AD POLOSA, SAPIENZA UNIVERSITY OF ROME

ON ELEMENTARY AND COMPOSITE PARTICLES: THE CASE OF EXOTIC HADRONS.

BASED ON WORK IN COLLABORATION WITH: A. ESPOSITO, D. GERMANI, B. GRINSTEIN, A. GLIOTI, L. MAIANI, F. PICCININI, A. PILLONI, R. RATTAZZI, V. RIQUER, M. TARQUINI

INTRODUCTION

- In what follows by 'elementary' hadron h we mean $q\bar{q}$, qqq or $qq\bar{q}\bar{q}\ldots$
- A 'composite' hadron or **'molecule'** is a mesonic or baryonic binary bound state hh'.
- Since the discovery of $X(3872)$, over 20 experimentally well established resonances have been studied.
  Most of them cannot be described as standard hadrons,
  might be either **molecules** or $qq\bar{q}\bar{q}\ldots$
- These particles are observed either in B meson or baryon decays, or in prompt pp collisions.

THE $X(3872)$ FINE TUNING

- If interpreted as a molecule, the $X(3872)$ should be a $D^0\bar{D}^{*0}$ bound state, with $J^{PC} = 1^{++}$ and $B \lesssim 100$ keV!
  Such a small value of B makes the X an outlier wrt to other X, Y, Z states, a champion in fine tuning.

- Does such a small B arise from a **tuning** of the strong interactions in the $D\bar{D}^*$ system ("molecule") making a large (and positive) so that $B = 1/(2ma^2)$ is so small?

- Most of the states are found within 10-20 MeV from meson-meson thresholds – most with central values above threshold but within Γ.

ELEMENTARY AND COMPOSITE DEUTERON

See Weinberg Phys. Rev. 137, B672 (1965)

Consider (like in the Lee model) the deuteron state

$$|d\rangle = \sqrt{Z}\,|\mathfrak{d}\rangle + \int_{\boldsymbol{k}} C_{\boldsymbol{k}}\,\underbrace{|np(\boldsymbol{k})\rangle}_{|\alpha\rangle}$$

where $\langle\alpha|\mathfrak{d}\rangle = 0$ and

$$\langle\mathfrak{d}|d\rangle = \sqrt{Z}$$

ELEMENTARY AND COMPOSITE DEUTERON

The state $|d\rangle$ is normalized

$$|d\rangle = \sqrt{Z}\,|\mathfrak{d}\rangle + \int_{\boldsymbol{k}} C_{\boldsymbol{k}} \underbrace{|np(\boldsymbol{k})\rangle}_{|\alpha\rangle}$$

The normalization $\langle d|d\rangle = 1$ gives

$$Z + \int_{\boldsymbol{k}} |C_{\boldsymbol{k}}|^2 = 1$$

and from the completeness relation

$$1 = |\mathfrak{d}\rangle\langle\mathfrak{d}| + \int |\alpha\rangle\langle\alpha|$$

$$\boxed{1 - Z = \int |\langle\alpha|d\rangle|^2 d\alpha}$$

See Weinberg Phys. Rev. 137, B672 (1965)

KÄLLÉN-LEHMAN REPRESENTATION

In the KL repres., the complete propagator of the bare field $\mathbf{\Phi}$ can be written in the spectral form

$$\Delta'(p) = \int_0^\infty \frac{\rho(\mu^2)}{p^2 + \mu^2 - i\epsilon}\, d\mu^2$$

$$\theta(p_0)\, \rho(-p^2) = \sum_n \delta^4(p - p_n)\, |\langle 0 | \Phi(0) | n \rangle|^2$$

$$|n\rangle = |\boldsymbol{k}\rangle \text{ or } |n\rangle = |\boldsymbol{k}_1, \boldsymbol{k}_2\rangle \ldots$$

and the Lehman sum rule can be proved

$$\rho(\mu^2) = Z\, \delta(\mu^2 - m^2) + \sigma(\mu^2)$$

$$\boxed{1 - Z = \int \sigma(\mu^2)\, d\mu^2}$$

ELEMENTARY VS COMPOSITE PARTICLES

$$\Delta'(p) = \underbrace{\frac{Z}{p^2 + m^2 - i\epsilon}}_{\mathscr{L} \supset \Phi\Delta^{-1}\Phi} + \dots$$

The fields of **elementary** particles appear in $\mathscr{L}$. The quadratic part of the action can be inverted to get the propagator.

As opposite, a **composite** particle is one whose field $\mathbf{\Phi}$ does *not* appear in $\mathscr{L}$: it can be created/destroyed by operators constructed by (functions of) other fields, e.g. those ones which do appear in $\mathscr{L}$.

KÄLLÉN-LEHMAN REPRESENTATION

In *perturbation theory* we would expect meson-meson pairs

$$\sigma(\mu^2) = 0 \quad \text{if} \quad \mu^2 < 4m^2$$

but in the real theory there could be bound states which appear below the meson-meson threshold.

What If the bound state sinks right at the one-meson state?

What we learn from this is that there is an upper bound to the coupling of the field $\mathbf{\Phi}$ to multiparticle states

$$g^2 = \int \sigma(\mu^2)\, d\mu^2 = 1 - Z \leq 1$$

ELEMENTARY AND COMPOSITE DEUTERON

The essential point in the Weinberg work is to show that the coupling

$$g_W^2 \equiv |\langle \alpha | V | d \rangle|^2$$

can be related to Z with the following formula

$$g_W^2 = \frac{2\pi\sqrt{2mB}}{m^2}(1 - Z)$$

where B is the binding energy of the deuteron.

This in turn leads to a relation between Z and the scattering observable r_0 – the effective range.

See Weinberg Phys. Rev. 137, B672 (1965)

DETERMINATION OF g_W

The 'unspecified' interaction V is introduced together with the free particle hamiltonian having 'bare elementary particle' eigenstates

$$(H_0 + V)|X\rangle = -B|X\rangle \qquad H_0|\alpha\rangle = E(\alpha)|\alpha\rangle \quad H_0|\mathfrak{X}\rangle = E_{\mathfrak{X}}|\mathfrak{X}\rangle$$

$$1 - Z = \int |\langle\alpha|X\rangle|^2 d\alpha = \int \frac{|\langle\alpha|V|X\rangle|^2}{(E(\alpha)+B)^2} d\alpha \equiv \int \frac{g_W^2}{(E(\alpha)+B)^2} d\alpha$$

$$d\alpha = \frac{1}{4\pi^2}(2m)^{3/2}\sqrt{E}\, dE$$

$$\int_0^\infty \frac{\sqrt{E}}{(E+B)^2} dE = \frac{\pi}{2\sqrt{B}}$$

THE X LIKE THE DEUETRON

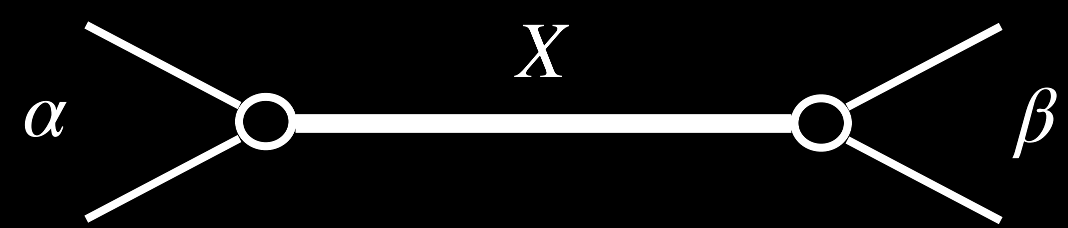
The state $|X\rangle$ is normalized as $|d\rangle$ and

$$|X\rangle = \sqrt{Z}\,|\mathfrak{X}\rangle + \int_{\boldsymbol{k}} C_{\boldsymbol{k}} \underbrace{|D\bar{D}^*(\boldsymbol{k})\rangle}_{|\alpha\rangle}$$

$$g_W^2 \equiv |\langle\alpha|V|d\rangle|^2 = \frac{2\pi\sqrt{2mB}}{m^2}(1-Z)$$

Now we want to plug this expression of g_W in the polar expression for the $D\bar{D}^*$ scattering amplitude (computed for $E, B \sim 0$) and compare to the standard scattering amplitude to find a relation between Z and r_0.

POLAR AMPLITUDE



Consider the $D\bar{D}^*$ scattering. The X particle couples to the 2-particle state $|\alpha\rangle, |\beta\rangle = |D\bar{D}^*\rangle$ with some coupling $g^2 \propto 1 - Z$.
The scattering amplitude, assuming D^* stable, is found to be

$$f(\alpha \to \beta) = -\frac{1}{8\pi E}\frac{g^2}{p^2 + m_X^2 - i\epsilon}$$

where $g^2 \equiv (8m\, m_X^2) \times g_W^2$ and $m =$ reduced mass $D\bar{D}^*$.

THE POLAR FORMULA FOR $f(\alpha \to \beta)$

$$f(\alpha \to \beta) = -\frac{1}{2\pi E}\sqrt{\frac{k' E_1' E_2' E_1 E_2}{k}}\, M_{\beta\alpha} = -\frac{1}{8\pi E}(2m_D)(2m_{D*})\, M_{\beta\alpha}$$

$$= -\frac{1}{8\pi E}(2m_D)(2m_{D*})(2m_X)\, M_{\beta X}\frac{1}{p^2 + m_X^2 - i\epsilon}M_{X\alpha}$$

$$= -\frac{1}{8\pi E}8mm_X^2\frac{|\langle D\bar{D}^* | V | X\rangle|^2}{p^2 + m_X^2 - i\epsilon} =$$

$$= -\frac{1}{8\pi E}8mm_X^2\frac{g_W^2}{p^2 + m_X^2 - i\epsilon} =$$

$$= -\frac{1}{8\pi E}\frac{g^2}{p^2 + m_X^2 - i\epsilon} \qquad g^2 \equiv 8mm_X^2 \times g_W^2$$

In Weinberg's treatment $g_W \equiv |\langle \alpha | V | X \rangle|$.
Thus $[g_W] = 1/\sqrt{E}$ and $[g] = E$.

