Search for isoscalar $bc\bar{u}\bar{d}$ tetraquarks using lattice QCD

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T_{bc}: LHCb meets theory
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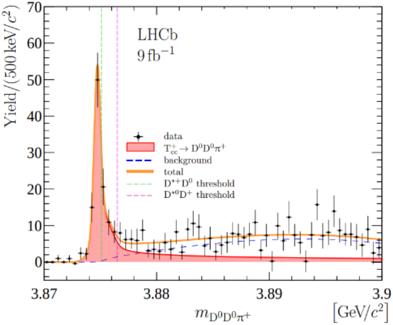
:ONLINE:

Funding agencies/computational resources

with **A. Radhakrishnan** and **N. Mathur**.
Based on article **arXiv:2307.14128** + an ongoing study

Motivation from experiments, T_{cc}^+



LHCb: 2109.01038, 2109.01056

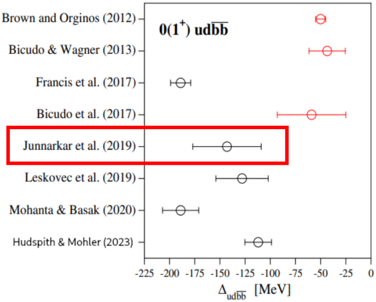
$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$

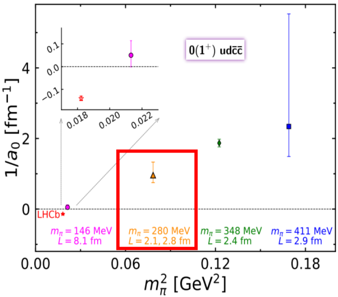
$$\begin{aligned} \delta m_{\rm pole} &= -360 \pm 40\,^{+4}_{-0}\ \mathrm{keV}/c^2\,, \\ \Gamma_{\rm pole} &= 48 \pm 2\,^{+0}_{-14}\ \mathrm{keV}\,. \end{aligned}$$

- The doubly charmed tetraquark T_{cc}^+, $I = 0$ and favours $J^P = 1^+$. Nature Phys., Nature Comm. 2022
  Striking similarities with the longest known heavy exotic, X(3872).
- No features observed in $D^0D^+\pi^+$: possibly not $I = 1$.
- Many more exotic tetraquark candidates discovered recently, T_{cs}, $T_{c\bar{s}}$, $X(6900)$.
  Prospects for T_{bc} in the near future. See talk by Ivan Polyakov
- Doubly heavy tetraquarks: theory proposals date back to 1980s.

c.f. Ader&Richard PRD25(1982)2370

Motivation from lattice, T_{bb} and T_{cc}





- Isoscalar axialvector channel $I(J^P) = 0(1^+)$.
- Deeper binding in doubly bottom tetraquarks $\mathcal{O}(100 MeV)$.

  Fig: Hudspith&Mohler 2023

  Red box: Our previous work on QQ tetraquarks: Junnarkar et al. PRD 2019
- Shallow bound state in doubly charm tetraquarks $\mathcal{O}(100 keV)$.

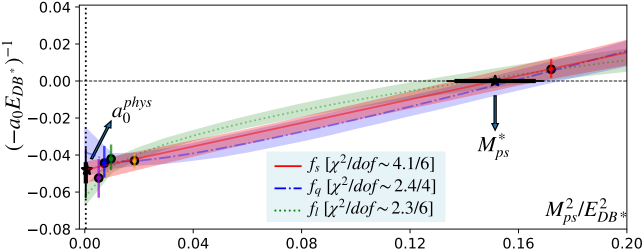
  Fig: HALQCD 2023

  Red box: T_{cc} and its quark mass dependence, see talk by Sasa Prelovsek
- No conclusive results in the bottom-charm tetraquark sector.
  Other updates from other calculations →

  see talk by Pedro Bicudo and Randy Lewis

  Meinel *et al* PRD 2022, Hudspith *et al* PRD 2020

Take away message: T_{bc} $(I)J^P = (0)1^+$ bound state



- Light quark mass ($m_{u/d}$ or M_{ps}) dependence
  indicates a real bound state at physical pion mass.
- DB^* scattering length[1] and binding energy (w.r.t. E_{DB^*}) in the continuum limit

$$a_0^{phys} = 0.57(^{+4}_{-5})(17)\ \text{fm} \quad \text{and} \quad \delta m_{T_{bc}} = -43(^{+6}_{-7})(^{+14}_{-24})\ \text{MeV}$$

