

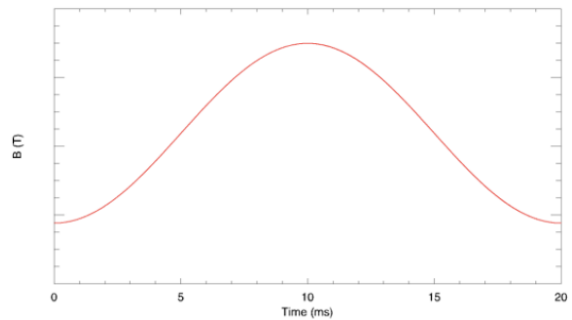
# Longitudinal Problem Set

Dr Rob Williamson  
rob.williamson@stfc.ac.uk

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(a) ISIS Synchrotron



(b) ISIS dipole field as a function of time

**Problem 1.1.** ISIS is a synchrotron that accelerates protons from a kinetic energy of 70 to 800 MeV on the rising edge of a 50 Hz sinusoidal main dipole field (see figure (b) above). Given the main dipole field at 70 MeV is 0.176 T calculate: the magnetic rigidity at that energy and the bending radius  $\rho$ . When at top energy (800 MeV), what is the corresponding magnetic rigidity and dipole field?

**Problem 1.2.** If 26.8% of the circumference is taken up with dipoles what is the mean radius  $R$  of the synchrotron? Calculate the revolution frequency at 0 ms (70 MeV) and 10 ms (extraction at 800 MeV).

**Problem 1.3.** Calculate the following parameters at 0, 5 and 10 ms

- Momentum
- Kinetic Energy
- Relativistic parameters  $\beta, \gamma$
- What does  $\gamma_t$  have to be for ISIS to remain below transition throughout acceleration?

**Problem 1.4.** What is the minimum RF voltage required as a function of time (0 - 10 ms) to accelerate a proton at ISIS? Why do we need more?

**Problem 1.5.** Given a mean dispersion of 1 m, what is the transition gamma,  $\gamma_t$ ? What transition kinetic energy does that correspond to? Calculate the slip factor,  $\eta$ , at 0, 5 and 10 ms.

From the longitudinal equation of motion on slide 29 one can derive the symplectic mapping equations:

$$\Delta E_{n+1} = \Delta E_n + V_1(\sin \phi_n - \sin \phi_s) \quad (1)$$

$$\phi_{n+1} = \phi_n - \frac{2\pi h \eta}{E_0 \beta^2 \gamma} \Delta E_{n+1} \quad (2)$$

from the  $n^{\text{th}}$  to the  $(n+1)^{\text{th}}$  turn where  $V_1$  is the peak gap voltage per turn and  $E_0$  is the rest energy. These can be used to follow a particle's trajectory in the longitudinal phase space  $(\Delta E, \phi)$  turn by turn.

**Problem 2.1.** Write a simulation program in the programming language of your choice that assumes several initial particle co-ordinates  $(0, \phi)$  in the range  $0 < \phi < \pi$  and calculates and applies the mapping equations over  $n$  turns. Assume a **constant energy** ( $t = 0$  ms, 70 MeV,  $\phi_s = 0$ ) and use the parameters you calculated in problems 1.3 and 1.5. (Other key parameters:  $h = 2$ ,  $V_1 = 19$  kV per turn and a sensible number for  $n$ .) Track four more trajectories with start co-ordinates  $(0.5 \text{ MeV}, \pi)$ ,  $(-0.5 \text{ MeV}, -\pi)$ ,  $(1 \text{ MeV}, \pi)$ ,  $(-1 \text{ MeV}, -\pi)$  and plot out all the longitudinal phase space trajectories on one graph.

**Problem 2.2.** For the above problem (2.1), pick out the three different types of trajectory and briefly describe the motion in each. Compare to the analogous situation of a simple pendulum.

**Problem 2.3.** Now assume we're half-way through acceleration ( $t = 5$  ms) and use the parameters associated with this time-point from problems 1.3 and 1.5. Again, make the assumption that  $\eta$ ,  $E_0$ ,  $\beta$  and  $\gamma$  are constant over the  $n$  turns you are tracking over. (Other key parameters:  $h = 2$ ,  $V_1 = 150$  kV per turn and a sensible number for  $n$ ). You'll need to calculate  $\phi_s$ .

Assume several initial particle co-ordinates  $(0, \phi)$ , this time in the range  $-\pi < \phi < \pi$  and ensure you track a particle with start co-ordinates  $(0, \pi - \phi_s - 0.02)$  to approximately trace out the separatrix. Plot out all the particles' longitudinal phase space trajectories over  $n$  turns on one graph.

**Problem 2.4.** Calculate the RF bucket height,  $\Delta E_{\text{max}}$  (energy acceptance) using equation 36 on slide 45 for the cases in 2.1 and 2.3. Compare with the bucket height from your separatrix particle trajectories. (Note that  $E_s$  in the equation is the total beam energy: rest mass and kinetic energy.)

**Hand in a copy of your program(s) with its (their) output. Marks are available for the code itself. Do get in touch if you have any questions or are having any issues.**

### OPTIONAL PROBLEM...for those who are interested

**Problem 3.1.** What if we now introduce a second harmonic RF system at twice the frequency of the first ( $h = 4$ ) such that we now have the energy gain per turn

$$\Delta E_{\text{turn}} = e(V_1 \sin \phi_s + V_2 \sin(2\phi_s + \theta)), \quad (3)$$

where  $V_2$  is the second harmonic RF voltage and  $\theta$  is the phase difference between the two RF systems. How does this change our mapping equations? Try putting this into your tracking code with  $V_2 = V_1$  for the case  $\phi_s = 0$ . How does the RF bucket change with

- a.  $\theta = \pi$ ?
- b.  $\theta = 0$ ?