THE POLAR FORMULA AT $B \sim 0$ AND $E \sim 0$

Neglecting terms of order B^2 and E^2 ($E = k^2/2m$) one finds in the case of the X

$$f(\alpha \to \beta) = -\frac{1}{8\pi m_X}\frac{g^2}{(p_D + p_{D^*})^2 + m_X^2 - i\epsilon} \simeq -\frac{1}{16\pi m_X^2}\frac{g^2}{E+B}$$

ADP Phys. Lett. B746, 248 (2015)

This can be compared to the *effective range expansion* of f

$$f = \frac{1}{-\varkappa_0 + \frac{1}{2}r_0 k^2 - ik}$$

where $\varkappa_0 = 1/a$. When $k = i\sqrt{2mB} \equiv i\varkappa$ ('matching' the shallow bound state in the attractive potential) we require be at the pole

$$\left(-\varkappa_0 + \frac{1}{2}r_0 k^2 - ik\right)_{k=i\varkappa} = 0$$

DETERMINATION OF r_0

The latter condition gives

$$-\varkappa_0 = -\varkappa + \frac{1}{2} r_0 \varkappa^2$$

to be substituted back in f

$$f = \frac{1}{\frac{r_0}{2}(k^2+\varkappa^2) - (\varkappa + ik)} = \frac{1}{\frac{r_0}{2}(k^2+\varkappa^2) - \frac{(\varkappa+ik)(\varkappa-ik)}{(\varkappa-ik)}} =$$

$$= -\frac{1}{\frac{r_0}{2}(k^2+\varkappa^2) - \frac{1}{2\varkappa}(k^2+\varkappa^2)} = -\frac{\varkappa}{m(1-r_0\varkappa)} \frac{1}{E+B}$$

So from a comparison with our "polar" f

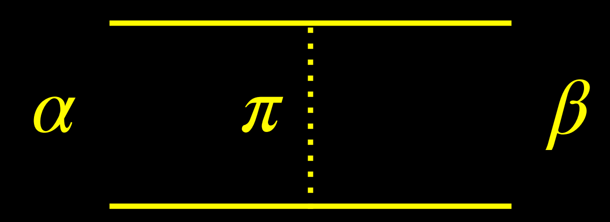
$$\frac{16\pi m_X^2}{m} \frac{\varkappa}{1-r_0\varkappa} = \frac{2\pi\varkappa}{m^2}(1-Z) \times (8mm_X^2)$$

Esposito, Maiani, Pilloni, ADP, Riquer, 2108.11413, *Phys. Rev. D*105 (2022) 3, L031503

r_0 AND a FORMULAE

Solving the previous formula for r_0

$$r_0 = -\frac{Z}{1-Z}R + O\left(\frac{1}{\Lambda}\right)$$



$$R = \frac{1}{\varkappa} = \frac{1}{\sqrt{2mB}}$$

(B = binding energy)

The (positive!) scattering length is obtained using the expression of r_0 given above into $\left(-\varkappa_0 + \frac{1}{2}r_0 k^2 - ik\right)_{k=i\varkappa} = 0$

$$a = \frac{2(1-Z)}{2-Z}R + O\left(\frac{1}{\Lambda}\right)$$

(scattering length > 0)

LANDAU ARGUMENT: $Z = 0$ MOLECULE

The potential scattering of two slow particles $(kR \ll 1)$ described by an attractive potential U, with range R, featuring a shallow bound state at $-B$ has a **universal** scattering amplitdue

$$f(ab \to ab) = -\frac{1}{\sqrt{2m}} \frac{\sqrt{B} - i\sqrt{E}}{E + B}$$

obtained by $\cot \delta_0 = -\sqrt{B/E}$. This is independent on the details of V and affected only by the value of B. A comparison with the pole formula

$$f(\alpha \to \beta) \simeq -\frac{1}{16\pi m_X^2} \frac{g^2}{E + B}$$

can be done at $k = i\varkappa$ where the numerator in the first is $2\sqrt{B}$

ADP Phys. Lett. B746, 248 (2015)

LANDAU ARGUMENT: $Z = 0$ MOLECULE

Therefore by comparing

$$f = -\frac{1}{\sqrt{2m}}\frac{2\sqrt{B}}{E+B}$$

with the pole formula for $f(\alpha \to \beta)$ we get

$$g^2 = \frac{16\pi m_X^2}{m}\sqrt{2mB} = 8mm_X^2 \times (g_W)_{Z=0}$$

In the Landau treatment $Z = 0$ and $r_0 = 0$ so it is impossible to establish if there is or not the 'elementary component'.

Most likely Landau had in mind the Heisenberg Nuclear Democracy argument.

L.D. Landau, JETP 39, 1865 (1960)

THE UNIVERSAL WAVEFUNCTION $\psi(r)$

Insert g^2 back in the pole formula

$$f(\alpha \to \beta) \simeq -\frac{1}{16\pi m_X^2}\frac{g^2}{E+B} = -\frac{\sqrt{2mB}}{m}\frac{1}{E+B}$$

In scattering theory the coefficient

$$\frac{\sqrt{2mB}}{m} \equiv \frac{A_0^2}{2m}$$

where A_0 is the coefficient of the stationary state corresponding to the bound state

$$\chi(r) = A_0 \exp(-r\sqrt{2mB})$$

so including the $Y_0^0 = 1/\sqrt{4\pi}$ we get

$$\psi(r) = \left(\frac{2mB}{4\pi^2}\right)^{1/4} \frac{\exp(-r\sqrt{2mB})}{r}$$

THE UNIVERSAL WAVEFUNCTION $\psi(r)$

The wf found (for small B values) is **universal**, i.e. it does not depend on the details of the potential.
For small B it is expected to be broader than the potential range, so a $\lambda\delta^3(\boldsymbol{r})$ potential might be used. Indeed the

$$\psi(r) = \left(\frac{2mB}{4\pi^2}\right)^{1/4} \frac{\exp(-r\sqrt{2mB})}{r}$$

E. Braaten and M. Kusunoki, PRD69, 074005 (2004)

can be found as the $E = -B$ bound state wf of the $\lambda\delta^3(\boldsymbol{r})$ potential provided that the (renormalized) coupling is

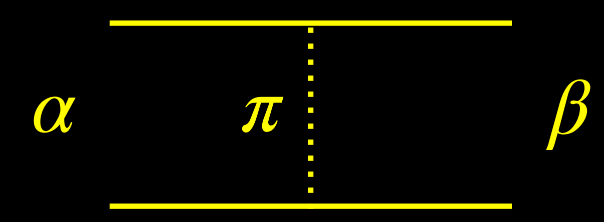
$$\lambda = \frac{2\pi}{m\sqrt{2mB}}$$

R. Jackiw, `Diverse topics in Theoretical and Mathematical Physics`, World Scientific

BACK TO r_0 AND a FORMULAE — NUMBERS

Solving the previous formula for $\boldsymbol{r_0}$

$$r_0 = -\frac{Z}{1-Z}R + O\left(\frac{1}{\Lambda}\right)$$



$$R = \frac{1}{\varkappa} = \frac{1}{\sqrt{2mB}}$$

($\boldsymbol{B} =$ binding energy)

The (positive!) scattering length is obtained using the expression of $\boldsymbol{r_0}$ just obtained into $\left(-\varkappa_0 + \frac{1}{2}r_0k^2 - ik\right)_{k=i\varkappa} = 0$

$$a = \frac{2(1-Z)}{2-Z}R + O\left(\frac{1}{\Lambda}\right)$$

(scattering length > 0)

THE Λ SCALE

In the case of the deuteron $\boldsymbol{d}$

$$\Lambda = m_\pi \Rightarrow \frac{1}{\Lambda} \simeq 1 \text{ fm}$$

because the pion can be integrated out given that

$$m_n - m_p \ll m_\pi$$

In the case of the $\boldsymbol{X}$, pion interactions between $\boldsymbol{D}$ and $\boldsymbol{\bar{D}^*}$ (u-channel)

$$\Lambda^2 = m_\pi^2 - \underbrace{(m_{D^*} - m_D)^2}_{q_0^2} \simeq (44 \text{ MeV})^2$$

giving

$$\frac{1}{\Lambda} \simeq 4.5 \text{ fm}$$

THE SIGN OF r_0 IN A ATTRACTIVE V

Scattering in the presence of shallow bound states generated by *purely attractive potentials* in NRQM are characterized by

$$r_0 \geq 0$$

even if there is a repulsive core, but in a *very* narrow region around the origin. Therefore the 1 fm estimated above is +1 fm

$$r_0 \simeq -\frac{Z}{1-Z}R + 1\ \text{fm} = r_0^{\text{exp.}} = +1.74\ \text{fm}$$

So we conclude that $Z \simeq 0$. The deuteron is a molecule! Only a "large" (wrt 1 fm) and negative r_0 would have been the token of the elementary deuteron.

Esposito, Maiani, Pilloni, ADP, Riquer, 2108.11413, *Phys. Rev. D*105 (2022) 3, L031503

DATA ON X: LHCB ANALYSIS

arXiv:2005.13419

For small kinetic energies

$$f(X \to J/\psi\pi\pi) = -\frac{(2N/g)}{(2/g)(E - m_X^0) - \sqrt{2\mu_+\delta} + E\sqrt{\mu_+/2\delta} + ik}$$

$$-\frac{1}{a} = \frac{2m_X^0}{g} + \sqrt{2\mu_+\delta} \simeq -6.92 \text{ MeV}$$ positive a

$$r_0 = -\frac{2}{\mu g} - \sqrt{\frac{2\mu_+}{2\mu^2\delta}} \simeq -5.34 \text{ fm}$$ negative r_0

using $E = k^2/2\mu$, μ being the reduced mass of the neutral $D\bar{D}^*$ pair, and taking g (LHCb) and m_X^0 (stable determination) from the experimental analysis. Since g can be larger, $r_0 \leq -2$ fm.

DETERMINATION OF Z

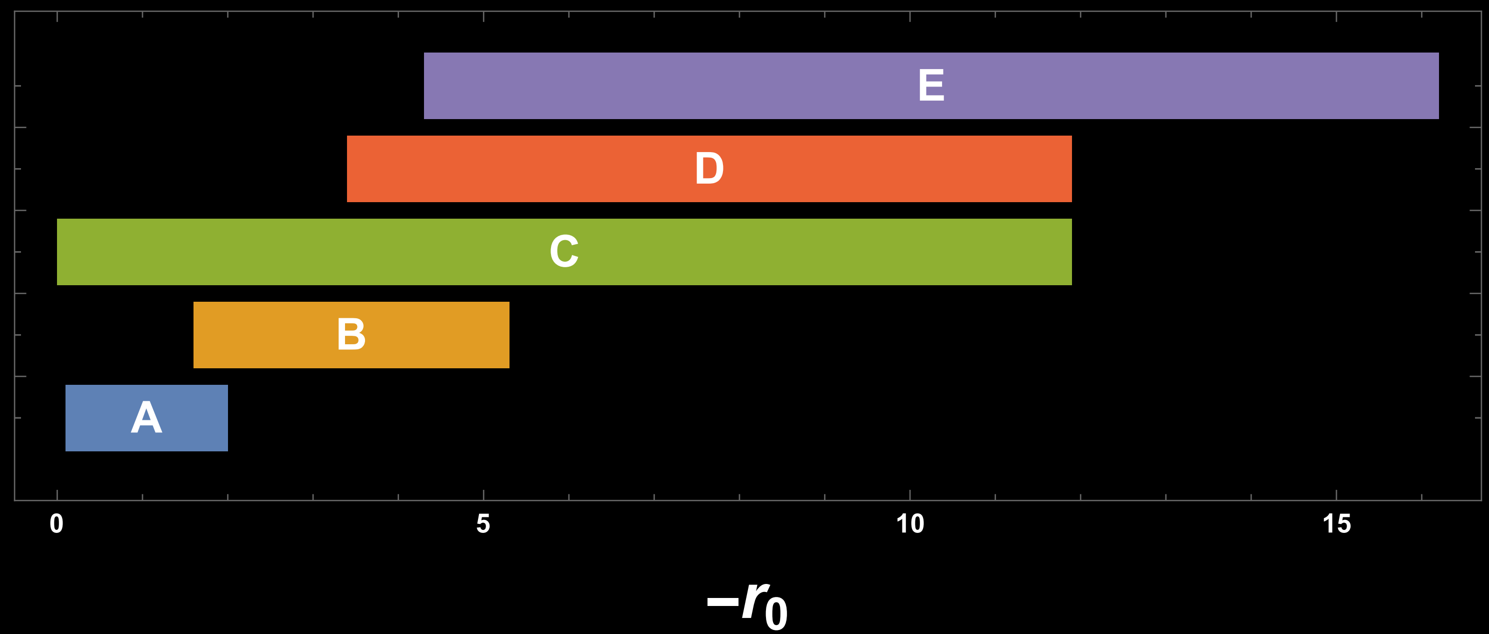
Neglect for the moment $O(1/\Lambda)$ corrections

$$r_0 = -\frac{Z}{1-Z}R = -5.34 \text{ fm}$$
$$a = \frac{2(1-Z)}{2-Z}R = 197/6.92 \text{ fm}$$

