Introduction to plasma wakefield acceleration

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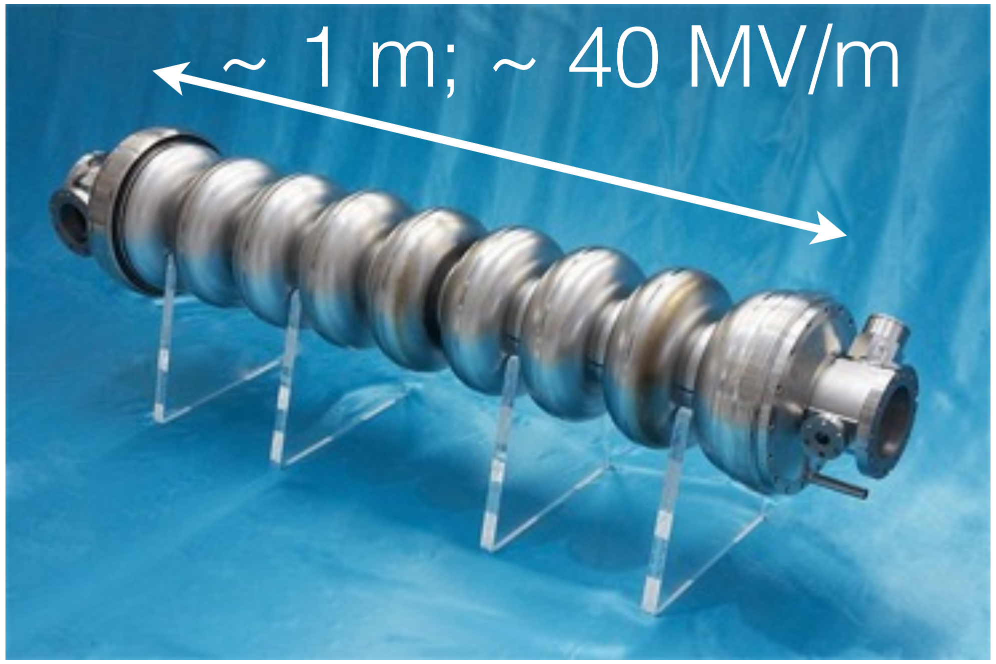
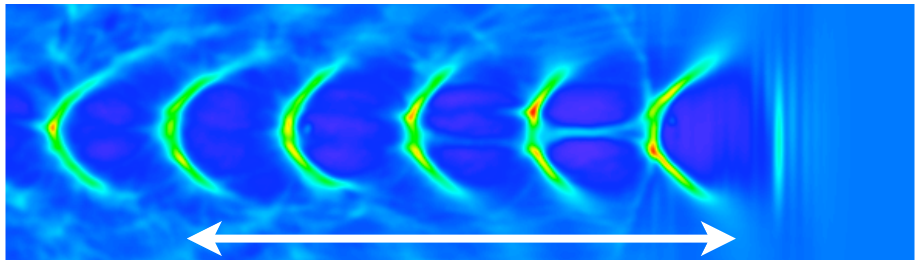
plasma as an accelerator



a section of RF cavity

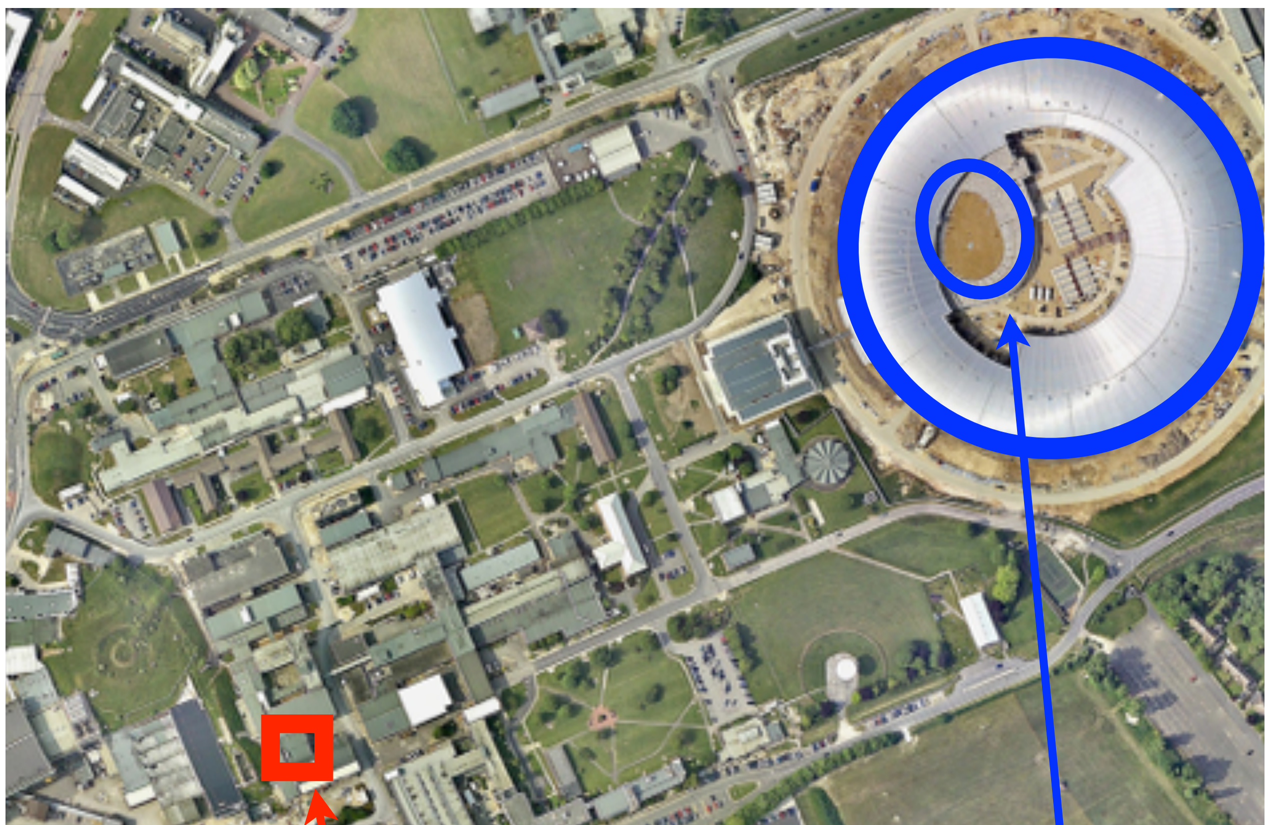


~ 50 μm; ~ 100 GV/m

a plasma wave

- Conventional Accelerators are large (100 metres) and expensive 10-100M$
- Conventional accelerators cannot achieve better than a few 10 MV/m or you get breakdown
- Plasma waves are a possible alternative - providing a route to university scale accelerators and radiation sources

why do we want to use a laser-plasma accelerator?

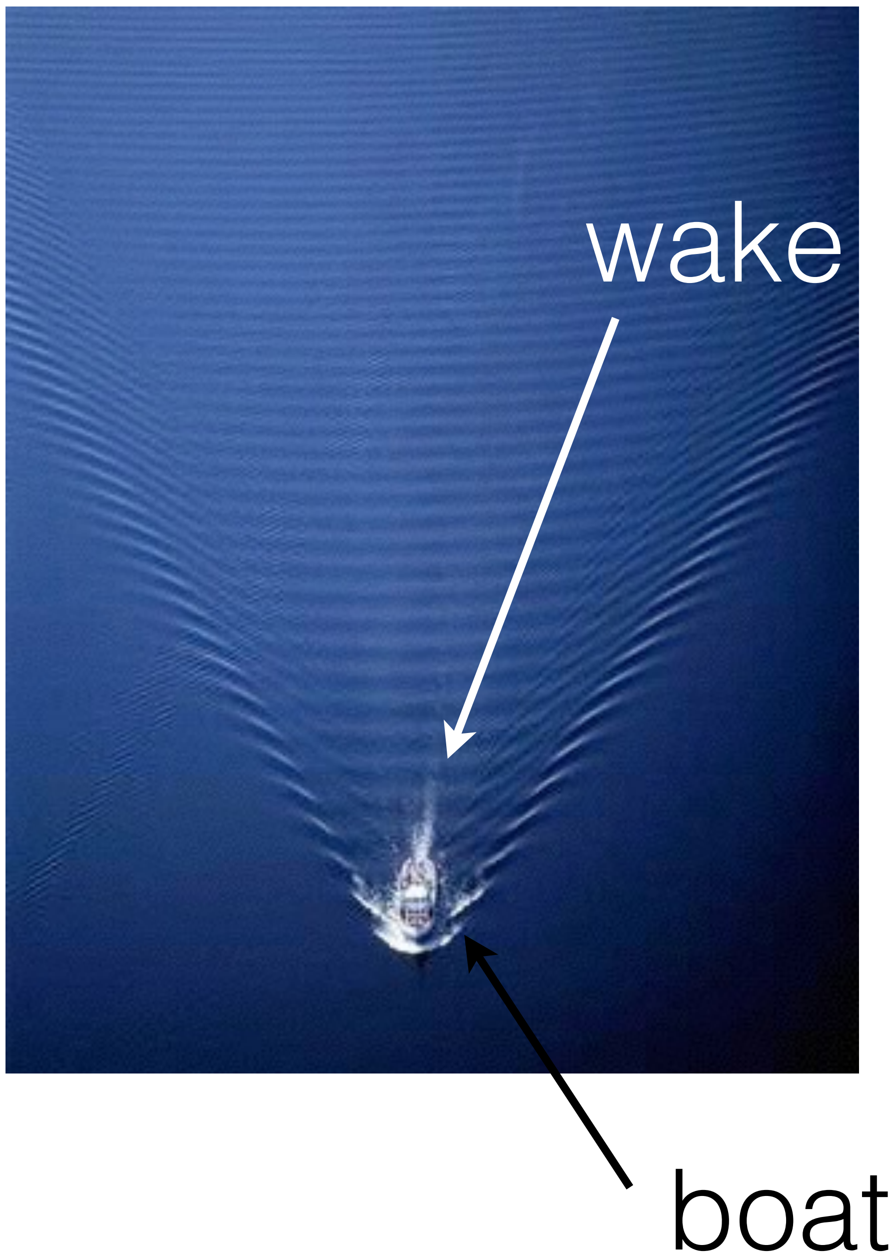


Diamond light source
3 GeV electron beam ~ £300 M

Astra Gemini Laser
1 GeV electron beam ~ £3 M

- Conventional particle accelerators are large and expensive machines
- Plasma based accelerators are a possible compact alternative
- in particular we are now quite good at accelerating electrons to ~ 1 GeV with ~ 100 TW lasers

Wakefield acceleration



- when a boat travels through water it produces a wave behind it - a ‘wake’
- the phase velocity of the wave is just the speed of the boat
- so we can use a laser pulse travelling at close to c in a plasma to drive a strong wave behind it.
- The wave in this case is an electron plasma oscillation

$$\omega_p = \left(\frac{n_0 e^2}{m_e \epsilon_0}\right)^{\frac{1}{2}}$$

- Because these are high frequency oscillations the ions do not move and we can have very strong electric fields

Driving Force

- For laser wakefield accelerators wake driven by ponderomotive force

$$\frac{d\mathbf{p}}{dt} = -\frac{e^2}{2m_e\omega_0^2}\nabla\langle E^2\rangle = -\frac{e^2}{2m_e}\nabla\langle A^2\rangle = -\frac{1}{2}m_e c^2\nabla\langle a^2\rangle$$

- For particle beam drivers wake driven by space charge field of drive bunch

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = -e\mathbf{E}$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2\right)\frac{n_1}{n_0} = -\frac{c^2}{2}\frac{\partial^2 a_{\mathrm{laser}}^2}{\partial x^2} - \omega_p^2\frac{n_{\mathrm{beam}}}{n_0}$$

Ponderomotive Force

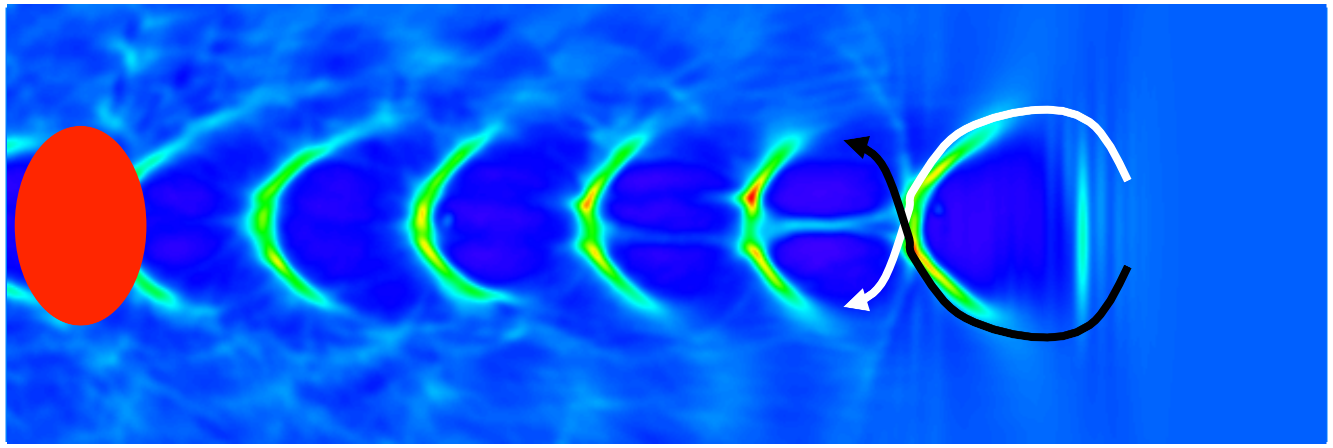
- This simple derivation was for low intensity ($a_0 < 1$) also called non-relativistic intensities ($I < 10^{18}$ Wcm^{-2}).
- How do we extend to high intensities?
- method 1) just replace $m_e c^2$ with $\gamma m_e c^2$- but do it at the right stage

$$\mathbf{F}_p = -\frac{e^2}{2\langle\gamma\rangle m_e \omega_0^2}\nabla\langle E^2\rangle = -\frac{1}{2}m_e c^2 \frac{1}{\langle\gamma\rangle}\nabla\langle a^2\rangle$$

- method 2) do it properly solving the equation of motion relativistically (see Quesnel + Mora Phys Rev E 1998)

$$\mathbf{F}_p = -\frac{1}{2}m_e c^2 \frac{1}{\langle\gamma\rangle}\nabla\langle a^2\rangle$$

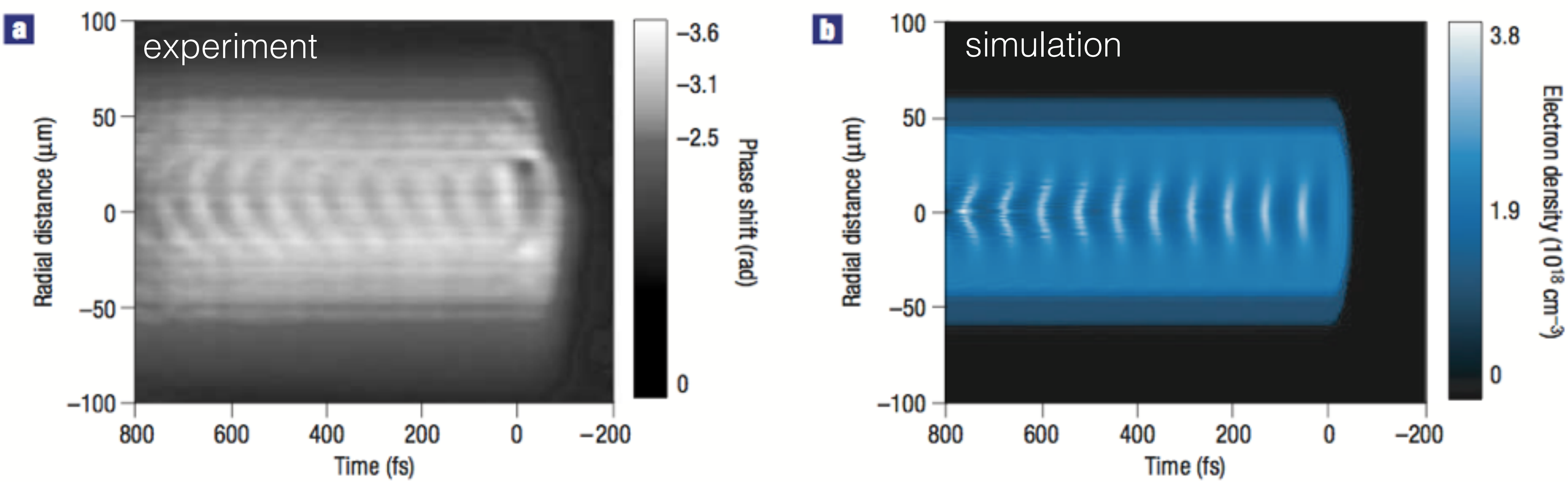
Driving relativistic plasma waves



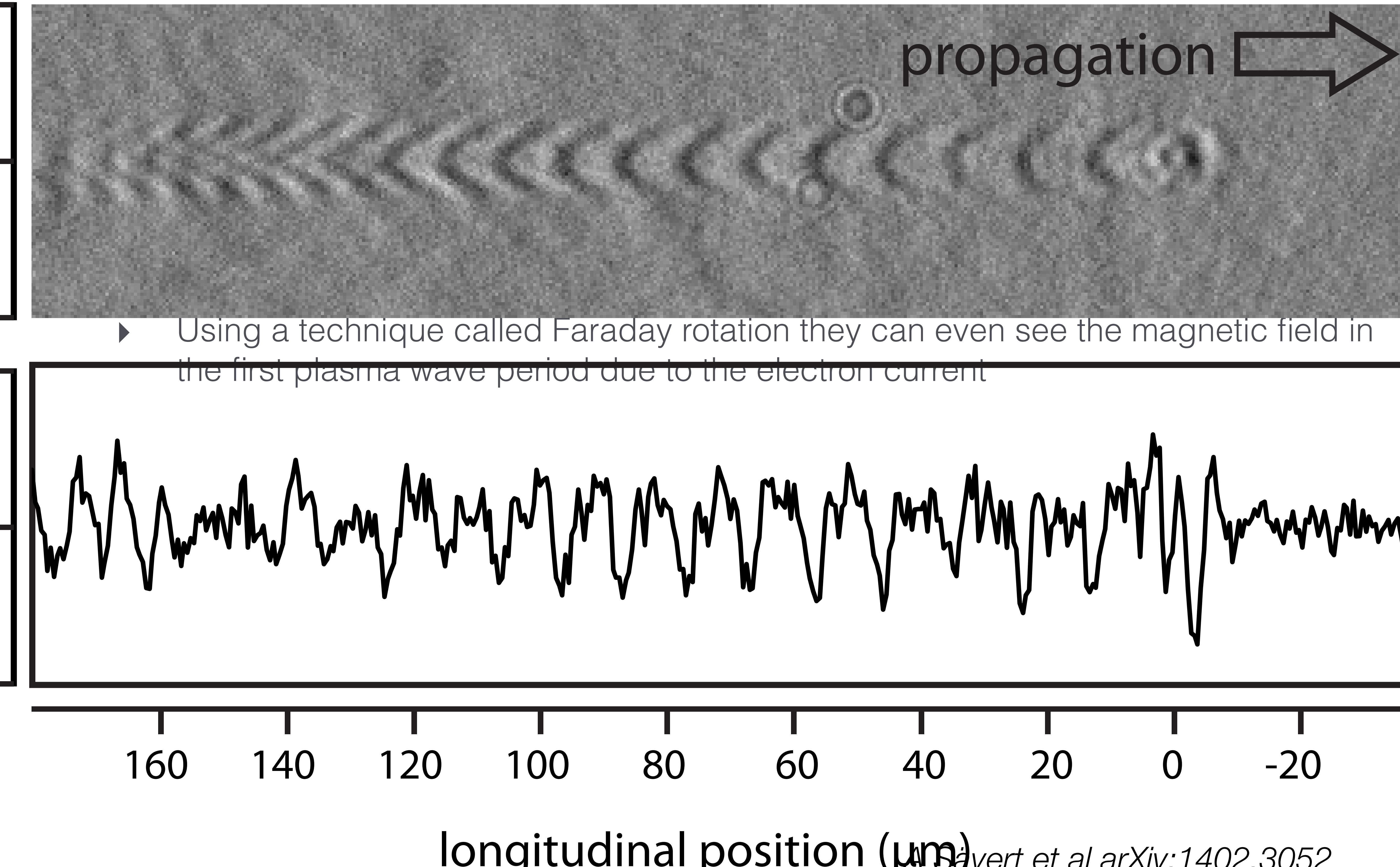
- The drive pulse of an intense laser pulse pushes away electrons just like a boat pushes away the water
- The much heavier ions are left behind - this charge separation makes a very large electric field
- As the electrons rush back to their original position they overshoot forming a plasma wave
- Plasma wave amplitude is largest if the drive duration is less than the plasma wavelength $c\tau_L < \lambda_p$

