_{i_1} N_{i_2} & \text{for } i_1 \neq i_2 \\ (N_{i_1} - 1) N_{i_2} & \text{for } i_1 = i_2 \end{cases}$$

Gaunt's sum rules

**J. R. Gaunt and W. J. Stirling,  
JHEP 03, 005 (2010)**

**APSR:** Since  $f_n^A(\xi) = \int d^2 p_{t,N} \rho_A^N(\xi, p_{t,N})$  is peaked around  $1/A$

$$\int_0^A dx_1 \int_0^{A-x_1} dx_2 \tilde{F}_{i_1 i_2}^{A,1}(x_1, x_2, 0) \sim \sum_{n=N,P} \int d\xi f_n^A(\xi)$$

$$\begin{cases} (N_{i_1}^n - 1) N_{i_2}^n & i_1 = i_2 \\ N_{i_1}^n N_{i_2}^n & i_1 \neq i_2 \end{cases}$$

Gaunt's sum rules for the nucleon DPD: numbers of quarks with given flavor  $i$  in the nucleon  $n$

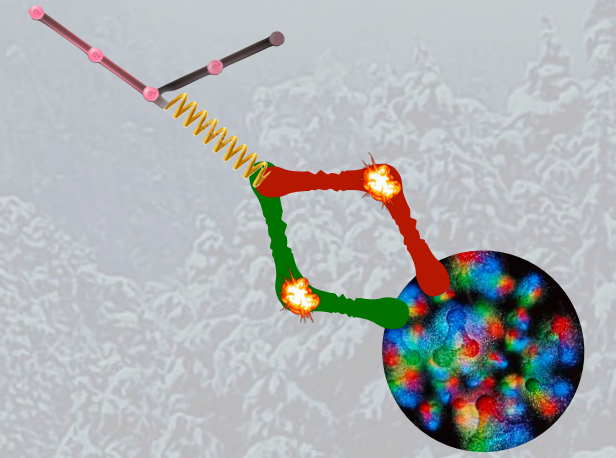
Normalized to 1



# DPS in $\gamma A$ collisions with light nuclei?

M.R. in progress

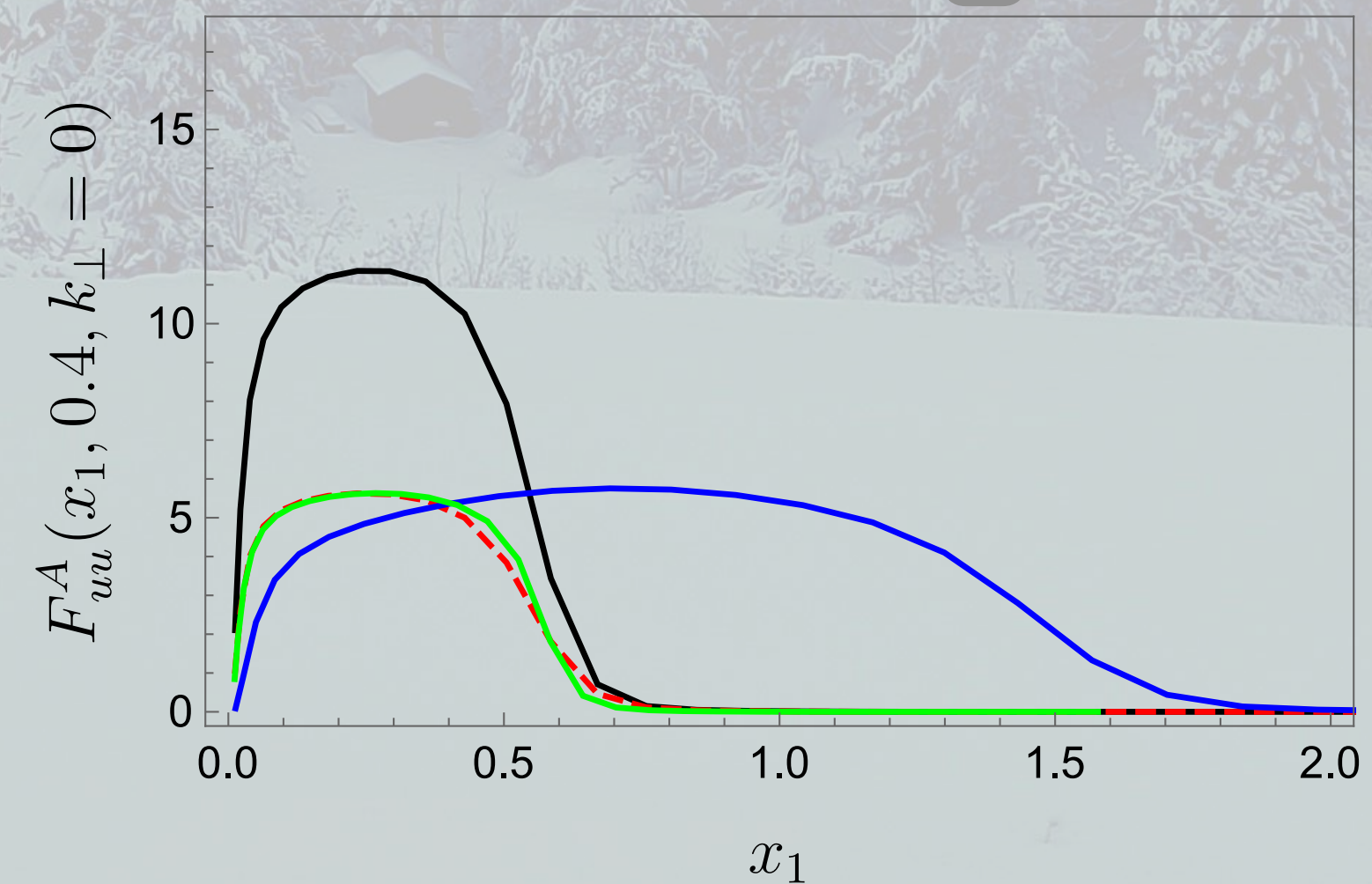
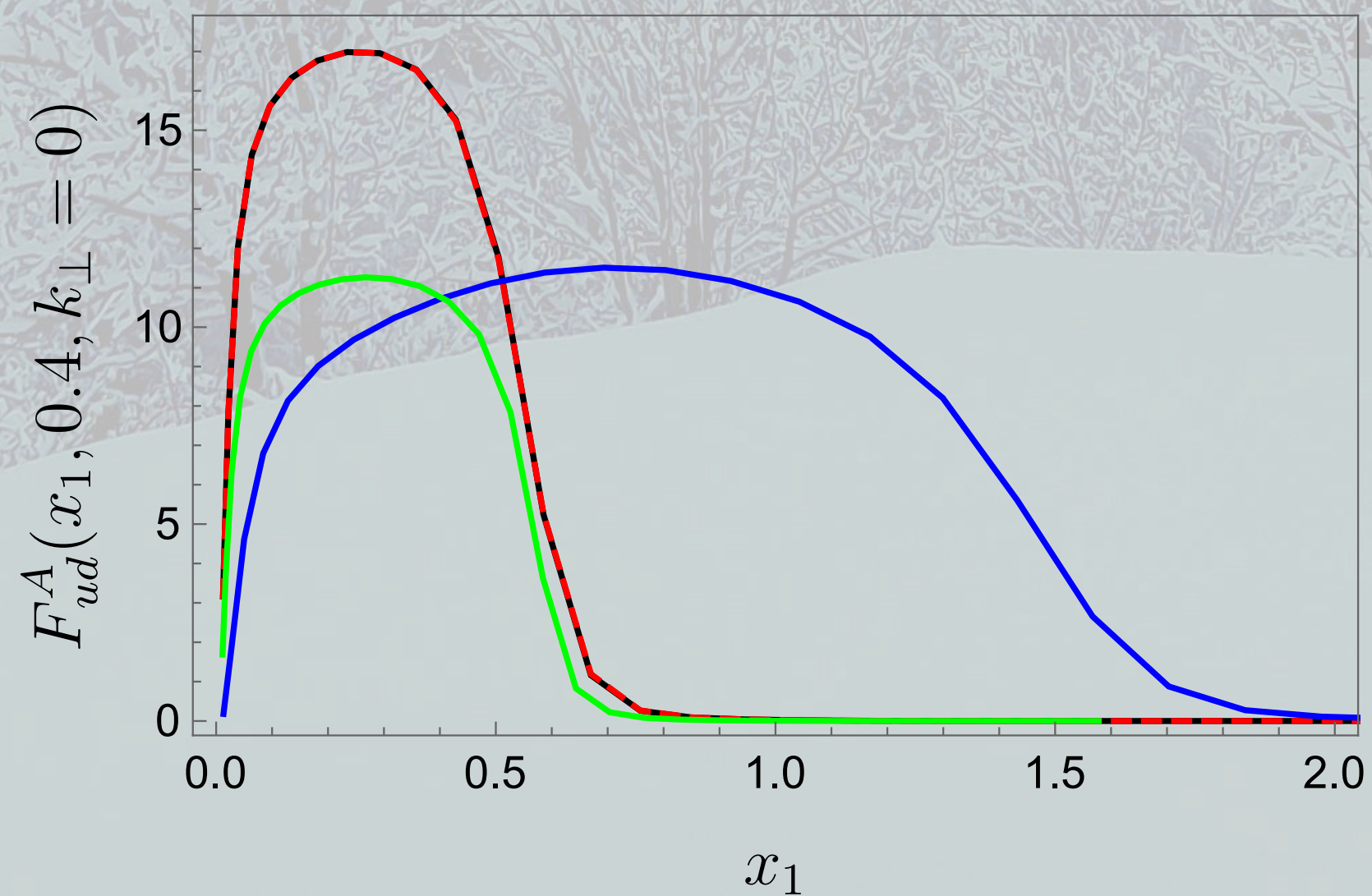
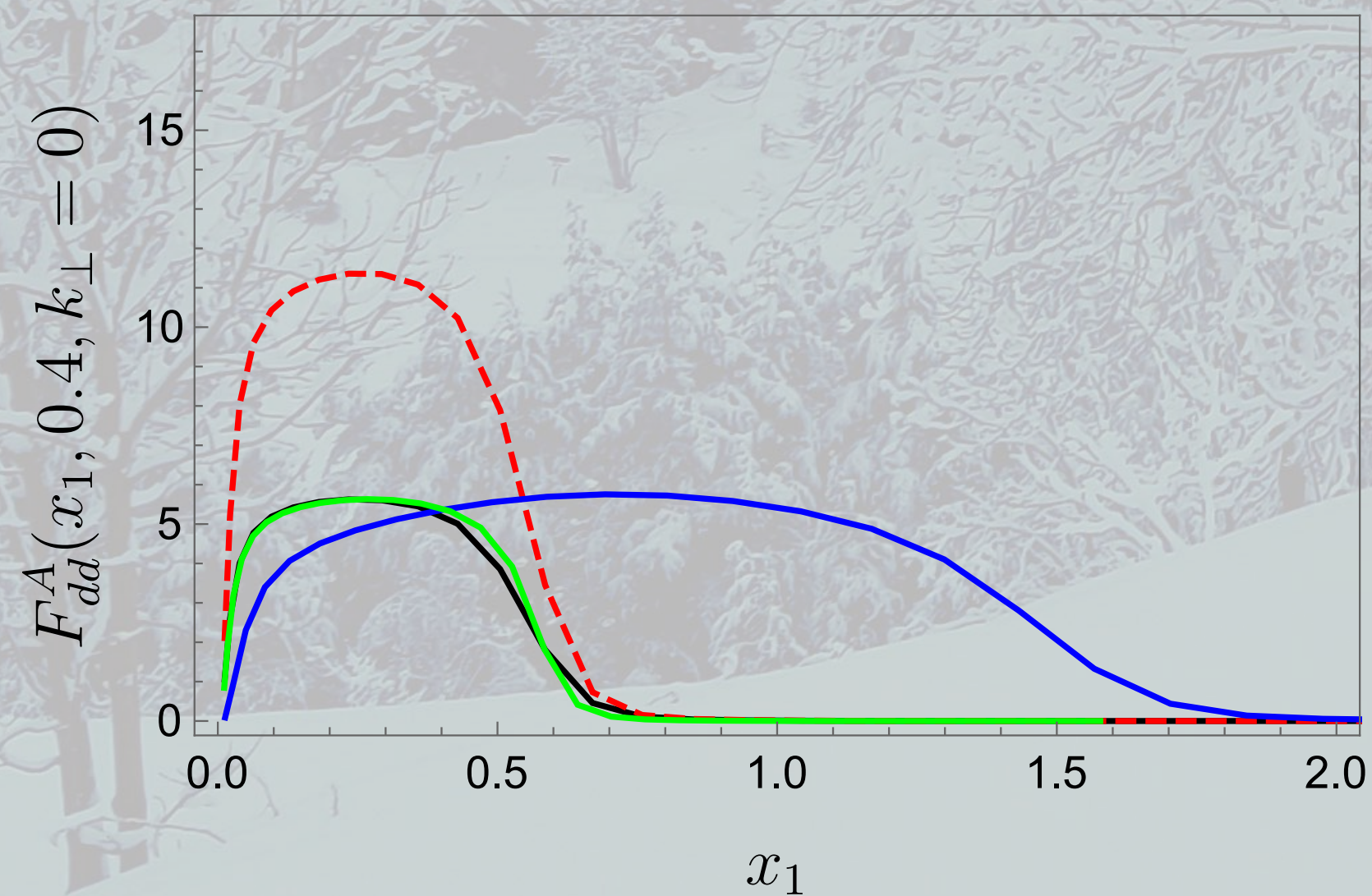
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- Only for valence quarks. We used a relativistic constituent quark model for the nucleon DPD
- verified approximated partial sum rules (numerically)