Gives $Z = 0.15 \neq 0!$ and $B = 20$ keV

Including ± 5 fm makes quite a difference depending on the sign.
In the case of -5 fm we might have $Z = 0$ even with $r_0^{\text{exp}} = -5.32$ fm!
In the case of $+5$ fm, a negative experimental r_0 is the proof of the compact state.
However we shall see that in the molecular case $O(1/\Lambda) \to -0.2$ MeV

$(-r_0)$ ACCORDING TO SOME ESTIMATES



A: Baru et al., 2110.07484

B: Esposito et al., 2108.11413

C: LHCb, 2109.01056

D: Maiani & Pilloni GGI-Lects

E: Mikhasenko, 2203.04622

r_0 FROM LATTICE

M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 2202.10110

Applying the lattice Lüscher method, the authors study the $D\bar{D}^*$ scattering amplitude and make a determination of the scattering length and of the effective range for $\mathcal{T}_{cc}$

$$a = -1.04(29)\ \text{fm}$$
$$r_0 = +0.96^{+0.18}_{-0.20}\ \text{fm}$$

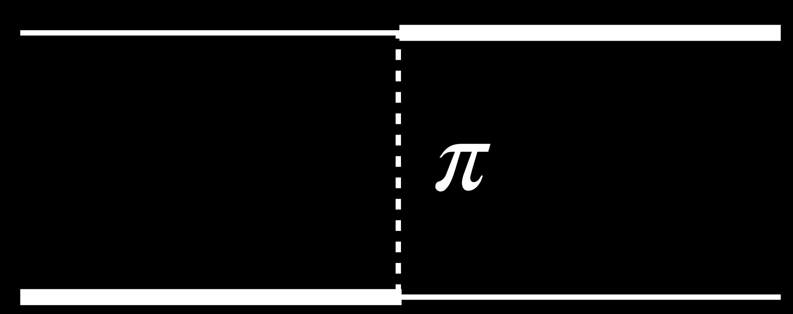
The mass of the pion is $m_\pi = 280$ MeV, to keep the D^* stable. This result, for the moment, is compatible with a *virtual state* because of the negative a – like the singlet deuteron.
As for LHCb (2109.01056 p.12)

$$a = +7.16\ \text{fm}$$
$$-11.9 \leq r_0 \leq 0\ \text{fm}$$

r_0 IN THE MOLECULAR PICTURE

$$H_{DD*} = \frac{\boldsymbol{p}_{D*}^2}{2m_{D*}} + \frac{\boldsymbol{p}_D^2}{2m_D} - \lambda_0\, \delta^3(\boldsymbol{r})$$

A perturbation to the $\delta^3(\boldsymbol{r})$ potential derives from



Esposito, Glioti, Germani, ADP, Rattazzi, Tarquini, PLB847, 138285 (2023).

Potential **=** FT of the propagator in NR approximation

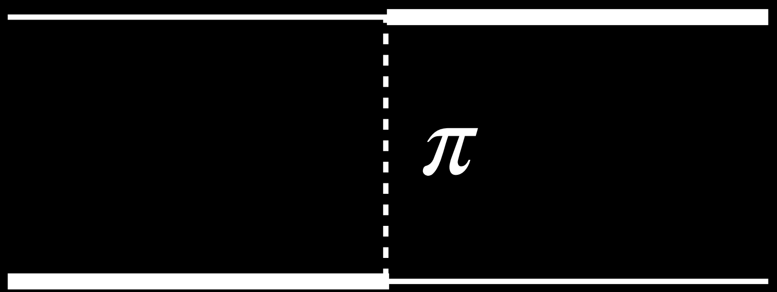
$$\int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{q^2 + m_\pi^2 - i\epsilon}\, d^3q \xrightarrow[\text{NR}]{} \int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2 - \mu^2 - i\epsilon}\, d^3q \approx \int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2 - i\epsilon}\, d^3q$$

$$\int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2 - i\epsilon}\, d^3q = -\frac{(2\pi)^3}{4\pi}\left(\frac{3\hat{r}_i \hat{r}_j}{r^3} - \frac{\delta_{ij}}{r^3} - \frac{4\pi}{3}\delta^3(\boldsymbol{r})\right)$$

r_0 IN THE MOLECULAR PICTURE

$$H_{DD*} = \frac{\boldsymbol{p}_{D*}^2}{2m_{D*}} + \frac{\boldsymbol{p}_D^2}{2m_D} - \lambda_0\,\delta^3(\boldsymbol{r})$$

A perturbation to the $\delta^3(\boldsymbol{r})$ potential derives from

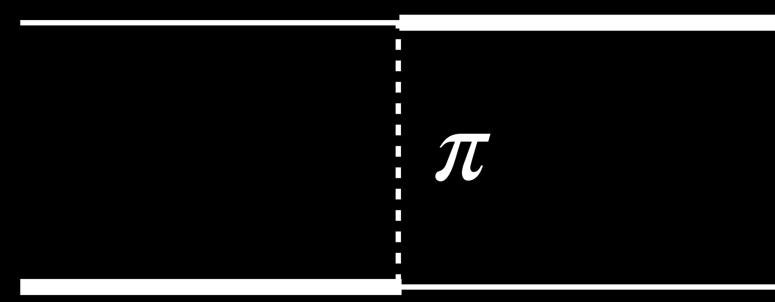


Esposito, Glioti, Germani, ADP, Rattazzi, Tarquini, PLB847, 138285 (2023).

In $\boldsymbol{S}$-wave we have to include the condition $\langle \hat{r}_i \hat{r}_j \rangle = \frac{1}{3}\delta_{ij}$ which, for $\boldsymbol{\mu} = \boldsymbol{0}$, leaves only an extra $\boldsymbol{\delta^3(r)}$ potential term.

But $\mu^2 = (m_{D*} - m_D)^2 - m_\pi^2 \simeq 44$ MeV, and this requires an extra, complex potential term.

THE COMPLEX POTENTIAL



Esposito, Glioti, Germani, ADP, Rattazzi, Tarquini, PLB847, 138285 (2023).

Keep μ finite! Are the corrections to r_0 of the size $O(1/m_\pi)$ or $O(1/\mu)$?

$$V_w = -\frac{g^2}{2f_\pi^2}\int \frac{q_i q_j\, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2 - \mu^2 - i\epsilon}\frac{d^3q}{(2\pi)^3} = -\underbrace{\frac{g^2}{6f_\pi^2}}_{\beta}\left(\delta^3(r) + \mu^2\frac{e^{i\mu r}}{4\pi r}\right)\delta_{ij}$$

The contraction with polarizations $e_i^{(\lambda)}\bar{e}_j^{(\lambda')}$ gives $\delta_{\lambda\lambda'}$.

As for the $\delta^3(r)$ potential, it has not the right coefficient to have a bound state at $E = -B$. But an overall λ can be defined appropriately to make give such a bound state.

THE COMPLEX POTENTIAL

So we divide $\boldsymbol{V}$ into

$$V = V_s + V_w = -\underbrace{\left(\lambda_0 + 4\pi\beta\right)}_{\lambda}\delta^3(\boldsymbol{r}) - \alpha\mu^2\frac{e^{i\mu r}}{r}$$

To compute any amplitude, all orders in $\boldsymbol{V_s}$ are needed, and possibly only the first order in $\boldsymbol{V_w}$.

Can we find $\boldsymbol{r_0}$ as a result of the correction to $\boldsymbol{f}$ due to the complex potential?

DISTORTED WAVE BORN APPROXIMATION

$$f = \frac{1}{k\cot\delta(k) - ik} = f_s + f_w = \frac{1}{-\frac{1}{a} - ik} + f_w$$

$$f_w = -\frac{2m}{4k^2}\int V_w(r)\,\chi_s^2(r)\,dr$$

Where $\chi_s(r)$ are scattering w.f. of the $\delta^3(\boldsymbol{r})$ potential, and $\boldsymbol{m}$ is the invariant $\boldsymbol{DD^*}$ mass. Thus r_0 is determined by the $\boldsymbol{k}^2$ coefficient in the *double expansion* around $k = 0$ and $\alpha = 0$ of the expression

$$f^{-1} = \left(\frac{1}{-\frac{1}{a} - ik} - \frac{2m}{4k^2}\int V_w(r)\,\chi_s^2(r)\,dr\right)^{-1}$$

CALCULATION OF r_0 (DWBA)

