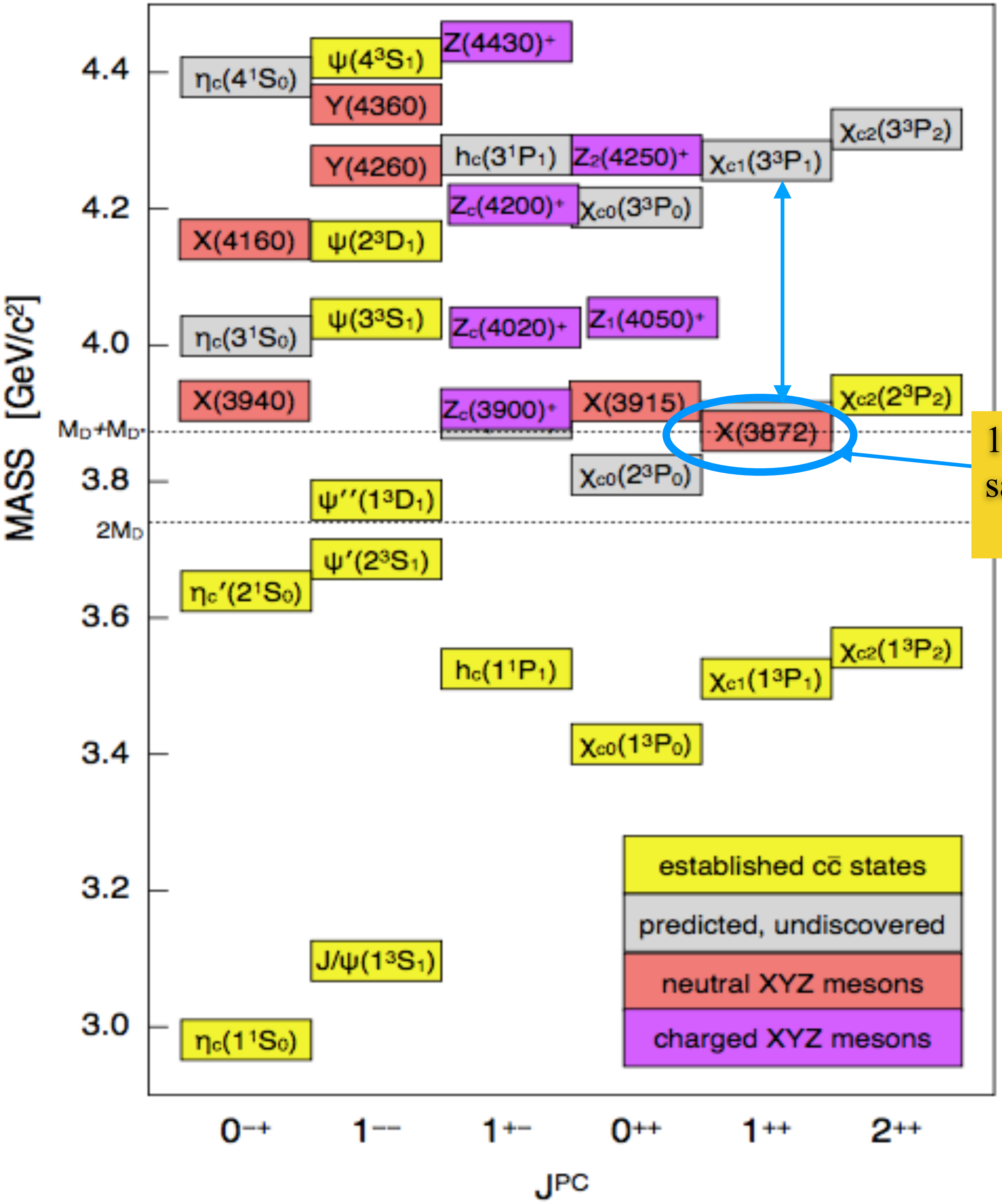
$T_{QQ\bar{q}\bar{q}}$ mass and decays: Quark model and Born-Oppenheimer approximation in QCD

Luciano Maiani
Roma University La Sapienza and INFN, Roma, Italy

1. Expected and Unexpected Charmonia

figure by:
S. L. Olsen (2015)
arXiv:1511.01589



1st Unexpected (2003): same q# as $\chi_{c1} 3^3P_1$ but considerably lighter

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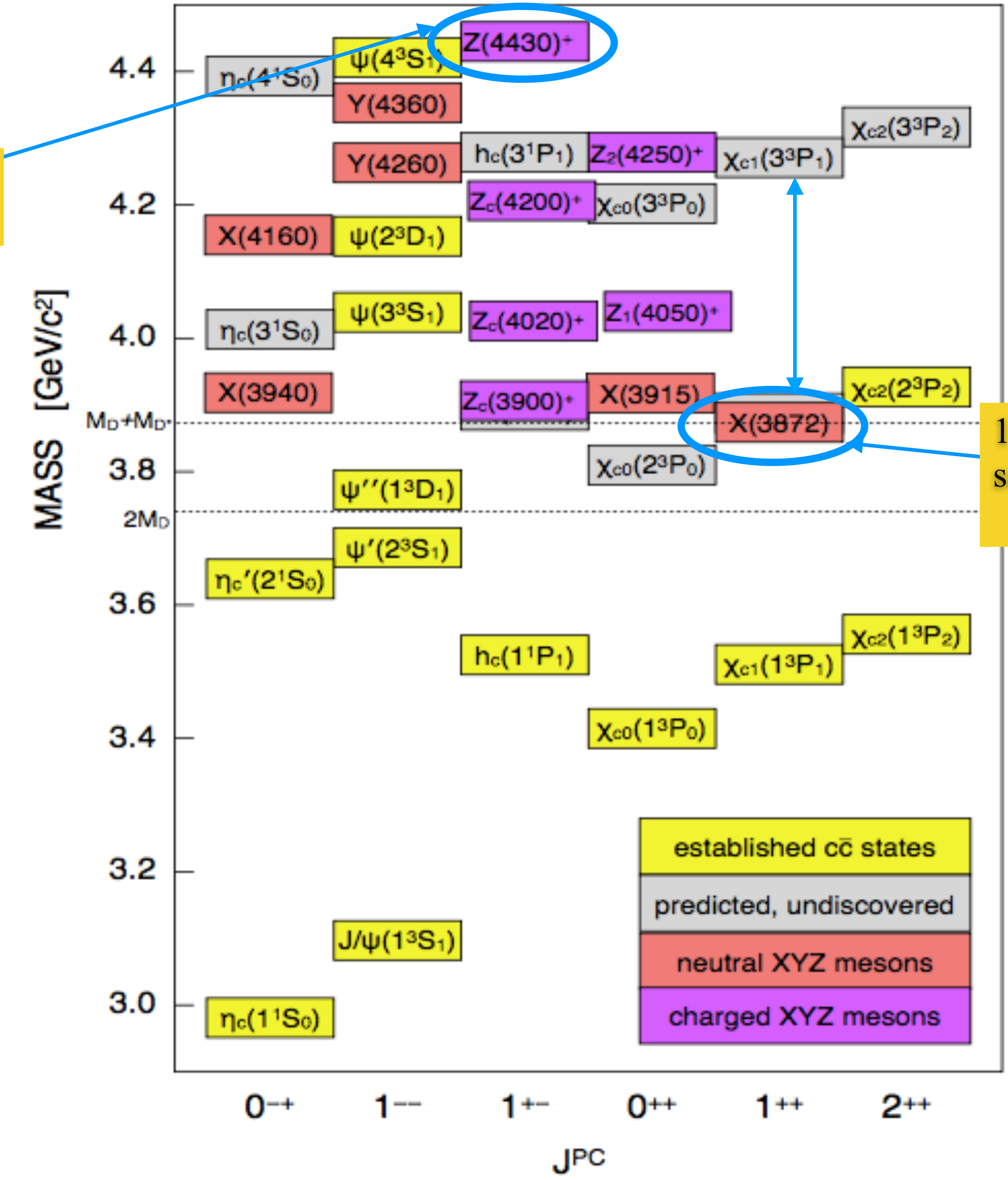

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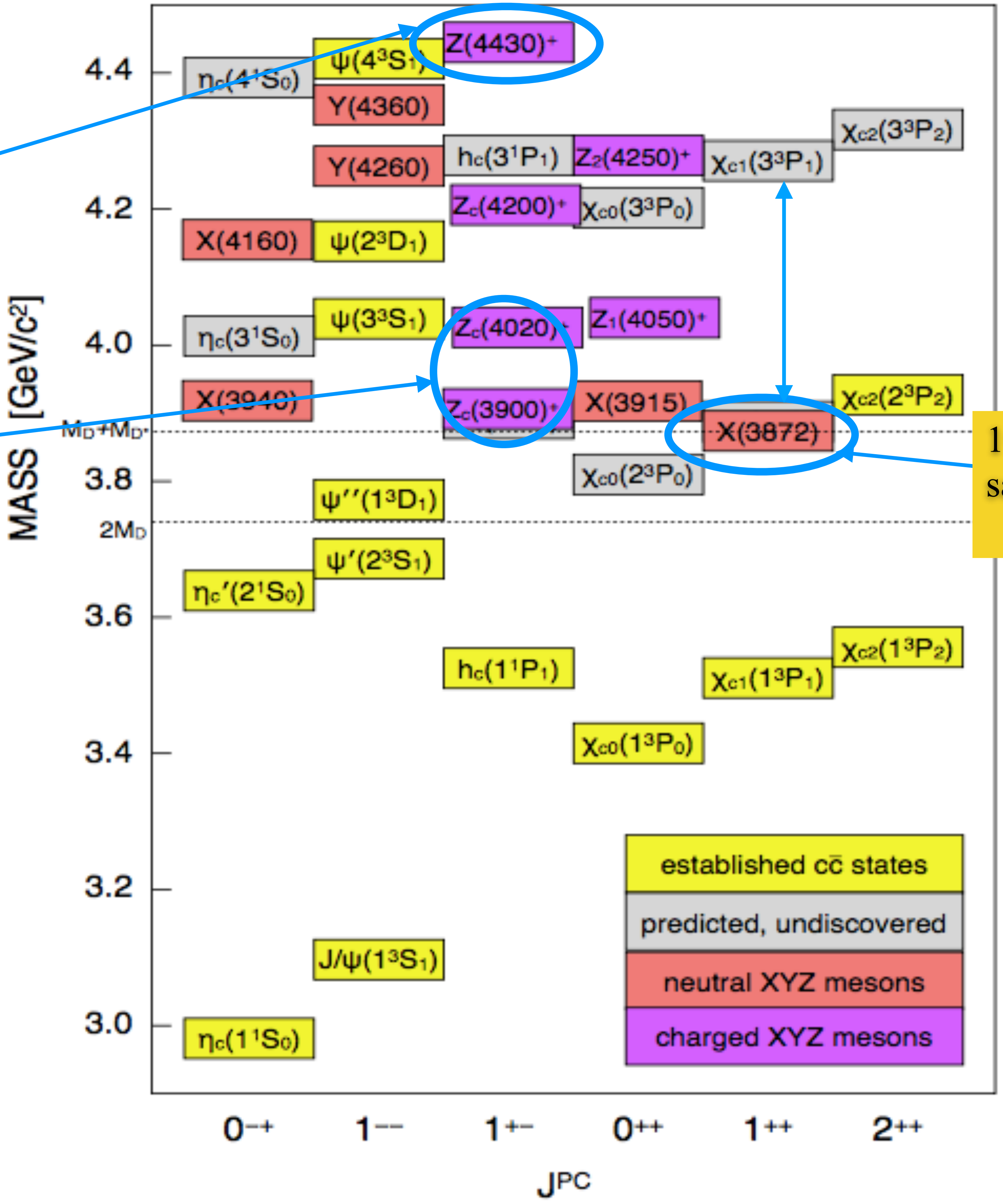

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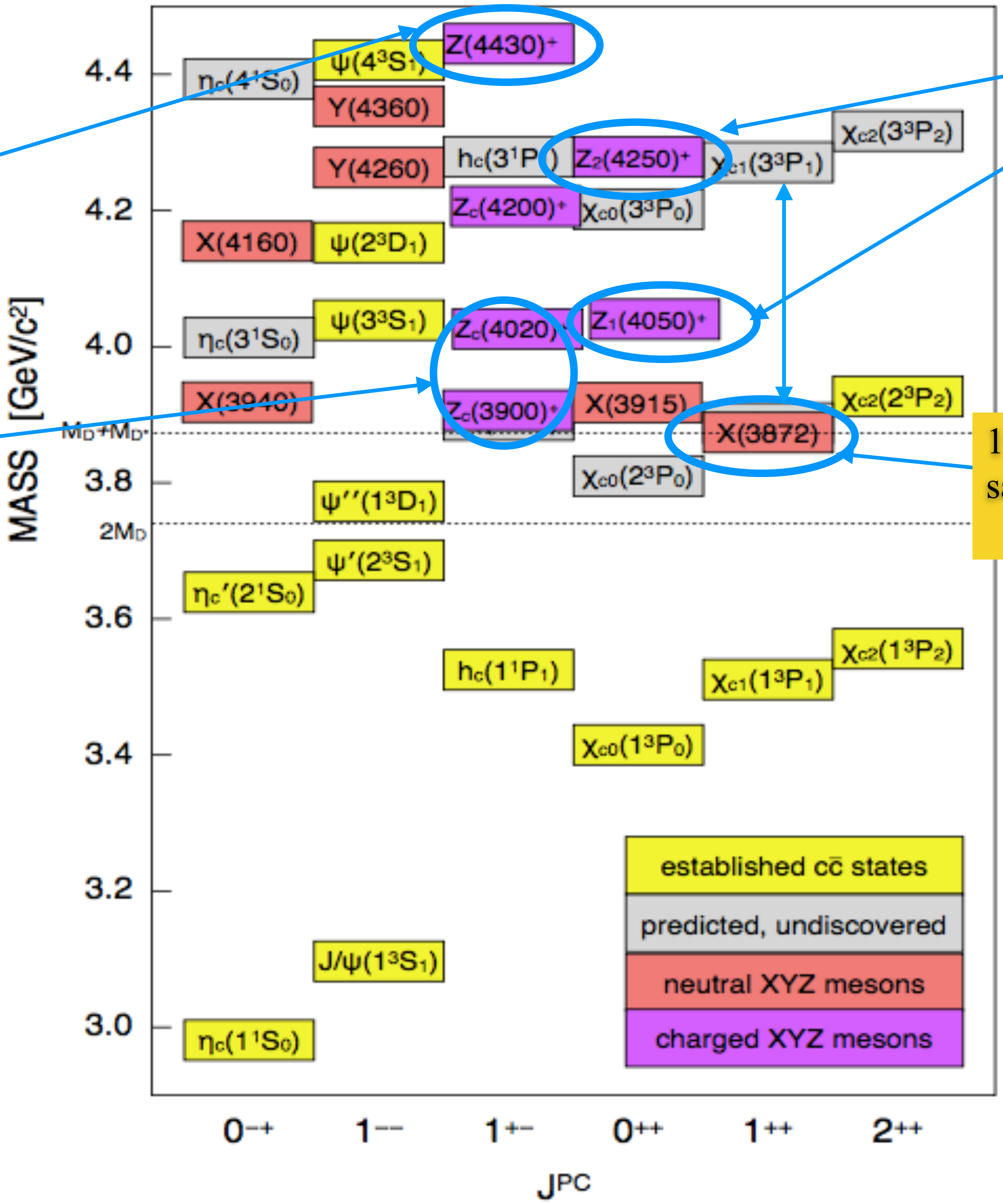

recent additions:
more than coincidence?
or
an almost filled multiplet?

2nd Unexpected (2007)
a radial excitation?

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Terminology

I. Bigi, L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer,
Four-quark mesons in non-leptonic B decays"
Phys. Rev. D **72** (2005), 114016; [arXiv:hep-ph/0510307 [hep-ph]].

- R. Jaffe (1977): light scalar mesons = *diquark-antidiquark states*
- Maiani et al. (2005) $X(3872)$ = *diquark-antidiquark* state
- Bigi et al. (2005) hidden charm exotic mesons=*4-quark states(for brief)*
- later *tetraquarks*", then "*compact tetraquarks*"…
- …to distinguish from molecular model: 4-quarks segregated into two mesons and bound in a *hadron molecule* by pion exchange (like atomic nuclei);

- Bigi et al. were the first to suggest that non-leptonic B meson decays could provide an efficient source of exotic hadrons, *a process much used in later experiments to discover tetra- and pentaquarks*.

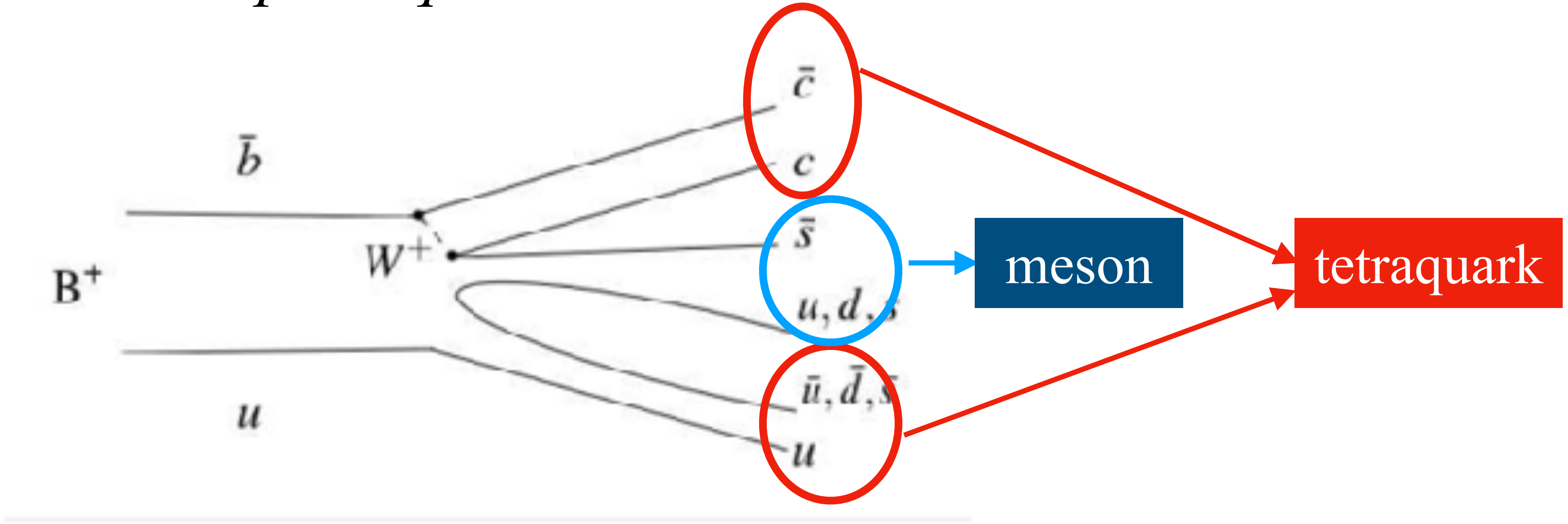
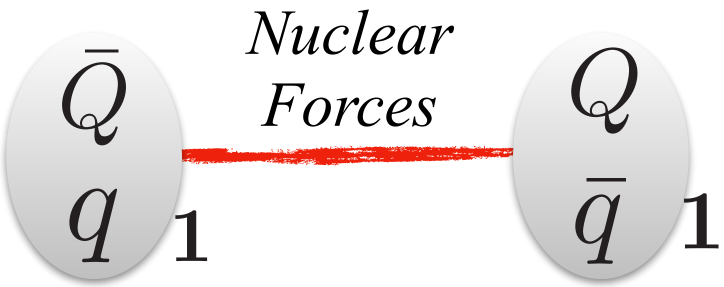


FIG. 1: Quark diagram for $B^+ \to K + X$, with $X = (c\bar{c}q\bar{q}')$.