$$0 < x_i < A$$

— 3He — 4He - - - 3H — 2H

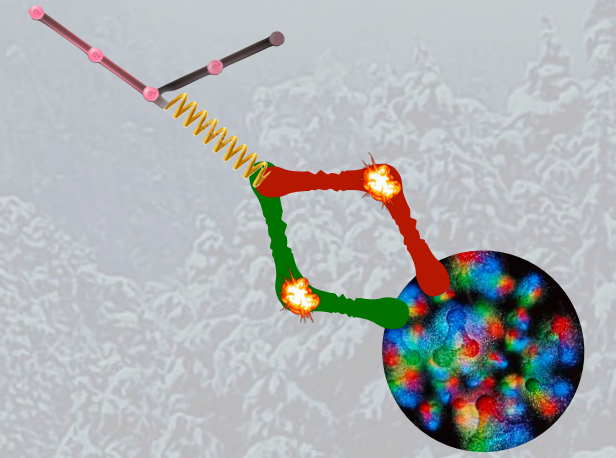




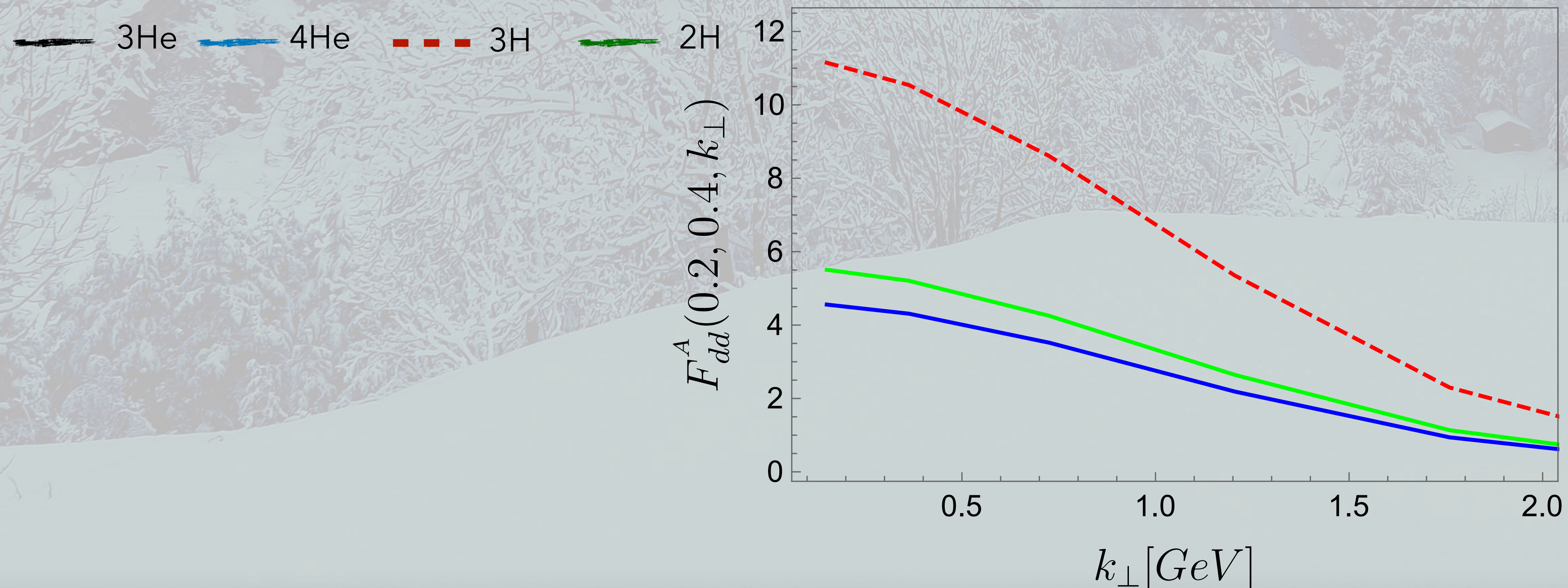
# DPS in $\gamma A$ collisions with light nuclei?

M.R. in progress

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- Only for valence quarks. We used a relativistic constituent quark model for the nucleon DPD
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# DPS in $\gamma A$ collisions with light nuclei?

M.R. in progress

For example in DPS2:

$$\begin{aligned} \tilde{F}_{a_1 a_2}^2(x_1, x_2, \vec{k}_\perp) &\propto \int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 p_{ti}}{\xi_i} \delta\left(\sum_i \xi_i - A\right) \delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right) \psi_A^*(\xi_1, \xi_2, \mathbf{p}_{t1}, \mathbf{p}_{t2}) \psi_A\left(\xi_1, \xi_2, \mathbf{p}_{t1} + \vec{k}_\perp, \mathbf{p}_{t2} - \vec{k}_\perp\right) \\ &\times G_{a_1}^{N_1}\left(\frac{x_1}{\xi_1}, |\vec{k}_\perp|\right) G_{a_2}^{N_2}\left(\frac{x_2}{\xi_2}, |\vec{k}_\perp|\right); \\ &\stackrel{\xi_i \sim 1}{\sim} G_{a_1}^{N_1}(x_1, |\vec{k}_\perp|) G_{a_2}^{N_2}(x_2, |\vec{k}_\perp|) \\ &\times \left[ \int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 p_{ti}}{\xi_i} \delta\left(\sum_i \xi_i - A\right) \delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right) \psi_A^*(\xi_1, \xi_2, \mathbf{p}_{t1}, \mathbf{p}_{t2}) \psi_A\left(\xi_1, \xi_2, \mathbf{p}_{t1} + \vec{k}_\perp, \mathbf{p}_{t2} - \vec{k}_\perp\right) \right] \end{aligned}$$

Nuclear 2-body form factor  $F_2(\vec{k}_\perp, -\vec{k}_\perp)$



# DPS in $\gamma A$ collisions with light nuclei?

M.R. in progress

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Nuclear 2-body form factor  $F_2(\vec{\mathbf{k}}_{\perp}, -\vec{\mathbf{k}}_{\perp})$

Calculated  $F_2(\vec{\mathbf{k}}_2, \vec{\mathbf{k}}_1)$  for  ${}^3\text{He}$  and  ${}^4\text{He}$  in:

**V. Guzey, M.R., S. Scopetta, M. Strikman and M. Viviani et al**, "Coherent  $J/\psi$  electroproduction on  $\text{He4}$  and  $\text{He3}$  at the EIC: probing Nuclear shadowing one nucleon at a time", PRL 129 (2022) 24, 242503



# DPS in $\gamma A$ collisions with light nuclei?

M.R. in progress

For example in DPS2:

$$\begin{aligned} \tilde{F}_{a_1 a_2}^2(x_1, x_2, \vec{k}_\perp) &\propto \int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 p_{ti}}{\xi_i} \delta\left(\sum_i \xi_i - A\right) \delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right) \psi_A^*(\xi_1, \xi_2, \mathbf{p}_{t1}, \mathbf{p}_{t2}) \psi_A(\xi_1, \xi_2, \mathbf{p}_{t1} + \vec{k}_\perp, \mathbf{p}_{t2} - \vec{k}_\perp) \\ &\times G_{a_1}^{N_1}\left(\frac{x_1}{\xi_1}, |\vec{k}_\perp|\right) G_{a_2}^{N_2}\left(\frac{x_2}{\xi_2}, |\vec{k}_\perp|\right) \\ &\stackrel{\xi_i \sim 1}{\sim} G_{a_1}^{N_1}(x_1, |\vec{k}_\perp|) G_{a_2}^{N_2}(x_2, |\vec{k}_\perp|) \\ &\times \left[ \int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 p_{ti}}{\xi_i} \delta\left(\sum_i \xi_i - A\right) \delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right) \psi_A^*(\xi_1, \xi_2, \mathbf{p}_{t1}, \mathbf{p}_{t2}) \psi_A(\xi_1, \xi_2, \mathbf{p}_{t1} + \vec{k}_\perp, \mathbf{p}_{t2} - \vec{k}_\perp) \right] \end{aligned}$$

**WE HAVE A LINK BETWEEN  
2 DIFFERENT PROCESSES!**

Nuclear 2

Calculated  $F_2(\vec{k}_2, \vec{k}_1)$  for  ${}^3\text{He}$  and  ${}^4\text{He}$

V. Guzey, M.R., S. Scopetta, M. Strikman and ...  
*He4 and He3 at the EIC: probing Nuclear shadowing*

... electroproduction on  
... JHEP 129 (2022) 24, 242503



# DPS in $\gamma A$ collisions with light nuclei?

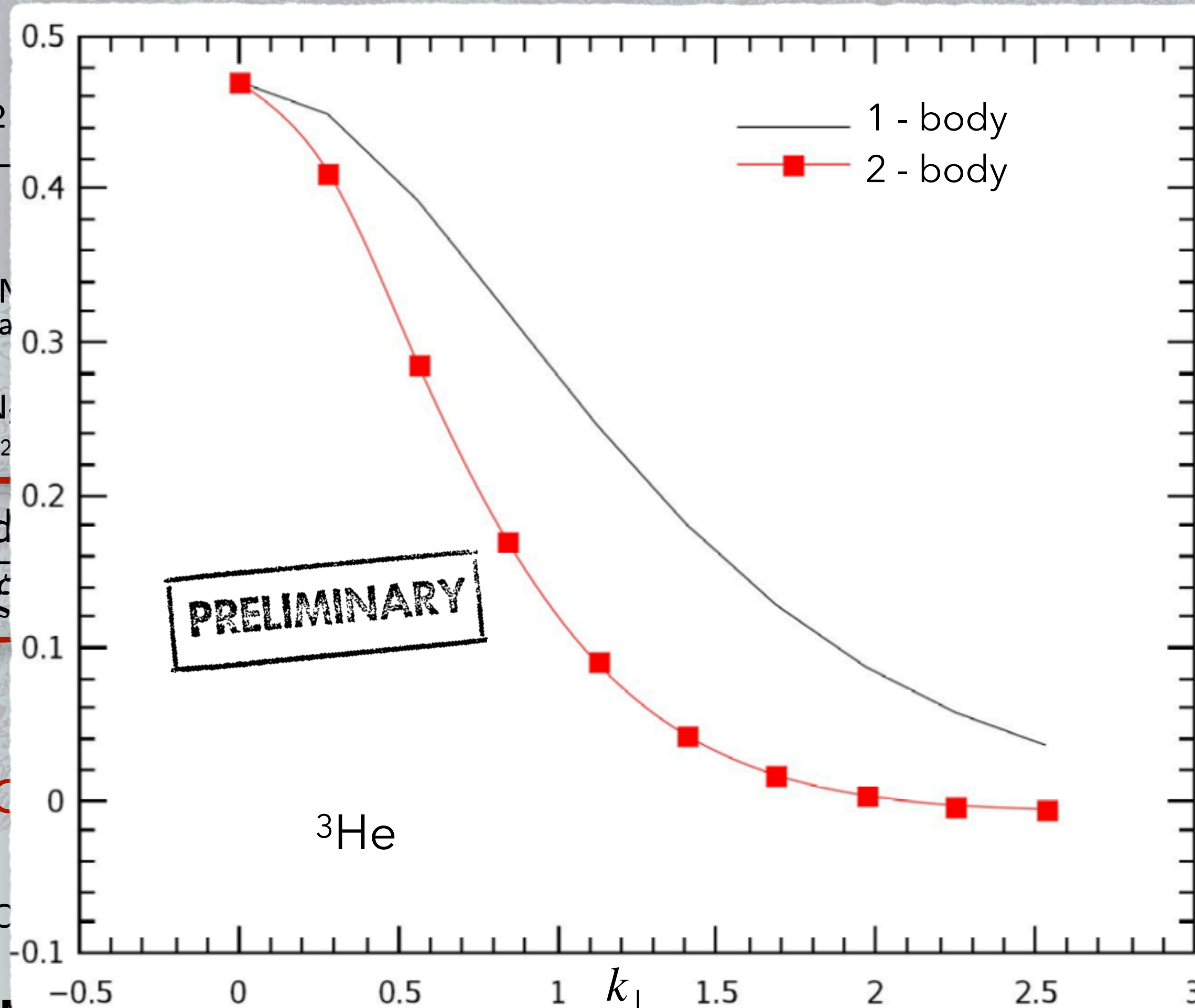
M.R. in progress

For example in DPS2:

$$\tilde{F}_{a_1 a_2}^2(x_1, x_2, \vec{k}_\perp) \propto \int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 \xi_i}{\xi_i} \times G_{a_1}^{N_1} \left( \frac{x_1}{\xi_1}, |\vec{k}_\perp| \right) G_{a_2}^{N_2} \left( \frac{x_2}{\xi_2}, |\vec{k}_\perp| \right) \times \left[ \int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 \xi_i}{\xi_i} \right]$$

Nuc

Calculated  $F_2(\vec{k}_2, \vec{k}_1)$  for



$(p_{t1} + \vec{k}_\perp, p_{t2} - \vec{k}_\perp)$

$(p_{t1} + \vec{k}_\perp, p_{t2} - \vec{k}_\perp)$

$(\vec{k}_\perp)$

V. Guzey, M.R., S. Scopetta, M. Strikman and M. Viviani et al, "Coherent  $J/\psi$  electroproduction on He4 and He3 at the EIC: probing Nuclear shadowing one nucleon at a time", PRL 129 (2022) 24, 242503



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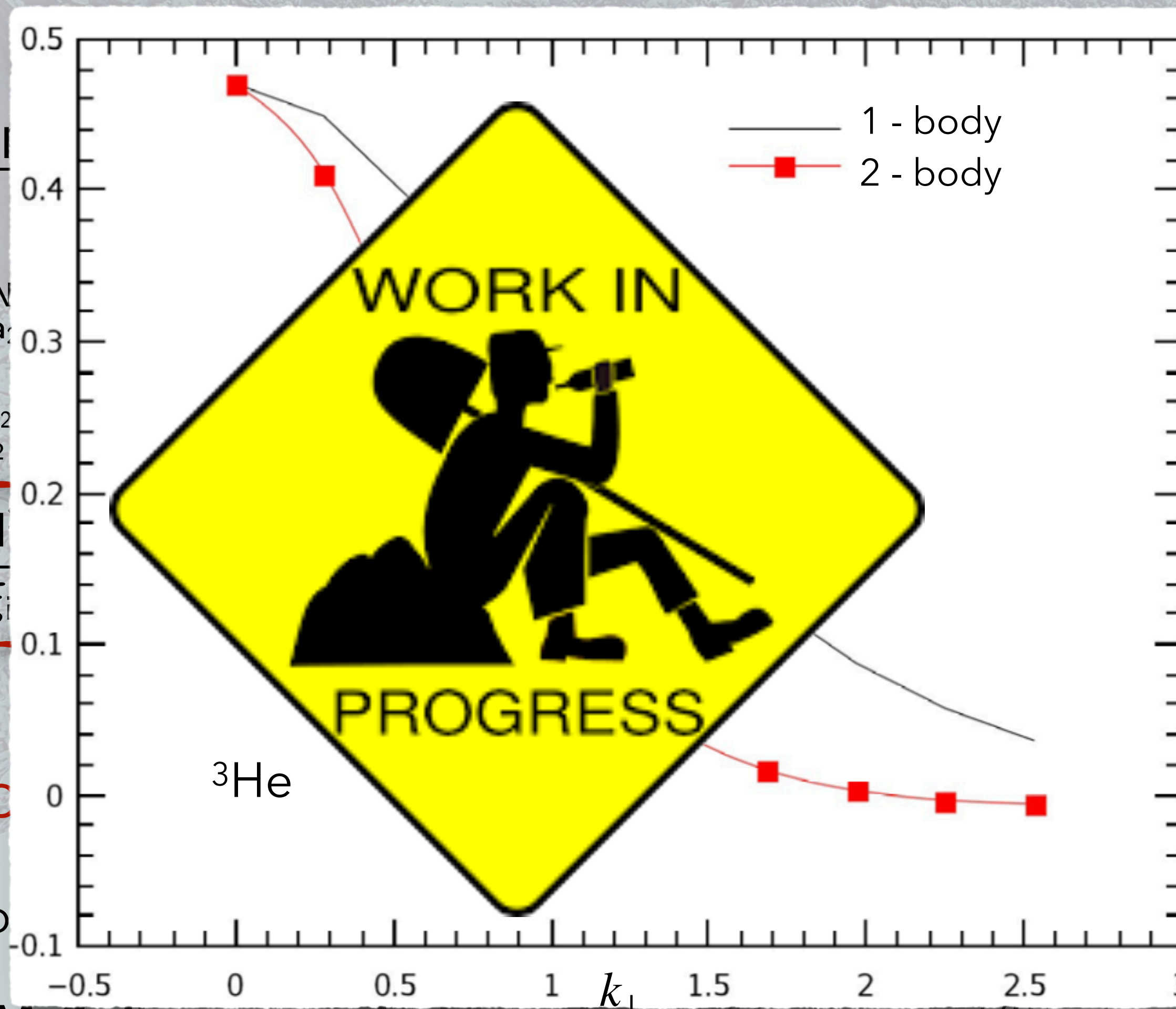
M.R. in progress