$$r_0 = 2m\alpha\left(\frac{2}{\mu^2 a^2} + \frac{8i}{3\mu a} - 1\right)$$

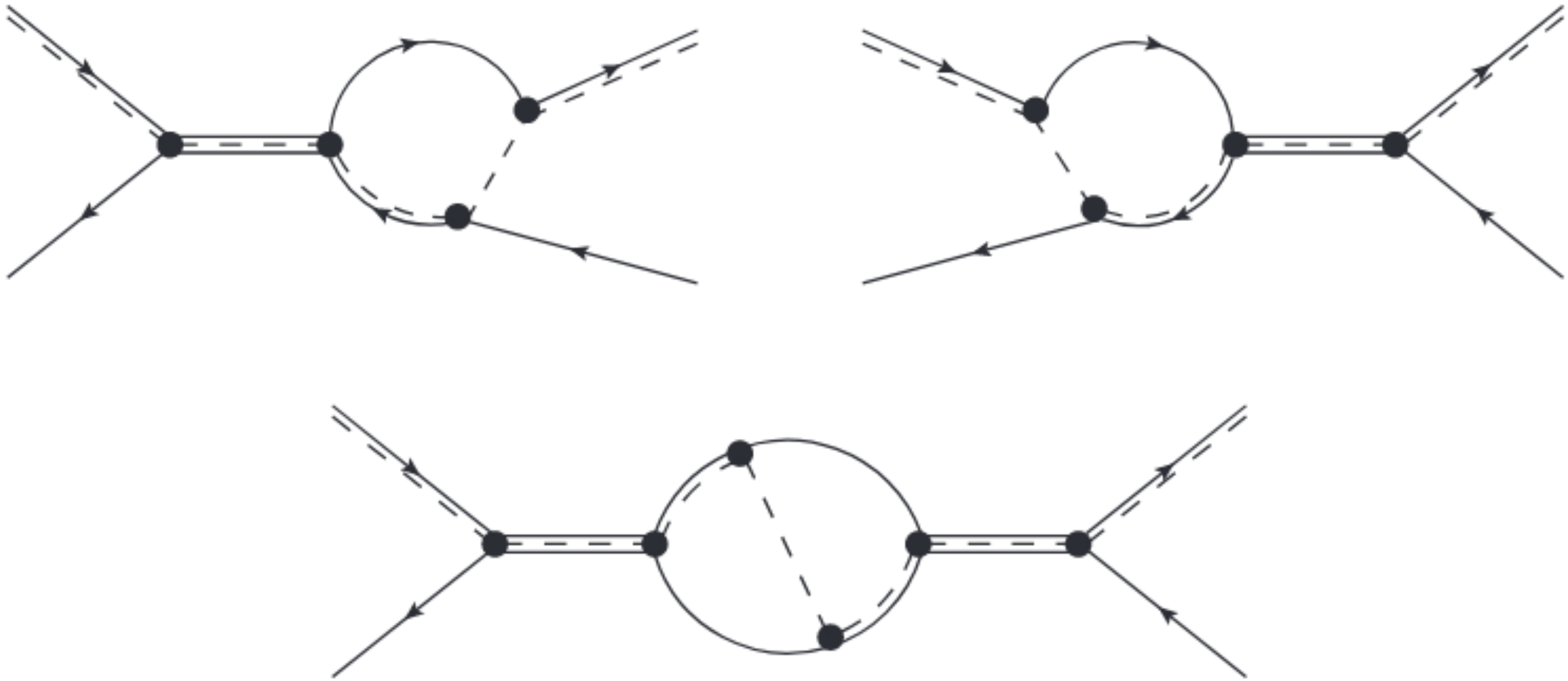
Esposito, Glioti, Germani, ADP, Rattazzi, Tarquini, PLB847, 138285 (2023).

$$-0.20 \text{ fm} \lesssim \text{ Re } r_0 \lesssim -0.15 \text{ fm}$$

$$0 \text{ fm} \lesssim \text{ Im } r_0 \lesssim 0.17 \text{ fm}$$

$$\alpha = \frac{g^2}{24\pi f_\pi^2} = \frac{5\times 10^{-4}}{\mu^2}$$

These results agree, analytically, with what found by Braaten et al. using EFT. It turns out that the real part of r_0 is just a tiny (negative!) fraction of a Fermi. This confirms the fact that the Weinberg criterion can be extended to the $X(3872)$ too.



Braaten, Galilean invariant XEFT, Phys. Rev. D 103, 036014 (2021), arXiv:2010.05801 [hep-ph]

RADIATIVE DECAYS OF THE X

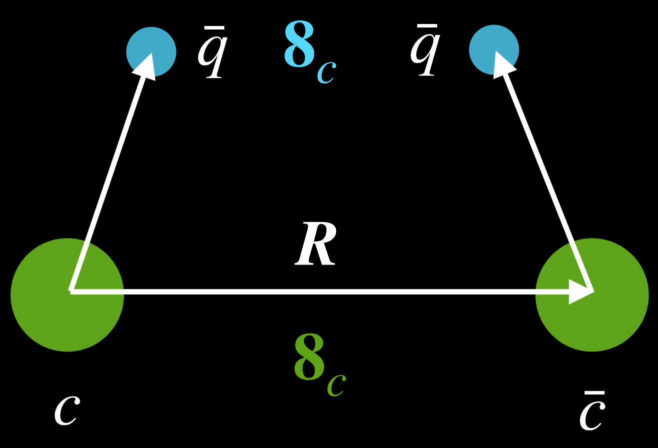
There are experimental indications that the ratio

$$\mathcal{R} = \frac{\mathcal{B}(X \to \psi' \gamma)}{\mathcal{B}(X \to \psi \gamma)}$$

is of order **1 or larger**. We find that this cannot be done with the universal $\psi(r)$ we discussed above giving

$$\mathcal{R}(R_0 = 10 \text{ fm}) \simeq 0.036$$

The situation is completely different for a compact tetraquark



B. Grinstein, L. Maiani, ADP *in preparation*

RADIATIVE DECAYS OF THE X

The fast motion of light quarks, in the field of heavy quarks (slow), generates an effective potential $V(R)$ which in turn regulates the slower motion of heavy quarks – and can be used to calculate the spectrum. This is the Born-Oppenheimer approximation.

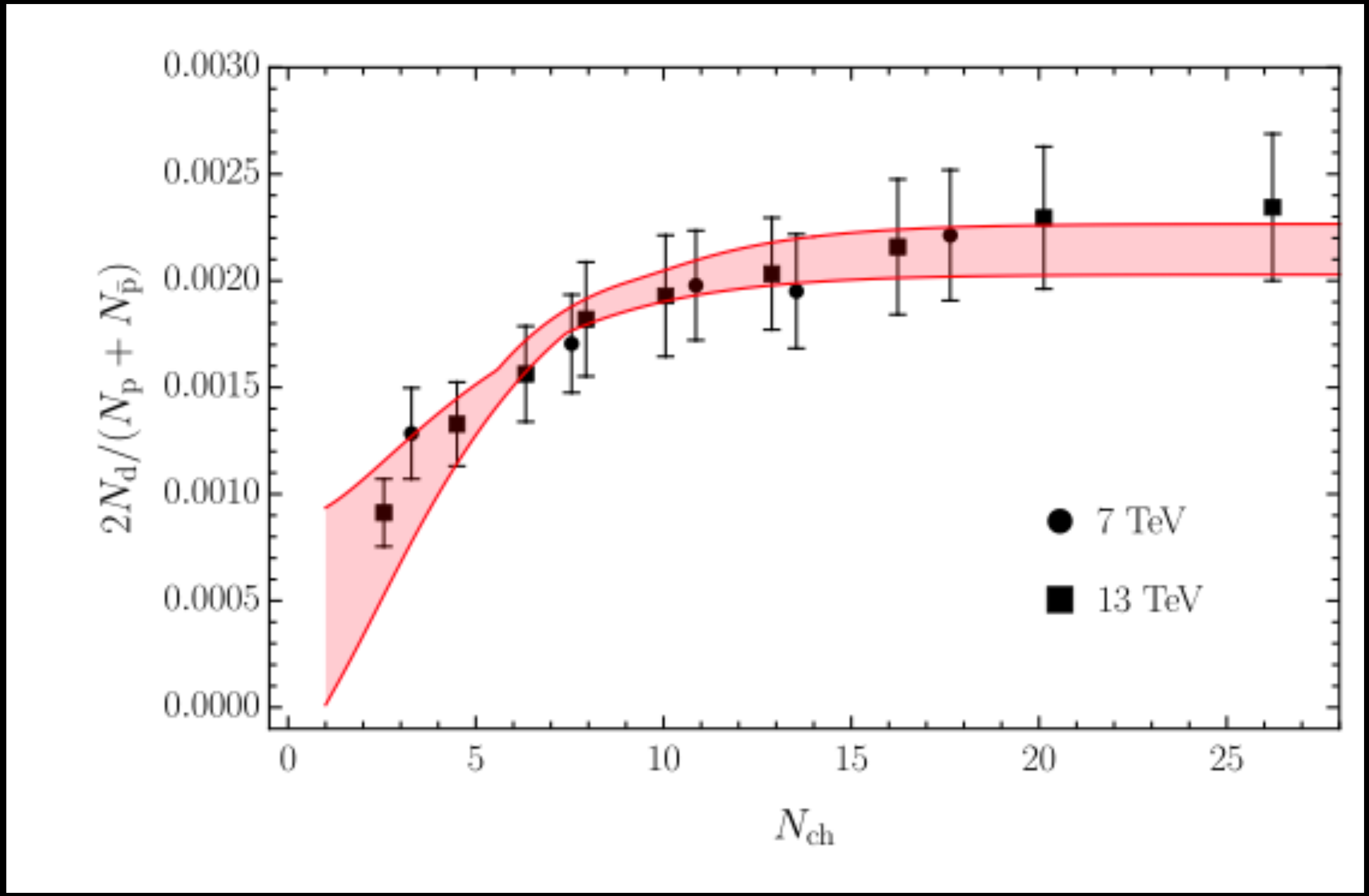
The light quarks have to meet in one point to make the photon: this is hard in a large molecule (small B), given that the size of D mesons is fixed. However this overlap is way more probable in a compact tetraquark where the cq and $\bar{c}\bar{q}$ orbitals can be larger.

$$\mathscr{R} \gtrsim 1 \text{ fm}$$

B. Grinstein, L. Maiani, ADP *in preparation*

DOES THE X(3872) BEHAVE AS THE DEUTERON?

ALICE: 1902.09290; 2003.03184



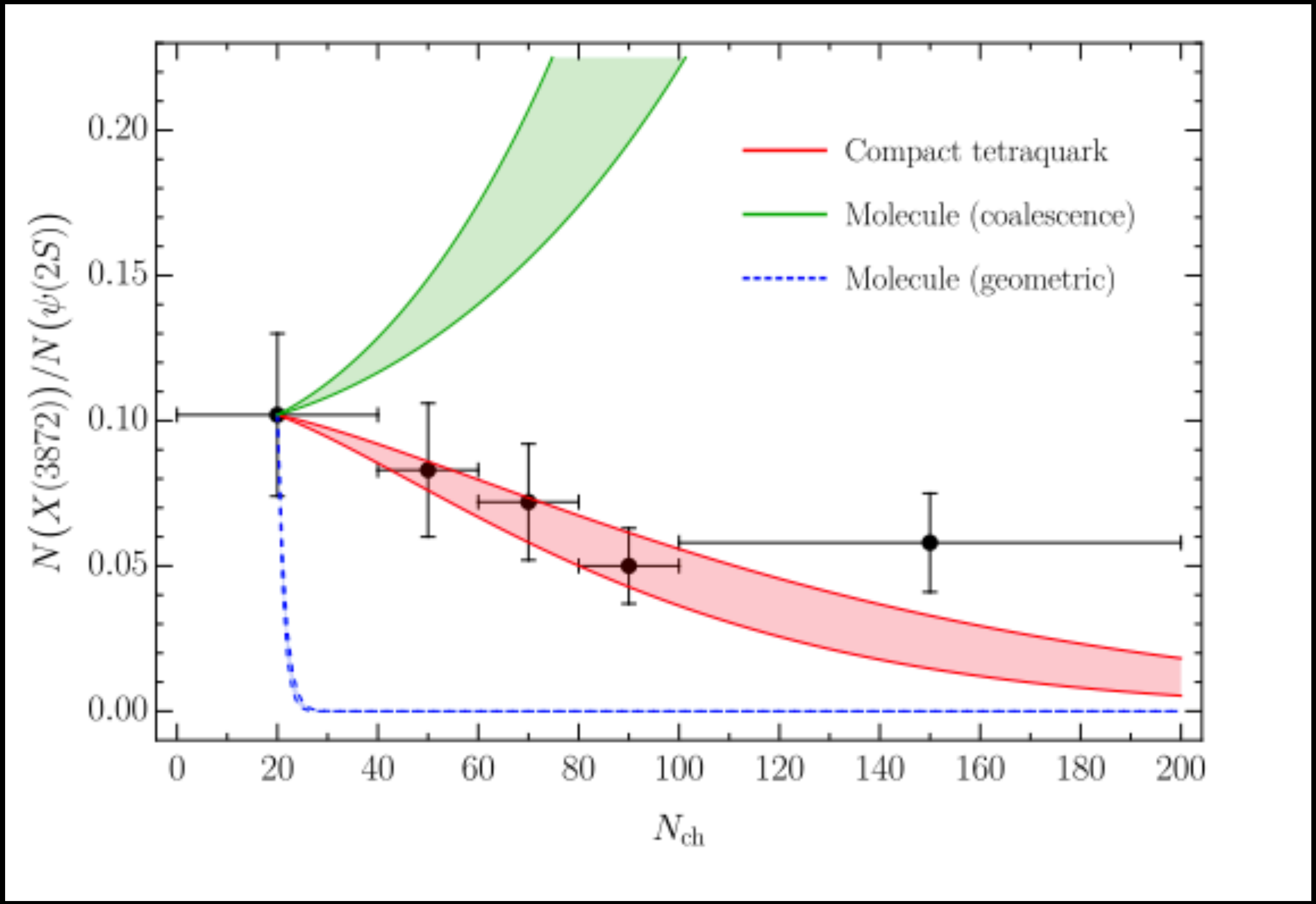
Esposito, Ferreiro, Pilloni, ADP, Salgado *Eur. Phys. J. C* 81 (2021) 669

