Lattice QCD studies of heavy-heavy-light-light Tetraquarks

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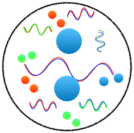
Overview

- Brief introduction to lattice studies
- Previous work
- Improvements in current analysis: Box sink method
  Multi-exponential fits
- Preliminary results
- Conclusion and outlook

Brief intro to lattice

$$\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A e^{-S[\bar{\psi},\psi,A]}$$



- Problem: QCD non-perturbative at low energies.
- Need to solve path integral. Discretise Euclidean action on a 4D lattice of space-time points with spacing a.
- Many discretisations of the action exist. We use $N_f = 2 + 1$ Wilson-Clover action [PACS-CS *0911.2561*].
- Calculate correlation functions by inserting operators and performing MC integration over quark and gluon fields.
- Calculations very expensive. Grows with smaller a, needed for larger m_h.
- Solution - use NRQCD for heavy mass, effective for $m_c \lesssim m_h$. NRQCD is hard to extrapolate to continuum.

Previous work: chiral analysis on $q_1 q_2 \bar{b}\bar{b}$

Our previous work [1607.05214] considered $J^P = 1^+$ $T_{ud\bar{b}\bar{b}}$ and $T_{\ell s\bar{b}\bar{b}}$ for three values of the pion mass (164, 299, 415 MeV). Two operators were used:

$$M(x) = \bar{b}_a^\alpha(x)\gamma_5^{\alpha\beta}u_a^\beta(x)\bar{b}_b^\kappa(x)\gamma_i^{\kappa\rho}u_b^\rho(x) - u \leftrightarrow d$$

$$D(x) = (u_a^\alpha(x))^T(C\gamma_5)^{\alpha\beta}d_b^\beta(x)\bar{b}_a^\kappa(x)(C\gamma_i)^{\kappa\rho}(\bar{b}_b^\rho(x))^T$$

Wall sources were used to generate the propagators. Local sinks were used.

Previous work: chiral analysis on $q_1 q_2 \bar{b}\bar{b}$

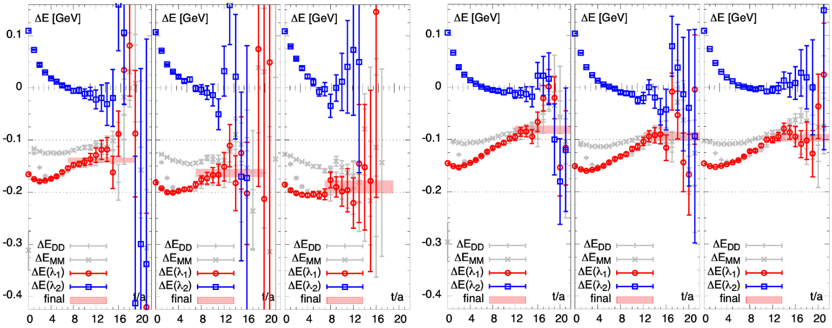
A GEVP analysis was performed using $G_{\mathcal{O}_1\mathcal{O}_2}(t) = \frac{G_{\mathcal{O}_1\mathcal{O}_2}(t)}{C_{PP}(t)V_{PP}(t)}$

$$F(t) = \begin{pmatrix} GDD(t) & GDM(t) \\ GMD(t) & GMM(t) \end{pmatrix}$$

$$F(t)\nu = \lambda(t)F(t_0)\nu$$
$$\lambda(t) = Ae^{-\Delta E(t-t_0)} = (1+\delta)e^{-\Delta E(t-t_0)}$$

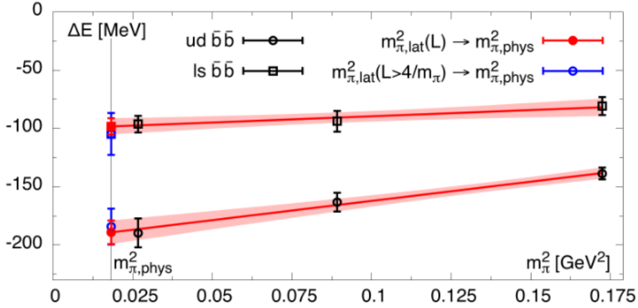
The 2×2 matrix gives 2 Eigenvalues. The ground state, and a mixture of all excited states. Extract ΔE from a single exponential fit to $\lambda(t)$.