- The critical M_{ps} at which T_{bc} becomes unbound

$$M_{ps}^* = 2.73(21)(19)\ \text{GeV}$$

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[1] Note the sign convention used: $[k cot\delta_0 \sim -1/a_0]$

Complexity involved in lattice hadron spectroscopy [No review]

- ✿ Deeply bound strong interaction stable hadrons:
  - Straightforward extraction from C_{2pt}, upto exponential volume corrections.
  - Chiral, continuum, and infinite volume extrapolations.

- ✿ Shallow bound states w.r.t. strong decay thresholds, near-threshold resonances and hadronic poles.
  - Involve inverse power law volume corrections.
  - Require rigorous finite volume scattering analysis *á la* Lüscher.
  - Complex calculations, but doable. Many lattice calculations exist by now.

- ✿ Highly excited resonances with multiple strong decay final state configurations and possible partial wave mixing scenarios.
  - Quite complex calculations: A handful of calculations exists mostly from HSC.
  - Three particle scattering channels.

c.f. Edwards Lattice 2019

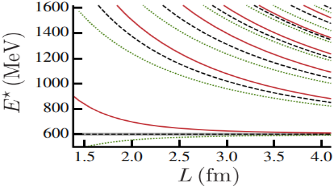
- ✿ Problem at hand (T_{bc} in DB^* scattering):

  Assume elastic (single scattering channel) approach.
  Assume no higher partial wave effects.
  Aim: To extract signatures of DB^* interactions, if any exists.

Finite volume signatures for hadron-hadron interactions

- Finite volume spectrum of two noninteracting particles:
  - Energy momentum dispersion relations.
  - $E_{cm} = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2}$. [Black dashed]
  - Note momenta is quantized in finite volume, $\mathbf{p} = \frac{2\pi}{L}\mathbf{n}$, so is E_{cm}.

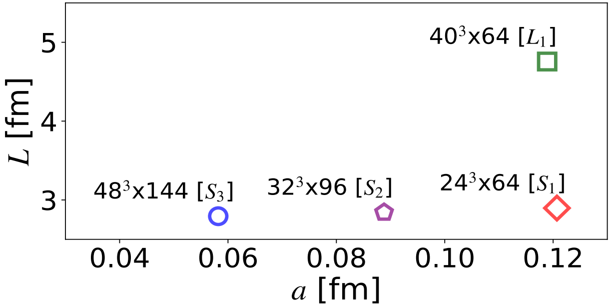
- Interactions reflected as phase shifts in the momentum p.
  Alternatively as deviations in energy spectrum from noninteracting scenario.
  Negative energy shifts ($E^i_{cm} - E^{ni}_{cm} < 0$) $\Rightarrow$ Attractive interactions [Green dotted]
  Positive energy shifts ($E^i_{cm} - E^{ni}_{cm} > 0$) $\Rightarrow$ repulsive interactions [Red solid]