Esposito, Piccinini, Pilloni, ADP *J.Mod.Phys.* 4 (2013) 1569-1573

Guerrieri, Piccinini, Pilloni, ADP *Phys.Rev.D* 90 (2014) 3, 034003

Number of deuterons as a function of the multiplicity computed with Boltzmann equation in a coalescence model.

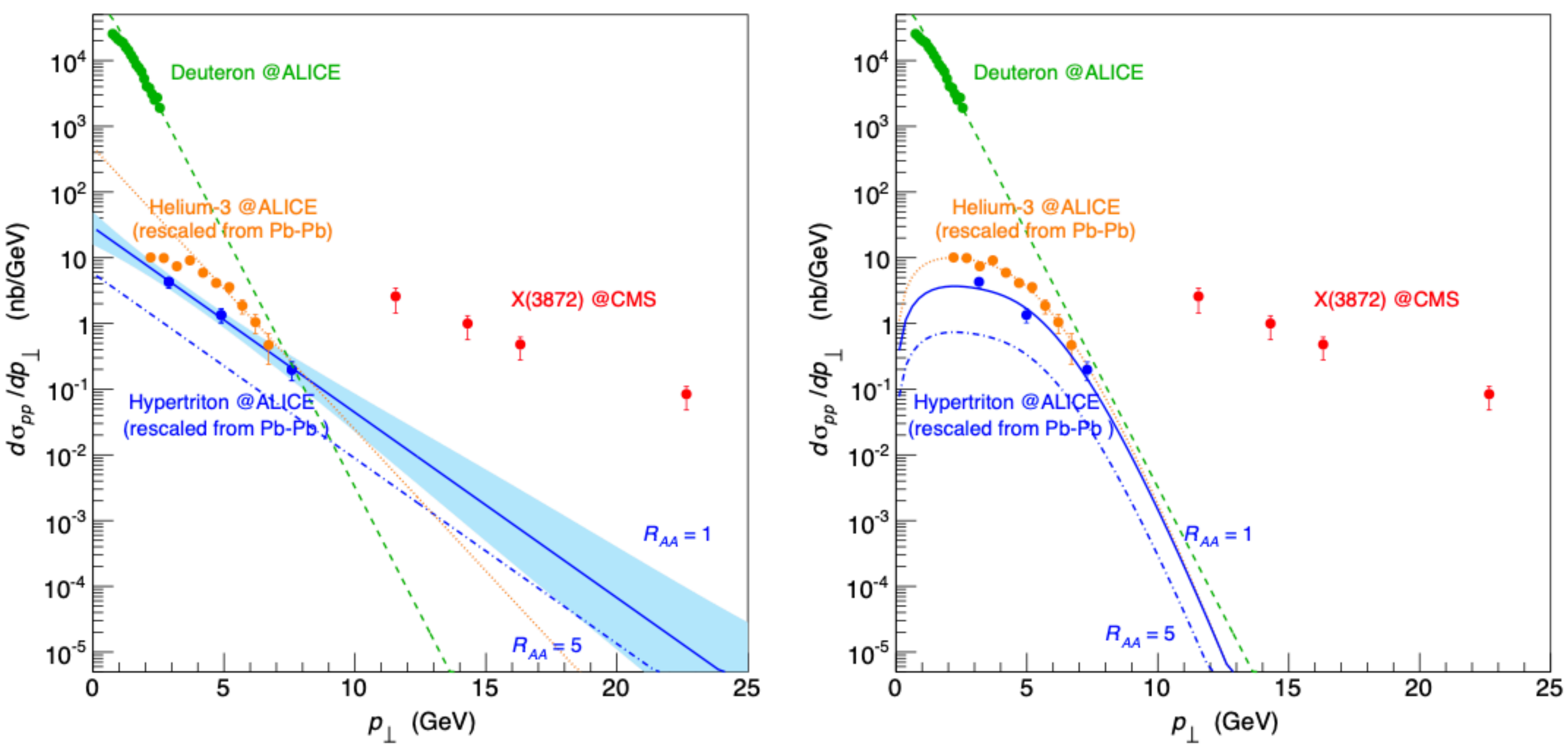
DOES THE X(3872) BEHAVE AS THE DEUTERON?



Esposito, Ferreiro, Pilloni, ADP, Salgado *Eur. Phys. J. C* 81 (2021) 669

The coalescence picture predicts a behavior (green band) qualitatively different from data.

NUCLEI AT HIGH p_T ?



Esposito, Guerrieri, Maiani, Piccinini, Pilloni, ADP, Riquer, *Phys. Rev. D* 92 (2015) 3, 034028

RELATIVE MOMENTA IN MOLECULES

$$\psi(r) = \left(\frac{2mB}{4\pi^2}\right)^{1/4} \frac{e^{-r\sqrt{2mB}}}{r}$$

$$V(r) = \lambda \frac{\delta(r)}{4\pi r^2}$$

Quantum Virial Theorem $\quad -2\langle T\rangle - 3\langle V\rangle = 0$

$$\langle T\rangle = -\frac{3}{2}\langle V\rangle = \frac{3\lambda}{2\pi}(2mB)^{3/2}$$

$$-B = \langle H\rangle = \langle T\rangle + \langle V\rangle = -\frac{\lambda}{2\pi}(2mB)^{3/2}$$

$$\langle p^2\rangle \approx 3(2mB) \qquad p \approx 1.7 \cdot 14 \simeq 24 \text{ MeV}$$

Using $\lambda = 2\pi/m\sqrt{2mB}$ (giving b.s. at $-B$ from $\lambda\delta^3(\mathbf{r})$) gives ~ 48 MeV.

$$\langle V \rangle_\psi = \lambda \int_0^\infty \frac{e^{-\alpha r}}{r^2} \frac{\delta(r)}{4\pi r^2} 4\pi r^2 \, dr = \frac{\lambda}{\epsilon} \int_0^\epsilon \frac{e^{-\alpha r}}{r^2} dr$$

Derive twice wrt α and do the integral

$$\frac{1}{\epsilon} \int_0^\epsilon e^{-\alpha r} dr = \frac{1 - e^{-\alpha\epsilon}}{\alpha\epsilon} \to 1$$

when $\epsilon \to 0$, thus the result is

$$\lambda \frac{\alpha^2}{2} + C$$

and $C = 0$ since $\langle V \rangle_\psi = 0$ when $\lambda = 0$

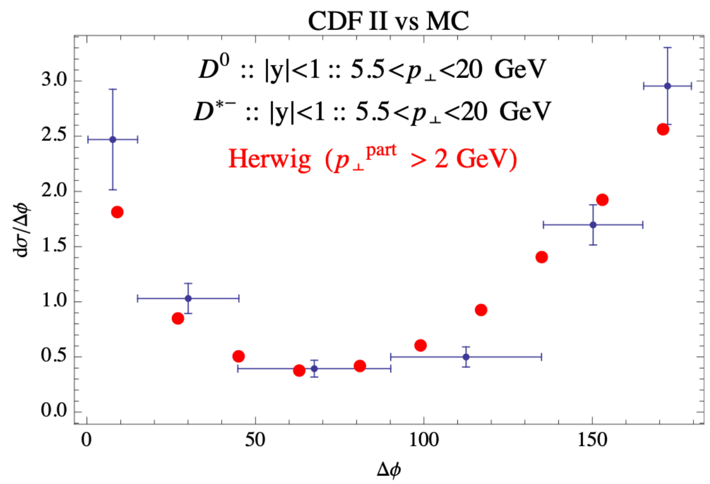


FIG. 1: The $D^0 D^{*-}$ pair cross section as function of $\Delta\phi$ at CDF Run II. The transverse momentum, $p_\perp$, and rapidity, y, ranges are indicated. Data points with error bars, are compared to the leading order event generator Herwig. The cuts on parton generation are $p_\perp^{\rm part} > 2$ GeV and $|y^{\rm part}| < 6$. We have checked that the dependency on these cuts is not significative. We find that we have to rescale the Herwig cross section values by a factor $K_{\rm Herwig} \simeq 1.8$ to best fit the data on open charm production.