Previous work: chiral analysis on $q_1 q_2 \bar{b}\bar{b}$



ΔE for the first and second GEVP Eigenvalues, relative to BB^* ($B_s B^*$) for $udbb$ [left] ($lsbb$ [right]). Plateaus are quite short and appear to rise.

Previous work: chiral analysis on $q_1 q_2 \bar{b}\bar{b}$



Ensemble	$\Delta E_{ud\bar{b}\bar{b}}$[MeV]	$\Delta E_{\ell s\bar{b}\bar{b}}$[MeV]
E_H	-139(5)	-81(8)
E_M	-163(8)	-94(9)
E_L	-190(12)	-96(7)
Phys	-189(10)(3)	-98(7)(3)

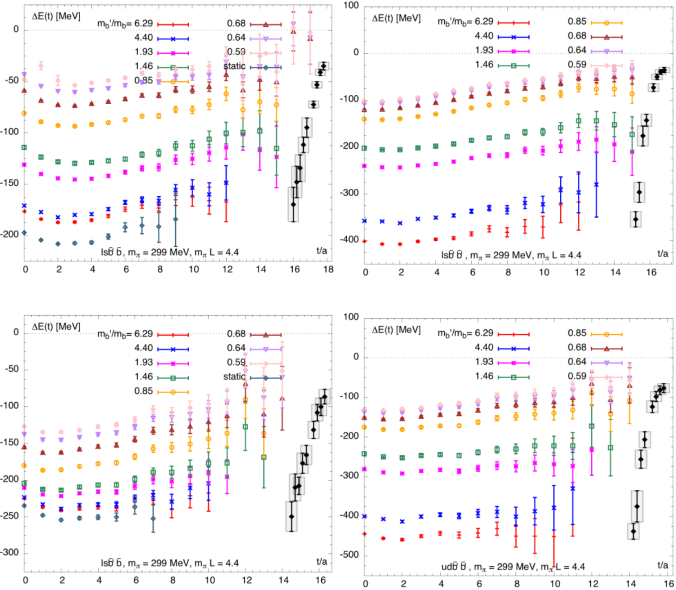
Bindings were extrapolated in m_π, with ΔE of 189(10)MeV and 98(7)MeV for $T_{ud\bar{b}\bar{b}}$ and $T_{\ell s\bar{b}\bar{b}}$ respectively.

Previous work: m_h analysis on $q_1 q_2 \bar{h}\bar{h}$ and $q_1 q_2 \bar{b}\bar{h}$

Next step in [1810.10550] :

- Uses same ensembles with $m_\pi = 164$, 299 and 415 MeV, looking for $J^P = 1^+$ tetraquark.
- Variable heavy mass $am_h \in \{0.9, 1.0, 1.2, 1.6, 3.0, 4.0, 8.0, 10.0\}$
- Four channels: $ud\bar{b}\bar{h}$, $ud\bar{h}\bar{h}$, $\ell s\bar{b}\bar{h}$, $\ell s\bar{h}\bar{h}$.
- Again, use GEVP and wall-local correlators. For $q_1 q_2 \bar{b}\bar{h}$ case, GEVP is 3×3.

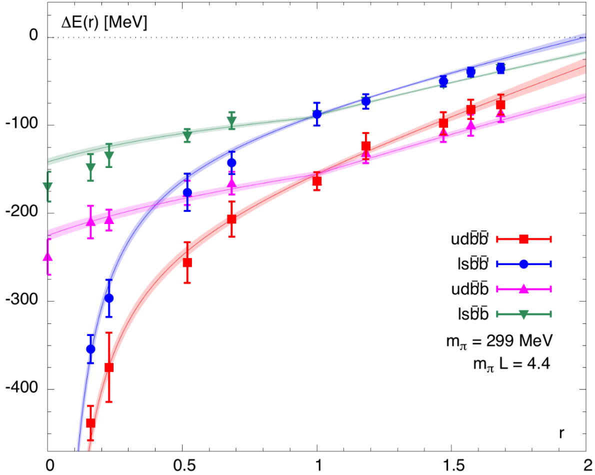
Previous work: m_h analysis on $q_1 q_2 \bar{h}\bar{h}$ and $q_1 q_2 \bar{b}\bar{h}$



Plateaus are better but often appear to be rising.

