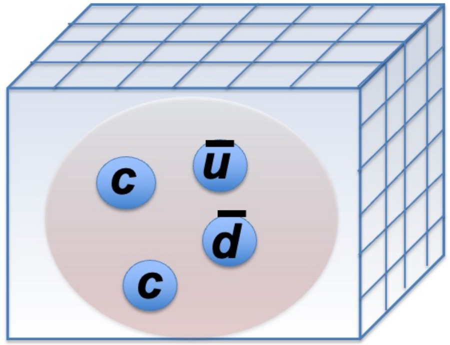
# $T_{cc}$ and its quark mass dependence from lattice QCD




Sasa Prelovsek

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Hunting for $T_{bc}$ , CERN 5th October

Disclaimer:

- I will not discuss lattice simulations of Tbc (see talk of Padmanath and others for Tbc)
- I will review several recent lattice simulations of Tcc
  and make some simplistic arguments on what these might imply for Tbc

# Doubly heavy tetraquarks

$QQ'\bar{q}\bar{q}'$

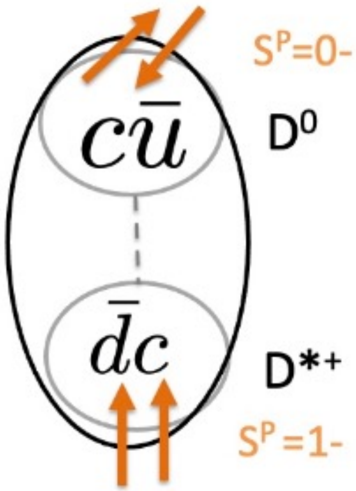
*Q=c,b   q=u,d,s*

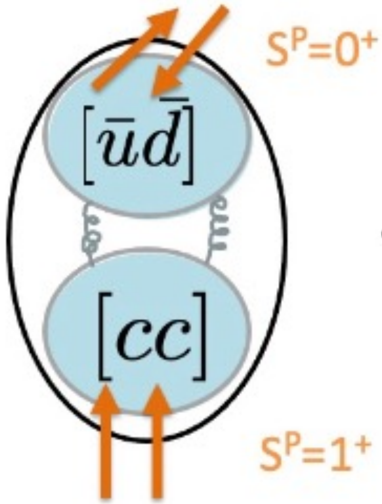
- Exotic hadrons
- Which states exist? flavor, $J^P$
- Mass ? Strongly stable ?
- Binding mechanism ?

exchange of

light mesons

$\pi, \rho, \pi\pi$ ?



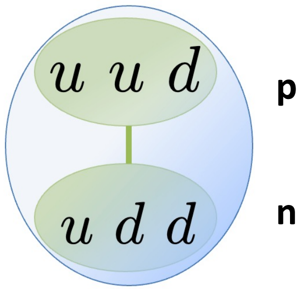




"molecular"

deuterium

$\pi, ..$

$E_b = -2.2$ MeV



$bbar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $I=0,\ J^P=1^+$ The only ones expected significantly below strong-decay thresholds $BB^*_{(s)}$

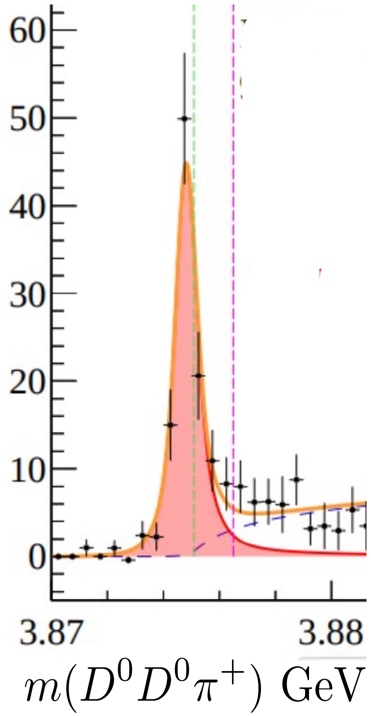
Other $QQ'\bar{q}\bar{q}'$ and $J^P$ $bc\bar{q}\bar{q}',\ cc\bar{q}\bar{q}'$ q=u,d,s

Theoretically expected near or above threshold

States near or above threshold have to be identified as poles in scattering T(E): much more challenging

$cc\bar{d}\bar{u}$

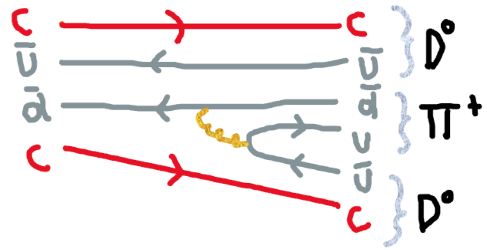
The longest lived exotic hadron ever discovered I=0, $J^P=1^+$ (most likely)




$$\delta m = m - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{pole} = -0.36 \pm 0.04 \text{ MeV}$$

LHCb 2109.01038, 2109.01056, Nature Physics



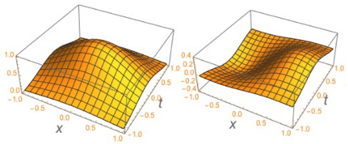

Omitting $D^* \to D\pi,\ T_{cc} \to DD\pi$

$T_{cc}$ would-be a bound state

QCD: $\mathcal{L}_{QCD} = \frac{1}{4} G_a^{\mu\nu} G_a^{\mu\nu} + \bar{q} i \gamma_\mu (\partial^\mu + i g_s G_a^\mu T^a) q - m_q \bar{q} q$  $g_s \not\ll 1$ at hadronic energy scale

# Lattice QCD

$$\langle C \rangle = \int DG\, Dq\, D\bar{q}\, C\, e^{-S_{QCD}/\hbar}$$



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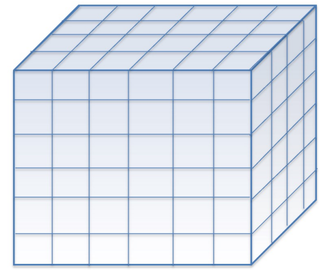


Main quantity extracted: finite-volume eigen-energies $E_n$  $\hat{H}|n\rangle = E_n |n\rangle$

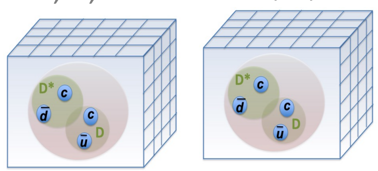
$\sum_n |n\rangle\langle n|$  $e^{-iE_n t_M}$ Eucledian time

$$C_{ij}^{2pt}(t) = \langle 0 | \mathcal{O}_i(t)\ \mathcal{O}_j^+(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle\ e^{-E_n t_E} \langle n | \mathcal{O}_j^+ | 0 \rangle$$

often "non-precision" studies:

single a, $m_{u/d} > m_{u/d}^{phy}$ , $m_\pi > 140$ MeV

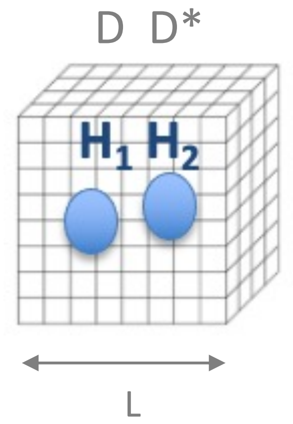
t, $J^P$, I  t=0, $J^P$, I

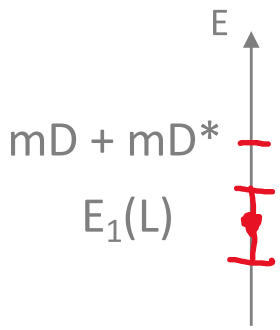



$$\mathcal{O} = \mathcal{O}(q, G)$$


- for strongly stable state well below threshold : $E_n(P=0) = m$  talk by R. Lewis (Tbb,...)
- resonances (Luscher's relation) $E_n^{cm} \rightarrow T(E_n^{cm})$  this talk (Tcc)  talk by Padmanath (Tbc)
- static potentials: $E_n \rightarrow V(r)$  talk by P. Bicudo

# Why $m(T_{cc})$ can not be extracted as $m=E_1$ ?




- eigenstate with $E_1 < mD + mD^*$ could correspond to a essentially free pair of D(0) and D*(0)

  whose energy is slightly shifted down due to feeble attraction on a finite lattice

- $E_1 < mD + mD^*$ by itself does not imply there is a (virtual) bound state or resonance

