Tutorial questions on synchrotron radiation

For the tutorial, below are four questions on synchrotron radiation. Each question relates to one of the four lectures, and in each case consists of several parts. The theory needed to answer the questions should be contained within the relevant lecture notes, however, please let me know if you have problems.

For those of you who are required to submit answers for marking, please email me your solutions to ian.martin@diamond.ac.uk before the start of the tutorial (i.e. before 14:00 on 30/11/2023).

Ian Martin (23/11/2023)

Q1) Consider an electron storage ring, with momentum 2.5 GeV/c, average current 400 mA and a bend radius of $\rho = 8.0$ m.

a) What is the required magnetic field in the dipole to give this bend radius [2 mark]?
b) What are the instantaneous radiated power and energy loss per turn for a single electron travelling through the bending magnet [2 marks]?
c) What is the average power lost by the electron beam that must be replaced by the RF cavities [2 marks]?
d) What is the critical photon energy and the critical angle at that energy for the bending magnet radiation [3 marks]?
e) What would be the energy loss per turn if the ring contained protons instead of electrons [2 marks]?

Q2) An undulator of length L = 1.76 m, period λ_u = 22.0 mm and peak-field B_0 = 0.95 T is installed into a 3.0 GeV electron storage ring.

a) Starting from the Lorentz Force and assuming a magnetic field variation of the form $B_y(z) = B_0 \sin(k_u z)$, use the electron equations of motion to show that the fundamental, on-axis resonant wavelength of the radiation will be $\lambda_r = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$, where $K = \frac{eB_0\lambda_u}{2\pi m_e c}$ [5 marks]
b) Calculate the corresponding on-axis photon energy and linewidth for the fifth harmonic [3 marks]
c) Calculate the resonant wavelength (first harmonic) at an angle of 100 µrad [1 mark]
d) Calculate the vertical tune-shift assuming a 2.0 m average vertical beta function at the insertion device [2 marks].

Q3) Consider a 3.5 GeV electron storage ring with circumference 600 m and bending magnets with 0.75 T field strength.

a) Starting from the synchrotron radiation integrals, what is the energy loss per turn of the ring [3 marks]?
b) Assuming the horizontal and vertical damping partition numbers are equal, what are the horizontal, vertical and longitudinal damping times of the ring [3 marks]?
c) The ring is found to suffer from a vertical coupled-bunch instability with an un-damped rise time of 15 ms. If the beam is to remain naturally stable, what would the minimum energy loss per turn need to be [2 marks]?
d) If the additional energy loss per turn is to be provided by a damping wiggler, what does the additional contribution to the 2nd synchrotron radiation integral (ΔI_2) need to be [3 marks]?
e) Assuming 2.5 m of space is available for the damping wiggler, what peak field is required [3 marks]?

Q4) For a 2.4 GeV triple-bend achromat electron storage ring consisting of 20 TBA cells and with a dipole field of 1.0 Tesla, calculate the following:

a) Instantaneous power radiated by a single electron [2 marks]
b) Mean photon energy [3 marks]
c) Total number of photons emitted per second, on average. [2 marks]
d) Equilibrium relative energy spread for the electron beam (assuming the longitudinal damping partition number $J_\epsilon = 2$) [4 marks]
e) The theoretical minimum emittance assuming $J_x = 1$ and that the ring is tuned for non-achromatic conditions [3 marks]

<u>Fundamental Constants</u>

Speed of light	= 299792458 m/s
π	= 3.141593
Permittivity of free space (ε_0)	= 8.854187E-12
Electron mass (m_e)	= 9.10938E-31 kg (510.999 keV/c^2)
Proton mass (m_e)	= 1.6726E-27 kg (938.28 MeV/c^2)
Electron charge	= 1.602176E-19 C
Classical electron radius	= 2.8179403E-15 m (e^2/4/pi/ε_0/m_e/c^2)
Reduced Planks constant (ħ)	= 1.054571e-34