



Science and  
Technology  
Facilities Council

ISIS Neutron and  
Muon Source

# Accelerator Physics

## Lecture 8: Longitudinal Dynamics

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- Acceleration and Energy Gain
- Time Varying Fields
- The Synchrotron
- Energy Ramping
- Principle of Phase Stability
- Off-Energy Particles
- Longitudinal Dynamics
- Longitudinal Hamiltonian
- Small Amplitude Oscillations
- Synchrotron Oscillations
- Large Amplitude Oscillations
- Summary

## Acceleration and Energy Gain, 1

- To accelerate we require a force **in the direction of motion!**
- Newton-Lorentz force on a charged particle:

$$\vec{F} = \frac{d\vec{p}}{dt} = q \left( \vec{E} + \vec{B} \times \vec{v} \right) \quad (1)$$

- Second term is always perpendicular to motion: **no acceleration**
- Hence to accelerate along the direction of motion we need an electric field in that direction.

$$\frac{dp}{dt} = qE_z \quad (2)$$

## Acceleration and Energy Gain, 2

- In relativistic dynamics energy and momentum are linked,

$$E^2 = E_0^2 + p^2 c^2, \quad dE = v dp \quad (3)$$

- The rate of **energy gain per unit length** of acceleration is therefore,

$$\frac{dE}{dz} = v \frac{dp}{dz} = \frac{dp}{dt} = qE_z \quad (4)$$

- And the **kinetic energy gained** from the electric field along  $z$  is,

$$\begin{aligned} dW &= dE = qE_z dz \\ \therefore W &= q \int E_z dz = qV \end{aligned} \quad (5)$$

- Accelerator physics typically uses units of **electron volts** for energy.
- **1 eV (electron volt)** is the kinetic energy lost (or gained) by a particle of unit charge when accelerated from rest through a potential difference of one volt in vacuum.
- Some useful conversions:

	$1 \text{ eV}$	$1.602 \times 10^{-19} \text{ J}$	
	$1 \text{ eV}/c^2$	$1.783 \times 10^{-36} \text{ kg}$	
electron	$9.109 \times 10^{-31} \text{ kg}$		$0.511 \text{ MeV}/c^2$
proton	$1.673 \times 10^{-27} \text{ kg}$		$938.272 \text{ MeV}/c^2$

## Methods of Acceleration

- Electrostatic fields are limited by insulation problems and magnetic fields don't accelerate
- **For circular machines DC acceleration is impossible** as  $\oint \vec{E} \cdot d\vec{s} = 0$
- From Maxwells equations,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \oint \vec{E} \cdot d\vec{s} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A} \quad (6)$$

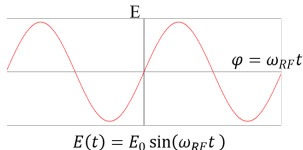
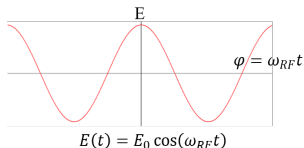
a **time varying magnetic field generates an electric field.**

- Therefore, use time varying fields which for most accelerator applications are at RF frequencies.



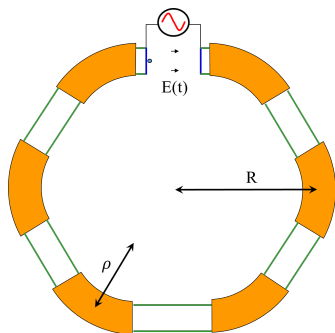
## Phase Conventions

1. For **linear accelerators**, the origin of time is taken as the **positive crest** of the RF voltage.
2. For **circular accelerators**, the origin of time is taken as the **positive gradient zero crossing** of the RF voltage.



3. We will stick to the circular accelerator convention.

## The Synchrotron, 1



- Constant orbit during acceleration
- Revolution frequency increases with energy
- RF cavity frequency increases with energy
- Magnetic field strength increases to maintain orbit radius
- Synchronism condition:

$$T = hT_{RF} = \frac{2\pi R}{v} \quad (7)$$



The **synchrotron** is so called because the accelerating RF cavities and the magnetic fields all have to work in synchronism in order for it to work. There is a **synchronous RF phase** for which the energy gain is precisely what is required to match the increase in magnetic field each turn. This implies the following conditions:

- Energy gain per turn,  $\Delta E_{\text{turn}} = eV \sin \phi_s$
- Synchronous particle
- RF synchronism  $\omega_{\text{RF}} = h\omega$
- Constant orbit
- $B\rho = p/e$ , implying a varying magnetic field

## Examples of Synchrotrons



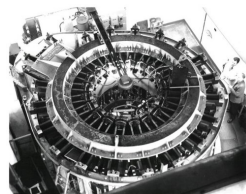
ISIS Spallation Source, UK



LHC, CERN, Switzerland



Diamond Light Sources, UK



Glasgow Synchrotron, UK

The momentum and magnetic field must increase following the magnetic rigidity equation

$$p = eB\rho \quad \Rightarrow \quad \frac{dp}{dt} = e\rho \frac{dB}{dt} \quad (8)$$

$$\begin{aligned} \Delta p_{\text{turn}} &= e\rho \frac{dB}{dt} T \\ &= e\rho \frac{dB}{dt} \frac{2\pi R}{v} \end{aligned} \quad (9)$$