No consensus, yet , on the structure of Hidden Charm Tetraquarks



Hadron Molecule

F-K. Guo, C. Hanhart, U-G Meißner, Q. Wang, Q. Zhao, and B-S Zou, arXiv 1705.00141 (2017)

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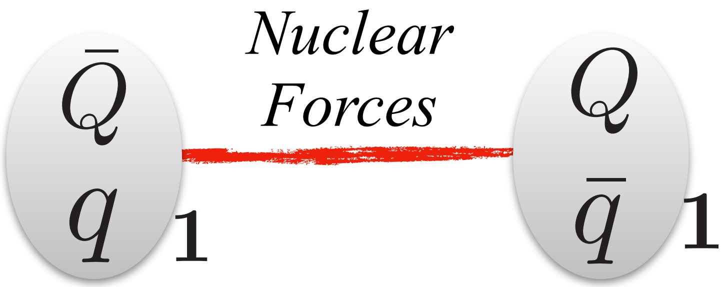

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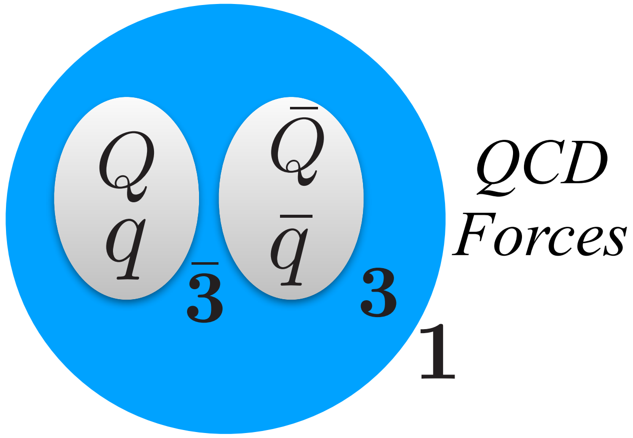

Compact Tetraquark

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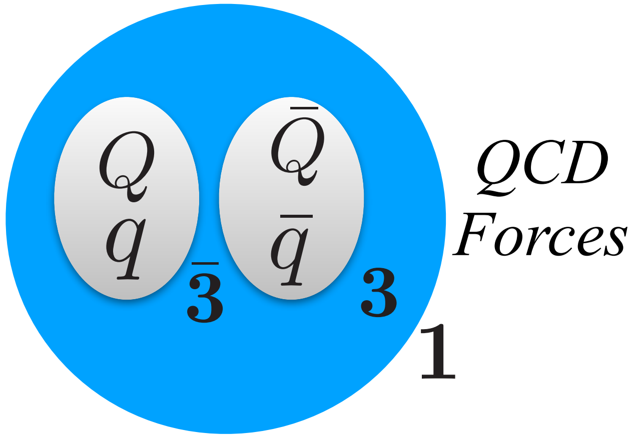

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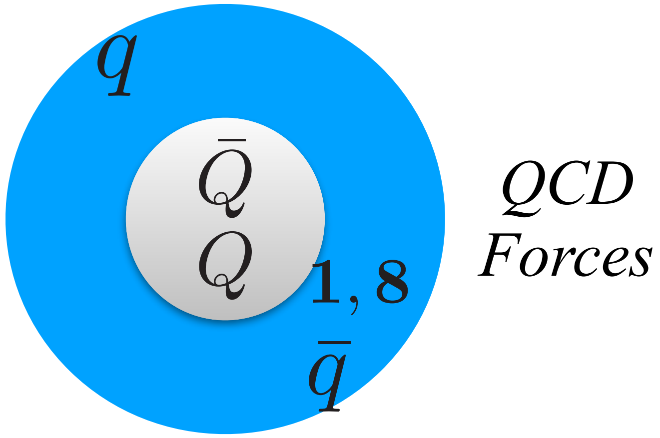

HadroCharmonium (1)
Quarkonium Adjoint Meson (8)

S. Dubynskiy, S. and M. B.Voloshin, Phys. Lett. **B 666**,(2008) 344.

E. Braaten, C. Langmack and D. H. Smith, Phys. Rev. **D 90** (2014) 01404

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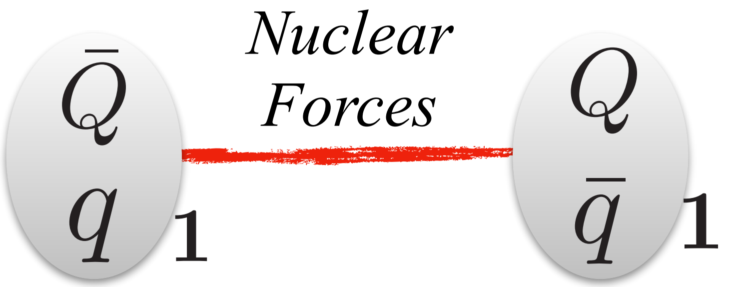

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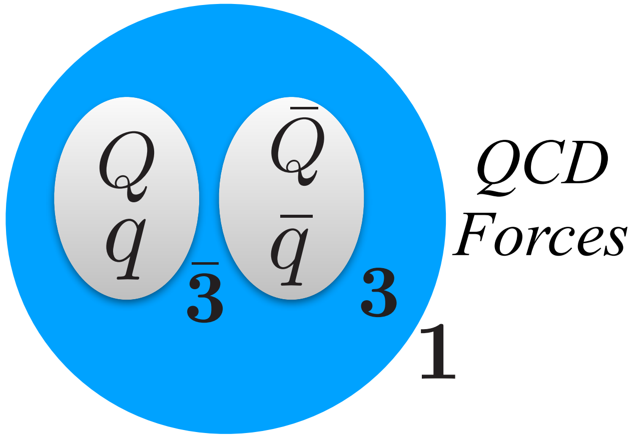

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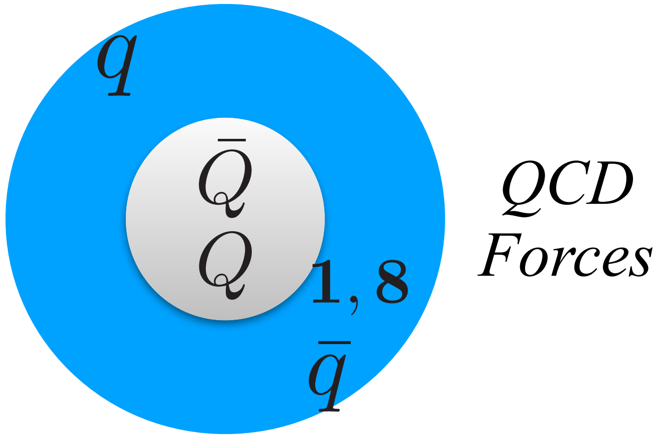

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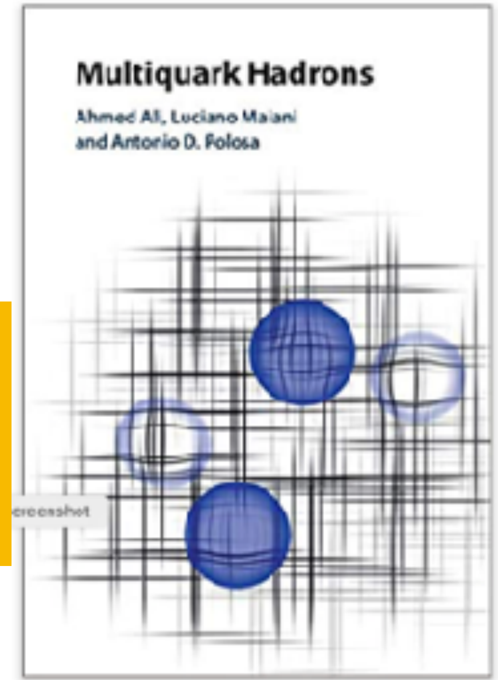
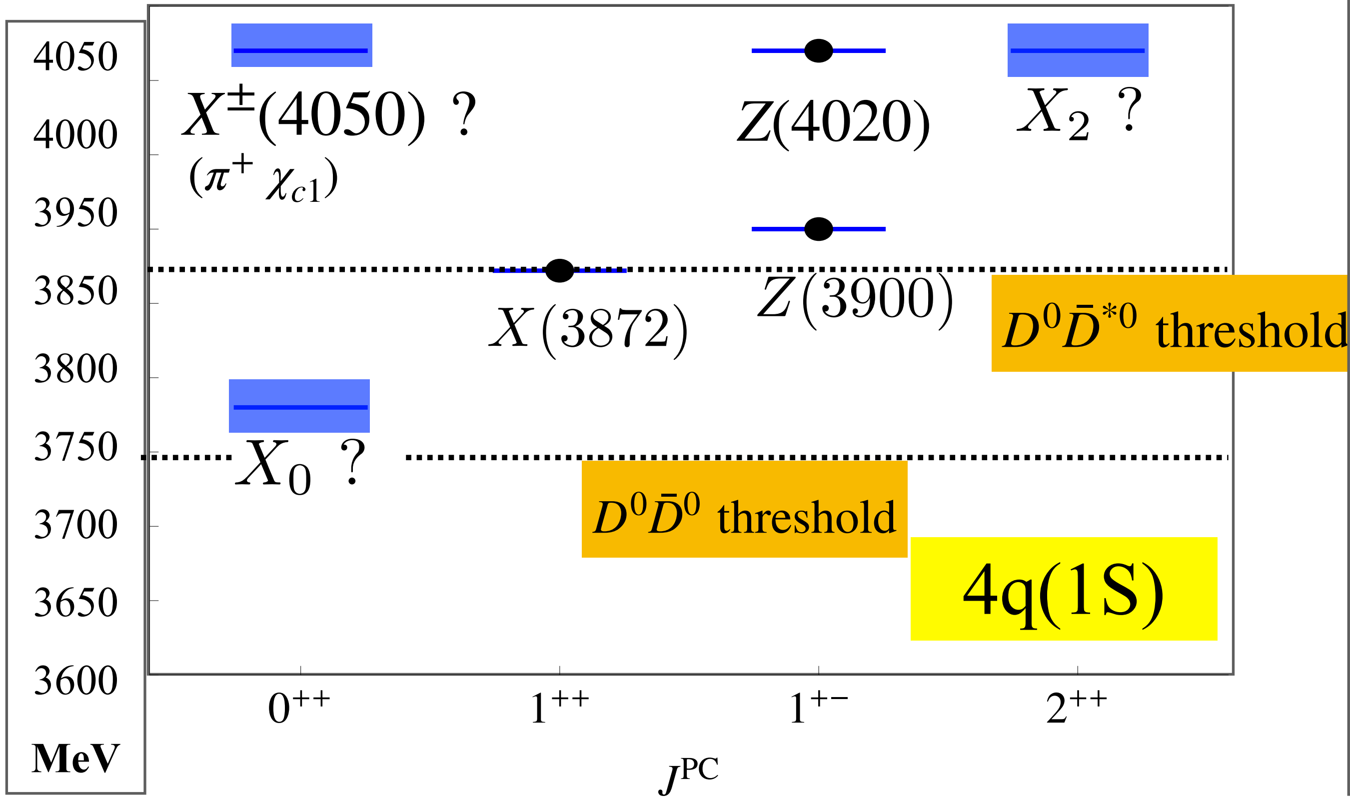
More variations in recent literature
For a review, see:
A. Ali, L. Maiani and A.D. Polosa, *Multiquark Hadrons*, Cambridge University Press (2019)

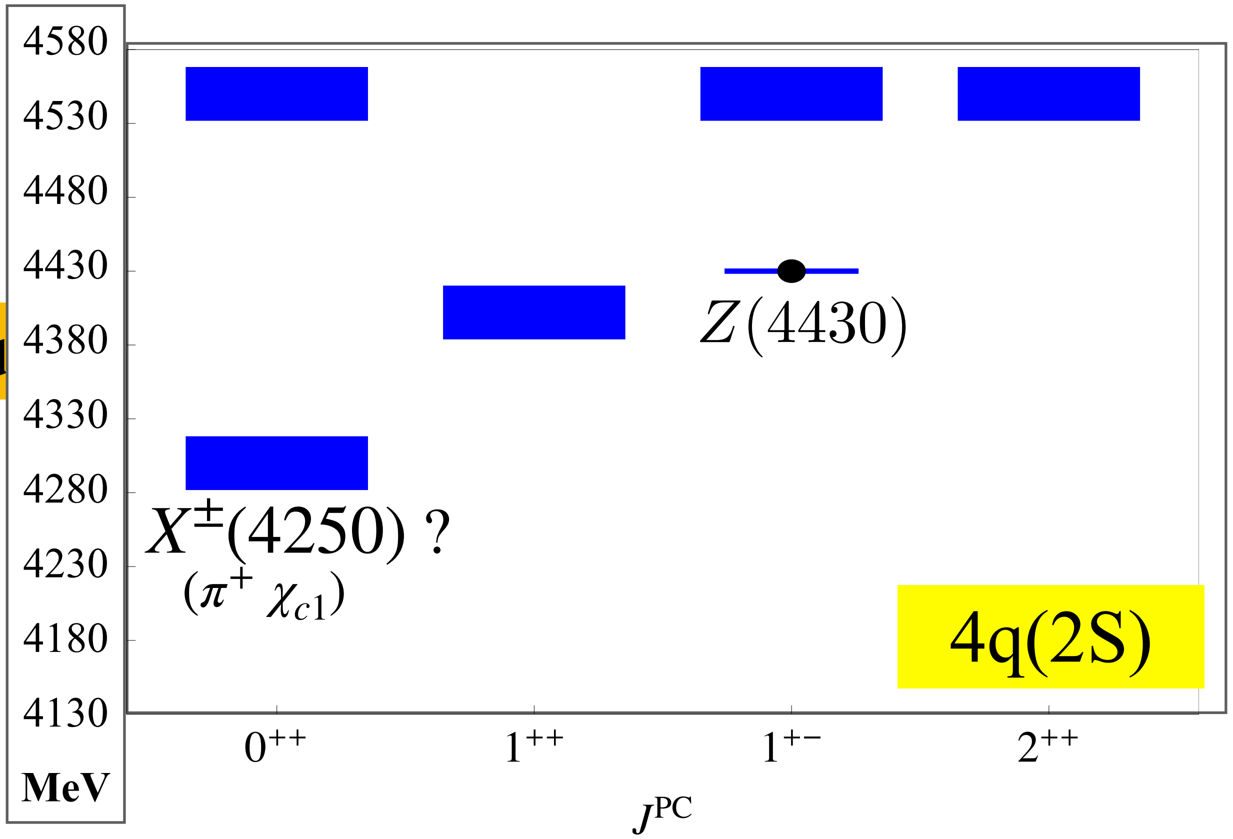


Multiquark Hadrons
Ahmed Ali, Luciano Maiani and Antonio D. Polosa

2. Compact tetraquarks $[cq][\bar{c}\bar{q}']$: the first multiplets

- The spectrum of 1S ground states is characterised by two quantities:
  - the diquark mass, $m_{[cq]}$
  - the spin-spin interaction inside the diquark or the antidiquark, κ_{cq}.
- The first radially excited, 2S, states are shifted up by a common quantity, the radial excitation energy, ΔE_r expected to be mildly dependent on the diquark mass: $E_r(cq) \sim E_r(cs)$



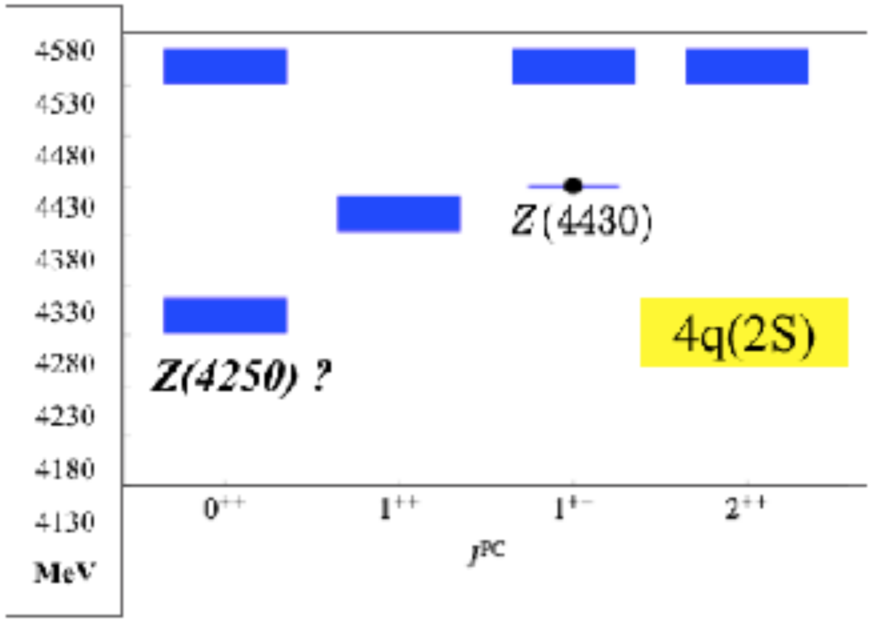


$$m_{[cq]} = 1980 \text{ MeV}$$

$$\kappa_{cq} = 67 \text{ MeV}$$

$$\Delta E_r(cq) = 530 \text{ MeV}$$

- Assignement of S-wave tetraquark multiplets of the first found “unexpected charmonia”



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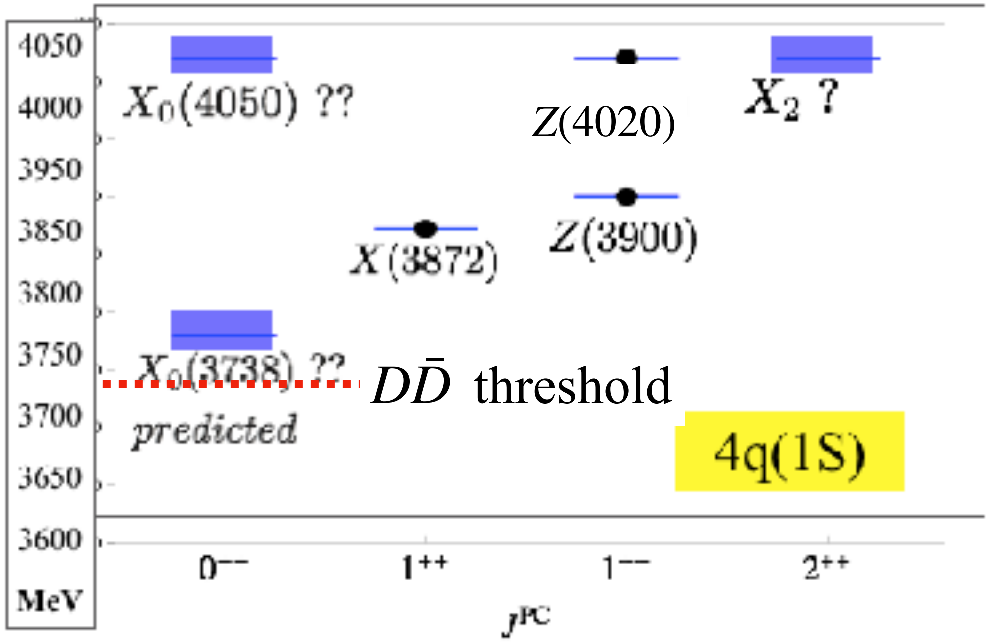