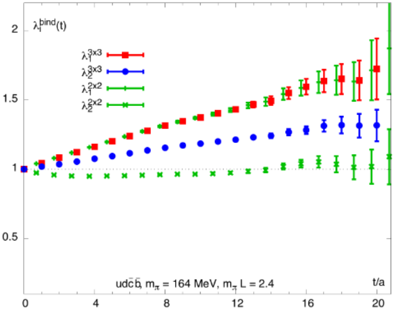
Note $b' \equiv h$.

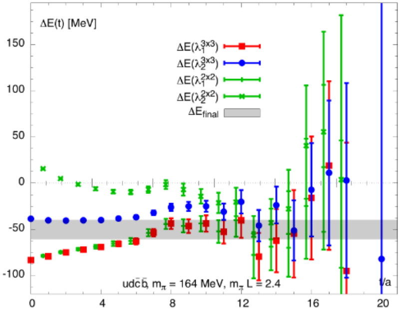
Previous work: m_h analysis on $q_1 q_2 \bar{h}\bar{h}$ and $q_1 q_2 \bar{b}\bar{h}$



We get $q_1 q_2 \bar{b}\bar{h}$ point at $m_h = \infty$ from static propagator.

Previous work: m_h analysis on $q_1q_2\bar{h}\bar{h}$ and $q_1q_2\bar{b}\bar{h}$





Work also carried out at charm mass ($ud\bar{b}\bar{c}$). Ground state is unaffected by 2×2 or 3×3, but plateaus still short. Find a binding of 38(23)MeV.

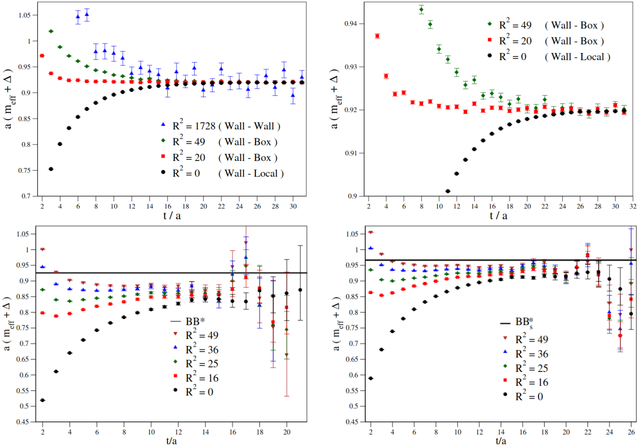
Previous work: src snk analysis on $q_1 q_2 \bar{h}_1 \bar{h}_2$

Most recent work in [2006.14294] :

- Uses meson-meson and diquark-diquark operators
- Large range of $J^P = 1^+$ tetraquarks: $udcb$, $udsb$, $udsc$, $udbb$, $lsbb$, $ucbb$, $scbb$, $uscb$.
- Looked at $J^P = 0^+$ too.
- Used GEVP method and a single ensemble with $m_\pi = 192$MeV
- Key difference - introduction of box sinks

$$S^{B,R} = \frac{1}{N} \sum_{r^2 \leq R^2} S(x + r, t)$$

Previous work: src snk analysis on $q_1q_2\bar{h}_1\bar{h}_2$



Box sink for B_c (top), $ud\bar{b}\bar{b}$(bot. l) and $ls\bar{b}\bar{b}$(bot. r). Different radii change direction of convergence.