From equation 3 we have  $dE = v dp$ , therefore

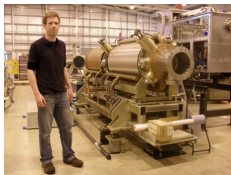
$$\begin{aligned} \Delta E_{\text{turn}} &= v \Delta p_{\text{turn}} \\ &= 2\pi R e \rho \frac{dB}{dt} \end{aligned} \quad (10)$$

## RF Acceleration

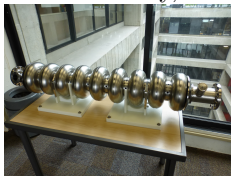
The energy gain is provided by the RF voltage,

$$\Delta E_{\text{turn}} = 2\pi R e \rho \frac{dB}{dt} = eV \sin \phi_s \quad (11)$$

$$\phi_s = \arcsin \left( 2\pi R \rho \frac{\dot{B}}{V} \right) \quad (12)$$



ISIS RF cavity, h=2



9 cell SC cavity for ILC

where  $\phi_s$  = **synchronous phase**. Each **synchronous particle** satisfies the rigidity equation (eqn 8). They have the nominal energy and follow the nominal trajectory.

- **Acceleration increases the revolution frequency**, so the RF frequency has to follow

$$f = \frac{f_{RF}(t)}{h} = \frac{v(t)}{2\pi R} = \frac{1}{2\pi R} \frac{p(t)c^2}{E(t)} = \frac{1}{2\pi R} \frac{ec^2\rho B(t)}{E(t)} \quad (13)$$

- Using the relativistic equation  $E^2 = (m_0c^2)^2 + p^2c^2$  we find the RF frequency must **follow the magnetic field** with

$$\frac{f_{RF}(t)}{h} = \frac{c}{2\pi R} \sqrt{\frac{B(t)^2}{B(t)^2 + (m_0c^2/ec\rho)^2}} \quad (14)$$

- When  $B$  becomes large in comparison to  $m_0c^2/ec\rho$  (corresponding to  $v \rightarrow c$ ) the frequency tends to  $c/2\pi R$

## Revolution Frequency Increase

We've seen that the revolution and RF frequency change during acceleration depending on the particle type and the magnetic field ramp. This is more **important at lower energies and for heavier particles.**

<b>PSB</b>	50 MeV - 1.4 GeV	602 kHz - 1746 kHz	190%
<b>PS</b>	1.4 GeV - 25.4 GeV	437 kHz - 477 kHz	9%
<b>SPS</b>	25.4 GeV - 450 GeV	43.45 kHz - 43.478 kHz	0.06%
<b>LHC</b>	450 GeV - 7 TeV	11.245 kHz	$2 \times 10^{-6}$

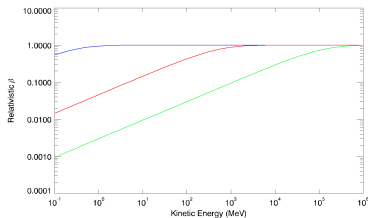
In lower energy circular accelerators the RF system needs more flexibility.

# Particle Types and Acceleration

The specific accelerating technology depends upon the evolution of the particle velocity

- **Electrons** reach a constant velocity ( $\sim c$ ) at low energy
- **Protons** and heavy ions require much more energy to reach a constant velocity
- RF resonators will be optimised for different velocities/frequencies
- Magnetic field follows the momentum increase

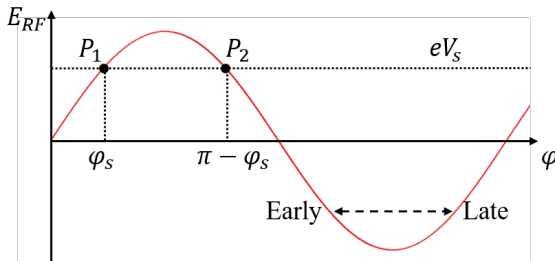
$$E = \gamma m_0 c^2, \gamma = \frac{E}{E_0} = \frac{1}{\sqrt{1 - \beta^2}}$$



**Electron**, 0.511 MeV; **Proton**, 938 MeV;  
**Uranium-238**, 222 GeV

## Phase stability in a Linac - 1

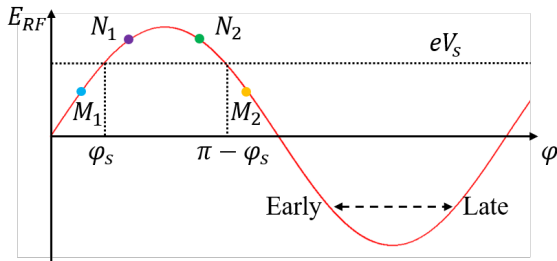
- Consider a series of gaps, operating in the  $2\pi$  mode
- $2\pi$  mode implies  $\vec{E}$  is the same in all gaps at any given time
- $eV_s = eV \sin \phi_s$ , the energy gain required for a particle to reach the next gap with the same RF phase:  $P_1, P_2$