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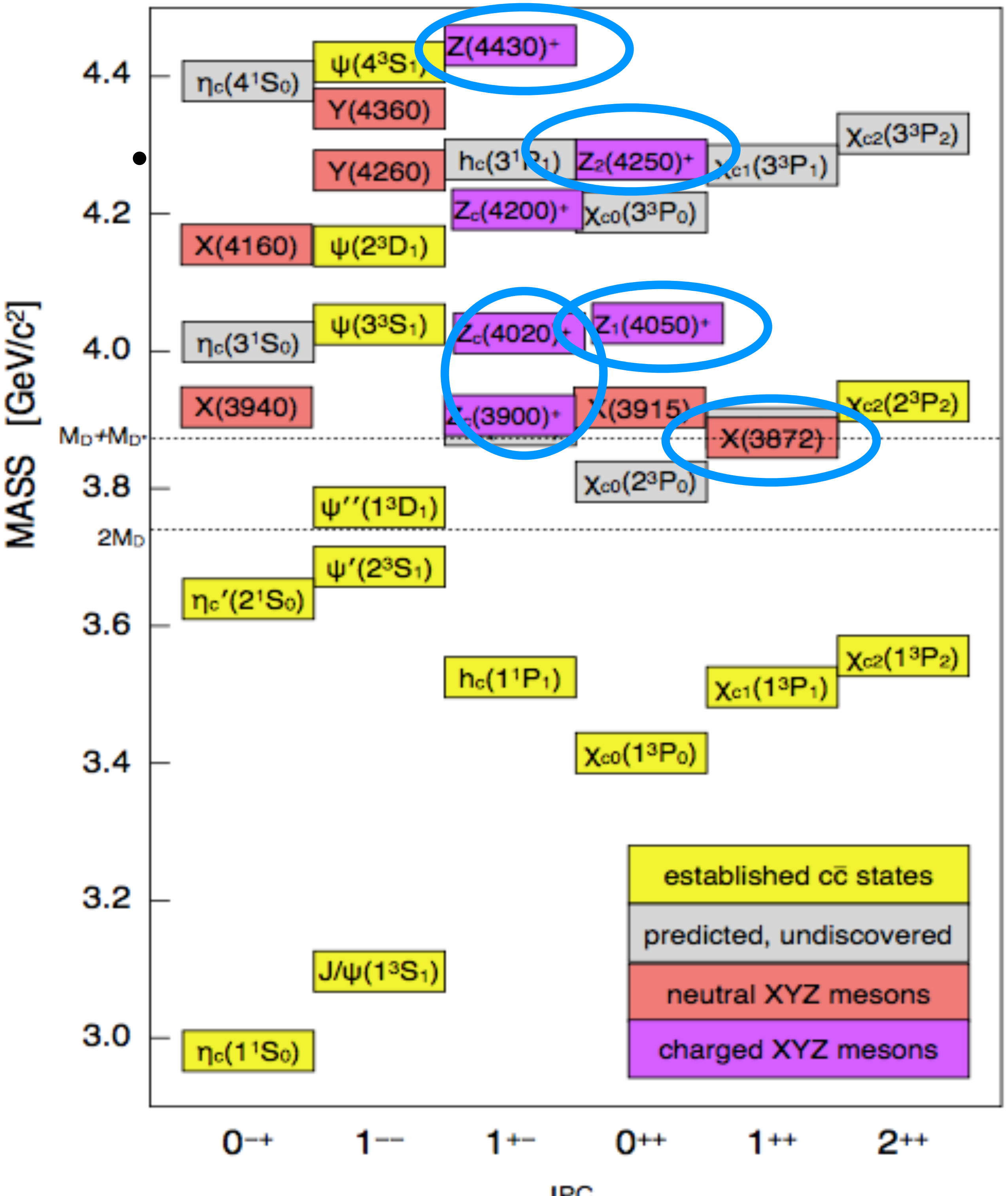

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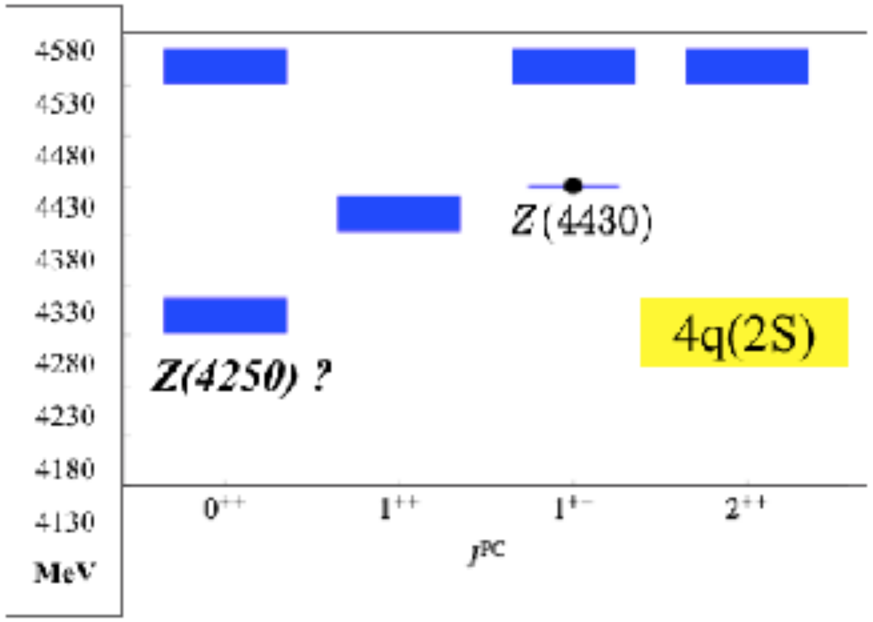

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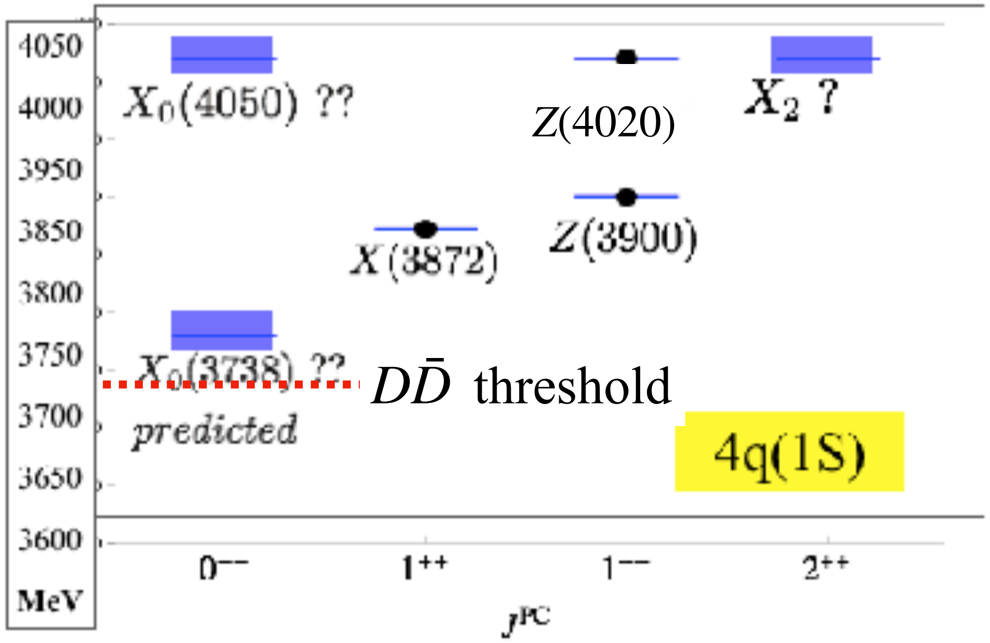

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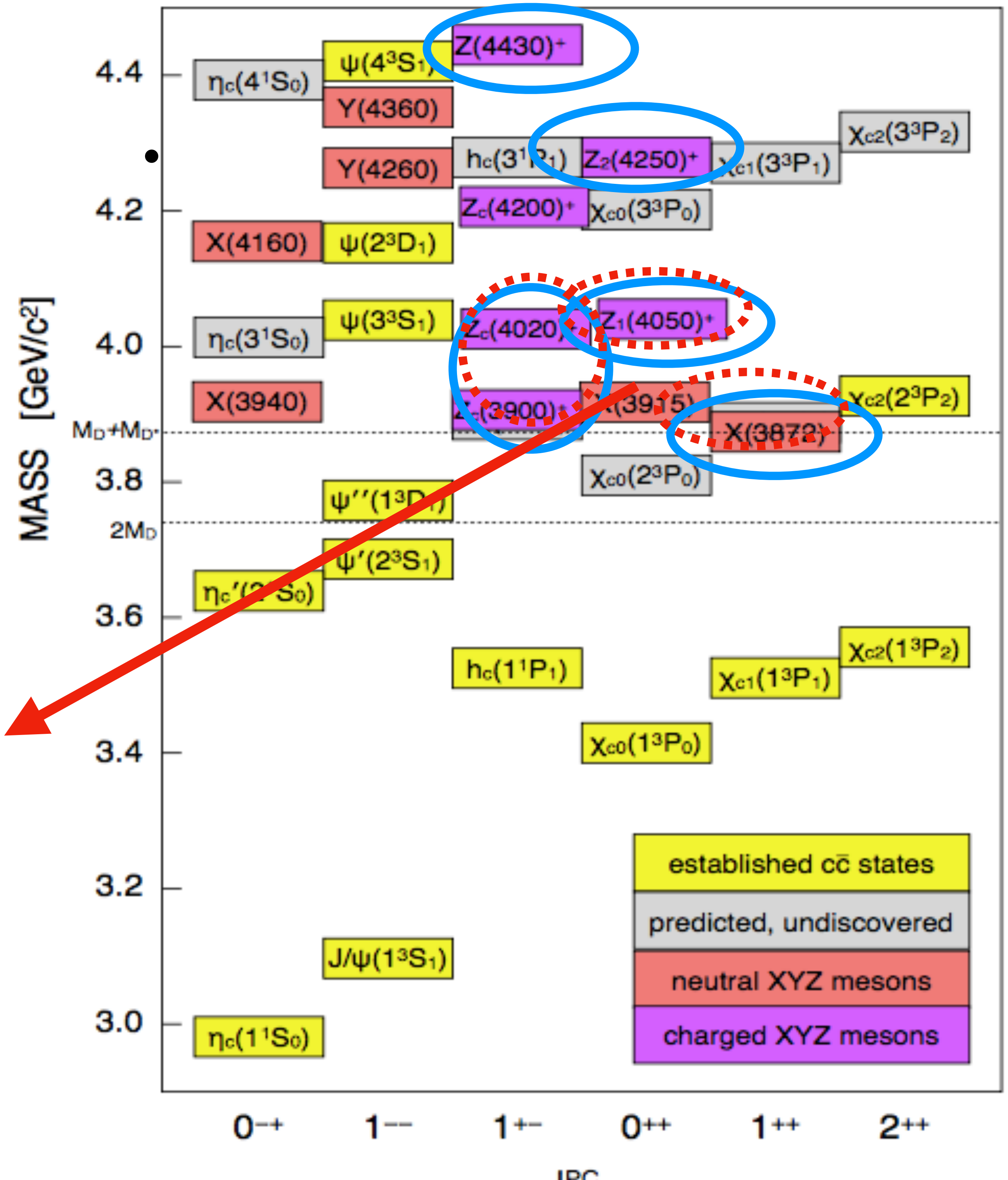

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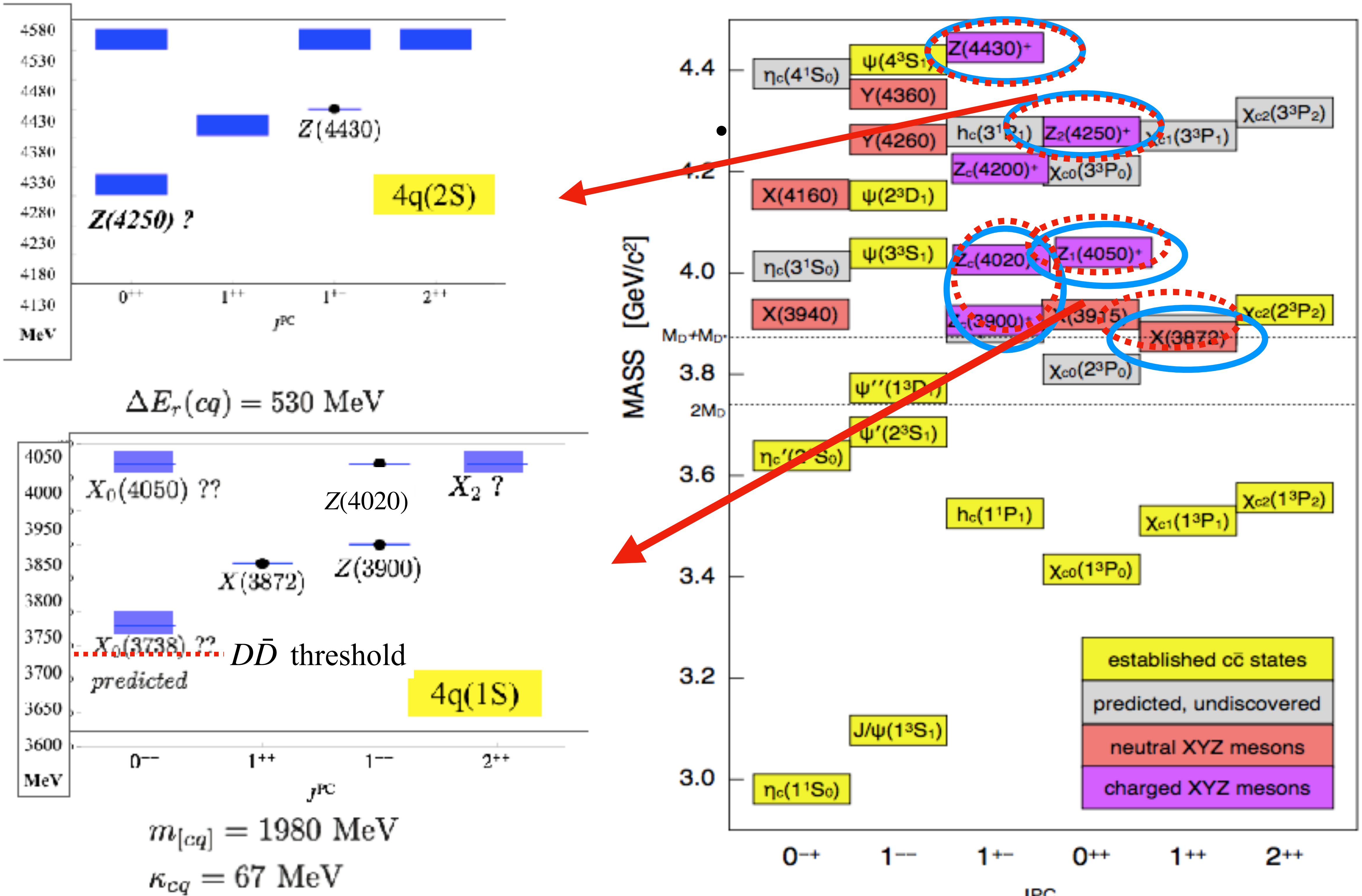

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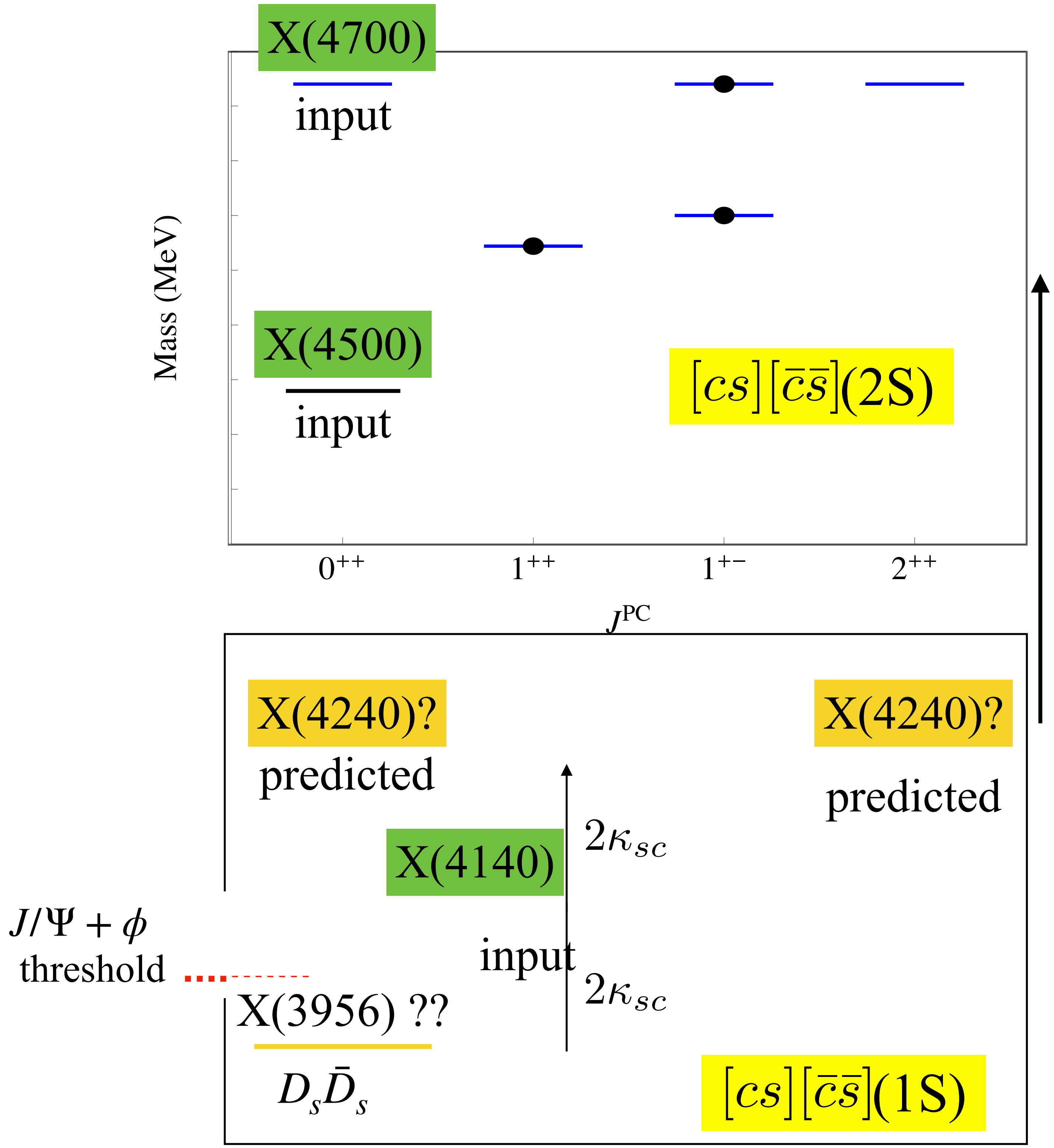
J/Ψ-φ structures and S-wave tetraquarks (2016)



$$\Delta m = m_{cs} - m_{cq} = 129\ \text{MeV};$$
$$\kappa_{sc} = 50\ \text{MeV}\ \ (\kappa_{qc} = 67\ \text{MeV})$$
$$\text{radial excit.} = 460\ \text{MeV}$$
$$[Z(4430) - Z(3900) = 530\ \text{MeV}]$$

Radial Excitation

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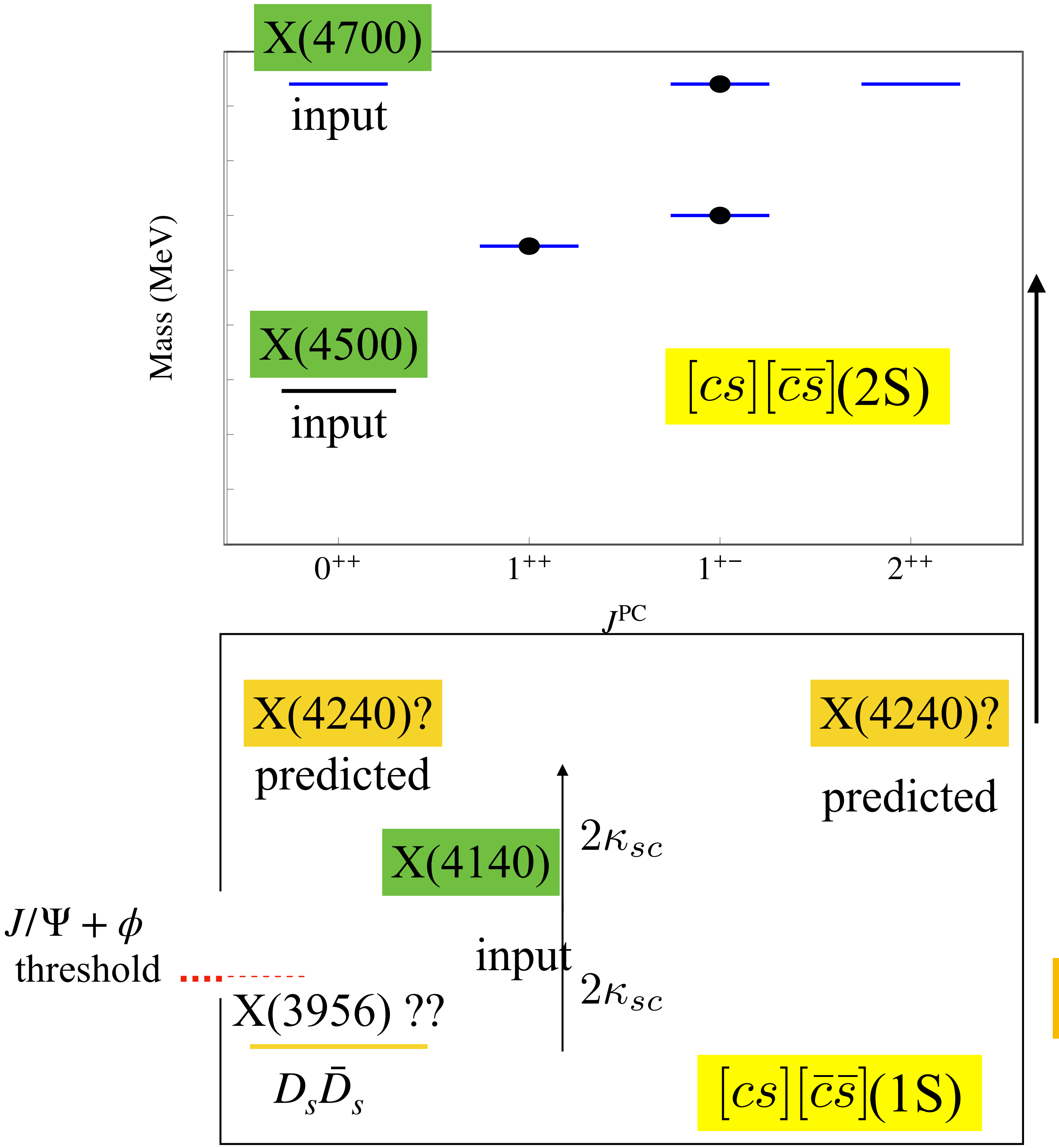