Driving Plasma waves

- The picture of wakefield I have shown so far is from a particle-in-cell numerical simulation
- But is it possible to “see” the plasma wave directly in experiments?
  - Yes! This is using a technique called Fourier domain holography (Matlis Nature Physics 2006)



Driving Plasma waves



- Using a technique called Faraday rotation they can even see the magnetic field in the first plasma wave period due to the electron current

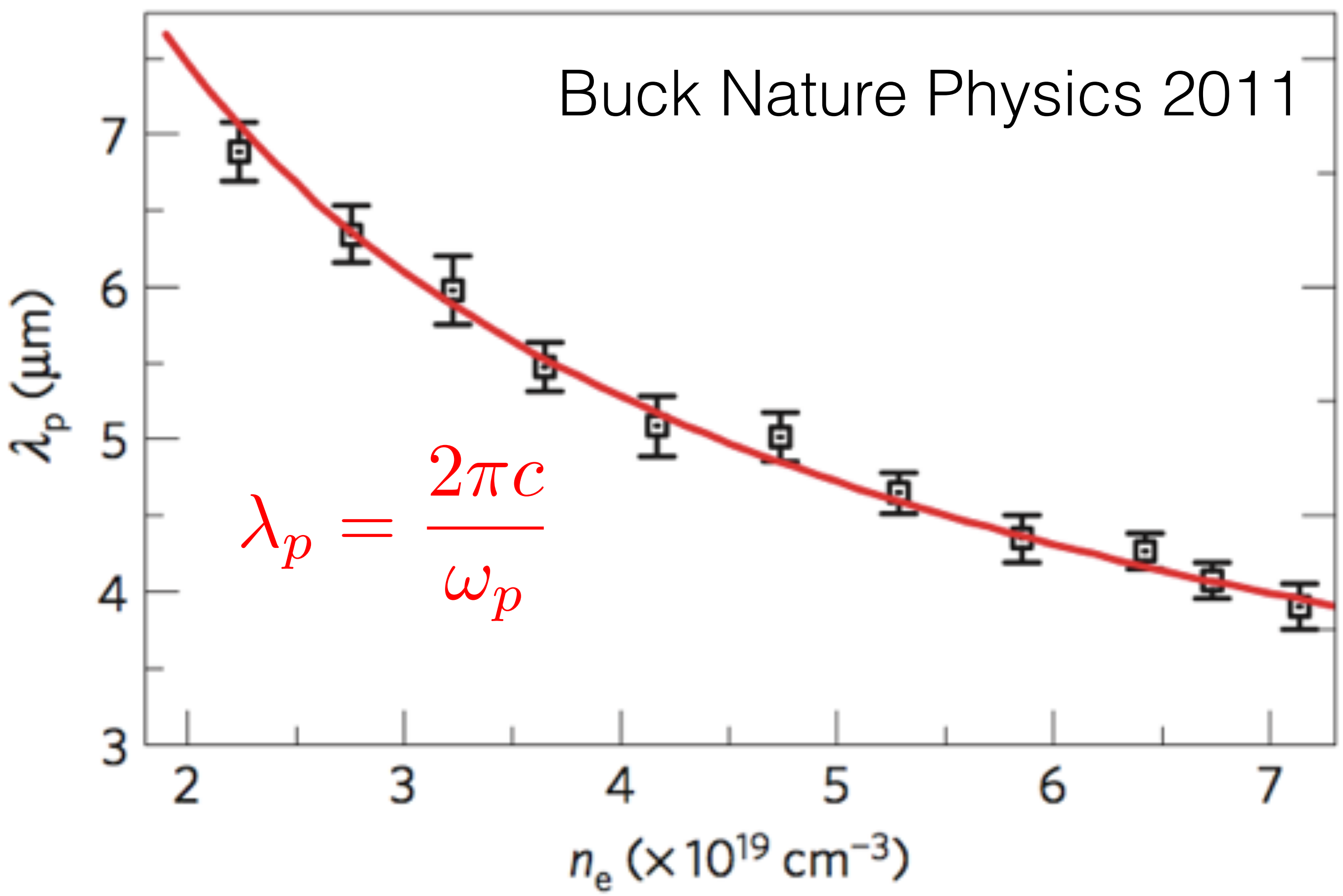
A Sävert et al arXiv:1402.3052

Phase velocity and wavelength of plasma waves

- The laser pulse speed determines the wavelength and phase velocity.
- Think of each electron as a separate oscillator, that is set in motion by the laser when the laser gets to it.
- If the first electron (at $z = 0$) is set in motion at $t = 0$, the next electron (at $z = \Delta z$) will start oscillating at $t = \Delta t = \Delta z / v_g$ where is the velocity of the laser pulse in the plasma (group velocity)
- there will be a wave with a phase velocity of $v_p = \Delta z / \Delta t = v_g$
- The wavelength will therefore be

$$\lambda_p = \frac{2\pi v_g}{\omega_p} \simeq \frac{2\pi c}{\omega_p}$$

Phase velocity and wavelength of plasma waves



- The wavelength of plasma waves is also experimentally verifiable

$\lambda_p \simeq 10\ \mu m$ at $n_e \approx 10^{19}$ cm^{-3}

(for λ = 800 nm laser)

Dephasing

- electrons travel slightly faster than the wave - eventually they stop being accelerated, this is called “dephasing”

Limits to Acceleration: 1) Dephasing

- Relativistic electrons ($v_e/c = \beta_e \rightarrow 1$) accelerating in the wave will move ahead of the wave which is moving at

$$\beta_p = \frac{v_g}{c} = \left(1 - \frac{n_e}{n_c}\right)^{\frac{1}{2}}$$

- The time it takes the electron to move half a plasma wave out of phase (i.e. from accelerating to decelerating field) is:

$$t_d = \frac{\lambda_p}{2c\,(\beta_e - \beta_p)} \approx \frac{\lambda_p}{c}\frac{n_c}{n_e} \qquad L_{dp} = \lambda_p \frac{n_c}{n_e}$$

- Dephasing length < 8 mm at $n_e = 4 \times 10^{18}$ cm^{-3}

- Dephasing is the fundamental limit to energy gain in LWFA

Limits to Acceleration: 2) pump depletion

- Creating the plasma wave takes energy - this must come from the drive pulse.

$$U_{\text{plasma}} = \frac{1}{4}\epsilon_0 E_{z0}^2$$ plasma wave electric field energy density $$E_{z0} = \delta \, \frac{m_e c \omega_p}{e}$$

$$W_{\text{plasma}} = U_{\text{plasma}} A L$$ energy in plasma wave cross section *A*, length *L*

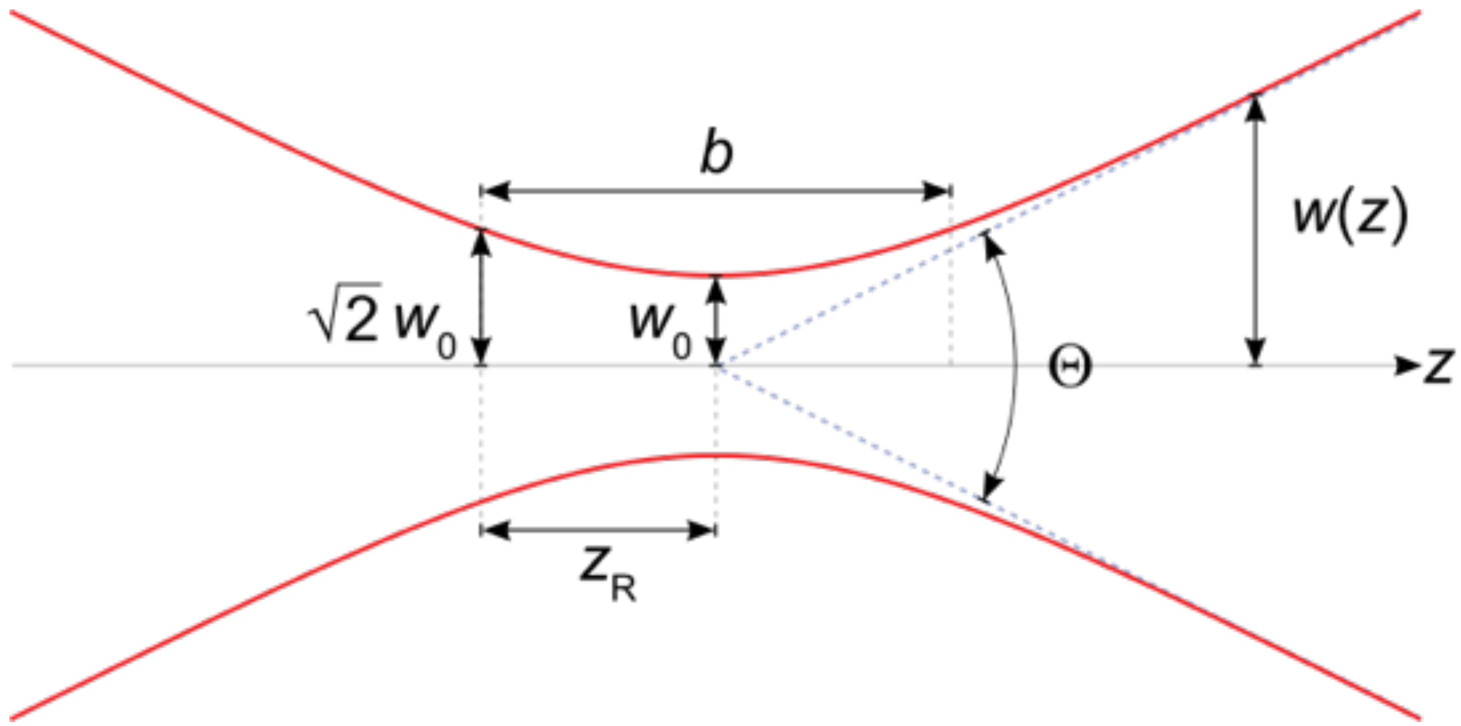
$$U_{\text{laser}} = \frac{1}{2}\epsilon_0 E_{L0}^2$$ laser electric and magnetic field energy density $$E_{L0} = a_0 \, \frac{m_e c \omega_0}{e}$$

$$W_{\text{laser}} = U_{\text{laser}} A c \tau_L$$ energy in laser pulse wave cross section *A*, duration τ $$c\tau = \epsilon \, \lambda_p$$

$$L_{pd} = 2\epsilon \left(\frac{a_0}{\delta}\right)^2 \frac{n_c}{n_e} \lambda_p$$

we can tailor parameters so pump depletion > dephasing

Limits to acceleration: 3) diffraction

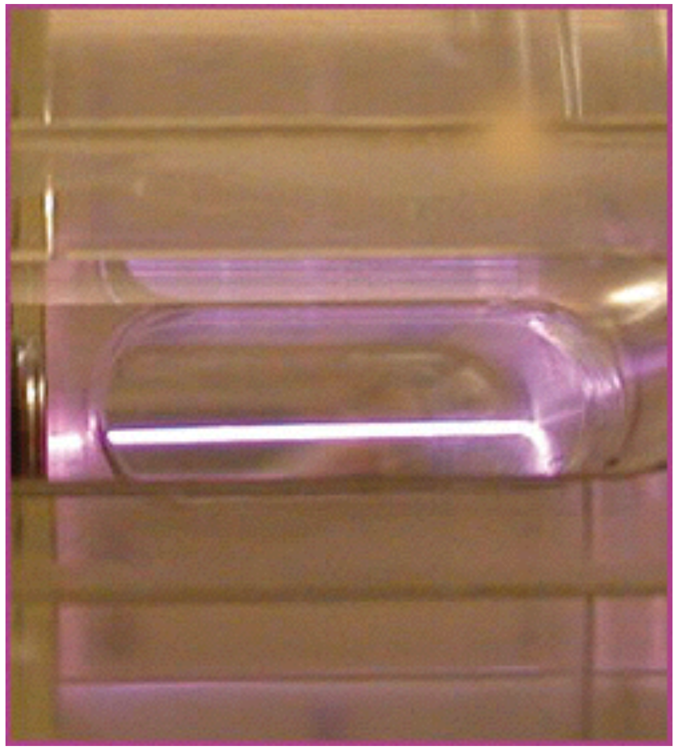


- We need to keep the laser intense over the entire interaction
- Distance over which a laser diffracts in vacuum is the Rayleigh Range

$$z_R = \frac{\pi w_0^2}{\lambda_0}$$

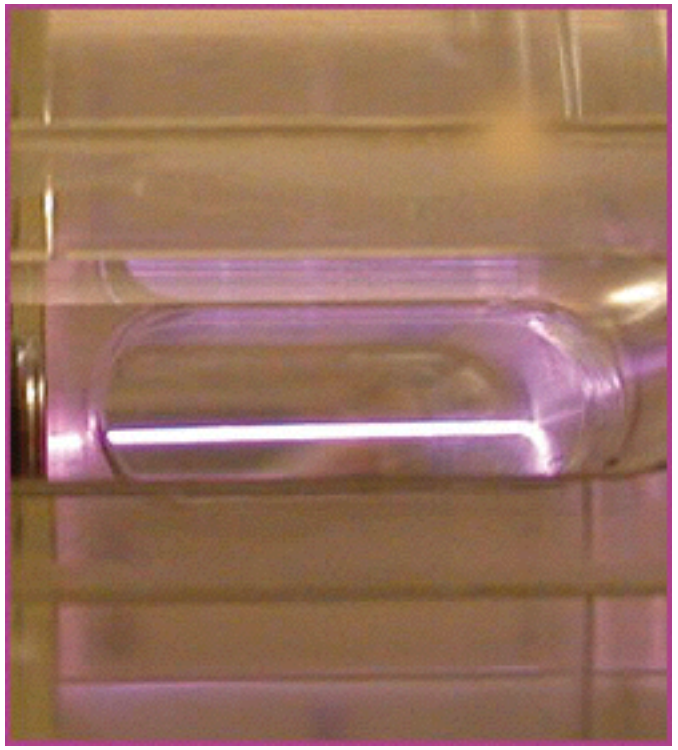
- For z_R = 1 cm we need focal spots ~ 50 μm - difficult to make very intense focal spot this large
  - (e.g. you need P > 90 TW for a_0 = 1)

Limits to acceleration: 3) diffraction



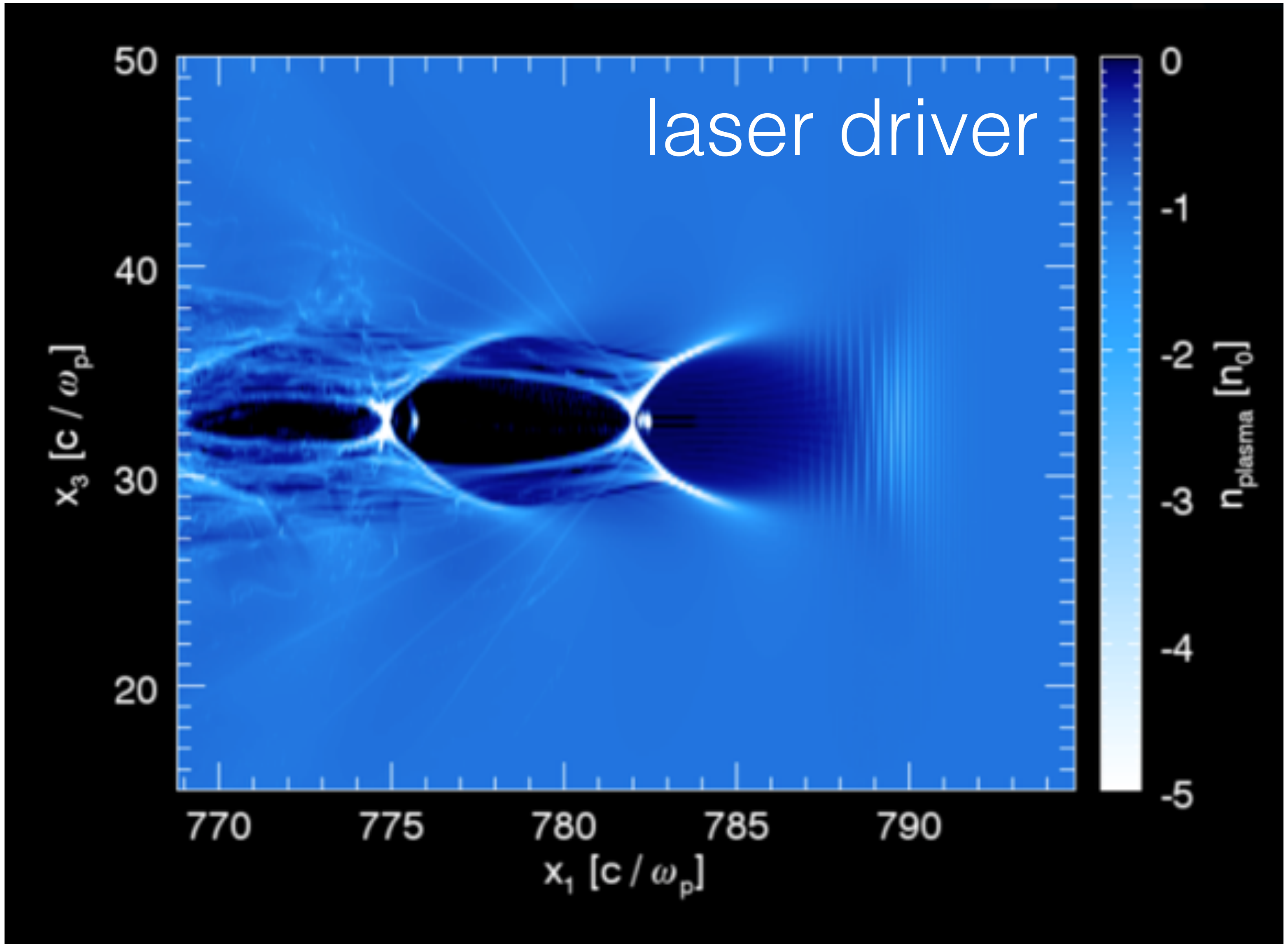
- To overcome diffraction we need to guide the laser - an optical fibre
- Can’t use a normal optical fibre - it will damage!
  - plasma waveguide - plasma density minimum on axis
- Pre-formed plasma waveguides (Hooker group)
- Self-guiding - pulse forms its own waveguide