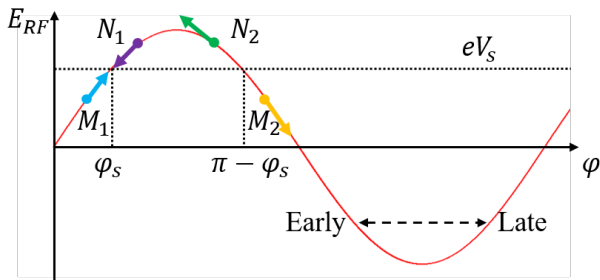


## Phase stability in a Linac - 2

- Consider a series of gaps, operating in the  $2\pi$  mode
- $2\pi$  mode implies  $\vec{E}$  is the same in all gaps at any given time
- $eV_s = eV \sin \phi_s$ , the energy gain required for a particle to reach the next gap with the same RF phase:  $P_1, P_2$



## Phase stability in a Linac - 3



- With increasing energy comes an increase in velocity
- $M_1$  and  $N_1$  move toward the synchronism  $\Rightarrow$  **STABLE**
- $M_2$  and  $N_2$  move away from synchronism  $\Rightarrow$  **UNSTABLE**
- **N.B. Ultra-relativistic particles no longer gain velocity**

## Off-Energy Particles

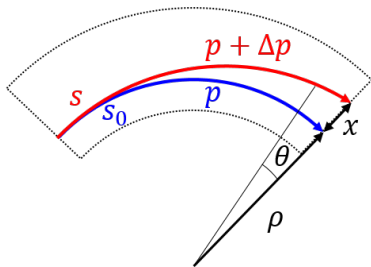
If a particle is slightly **off the design momentum** it will have a **different orbit**.

- Path length of an orbit displaced by  $x$

$$ds_0 = \rho d\theta \quad ds = (\rho + x)d\theta$$

- Relative difference in path length ( $D_x = \text{dispersion}$ )

$$\frac{dl}{ds_0} = \frac{ds - ds_0}{ds_0} = \frac{x}{\rho} = \frac{D_x}{\rho} \frac{dp}{p}$$



## Momentum Compaction

- Integrating leads to the total path length change

$$\Delta C = \oint dl = \oint \frac{x}{\rho(s_0)} ds_0 = \oint \frac{D_x(s_0)}{\rho(s_0)} \frac{dp}{p} ds_0 \quad (15)$$

note that since  $D_x$  is usually positive the total path length increases for higher energy particles.

- Momentum compaction factor**,  $\alpha_c$  is defined as

$$\alpha_c \equiv \frac{dL/L}{dp/p} = \frac{1}{L} \oint \frac{D_x(s_0)}{\rho(s_0)} ds_0 \approx \frac{1}{C} \sum_i \langle D_x \rangle_i \theta_i \quad (16)$$

where  $\langle D_x \rangle_i$  and  $\theta_i$  are the average dispersion and the bending angle of the  $i^{\text{th}}$  dipole.

- **Off-momentum particles** have different revolution frequencies to on-momentum particles due to different orbit lengths and velocities

$$f_r = \frac{\beta c}{2\pi R} \quad \Rightarrow \quad \frac{df_r}{f_r} = \frac{d\beta}{\beta} - \frac{dR}{R} = \frac{d\beta}{\beta} - \alpha_c \frac{dp}{p} \quad (17)$$

- Calculate  $d\beta/\beta$  as a function of  $dp/p$

$$p = \gamma m_0 \beta c \quad \Rightarrow \quad \frac{dp}{p} = \frac{d\beta}{\beta} + \frac{d\gamma}{\gamma} = (1 - \beta^2)^{-1} \frac{d\beta}{\beta} = \gamma^2 \frac{d\beta}{\beta} \quad (18)$$

- Putting these two equations together (eqns 17 and 18) we get the relative change in revolution frequency

$$\frac{df_r}{f_r} = \left( \frac{1}{\gamma^2} - \alpha_c \right) \frac{dp}{p} = \eta \frac{dp}{p} \quad (19)$$

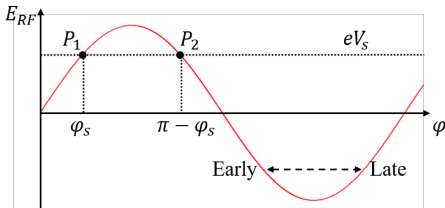
where  $\eta = \gamma^{-2} - \alpha_c = \gamma^{-2} - \gamma_t^{-2}$  is the **slip factor**

- Annoyingly in some references  $\eta$  is defined with a minus sign so **be careful!**
- Transition energy** is when  $\gamma = \gamma_t = \alpha_c^{-1/2}$  and  $\eta = 0$ . At this energy the revolution frequency is independent of momentum deviation.
- Below transition** a higher momentum particle has a higher  $f_r$  than the synchronous particle, **above transition** the converse is true.