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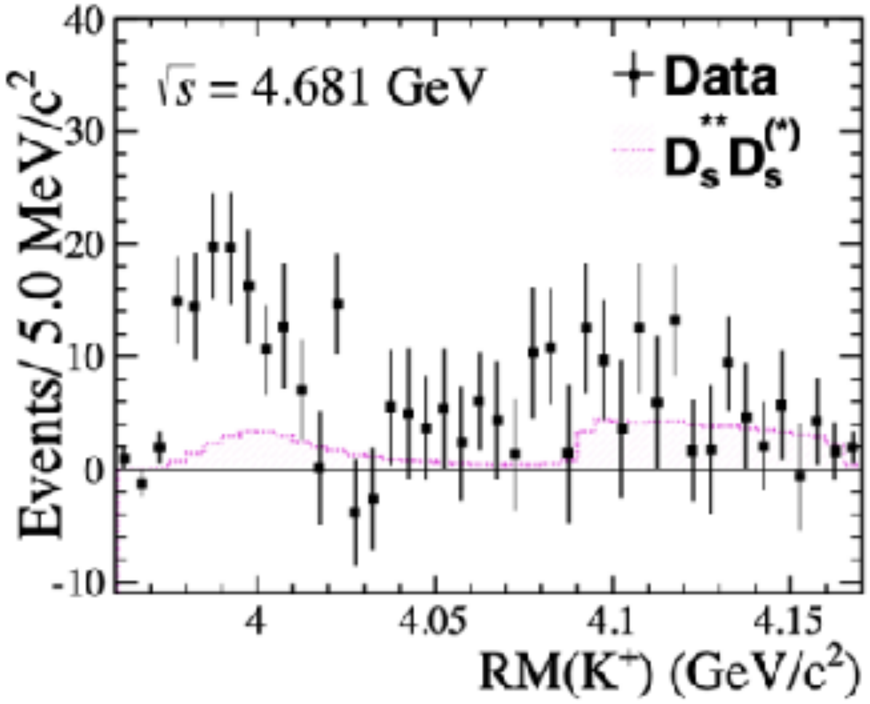
NOTE :

$$X(4140) - X(3872) \sim 270 \text{ MeV};$$
$$\phi(1020) - \rho(770) \sim 244 \text{ MeV}$$

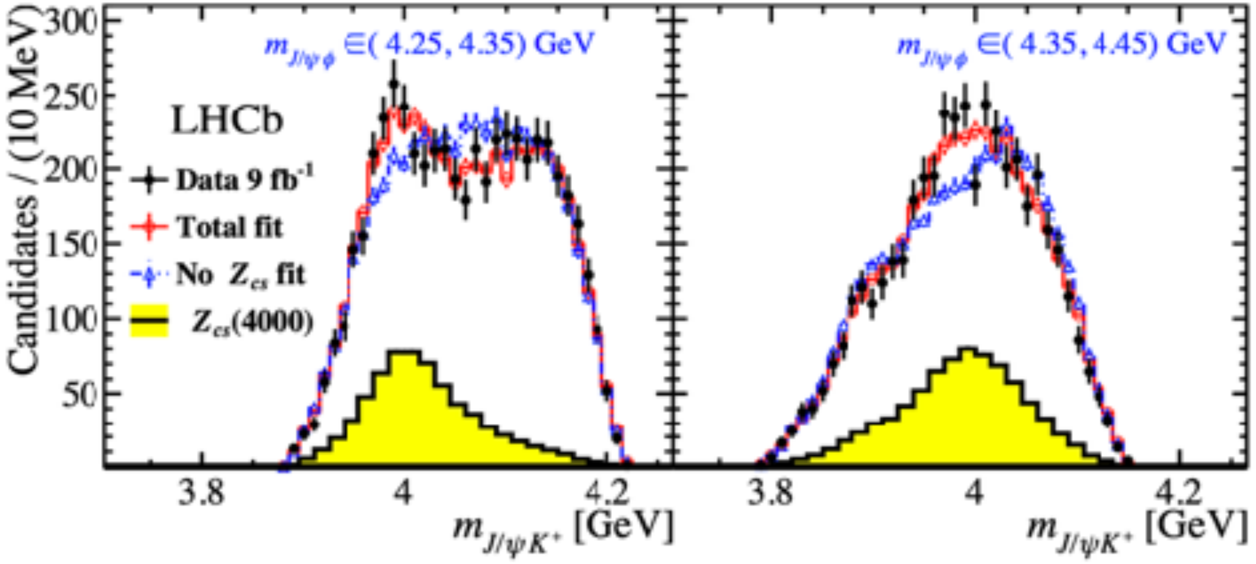
Z_{cs} predicted at $3872 + 135 = 4007$ MeV

2. Strange Exotics

BES III(2021): $e^+e^- \to K^+ + Z_{cs}^-(3985) \to K^+(D_s^* D^0 + D_s^- D^{*0})$



LHCb (2021): $B \to \Psi + K^+ + \phi \to Z_{cs}(4003) + \phi$



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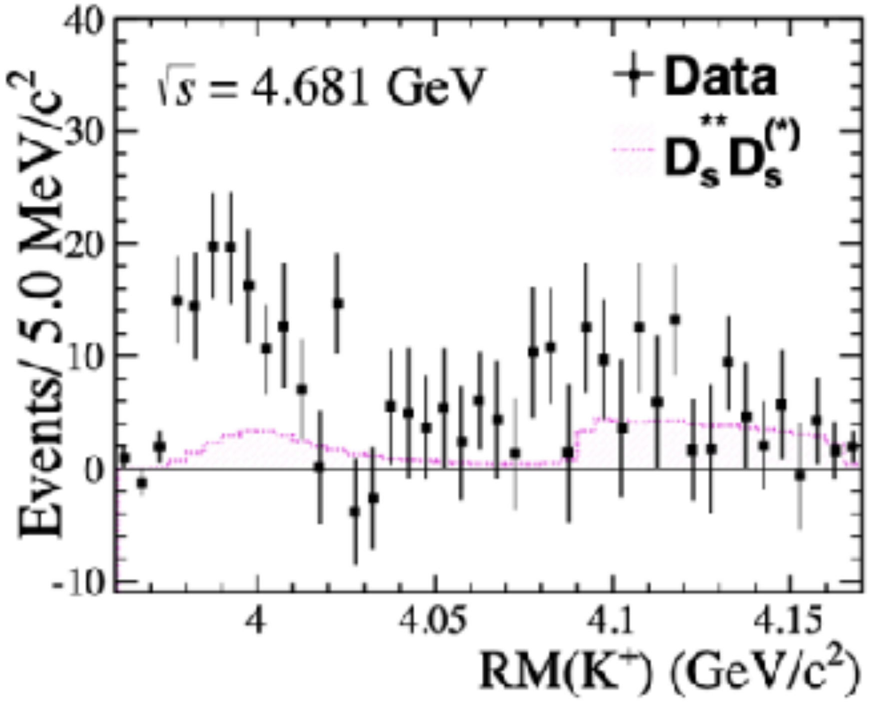

Inconsistent widths: not the same particle !!!

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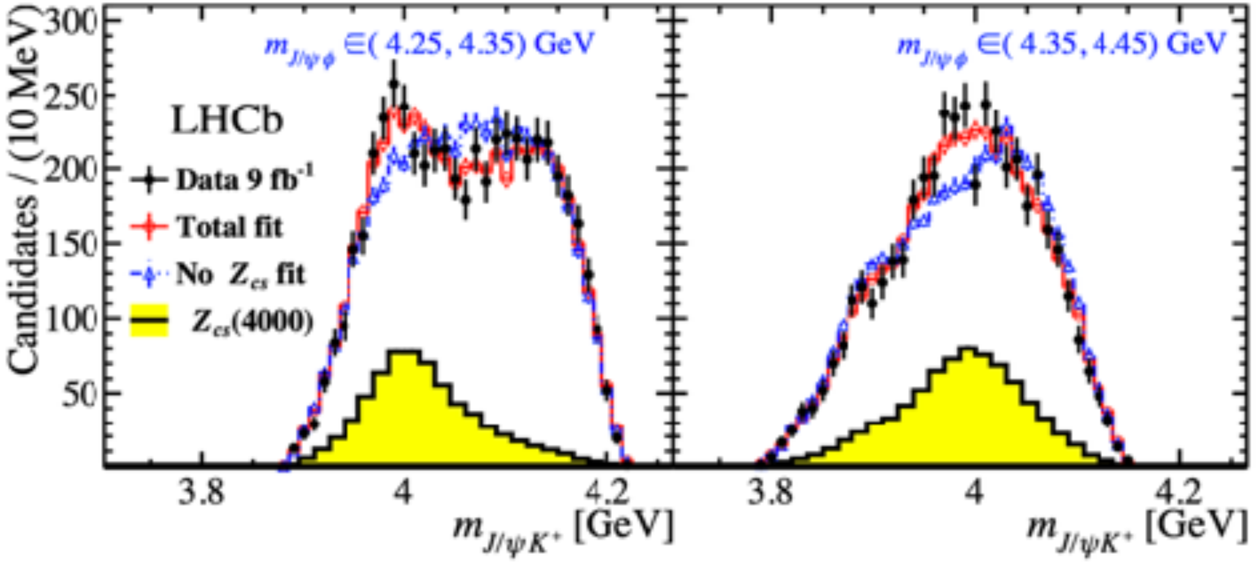

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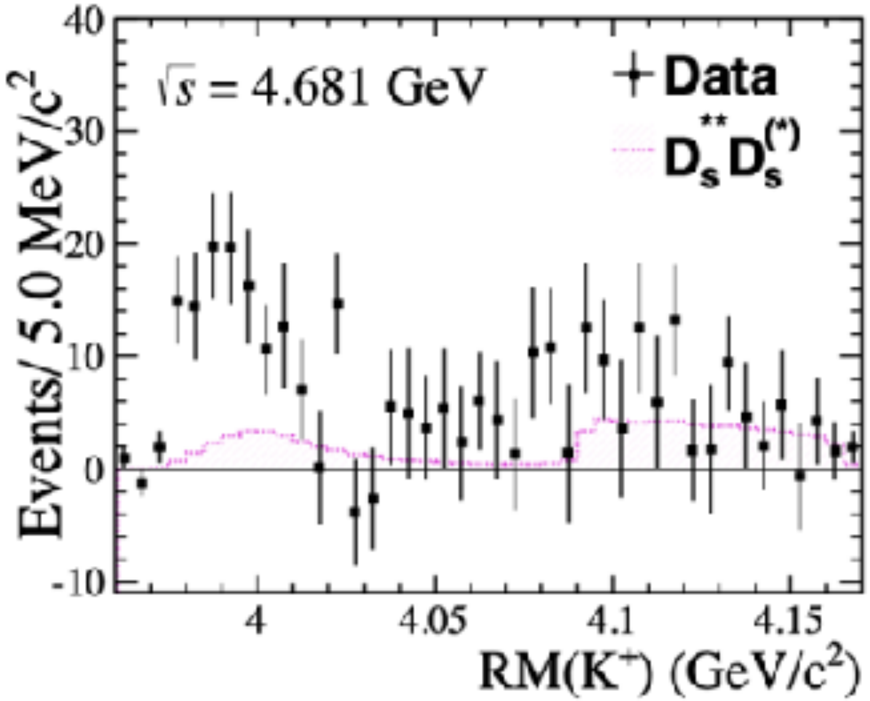

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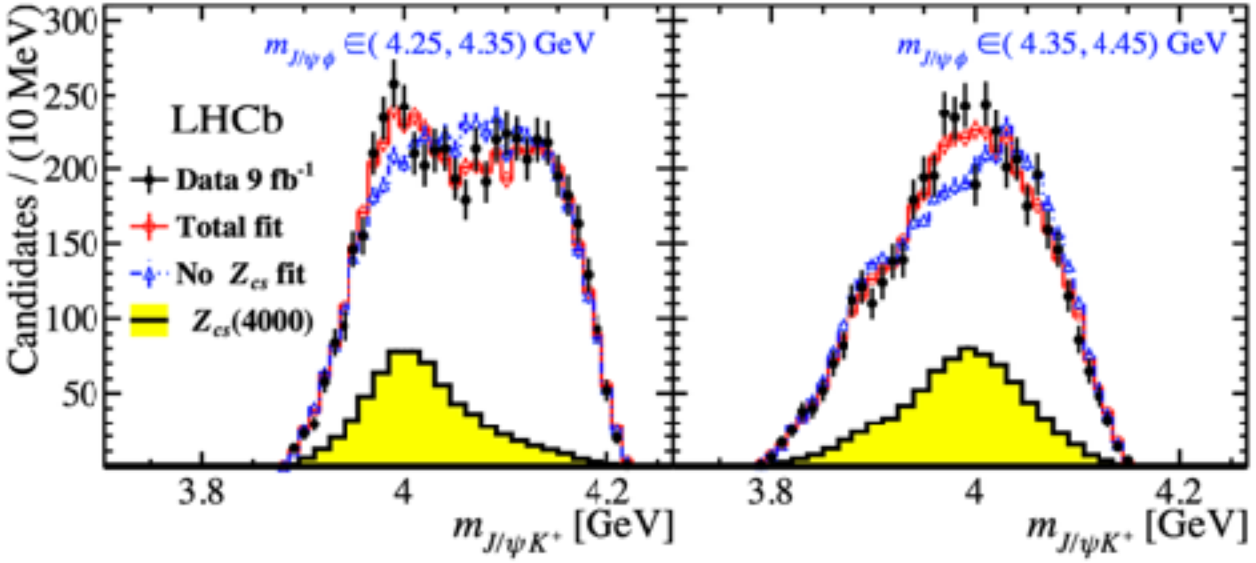
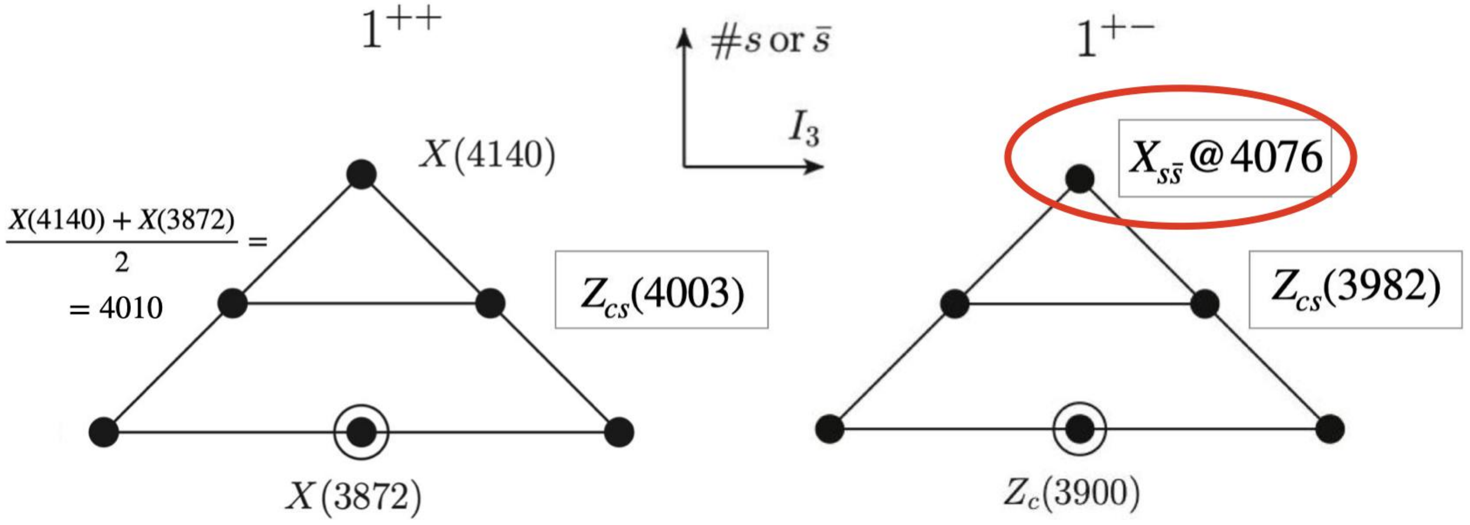


L. Maiani, A. D. Polosa and V. Riquer, Sci. Bull. **66** (2021), 1616, arXiv:2103.08331

Predicted in 2017:

$$\frac{X(4140) + X(3872)}{2} = 4007$$



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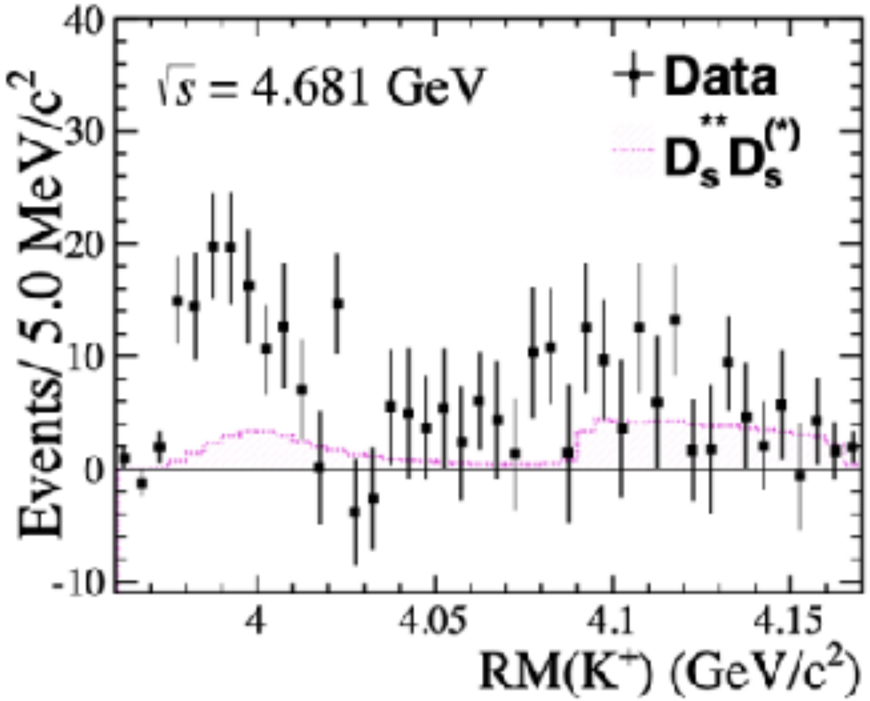