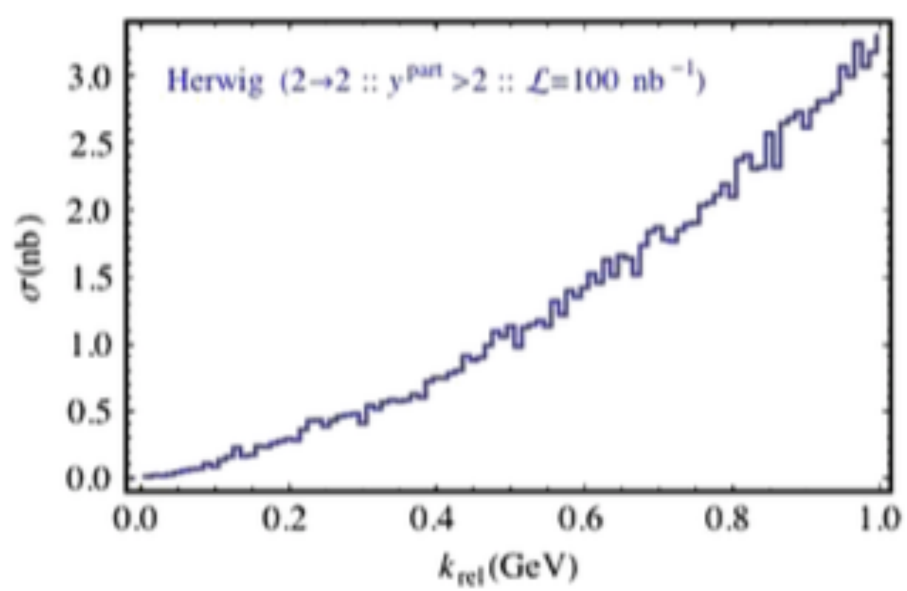
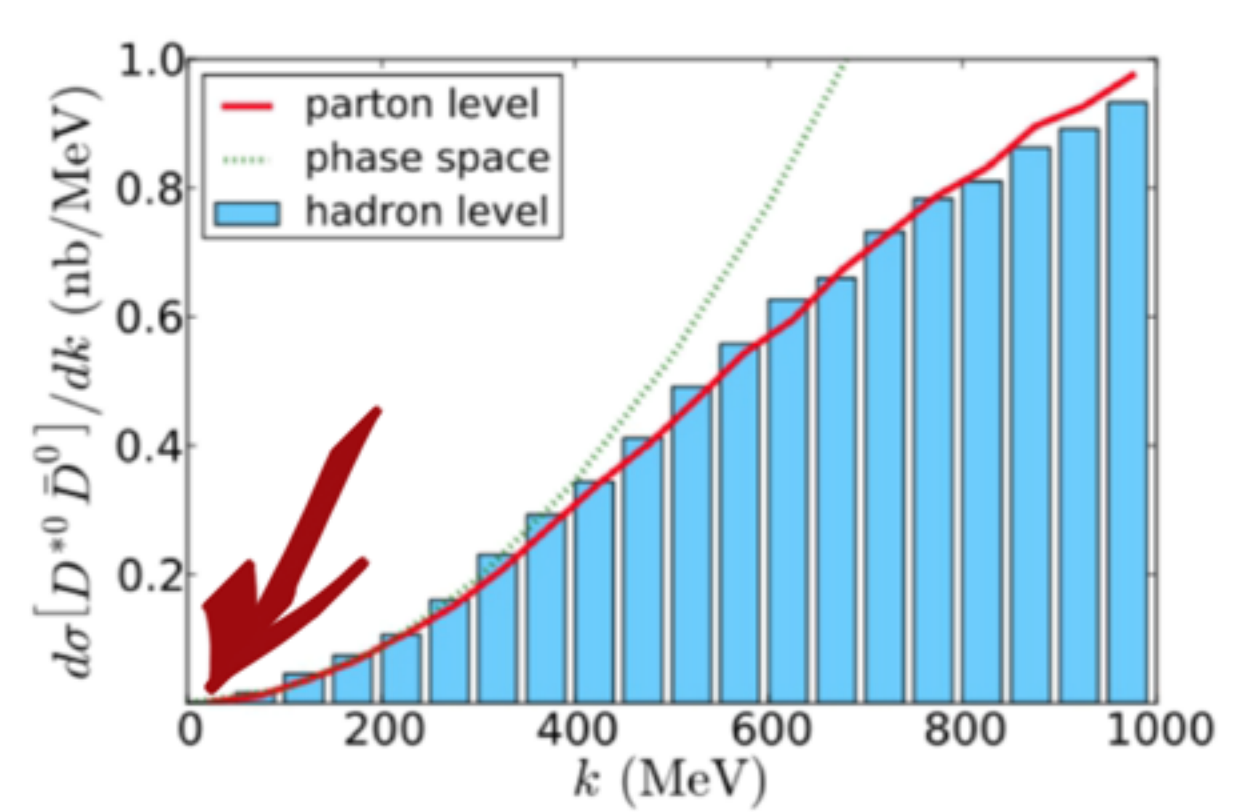


FIG. 3 (color online). The integrated cross section obtained with HERWIG as a function of the center of mass relative momentum of the mesons in the $D^0\bar{D}^{*0}$ molecule. This plot is obtained after the generation of 55×10^9 events with parton cuts $p_\perp^{\rm part} > 2$ GeV and $|y^{\rm part}| < 6$. The cuts on the final D mesons are such that the molecule produced has a $p_\perp > 5$ GeV and $|y| < 0.6$.

Bignamini, Grinstein, Piccinini, ADP, Sabelli, PRL103 (2009) 162001



Braaten and Artoisenet, PRD81103 (2010) 114018

INTERPRETATION OF HIGH p_T DATA

We might conclude that the X **does not** look like a deuteron – or a 'deuson' $D\bar{D}^*$.

On the other hand it **does!** (Anomalously) small binding energy, isospin violations in $J/\psi(\rho/\omega)$ decays, absence for the time being, of charged partners, evoke the deuson!

THE EQUAL SPACING RULE

In the vector mesons octet

$$K^* \approx (\phi + \rho)/2$$

The analog of ϕ in the hidden charm tetraquarks is

$$X(1^{++}) = [cs][\bar{c}\bar{s}]$$

$X(4140)$ seen in $J/\psi\phi$

To first order in SU(3) flavor symmetry breaking we might predict

$$Z_{cs} \stackrel{!}{=} (X(4140) + X(3872))/2 = 4009 \text{ MeV}$$

A Z_{cs} has been observed at 4003 MeV.

Maiani, ADP, Riquer, Sci. Bulletin 66, 1616 (2021)

THE EQUAL SPACING RULE

L. Maiani et al. *Science Bulletin 66 (2021) 1616–1619*

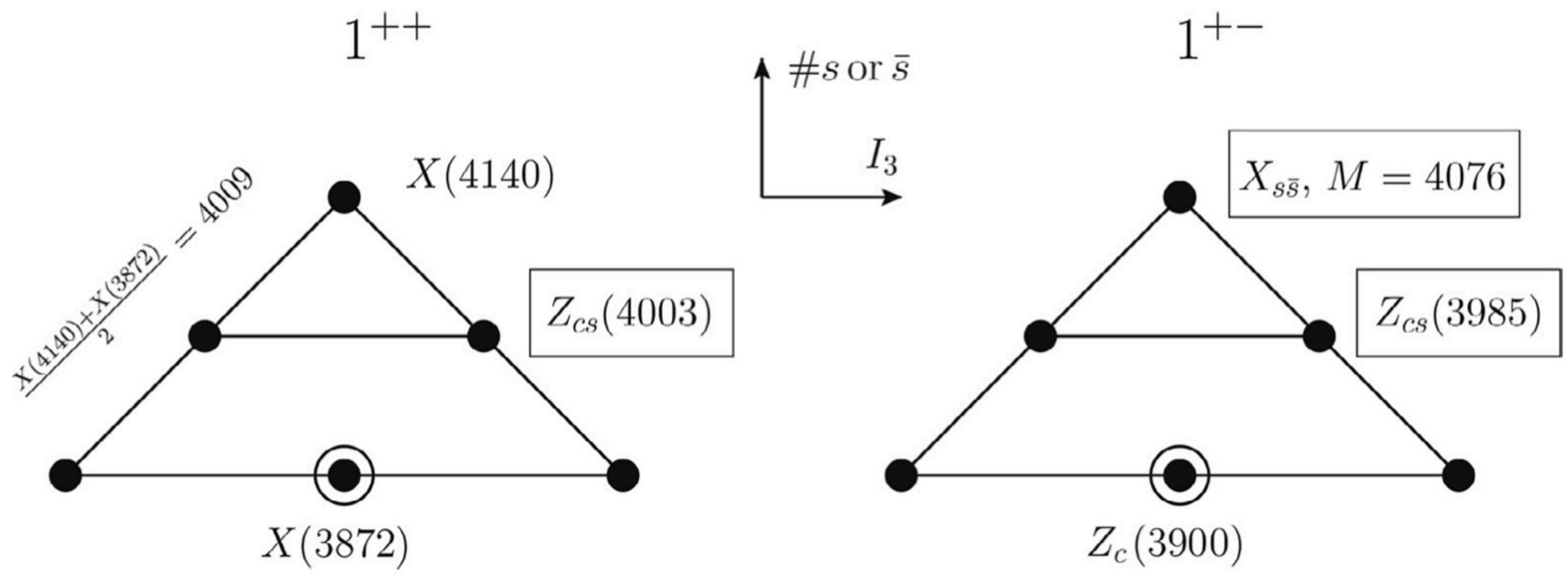


Fig. 1. The hidden charm-strange resonances and the missing $X_{s\bar{s}}$ tetraquark with its predicted mass are given in the boxes. The $SU(3)_f$ prediction for the mass of the strange state of the $X(3872) - X(4140)$ nonet is $M = 4009$ MeV to be compared with the Z_{cs} of Solution 1 at 4003 MeV. The upper state on the right panel has not yet been observed. By $\mathscr{C} = \pm 1$ nonets we refer to the sign of charge conjugation of the neutral-non-strange members, see Eq. (9).

Z_{cs} AND NEGATIVE CHARGE CONJUGATION

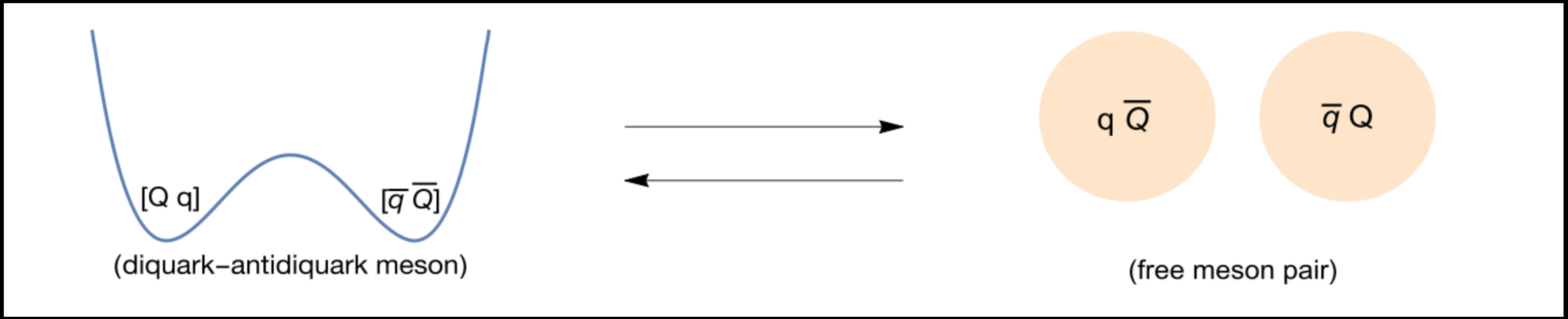
Observed by LHCb in the decay

$$B^+ \to \phi + Z_{cs}^+(4003) \to \phi + K^+ + J/\psi$$

In the diquark-antidiquark model we predict that $M(X(1^{++})) = M(Z(1^{+-}))$. Using the same spacing rules, given the $Z(3900)$ and the recently discovered $Z_{cs}(3985)$ we predict a $Z_{ss}(\simeq 4076)$

Maiani, ADP, Riquer, Sci. Bulletin 66, 1616 (2021)

`SEGREGATED` DIQUARKS



Maiani, ADP, Riquer PLB 778 (2018) 247

Maiani, Piccinini, ADP, Riquer PRD71 (2005) 014028

If $X^{\pm}$ is degenerate with X^0 it can't decay in $D^{\pm}\bar{D}*$ – it is forced to decay in $J/\psi\rho^{\pm}$, tunneling the heavy quark at a higher price in rate.