## Phase stability in a Synchrotron - 1

The definition of the slip factor,  $\eta$  (equation 19), an increase in momentum:

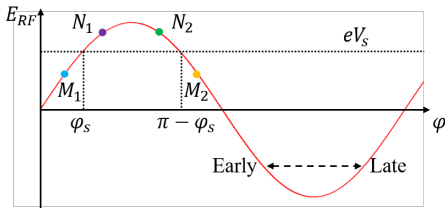
- **Below transition** ( $\eta > 0 \Rightarrow \gamma < \gamma_t$ ) gives a **higher revolution frequency** (increase in velocity dominated)
- **Above transition** ( $\eta < 0 \Rightarrow \gamma > \gamma_t$ ) gives a **lower revolution frequency** as  $v \approx c$  and a longer path (momentum compaction dominated)



## Phase stability in a Synchrotron - 2

The definition of the slip factor,  $\eta$  (equation 19), an increase in momentum:

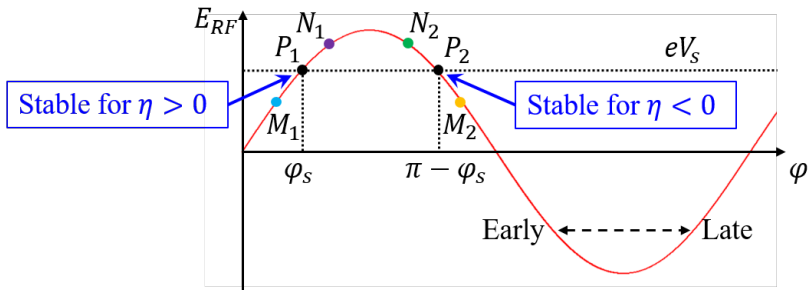
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## Phase stability in a Synchrotron - 3

$$\eta = \frac{1}{\gamma^2} - \alpha_c = \frac{1}{\gamma^2} - \frac{1}{\gamma_t^2}$$



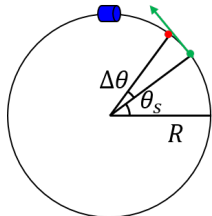
- The acceleration of charged particles in circular machines involves the coupled variables of energy and phase. The dynamics is often referred to as **synchrotron motion**.
- As there is a well defined **synchronous particle** ( $\phi_s, E_s$ ) it is best to consider particle coordinates with respect to that particle.
- Therefore we introduce a series of reduced variables:

$$\begin{aligned}\Delta E &= E - E_s, & \text{particle energy} \\ \Delta p &= p - p_s, & \text{particle momentum} \\ \Delta \phi &= \phi - \phi_s, & \text{particle RF phase} \\ \Delta \theta &= \theta - \theta_s, & \text{azimuthal angle} \\ \Delta f_r &= f_r - f_{r,s}, & \text{revolution frequency}\end{aligned}$$

## First Energy-Phase Equation

The RF phase coordinate is related to the azimuth by  $\Delta\phi = \phi - \phi_s = -h\Delta\theta$ , or

$$\Delta\omega = \frac{d\Delta\theta}{dt} = -\frac{1}{h} \frac{d\Delta\phi}{dt} = -\frac{1}{h} \frac{d\phi}{dt} \quad (20)$$



From the definition of the slip factor (equation 19) and the relation between energy and momentum (equation 3) we get the **first energy phase equation**:

$$\frac{d\phi}{dt} = -h\omega_r\eta \frac{dp}{p} = -\frac{h\omega_r^2\eta}{\beta^2 E} \left( \frac{\Delta E}{\omega_r} \right) \quad (21)$$

## Second Energy-Phase Equation - 1

- The energy gain per turn has already been defined as  $\Delta E_{\text{turn}} = eV \sin \phi_s$  (equation 11).
- So the rate of energy gain is  $\dot{E} = f_r eV \sin \phi$
- The rate of relative energy change with respect to the synchronous particle is

$$\Delta \left( \frac{\dot{E}}{\omega_r} \right) = \frac{eV}{2\pi} (\sin \phi - \sin \phi_s) \quad (22)$$

- Expanding the L.H.S. to first order

$$\Delta(\dot{E}T_r) \cong \dot{E}\Delta T_r + T_{r,s}\Delta\dot{E} = \Delta E\dot{T}_r + T_{r,s}\Delta\dot{E} = \frac{d(T_{r,s}\Delta E)}{dt} \quad (23)$$

## Second Energy-Phase Equation - 2

- This leads to the **second energy-phase equation**

$$\frac{d}{dt} \left( \frac{\Delta E}{\omega_r} \right) = \frac{eV}{2\pi} (\sin \phi - \sin \phi_s) \quad (24)$$

- Combining these two equations leads to the **longitudinal equation of motion**

$$\frac{d}{dt} \left( \frac{\beta^2 E}{h\eta\omega_r^2} \frac{d\phi}{dt} \right) + \frac{eV}{2\pi} (\sin \phi - \sin \phi_s) = 0 \quad (25)$$

- This second order differential equation is non-linear. Also, the parameters within the bracket (in general) vary slowly in time