- Scattering amplitude T(E) has to be extracted

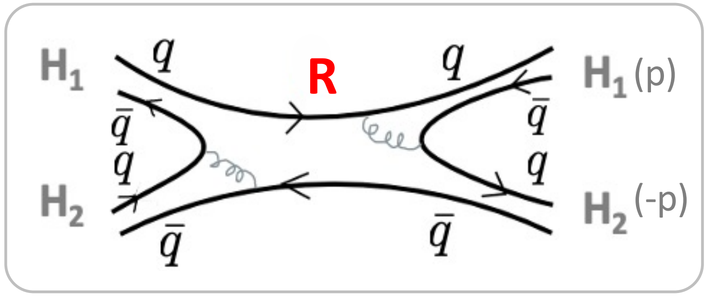
- pole in T(E) indicates a presence of Tcc

applies for states near or above threshold

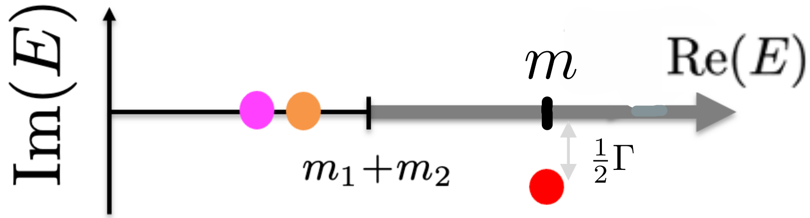
# Resonances R→ $H_1$ $H_2$ , bound states near threshold

E:= $\sqrt{m_1^2+p^2}+\sqrt{m_2^2+p^2}$

scattering amplitude T(E)




$$S(E) = e^{2i\delta(E)} = 1 + 2i\,\frac{2p}{E}\,T(E) \rightarrow \boxed{T \propto \frac{1}{p\cot\delta - ip}}$$



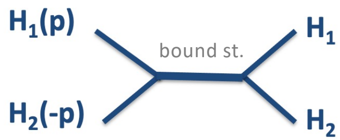

Virtual bound st. | Bound st. | Resonance

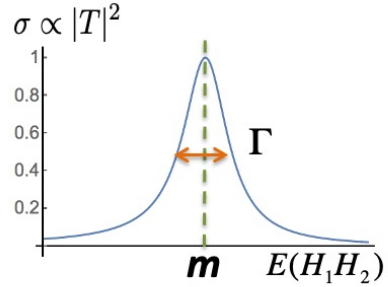
p = - i |p|, sheet II | p = i |p|, sheet I | sheet II

$p^2<0$

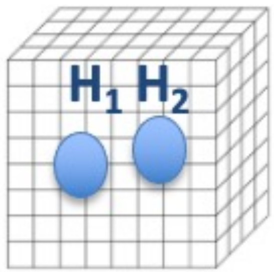
$$T(E) \propto \frac{1}{E^2 - m^2}$$

$$T(E) \propto \frac{1}{E^2 - m^2 + iE\Gamma}$$




Scattering amplitude T(E) from lattice QCD





$$\sigma(E) \propto |T(E)|^2$$

real E | for real E | for complex E

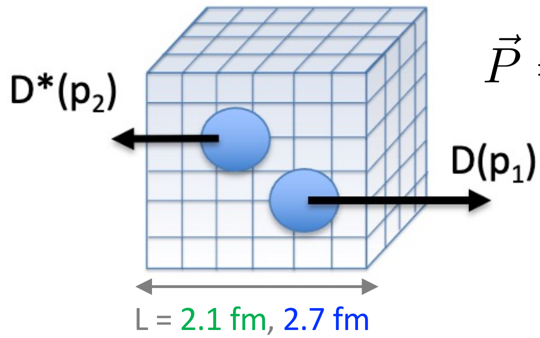
$$E \rightarrow T(E) \rightarrow T(E^c)$$

analytic relation:
Luscher 1991
generalizations by many authors

analytic contin.
to complex E

# $T_{cc}$

$cc\bar{u}\bar{d}$ $\quad I=0,\ J^P=1^+$

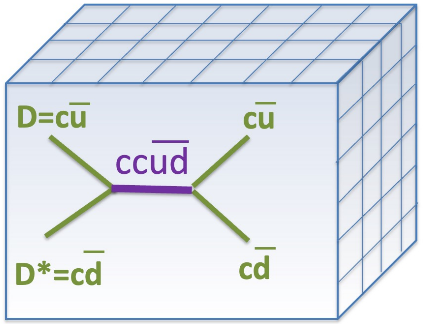



$\vec{P} = \vec{p}_1 + \vec{p}_2$

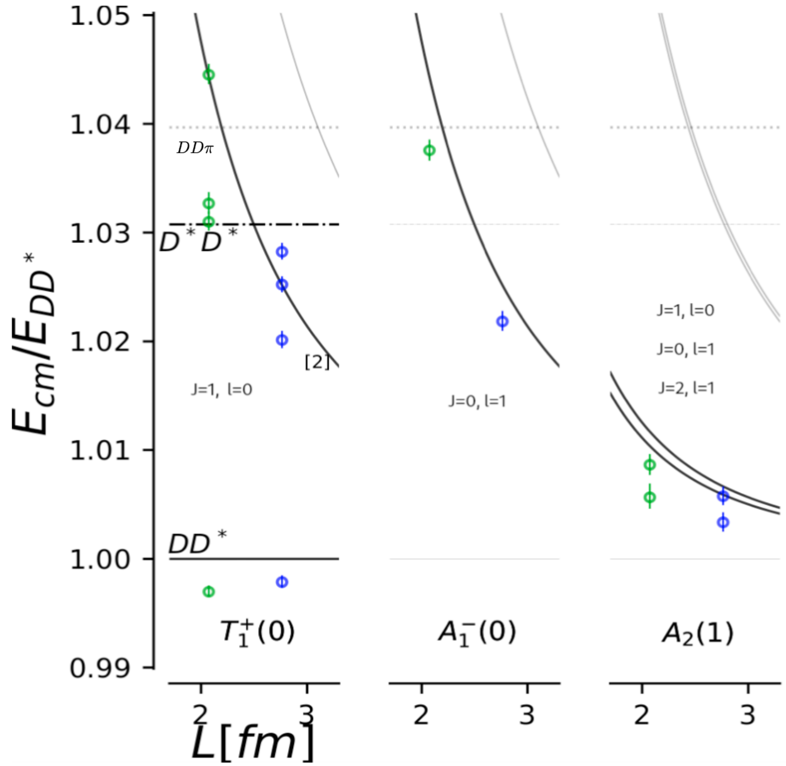
D $\quad$ D*

$$\mathcal{O} = (\bar{u}\gamma_5 c)_{\vec{p}_1}\ (\bar{d}\gamma_i c)_{\vec{p}_2} - (\vec{p}_1 \leftrightarrow \vec{p}_2) \qquad \vec{p}_{1,2} = \vec{n}_{1,2}\ \tfrac{2\pi}{L}$$

$$(\bar{u}\gamma_5\gamma_t c)_{\vec{p}_1}\ (\bar{d}\gamma_i\gamma_t c)_{\vec{p}_2}$$




$\vec{P}=0 \qquad \vec{P} = \frac{2\pi}{L}\ (0,0,1)$




$E_{DD^*} \equiv m_D + m_{D^*}$

Padmanath & SP, PRL2022, $m_\pi \approx 280$ MeV

$D^* \not\to D\pi,\ T_{cc} \not\to DD\pi$
$DD\pi$ above analyzed region

these applies to all available lattice studies of Tcc

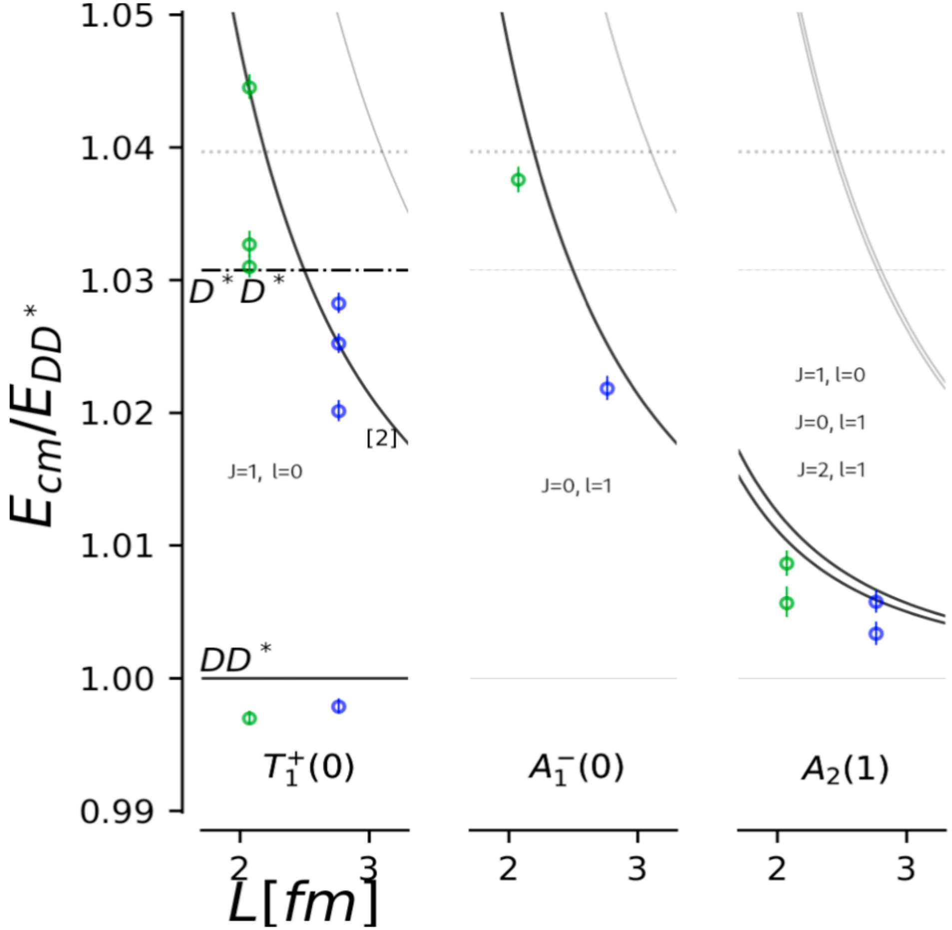
E < $E^{non.int.}$ (lines) : $\quad E^{n.i.} = \sqrt{m_D^2 + \vec{p}_1^{\,2}} + \sqrt{m_{D^*}^2 + \vec{p}_2^{\,2}}$
$\vec{p}_i = \vec{n}_i \frac{2\pi}{L}$