Previous work: src snk analysis on $q_1 q_2 \bar{h}_1 \bar{h}_2$

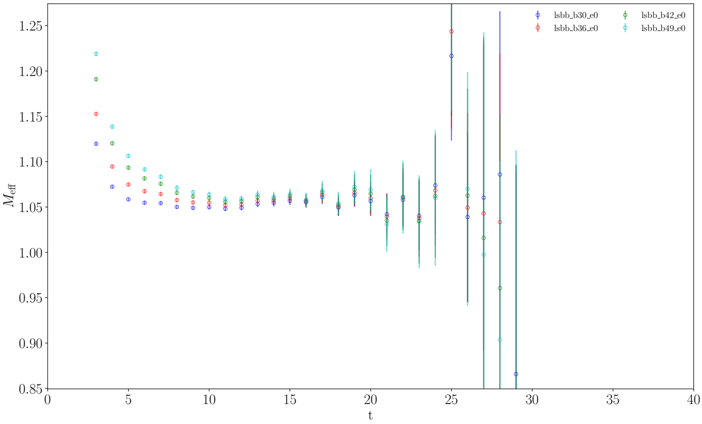
- Only find deep binding in *udbb* and *lsbb* channels 113 and 36 MeV respectively
- Did not find measureable binding in *udcb* - not deeply bound
- Shallow binding can't be ruled out without FV (Luscher) analysis.

Current work on $q_1 q_2 \bar{h}\bar{h}$ (PRELIMINARY)

Current work builds on [2006.14294] , and applys it to chiral and m_h analyses.

- Uses a large number of (local) operators and four box sink radii
- Focus on $J^P = 1^+$ $udhh$ and $lshh$ tetraquarks
- Carry out heavy mass analysis with nine m_h values
- Also chiral extrapolation with six m_π values
- Use multi-exponential fits to extract energies

Current work on $q_1 q_2 \bar{h}\bar{h}$ (PRELIMINARY)



We find the box sink method leads to improved plateaus.

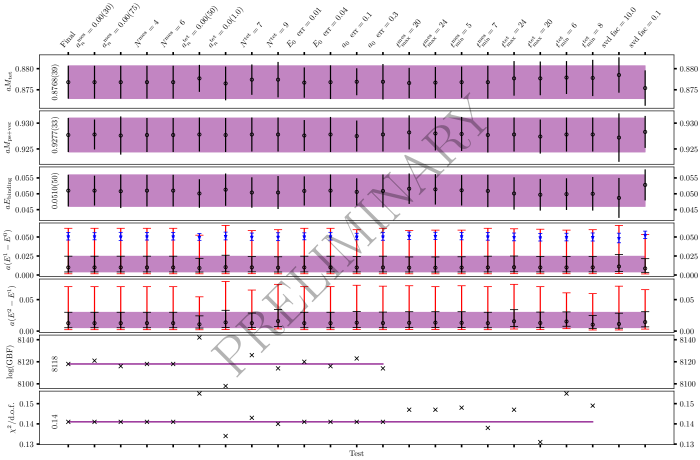
Current work on $q_1 q_2 \bar{h}\bar{h}$ (PRELIMINARY)

We perform correlated, simultaneous fits to the meson and tetraquark data, across all the source-sink combinations, for all sink radii.

$$C_2^{\text{mes/tet}} = \sum_n^N a_n^{\text{src}} a_n^{\text{snk}} \left(e^{-E_n^{\text{mes/tet}} t} \pm e^{-E_n^{\text{mes/tet}}(T-t)}\right)$$

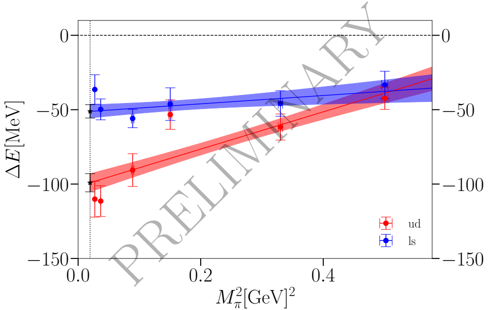
Bayesian fitting approach, with χ^2 and the Gaussian Bayes factor to judge fit quality.