Briceño, Dudek, and Young, RMP 2018

- **Determine energy shifts in DB^* scattering near threshold, if any exists.**

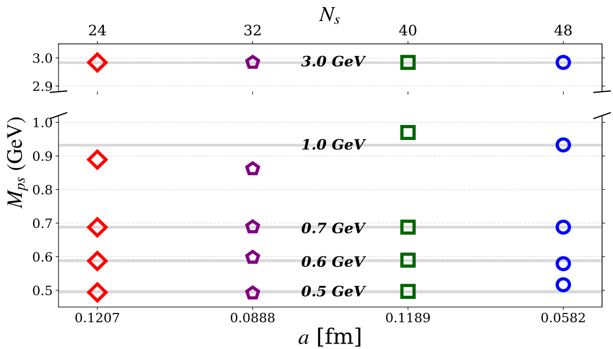
Lattice setup



- MILC dynamical ensembles with $N_f = 2+1+1$ HISQ fields.
- Valence quark fields with masses ranging from light to charm: overlap action
  No $\mathcal{O}(am)$ errors.
- Bottom quark evolution using a NRQCD Hamiltonian.
  tuned using kinetic mass of 1S bottomonium spin averaged $\overline{M}^{\bar{b}b}$ Mathur *et al* Lattice 2016
- Several publications based on this lattice setup.

Chakraborty, Junnarkar, Mathur, Mondal, MP, PRD/PRL 2018-2023

Valence light quark masses ($m_u = m_d = m_l$) studied



- One quark mass at the charm point ($M_{ps} \sim 3.0$ GeV). Basak *et al* Lattice 2014
  tuned using kinetic mass of 1S charmonium spin averaged $\overline{M}^{\bar{c}c}$

- Another at the strange point ($M_{ps} \sim 0.7$ GeV). Chakraborty *et al* PRD 2015
  tuned using the fictituous pseudoscalar $\bar{s}s$

- Three other quark masses approximately corresponding to pseudoscalar masses,
  $M_{ps} \sim 0.5,\ 0.6,\ \mathit{and}\ 1.0$ GeV.

Correlation functions and Interpolators [Axialvector]

- Focus on the T_{1g} finite volume irrep in the rest frame.

- Two point correlations computed as

$$\mathcal{C}_{ij}(t) = \sum_{\mathbf{x}} \left\langle \mathcal{O}_i(\mathbf{x},t)\mathcal{O}_j^\dagger(0) \right\rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E^n} e^{-E^n t},$$

  with wall smearing for quark fields at source.

- Focus only on the ground state energy splitting. Relevant low lying two meson thresholds

$$\begin{array}{rrcl}
DB^* \ [included] : & E^{phys}_{et} & \sim & 7.190 GeV \\
BD^* \ [included] : & E^{phys}_{it1} & \sim & 7.290 GeV \\
D^*B^* \ [excluded] : & E^{phys}_{it2} & \sim & 7.334 GeV
\end{array}$$

- Local 2 two-meson-like interpolators and one diquark-antidiquark-like interpolator

$$\begin{aligned}
\mathcal{O}_1(x) &= [\bar u \gamma_i b][\bar d \gamma_5 c](x) - [\bar d \gamma_i b][\bar u \gamma_5 c](x), \\
\mathcal{O}_2(x) &= [\bar u \gamma_5 b][\bar d \gamma_i c](x) - [\bar d \gamma_5 b][\bar u \gamma_i c](x), \\
\mathcal{O}_3(x) &= [(\bar u^T \Gamma_5 \bar d - \bar d^T \Gamma_5 \bar u)(b \Gamma_i c)](x).
\end{aligned}$$

Spectrum extraction

- $\mathcal{C}_{ij}(t)$ are solved for the generalized eigenvalue problem [GEVP]

$$\mathcal{C}(t)v^n(t) = \lambda^n(t)\mathcal{C}(t_0)v^n(t)$$

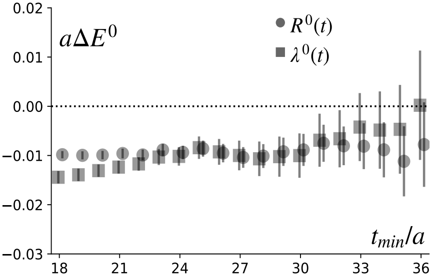
- Fits to the eigenvalue correlators [λ^n] and the ratio of eigenvalue correlators with a non-interacting correlator [$R^n(t) = \frac{\lambda^n(t)}{\mathcal{C}_{m_1}(t)\mathcal{C}_{m_2}(t)}$].