## Longitudinal Hamiltonian

- The two energy-phase equations can also be derived from a **Hamiltonian**,  $\mathcal{H}$  (the total energy in the system) in canonical variables  $(\phi, W)$  ( $W = \Delta E/\omega_r$ )

$$\mathcal{H}(\phi, W) = \frac{h\omega_0^2\eta}{2\beta^2 E_s} W^2 + \frac{q}{2\pi} U(\phi) \quad (26)$$

where  $U(\phi) = \int_{\phi_s}^{\phi} [V(\phi') - V(\phi_s)] d\phi'$  is the potential energy

- The two energy-phase equations are then derived from the Hamiltonian by

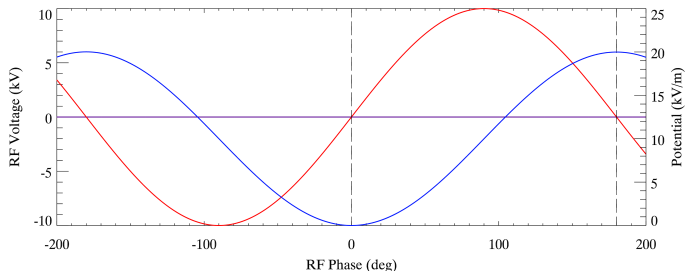
$$\frac{dW}{dt} = -\frac{\partial\mathcal{H}(\phi, W)}{\partial\phi} \quad \frac{d\phi}{dt} = \frac{\partial\mathcal{H}(\phi, W)}{\partial W} \quad (27)$$

## Single Harmonic RF - 1

Let's take the simple example we've been working with, single harmonic RF with  $V(\phi) = V_1 \sin \phi$

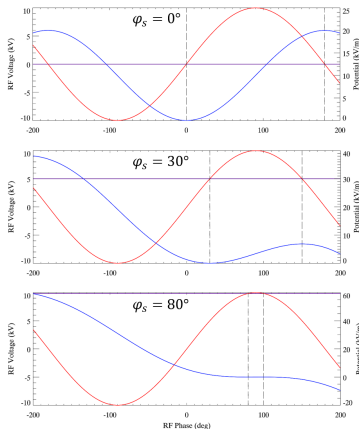
- The potential is:

$$U(\phi) = V_1 [\cos \phi_s - \cos \phi - (\phi - \phi_s) \sin \phi_s] \quad (28)$$



## Single Harmonic RF - 2

- What we have is a **potential well** created by the RF cavity voltage
- As the synchronous phase changes, or the amount of acceleration required to maintain synchronism changes, the shape of the well changes
- What does the Hamiltonian look like?

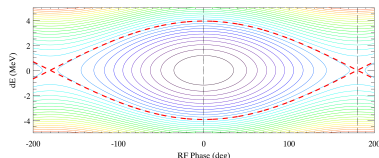
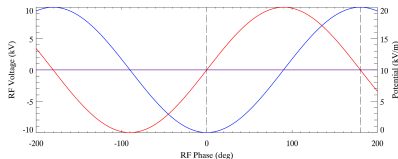




## Single Harmonic RF - 3

$$\mathcal{H}(\phi, W) = \frac{h\omega_0^2\eta}{2\beta^2 E_s} W^2 + \frac{qV_1}{2\pi} [\cos \phi_s - \cos \phi - (\phi - \phi_s) \sin \phi_s] \quad (29)$$

- How does this help?
- Contours of constant  $\mathcal{H}$  are **particle trajectories**,  $\mathcal{H}$  is conserved
- Let's consider some particles near to  $\phi_s \dots$



## Small Amplitude Oscillations - 1

Rearranging the longitudinal EOM (eqn 25) assuming constant  $\beta$ ,  $E$ ,  $\omega_r$  and  $\eta$ :

$$\ddot{\phi} + \frac{\Omega_s^2}{\cos \phi_s} (\sin \phi - \sin \phi_s) = 0 \quad \text{with} \quad \Omega_s^2 = \frac{h\eta\omega_r^2 eV \cos \phi_s}{2\pi\beta^2 E_s} \quad (30)$$

- Consider **small deviations** in phase from reference

$$\begin{aligned} \sin \phi - \sin \phi_s &= \sin(\phi_s + \Delta\phi) - \sin \phi_s \\ &\cong \Delta\phi \cos \phi_s \end{aligned} \quad (31)$$

- Thereby reducing the motion to a **harmonic oscillation**

$$\ddot{\phi} + \Omega_s^2 \Delta\phi = 0 \quad (32)$$

where  $\Omega_s$  is the **synchrotron angular frequency**

## Small Amplitude Oscillations - 2

- The **synchrotron tune**  $Q_s = \Omega_s/\omega_r$  is the number of synchrotron oscillations per revolution
- Typical values are  $\ll 1$ ,  $10^{-3}$  for proton synchrotrons and  $10^{-1}$  for electron storage rings
- It also reveals a **stability condition** for  $\phi_s$  as