Limits to acceleration



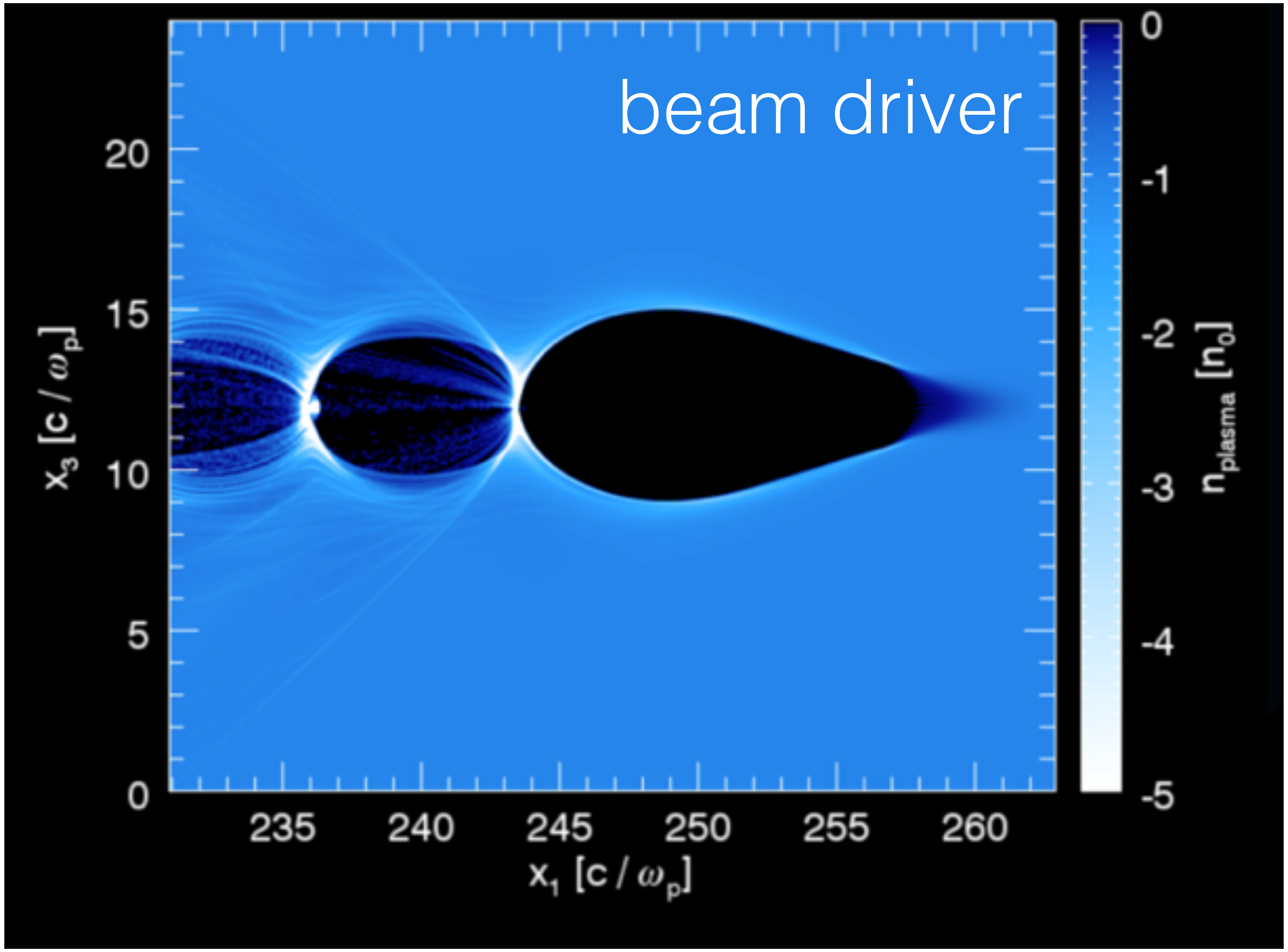
- Currently laser driven experiments are mostly limited by dephasing…

The blow-out regime



- If the drive beam is strong enough then it can completely expel *all* the electrons from near the laser pulse - we call this the blow-out or bubble regime

The blow-out regime



- If the drive beam is strong enough then it can completely expel *all* the electrons from near the laser pulse - we call this the blow-out or bubble regime

The bubble regime: the bubble size

- we can estimate r_b by balancing the ponderomotive force and space charge force of the ionic bubble

$$F_p = -\frac{1}{2} m_e c^2 \nabla \frac{a^2}{\gamma}$$

$$\nabla \cdot \mathbf{E} = \frac{-e(n_e - n_i)}{\epsilon_0} = \frac{e n_0}{\epsilon_0}$$

$$\gamma \approx \sqrt{1 + a^2} \approx a$$

$$E(r) \approx e n_0 r / \epsilon_0$$

$$F_p \approx m_e c^2 a_0 / w_0$$

$$F_{sc} \approx -e^2 n_0 r / \epsilon_0$$

$$m_e c^2 \frac{a_0}{w_0} - \frac{e^2 n_0 r_b}{\epsilon_0} = 0$$

$$r_b \approx \frac{a_0}{k_p^2 w_0}$$

it turns out that the situation is best if the laser spot size is matched to the bubble so we have:

$$r_b \approx 2\sqrt{a_0} \frac{c}{\omega_p}$$

The bubble regime: the field strength

- Using the equation for the electric field $E(r) = en_0 r/\epsilon_0$
- And the blow-out radius $r_b \approx 2\sqrt{a_0}\frac{c}{\omega_p}$
- we can estimate the field strength of the bubble - it is:

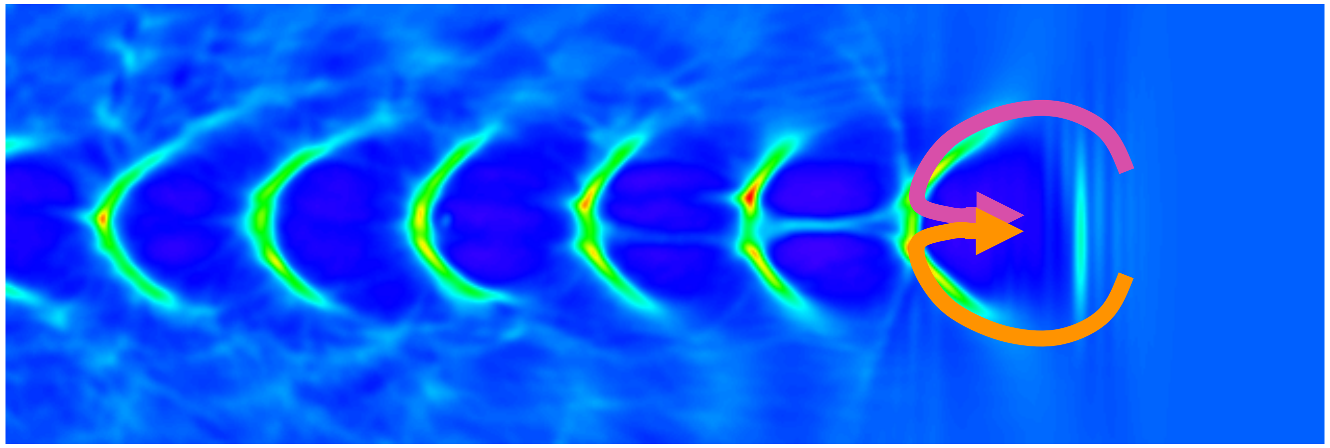
$$E_{max} \approx \sqrt{a_0}\frac{m_e c\omega_p}{e}$$

- For $a_0 \approx 3$ and a plasma density of $n_0 = 4 \times 10^{18}$ cm^{-3} we get a maximum field of 330 GV/m !
- Combining this with the dephasing length we would get a maximum electron energy of 2.4 GeV
  - this is an overestimate as non-linear effects make the group velocity a bit slower

Injecting electrons into the wave

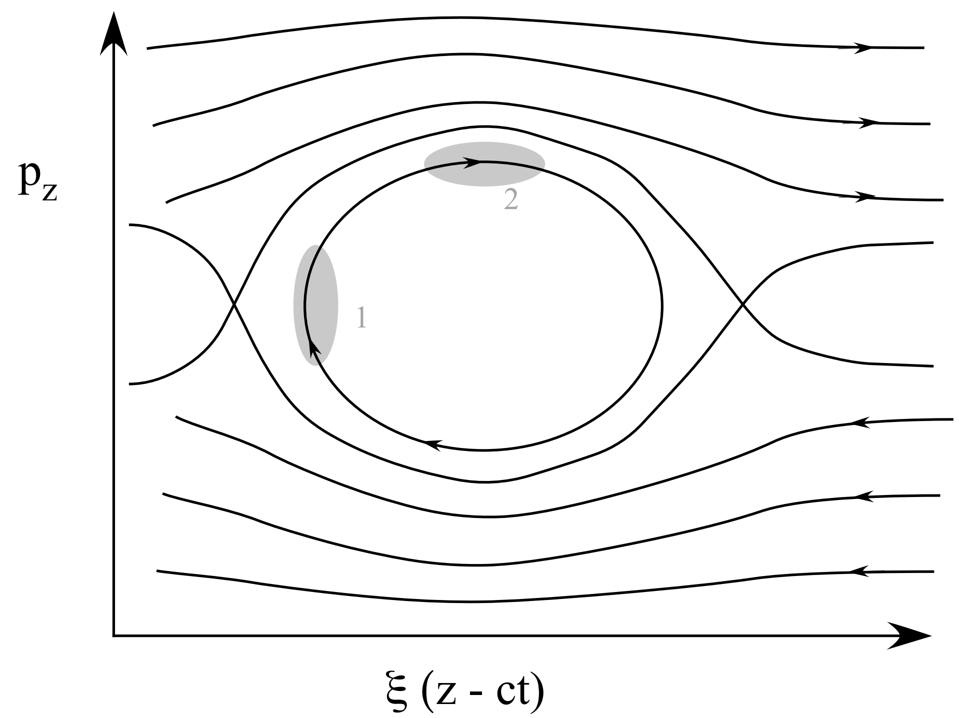
- For a surfer to “catch a wave” he must swim to get up to speed before the wave arrives
- if he is too slow the wave will just pass over him
- we must find a way of accelerating electrons up to the correct speed for them to be trapped by the wave and accelerated

self-injection



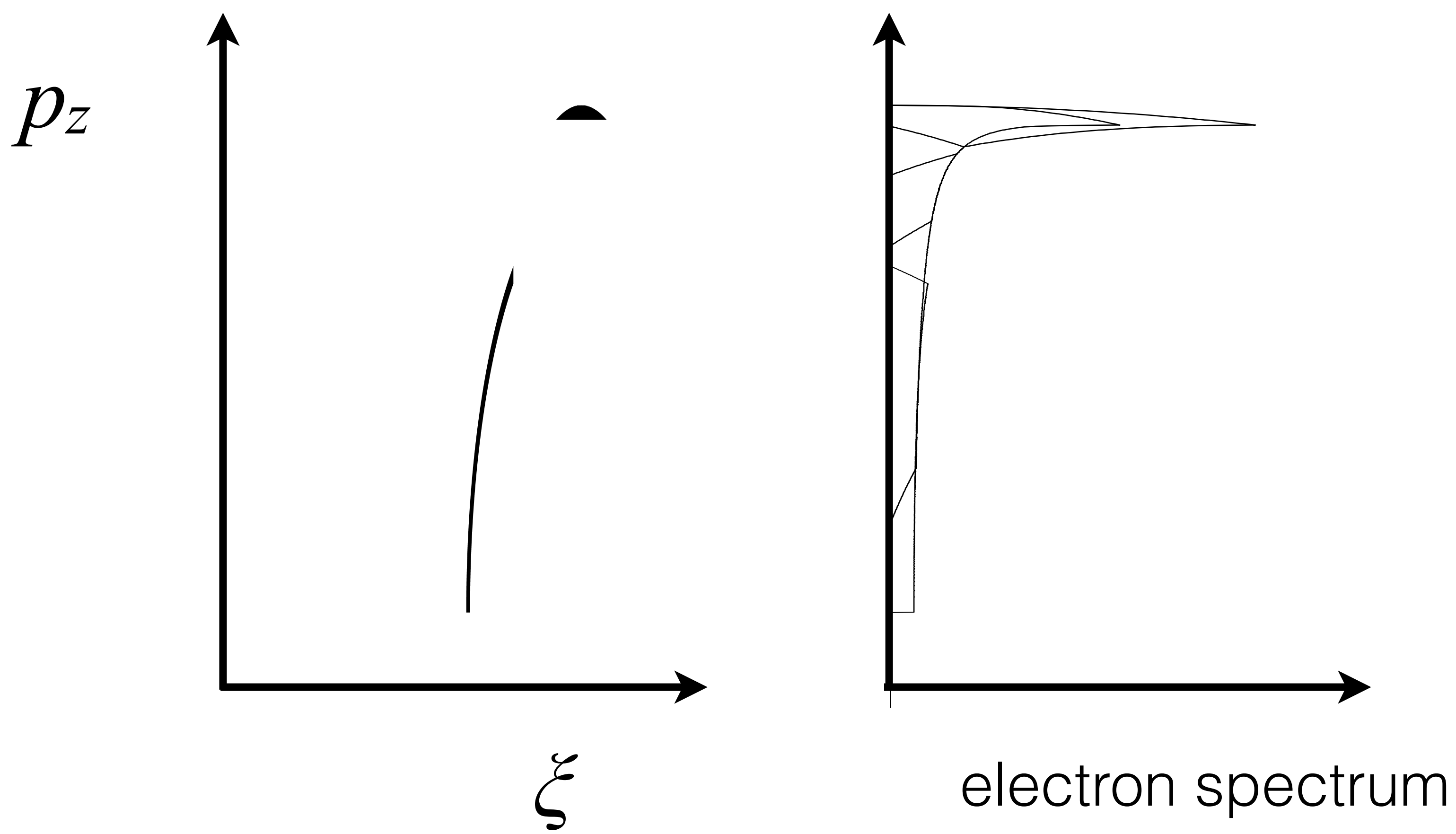
- Nature is kind to us - when the wakefield has a large enough amplitude some electrons can be trapped
- They are all injected at the back of the bubble so can be accelerated to the same energy - quasi-monoenergetic electron beams

self-injection



- this a plot of the longitudinal position ($\xi = z\text{-}ct$) in the wave against the longitudinal momentum p_z (called the p_z–ξ phase space)
  - The black arrows show electron trajectories
  - Trapped electrons follow closed orbits
- self-injection in the bubble only happens over a small range of ξ at the back of the bubble
- phase space rotation exchanges initial spread in p_z for spread in ξ

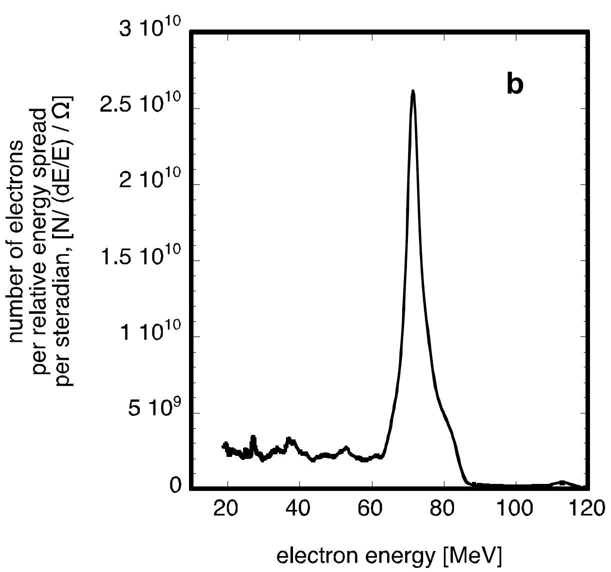
self-injection



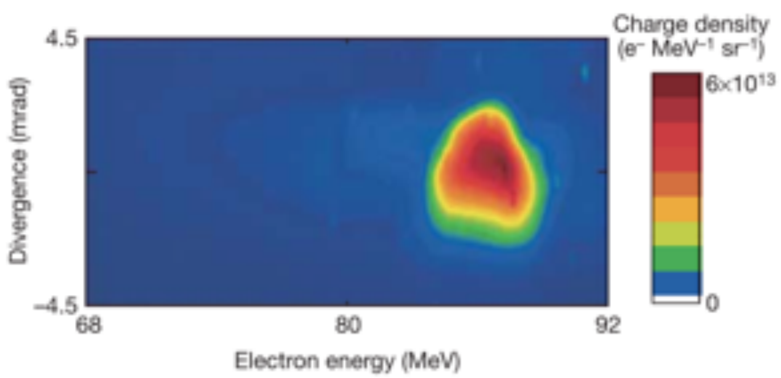
- This animation demonstrates how phase space rotation changes the electron spectrum

what sort of electron beams can we get?