MP *et al* Lattice 2021

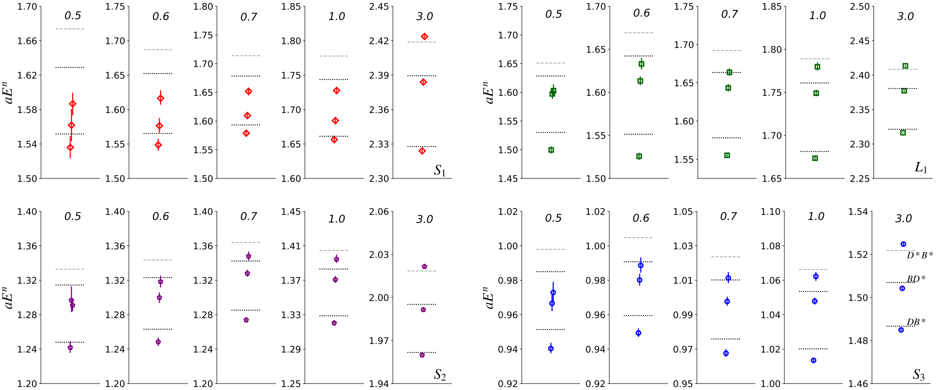
- Fits to the ground state in the finest ensembl with $M_{ps} \sim 0.7$ GeV in terms of energy splittings from $M_{B^*} + M_D$.

$$\Delta E^0 = E^0 - M_{B^*} - M_D$$

- t_{min} dependence of energy estimates from fits to $R^0(t)$ and $\lambda^0(t)$ →

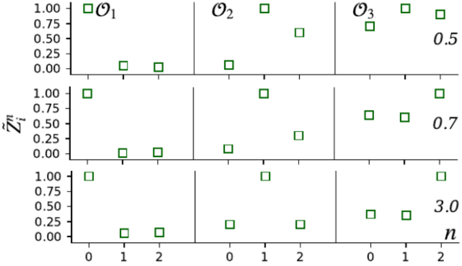


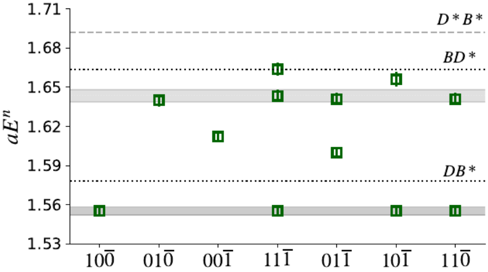
Finite volume spectrum



- Similar excited state pattern for all ensembles, for any given pseudoscalar mass.
- Statistically significant negative energy shifts:
  attractive interaction between the mesons involved.
  consistent with our preliminary results presented in Mathur&MP Lattice 2021.
- Additive energy offsets inherent to NRQCD.

Operator state overlaps and operator basis dependence



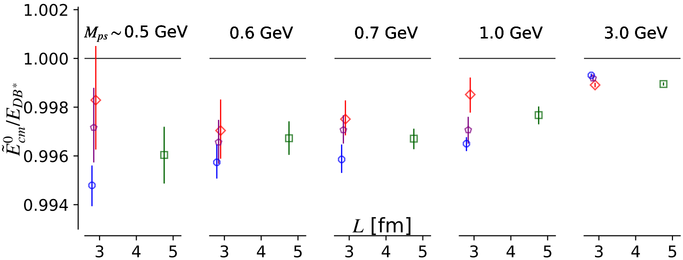


- Ground state very well determined by the DB^*-like operator $\mathcal{O}_1$.
- Excited states shows dominant two-meson and diquark-antidiquark Fock components.
  Decreasing diquark-antidiquark Fock component with increasing $m_{u/d}$.
  Consistent with phenomenological expectations.