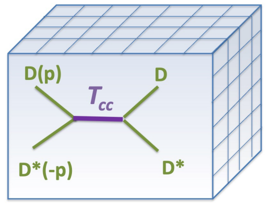
attractive interaction between D and D*

# $T_{cc}$ Eigen-energies and scattering amplitude

Padmanath, S.P.: 2202.10110, *PRL 2022*

at $m_\pi \approx 280\ MeV$



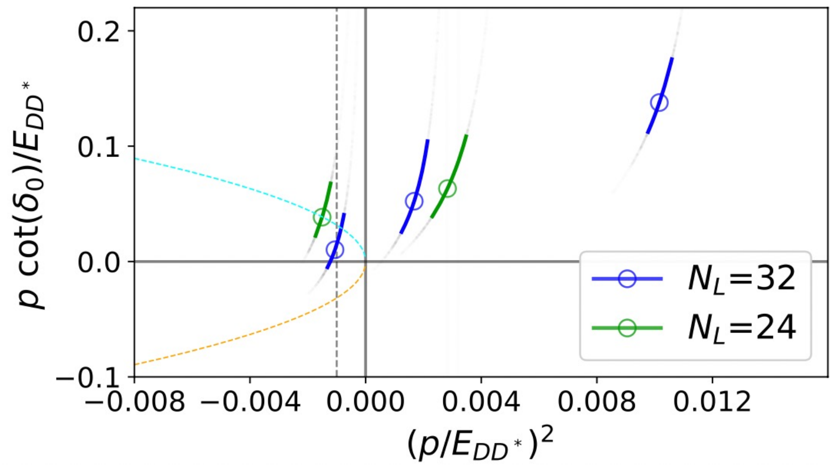




Luscher's relation

E -> T(E), $\delta(E)$

$$T = \frac{E}{2}\frac{1}{p\cot\delta - ip}$$

partial wave l=0




$$E_{DD^*} \equiv m_D + m_{D^*}$$

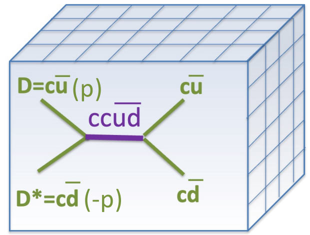
# $T_{cc}$

dependence on $m_{u/d}$

$cc\bar{d}\bar{u}$

I=0, $J^P$=1+

$$T \propto \frac{1}{p\cot\delta - ip}$$

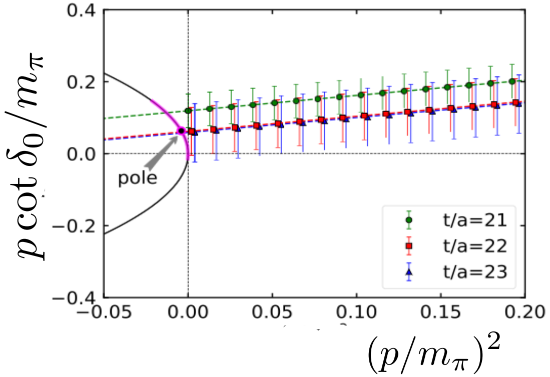



D* is stable at these $m_\pi$

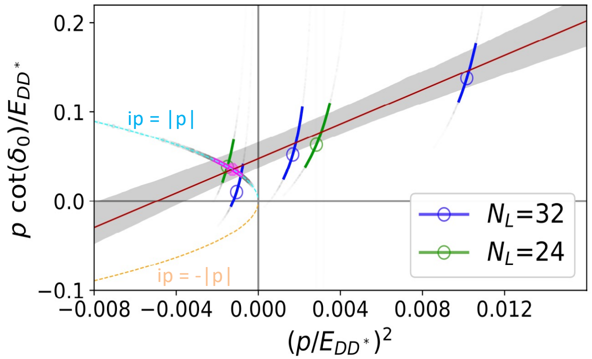
$T(E) \propto \frac{1}{E^2 - m^2}$ for E~m

LHCb

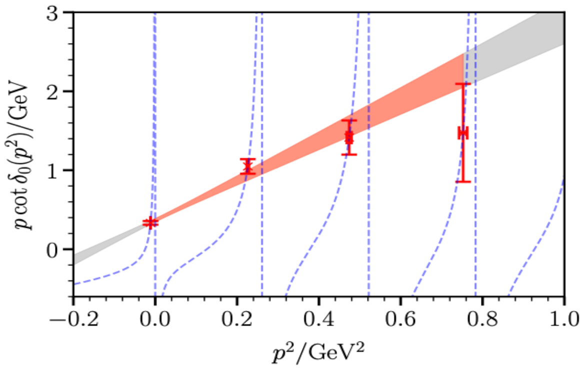
HALQCD method, 2302.04505, $m_\pi \approx 146$ MeV




Padmanath, SP: 2202.10110, PRL, $m_\pi \approx 280$ MeV




CLQCD 2206.06185, PLB, $m_\pi \approx 348$ MeV




- 0.36(4) MeV - 0.045(77) MeV $-9.9(^{+3.6}_{-7.2})$ MeV : binding energy $\delta m$

bound st.

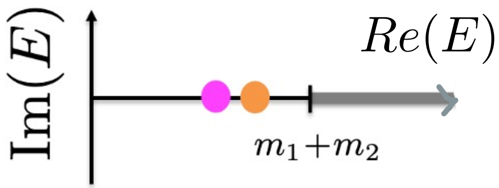
virtual bound st. pole

$$p\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2}\, r_0\, p^2$$

$$\frac{1}{a_0} + \frac{1}{2}\, r_0\, p^2 - i\,p = 0$$

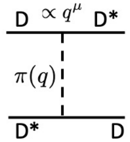
p = - i |p| for all three simulations

pole



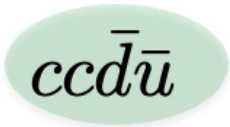

Virtual bound st. p = - i |p|

Bound st. p = i |p|




- possible effects of left-hand cuts discussed later in this talk
- their effects are omitted till then:
  - effective range approx. assumed
  - poles extracted

# Dependence on $m_{u/d}$ and $m_c$




Padmanath, S.P.: 2202.10110, *PRL Supplemental material*

in case of molecular binding mechanism

Simple arguments in QM

for arbitrary fully attractive potential

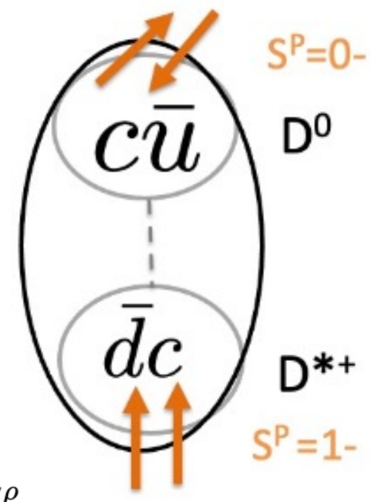
$$\pi, \rho, \pi\pi \;?$$

simplest example of attractive potential

$m_{u/d}$

$$V(r) \propto -\frac{e^{-m_{ex} r}}{r}$$

$$m_{ex}: \; m_\pi, m_\rho$$



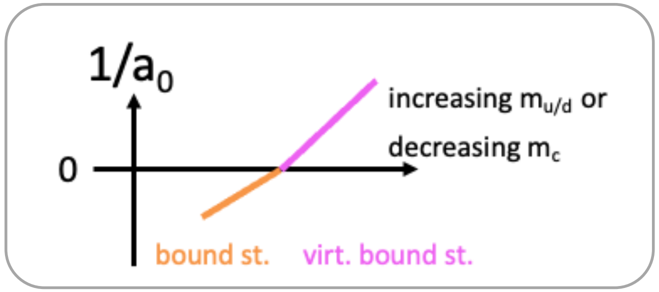

$m_c$

$$\hat{H}_{kin} = \frac{\hat{p}^2}{2\, m_{red}}$$

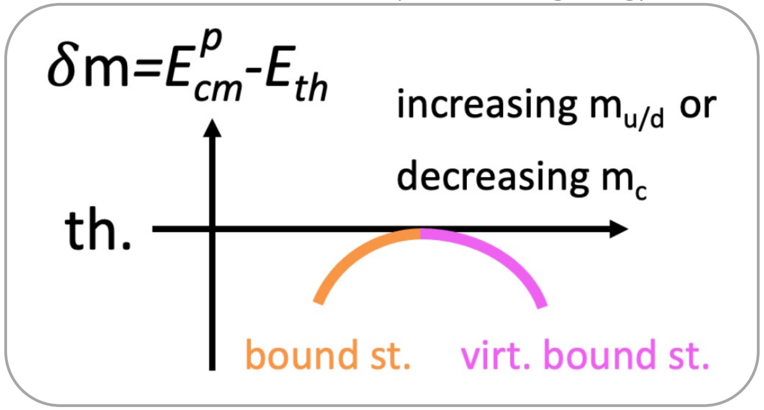
$$m_r \simeq \frac{m_D m_{D^*}}{m_D + m_{D^*}}$$