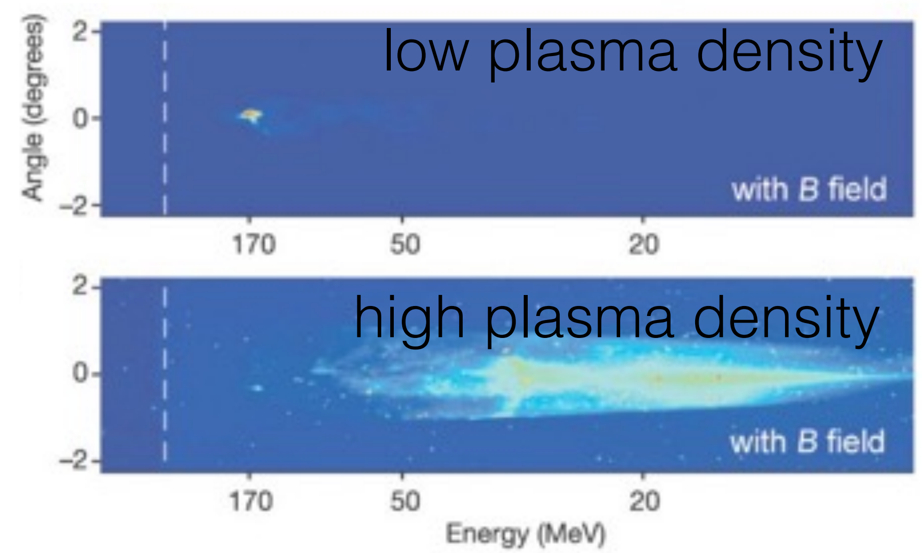
- Back in 2004 the Imperial College group, a group in the US (LBNL) and a group in France (LOA) were the first to report narrow energy spread beams from a laser wakefield accelerator



Mangles et al Nature 2004

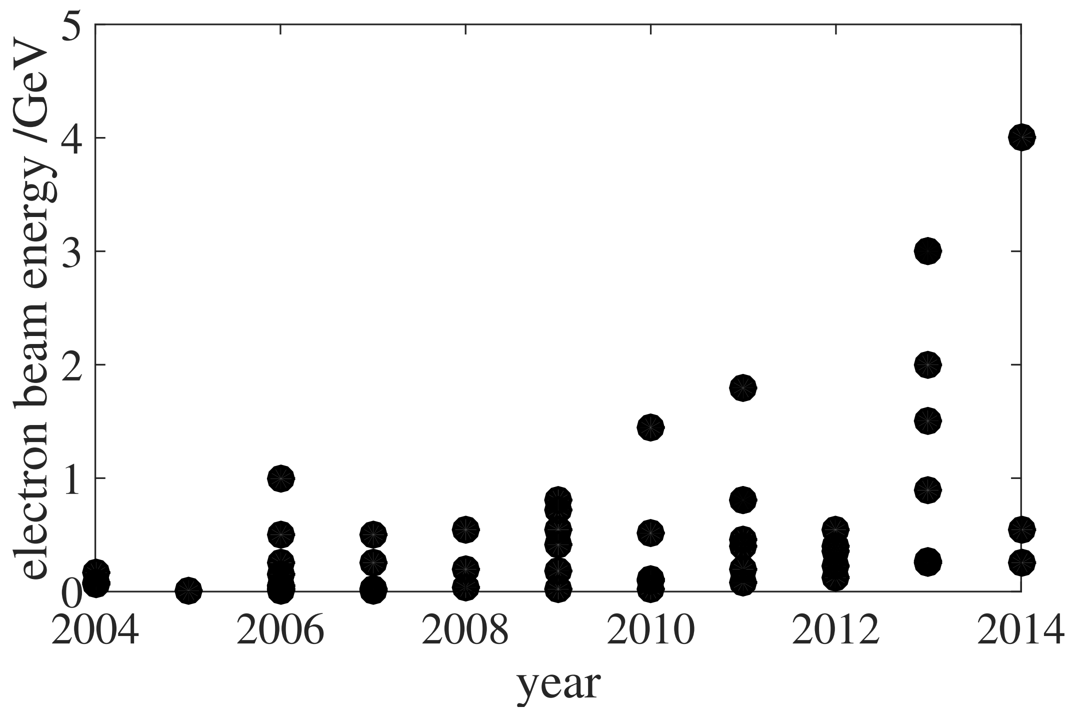


Geddes Nature 2004

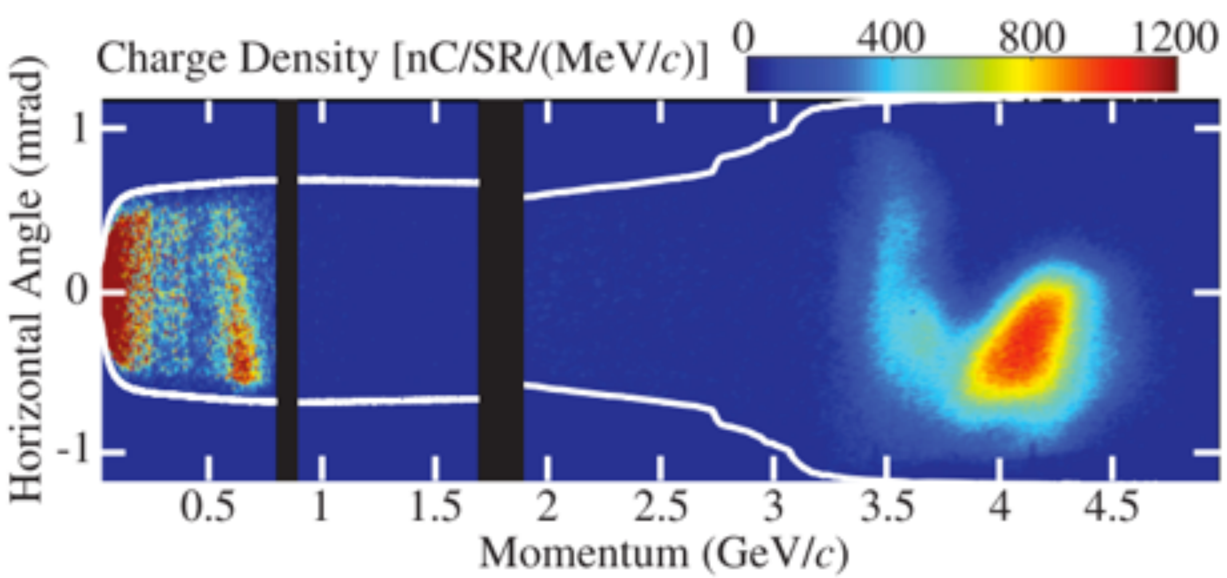


Faure et al Nature 2004

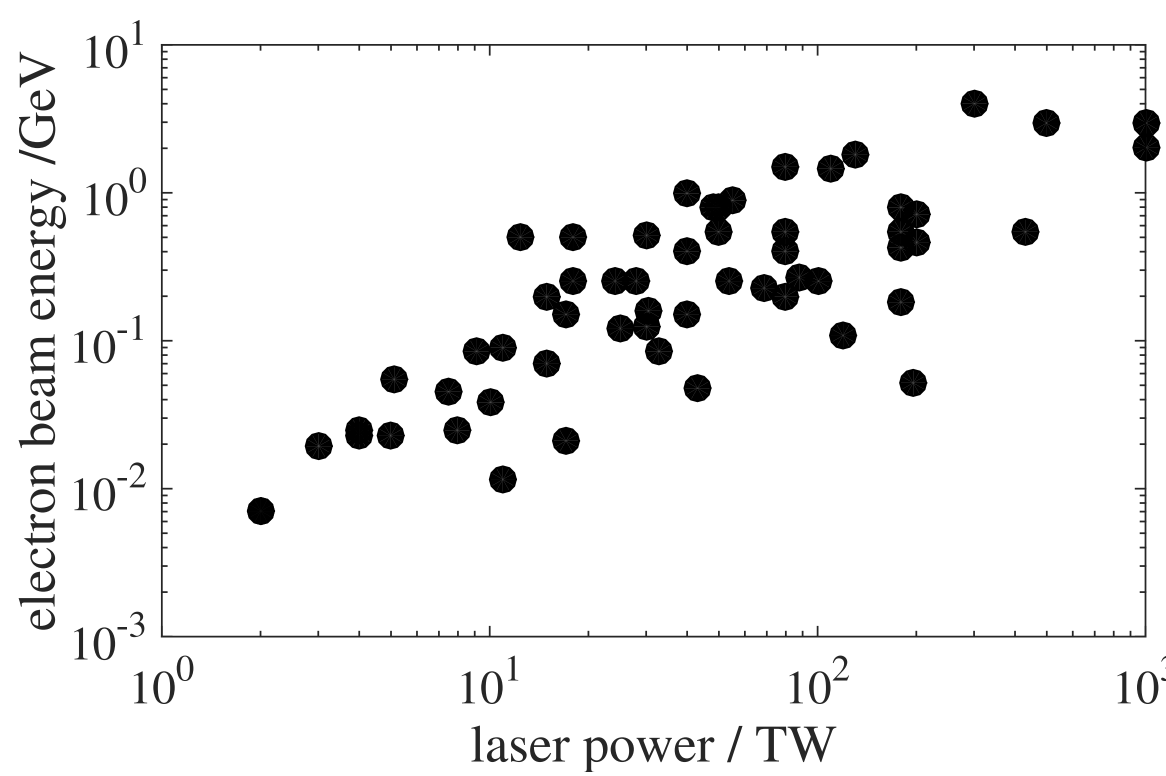
extending the energy range



- Progress in energy gain has been rapid
  - 0.2 MeV in 2004 to 4.2 GeV in 2014 : 20 x increase in 10 years
  - increase in beam energy due to increasing laser powers
  - allows operation at lower density (limited by dephasing)



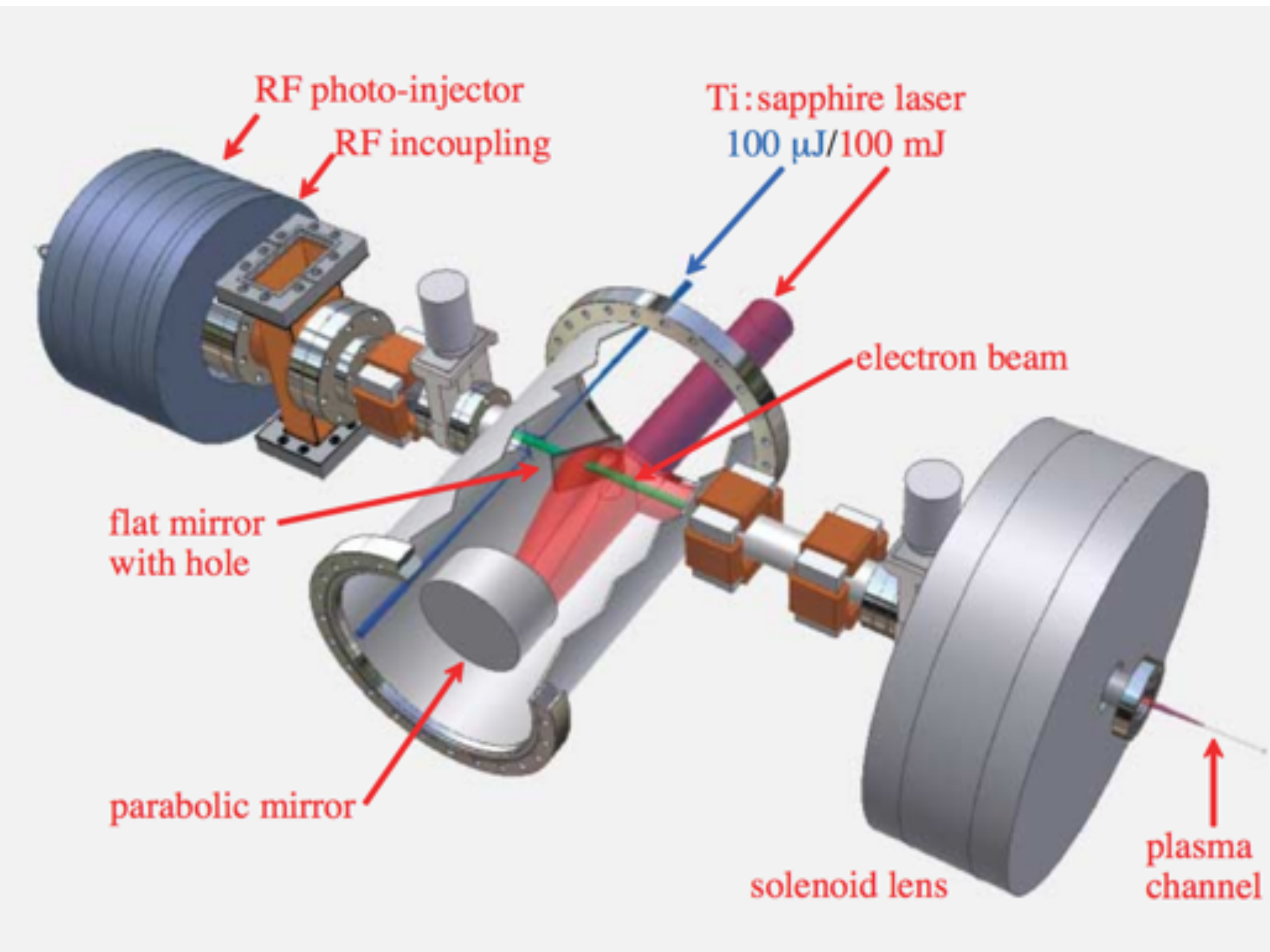
Leemans PRL 2014



advanced injection schemes

- This section will briefly discuss some other injection schemes:
  - external injection
  - colliding pulse injection
  - density down ramp injection (slow ramps and fast ramps)
  - ionisation injection

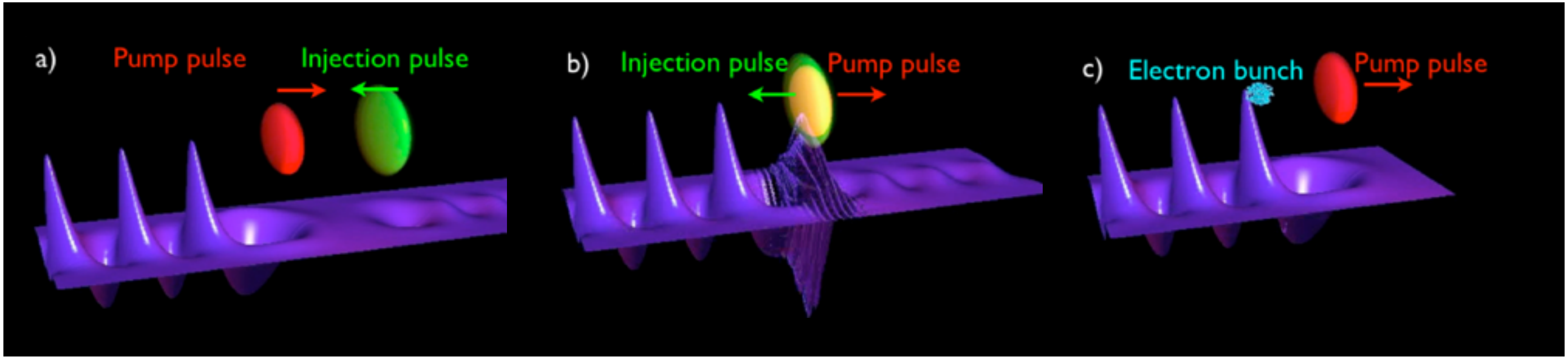
external injection



- uses a conventionally produced electron beam and tries to put it into the correct part of the plasma wave
- requires exquisite alignment and timing between the electron beam and the laser
- requires a very small, short electron bunch ($\sigma_z, \sigma_{x,y} < \lambda_p$)
- usually the idea is to operate in a linear or quasi-linear regime as this is thought to be more stable

van der Wiel M, Phil. Trans. R. Soc. A 2006

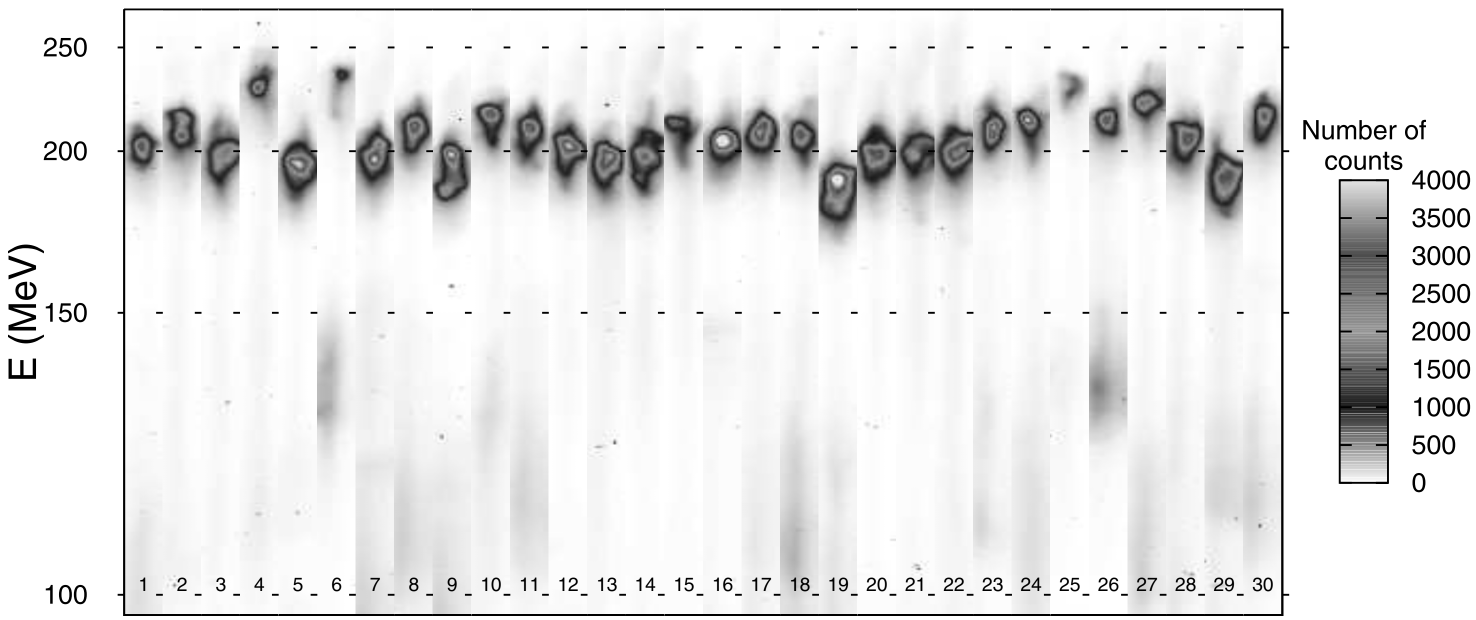
colliding pulse injection

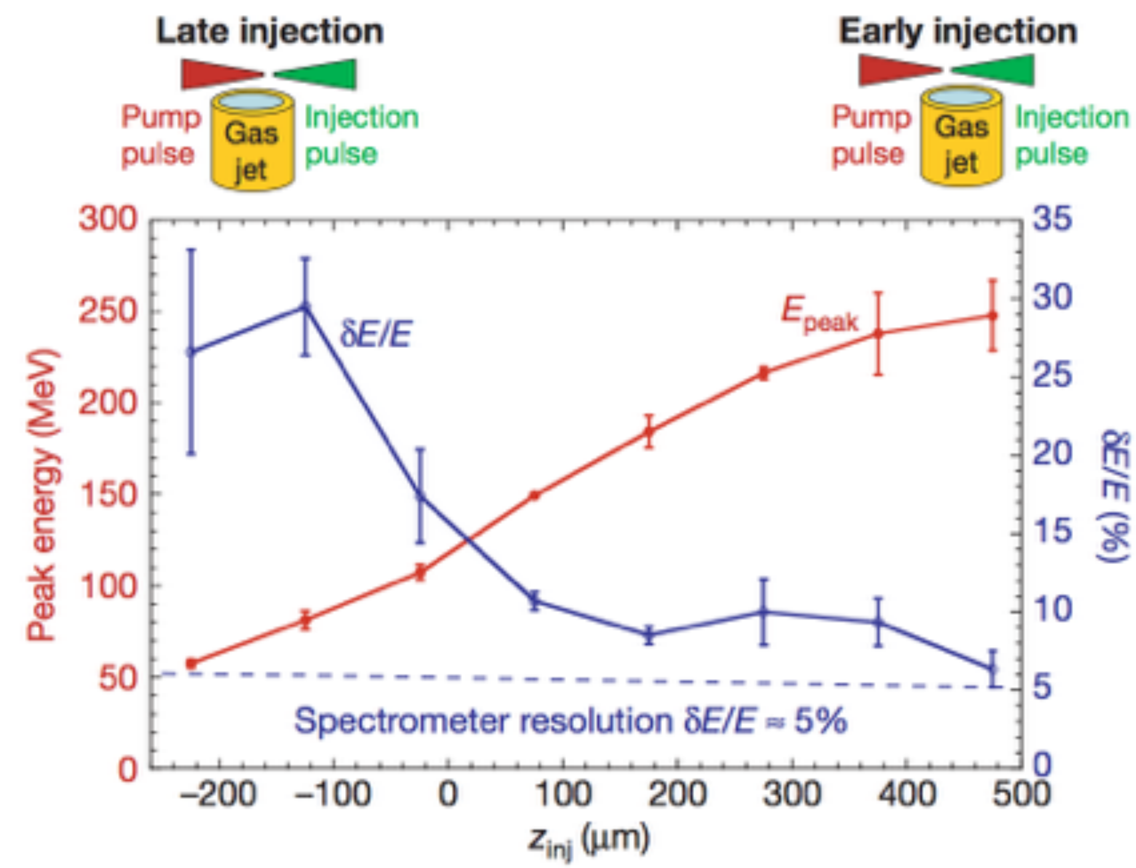


- A moderately intense laser drives a non-linear plasma wave (but below the self-injection threshold)
- A second laser pulse collides with the first - the resulting interaction or “beatwave” heats the plasma at the interaction point
- Electrons in this hot-spot can then become trapped in the plasma wave