For example in DPS2:

$$\tilde{F}_{a_1 a_2}^2(x_1, x_2, \vec{k}_\perp) \propto \int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 \perp_i}{\xi_i} \times G_{a_1}^{N_1} \left( \frac{x_1}{\xi_1}, |\vec{k}_\perp| \right) G_{a_2}^{N_2} \left( \frac{x_2}{\xi_2}, |\vec{k}_\perp| \right) \times \left[ \int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 \perp_i}{\xi_i} \times G_{a_1}^{N_1} \left( x_1, |\vec{k}_\perp| \right) G_{a_2}^{N_2} \left( x_2, |\vec{k}_\perp| \right) \right]$$

Calculated  $F_2(\vec{k}_2, \vec{k}_1)$  for

Nucleus



$(p_{t1} + \vec{k}_\perp, p_{t2} - \vec{k}_\perp)$

$(p_{t1} + \vec{k}_\perp, p_{t2} - \vec{k}_\perp)$

$(\vec{k}_\perp)$

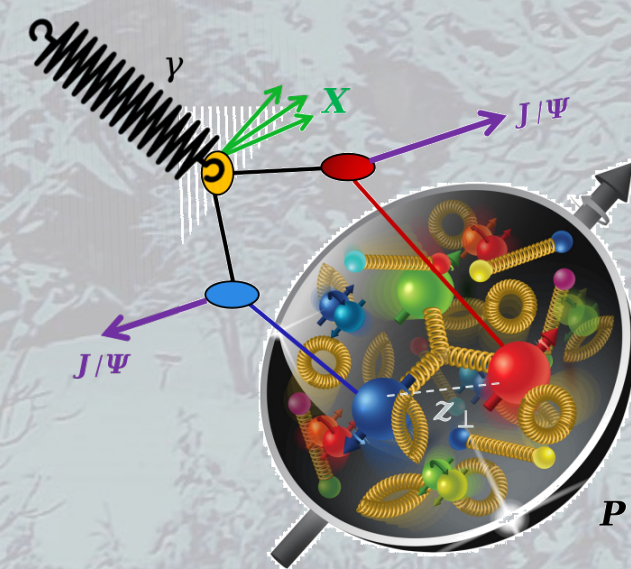
V. Guzey, M.R., S. Scopetta, M. Strikman and M. Viviani et al, "Coherent  $J/\psi$  electroproduction on He4 and He3 at the EIC: probing Nuclear shadowing one nucleon at a time", PRL 129 (2022) 24, 242503



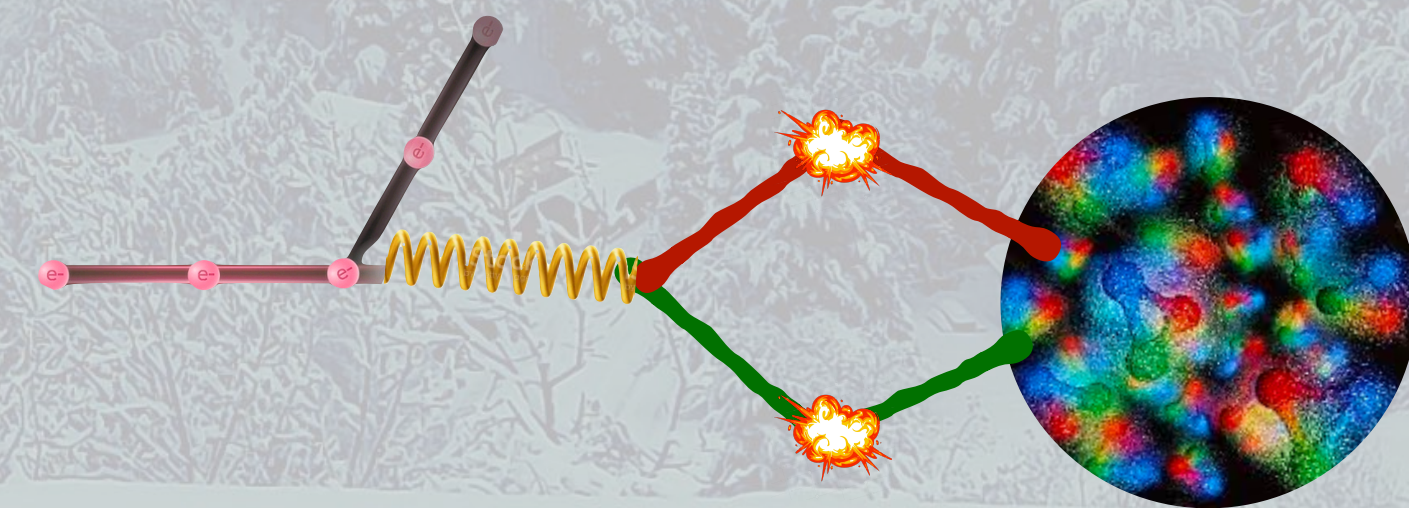
# CONCLUSIONS

- 1) We demonstrated DPS represents a new way to access new information of hadrons
- 2) Several experimental analyses and theoretical developments are on going
- 3) We proposed to consider DPS initiated via photon-proton interactions:

a) DPS@EIC



b) Nuclear DPS@EIC



- a) DPS contributes, in particular in the 4-jets photoproduction
- b) We have estimated SPS and DPS cross sections for quarkonium-pair photoproduction at the EIC using the NRQC framework
- c) The dependence of  $\sigma_{\text{eff}}^{\gamma P}(Q^2)$  on  $Q^2$  can unveil the mean distance of partons in the proton
- d) Quarkonium-pair photoproduction is a promising channel to probe the gluonic content of the photon structure

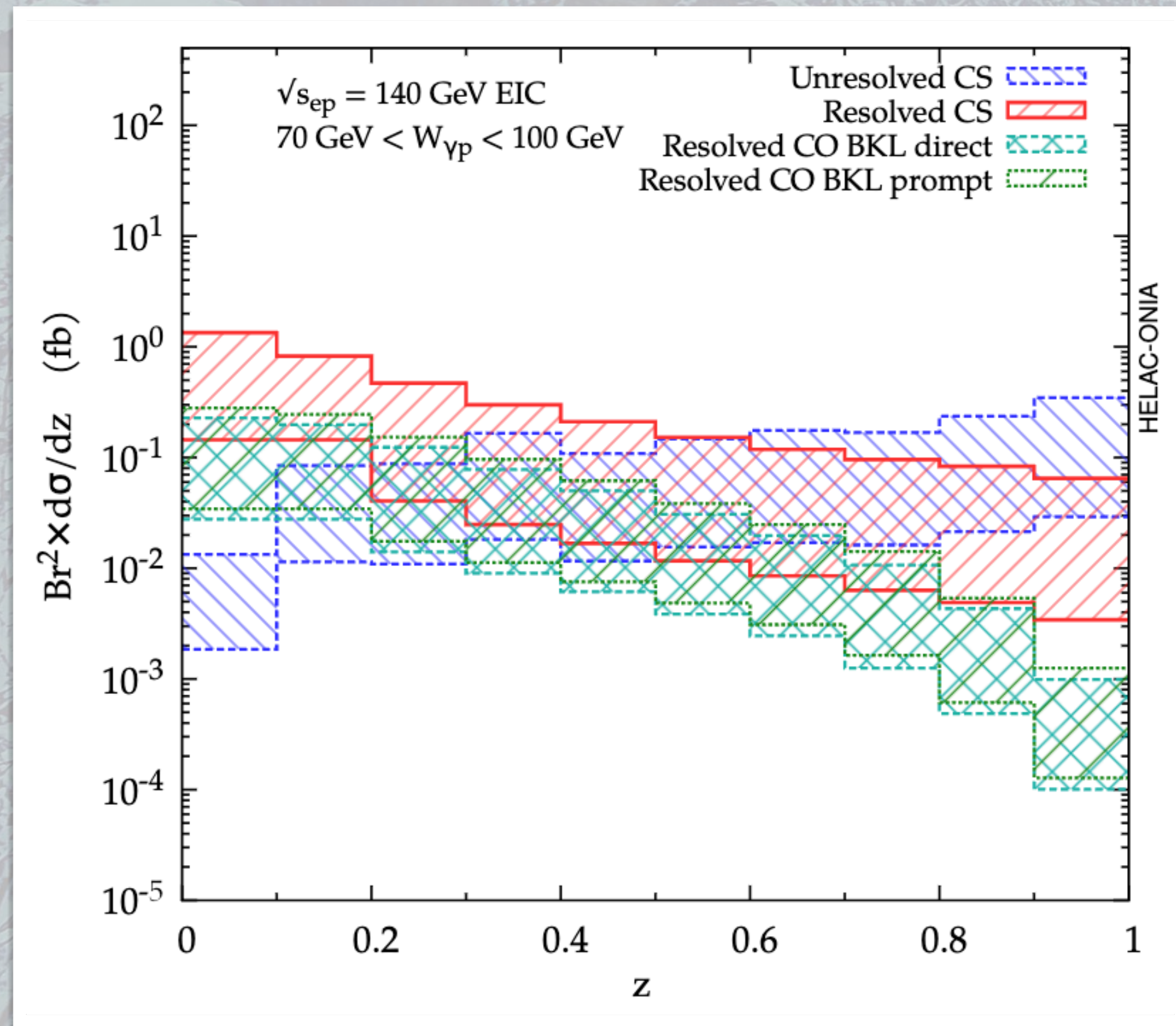


# CONCLUSIONS

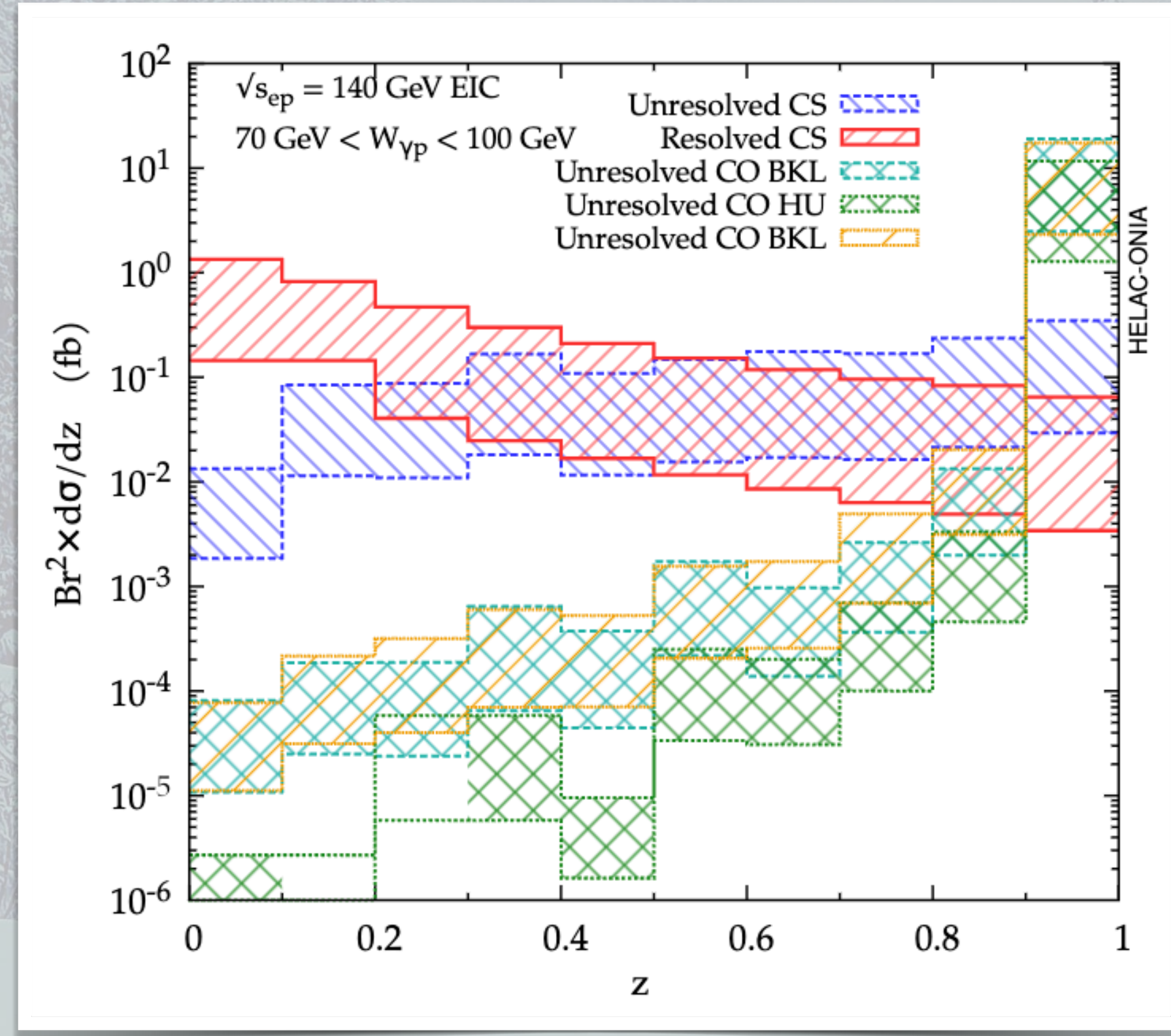


Thanks for the attention





$$\mu_0 = \frac{H_T}{2} = \frac{\sum_i \sqrt{p_{Ti}^2 + m_i^2}}{2}$$



CO negligible if we cut  $z < 0.9$  (to be checked)