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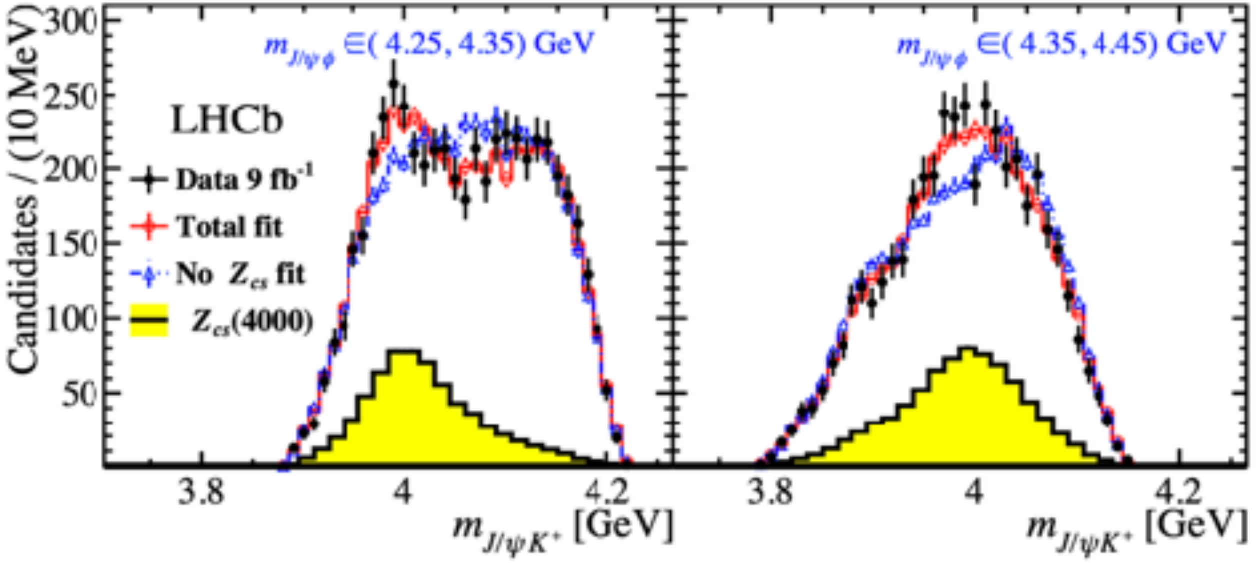

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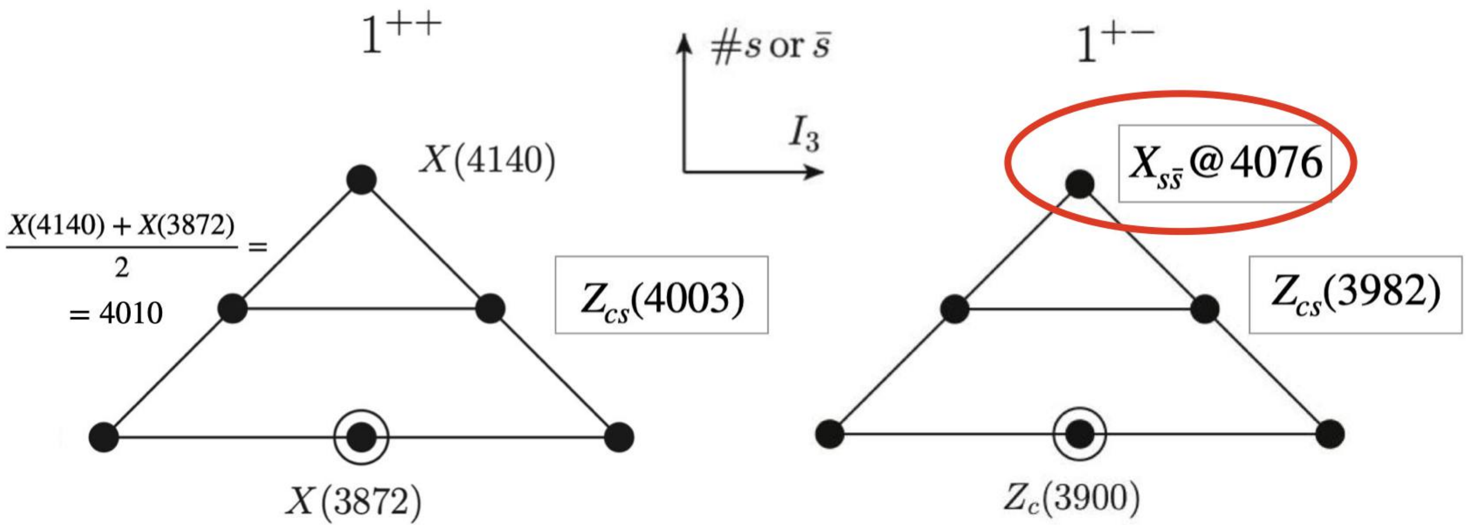
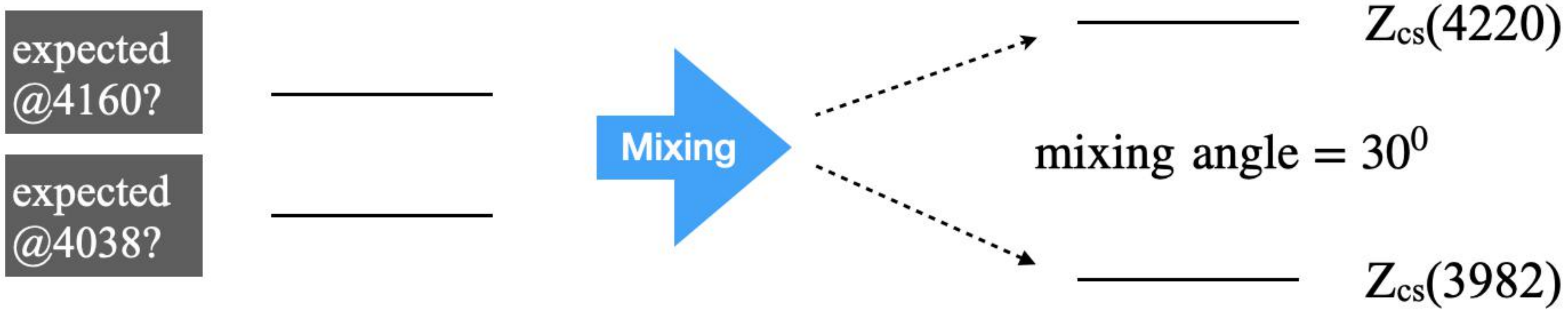


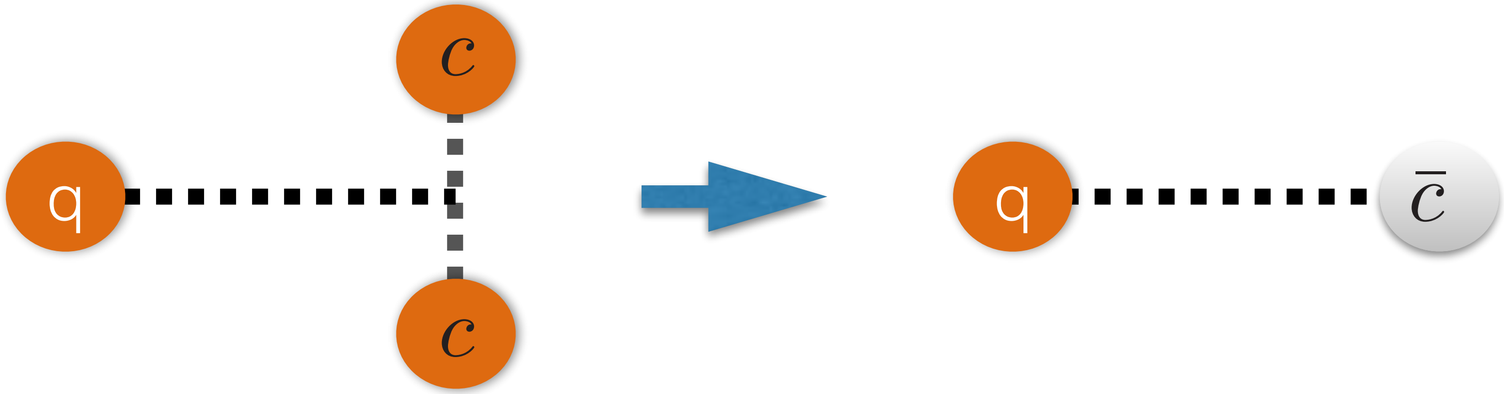
- ***LHCb sees a $Z_{cs}(4220)$, $J^P = 1^+$or 1^-***: is it too heavy for $Z_c(4020)$ ***?*** A bold proposal:



3. The new sensation: doubly heavy baryons and doubly heavy tetraquarks

Single heavy-doubly heavy quark symmetry: M. Savage, M. B. Wise, PLB 248,1990; N. Brambilla, A. Vairo and T. Rosch, PRD 72, 2005; T. Mehen, arXiv:1708.05020v3

- Doubly heavy baryons are related to single quark heavy mesons:



- QCD forces are mainly spin independent, so there is an approximate symmetry relating masses of DH baryons to SH mesons: e.g. $M(\Xi_{cc}^*) - M(\Xi) = \frac{3}{4}(M(D^*) - M(D))$

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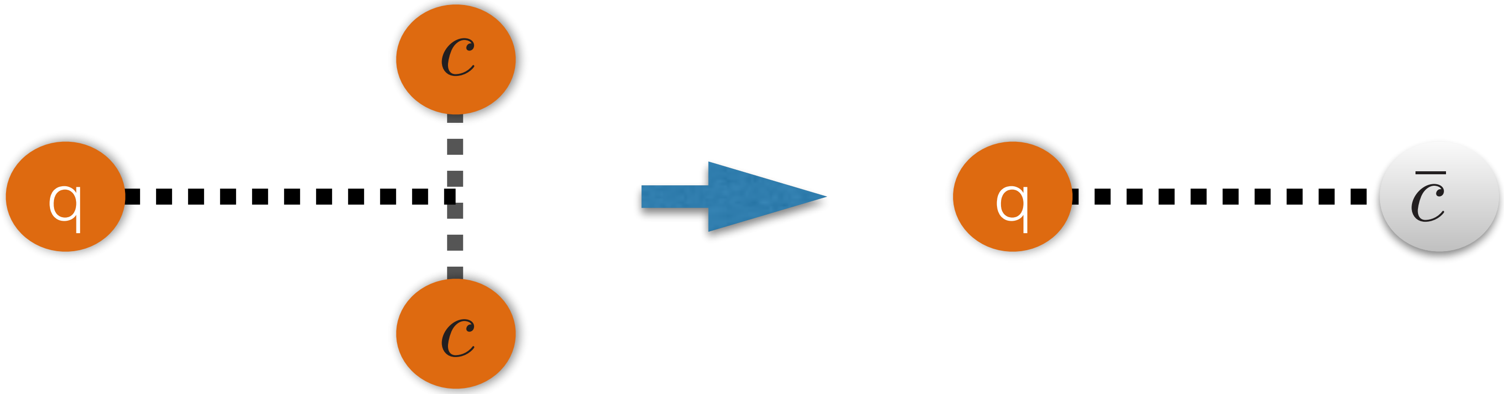

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- Doubly heavy tetraquarks have been anticipated long ago.

Esposito, M. Papinutto, A. Pilloni, A. D. Polosa, and N. Tantalo, Phys. Rev. D88, 054029 (2013)

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M. Karliner and J. L. Rosner, PRL **119** (2017) 202001. E. J. Eichten and C. Quigg, PRL **119** (2017) 202002.; S. Q. Luo et al. Eur. Phys. J. C **77** (2017) 709.

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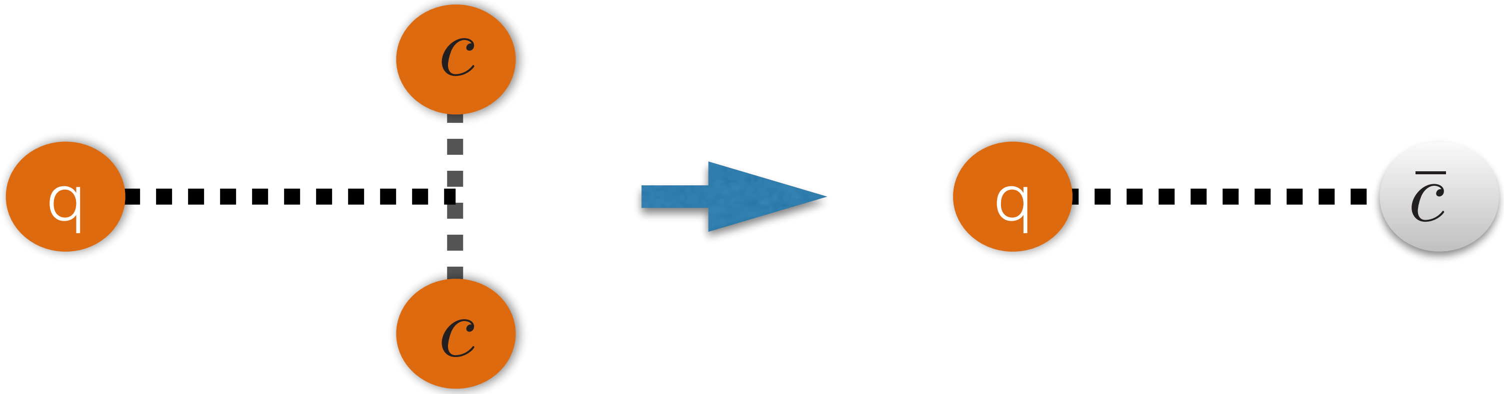

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- extended calculations of $\mathcal{T}_{cc}$, $\mathcal{T}_{cb}$ and $\mathcal{T}_{bb}$ mass in the Born-Oppenheimer approximation have been presented: analytical

L. Maiani, A. D.Polosa and V. Riquer, PRD **100** (2019) 074002,
L. Maiani, A. Pilloni, A. D.Polosa and V. Riquer, PL B **836** (2023), 137624

- and in Lattice QCD

see later for Reffs.

Quark model calculation, $T_{bb\bar{u}\bar{d}}$

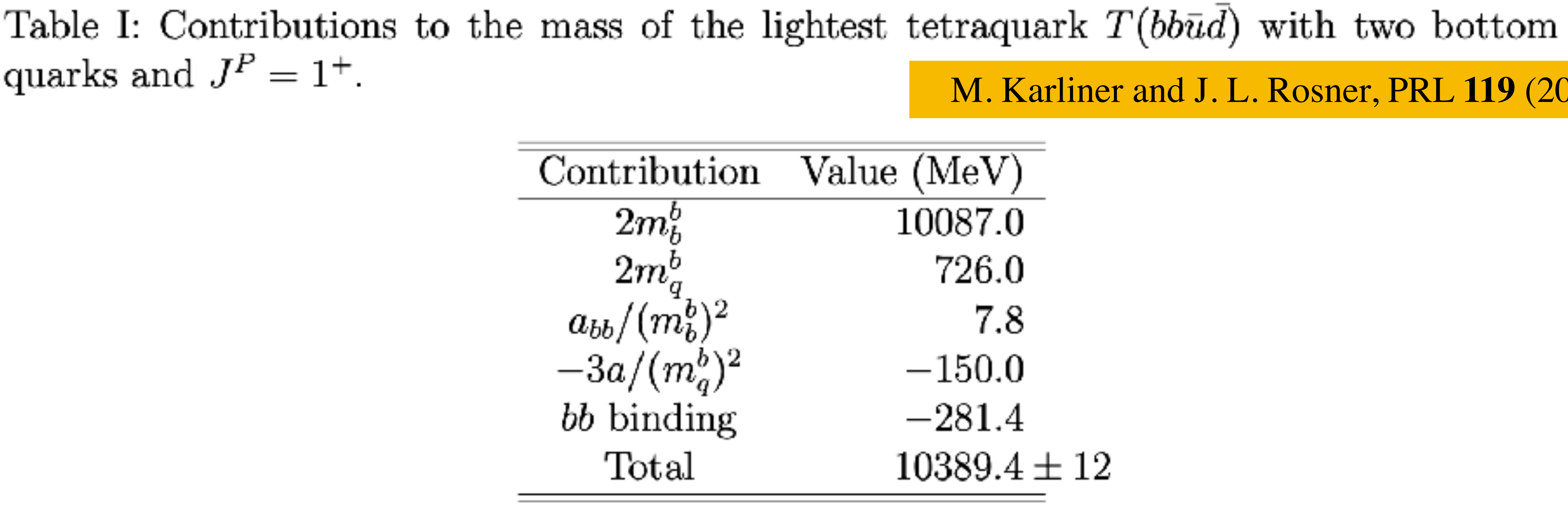
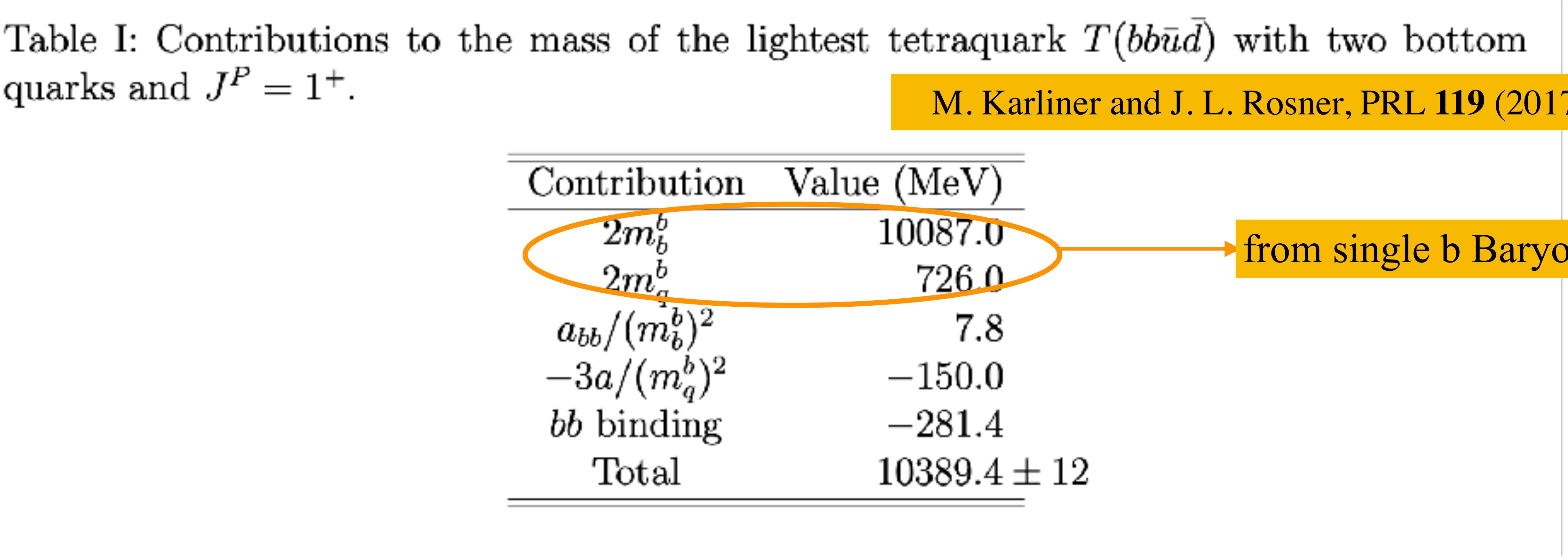


Table I: Contributions to the mass of the lightest tetraquark $T(bb\bar{u}\bar{d})$ with two bottom quarks and $J^P = 1^+$.