Faure, Nature 2006

colliding pulse injection

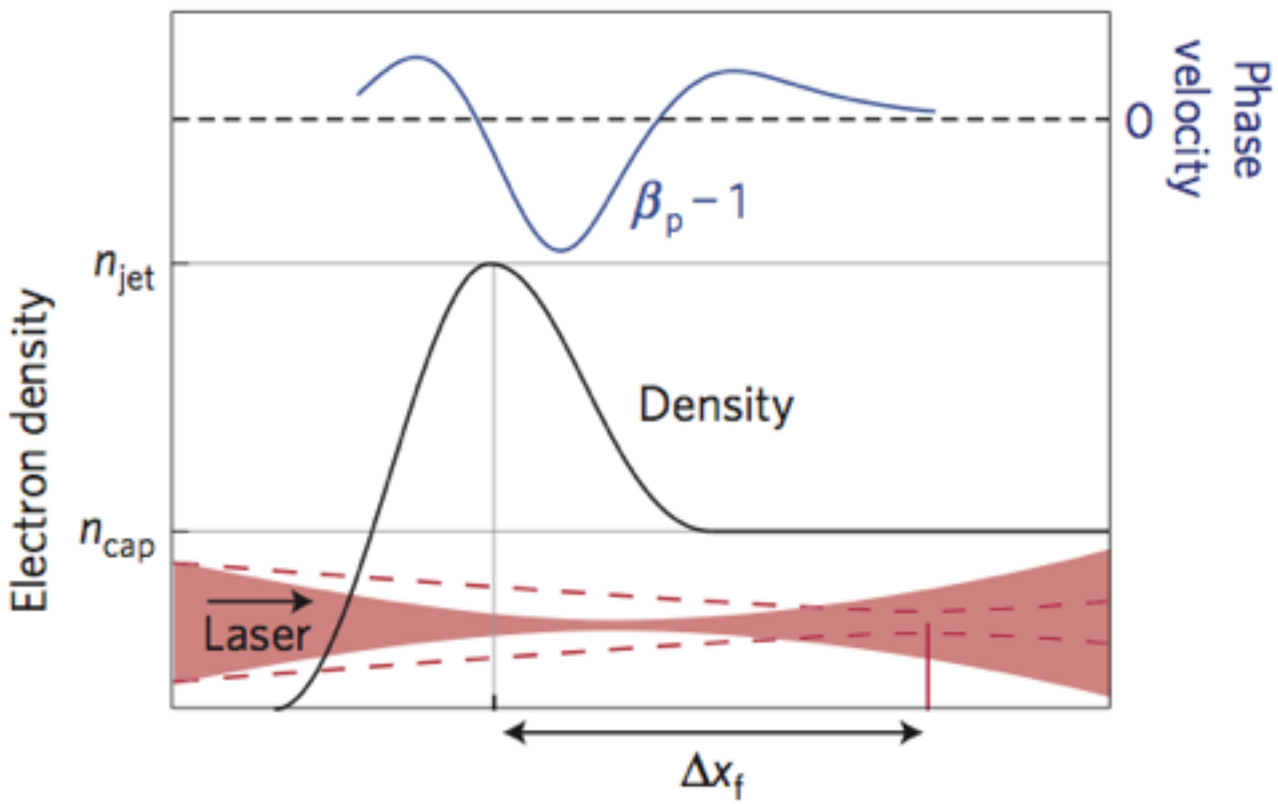


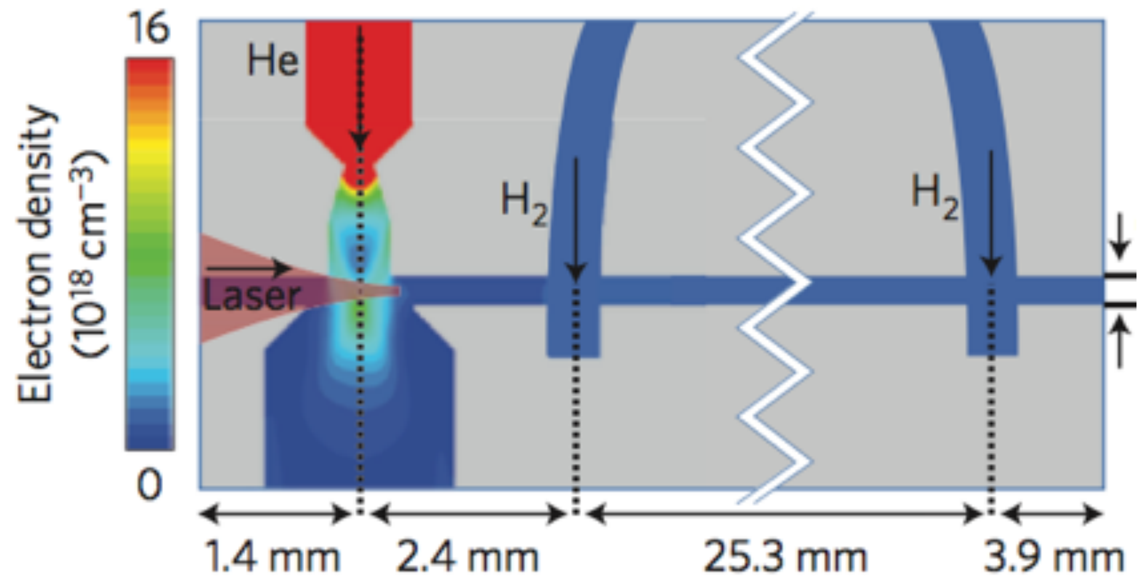


- Colliding pulses can produce very stable electron beams
- choosing the position of the collision in the gas jet allows the electron beam energy to be tuned

Faure, Nature 2006

density down-ramp injection

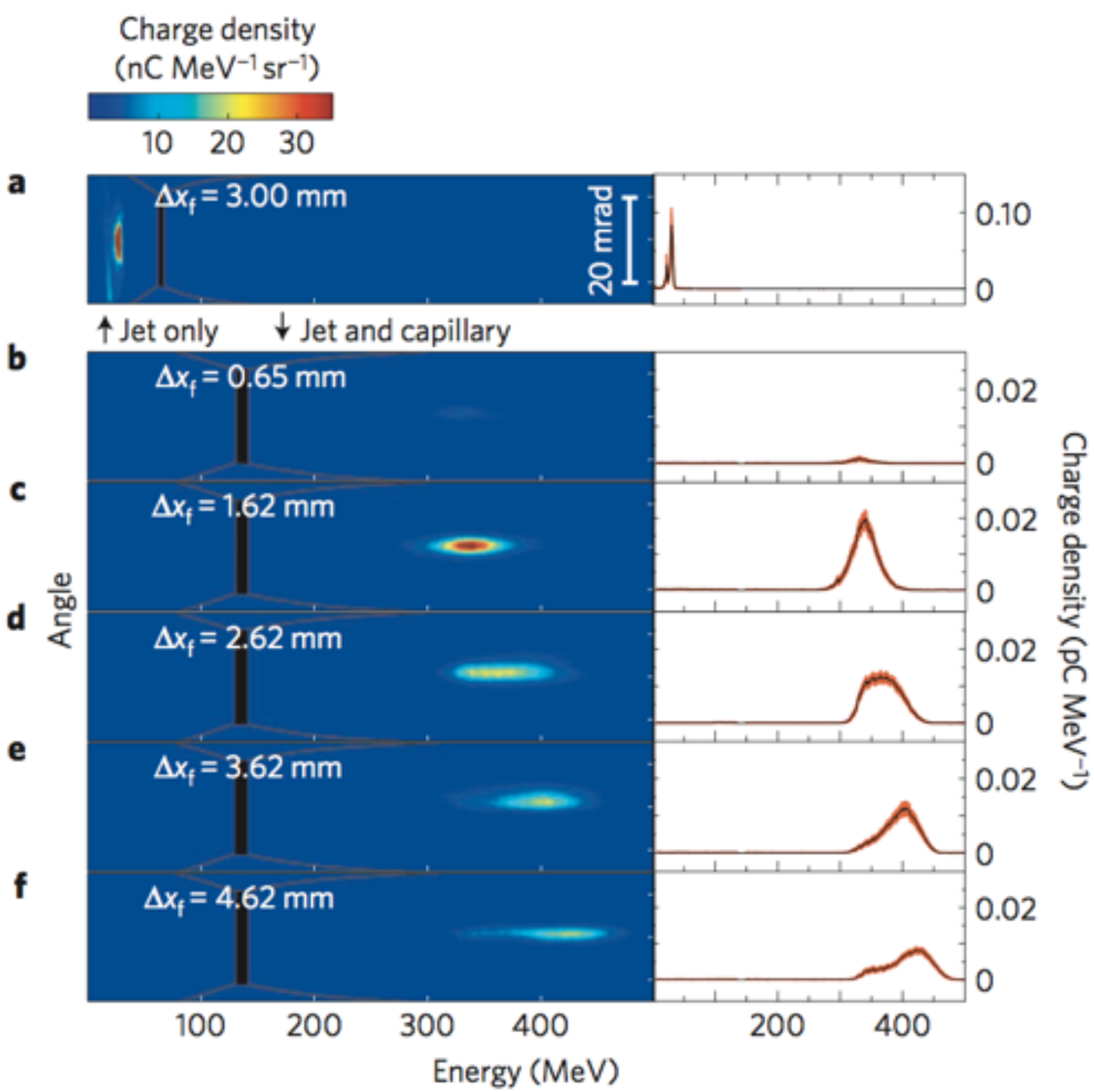




- By allowing laser to travel down a density down-ramp the plasma wave phase velocity can be made to slow down
- A slow phase velocity means electrons can be easily trapped
- If a constant density region is placed after the down-ramp these electrons can be accelerated to high-energy

Bulanov PRE 1998, Geddes PRL 2008, Gonsalves Nature Physics 2011

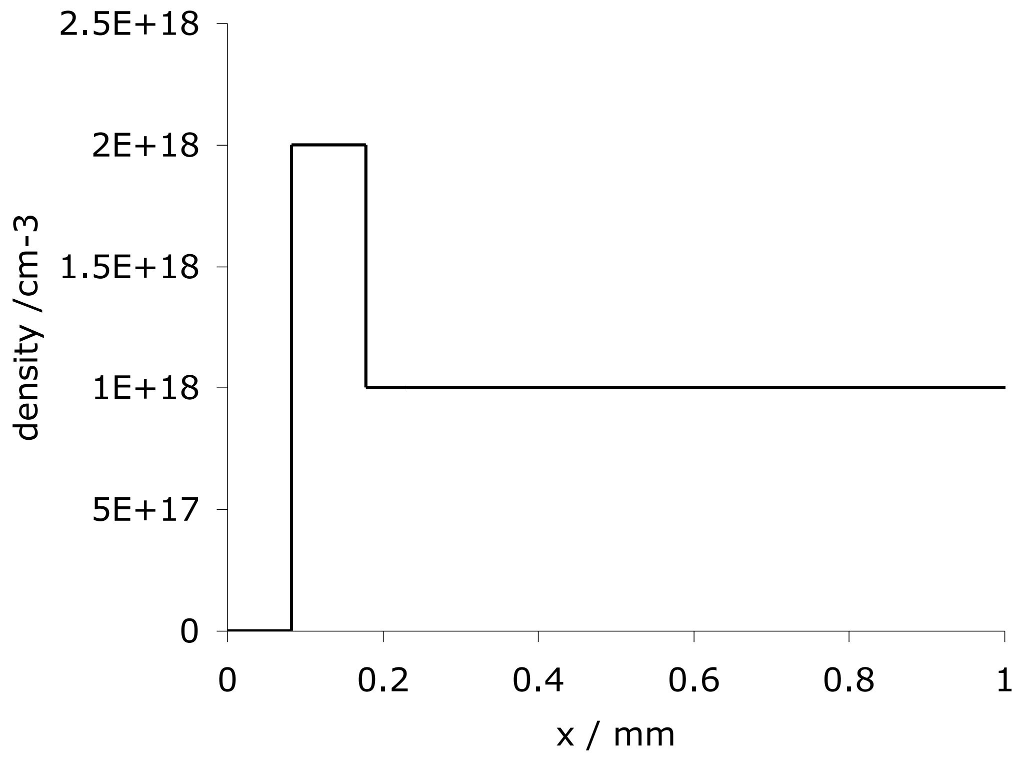
density down-ramp injection



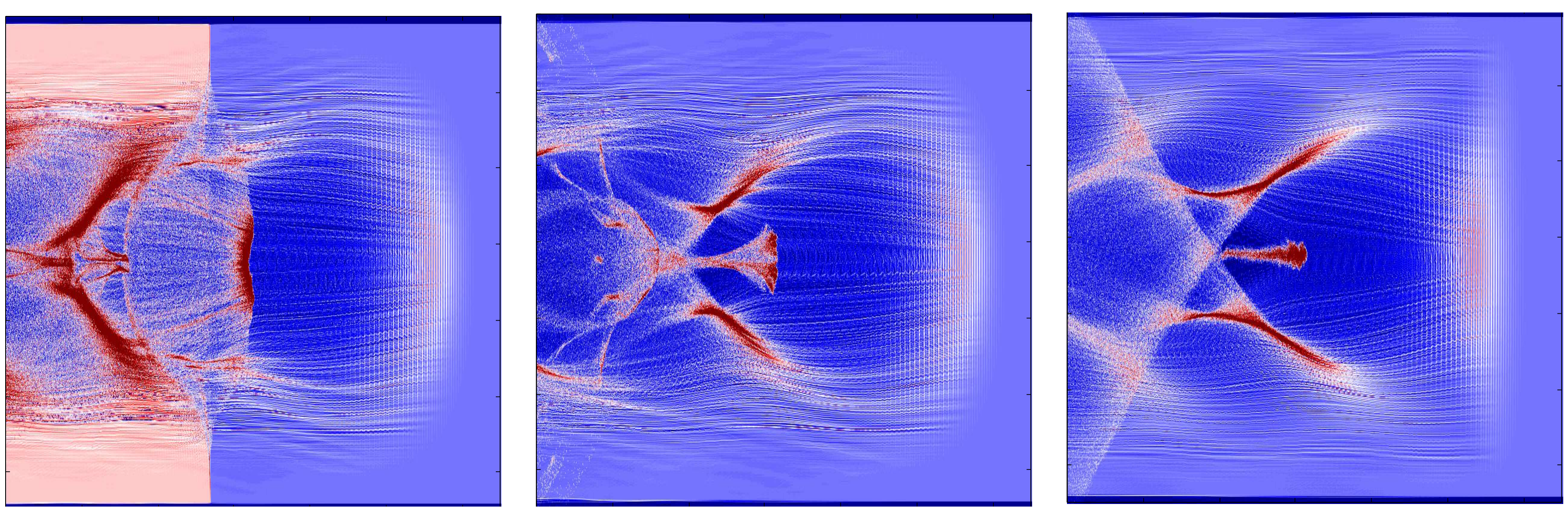
- By tuning the position of the laser focus relative to the down-ramp the position of injection can be controlled
- This allows some tunablilty in the electron beam energy
- Electrons produced from down-ramp can be remarkably stable (1 % level)