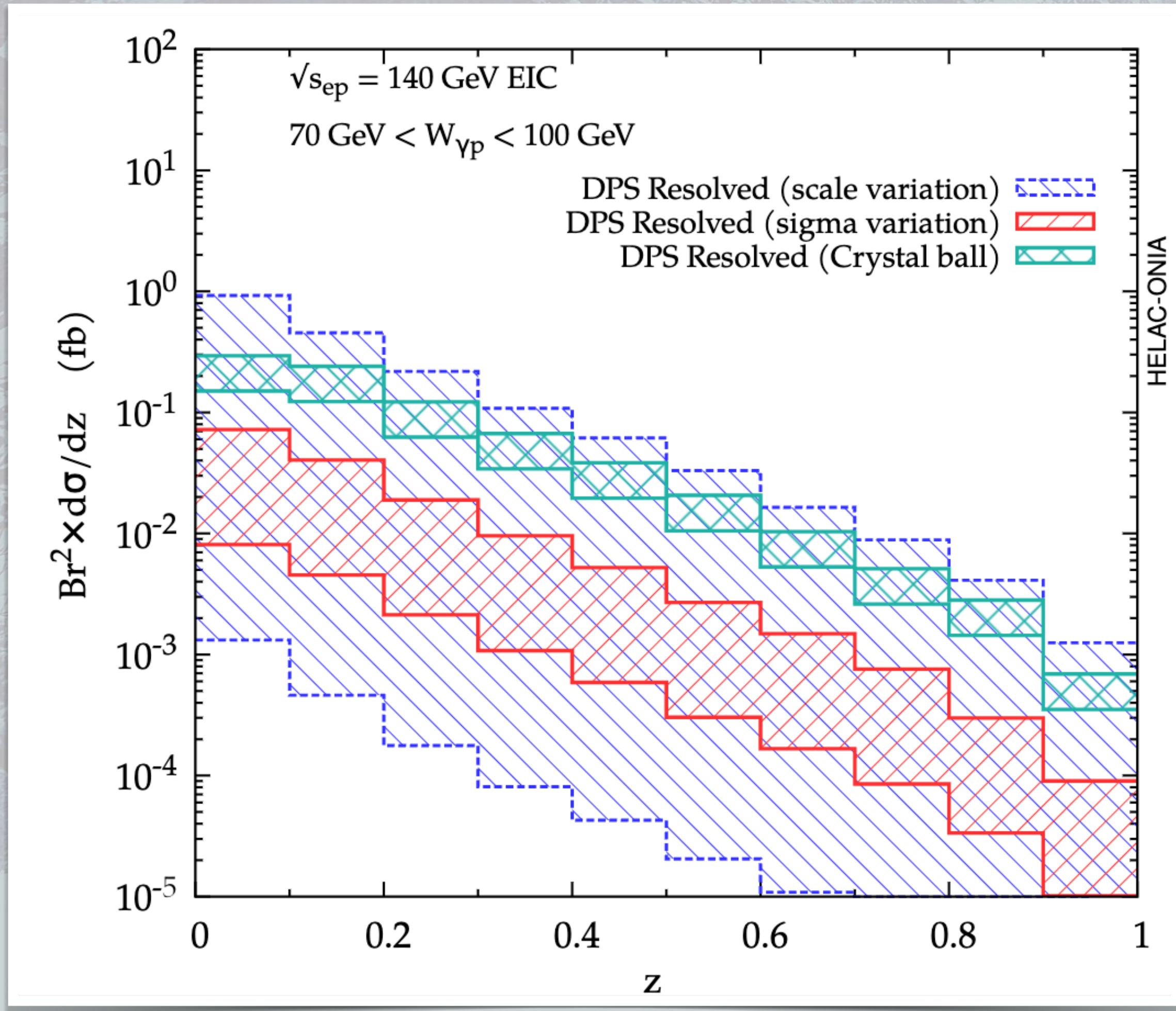
# Numerical Results

PRELIMINARY

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

We also considered to use the Crystal Ball parametrization of the square of the amplitude  $gg \rightarrow Q + X$

$$\begin{cases} \frac{\lambda^2 \kappa \hat{s}}{M_Q^2} \exp(-\kappa \frac{P_T^2}{M_Q^2}) & \text{when } P_T \leq \langle P_T \rangle \\ \frac{\lambda^2 \kappa \hat{s}}{M_Q^2} \exp(-\kappa \frac{\langle P_T \rangle^2}{M_Q^2}) \left(1 + \frac{\kappa}{n} \frac{P_T^2 - \langle P_T \rangle^2}{M_Q^2}\right)^{-n} & \text{when } P_T > \langle P_T \rangle \end{cases}$$





# Backup - Luminosity I

---

To test if in future a dependence of the effective cross section on the photon virtuality could be observed, we considered again the 4 JET photoproduction:

1) We divided the integral of the cross section on  $Q^2$  in two intervals:

$$Q^2 \leq 10^{-2} \quad \text{and} \quad 10^{-2} \leq Q^2 \leq 1 \quad \text{GeV}^2$$

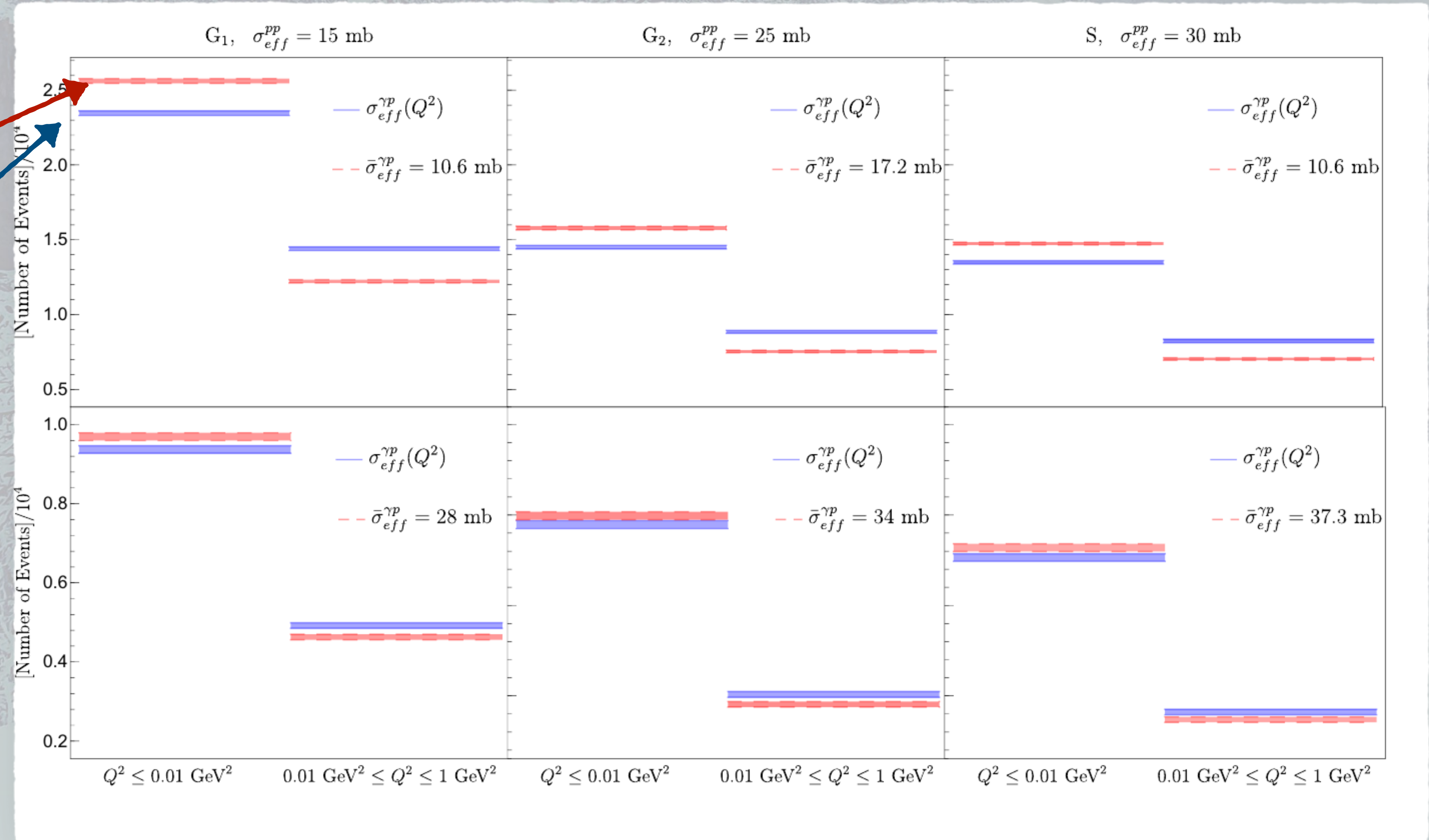
2) We have estimated for each photon and proton models a constant effective cross section (with respect to  $Q^2$ ) such that the total integral of the cross section on  $Q^2$  reproduce the full calculation obtained by means of  $\sigma_{\text{eff}}^{\gamma p}(Q^2)$

3) We estimate the minimum luminosity to distinguish the two cases



# Backup - Luminosity II

With an integrated luminosity of 200 pb<sup>-1</sup> we can separate:





# Backup - $\sigma_{\text{eff}}^{\gamma p}(Q^2 \rightarrow \infty)$

1) we show that high virtual behavior of the effective cross sections correctly follows the result in **J.R. Gaunt JHEP 01, 042 (2013)**, i.e.:

$$\sigma_{\text{eff}}^{\gamma p}(Q^2 \rightarrow \infty) = \sigma_{1v2}^{pp} = \left[ \int \frac{d^2 k_{\perp}}{(2\pi)^2} T_p(k_{\perp}) \right]^{-1}$$

2) In Ref. **M.Rinaldi and F.A: Ceccopieri JHEP 09, 097 (2019)**, we prove, in a general framework:

$$\frac{\pi}{2} \langle b^2 \rangle \leq \sigma_{\text{eff}}^{\gamma p}(Q^2 \rightarrow \infty) \leq 2\pi \langle b^2 \rangle$$

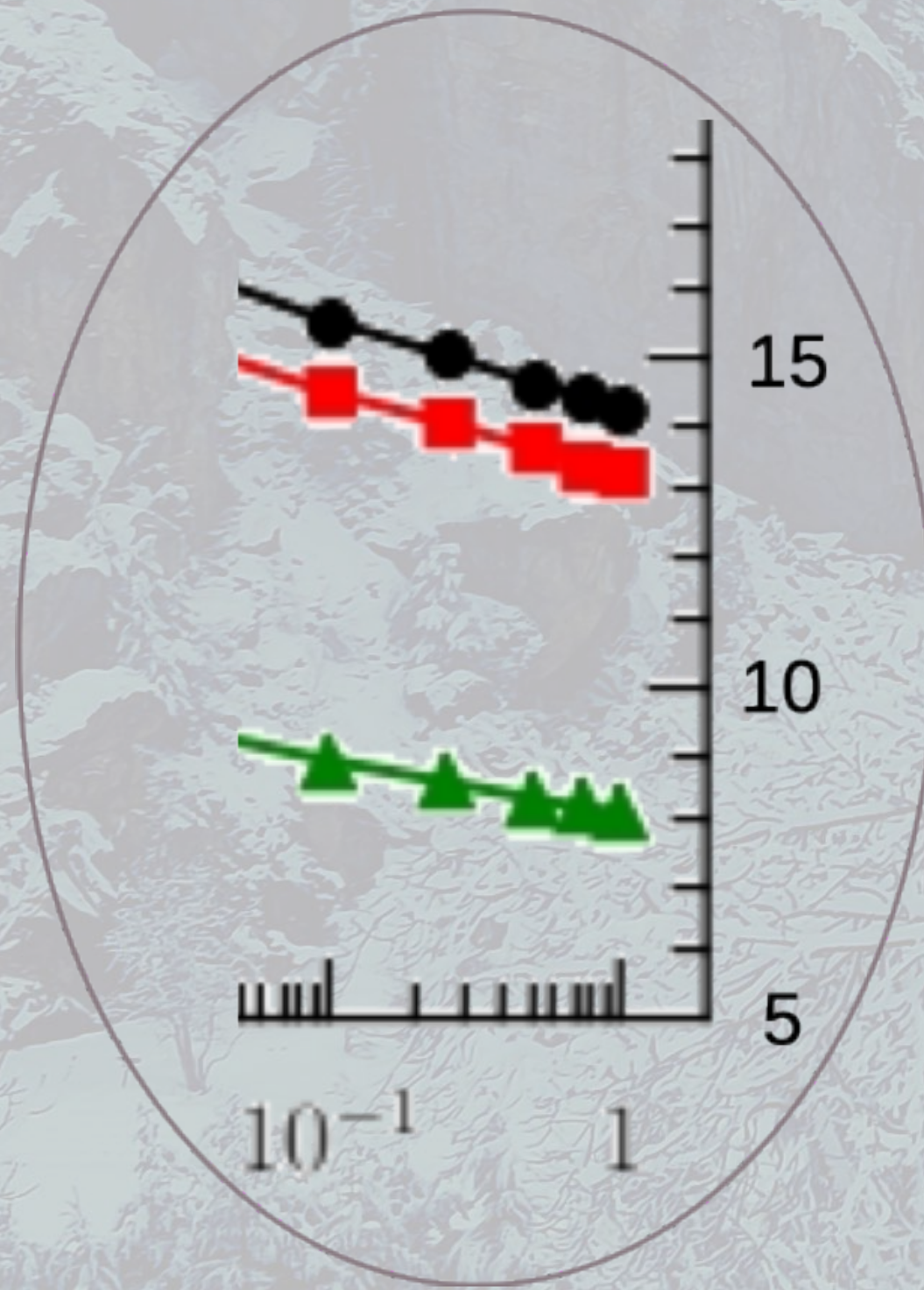
Being:  $\sigma_{\text{eff}}^{\gamma p}(Q^2 \rightarrow \infty) = \sigma_{\text{eff}}^{2v1}$

$$\frac{\sigma_{\text{eff}}^{pp}}{6} \leq \sigma_{\text{eff}}^{\gamma p}(Q^2 \rightarrow \infty) \leq 2 \sigma_{\text{eff}}^{pp}$$

Extracted from data



# Backup - $\sigma_{eff}^{\gamma p}(Q^2 \rightarrow \infty)$



$\sim 30/2$  mb  
 $\sim 25/2$  mb

$\sim 15/2$  mb

$$[\sigma_{eff}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi)^2} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$$

$$[\sigma_{eff}^{\gamma p}(Q^2)]^{-1} \underset{Q^2 \gg 1}{\sim} \int \frac{d^2 k_{\perp}}{(2\pi)^2} T_p(k_{\perp}) \times 1$$