Contribution	Value (MeV)
$2m_b^b$	10087.0
$2m_q^b$	726.0
$a_{bb}/(m_b^b)^2$	7.8
$-3a/(m_q^b)^2$	−150.0
bb binding	−281.4
Total	10389.4 ± 12

M. Karliner and J. L. Rosner, PRL **119** (2017) 202001

Quark model calculation, $T_{bb\bar{u}\bar{d}}$



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Contribution	Value (MeV)
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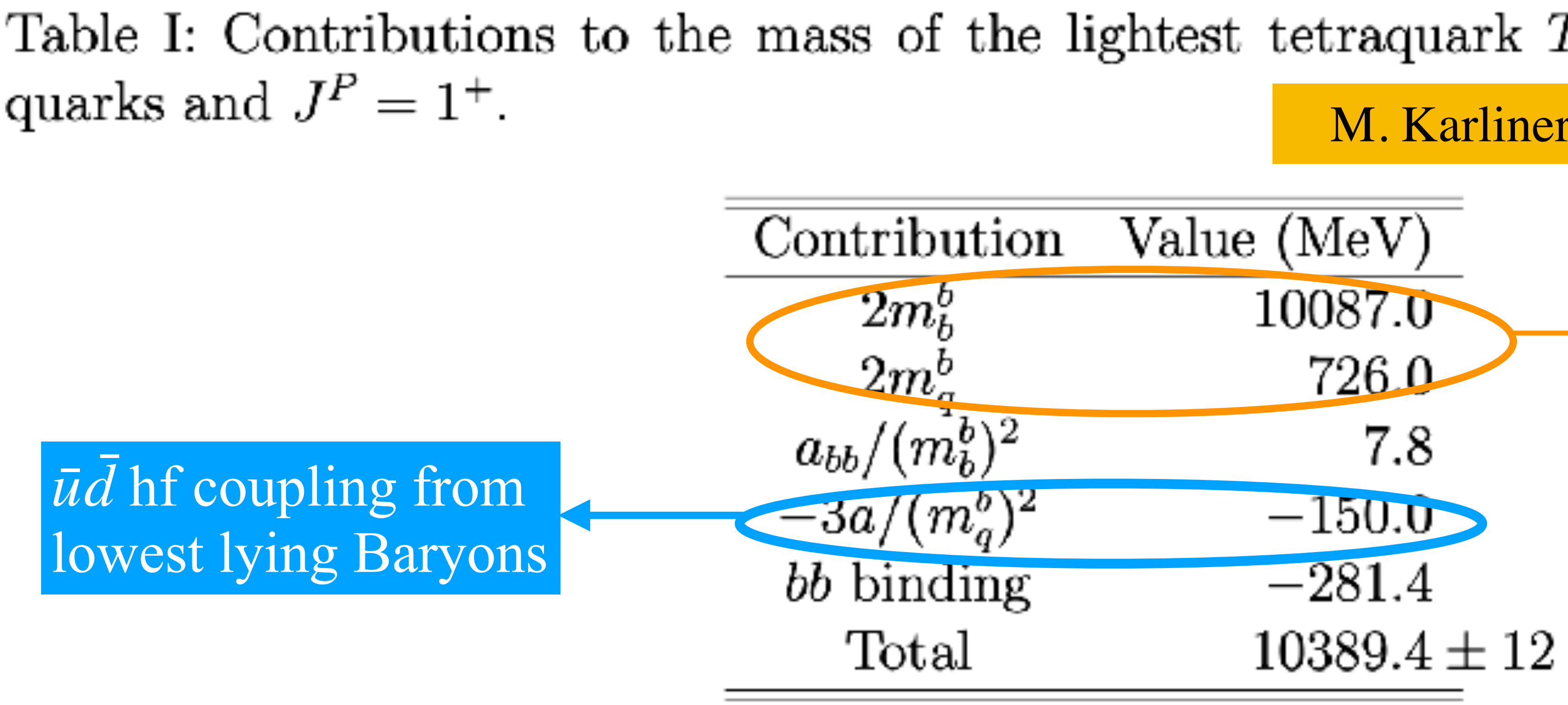

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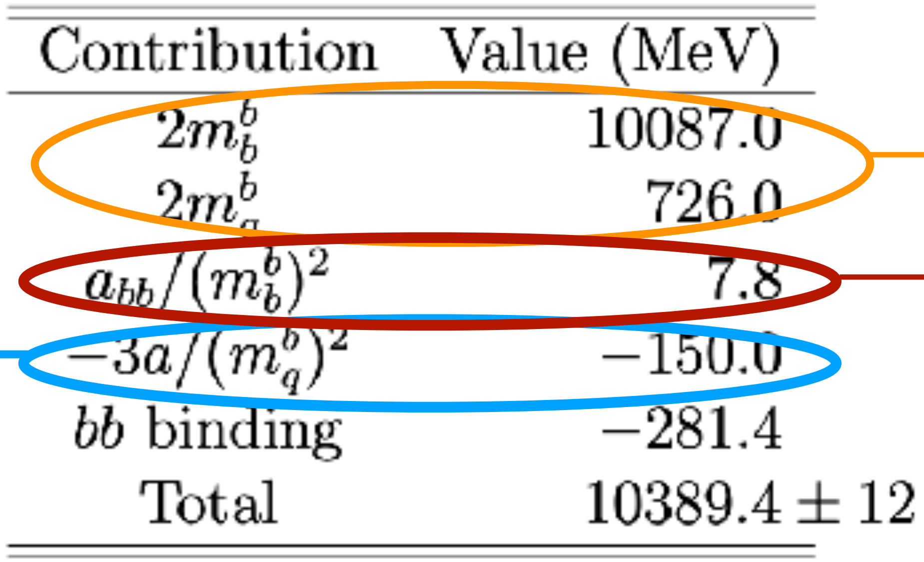

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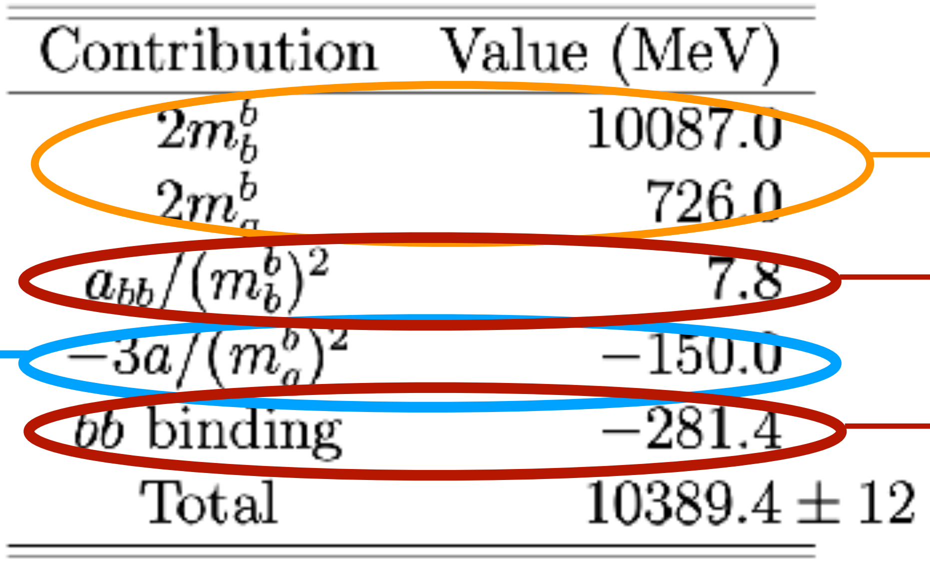

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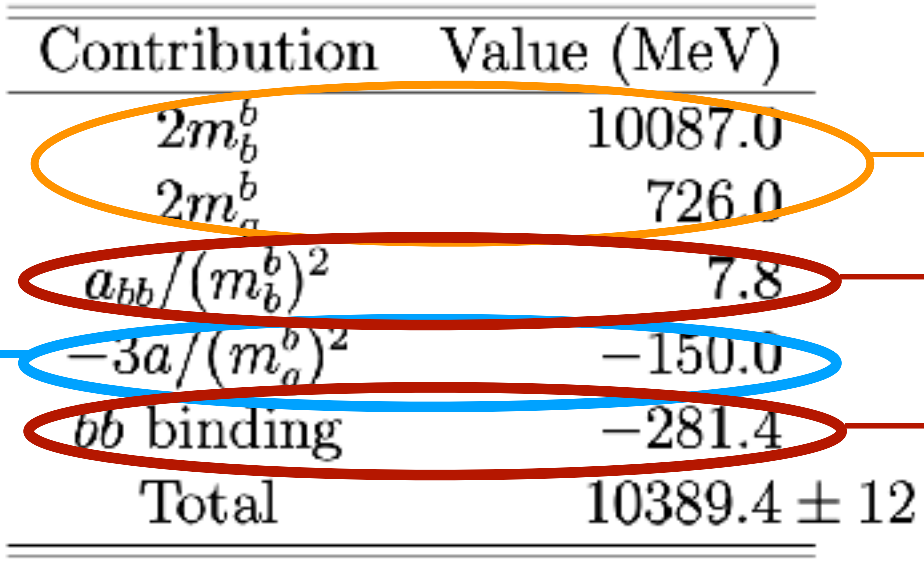

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- A similar estimate gives: $Q = T_{cc\bar{u}\bar{d}} - D\bar{D} = +\,140$

and a mass close to the $D\bar{D}^*$ threshold, as later observed.

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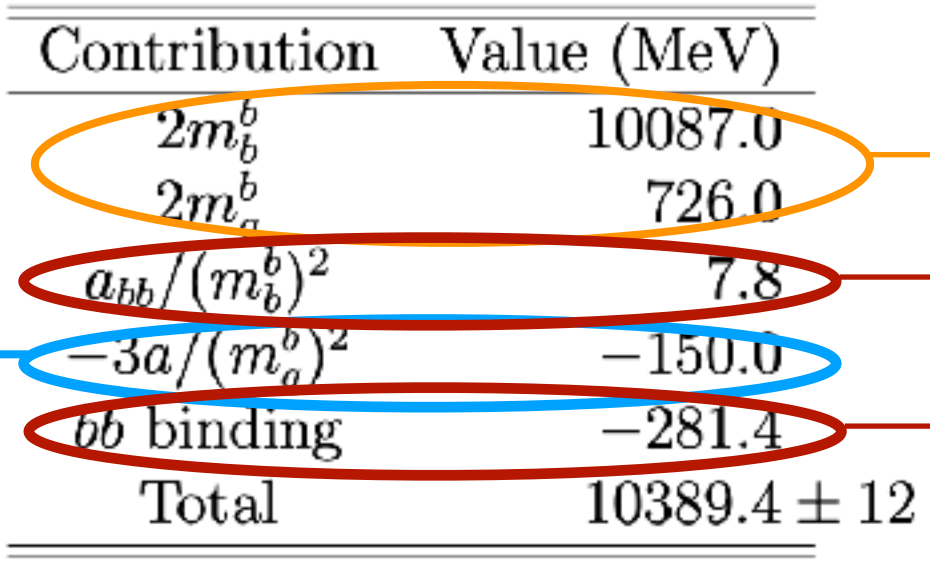

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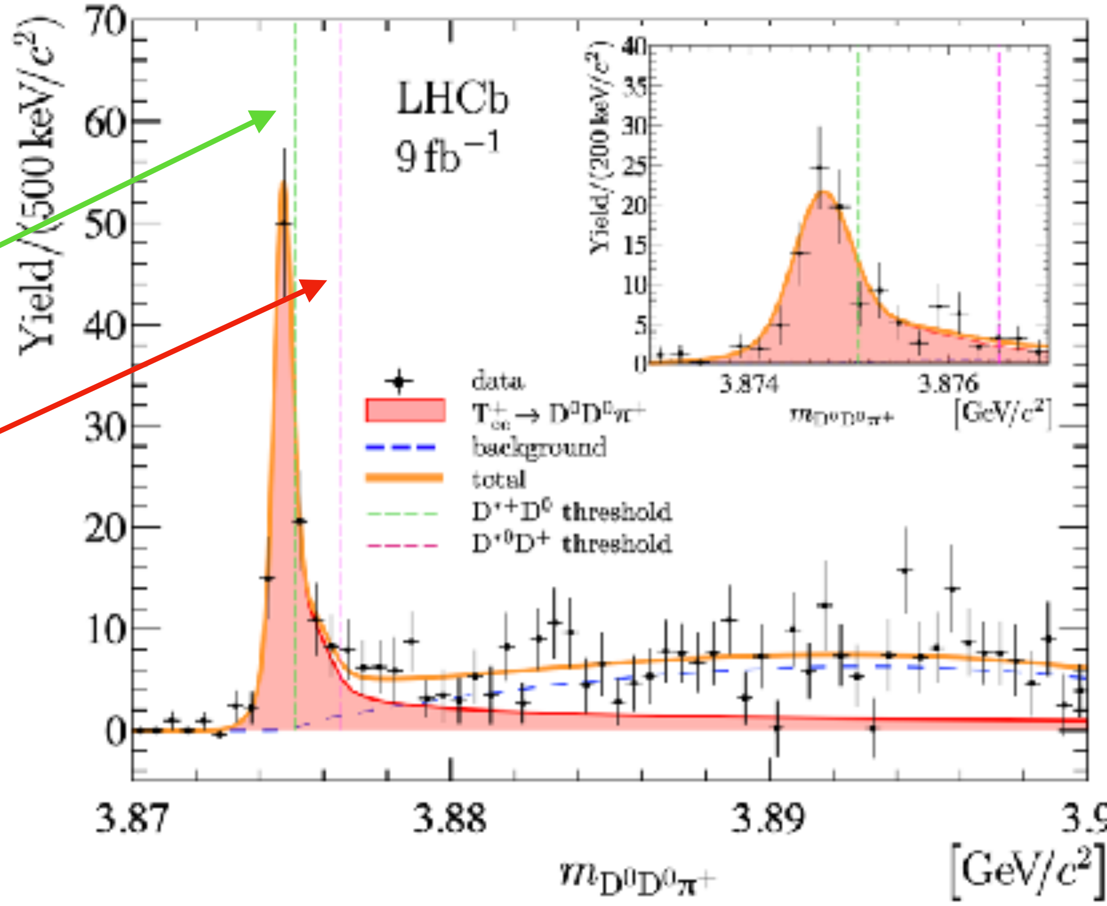
$$\text{Mass}: D^{*+}D^0 = 3875.1 \text{ MeV}$$

$$\text{Mass}: D^{*0}D^+ = 3876.5 \text{ MeV}$$

LHCb arXiv:2109.01056v2



The limit of large heavy quarks mass

- ***Most clearly stated by Eichten and Quigg***:
  - in the large mass limit, the heavy quarks go to short distance, where the coulomb-like QCD potential dominates
  - the QQ binding energy is then given by the QCD Rydberg, so that: $Q_{\text{value}} = M(T) - 2M(P) = -\frac{1}{2}(\frac{2}{3}\alpha_s)^2 \bar{M}_Q + \mathcal{O}(m, M^{-1})$
  - $\bar{M}_Q$ the reduced mass of the QQ pair
  - P is the pseudoscalar $(Q\bar{q})$ meson
- for $\bar{M}_Q$ large enough $T = (QQ\bar{q}\bar{q})$ is stable against strong as well as electromagnetic decays into $PP + \gamma$.
- but: is the b quark mass heavy enough for stability?

4. Born-Oppenheimer approximation in QCD, with heavy (QQ’) and light ($\bar{u}\bar{d}$)

1. Consider the heavy quarks as classical sources with fixed position and quantum numbers;
2. Compute the energy of the light quarks in the field of the heavy sources, $\epsilon_{light}(x_A, x_B)$;
3. ***The tetraquark binding energy is the eigenvalue of the Schrödinger equation for QQ’*** with potential $V_{TOT}(R)$;

$$\left(-\frac{\nabla^2}{2\bar{M}_{QQ'}} + V_{TOT}\right)\Psi = E_{BO}\Psi; \qquad V_{TOT}(R) = V_{bb}(R) + \epsilon_{light}(R)\ ;$$

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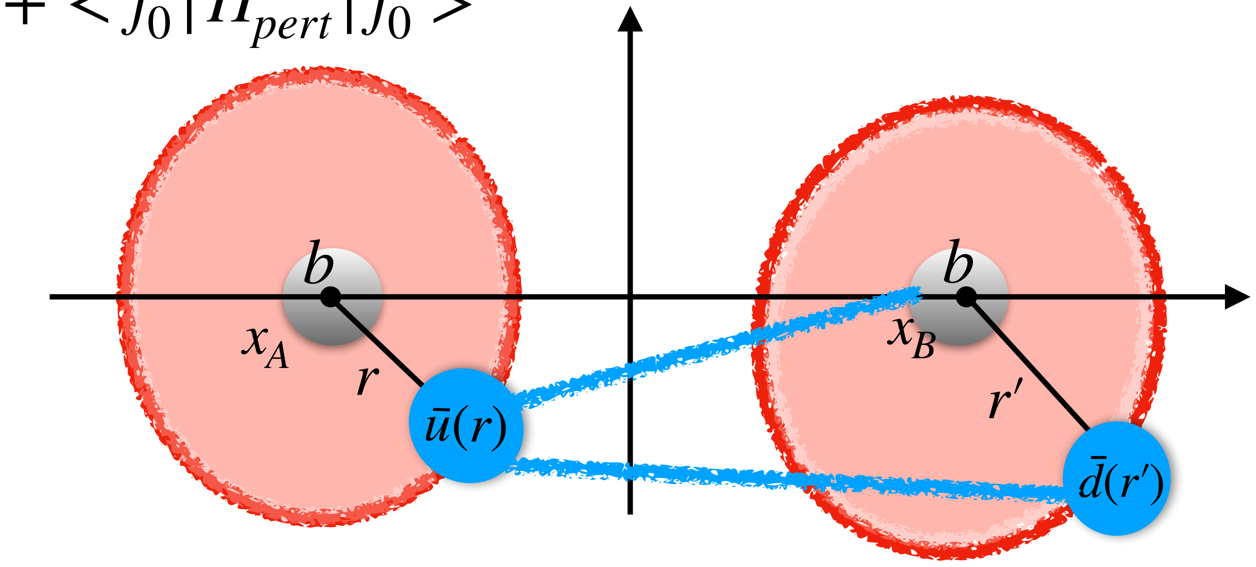
- Perturbation theory:
  - interactions $b - \bar{u}$, or $b - \bar{d}$ (black lines in Fig.) make bound states (orbitals). We take as ground state:

$$f_0 = \frac{\psi_{\bar{u}}\phi_{\bar{d}} + \psi_{\bar{d}}\phi_{\bar{u}}}{\sqrt{2(1+S)}}; \quad \epsilon_0 = E_\psi + E_\phi$$

  - other interactions (one gluon exchange or spin-spin interactions (the blue lines in the Fig.) are treated as first order perturbations: $\epsilon_{light}(R) = \epsilon_0 + < f_0 | H_{pert} | f_0 >$



$$\psi_{\bar{u}}(r) = \frac{2\ A^{3/2}e^{-Ar}}{\sqrt{4\pi}} \qquad \phi_{\bar{d}}(r') = \frac{2\ A^{3/2}e^{-Ar'}}{\sqrt{4\pi}}$$

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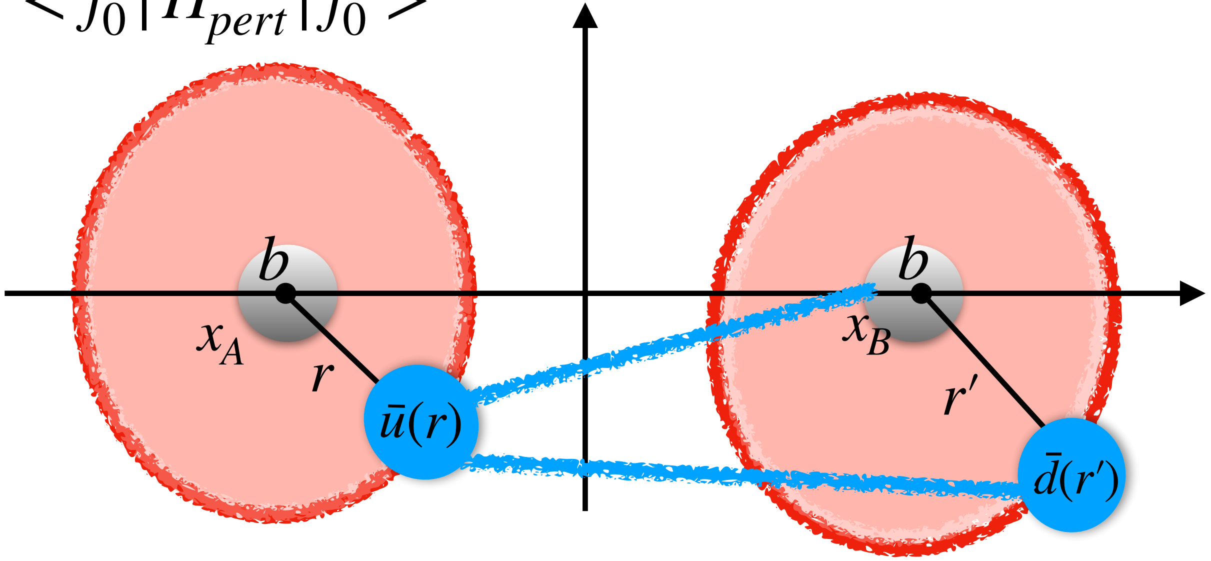
$$T = |(bb)_{\bar{\mathbf{3}}}, (\bar{u}\bar{d}_{\mathbf{3}}\rangle_{\mathbf{1}} =$$
$$= \sqrt{\frac{1}{3}}\,|(\bar{u}b)_{\mathbf{1}}, (\bar{d}b)_{\mathbf{1}}\rangle_{\mathbf{1}} - \sqrt{\frac{2}{3}}\,|(\bar{u}b)_{\mathbf{8}}, (\bar{d}b)_{\mathbf{8}}\rangle_{\mathbf{1}};$$