Gonsalves Nature Physics 2011

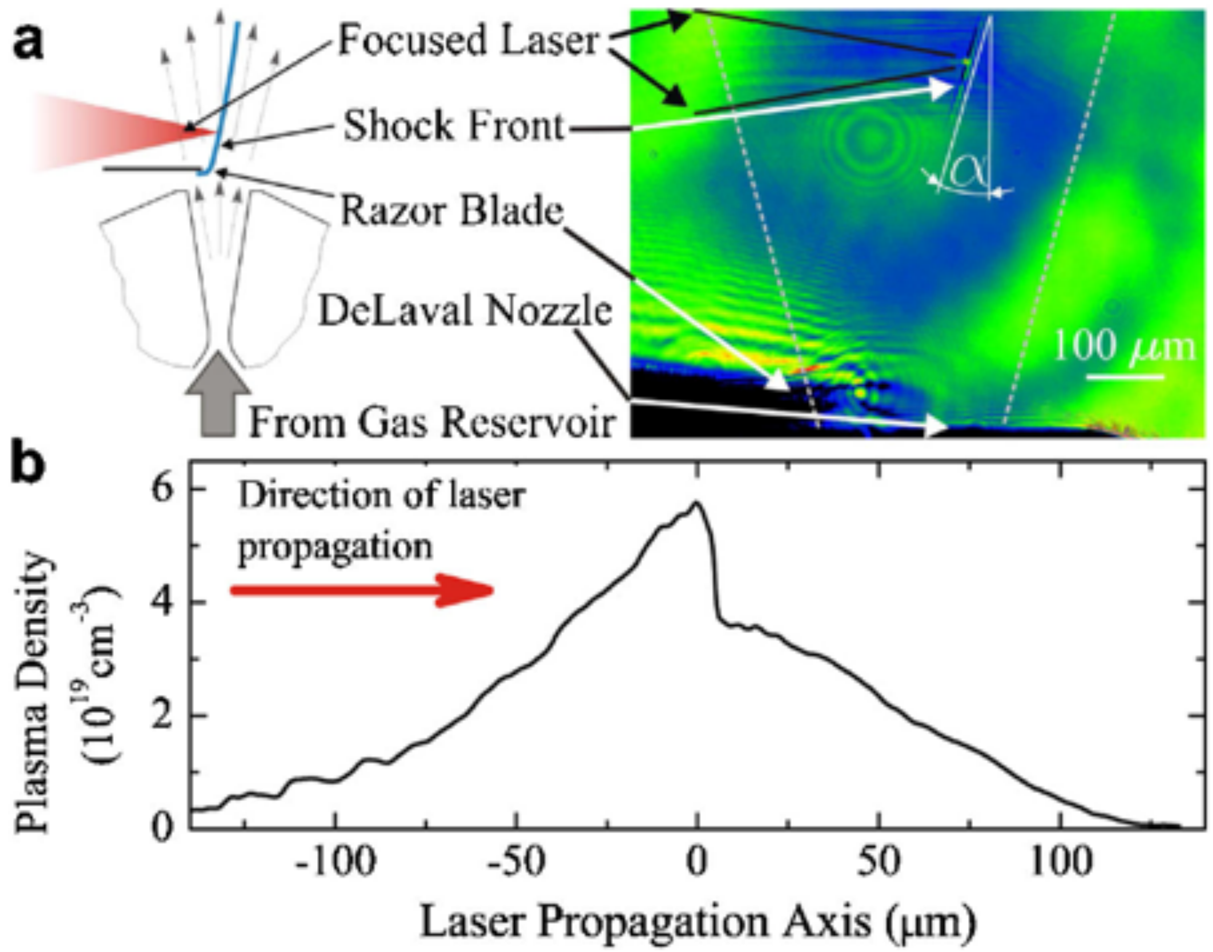
density down-ramp injection



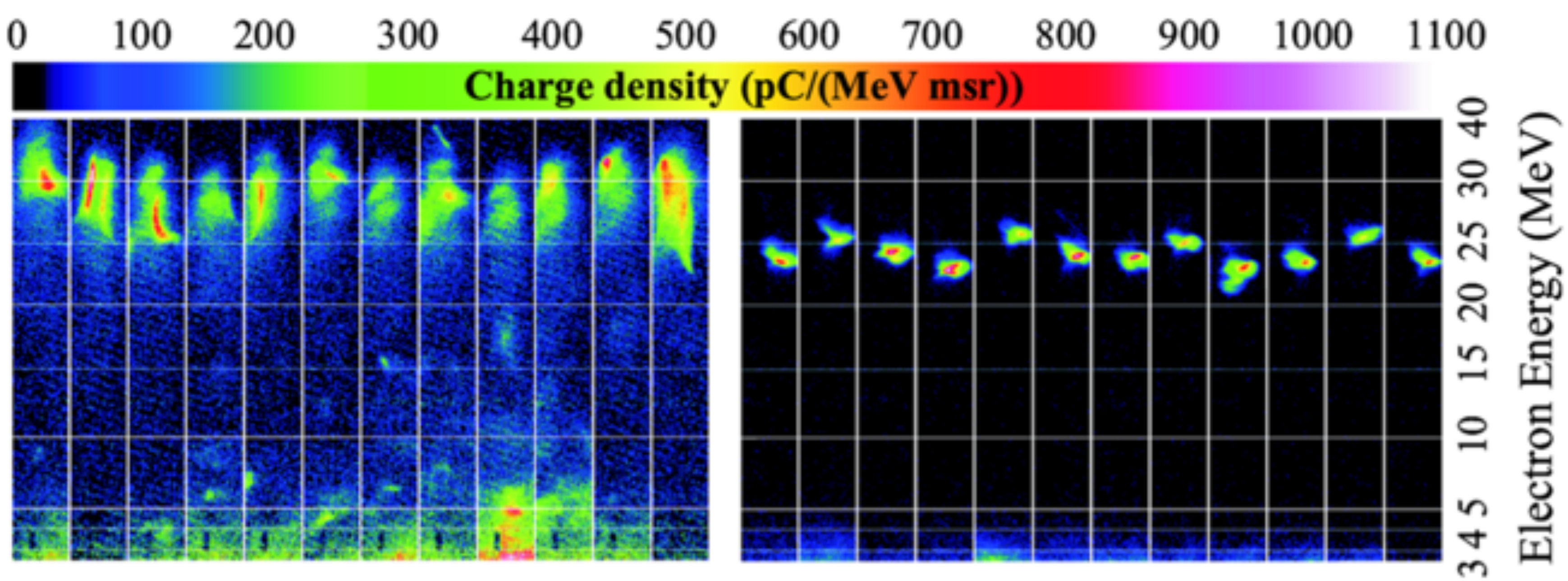
- An alternative down-ramp injection scheme uses a *sharp* down-ramp ($\sim \lambda_p$)
- This sharp density down-ramp injection works because electrons that are oscillating in the wave suddenly feel no restoring force and so are injected into the wave



density down-ramp injection



- These sharp profiles can be achieved using obstructed supersonic gas jets
- Density transition injection also produces pretty stable electron beams



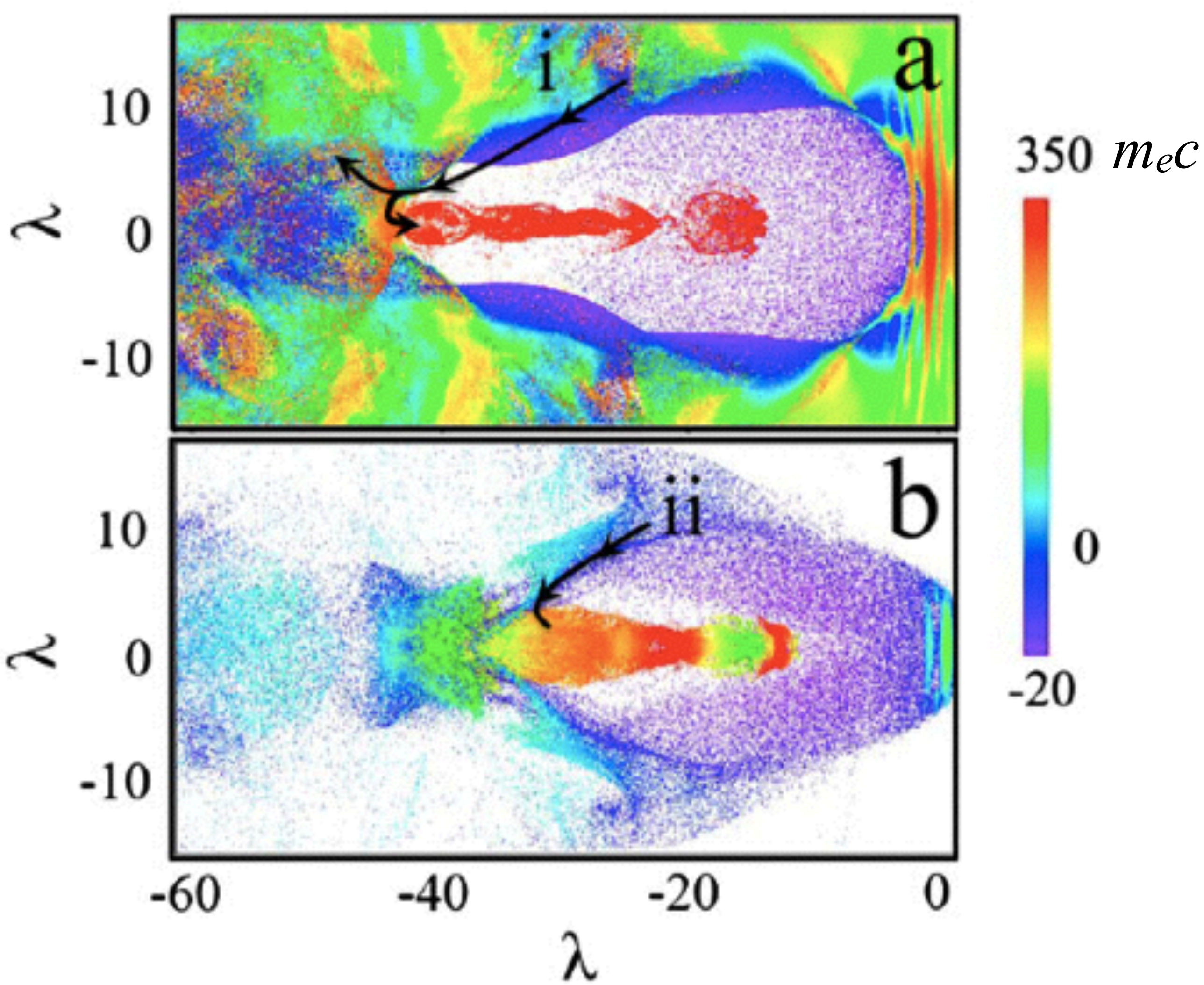
(a) Self injection

(b) Injection at density transition

Schmid PRSTAB 2010

ionisation injection

a) electrons from helium and outer shells of nitrogen



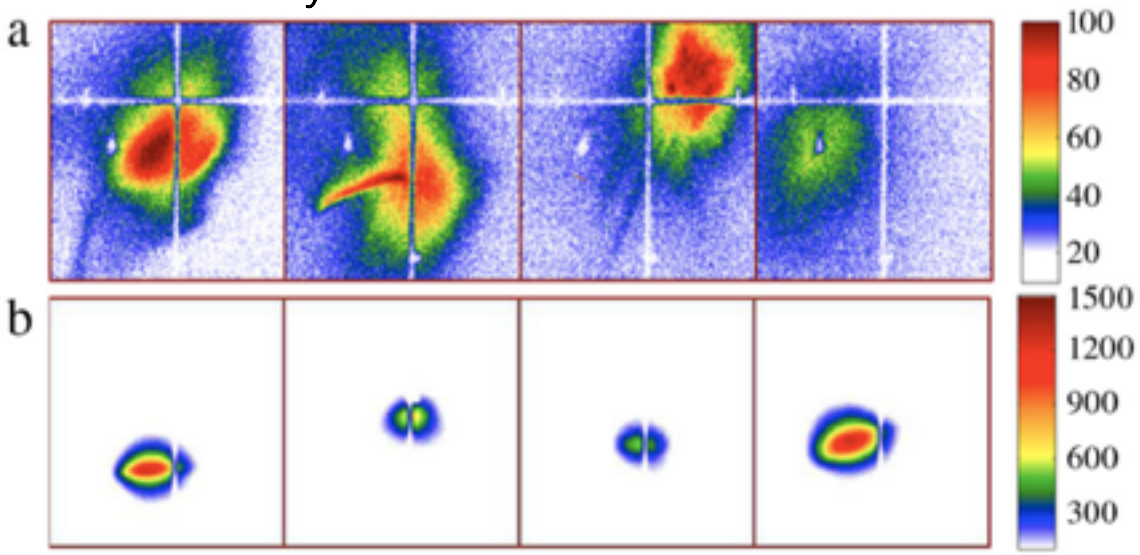
b) electrons from inner shells of nitrogen

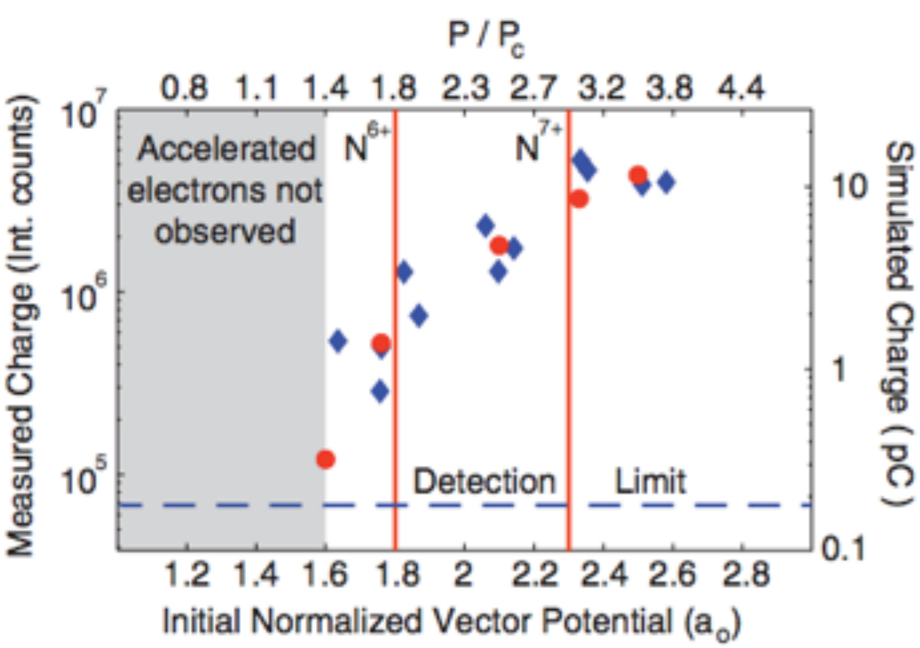
Pak PRL 2010 and McGuffey PRL 2010

- Gas mixture e.g. He:N_2 (95:5)
- Helium and outer electrons of nitrogen are ionised early and supply the electrons to create the plasma wave
- Inner electrons of Nitrogen are only ionized near the peak laser intensity - i.e. *inside* the bubble

ionisation injection

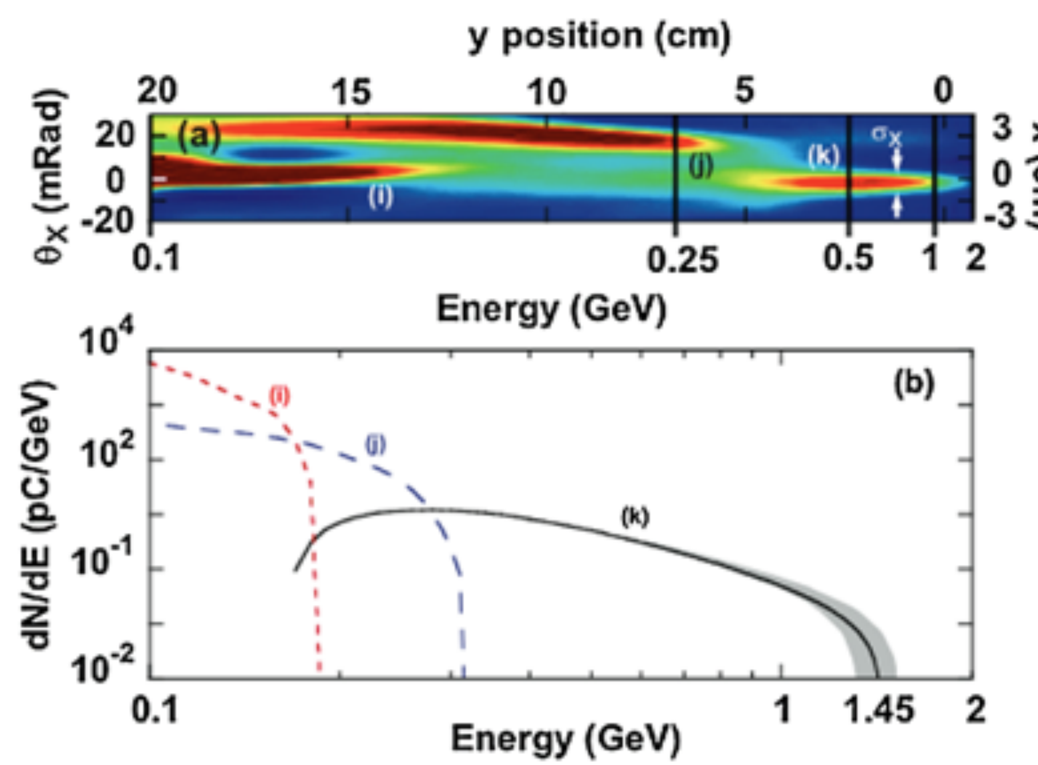
McGuffey PRL 2010





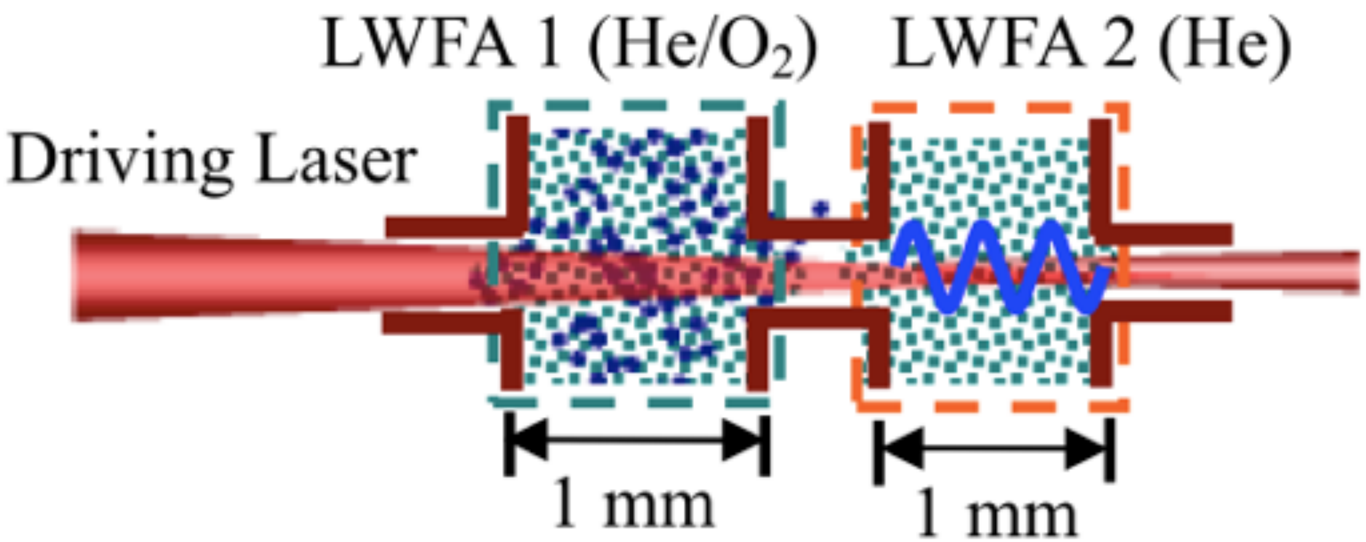
Pak PRL 2010

- These electrons that are "born" inside the bubble are much more easily trapped
  - this lowers the injection threshold
  - and increases the charge trapped
- Ionisation injection is continuous so leads to large energy spreads - beyond 1 GeV has been observed