For the proton models we have used:

$$\int \frac{d^2 k_{\perp}}{(2\pi)^2} T_p(k_{\perp}) \sim 2 \int \frac{d^2 k_{\perp}}{(2\pi)^2} T_p(k_{\perp})^2$$



$$\sigma_{eff}^{\gamma p}(Q^2 \gg 1 \text{ GeV}^2) \sim \sigma_{eff}^{pp}/2$$

Thus for QED:  $Q^2 \gg 1 \text{ GeV}^2$  almost approximates the asymptotic



# DPS in pA collisions

The DPS cross-section

$$d\sigma_{\text{DPS}}^{\text{ML}} = \frac{m}{2} \sum_{i,j,k,l} d\hat{\sigma}_{ik}^{\text{M}} d\hat{\sigma}_{jl}^{\text{L}} \int d^2b_{\perp} F_p^{ij}(x_1, x_2, \vec{b}_{\perp}) \int d^2B \left\{ \right.$$

the thickness function as a function of the impact parameter B

$$\bar{T}(\vec{b}_{\perp} + \vec{B}) \sim \bar{T}(\vec{B})$$

$$\bar{T}_N(B) = \int dz \underbrace{\rho_N(\sqrt{B^2 + z^2})}$$

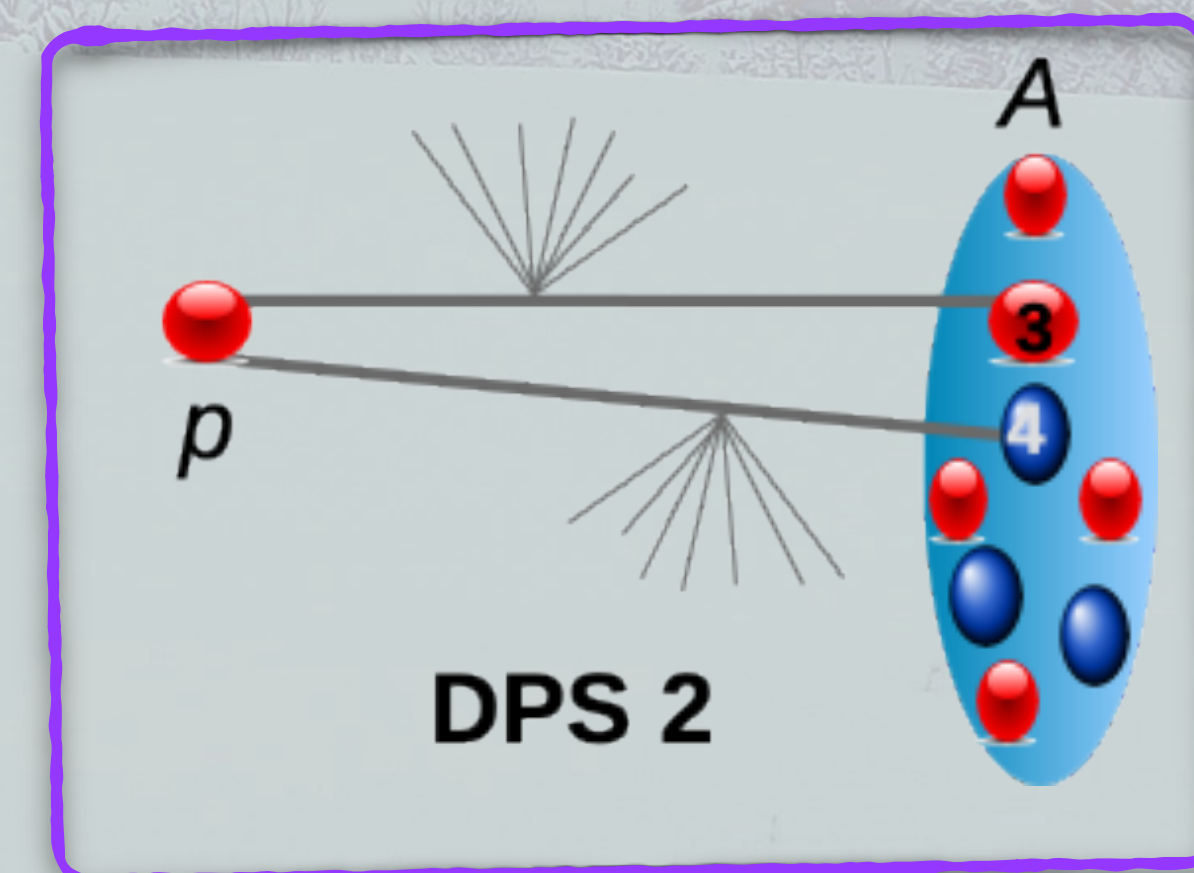
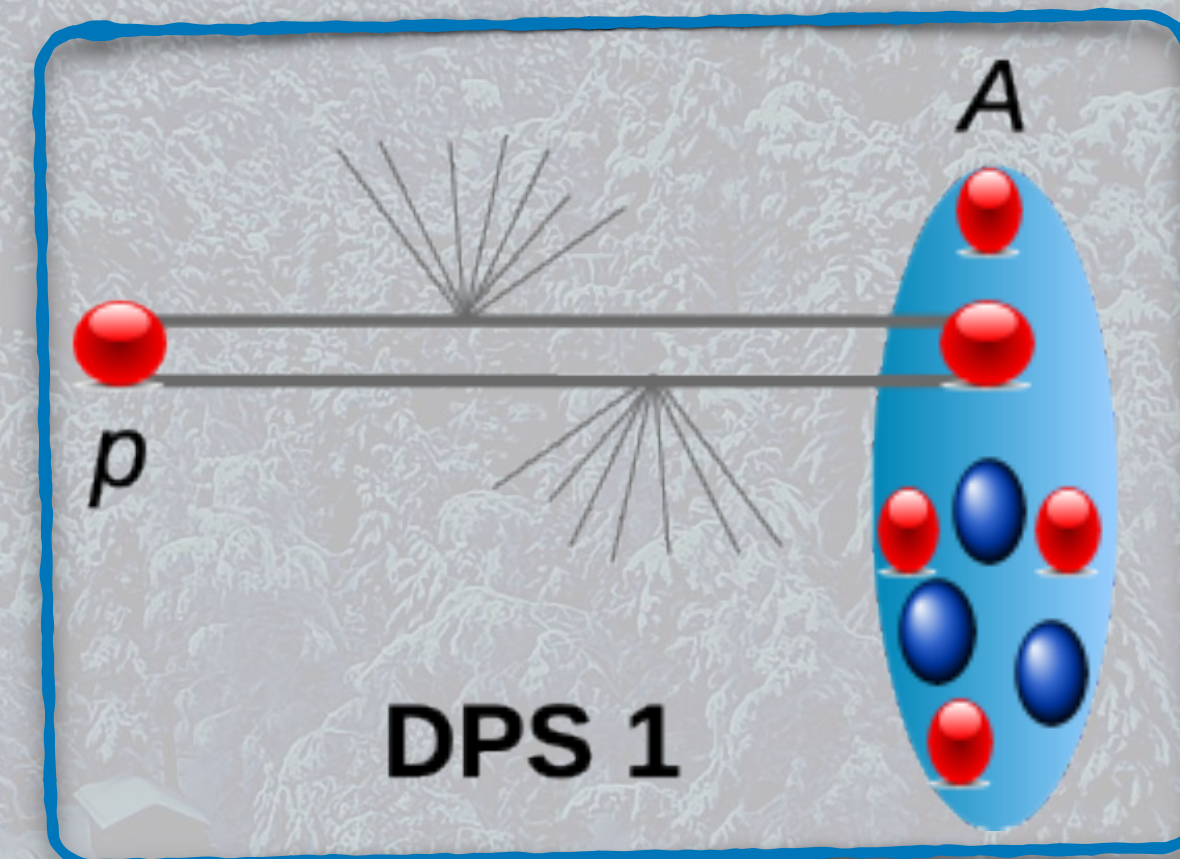
Wood-Saxon distribution for pb normalized to A

$$\sum_{N=p,n} F_N^{kl}(x_3, x_4, \vec{b}_{\perp}) \bar{T}_N(B)$$

+

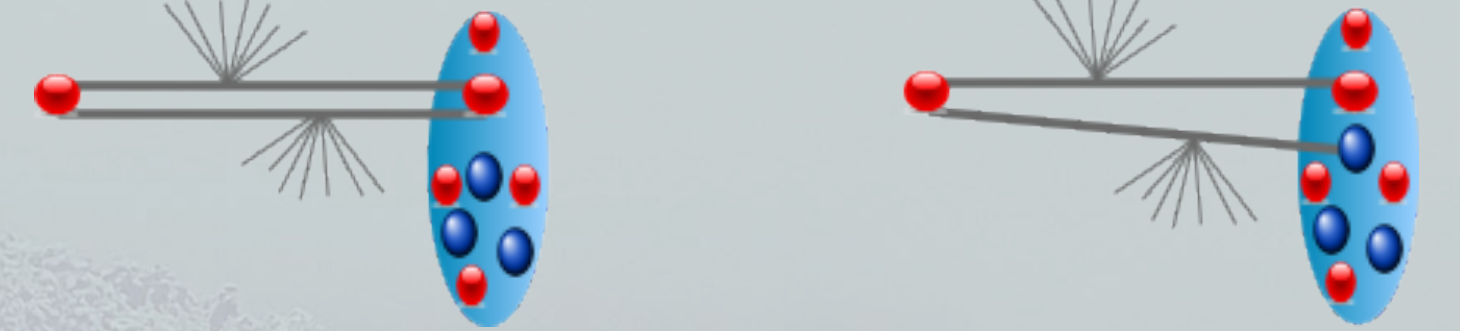
$$\sum_{N_3, N_4=p,n} f_{N_3/A}^k(x_3) f_{N_4/A}^l(x_4) T_{N_3}(B) T_{N_4}(B)$$

}





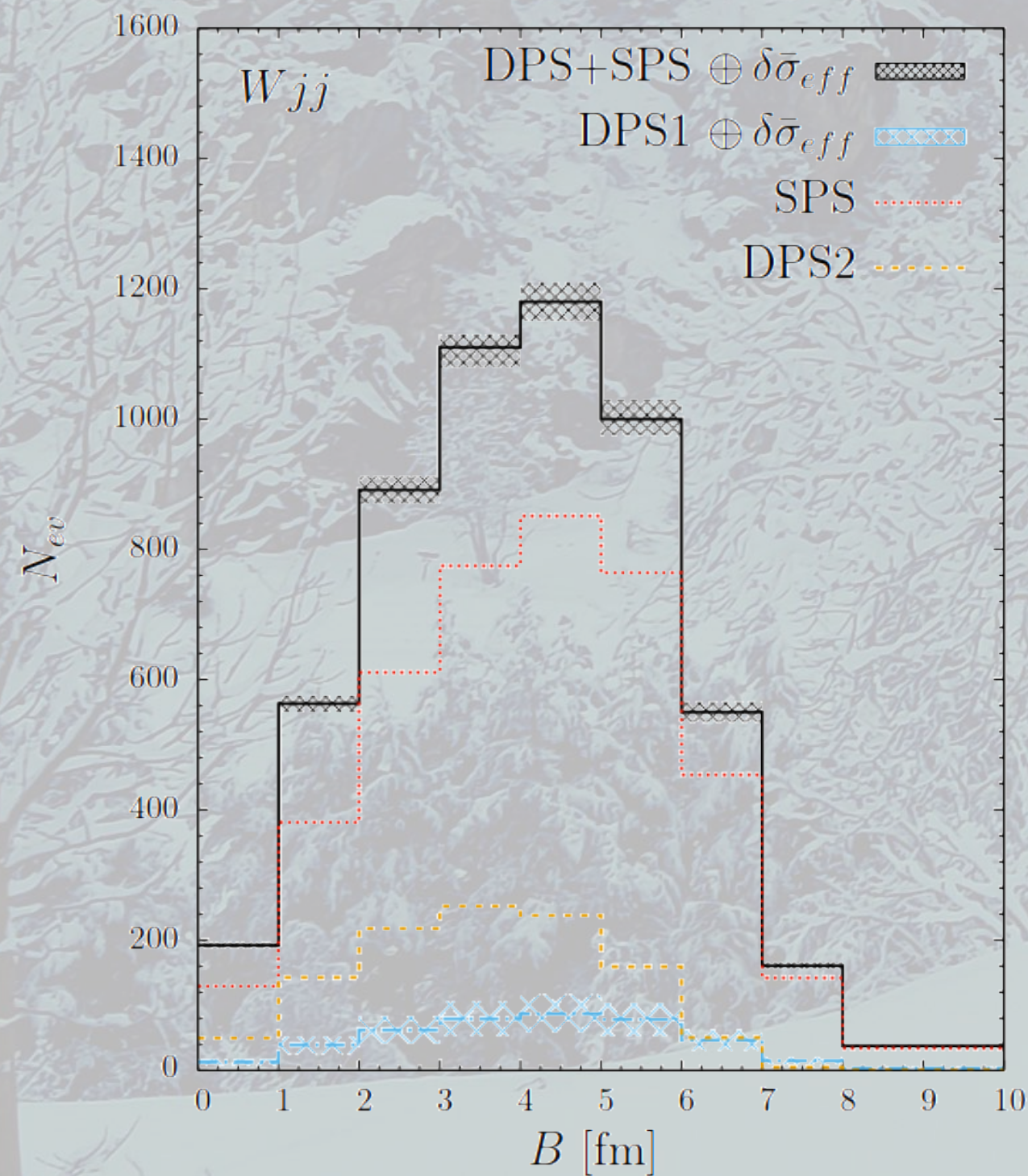
# DPS in pA collisions



Some examples of predictions:

## W+di-jets

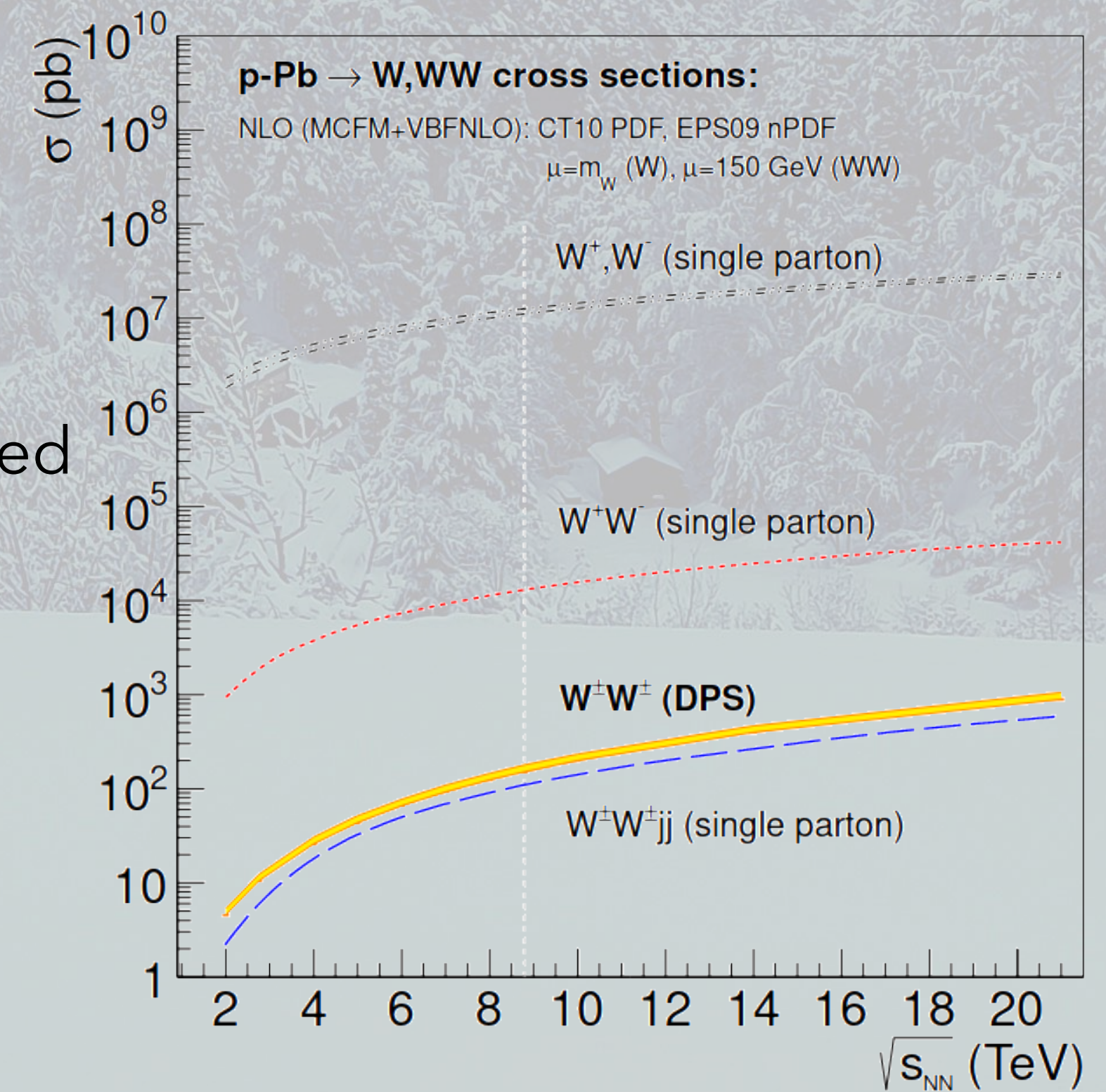
B. Blok and F. A. Ceccopieri EPJC (2020) 80, 278