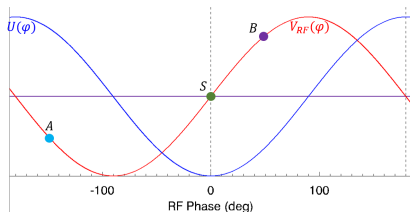
$$\Omega_s^2 > 0 \quad \Rightarrow \quad \eta \cos \phi_s > 0 \quad (33)$$

$$\gamma < \gamma_t \quad \eta > 0 \quad 0 < \phi_s < \pi/2$$

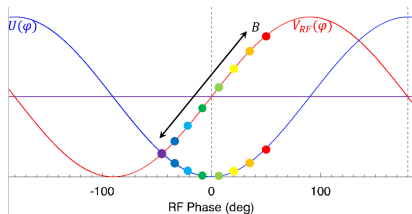
$$\gamma > \gamma_t \quad \eta < 0 \quad \pi/2 < \phi_s < \pi$$

## Synchrotron Oscillations - 1

- Consider the simple case of no acceleration ( $\phi_s = 0$ ), below transition ( $\gamma < \gamma_t$ )
- **Particle S** is synchronous
- **Particle A** is decelerated,  $f_r$  decreases so it arrives later (i.e. moves toward S)
- **Particle B** is accelerated,  $f_r$  increases so it arrives earlier (moves toward S)



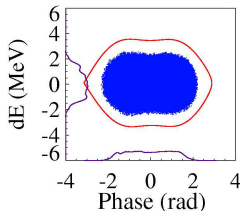
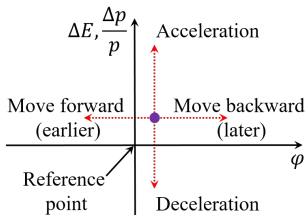
## Synchrotron Oscillations - 2



- The particle oscillates around the synchronous phase, so-called **synchrotron oscillations**
- The amplitude depends on the initial phase and energy
- **Synchrotron frequency** is much slower than the transverse (usually multiple revolutions per oscillation)
- The restoring force from the RF electric field is much smaller than the quadrupolar magnetic field

# Longitudinal Phase Space

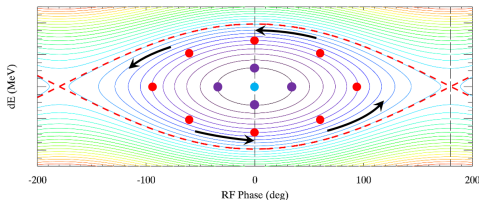
The energy-phase oscillations can also be observed in the longitudinal phase space we saw with the Hamiltonian



The particle trajectory in phase space describes the longitudinal motion.

**Longitudinal emittance** is the phase space area including all the particles

# Longitudinal Phase Space Oscillations



- Particles follow Hamiltonian contours oscillating around the synchronous point  $(\phi_s, E_s)$
- Energy is exchanged for RF phase like exchanges between kinetic and potential energy
- This is called **synchrotron motion**



These are all for below transition

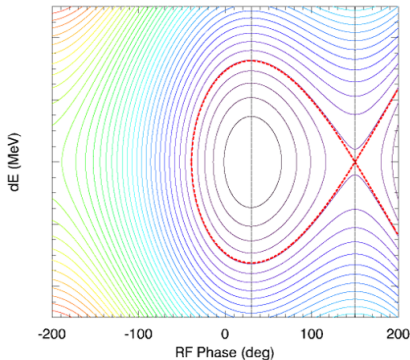
$\phi_s = 0^\circ$ , phase distribution

$\phi_s = 0^\circ$ , energy distribution



# Large Amplitude Oscillations

- When  $\Delta\phi$  is large the EOM is non-linear
- Move from elliptical orbits to hyperbolic close to UFP
- Can use Hamiltonian to calculate the separatrix

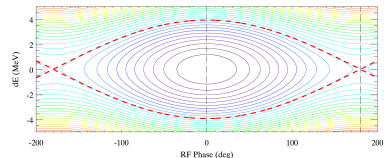
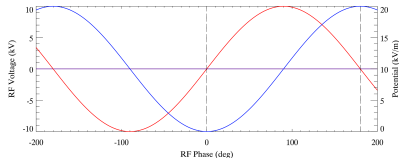


- First find the co-ordinate of UFP from  $\frac{dU}{d\phi}$ ,  $\phi = \pi - \phi_s$
- Calculate Hamiltonian of the separatrix from equation (29)

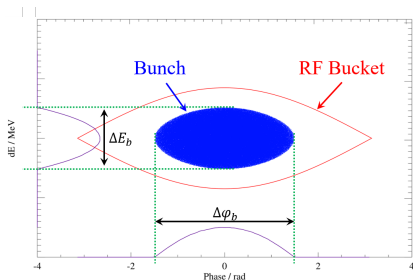
$$\begin{aligned}\mathcal{H}_{\text{sep}} &= \frac{qV_1}{2\pi} [\cos \phi_s - \cos(\pi - \phi_s) - (\pi - 2\phi_s) \sin \phi_s] \\ &= \frac{qV_1}{2\pi} [2 \cos \phi_s - (\pi - 2\phi_s) \sin \phi_s]\end{aligned}\quad (34)$$