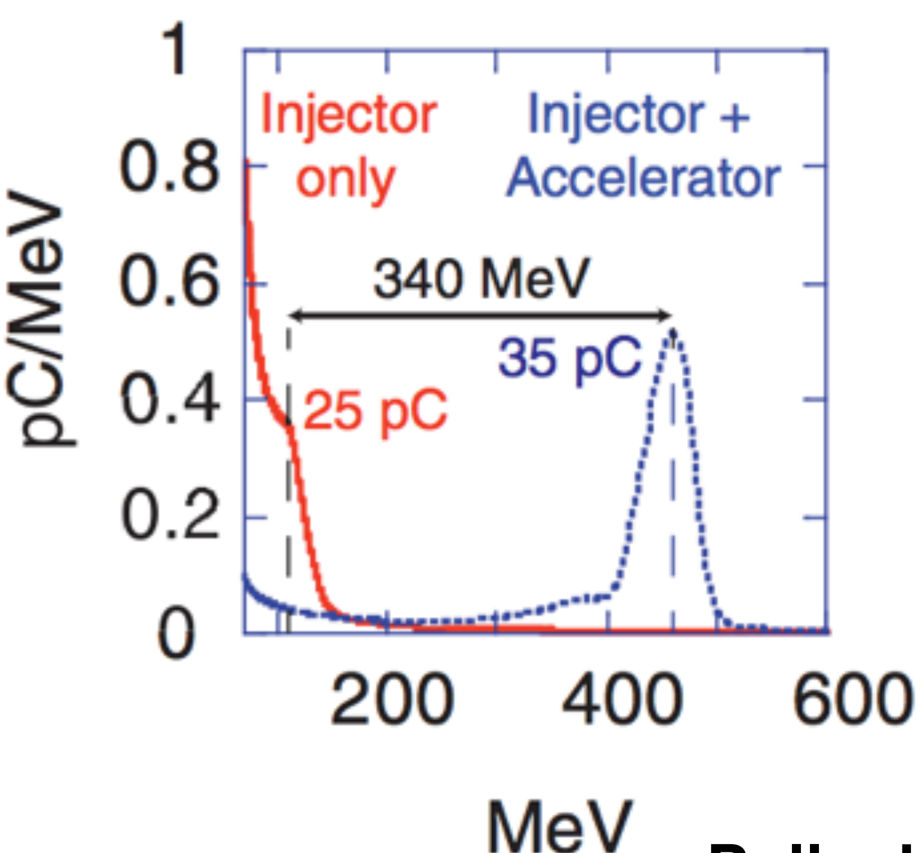


Clayton PRL 2010

ionisation injection



Liu PRL 2011



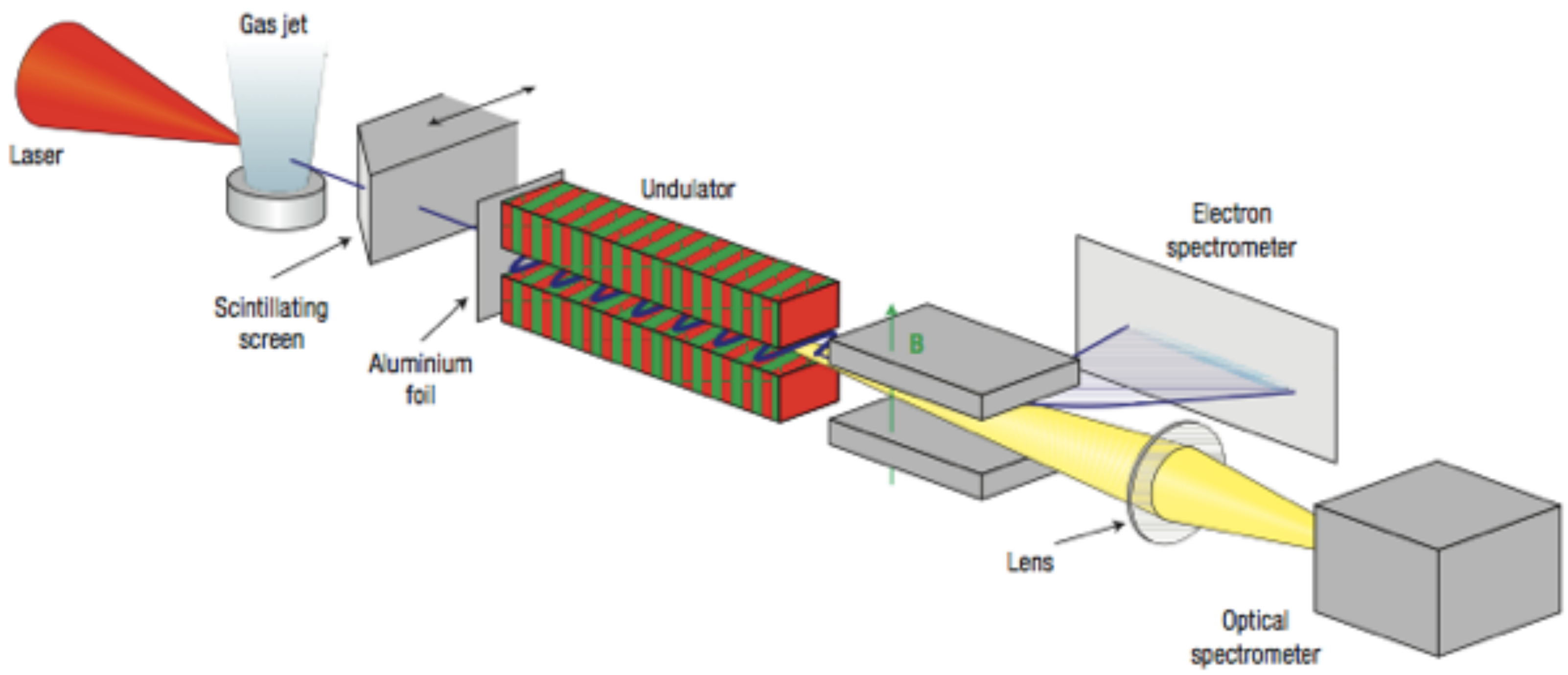
Pollock PRL 2011

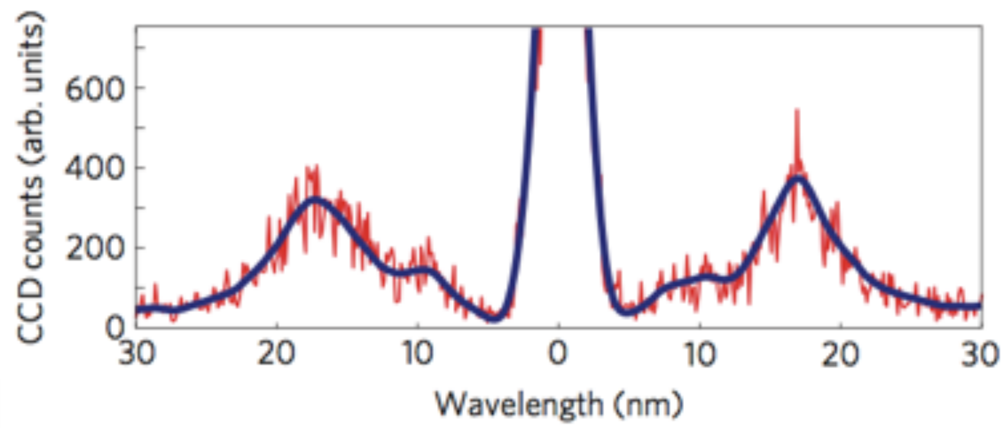
- By using a two-compartment gas cell:
  - the first compartment containing the injection gas,
  - the second only containing helium
- the continuous injection problem can be overcome

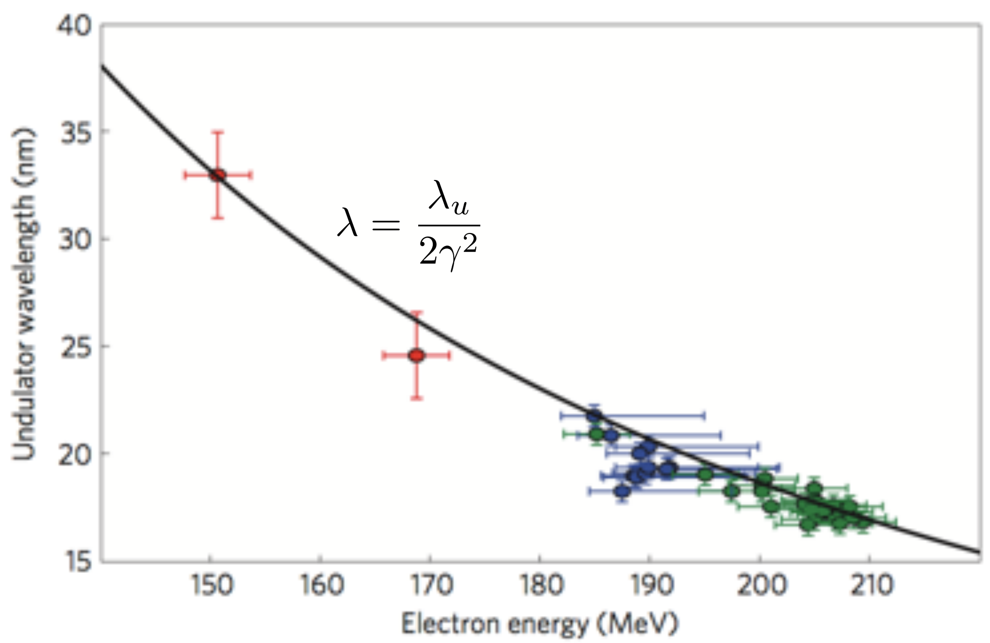
Wakefield accelerators as a radiation source

- Wakefield accelerators now reaching multi-GeV levels with ~100 TW lasers
- This is energy range for conventional synchrotrons
    - Can we use plasma wakefield accelerators as a light source?

LWFA as undulator radiation source



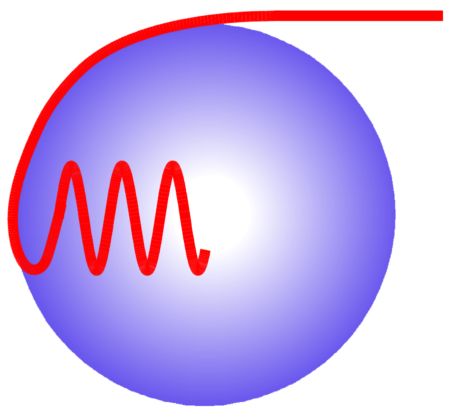




- MPQ/Oxford Collaboration
  - 0.5 cm undulator wavelength, electron energies up to 200 MeV
  - monochromatic radiation at ~20 nm = 60 eV

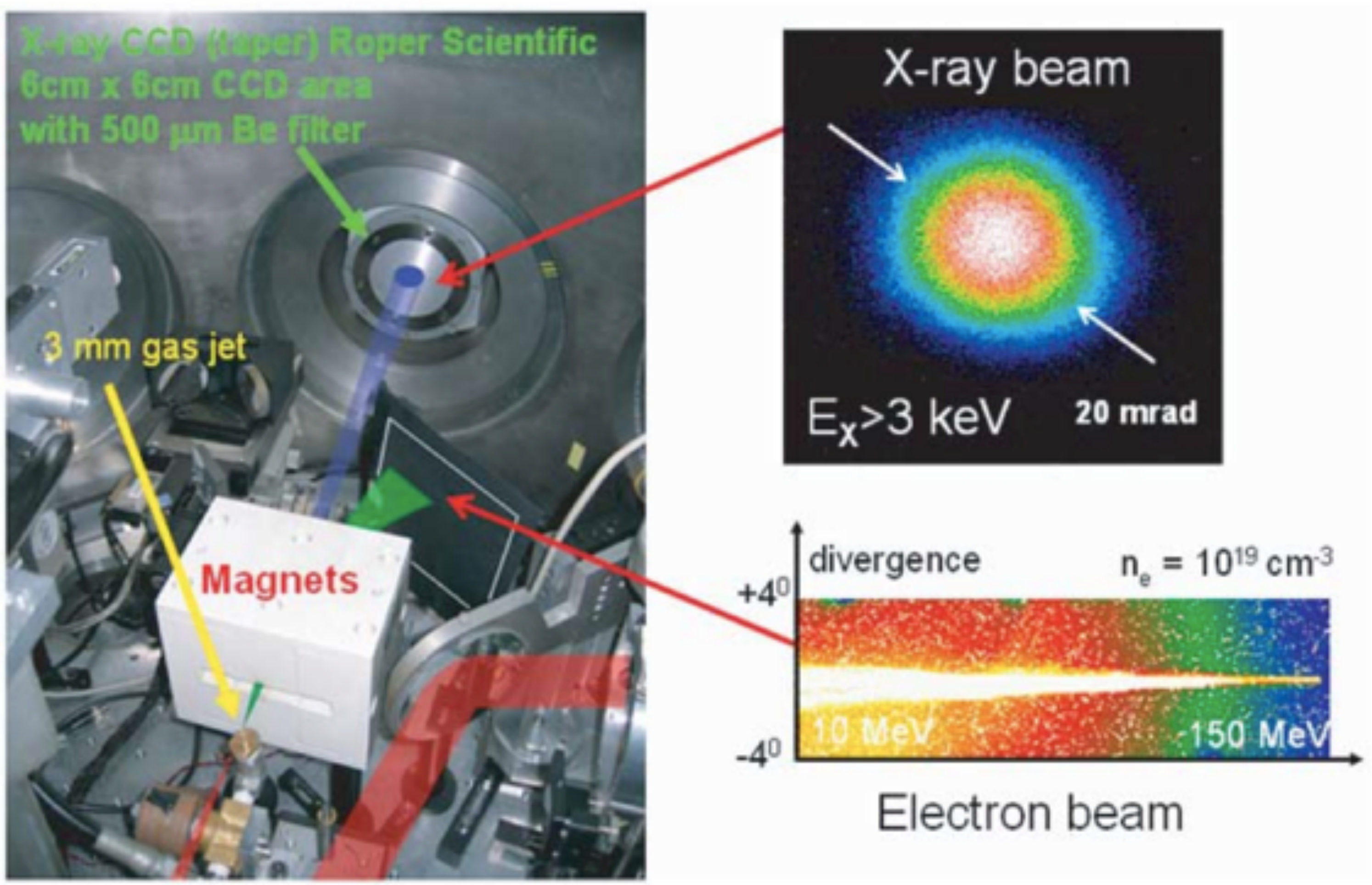
Schlenvoigt Nature Phys 2007 **Fuchs Nature Phys 2009**

LWFA as wiggler radiation source



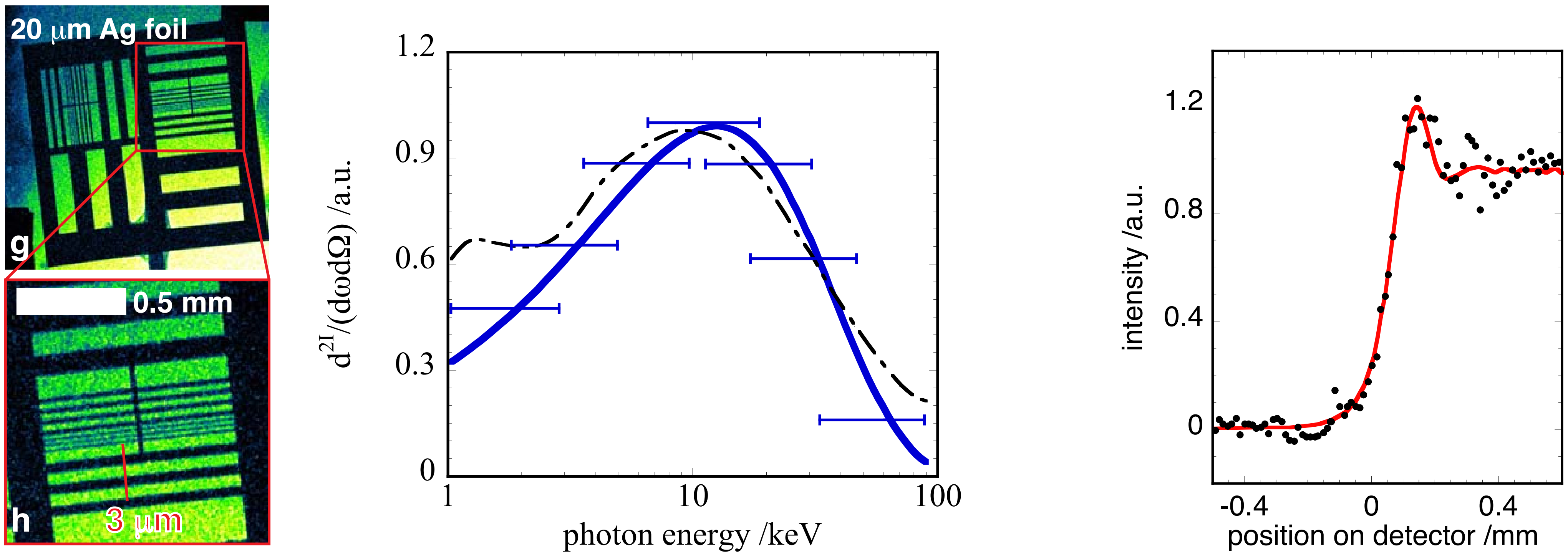
- the bubble has very strong focusing forces as well as accelerating ones
- this leads to transverse oscillations of the electron as it accelerates - called “betatron oscillations”
- frequency of oscillations is plasma frequency for radial oscillations of relativistic beam $\omega_B = \frac{\omega_p}{\sqrt{2\gamma}}$
- wavelength of this “plasma wiggler” can be very short $\lambda_B = \sqrt{2\gamma}\lambda_p$
- for $n_e = 10^{19}$ cm^{-3} and 200 MeV electrons it is 300 μm

LWFA as wiggler radiation source



- Betatron radiation was first observed from a LWFA by Rousse et al (PRL 2004)
- 30 TW laser
- broad band ~ 100 MeV electrons
- radiation at ~ 1 keV