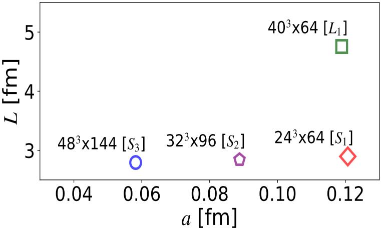
Junnarkar&Mathur&MP PRD 2019, Hudspith *et al* PRD 2020

- Consistent negative energy shift for ground state from full basis.
  Similar negative energy shift observed for first excited state in the full basis.
- Example shown for the case: $M_{ps} \sim 0.7$ GeV in the large volume ensemble.

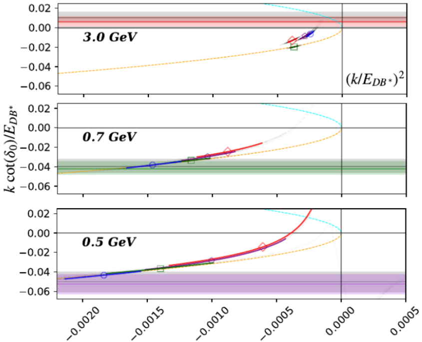
The ground state spectrum

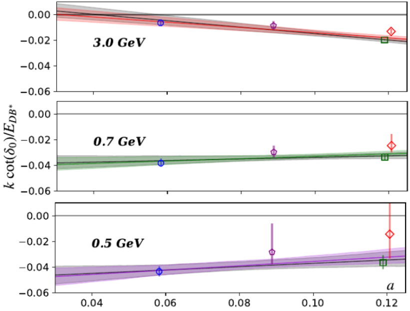


- Energy spectrum determined based on fits to $R^0(t)$.
  Automatic accounting for NRQCD additive correction.
- Energy reconstructed using $\tilde{E}^0 = \Delta E^0 + \overline{M}^{lat}_{B^*} + M^{lat}_{D}$
  where $\overline{M}^{lat}_{B^*} = M^{lat}_{B^*} - 0.5\overline{M}^{\bar{b}b,\ lat} + 0.5\overline{M}^{\bar{b}b,\ phys}$
- Consistent negative energy shifts. Mathur&MP Lattice 2021
  Decreasing magnitude with increasing $m_{u/d}$ or M_{ps}
- Non-trivial lattice spacing dependence.



Scattering amplitudes and continuum extrapolation



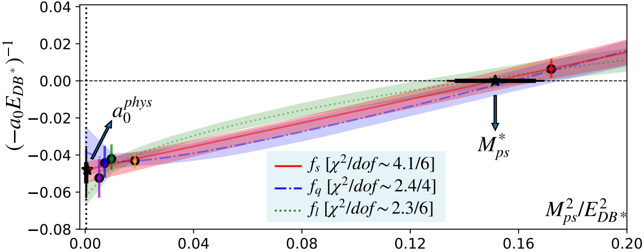


- Elastic DB^* scattering: finite volume analysis *à la* Lüscher. Briceño PRD 2014
  Only ground states used and only scattering length in an ERE. $[k cot\delta_0 \sim -1/a_0]$

- A linear lattice spacing dependence assumed for the fitted amplitude.

- Determined DB^* scattering length in the continuum limit for all M_{ps}.
  Results indicate attractive interaction between D and B^* mesons at all M_{ps}.

M_{ps} dependence of DB^* scattering length



- Light quark mass ($m_{u/d}$ or M_{ps}) dependence.

$$f_l(M_{ps}) = \alpha_c + \alpha_l M_{ps}, \quad f_s(M_{ps}) = \beta_c + \beta_s M_{ps}^2, \quad \text{and} \quad f_q(M_{ps}) = \theta_c + \theta_l M_{ps} + \theta_s M_{ps}^2.$$

indicates a real bound state at physical pion mass.