- Orbitals have triality zero.
- Soft gluons can screen the color of the orbital ***orbitals are not confined***!
- we must add a constant C, so that $V_{TOT}(R) + C \to 2M_b^{mes} + 2M_q^{mes}$ at $R \to \infty$
- that is $\epsilon_0 + C = 2(M_b^{mes} + M_q^{mes})$



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NOTE: Spin-spin interactions can be included in the BO potential (as in lattice QCD)

- $H_{pert}(\bar{u})$ is represented by the blue lines in the figure
- it includes gluon exchange, e.g.

$$-2/3\frac{\alpha_s}{|x_{\bar{u}}-x_{\bar{d}}|}$$

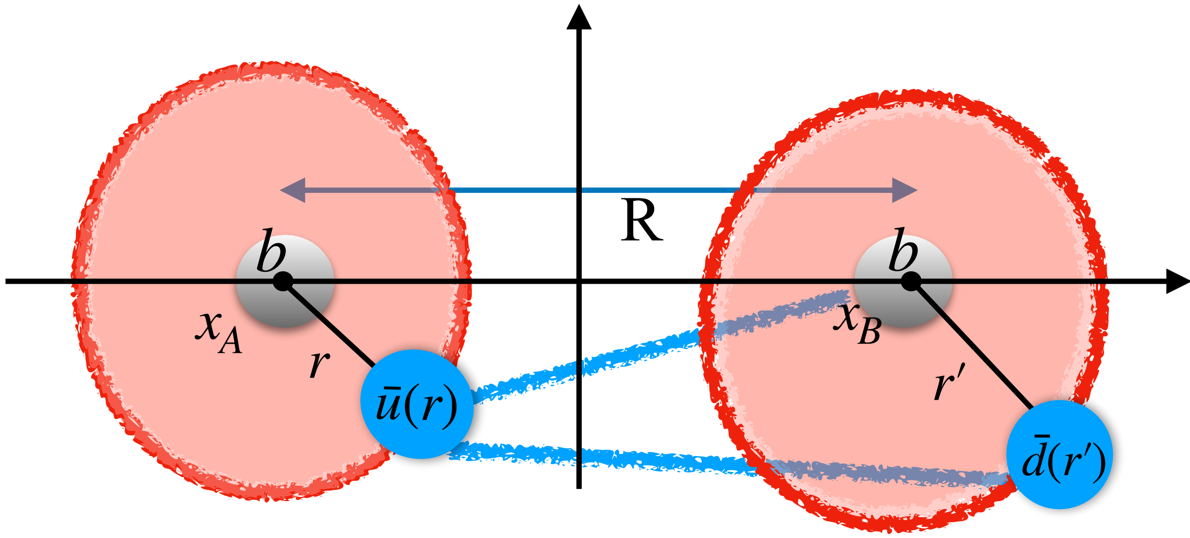
and spin-spin interaction

$$H_{hf\bar{q}\bar{q}}=\frac{8\pi\alpha_s}{9M_q^2}\;2(\mathbf{s}_{\bar{\mathbf{q}}}\cdot\mathbf{s}_{\bar{\mathbf{q}}})\delta^{(3)}(x_{\bar{u}}-x_{\bar{d}})$$

- Integrating the delta function with the wave functions of light quarks, one obtains a function of R:

$$V_{hf}(R)=<f_0|H_{hf\bar{q}\bar{q}}|f_o>(R)$$

this is the hf potential, to be added in the V_{BO} potential.



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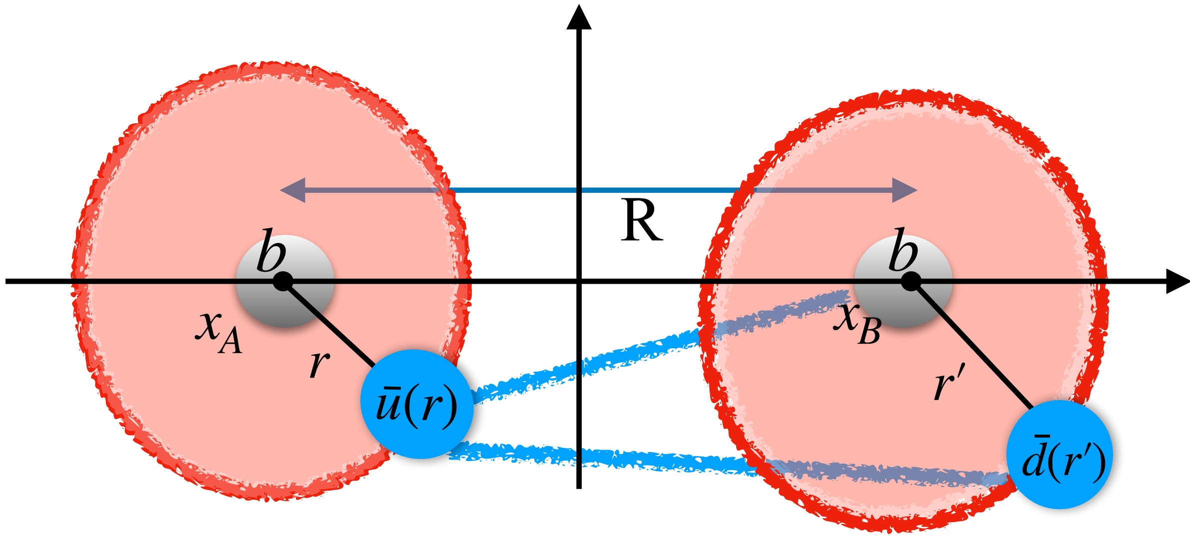

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- Numerically, we find:

$$E_{\text{with hf}}-E_{\text{no hf}}=\text{- 4.5 (-13.8) MeV}=-\frac{3}{2}(\kappa_{qq})_{\text{Tetraq}}$$

to be compared with $-\frac{3}{2}(\kappa_{qq})_{\text{Baryon}}=-150\ \text{MeV}$, had we taken the hf coupling of baryons (as in K&R).

- An explanation is that confinement in baryons keep light quarks closer w.r.t. tetraquarks with orbitals not confined.

The Born-Oppenheimer approximation satisfies the Eichten & Quigg criterion for increasing heavy quark mass

$$(-\frac{\nabla^2}{2\bar{M}} + V_{TOT})\Psi = E_{BO}\Psi$$

- write:

$$V_{TOT}(R) = V(R) + 2M_b^{mes} + 2M_q^{mes};$$

$$V = -\frac{2}{3}\frac{\alpha_s}{r} + < f_0 | H_{pert} | f_0 > (R), \text{ vanishes for } r \to +\infty$$

$$E_{BO} = E + 2M_b^{mes} + 2M_q^{mes}$$

- $Q_{\text{value}} = E + \frac{1}{2}\kappa''_{QQ} + 3\kappa_{Q\bar{q}}$

- E=eigenvalue of the BO Schroedinger equation without rest masses, negative of $\mathcal{O}[-(\frac{2}{3}\alpha_s)^2\bar{M}_Q]$;
- κ''_{QQ} (heavy quark hf interaction) is positive but subdominant, $\mathcal{O}[+(\frac{2}{3}\alpha_s)^4\bar{M}_Q]$
- $\kappa_{Q\bar{q}} > 0$ is the spin-spin interaction of the $(Q\bar{q})$ meson, $\mathcal{O}(M_q/M_Q)$

However: Is the b-quark mass heavy enough?

The mass of the lightest double heavy tetraquarks can be computed!

- Recent estimates of the mass of the lightest, double heavy tetraquarks indicate that the I=0, $bb\bar{u}\bar{d}$ tetraquark ***could be stable***. The table below gives a comparison of different theoretical results.
- ***Q-value is taken with respect to PS-PS threshold (not V-PS!)*** $M(D^*) - M(D) = 140$ MeV
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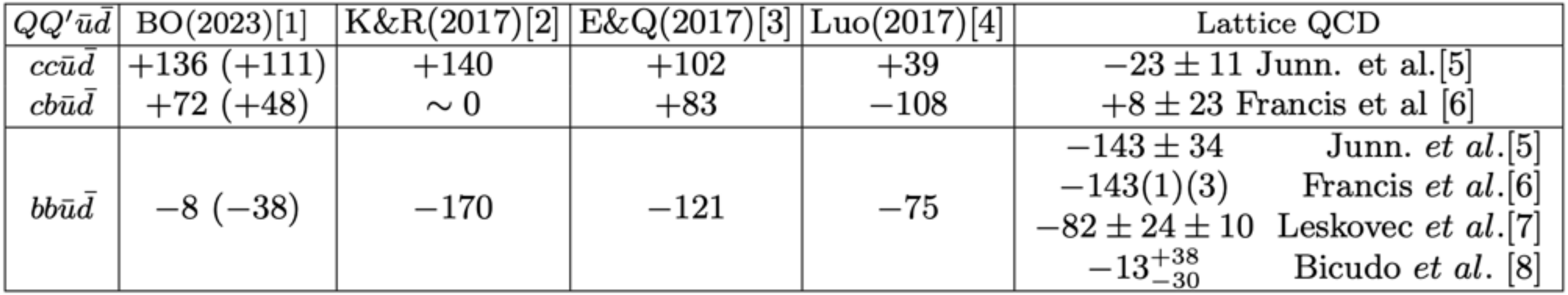

$QQ'\bar{u}\bar{d}$	BO(2023)[1]	K&R(2017)[2]	E&Q(2017)[3]	Luo(2017)[4]	Lattice QCD
$cc\bar{u}\bar{d}$	+136 (+111)	+140	+102	+39	-23 ± 11 Junn. et al.[5]
$cb\bar{u}\bar{d}$	+72 (+48)	~ 0	+83	−108	$+8 \pm 23$ Francis et al [6]
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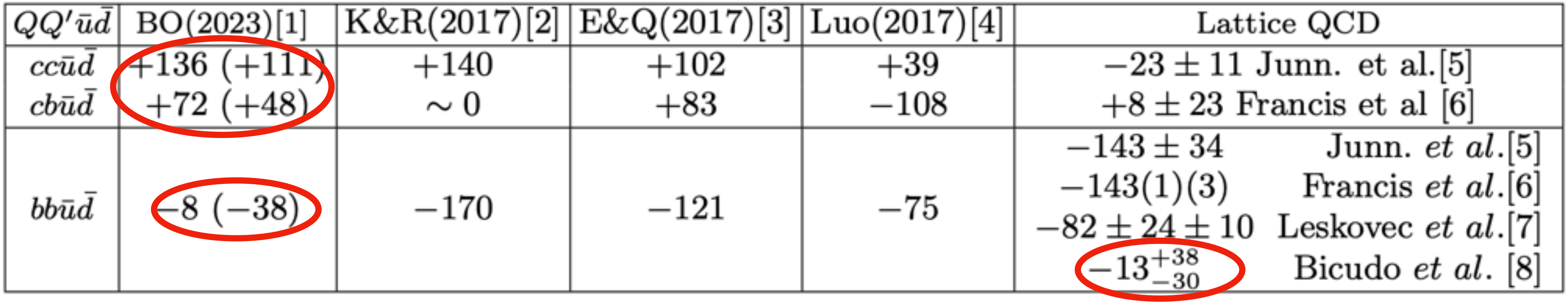

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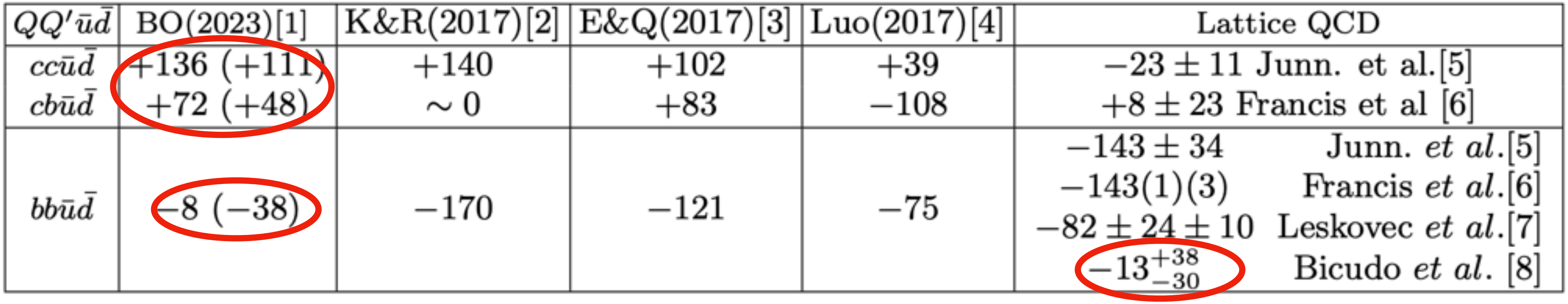

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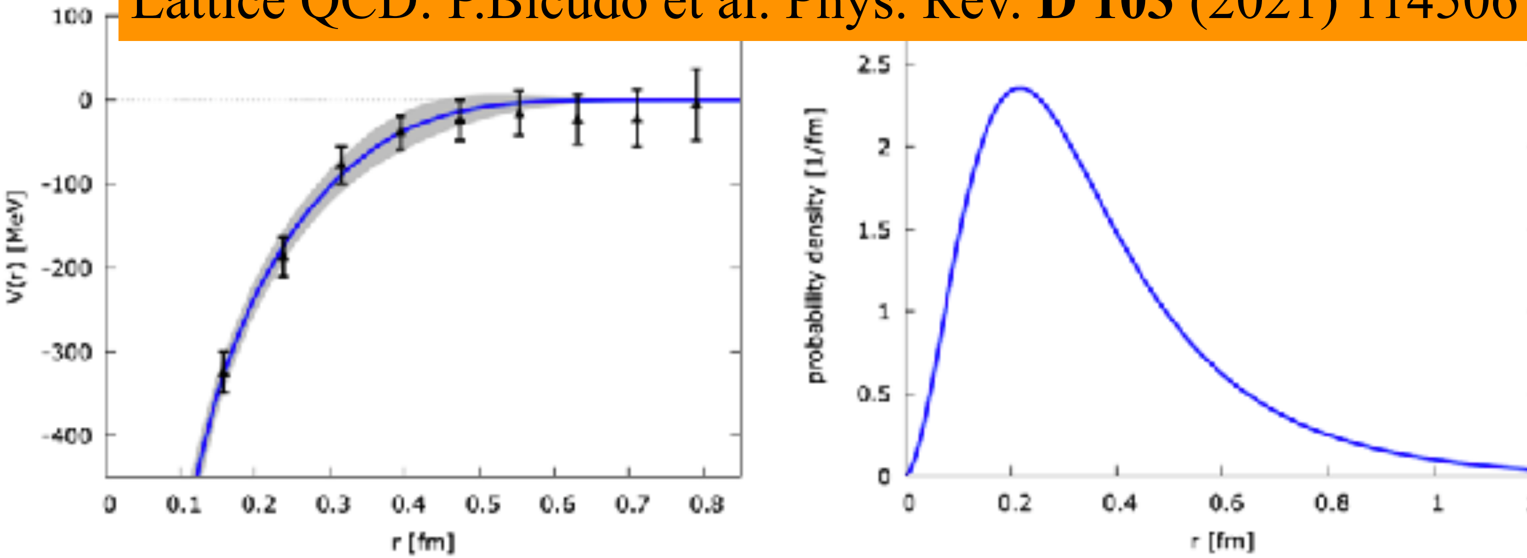
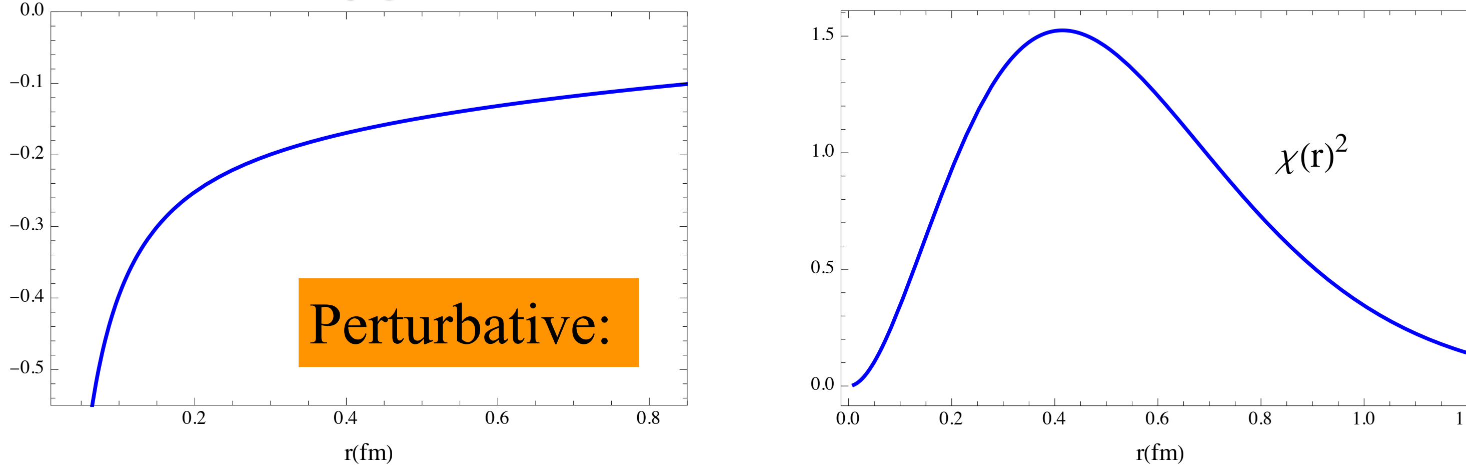
Born-Oppenheimer approximation: $M(\mathscr{T}_{cc}^{BO}) = 3871(3846)$[expt. 3875]

Comparison with BO in Lattice QCD

Lattice QCD: P.Bicudo et al. Phys. Rev. **D 103** (2021) 114506





Perturbative:

L. Maiani, A. D.Polosa and V. Riquer, PR D**100** (2019) 074002

Comparison with BO in Lattice QCD

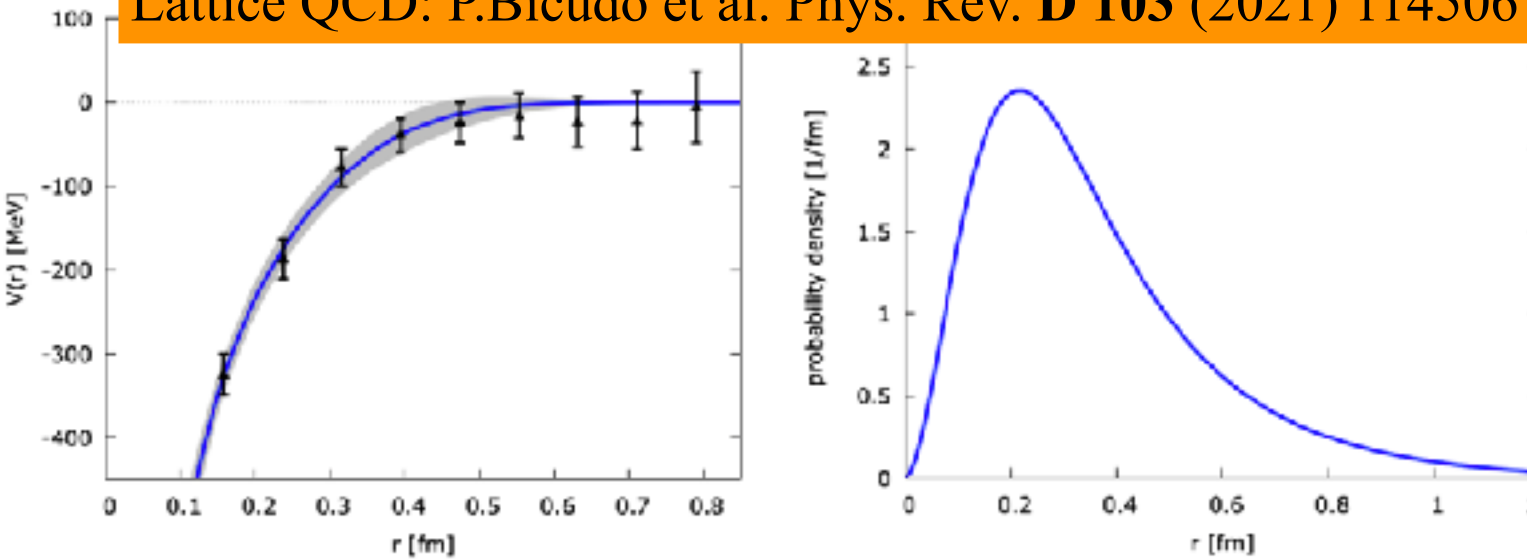
For $R \to +\infty$ $V_{BO}(R)$ vanishes in both cases: why not confined?