- SPS dominant  
 - DPS2 bigger than DPS1 has expected

## Same sign WW

D. D'Enterria and Snigirev, PLB 718 (2013) 1395-1400





# DPS in $\gamma A$ collisions with light nuclei?

M.R. in progress

For example in DPS2:

$$\begin{aligned} \tilde{F}_{a_1 a_2}^2(x_1, x_2, \vec{k}_\perp) &\propto \int \frac{1}{\xi_1 \xi_2} \prod_{i=1}^{i=A} \frac{d\xi_i d^2 p_{ti}}{\xi_i} \delta\left(\sum_i \xi_i - A\right) \delta^{(2)}\left(\sum_i \mathbf{p}_{ti}\right) \psi_A^*(\xi_1, \xi_2, \mathbf{p}_{t1}, \mathbf{p}_{t2}) \psi_A(\xi_1, \xi_2, \mathbf{p}_{t1} + \vec{k}_\perp, \mathbf{p}_{t2} - \vec{k}_\perp) \\ &\times G_{a_1}^{N_1}\left(\frac{x_1}{\xi_1}, |\vec{k}_\perp|\right) G_{a_2}^{N_2}\left(\frac{x_2}{\xi_2}, |\vec{k}_\perp|\right); \end{aligned}$$

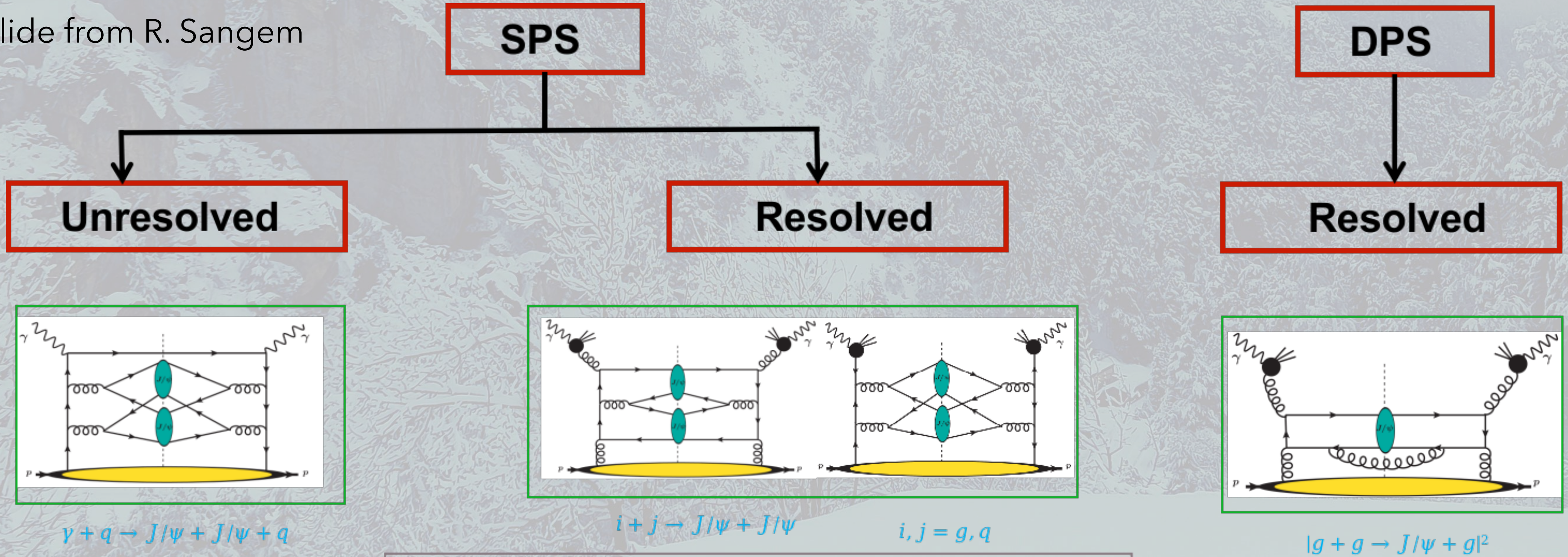
if we approximate:  $\xi_i \sim 1$  we get:



# Di $J/\psi$ photo-production@EIC

F. A. Ceccopieri, H. S. Shao, J. P. Lansberg, M. R. and R. Sangem in prep.

\*Slide from R. Sangem



Range of cross sections in CSM = 100 GeV

$\sigma_{SPS}^{(J/\psi, J/\psi)} \times Br^2 = 4 - 30 \text{ fb}$	}	(Resolved) $\sigma_{eff}^{vp} = 10 \text{ mb}$ for DPS
$\sigma_{DPS}^{(J/\psi, J/\psi)} \times Br^2 = 0.2 - 5 \text{ fb}$		
$\sigma_{SPS}^{(J/\psi, J/\psi)} \times Br^2 = 2 - 12 \text{ fb}$		(Unresolved)





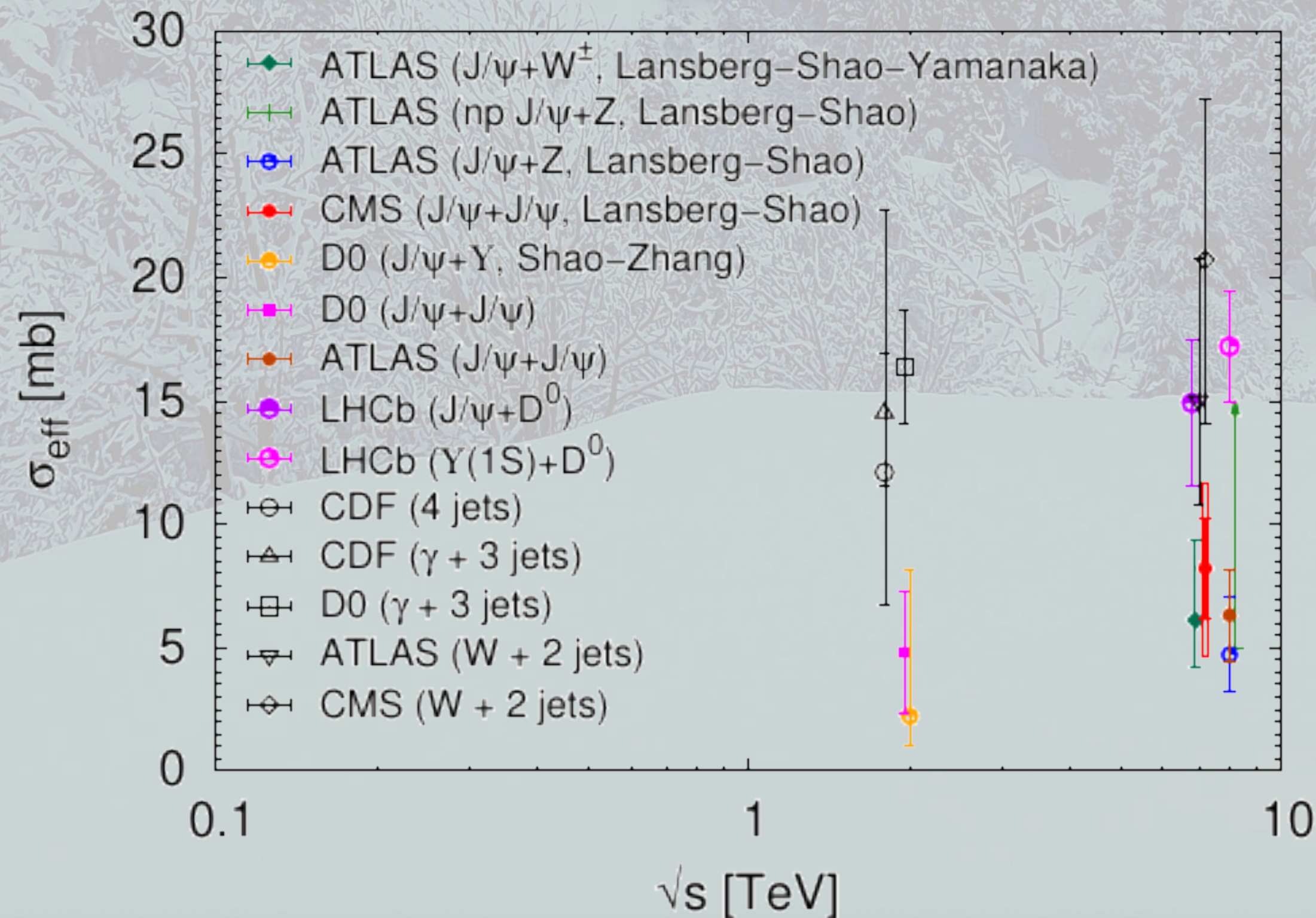
# Some Data and Effective Cross Section

$$\sigma_{\text{eff}}^{\text{pp}} = \frac{m}{2} \frac{\sigma_A^{\text{pp}} \sigma_B^{\text{pp}}}{\sigma_{\text{DPS}}^{\text{pp}}}$$

Differential X-section single parton scattering for the process:  $pp \rightarrow A(B) + X$

Differential X-section double parton scattering for the process:  $pp \rightarrow A + B + X$

POCKET FORMULA



First observation of same sign WW via DPS:

$$\sigma_{\text{eff}} = 12.2^{+2.9}_{-2.2} \text{ mb}$$

[CMS coll.], PRL 131 (2023) 091803

$$\sigma^{\text{DPS}} \sim 6.28 \text{ fb}$$



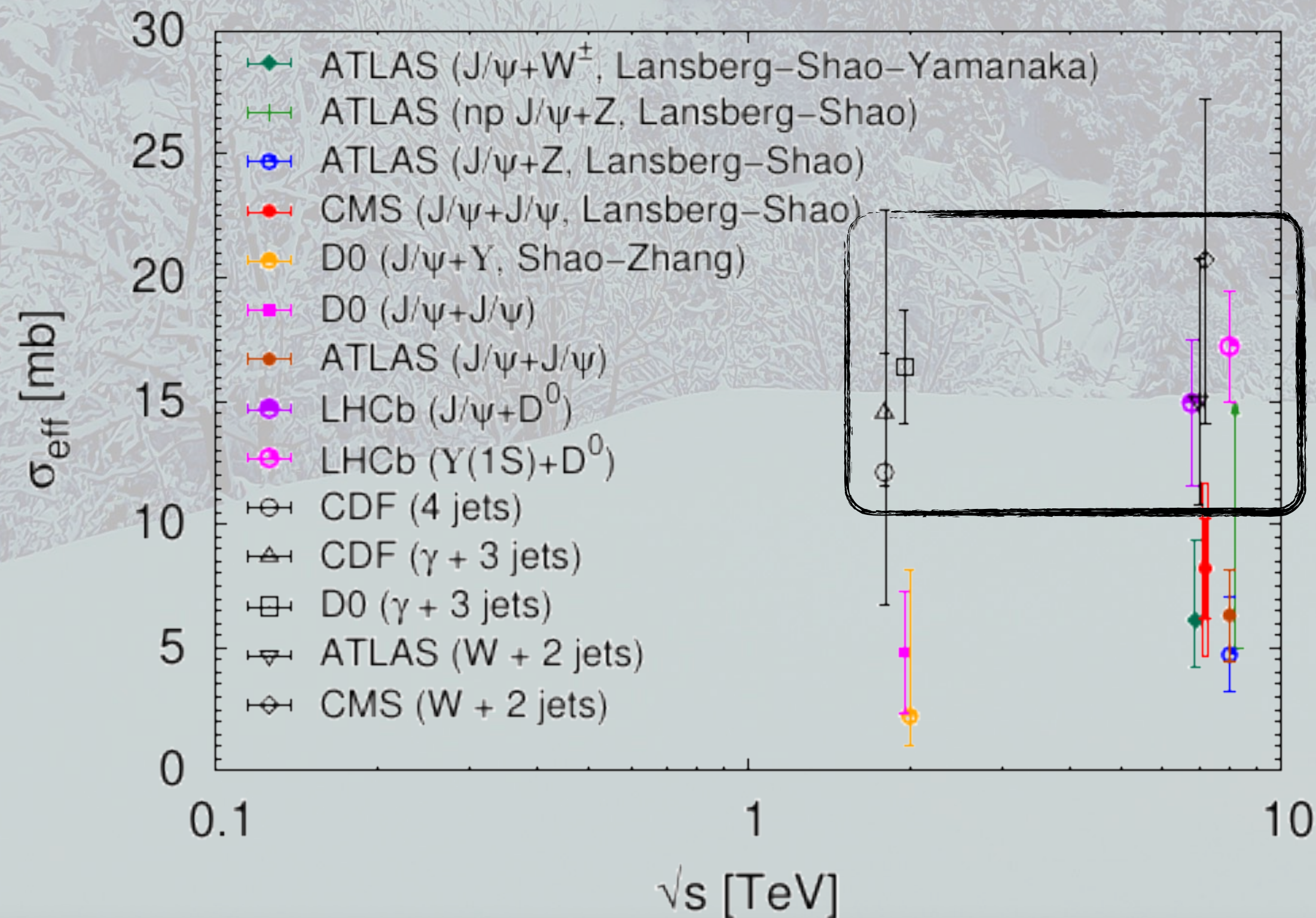
# Some Data and Effective Cross Section

$$\sigma_{\text{eff}}^{pp} = \frac{m}{2} \frac{\sigma_A^{pp} \sigma_B^{pp}}{\sigma_{\text{DPS}}^{pp}}$$

→ Differential X-section single parton scattering for the process:  $pp \rightarrow A(B) + X$   
→ Differential X-section double parton scattering for the process:  $pp \rightarrow A + B + X$

**POCKET FORMULA**

Results for W, Jet productions...