- DB^* scattering length and binding energy in the continuum limit

$$a_0^{phys} = 0.57\left(^{+4}_{-5}\right)(17)\text{ fm} \quad \text{and} \quad \delta m_{T_{bc}} = -43\left(^{+6}_{-7}\right)\left(^{+14}_{-24}\right)\text{ MeV}$$

- The critical M_{ps} at which T_{bc} becomes unbound

$$M_{ps}^* = 2.73(21)(19)\text{ GeV}$$

Scalar T_{bc} tetraquarks: DB scattering

- Focus on the A_{1g} finite volume irrep in the rest frame.

- Two point correlations computed as

$$\mathcal{C}_{ij}(t) = \sum_{\mathbf{x}} \left\langle \mathcal{O}_i(\mathbf{x},t)\mathcal{O}_j^\dagger(0) \right\rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E^n} e^{-E^n t},$$

  with wall smearing for quark fields at source.

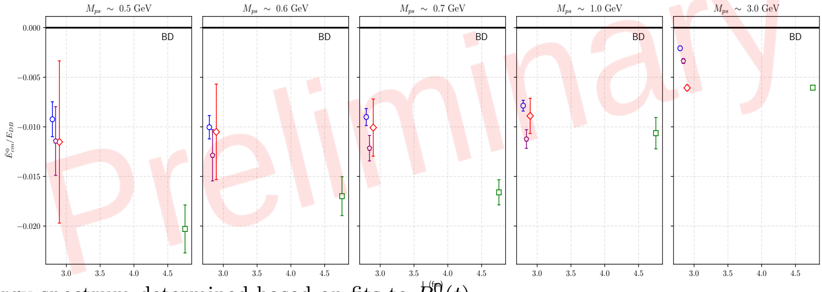
- Current efforts on determining the lowest two energy levels.
  Relevant low lying two meson thresholds (No relevant trivial three particle channels)

$$\begin{array}{rl} DB\ [included]: & E_{et}^{phys} \sim\ 7.145GeV \\ D^*B^*\ [excluded]: & E_{it1}^{phys} \sim\ 7.334GeV \end{array}$$

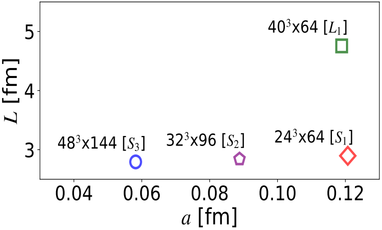
- Local two-meson-like interpolator and one diquark-antidiquark-like interpolator

$$\begin{aligned} \mathcal{O}_1(x) &= [\bar{u}\gamma_5 b][\bar{d}\gamma_5 c](x) - [\bar{d}\gamma_5 b][\bar{u}\gamma_5 c](x), \\ \mathcal{O}_2(x) &= [(\bar{u}^T C\gamma_5 \bar{d} - \bar{d}^T C\gamma_5 \bar{u})(bC\gamma_5 c)](x). \end{aligned}$$

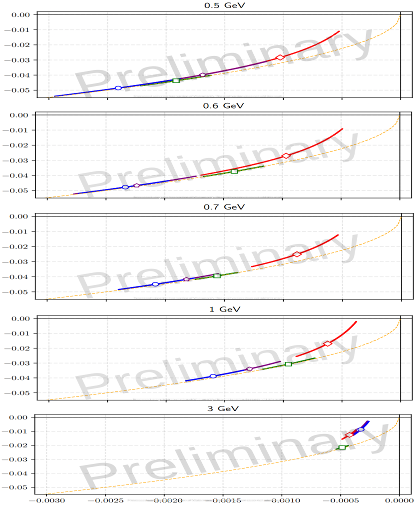
The ground state spectrum: Scalar T_{bc}



- Energy spectrum determined based on fits to $R^0(t)$.
  Automatic accounting for NRQCD additive correction.
- Energy reconstructed using $\tilde{E}^0 = \Delta E^0 + \overline{M}_B^{lat} + M_D^{lat}$
  where $\overline{M}_B^{lat} = M_B^{lat} - 0.5\overline{M}^{\bar{b}b,\ lat} + 0.5\overline{M}^{\bar{b}b,\ phys}$
- Similar negative energy shifts as observed for 1^+.
  Decreasing magnitude with increasing $m_{u/d}$ or M_{ps}
- Non-trivial lattice spacing dependence.