$$p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2}\, r_0\, p^2$$

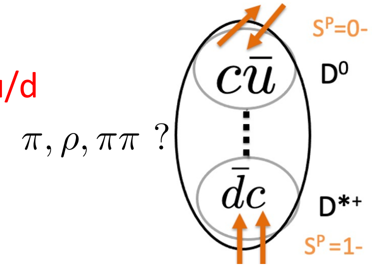
sketch of expected scattering lenght a0

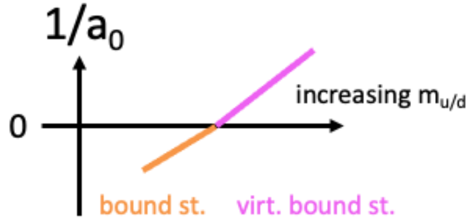


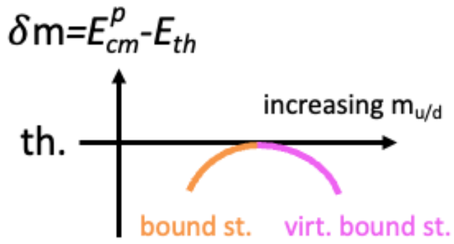

sketch of expected binding energy



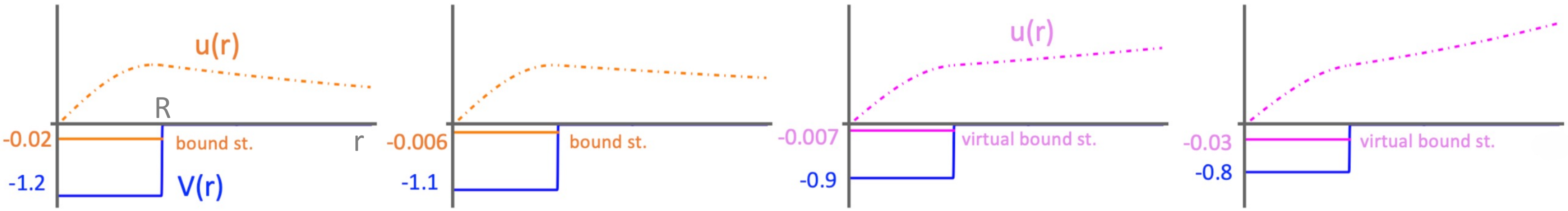
# Dependence on $m_{u/d}$

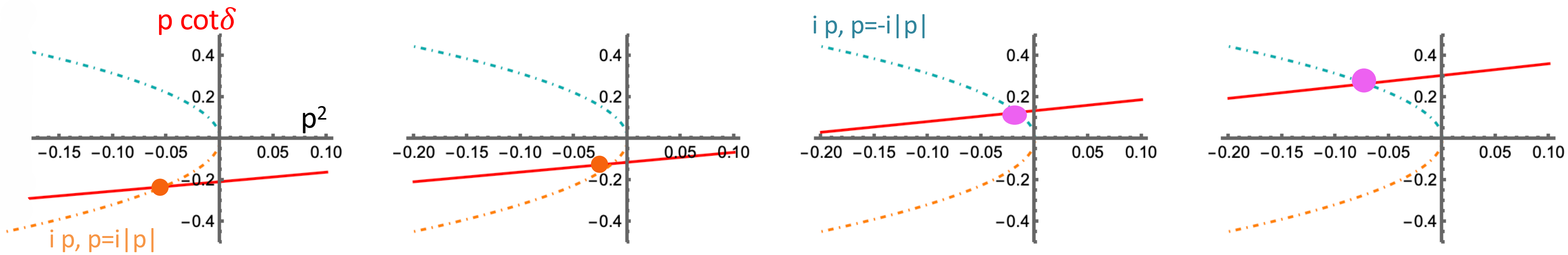


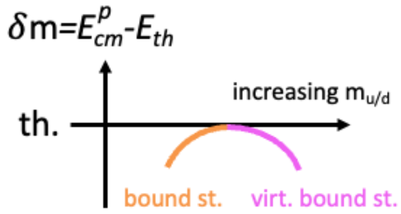

## Square well potential (analogous conclusion for other fully attractive shapes)



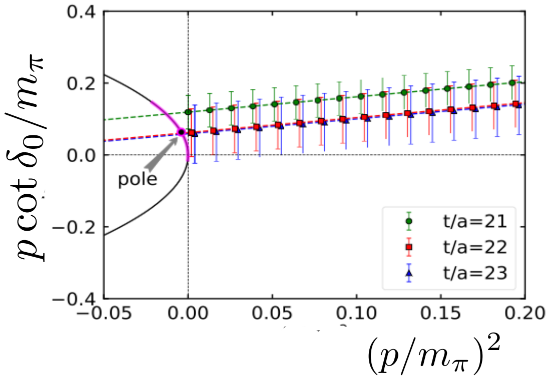

increasing $m_{u/d}$ , decreasing attraction

# $T_{cc}$ dependence on $m_{u/d}$

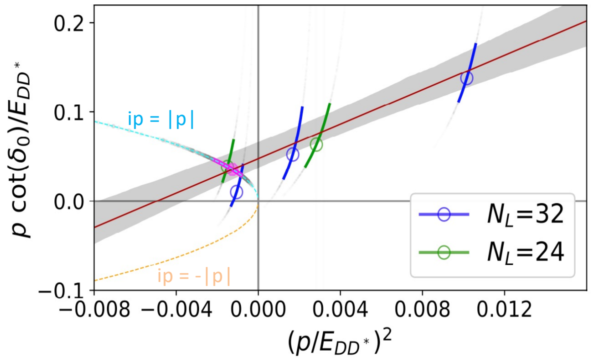



LHCb

HALQCD method, 2302.04505, $m_\pi \approx 146$ MeV




Padmanath, SP: 2202.10110, PRL, $m_\pi \approx 280$ MeV

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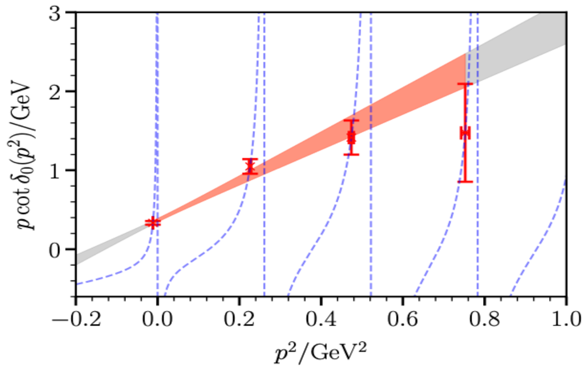
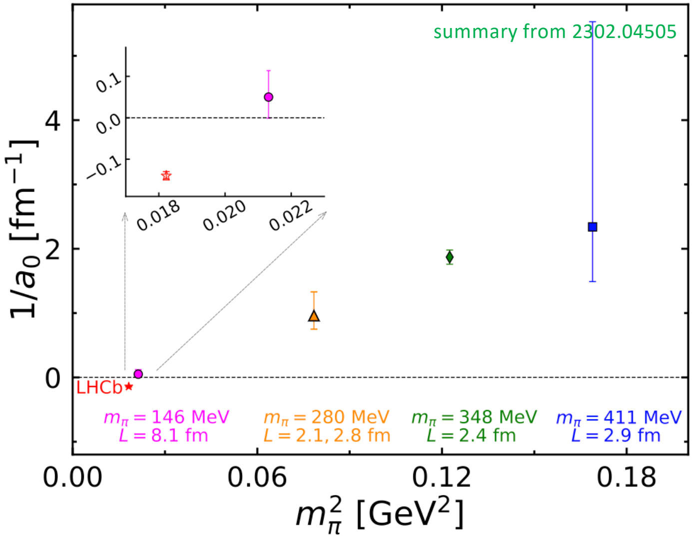
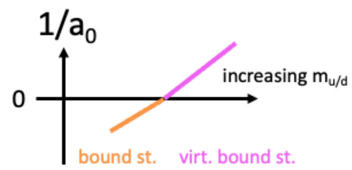