- Put back into the Hamiltonian to get separatrix equation

$$\Delta E_{\text{sep}} = \sqrt{\frac{qV\beta^2 E_s}{\pi h \eta} [\cos \phi_s + \cos \phi + (\phi - \pi + \phi_s) \sin \phi_s]}\quad (35)$$



- Restoring force is non-linear  $\Rightarrow$  speed depends on  $(\phi, \Delta E)$
- Two **fixed points**, unstable and stable
- Two clear regions (**libration and rotation**) separated by the **separatrix** passing through the **UFP**, at the maximum of  $U(\phi)$
- Oscillatory motion around the **SFP**, at the minimum of  $U(\phi)$
- Rotary motion beyond the separatrix, the **RF bucket**



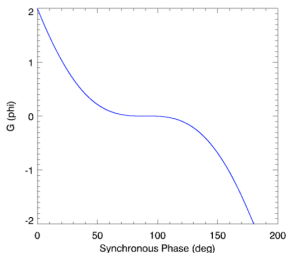
- Bunches of particles fill only a portion of the bucket area
- RF bucket area = **longitudinal acceptance** in units of **eVs**
- Bunch area = **longitudinal emittance**  
 $= 4\pi\sigma_{\Delta E}\sigma_{\Delta t}$
- **N.B. References can use different definitions for emittances!**

## Energy Acceptance

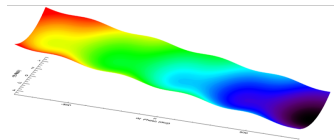
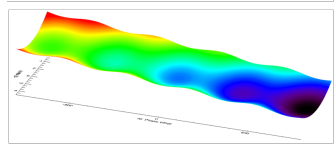
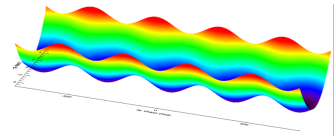
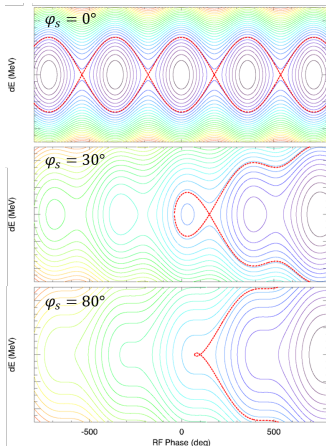
- It is clear the separatrix has a maximum at  $\phi = \phi_s$
- RF bucket height also referred to as **energy acceptance**

$$\left(\frac{\Delta E}{E_s}\right)_{\max} = \sqrt{\frac{qV\beta^2}{\pi h\eta E_s} [2\cos\phi_s + (2\phi_s - \pi)\sin\phi_s]} \quad (36)$$

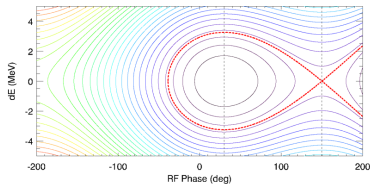
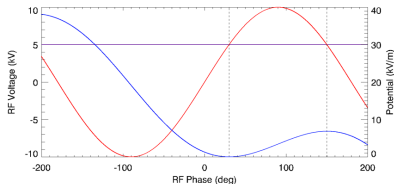
- It depends strongly on  $\phi_s$
- It becomes smaller when  $\phi_s$  is changing during acceleration
- A **higher voltage**  $\Rightarrow$  **larger acceptance**
- For **higher  $h$**  the same voltage produces a **smaller acceptance**



# Accelerating Bucket - 1



## Accelerating Bucket - 2



Examples (all below transition)

$\phi_s = 30^\circ$ , phase distribution

$\phi_s = 30^\circ$ , energy distribution

- Motion still divided into two clear regions
- Stable area (RF bucket) reduces in size

- How to accelerate charged particles ...
- What makes a synchrotron a synchrotron ...
- Momentum compaction, slip factors, dispersion, ...
- Transition, phase stability, synchronous phase, ...
- Deriving the equation of motion, ...
- $\mathcal{H}$ amiltonians, potentials, fixed points, ...
- RF buckets, separatrices, emittance, synchrotron tune, ...
- Longitudinal acceptance, energy acceptance, ...
- What next?





- Frank Tecker, **Longitudinal Beam Dynamics in Circular Accelerators**, CERN Accelerator School 2018, <https://cas.web.cern.ch/schools/constanta-2018>
- Edward J.N. Wilson, **An Introduction to Particle Accelerators**, Oxford University Press, 2001
- S.Y. Lee, **Accelerator Physics**, 2<sup>nd</sup> edition, World Scientific, 2007
- H. Wiedemann, **Particle Accelerator Physics I**, 2<sup>nd</sup> edition, Springer, 2003
- A. Chao, K.H. Mess, M. Tigner, F. Zimmerman, **Handbook of Accelerator Physics and Engineering**, 2<sup>nd</sup> edition, World Scientific, 2013
- Mario Conte, William W MacKay, **An Introduction to the Physics of Particle Accelerators**, 2<sup>nd</sup> edition, World Scientific, 2008
- Klaus Wille, **The Physics of Particle Accelerators: An Introduction**, Oxford University Press, 2005
- D.A. Edwards, M.J. Syphers, **An Introduction to the Physics of High Energy Accelerators**, Wiley-VCH, 2004
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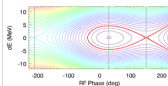
ISIS Neutron and  
Muon Source

