Transverse Beam Dynamics - Tutorial

JAI lectures 2023 - Michaelmas Term

1 Preliminary exercices

1. Watch this Iron Man clip and discuss the main accelerator physics concepts involved either if they are properly represented or not in the movie.
2. Go through the short questions posted during lectures and try to answer them.

2 To think about

1. How can we measure β^*(β-function at the IP) in the LHC?

We cannot measure it directly because we do not have BPMs a the IP. However using K-modulation technique, the strength of the last quadrupole before the IP is modulated. This modulation produces a measurable tune shift. The tune shift is linerly related to the β-function at the quadrupole location.

$$\Delta Q = \frac{\beta_q \Delta K}{4\pi}$$

By transporting the measured β-function at the quadrupole to the IP we can have an estimation of the β-function at that location.

1. What are the possible effects of ground motion in the beam?
2. What can we do if there is a small object partially blocking the beam aperture?

3 Exercise: Understanding the phase space concept

1. Phase Space Representation of a Particle Source:
   - Consider a source at position s_0 with radius w emitting particles. Make a drawing of this setup in the configuration space and in the phase space. Which part of the phase space can be occupied by the emitted particles?
   - Any real beam emerging from a source like the one above will be collimated. This can be modelled by assuming that a distance d away from the source there is an iris with opening radius $R = w$. Draw this setup in the configuration space and in the phase space. Which part of the phase space is occupied by the beam, right after the collimator?
2. Sketch the emittance ellipse of a particle beam in:

   (I) horizontal x-x' phase space at the position of a transverse waist,

   (II) when the beam is divergent, and

   (III) when the beam is convergent.

4 Exercise: Stability condition

Consider a lattice composed by a single 2 meters long quadrupole, with $f = 1$ m

- Prove that if the quadrupole is defocusing, then a lattice is not stable
- Prove that if the quadrupole is focusing, then the lattice is stable

5 Question: Moon Collider

In the science-fiction novel *Firstborn* written by Arthur C. Clarke, the Alephtron is described as a particle accelerator wrapping around the lunar equator. Let's consider our magnet technology at that time reaches 20 T at 20 m long dipoles. The goal is to produce collisions at 1 PeV (10^{15}eV) in the center of mass. ($R_{\text{moon}} = 1737$ km)

1. What is the minimum filling factor (fraction of the accelerator filled with dipoles) required in order to reach the desired energy with the technology available?

2. Enumerate two advantages and two disadvantages of building a particle accelerator on the surface of the Moon.

6 Exercise: Bump and Orbit Control

Given two kickers located at the two ends of a FODO cell with phase advance 45 degrees (the two kickers are located at L_{cell} distance from each other), compute the strengths of such kickers (in radians) in order to give the beam, initially at $(x_i,\, x'_i) = (0,\, 0)$, an arbitrary offset at the end of the cell while preserving its angle, $\left(x_f,\, x'_f\right) = (x_{\text{arbitrary}},\, 0)$.

7 Exercise: Solenoid-based transport system

Solenoids have the useful property of focusing the beam in both the x and y directions, but introduce coupling. The transfer matrix of a solenoid magnet fully shows the coupled nature of the motion:

$$M_{\text{solenoid}} = \begin{pmatrix} C^2 & \frac{SC}{K} & SC & \frac{S^2}{K} \\ -KSC & C^2 & -KS^2 & SC \\ -SC & -\frac{S^2}{K} & C^2 & \frac{SC}{K} \\ KS^2 & -SC & -KSC & C^2 \end{pmatrix}$$

In this matrix,

$$K = \frac{B_z}{2P/q} \qquad [1/\text{m}],$$
$$C = \cos(KL),$$
$$S = \sin(KL),$$

B_z is the solenoid magnetic field, and L is the solenoid length. A good way to remove coupling consists in following a solenoid with another solenoid of opposite magnetic field, $-B_z$. The transfer matrix of a cell composed by two such solenoids reads:

$$M = \begin{pmatrix} 2C^2-1 & \frac{2SC}{K} & 0 & 0 \\ -2KSC & 2C^2-1 & 0 & 0 \\ 0 & 0 & 2C^2-1 & \frac{2SC}{K} \\ 0 & 0 & -2KSC & 2C^2-1 \end{pmatrix}$$

Questions:

1. Why is this motion uncoupled?

2. Under which conditions a lattice composed of such cells can provide stable motion?

3. Imagine you have a periodic lattice composed by a large number of these cells. What is the phase advance introduced by each cell? **(Hint:** Remember the double-angle trigonometric formulæ: $\cos(2\theta) = 2\cos^2\theta - 1$, and $\sin(2\theta) = 2\sin\theta\cos\theta$)

4. Calculate the Twiss parameter β and α at the beginning of the cell.

5. Let's put some numbers: $B_z = 1$ T, $L = 1$ m, in a system devoted to transporting protons with rigidity 1 T·m. Calculate K, $\Delta\mu$, β, and α.