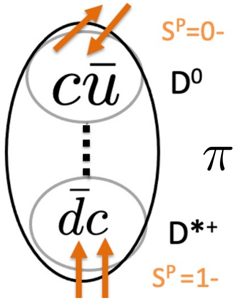

- 0.36(4) MeV  - 0.045(77) MeV  $-9.9(^{+3.6}_{-7.2})$ MeV : binding energy $\delta m$

bound st.  virtual bound st. pole



# $T_{cc}$ Exchange of which particles drives the attraction (within molecular picture)?



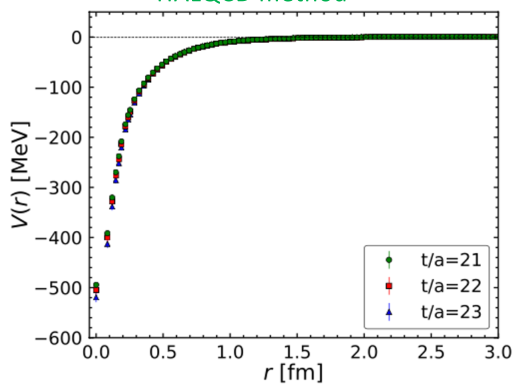

$\pi, \rho, \pi\pi$ ?

HALQCD, 2302.04505, $m_\pi \approx$146 MeV

$\pi$, $\rho$, $\pi\pi$ ?

$V(r) \simeq -\frac{e^{-2m_\pi r}}{r^2}$ r > 1 fm

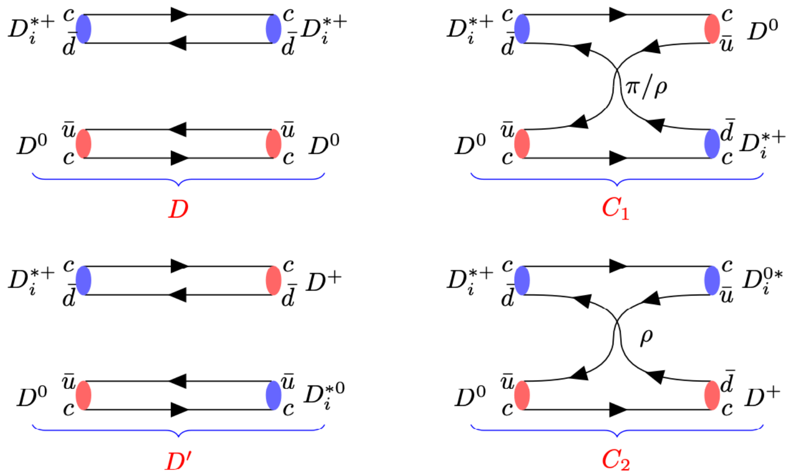
HALQCD method




CLQCD 2206.06185, PLB,
$m_\pi \approx$ 348 MeV

$\pi$, $\rho$, $\pi\pi$ ?

not excluded




I=0 attractive, I=1: repulsive

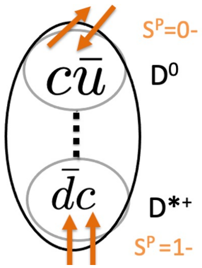
$C^{(I)}(p,t) = D - C_1(\pi/\rho) + (-)^{I+1}(D' - C_2(\rho))$

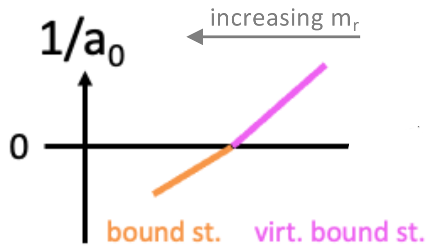
$C_2$ drives attraction in I=0 channel

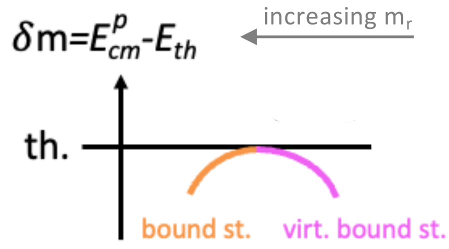
# Dependence on $m_Q$

$T_{cc}: \quad 1/m_r = 1/m_D + 1/m_{D*}$

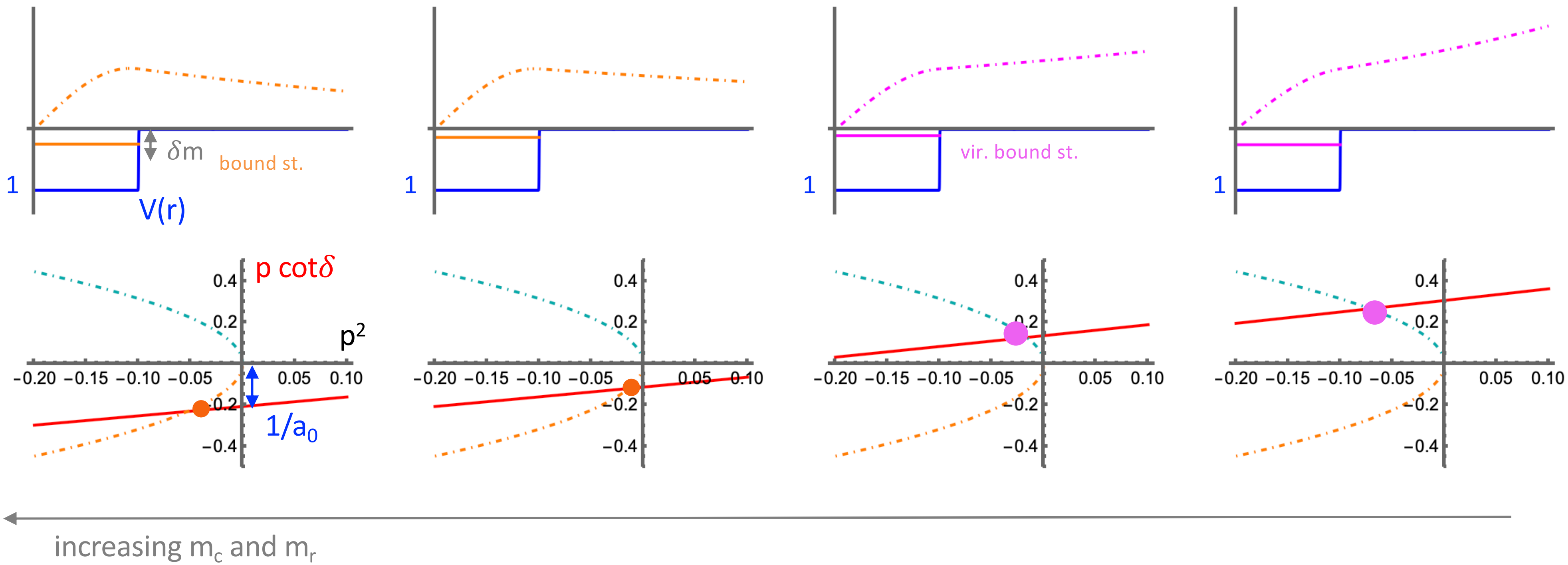
$T_{bc}: \quad 1/m_r = 1/m_D + 1/m_{B*}$



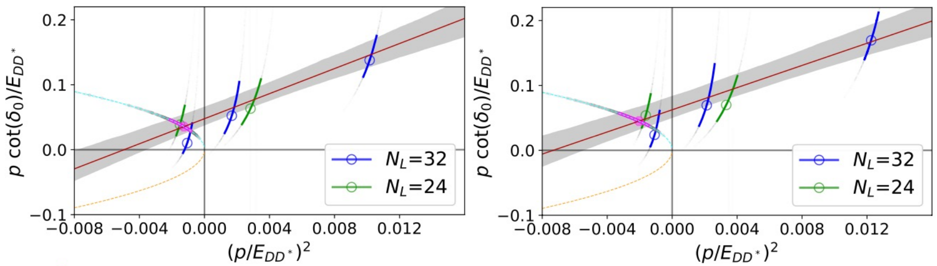




## Square well potential (analogous conclusion for other fully attractive shapes)




Tcc: Padmanath, S.P.: 2202.10110, *PRL*

| | $m_D$ [MeV] | $a^{(J=1)}_{l=0}$ [fm] | $\delta m_{T_{cc}}$ [MeV] | $T_{cc}$ |
|---|---|---|---|---|
| $m_c^{(h)}$ | 1927(1) | 1.04(29) | $-9.9^{+3.6}_{-7.2}$ | virtual bound st. |
| $m_c^{(l)}$ | 1762(1) | 0.86(0.22) | $-15.0(^{+4.6}_{-9.3})$ | virtual bound st. |




Collins, Nefediev, Padmanath, S.P.: 23xx.xxxxx:

further $m_c > m_c^{phy}$:

# Effect of [cc][ud] interpolators grows from c to b

Emmanuel Ortiz Pacheco, Collins, Leskovec, Padmanath, SP

(talk by Pacheco at lattice 2023)