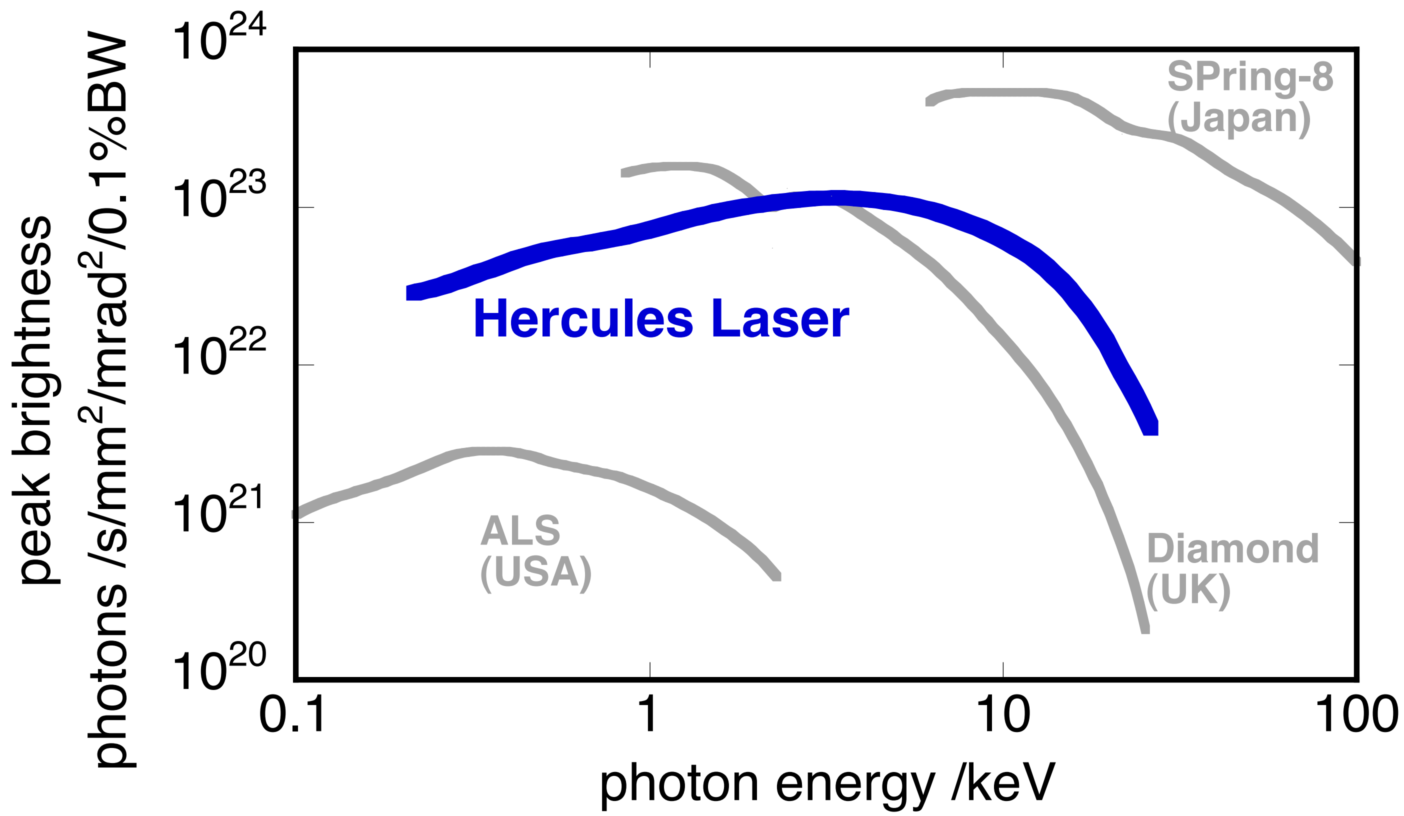
LWFA as wiggler radiation source



Imperial College /Michigan groups: Kneip Nature Phys 2010

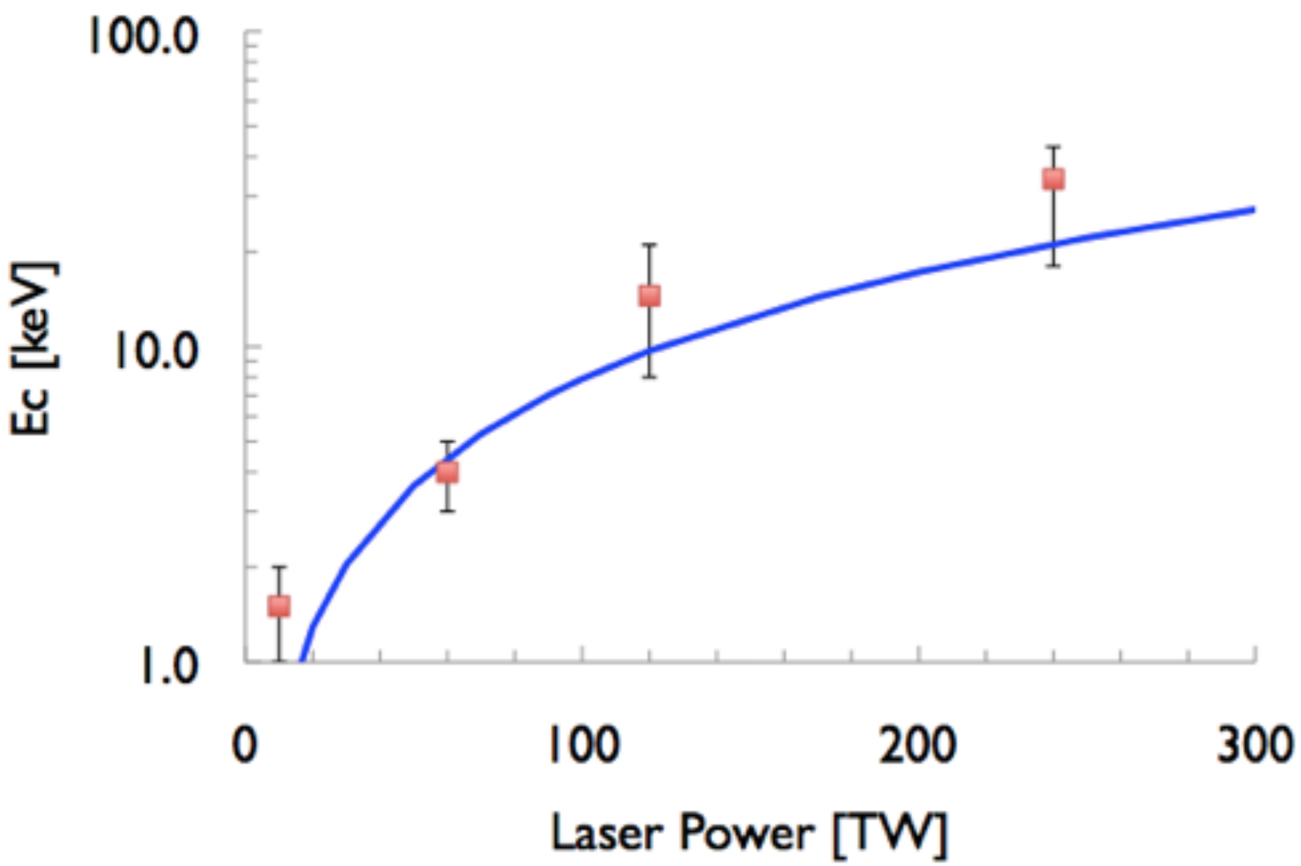
- higher laser power (70 TW)
- higher electron energies (400 MeV)
  - very small source (< 3 μm), very short duration (~ 30 fs)
  - x-rays at 10 keV

LWFA as wiggler radiation source



- Due to the small source size and short pulse, duration our x-ray source is very bright -
- peak brightness is comparable to conventional synchrotron (not average brightness though)

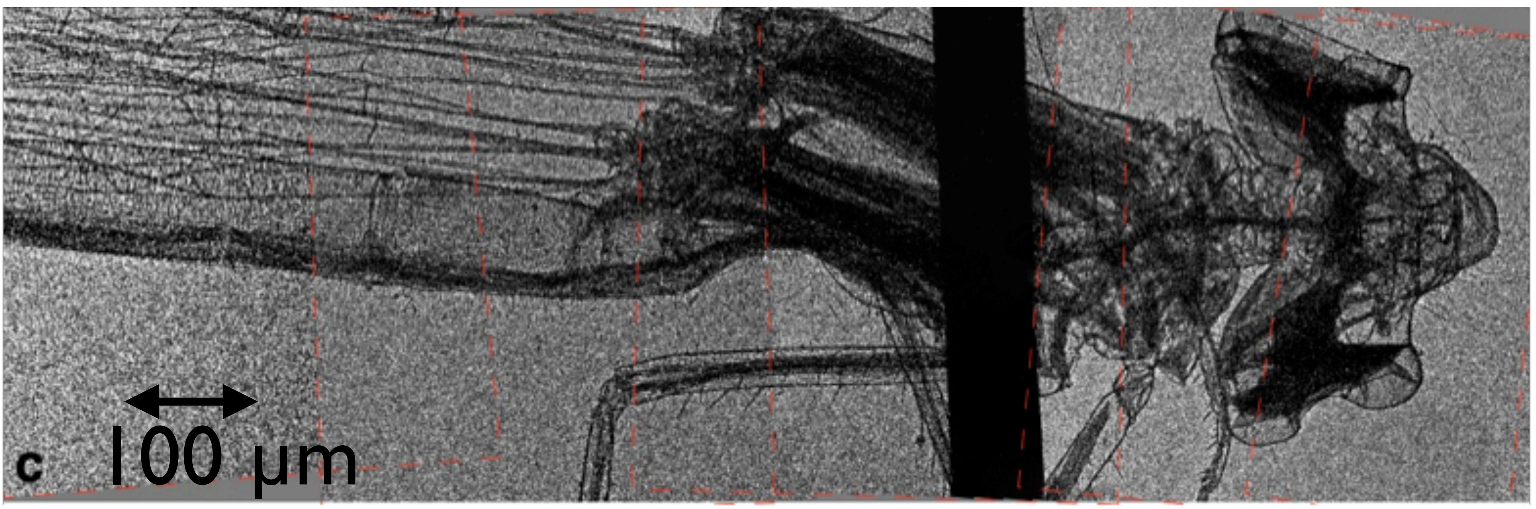
Scaling to higher photon energies



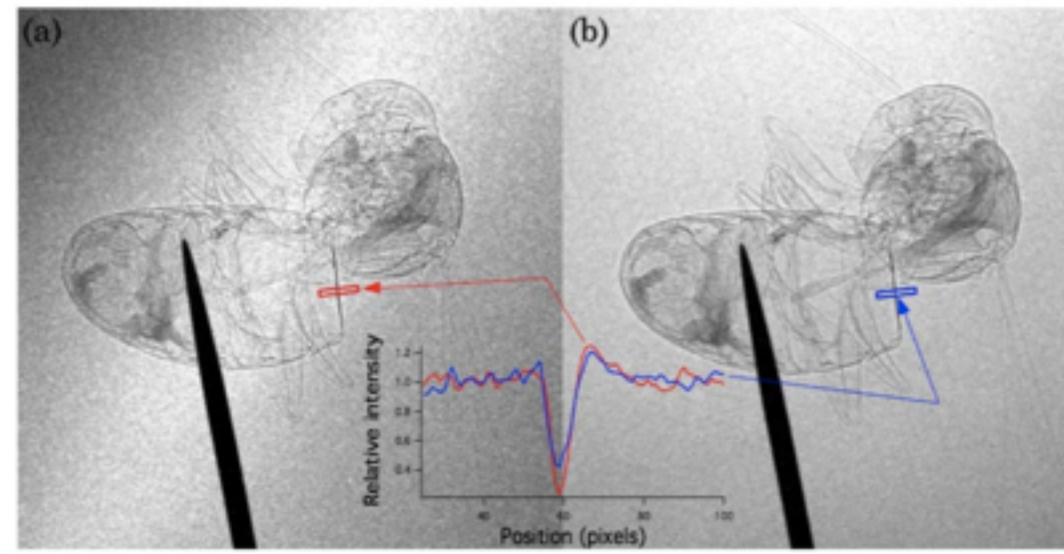
data from:
Mangles APL 2009 (Lund)
Kneip Nature Phys 2010 (Michigan)
Astra Gemini 2014 (to be published)

- x-ray radiation scales with laser power
- This is due to fact that we can accelerate higher energy electrons with higher power lasers

applications of betatron radiation

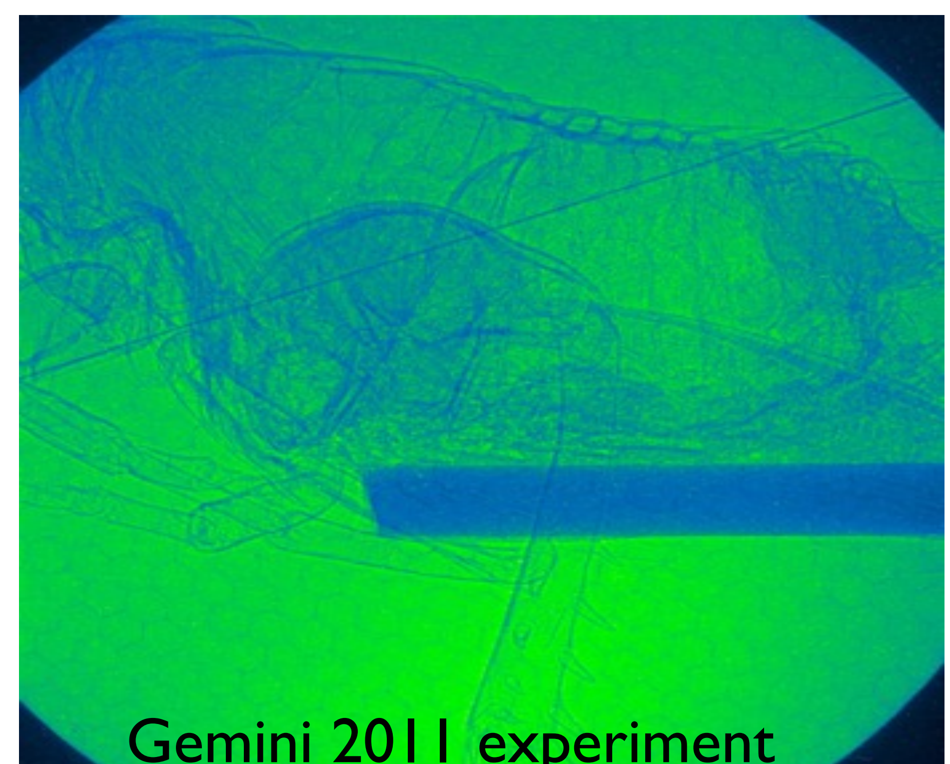


Kneip Applied Physics Letters 2011



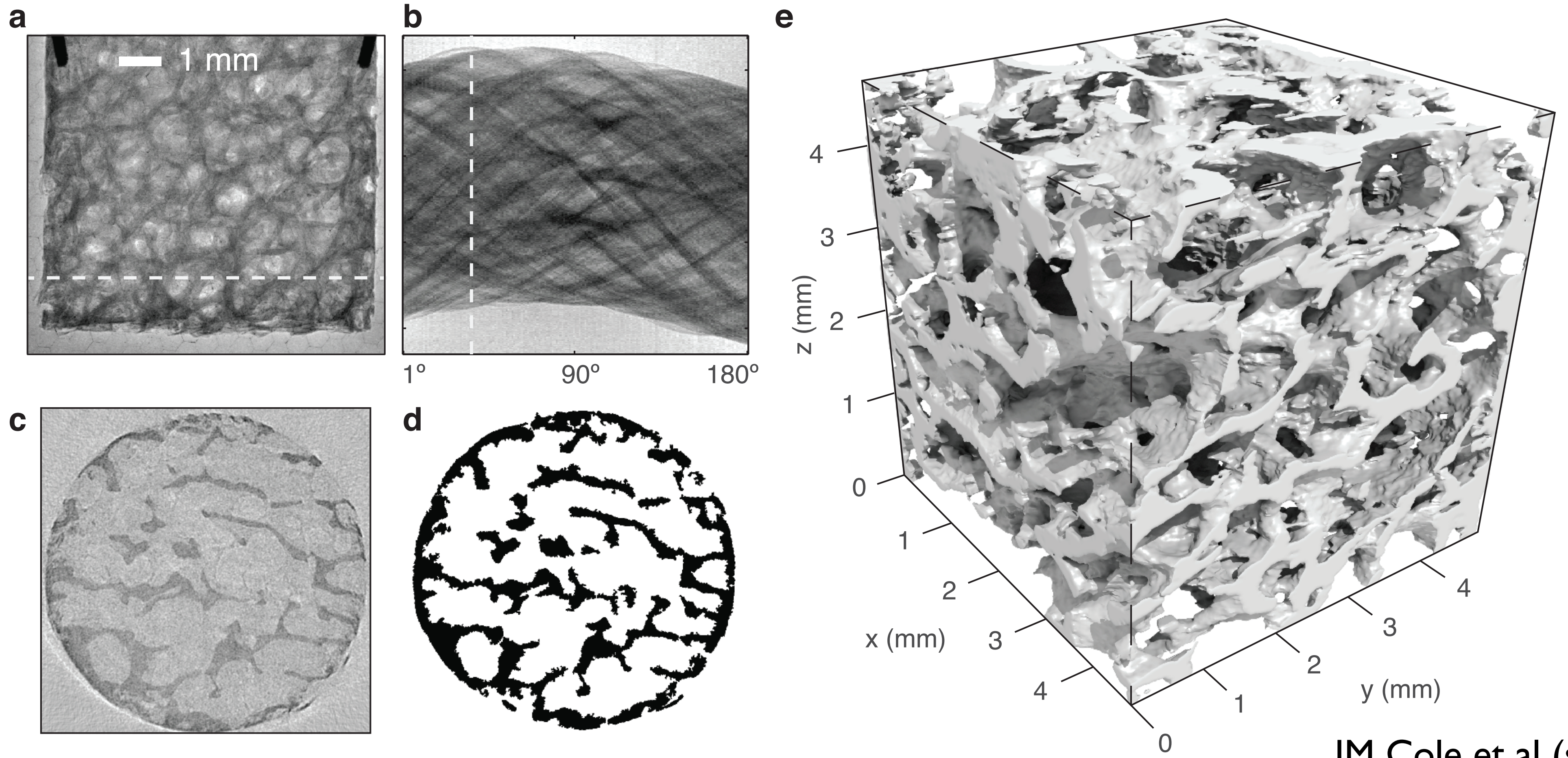
Fourmaux Optics Letters 2011

- high definition, high resolution imaging using phase contrast,
- possible because of the very small source size



Gemini 2011 experiment

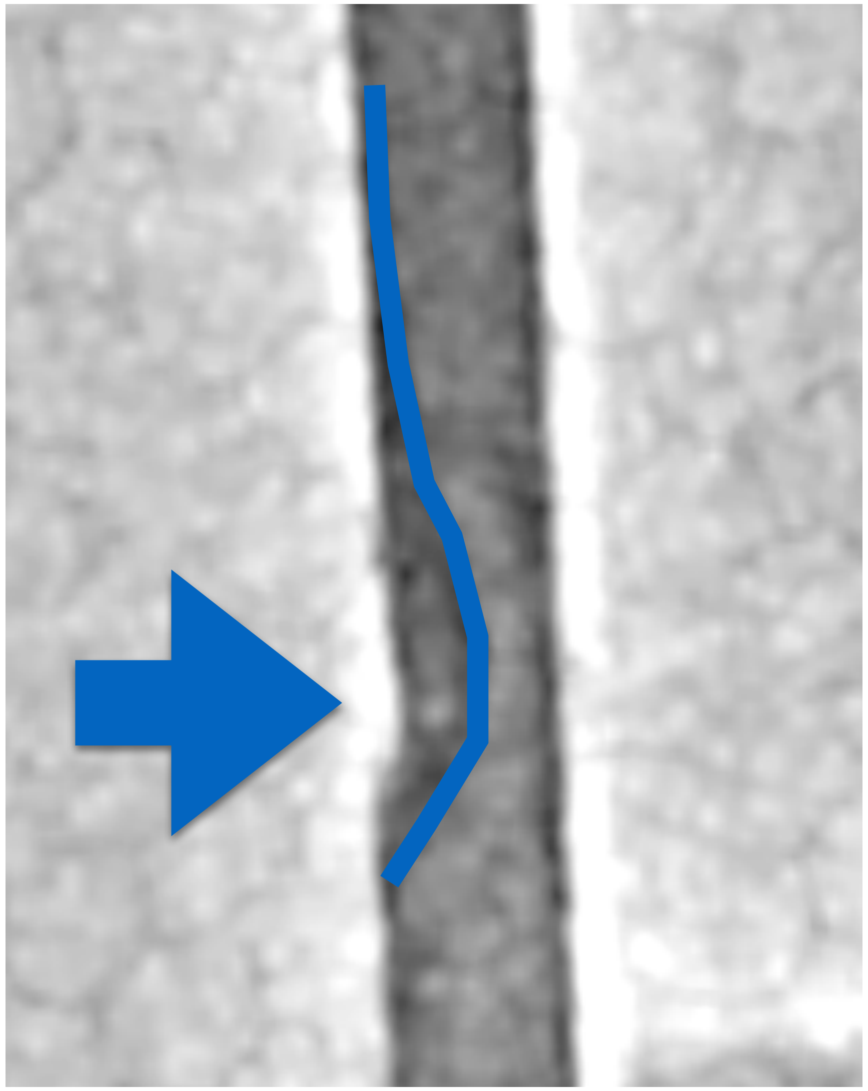
applications of betatron radiation



JM Cole et al (submitted) 2015

- Betatron sources now have properties needed to do useful medical imaging
  - e.g. tomography of human bone samples
    - high photon energy
    - small source size
    - reliability to take many shots per sample

applications of betatron radiation



- Ultra-short X-ray flash can be used to freeze rapid motion
- Direct imaging of shock waves travelling through matter (travelling at many km/s)
- Could help our understanding of the material properties inside stars and planets

Summary

- This lecture has covered:
  - introduction to laser wakefield acceleration
    - driving plasma waves with lasers
    - injecting electrons into plasma waves
  - introduction to x-ray generation in laser wakefield accelerators
    - undulators and wigglers
    - betatron radiation
- Any questions?

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