The $X^{\pm}$ might still be hiding in $J/\psi\rho^{\pm}$ decays.

This picture of `segregated diquarks` inspired the idea of `segregated hevay-quarks`, kept away by color repulsion in the octet.

CONCLUSIONS

- The field of exotic hadron spectroscopy is open, **rich of data and neat problems to solve.** It is the proof of our limited understanding of strong interactions.
- The field would greatly benefit from **input of "non-experts" from other fields**.

SOME MORE TECHNICAL CONCLUSIONS

- It would be useful to have new comparative studies on the r_0 of the X(3872) and of the $\mathcal{T}_{QQ}$ particles, and to agree on the way to extract information from data (not easy).
- It would be of great relevance to learn more, on the experimental side, about deuteron production at high p_T .
- Some states are produced promptly in pp collisions, some are not. There is no clear reason why!
- Are there loosely bound molecules $B\bar{B}^*$? Can we formulate more stringient bounds on $X^{\pm}$ particles?
- Derive Weinberg criterium in a modern language.
- More basically: are we on the right questions?

BACKUP

THE COMPLETE PROPAGATOR

Let $|\boldsymbol{k}\rangle$ be a one-particle state with mass $\boldsymbol{m}$.
Suppose $\langle\boldsymbol{k}|$ has a non-zero amplitude with $\Phi^\dagger(0)|0\rangle$.
Lorentz invariance requires

$$\langle 0|\Phi(0)|\boldsymbol{k}\rangle = \frac{N}{\sqrt{2E}} \qquad E = \sqrt{\boldsymbol{k}^2 + m^2}$$

Then, according to a **general result**, the complete propagator $\Delta'(p)$ of the **bare** field Φ has a **pole** at $-m^2$ with residue $Z = |N|^2 > 0$

$$\Delta'(p) = \frac{Z}{p^2 + m^2 - i\epsilon}$$

KÄLLÉN-LEHMAN REPRESENTATION

In the KL repres., the complete propagator of $\mathbf{\Phi}$ which may, or may not, be elementary, is

$$\Delta'(p) = \int_0^\infty \frac{\rho(\mu^2)}{p^2 + \mu^2 - i\epsilon}\, d\mu^2$$

where, on general grounds, the spectral function is defined by ($\rho = 0$ for $p^2 > 0$)

$$\theta(p_0)\,\rho(-p^2) = \sum_n \delta^4(p - p_n)\,|\langle 0\,|\,\Phi(0)\,|\,n\rangle\,|^2$$

and $|\,n\rangle = |\,\boldsymbol{k}\rangle$ or $|\,n\rangle = |\,\boldsymbol{k}_1, \boldsymbol{k}_2\rangle\ldots$ If we substitute $|\,n\rangle = |\,\boldsymbol{k}\rangle$ in the previous formula, we obtain

$$\rho(\mu^2) = Z\,\delta(\mu^2 - m^2)$$

THE COMPLETE PROPAGATOR

Indeed, considering only the one-particle states

$$\langle 0 | \Phi(0) | \boldsymbol{k} \rangle = \frac{N}{\sqrt{2E}} \qquad E = \sqrt{\boldsymbol{k}^2 + m^2}$$

$$\theta(p_0)\, \rho(-p^2) = \sum_n \delta^4(p - p_n) \, | \langle 0 | \Phi(0) | n \rangle |^2 = \int \delta^4(p - p_1) \frac{Z}{2E} d^3 p_1 + \dots$$

$$\int \delta^4(p - p_1) \frac{Z}{2E} d^3 p_1 = Z \int d^4 p_1 \theta(p_{10}) \delta(p_1^2 + m^2) \delta^4(p - p_1) = Z\theta(p_{10}) \delta(p_1^2 + m^2)$$

set $p_1^2 = -\mu^2$

SPECTRAL FUNCTION

However the spectral function also includes multiparticle states in $|n\rangle$. The contribution of states like $|\boldsymbol{k}_1, \boldsymbol{k}_2, \ldots\rangle$ is incorporated in the function $\sigma(\mu^2) \geq 0$

$$\rho(\mu^2) = Z\,\delta(\mu^2 - m^2) + \sigma(\mu^2)$$

and the Lehman Sum Rule can be proved

$$1 = Z + \int \sigma(\mu^2)\, d\mu^2$$

In the absence of coupling to multiparticle states we get the free particle propagator, $Z = 1$.

$Z = 0$ AS THE TOKEN OF THE COMPOSITE PARTICLE

The opposite case is $\mathbf{Z = 0}$: the coupling to multiparticle states is as strong as possible.

The function $\boldsymbol{\sigma}$ could be due to two-particle states only: the constituents of the composite particle described by $\mathbf{\Phi}$.

What we learn from this is that there is an upper bound to the coupling of the field $\mathbf{\Phi}$ to multiparticle states:

$$g^2 = \int \sigma(\mu^2)\, d\mu^2 = 1 - Z \leq 1$$

Non-relativistic quantum mechanics helps at making more useful steps forward on this discussion.

ANALYTICAL PROPERTIES OF SCATTERING

Asymptotic wf in a V which vanishes rapidly at infinity

$$\chi(r) = A(E)\,\exp(-r\sqrt{-2mE}) + B(E)\,\exp(+r\sqrt{-2mE})$$

$$\sqrt{-E} > 0 \quad \text{if} \quad E < 0$$

going from R^- to R^+ through a path in the upper half plane

$$\chi(r) = A(E)\,\exp(ikr) + B(E)\,\exp(-ikr)$$

$$k = \sqrt{2mE}$$

i.e. on the upper edge of the cut $\sqrt{-E} = -\,i\sqrt{E}$ or $ik \leftrightarrows -\sqrt{-2mE}$ (and everywhere $\Re\sqrt{-E} > 0$): ik becomes $-\sqrt{-2mE}$ on the left half of the real axis R^-.

ANALYTICAL PROPERTIES OF SCATTERING

Bound states correspond to wf vanishing at infinity

$$B(E) = 0$$

Real (S. eq has real eigenvalues) zeroes on $\boldsymbol{R}^-$. The scattering amplitude has a pole at a shallow level $\boldsymbol{E = -B}$ with $\boldsymbol{B > 0}$ (if $\boldsymbol{E = -B}$ is on the non-physical sheet one speaks of *virtual state*). Thus $ik = -\sqrt{2mB}$ or $k = i\sqrt{2mB} = i\varkappa$.

The two general relations hold

$$f = -\frac{A_0^2}{2m}\frac{1}{E+B}$$

$$\chi = A_0 \exp(-r\sqrt{2mB})$$

normalized wf of the corresponding stationary state.

DIMENSIONS OF g_W

$$\underbrace{g_W}_{E^{-1/2}} = |\langle \alpha | V | d \rangle| = \left| \int d^3x \, \psi_\alpha^*(x) \underbrace{V(x)}_{E} \underbrace{\psi_d(x)}_{E^{3/2}} \right|$$

\`NUCLEAR DEMOCRACY\`

"A proton could be obtained from a neutron and a pion, or from a Λ and a K, or from two nucleons and one anti-nucleon, and so on. Could we therefore say that a proton consists of continuous matter? […] *There is no difference in principle between elementary particles and compound systems.*"

–WERNER HEISENBER, 1975 TALK AT GERMAN PHYSICAL SOCIETY

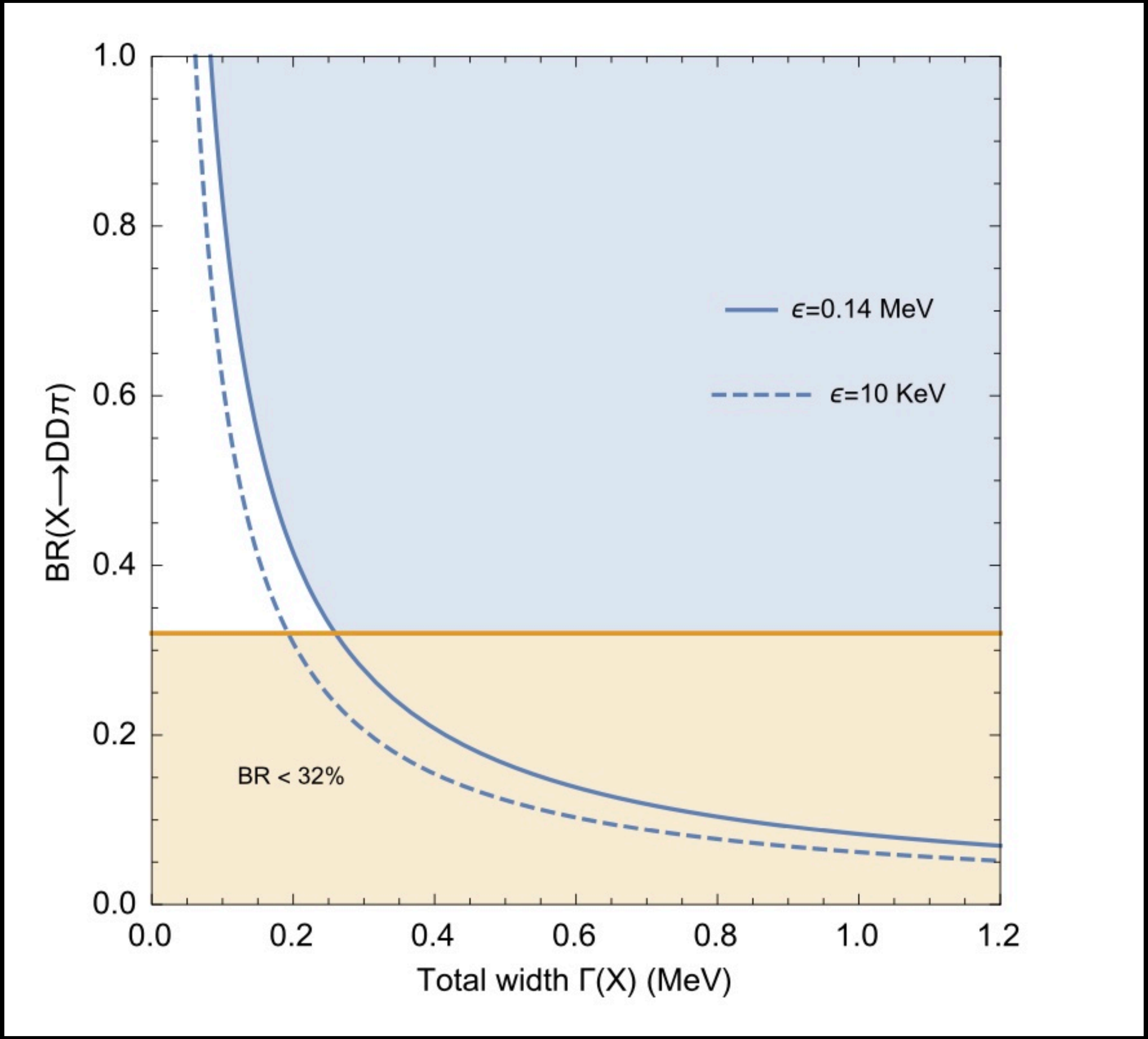
LANDAU ARGUMENT: Z=0 MOLECULE

This leads to

$$B = g_L^4 \frac{m^5}{512\pi^2 m_a^4 m_b^4}$$

The branching fraction $\mathscr{B}(X \to DD\pi)$ multiplied by Γ gives the partial width, that is determined by g, or by B, from the previous formula.
So we have hyperbolae in the $\mathscr{B}$ vs. Γ space, at fixed values of B.
All the hyperbolae with high values of B are excluded. So we have. to consider only those hyperbolae having B below 100 keV.
Once B, $\mathscr{B}$ are determined they have to cross on one of the (dashed) hyperbolae in the unshaded region.