DD* $O = (\bar{u}\gamma_5 c)_{\vec{p}}(\bar{d}\gamma_i c)_{-\vec{p}} - (u \leftrightarrow d)$

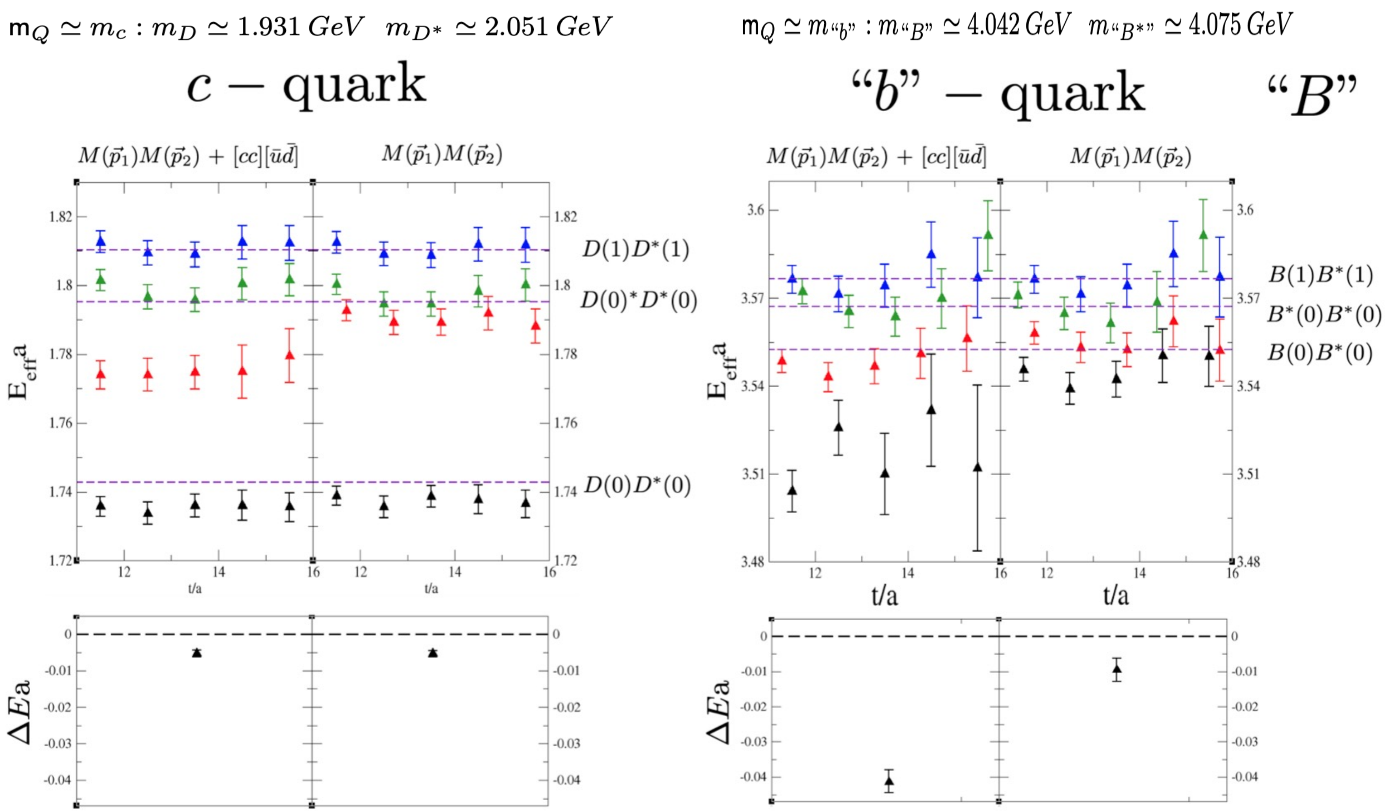
[cc][ud] $O = \sum_x [c(x)C\gamma_i c(x)][\bar{u}(x)C\gamma_5 \bar{d}(x)]e^{iPx}\ |_{P=0}$

effective eigen-energies

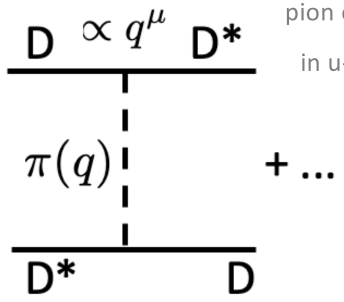
(as a function of lattice time)

energy shift

of the ground state



# Effects of pion exchange and left-hand cut




pion exchange
in u-channel

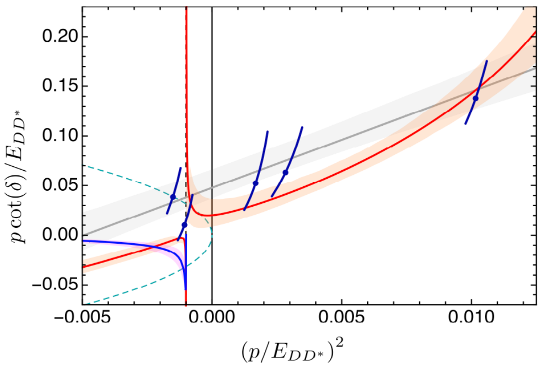
- pion exchange: suppressed near threshold due to derivative coupling
- CLQCD, HALQCD lattice studes: one-pion exchanges not crucial for existence of Tcc

- Du, Hanhart, Nefediev et al. 2303.09441: incorporate pion exchange and left-hand cut

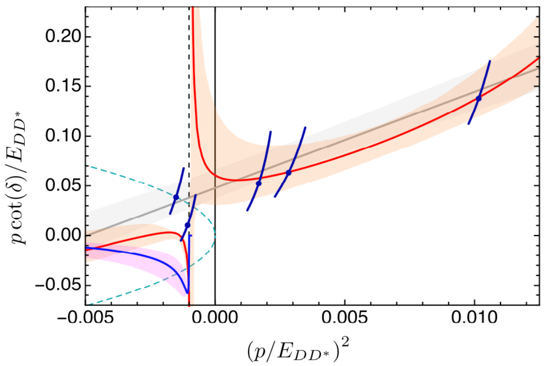
- pion exchange at large r: feeble attraction for $m_{D^*}-m_D>m_\pi \;:\; m_\pi=m_\pi^{phy}$

  feeble repulsion for $m_{D^*}-m_D<m_\pi \;:\; m_\pi=m_\pi^{lat}>m_\pi^{phy}$

- reanalysis our Tcc data : lattice data supports significant attraction at small r

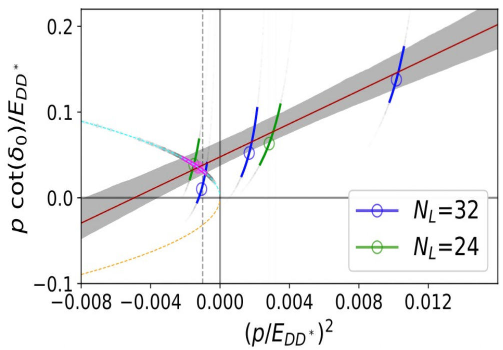



● two virtual bound states

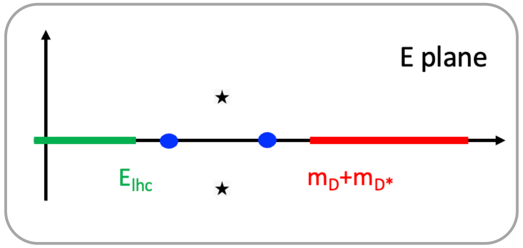



★ one narrow resonance

Padmanath, S.P.: 2202.10110, *PRL*




one virtual bound state

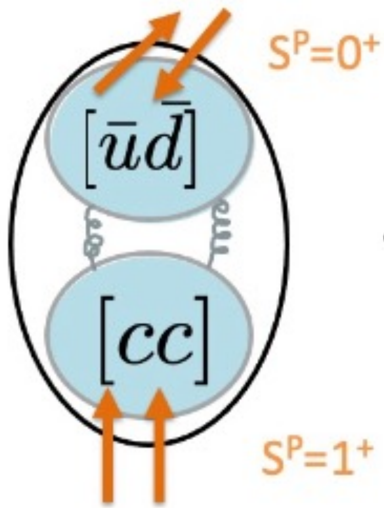



both conclusions support the presence of significant attraction at small distances and poles, likely related to Tcc

$cc\bar{d}\bar{u}$

# Dependence on $m_{u/d}$ and $m_c$

in case of diquark antidiquark binding mechanism

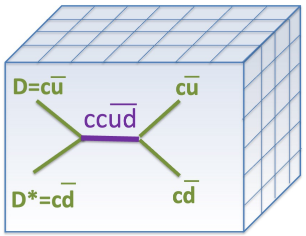



- [QQ] binding increases with increasing mQ
- [ud] binding decreases with decreasing $m_{u/d}$
- [QQ][ud] binding with respect to DD* threshold,
  dependence on $m_Q$ , $m_{u/d}$ : ??