Spare Slides

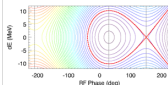
- Transition  $\Rightarrow$  velocity change and path length change compensate
- $f_r$  is independent from the momentum offset
- Crossing transition makes the previous  $\phi_s$  unstable
- RF needs to rapidly change its phase called a **phase jump**
- For example, PS (1.4 - 25.4 GeV) crosses transition at 6 GeV

## Crossing Transition

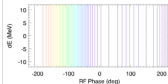
$$\eta = 0.5$$



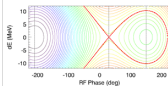
$$\eta = 0.1$$



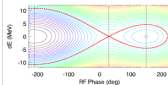
$$\eta = 0$$



$$\eta = -0.1$$

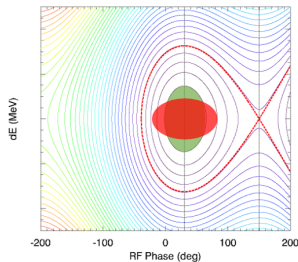


$$\eta = -0.5$$



## Beam Matching - 1

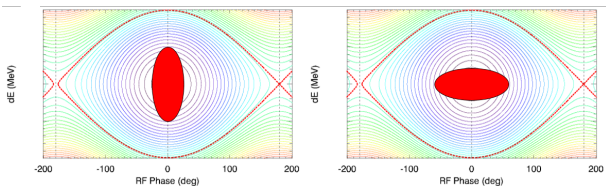
- How well can we confine and control particles?
- Want to put them within a given  $\mathcal{H}$  contour and keep them there
- Make the density a function of  $\mathcal{H}$  and it will conform to the contours in phase space
- It forms a **stationary distribution** (time independent) assuming  $\mathcal{H}$  is time independent or adiabatically varying



- Matched
- Unmatched

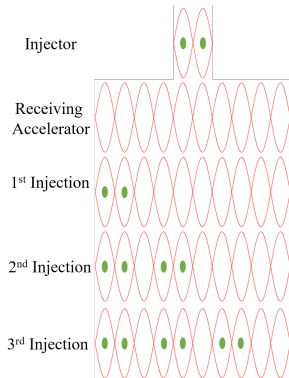
## Effect of a Mismatch

- Consider a short bunch with a large energy spread
- After a quarter synchrotron oscillation: a long bunch with a small energy spread

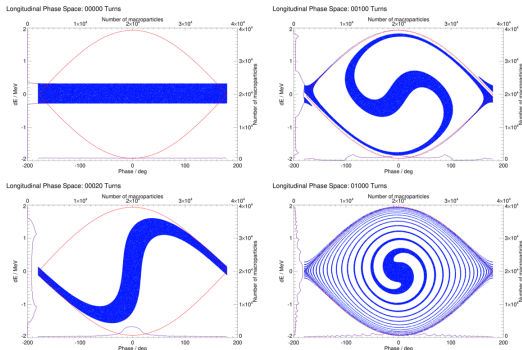


- For larger amplitudes the synchrotron motion is slower which leads to filamentation and emittance growth and possible beam loss
- [Matched example](#), [Mismatched example](#), [Phase error](#)

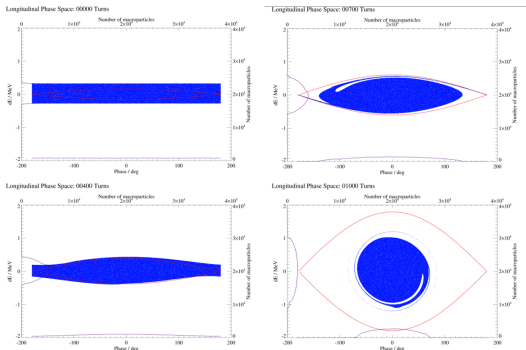
- Where are you injecting from?
- **Bunch to bucket transfer**  $\Rightarrow$   
match bunch from bucket of  
previous accelerator to next
- Particles always subject to  
longitudinal focusing
- Time structure of beam preserved
- No need for bunch capture,  
adiabatic or otherwise



- A debunched beam can be captured by the synchrotron RF
- With constant RF volts during injection the beam **filaments** to fill the bucket



- Increase the RF volts **adiabatically** (i.e. slowly with respect to the synchrotron motion)
- **Capture** a large portion in a relatively small emittance





- Another important factor for intense low/medium energy hadron beams is the effect of **space charge**
- The self field of the beam then becomes significant compared to the RF focusing field
- Depends on the beam distribution and, in general, is a time varying addition to the potential
- Makes analytical solutions, and understanding intense beams difficult!
- Look for stationary distributions **WITH space charge**
- Useful stationary distribution found by Hofmann and Pedersen