Scalar T_{bc}: DB scattering amplitude



- $k cot\delta_0/E_{DB}$ versus $(k/E_{DB})^2$.
- Elastic DB scattering: finite volume analysis *à la* Lüscher. Briceño PRD 2014
- Only ground states used and only scattering length in an ERE. $[k cot\delta_0 \sim -1/a_0]$
- Similar behaviour of observed scattered amplitudes.
- Ongoing efforts in extracting excited state info and continuum and chiral extrapolations.

Summary

- We have simulated DB^*-BD^* scattering on the lattice and determine DB^* scattering length.

- Transparent evidence for negative energy shifts and hence attractive interaction between D and B^* mesons.

  Uphold our results from Mathur&MP Lattice 2021

- Studied light quark mass ($m_{u/d}$ or M_{ps}) dependence from M_{ps} ∼0.5 to ∼ 3.0 GeV. Real bound state with binding energy

$$\delta m_{T_{bc}} = -43(^{+6}_{-7})(^{+14}_{-24}) \text{ MeV}$$

- The critical M_{ps} at which T_{bc} becomes unbound

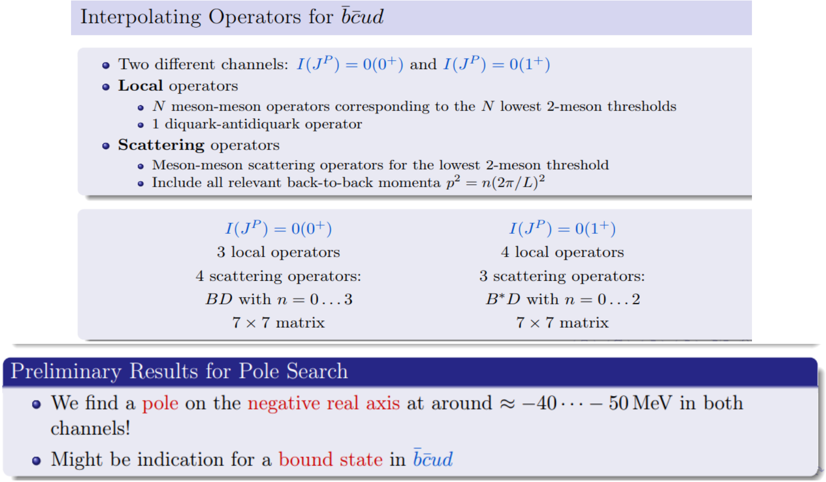
$$M^*_{ps} = 2.73(21)(19) \text{ GeV}$$

- We ignored effects from higher partial wave mixing and left hand cuts in our analysis.

- Ongoing efforts on scalar T_{bc}: DB scattering. Similar energy splittings are observed.

STAY TUNED ...

Meanwhile: Another lattice investigation



Interpolating Operators for $\bar{b}\bar{c}ud$

- Two different channels: $I(J^P) = 0(0^+)$ and $I(J^P) = 0(1^+)$
- **Local** operators
  - N meson-meson operators corresponding to the N lowest 2-meson thresholds
  - 1 diquark-antidiquark operator
- **Scattering** operators
  - Meson-meson scattering operators for the lowest 2-meson threshold
  - Include all relevant back-to-back momenta $p^2 = n(2\pi/L)^2$

$I(J^P) = 0(0^+)$	$I(J^P) = 0(1^+)$
3 local operators	4 local operators
4 scattering operators:	3 scattering operators:
BD with $n = 0 \ldots 3$	B^*D with $n = 0 \ldots 2$
7×7 matrix	7×7 matrix