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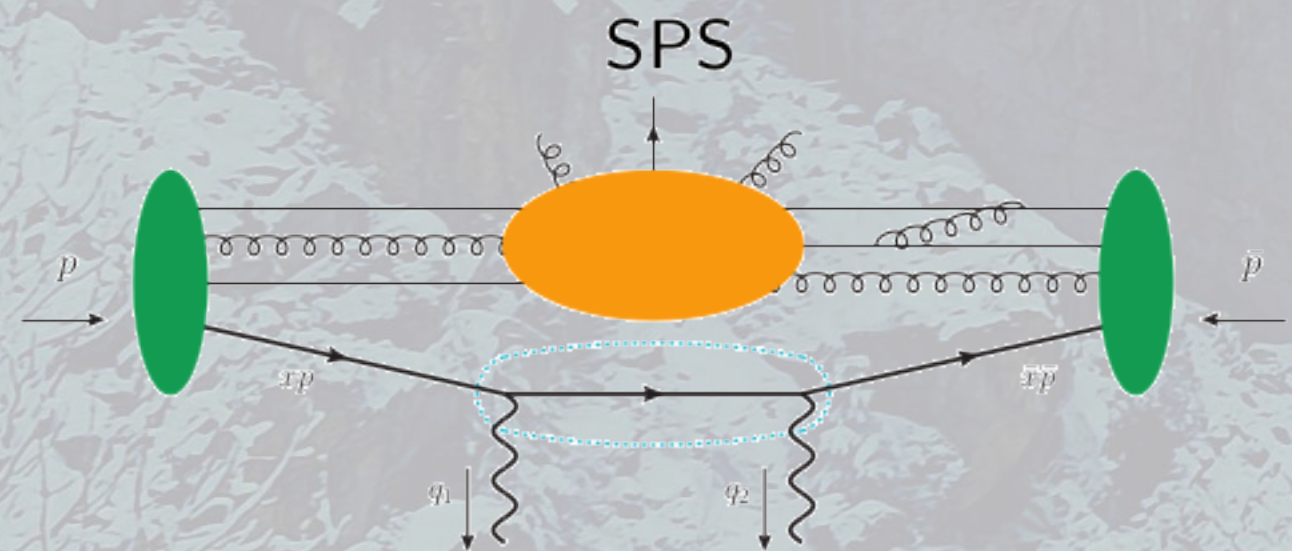
[CMS coll.], PRL 131 (2023) 091803

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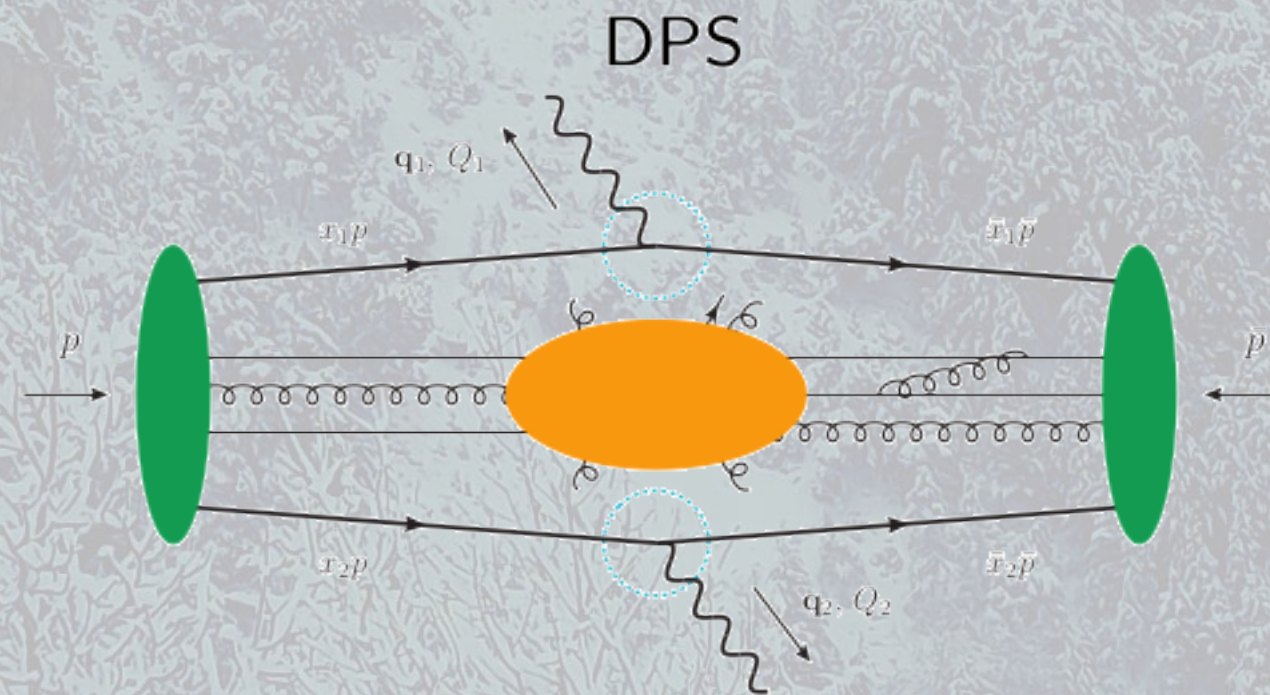


# Double Parton Scattering scales

Scale analysis of SPS and DPS processes



$$|\mathbf{q}_1^\perp + \mathbf{q}_2^\perp| \sim \Lambda \ll Q$$



$$|\mathbf{q}_1^\perp| \sim \Lambda \ll Q$$

$$|\mathbf{q}_2^\perp| \sim \Lambda \ll Q$$

First appearance in theory studies:

Politzer  
Paver, Treleani  
Mekhfi

Other ground-setting works:

Gaunt, Stirling  
Blok et al.  
Diehl et al.  
Manohar, Waalewijn  
Ryskin, Snigirev

...

where:

$$- Q = \min(Q_1, Q_2)$$

-  $\Lambda$  transverse momentum scale

$$- \Lambda_{\text{QCD}} \ll \Lambda \ll Q$$

Usually:

$$\frac{d^2\sigma_{\text{SPS}}}{d^2q_1 d^2q_2} \sim \frac{d^2\sigma_{\text{DPS}}}{d^2q_1 d^2q_2}$$

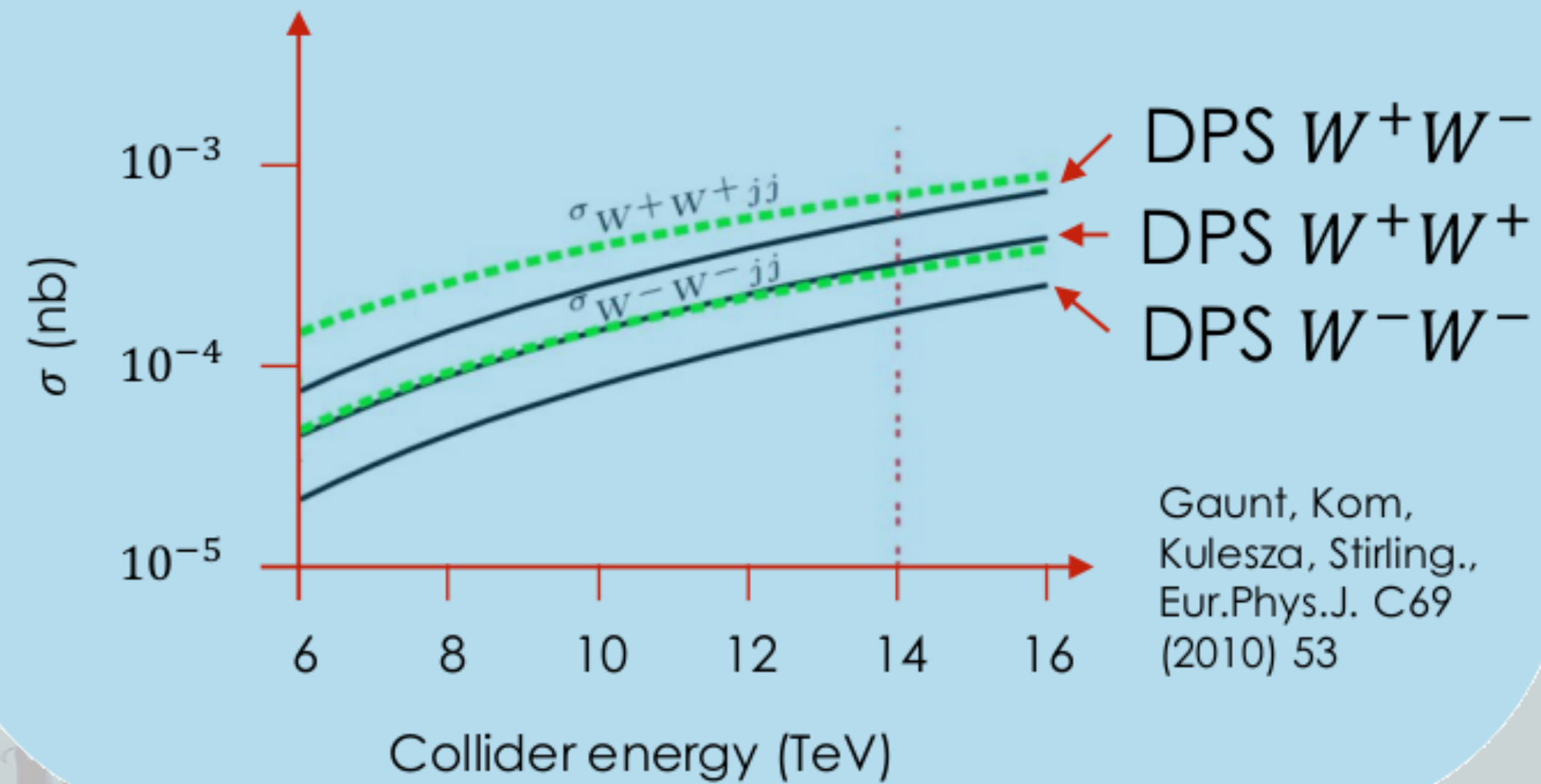
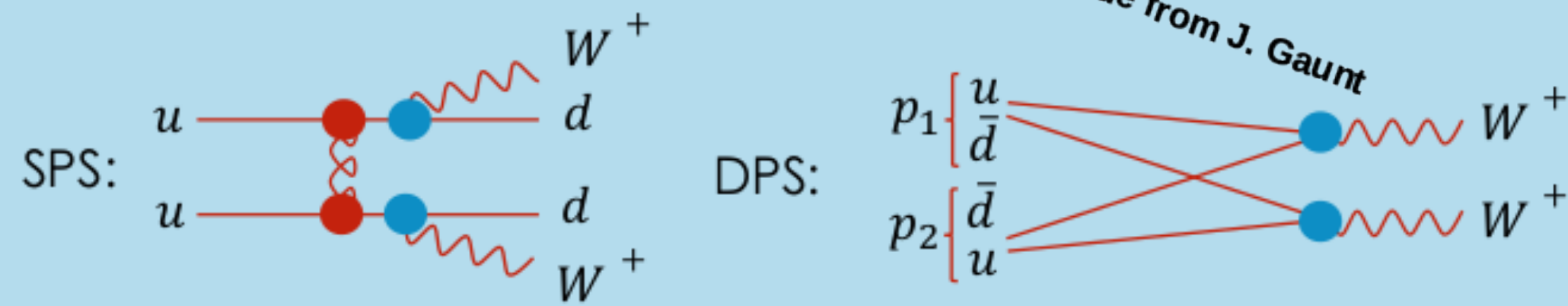
$$\frac{\sigma_{\text{DPS}}}{\sigma_{\text{SPS}}} \sim \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$

Nagar's slides MPI 2021

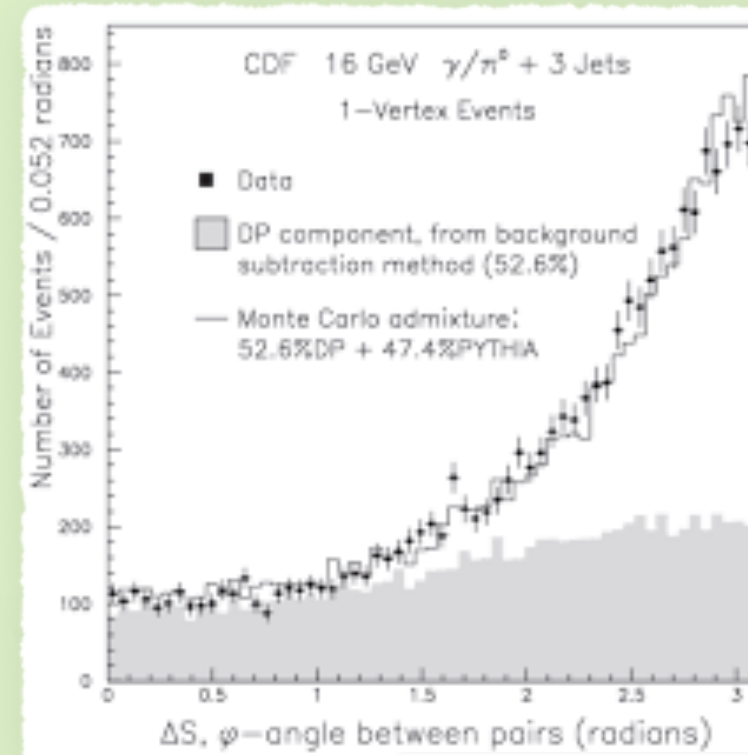


# Where and Why DPS?

DPS can give a significant contribution to processes where SPS is suppressed by small/multiple coupling constants:

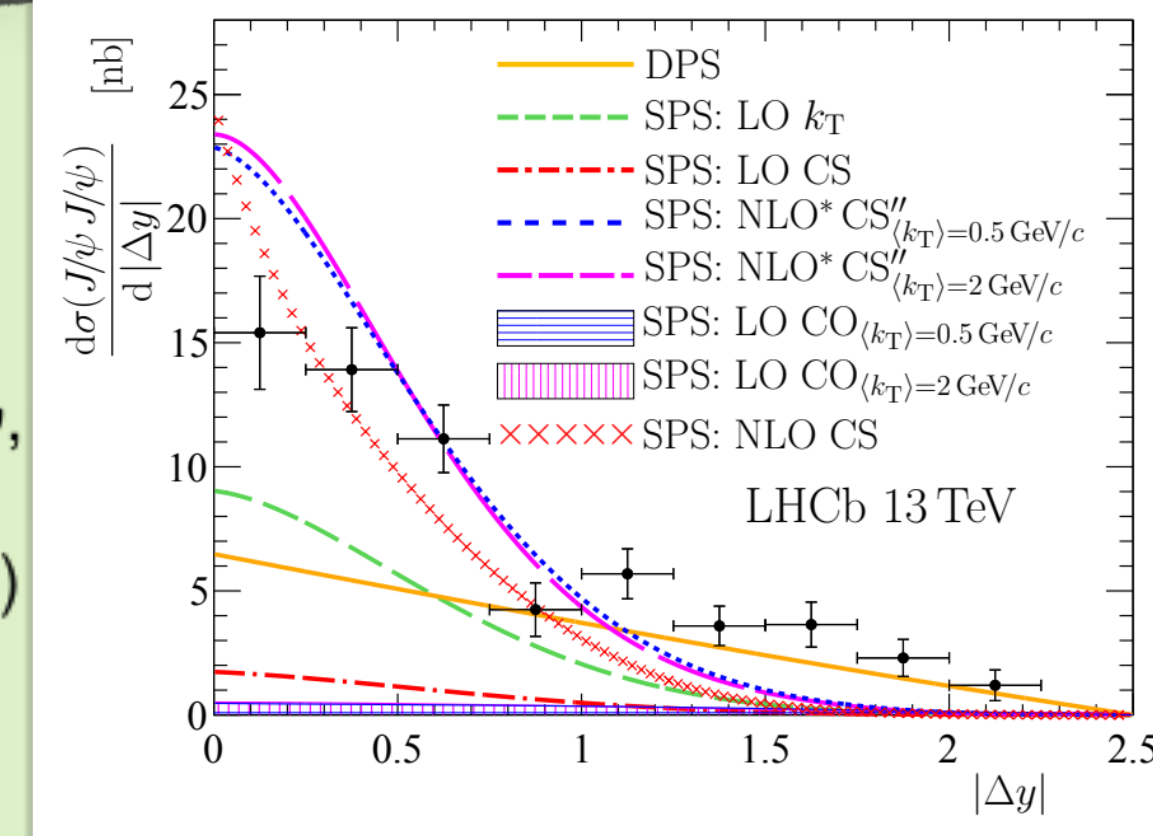


...or in certain phase space regions



CDF,  $\gamma + 3j$ ,  
Phys.Rev. D56  
(1997) 3811-3832

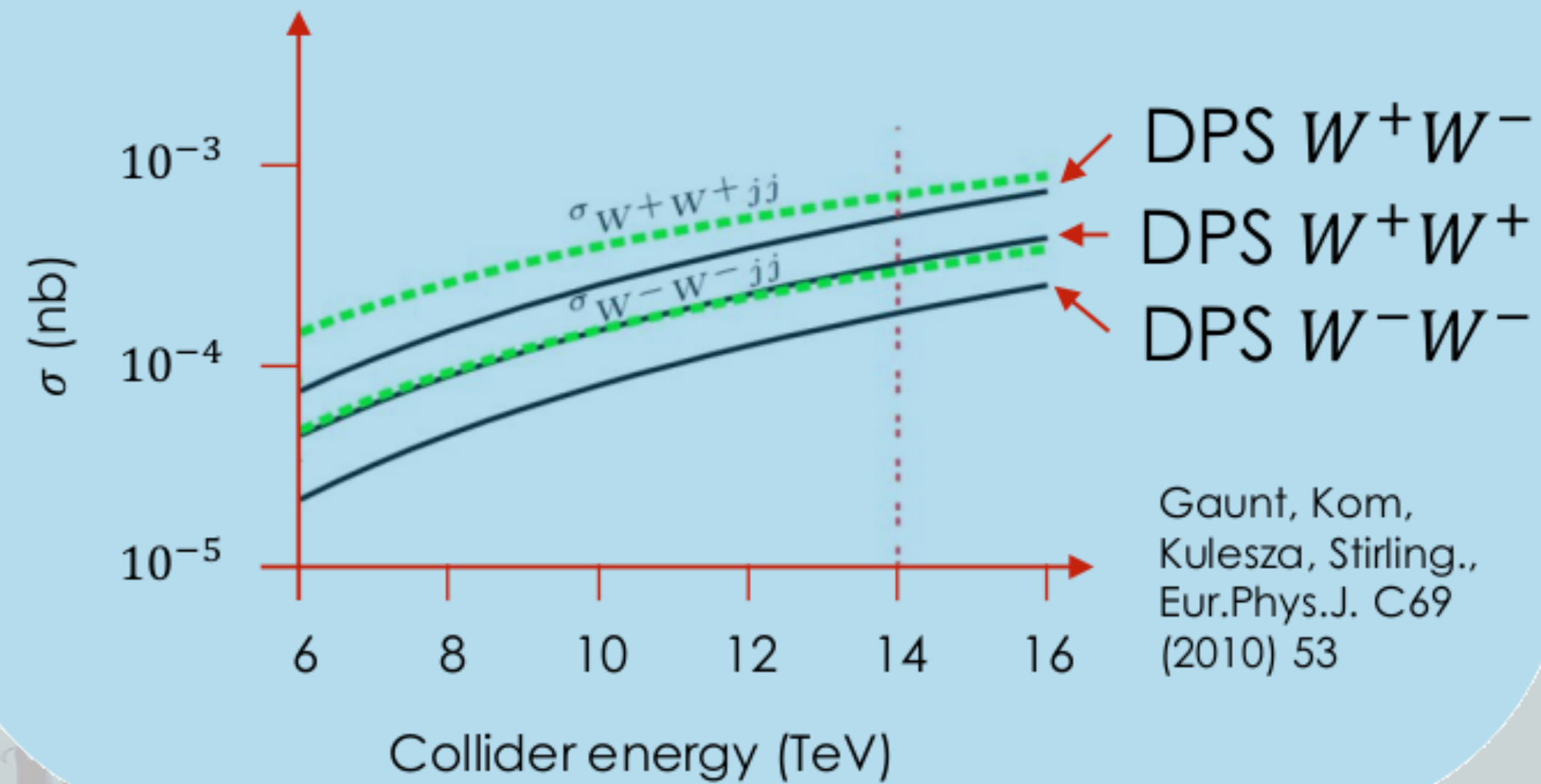
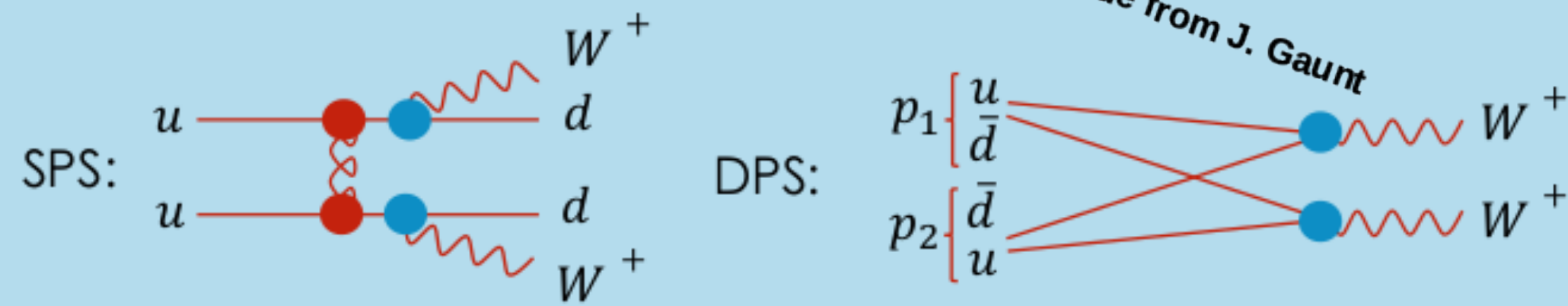
LHCb,  
double  $J/\psi$ ,  
JHEP 06,  
047, (2017)



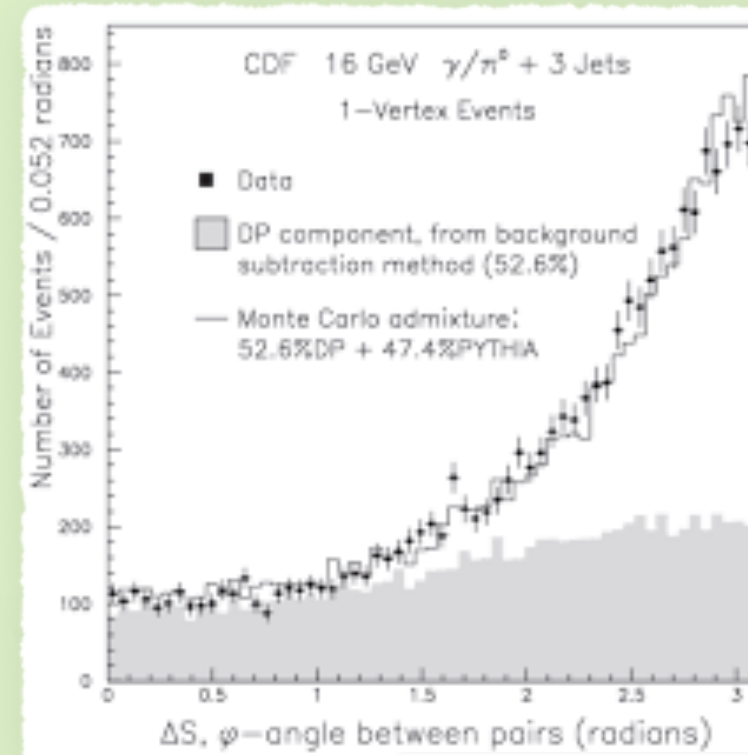


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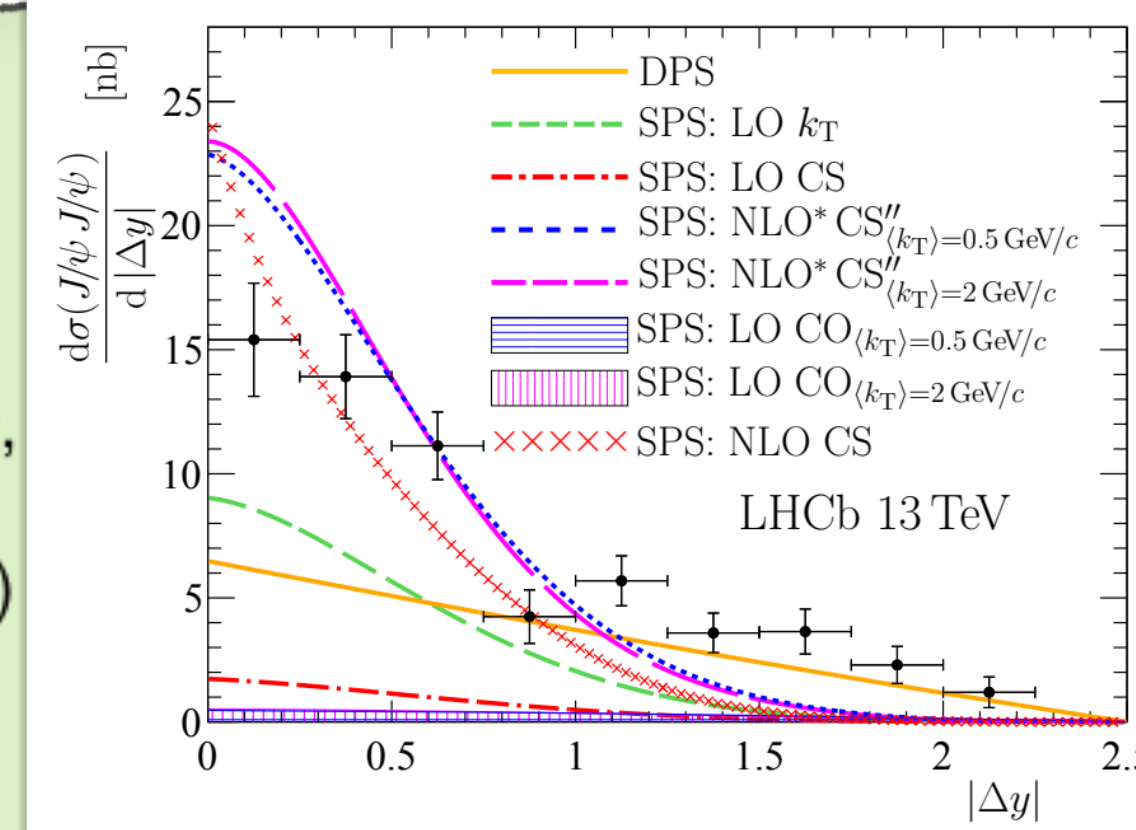


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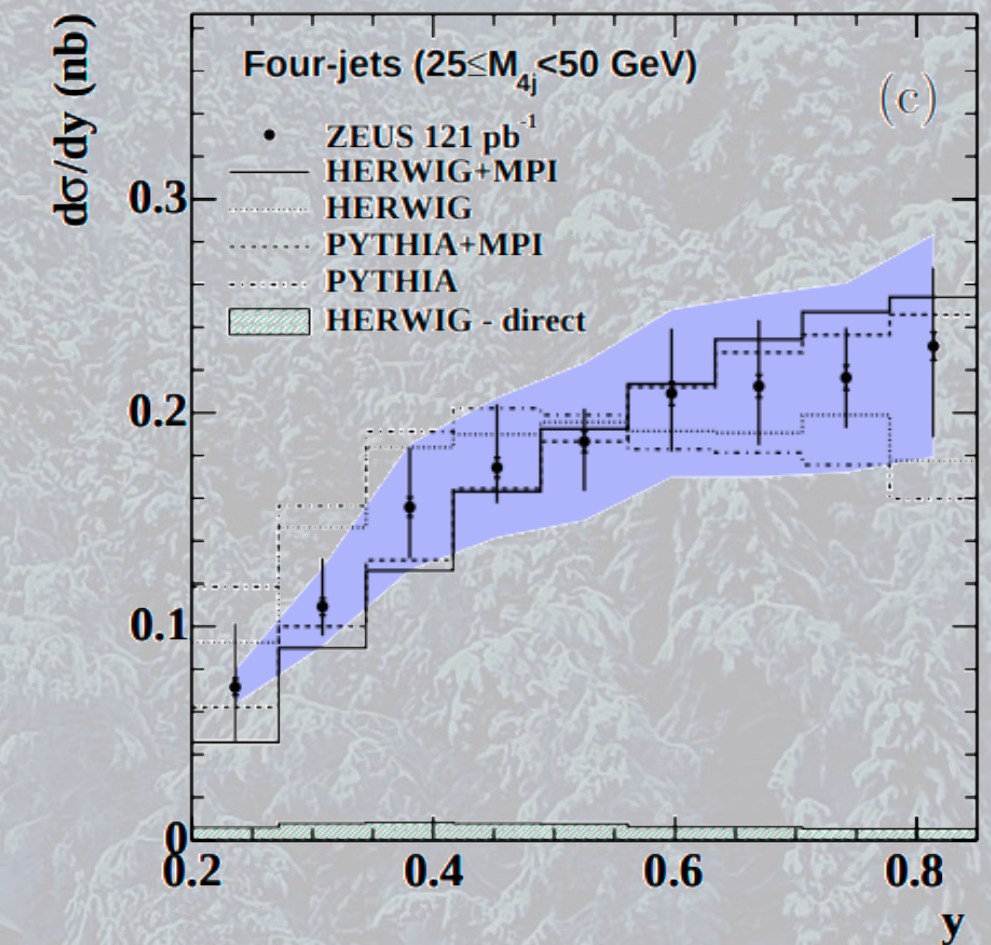


CDF,  $\gamma + 3j$ ,  
 Phys.Rev. D56  
 (1997) 3811-3832

LHCb,  
 double  $J/\psi$ ,  
 JHEP 06,  
 047, (2017)



in ep Colliders?



HERA data, ZEUS coll,  
 Nucl. Phys. B 729, 1 (2008)