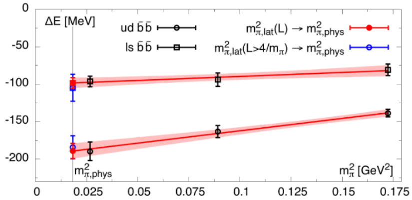
Current work on $q_1 q_2 \bar{h}\bar{h}$ (PRELIMINARY)



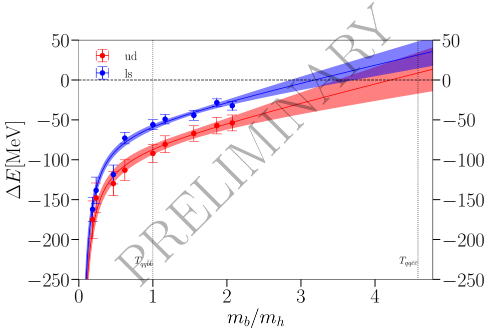
We find our fits to be very stable.

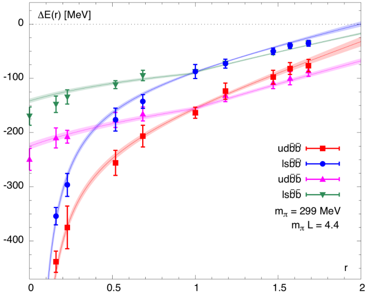
Current work on $q_1q_2\bar{h}\bar{h}$ (PRELIMINARY)





Current work on $q_1 q_2 \bar{h}\bar{h}$ (PRELIMINARY)





Binding shallower than has been found in the past.

Conclusion and outlook

Current work builds on [2006.14294]

- Previous work identified $J^p = 1^+$ $udbb$, $lsbb$ as strongly bound
- $udbc$ not strongly bound - requires Luscher method ([2205.13982] agrees)
- Box sink improve plateaus
- Preliminary analysis seems to suggest lower binding then previously, agrees with [2205.13982] on $\ell s\bar{b}\bar{b}$
- Future analysis of $udbc$ to come

Back up

Type ($\psi\phi\theta\omega$)	$I(J)^P$	Diquark-Antidiquark	Dimeson
$udcb/udsb/udsc$	$0(1)^+$	$D(\gamma_5,\gamma_i), D(\gamma_t\gamma_5,\gamma_i\gamma_t)$ $E(\gamma_5,\gamma_i), E(\gamma_t\gamma_5,\gamma_i\gamma_t)$	$M(\gamma_5,\gamma_i)-N(\gamma_5,\gamma_i)$ $M(I,\gamma_i\gamma_5)-N(I,\gamma_i\gamma_5)$ $O(\gamma_5,\gamma_i)-P(\gamma_5,\gamma_i)$ $O(I,\gamma_i\gamma_5)-P(I,\gamma_i\gamma_5)$ $\epsilon_{ijk}M(\gamma_j,\gamma_k)$
$udbb$	$0(1)^+$	$D(\gamma_5,\gamma_i),\ D(\gamma_t\gamma_5,\gamma_i\gamma_t)$	$M(\gamma_5,\gamma_i)-N(\gamma_5,\gamma_i)$ $M(I,\gamma_i\gamma_5)-N(I,\gamma_i\gamma_5)$
$lsbb/ucbb/scbb$	$\frac{1}{2}(1)^+$	$D(\gamma_5,\gamma_i),\ D(\gamma_t\gamma_5,\gamma_i\gamma_t)$	$M(\gamma_5,\gamma_i),\ M(I,\gamma_i\gamma_5)$ $N(\gamma_5,\gamma_i),\ N(I,\gamma_i\gamma_5)$ $\epsilon_{ijk}M(\gamma_j,\gamma_k)$
$uscb$	$\frac{1}{2}(1)^+$	$D(\gamma_5,\gamma_i),\ D(\gamma_t\gamma_5,\gamma_i\gamma_t)$ $E(\gamma_5,\gamma_i),\ E(\gamma_t\gamma_5,\gamma_i\gamma_t)$	$M(\gamma_5,\gamma_i),\ M(I,\gamma_i\gamma_5)$ $N(\gamma_5,\gamma_i),\ N(I,\gamma_i\gamma_5)$ $O(\gamma_5,\gamma_i),\ O(I,\gamma_i\gamma_5)$ $\epsilon_{ijk}M(\gamma_j,\gamma_k)$

TABLE I: $J^P = 1^+$ operators used in this work.