generic/robust predictions for $\delta$m and 1/a0 needed:

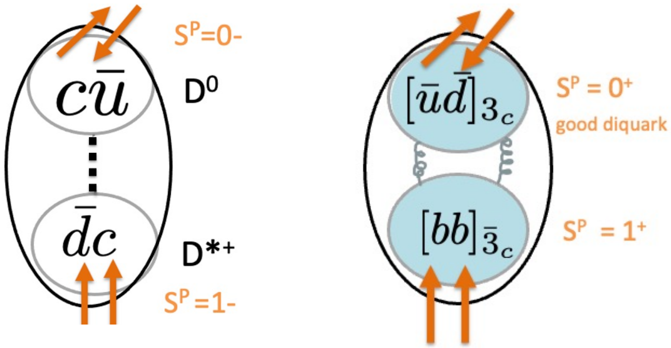
looking forward to test those from lattice

# Conclusions: $T_{cc}$ and its quark mass dependence from lattice




- Tcc is the longest-lived exotic hadron discovered in experiment
- lies near threshold -> has to be extracted from DD* scattering amplitude
- lattice studies find attraction
- attraction increases with decreasing pion mass
- attraction increases with increasing heavy quark mass

  this would (naively) imply that Tbc is more strongly bound than Tcc
- can quark-mass dependence be used to dissentangle which binding mechanism is dominant ?



# Backup

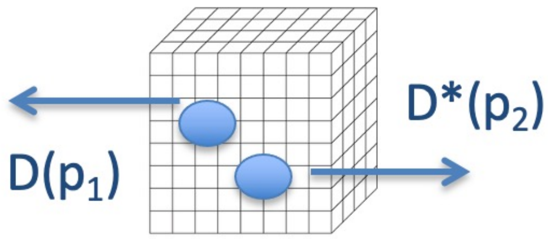
# Interpolators for Tcc

Example: P=0

$J^P=1^+$ -> cubic irrep $T_1^+$

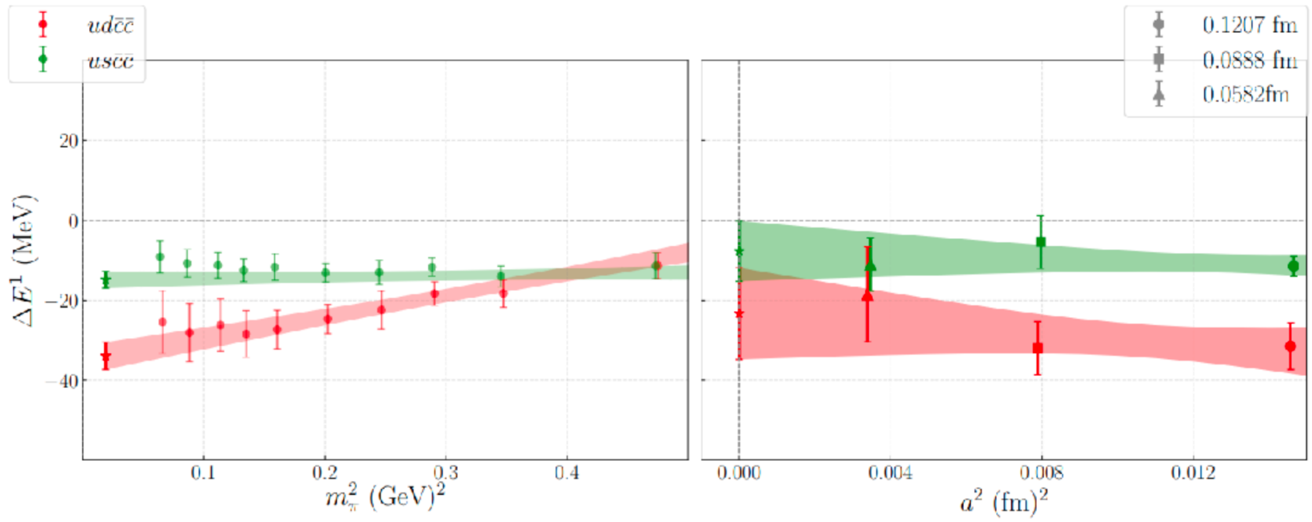
P=D, V=D*

$$
\begin{aligned}
O^{l=0} &= P(\{0,0,0\})V_z(\{0,0,0\})\\
O^{l=0} &= P(\{1,0,0\})V_z(\{-1,0,0\}) + P(\{-1,0,0\})V_z(\{1,0,0\})\\
&\quad + P(\{0,1,0\})V_z(\{0,-1,0\}) + P(\{0,-1,0\})V_z(\{0,1,0\})\\
&\quad + P(\{0,0,1\})V_z(\{0,0,-1\}) + P(\{0,0,-1\})V_z(\{0,0,1\})]\\
O^{l=2} &= P(\{1,0,0\})V_z(\{-1,0,0\}) + P(\{-1,0,0\})V_z(\{1,0,0\})\\
&\quad + P(\{0,1,0\})V_z(\{0,-1,0\}) + P(\{0,-1,0\})V_z(\{0,1,0\})\\
&\quad -2[P(\{0,0,1\})V_z(\{0,0,-1\}) + P(\{0,0,-1\})V_z(\{0,0,1\})]\\
O^{l=0} &= V_{1x}[0,0,0]V_{2_y}[0,0,0] - V_{1y}[0,0,0]V_{2x}[0,0,0]
\end{aligned}
$$



# Previous lattice QCD study of $T_{cc}$ channel

Junnarkar, Mathur, Padmanath, PRD 99, 034507 (2019), 1810.12285



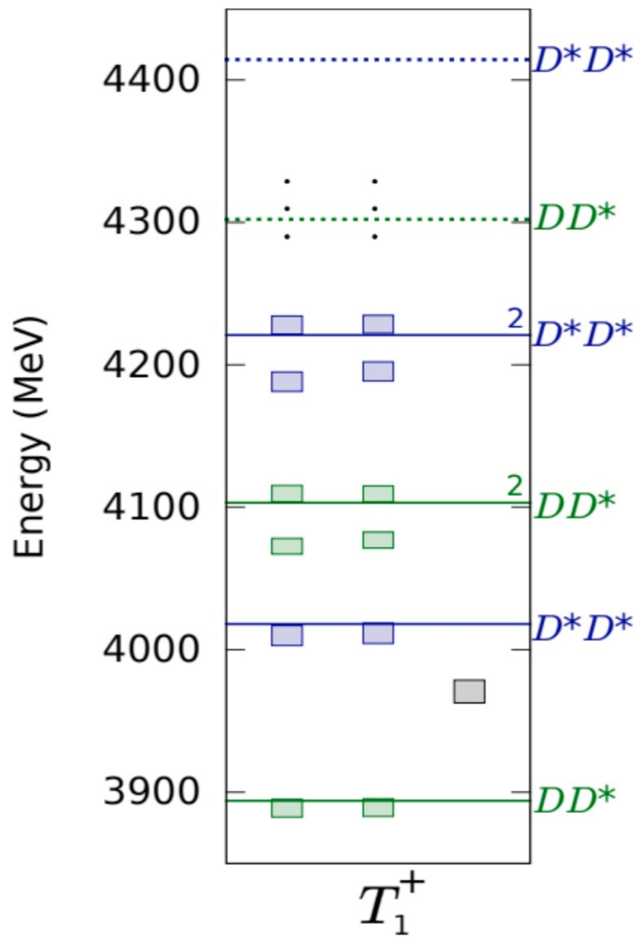

lowest finite-volume eigen-energy for P=0, $J^P=1^+$, I=0

- Study performed on LQCD ensembles with different lattice spacings. Single volume and only rest frame finite-volume irreps considered.
- Including a meson-meson and diquark-antidiquark interpolator. Diquark-antidiquark interpolators do not influence the low energy spectrum.
- The ground state energy subjected to chiral and continuum extrapolations.
- A finite-volume energy level 23(11) MeV below $DD^*$ threshold. No rigorous scattering analysis and no pole structure determined.

# Previous lattice QCD study of $T_{cc}$ channel

Hadron Spectrum, JHEP 11, 033 (2017), 1709.01417

finite-volume eigen-energies for P=0, $J^P=1^+$, I=0




- Single volume rest frame study on a relatively coarse lattice ($a_s \sim 0.12$ fm).
- Large basis of meson-meson and diquark-antidiquark interpolators.
- Diquark-antidiquark interpolators do not influence the low energy spectrum.
- No statistically significant energy shifts observed near $DD^*$ threshold.
  $\Rightarrow$ No scattering amplitude extraction.