LANDAU ARGUMENT



ADP Phys. Lett. B746, 248 (2015)

A DERIVATION OF THE DWBA FORMULA

$$f_{\text{Born}} = -\frac{m}{2\pi}\int V(r)\, e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{r}} d^3r$$

$$e^{i\boldsymbol{k}\cdot\boldsymbol{r}} = \sum_{\ell=0}^{\infty} i^{\ell} j_{\ell}(kr)(2\ell+1)P_{\ell}(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{r}})$$

Expand

$$e^{-i\boldsymbol{k}'\cdot\boldsymbol{r}} = \sum_{\ell=0}^{\infty} i^{\ell} j_{\ell}(k'r)(2\ell+1)(-1)^{\ell} P_{\ell}(\hat{\boldsymbol{k}}'\cdot\hat{\boldsymbol{r}})$$

$$\int P_{\ell}(\boldsymbol{n}_1\cdot\boldsymbol{n}_2)P_{\ell'}(\boldsymbol{n}_1\cdot\boldsymbol{n}_3)d\Omega_1 = \delta_{\ell\ell'}\frac{4\pi}{(2\ell+1)}P_{\ell}(\boldsymbol{n}_2\cdot\boldsymbol{n}_3)$$

$(-1)^{\ell} i^{2\ell} = +1$ for every ℓ, and $k = k'$ for elastic collisions

A DERIVATION OF THE DWBA FORMULA

So we get

$$f = -2m \sum_{\ell=0}^{\infty} (2\ell + 1) P_\ell(\cos\theta) \int V(r)(j_\ell(kr))^2 r^2 dr$$

To be compared with Holtsmark formula

$$f = \sum_{\ell=0}^{\infty} (2\ell + 1) P_\ell(\cos\theta) \frac{e^{i\delta_\ell} \sin \delta_\delta}{k}$$

giving

$$f_\ell = \frac{e^{i\delta_\ell} \sin \delta_\ell}{k} = -2m \int V(r)(j_\ell(kr))^2 r^2 dr$$

A DERIVATION OF THE DWBA FORMULA

$$\chi_\ell^{(0)}(r) = 2kr\, j_\ell(kr)$$

$$f_\ell = -\frac{2m}{4k^2}\int V(r)\,(\chi_\ell^{(0)}(r))^2\,dr$$

DWBA consists in replacing

$$f_w = -\frac{2m}{4k^2}\int_0^\infty V_w(r)\,(\chi_s(r))^2\,dr$$

Where we substituted $\chi^{(0)} \rightarrow \chi_s$

A RECIPE TO COMPUTE r_0

- Use $e^{-\mu r}$ in place of $e^{i\mu r}$ and in the final expression set $\mu \to -i\mu$
- Use the regularized* $\chi_s^I(r) = 2kr\left(\frac{e^{i\delta}\sin(kr+\delta)}{kr} - \frac{e^{i\delta}\sin\delta}{kr}\right)$

  for $r \in [0,\lambda]$ and $\chi_s^{II}(r) = 2kr\left(\frac{e^{i\delta}\sin(kr+\delta)}{kr}\right)$ for $r \in [\lambda, \infty[$
- The integral is finite. Use* $\delta = \cot^{-1}\left(-\frac{1}{ka_s}\right)$
- Double-expand the result around $k=0$ and $\alpha = 0$.
- Take the $\lambda \to 0$ limit
- Set $\mu \to -i\mu$

*R. Jackiw, `Delta Function Potentials in two- and three- dimensional quantum mechanics` in Diverse Topics in Theoretical and Mathematical Physics, World Scientific.
See also Gosdzynsky, Tarrach (https://doi.org/10.1119/1.16691) — suggested by Adam Szczepaniak.

THE IMAGINARY PART OF $V_w(r)$

How to take into account that there are unstable particles in the external legs of the amplitudes? We could add `by hand` the D^* decay width to $V_s + V_w$

$$-\frac{\nabla^2}{2m}\psi(r) - \left[\left(\lambda_0 + 4\pi\alpha\right)\delta^3(\mathbf{r}) + \alpha\mu^2\frac{e^{i\mu r}}{r} + i\frac{\Gamma}{2}\right]\psi(r) = E\,\psi(r)$$

Indeed the complex potential V_w alone will not allow any imaginary part in the positive spectrum $E > 0$ (exception made for ψs' exponentially blowing up). Note also that

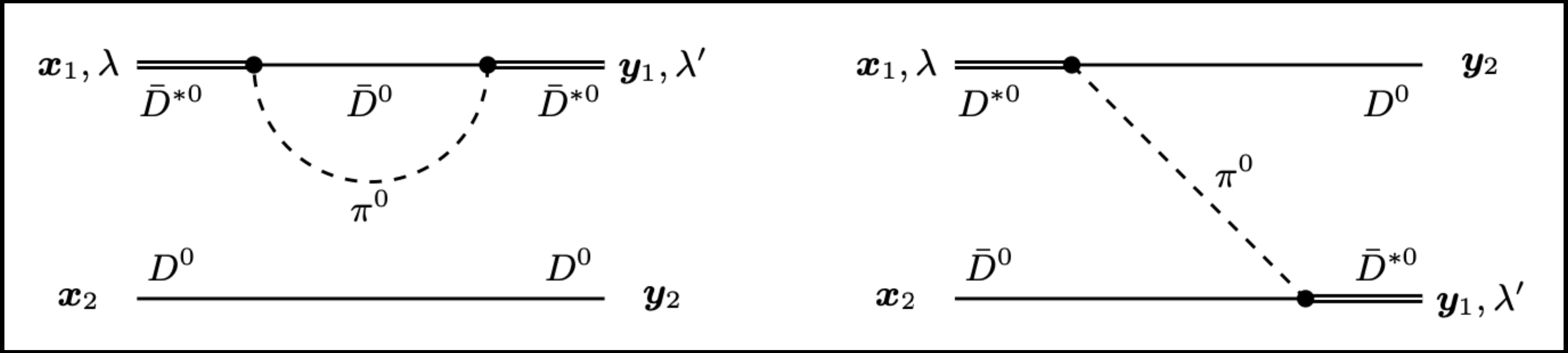
$$\left(\lim_{r\to 0}\Im(V(r)) = \lim_{r\to 0}\Im\,\alpha\mu^2\frac{e^{i\mu r}}{r} = \frac{g^2\mu^3}{24\pi f_\pi^2} \equiv \frac{\Gamma}{2}\right)$$

Esposito, Glioti, Germani, ADP, Rattazzi, Tarquini, PLB847, 138285 (2023).

THE IMAGINARY PART OF $V_w(r)$

The origin of the $i\Gamma/2$ is traced studying

$$H_{\rm eff} = H_{DD^*} + H_I^\dagger \frac{1}{E - H_{DD\pi} + i\epsilon} H_I$$



The asymptotic kinetic energy $k^2/2m$ equals $E + i\Gamma/2 \in R^+$

Esposito, Glioti, Germani,
ADP, Rattazzi, Tarquini,
PLB847, 138285 (2023).

THE EFFECTIVE RANGE EXPANSION

$$f = \frac{1}{k \cot \delta(k) - ik}$$

$$k \cot \delta = -\frac{1}{a} + \underbrace{\frac{1}{2}\Lambda^2 \sum_{n=0}^{\infty} r_n \left(\frac{p^2}{\Lambda^2}\right)^{n+1}}_{r(\Lambda)} = -\frac{1}{a} + \frac{1}{2} r_0 k^2 + \ldots$$

In NN scattering $|1/a| \ll \Lambda$ where we assume that baryons interact through a scalar particle with mass Λ and $|r_n| \sim 1/\Lambda$.

From the lineshape of the X one finds $1/a \sim 28$ MeV $< \mu < m_\pi$.

In doing a low momentum expansion we need $ak < 1$ or $k < 1/a$, i.e. much below the cutoff μ.

Better to expand in (k/Λ) retaining ka

$$f = -\frac{1}{(1-x)\left(\frac{1}{a} + ik\right)} = -\frac{(1 + x + x^2 + \ldots)}{\left(\frac{1}{a} + ik\right)}, \qquad x = \frac{r(\Lambda)}{\left(\frac{1}{a} + ik\right)}$$

THE SCATTERING LENGTH

The scattering length in the formula of r_0 is taken from data: it is a *renormalized scattering length* $a = a_R$.
The renormalization is required by the UV divergences appearing in the calculation of r_0 – due to scales $r < \epsilon$ cutoff.

$$\frac{a_s}{a_R} = 1 - (2\alpha\mu\mu_r)\left[\frac{1}{a_R\mu} + \gamma_E\mu a_R + 2i + \mu a_R\left(\log(\epsilon\mu) - i\frac{\pi}{2}\right)\right]$$

where $k\cot\delta = -1/a_s$

CORRECTION TO B

In the molecular hypothesis the unperturbed $H_0 + V_s$ features a bound state for $a_s > 0$, with $B = 1/(2ma_s^2)$ with w.f.

$$\psi_X(r) = \frac{1}{\sqrt{2\pi a_s}} \frac{e^{-r/a_s}}{r}$$

The shift in the binding energy of the X due to pion interactions is

$$\Delta B = \left(\psi_X, V_w \Psi_X\right) = \frac{\mu^2 \alpha}{2\pi a_s} \int^{\text{reg}} \frac{e^{i\mu r - 2r/a_s}}{r^3} d^3 r$$

This has an imaginary part which is the effect of binding in the $D\bar{D}^*$ system on D^* decay; we find a ≈ 1 keV effect.

THE VICINITY TO THRESHOLD

Mass central values only

$X(3872)$	$Z_c^{0\pm}(3900)$	$Z_c^{0\pm}(4020)$	$Z_b^{0\pm}(10610)$	$Z_b^{0\pm}(10650)$
$D^0\bar{D}^{*0}$	$D^0\bar{D}^{*0\pm}$	$D^{*0}\bar{D}^{*0\pm}$	$B^0\bar{B}^{*0\pm}$	$B^{*0}\bar{B}^{*0\pm}$
$\delta \approx 0$	$+28$	$+6.7$	$+5$	$+1.8$

(MeV)