Type ($\psi\phi\theta\omega$)	$I(J)^P$	Diquark-Antidiquark	Dimeson
$udcb/udsb/udsc$	$0(0)^+$	$E(\gamma_5,\gamma_5), E(\gamma_t\gamma_5,\gamma_t\gamma_5)$	$M(\gamma_5,\gamma_5)-N(\gamma_5,\gamma_5)$ $M(I,I)-N(I,I)$ $M(\gamma_i,\gamma_i)$
$uscb$	$\frac{1}{2}(0)^+$	$E(\gamma_5,\gamma_5),\ E(\gamma_t\gamma_5,\gamma_t\gamma_5)$	$M(\gamma_5,\gamma_5),\ M(I,I)$ $N(\gamma_5,\gamma_5),\ N(I,I)$ $M(\gamma_i,\gamma_i)$

$$D(\Gamma_1,\Gamma_2) = (\psi_a^T C\Gamma_1\phi_b)(\bar\theta_a C\Gamma_2\bar\omega_b^T),$$
$$E(\Gamma_1,\Gamma_2) = (\psi_a^T C\Gamma_1\phi_b)(\bar\theta_a C\Gamma_2\bar\omega_b^T - \bar\theta_b C\Gamma_2\bar\omega_a^T),$$
$$M(\Gamma_1,\Gamma_2) = (\bar\theta\Gamma_1\psi)(\bar\omega\Gamma_2\phi), \qquad N(\Gamma_1,\Gamma_2) = (\bar\theta\Gamma_1\phi)(\bar\omega\Gamma_2\psi),$$
$$O(\Gamma_1,\Gamma_2) = (\bar\omega\Gamma_1\psi)(\bar\theta\Gamma_2\phi), \qquad P(\Gamma_1,\Gamma_2) = (\bar\omega\Gamma_1\phi)(\bar\theta\Gamma_2\psi).$$

Back up

$$\Delta E = \frac{C_0}{2r} + C_1^{ud} + C_2^{ud}\,(2r) + 23 \text{ MeV}\, r \tag{1}$$

for the $ud\bar{b}'\bar{b}'$ case, where the first term represents the Coulomb binding contribution, the second the good-ud-diquark attraction, the third the residual heavy-light interactions in the tetraquark state and the fourth the two-meson threshold contribution. The numerical value appearing in the fourth term follows from the observed meson splittings. Similarly, for the $ud\bar{b}'\bar{b}$ case, one expects the form

$$\Delta E = \frac{C_0}{1+r} + C_1^{ud} + C_2^{ud}\,(1+r) + (34 \text{ MeV} - 11 \text{ MeV}\, r) \tag{2}$$

to provide a good representation for $m_{b'} > m_b$ and the form

$$\Delta E = \frac{C_0}{1+r} + C_1^{ud} + C_2^{ud}\,(1+r) + (34 \text{ MeV}\, r - 11 \text{ MeV}) \tag{3}$$

to provide a good representation for $m_{b'} < m_b$. The corresponding expectations for the cases involving an ℓs, rather than ud, good-diquark are

$$\Delta E = \frac{C_0}{2r} + C_1^{\ell s} + C_2^{\ell s}\,(2r) + 24 \text{ MeV}\, r \tag{4}$$

for $\ell s\bar{b}'\bar{b}'$,

$$\Delta E = \frac{C_0}{1+r} + C_1^{\ell s} + C_2^{\ell s}\,(1+r) + (34 \text{ MeV} - 12 \text{ MeV} r) \tag{5}$$

for $\ell s\bar{b}'\bar{b}$ with $m_{b'} > m_b$ and

$$\Delta E = \frac{C_0}{1+r} + C_1^{\ell s} + C_2^{\ell s}\,(1+r) + (36 \text{ MeV}\, r - 11 \text{ MeV}) \tag{6}$$

for $\ell s\bar{b}'\bar{b}$ with $m_{b'} < m_b$.

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