# HALQCD study of Tcc

Lyu, Aoki et al, 2302.04505

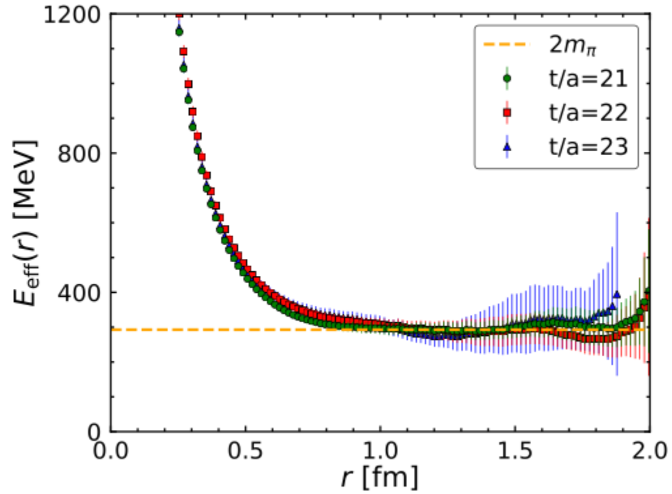
$$R(\boldsymbol{r},t) = \sum_{\boldsymbol{x}} \langle 0 | D^*(\boldsymbol{x}+\boldsymbol{r},t) D(\boldsymbol{x},t) \overline{\mathcal{J}}(0) | 0 \rangle \, / e^{-(m_{D^*}+m_D)t}$$

$$\left[\frac{1+3\delta^2}{8\mu}\partial_t^2 - \partial_t - H_0 + O(\delta^2\partial_t^3)\right] R(\boldsymbol{r},t) = \int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}') R(\boldsymbol{r}',t).$$

$$V(r) = R^{-1}(\boldsymbol{r},t)\left[\frac{1+3\delta^2}{8\mu}\partial_t^2 - \partial_t - H_0\right] R(\boldsymbol{r},t),$$

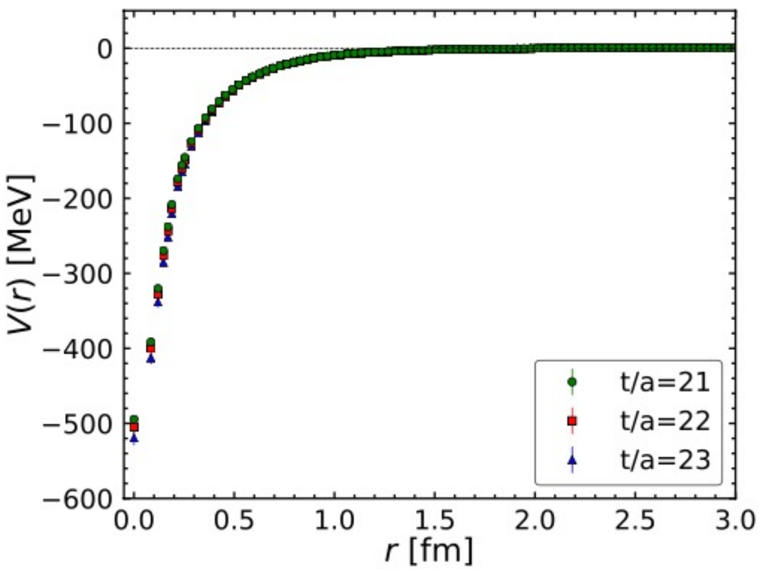
V(r) $\sim \frac{e^{-2m_\pi r}}{r^2}$ r > 1 fm

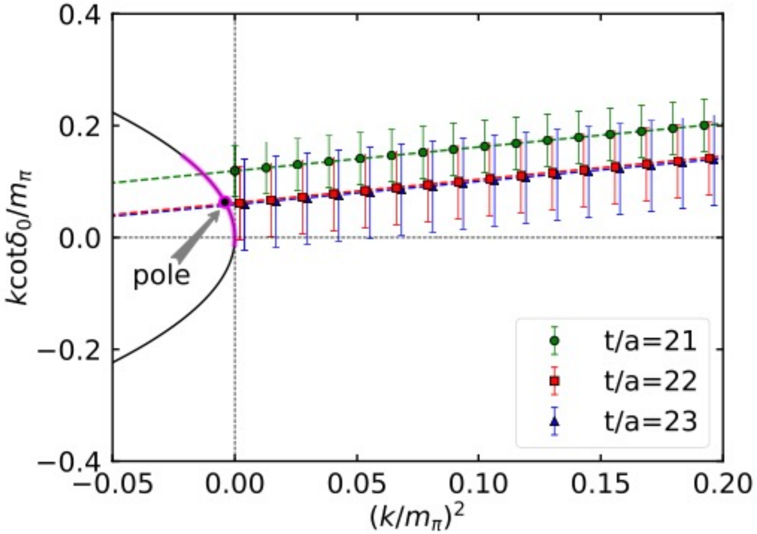
$$E_{\rm eff}(r) = -\frac{\ln[V(r)r^2/a_3]}{r}$$



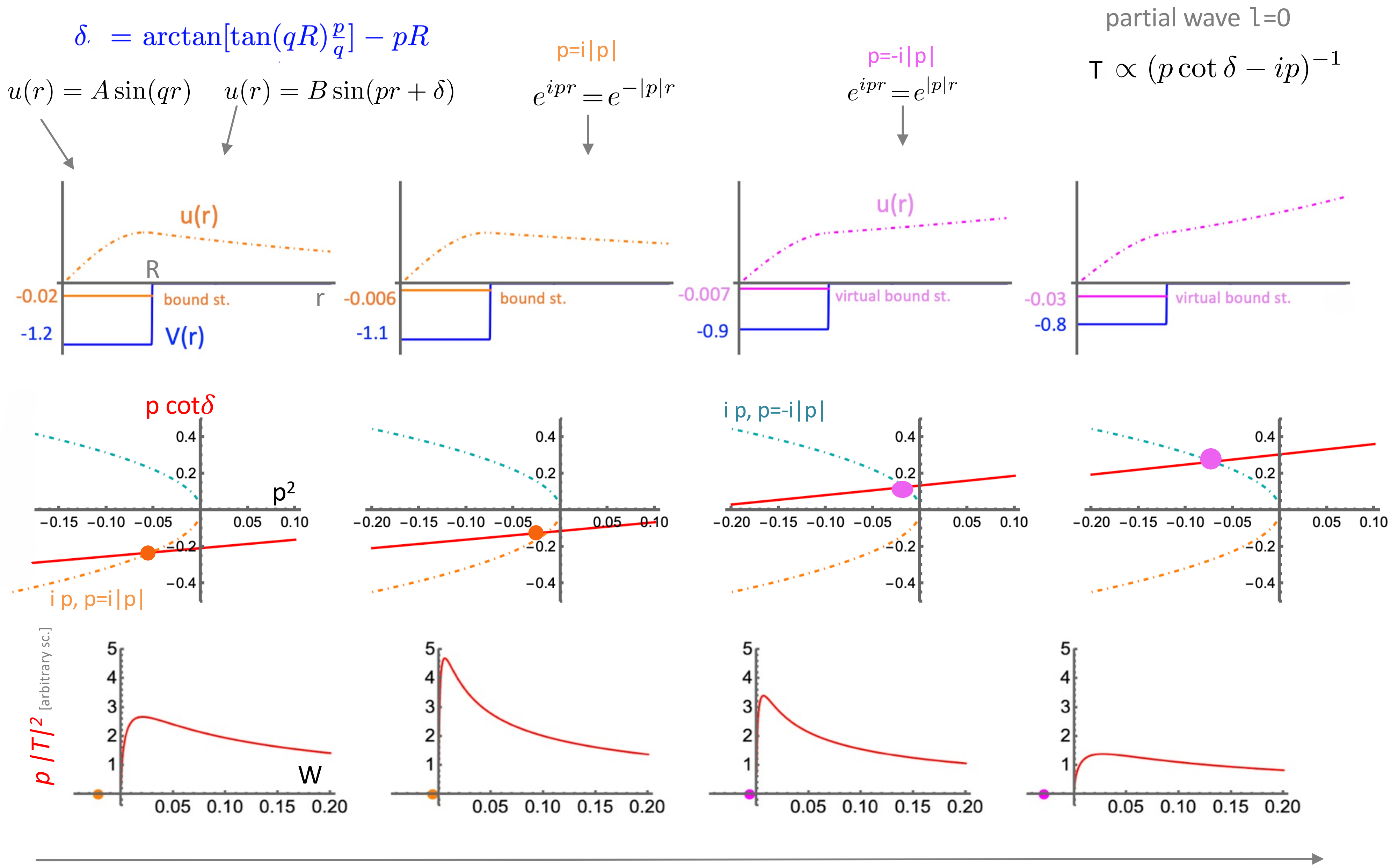

$$V^B_{\rm fit}(r;m_\pi) = \sum a_i e^{-(r/b_i)^2} + a_3(1-e^{-(r/b_3)^2})^n V_\pi^n$$

parameter set, $(a_1,a_2) = (-284(36), -201(60))$ in MeV, $a_3 = -45(12)$ MeV · fm$^2$, and $(b_1,b_2,b_3) = (0.15(2), 0.32(12), 0.49(24))$ in fm. Also, we find that



# Simplest Example: scattering in square-well potential in QM




increasing $m_{u/d}$ , decreasing attraction $V_0$ (or decreasing R)