Preliminary Results for Pole Search

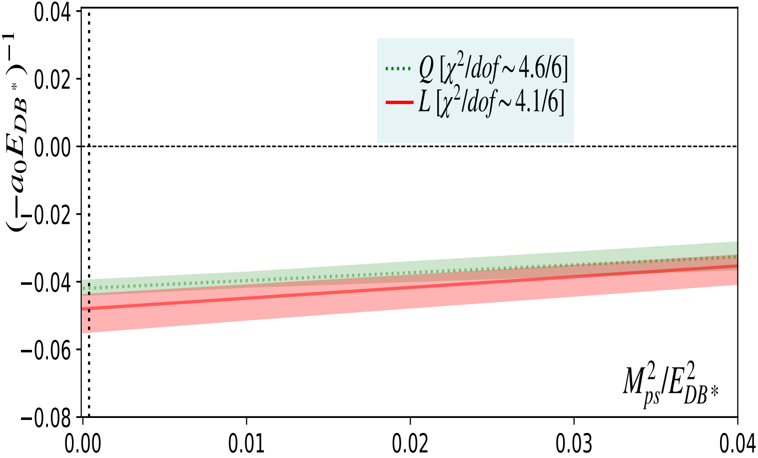
- We find a pole on the negative real axis at around $\approx -40 \cdots - 50\,\text{MeV}$ in both channels!
- Might be indication for a bound state in $\bar{b}\bar{c}ud$

Similar conclusions and similar binding energies.

Exotic Hadrons 2023, Martin Pflaumer

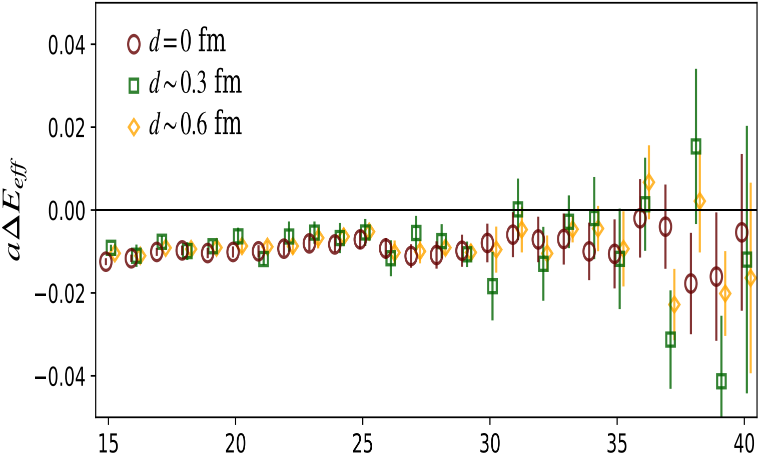
Thank you

Lattice spacing dependence



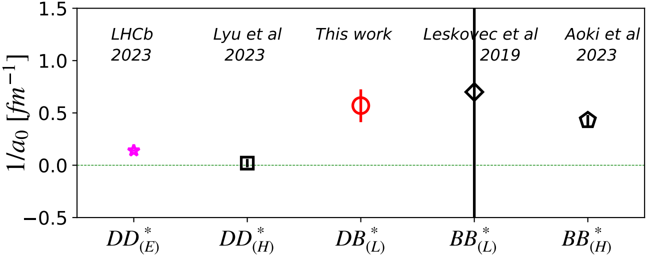
Conclusions are unaffected and consistent numbers in the physical limits.
Deviations are counted towards systematic uncertainties.

Sink smearing dependence



Conclusions remain robust and consistent numbers.
Larger uncertainties with large smearing widths.

Comparison of inverse scattering lengths DD^*, DB^*, BB^*



- Looking for any pattern between different doubly heavy systems (at physical M_{ps}).
  Lüscher formalism based analysis: This work, Leskovec *et al* 2019
  HALQCD procedure: Lyu *et al* 2023, Aoki *et al* 2023.

- Uncertainties from Lüscher-based procedures are typically large.
  Need more work to map such a dependence.