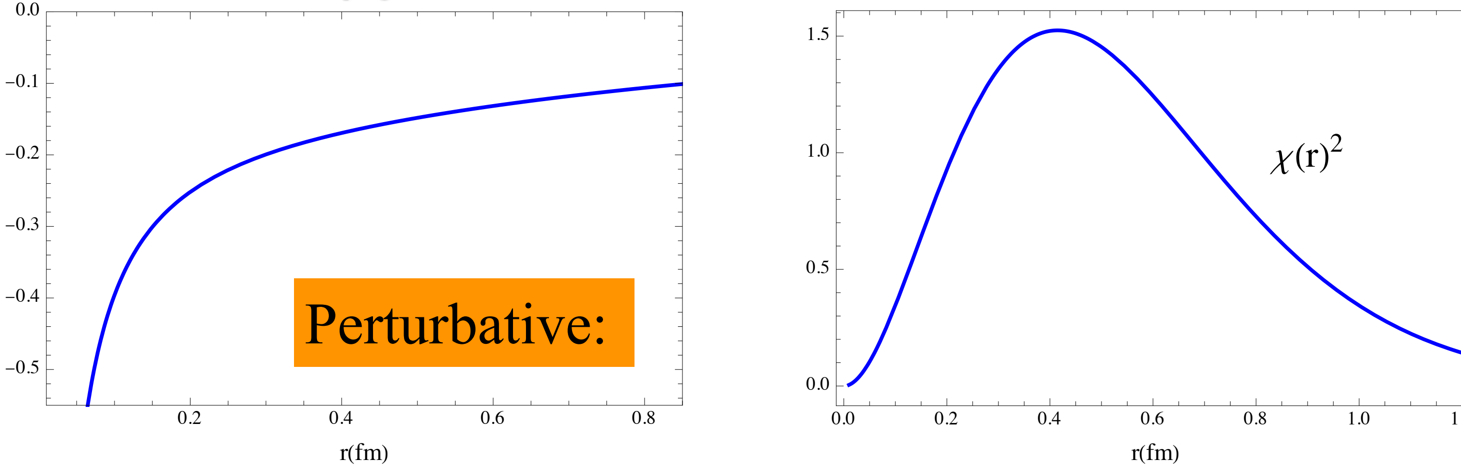
- At ∞, the $(b\bar{q})$ orbitals, ***B,*** are in a superposition of:

$B_{color\ \mathbf{8}} - B_{color\ \mathbf{8}}$ and $B_{color\ \mathbf{1}} - B_{color\ \mathbf{1}}$

- but in $B_{color\ \mathbf{8}} - B_{color\ \mathbf{8}}$ color can be screened by soft gluons from vacuum
- ***the asymptotic state is $B_{color\ \mathbf{1}} - B_{color\ \mathbf{1}}$ with vanishing interaction at $R \to \infty$***

Lattice QCD: P.Bicudo et al. Phys. Rev. **D 103** (2021) 114506





Perturbative:

L. Maiani, A. D.Polosa and V. Riquer, PR D**100** (2019) 074002

Comparison with BO in Lattice QCD

For $R \to +\infty$ $V_{BO}(R)$ vanishes in both cases: why not confined?

- At ∞, the $(b\bar{q})$ orbitals, ***B,*** are in a superposition of:

$B_{color\ \mathbf{8}} - B_{color\ \mathbf{8}}$ and $B_{color\ \mathbf{1}} - B_{color\ \mathbf{1}}$

- but in $B_{color\ \mathbf{8}} - B_{color\ \mathbf{8}}$ color can be screened by soft gluons from vacuum
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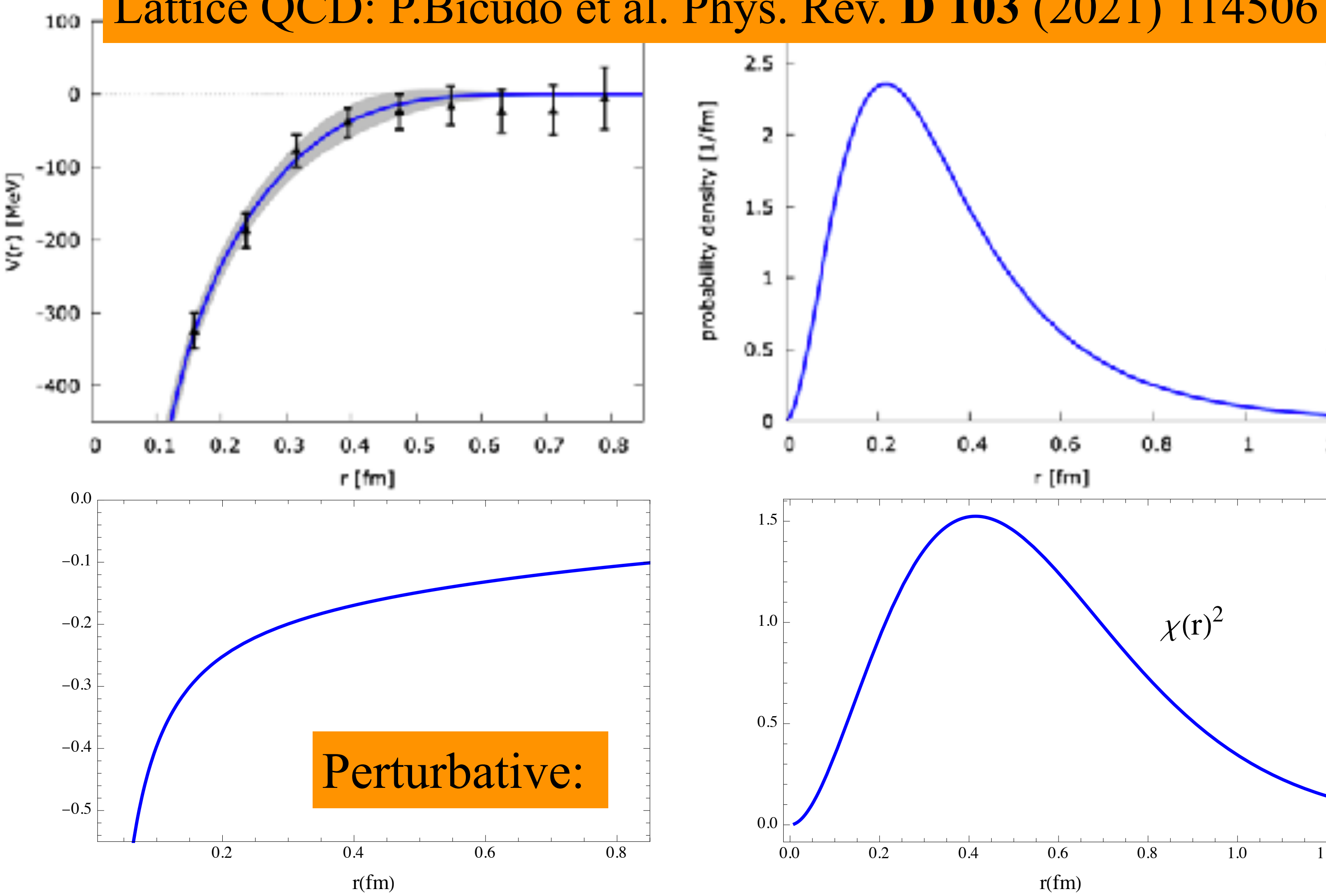
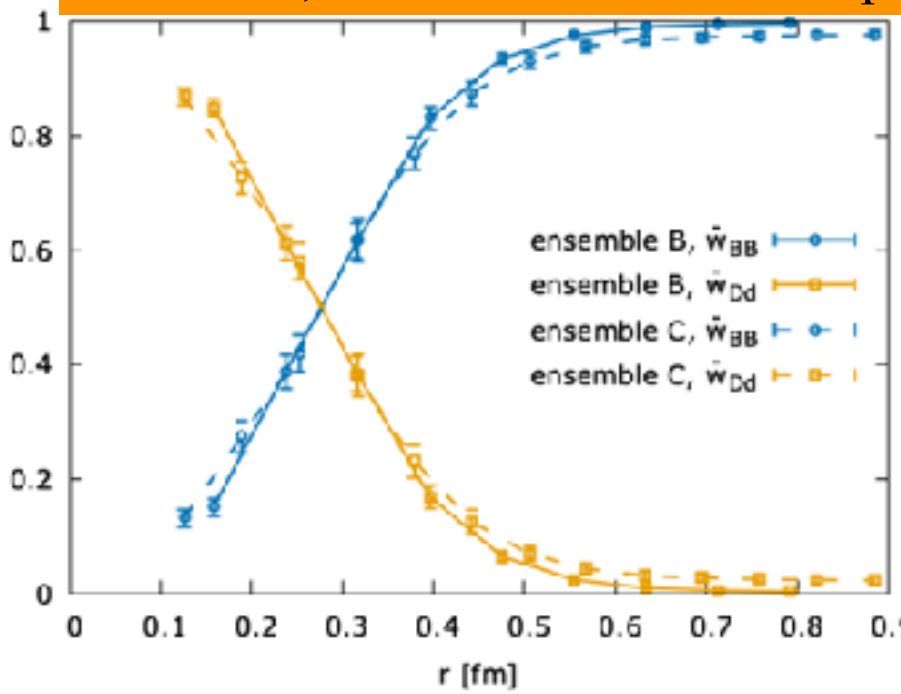


Perturbative:

- Projection of the lattice $\bar{q}\bar{q}$ result over Dd (yellow) or BB (blue) gives a picture of the space arrangement of light quarks at a given bb distance, R
- Mainly $[bb]_{\bar{\mathbf{3}}}[\bar{q}\bar{q}]_{\mathbf{3}}$ at the peak of the bb wave function ~0.2 fm.

L. Maiani, A. D.Polosa and V. Riquer, PR D**100** (2019) 074002



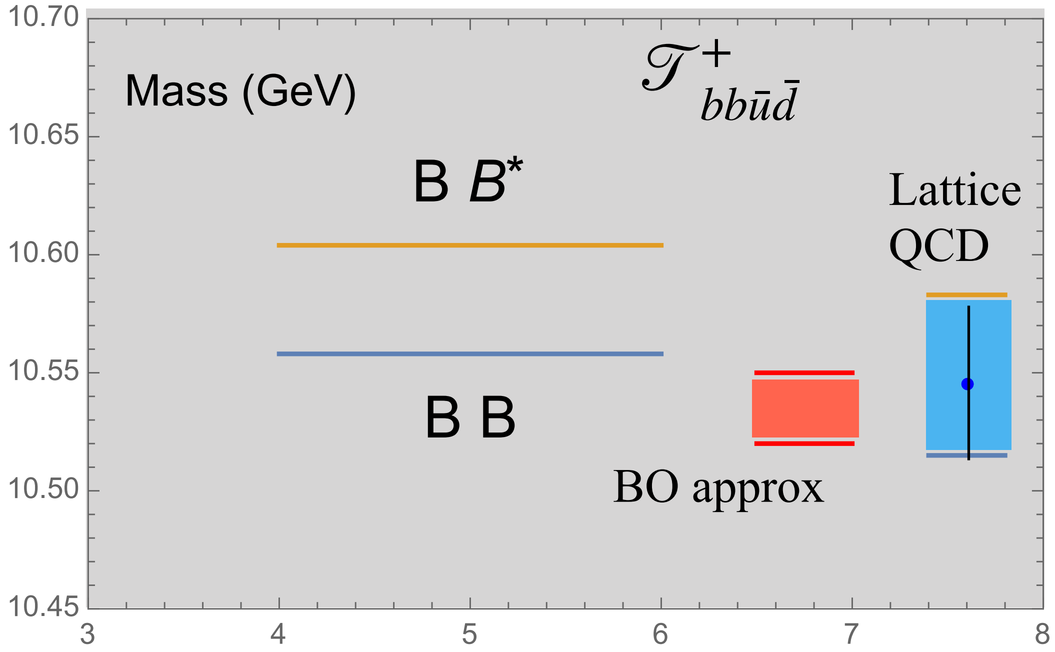
← B_1B_1 = Meson-Meson

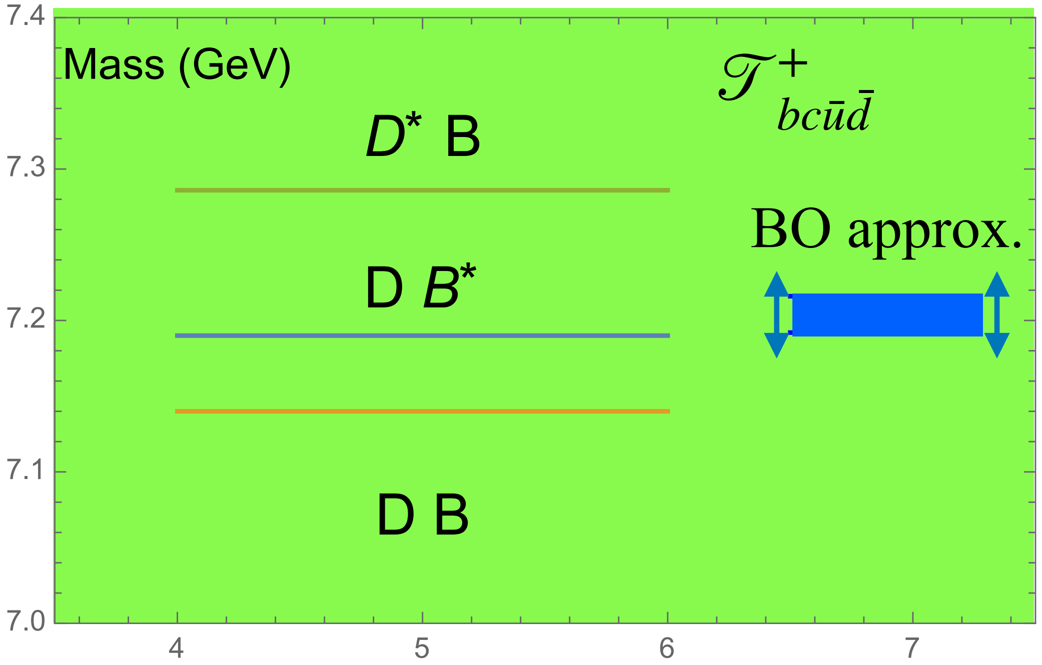
Lattice QCD

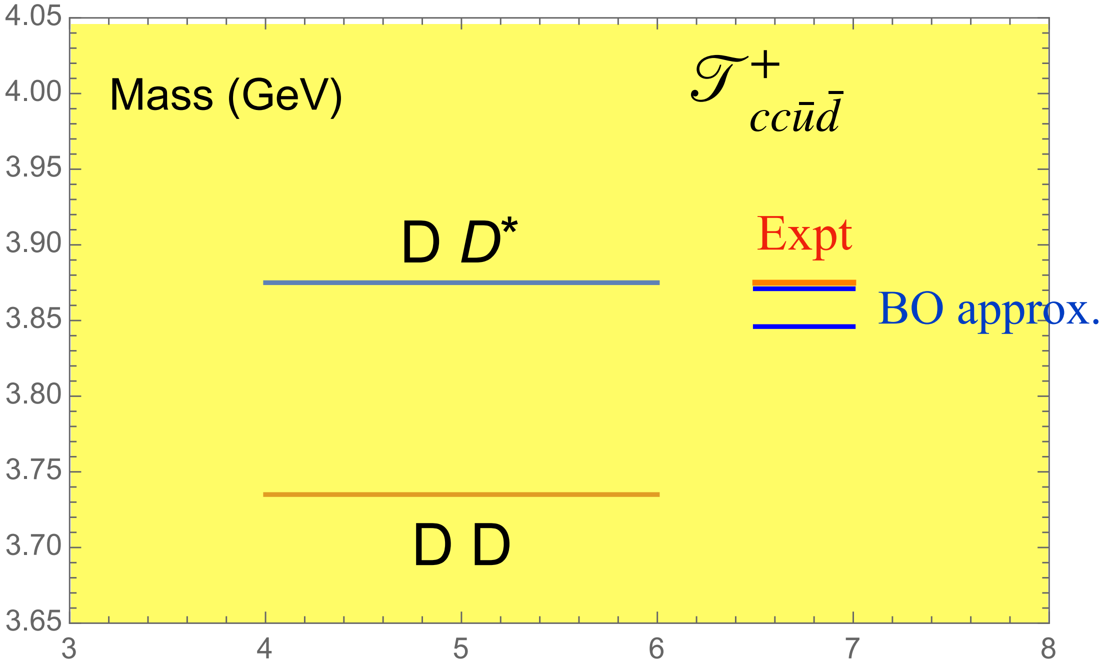
← $Dd = [bb][\bar{q}\bar{q}]$

Towards stability ?

	κ''_{QQ}	E	$\kappa_{Q\bar{q}}$ (P)	Q-value
cc	$+1.2$ $(+2.0)$	-74.8 (-100.2)	$+70$	$+135.8$ $(+110.8)$
bc	$+0.5$ $(+0.8)$	-67.3 (-91.5)	$+46.5$	$+72.4$ $(+48.4)$
bb	$+0.5$ $(+0.7)$	-77.3 (-107.4)	$+23$	-8.0 (-38.0)







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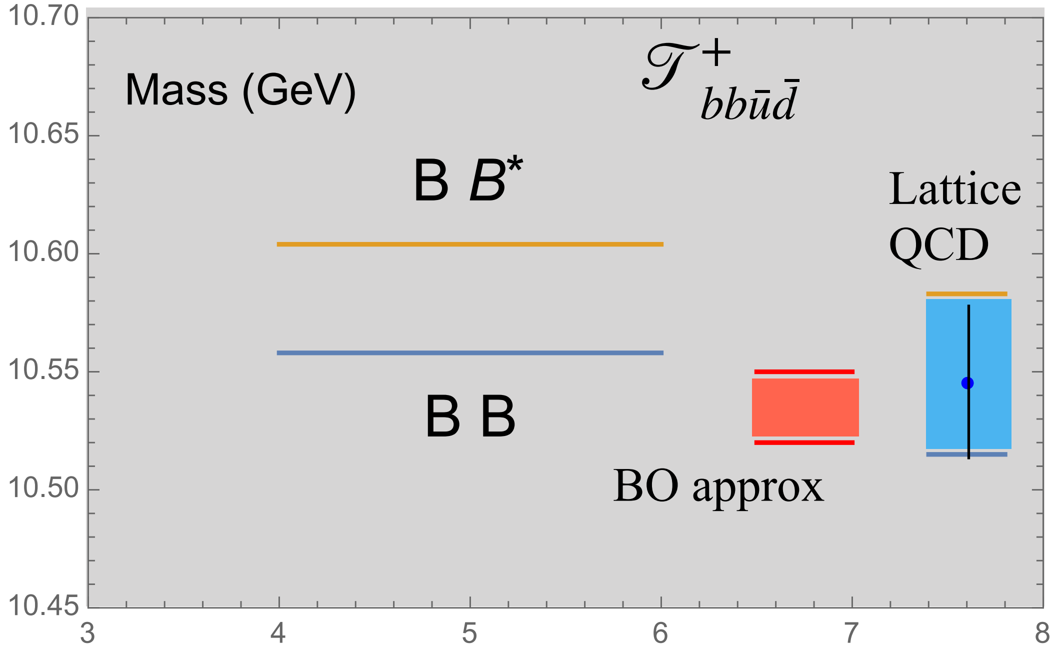
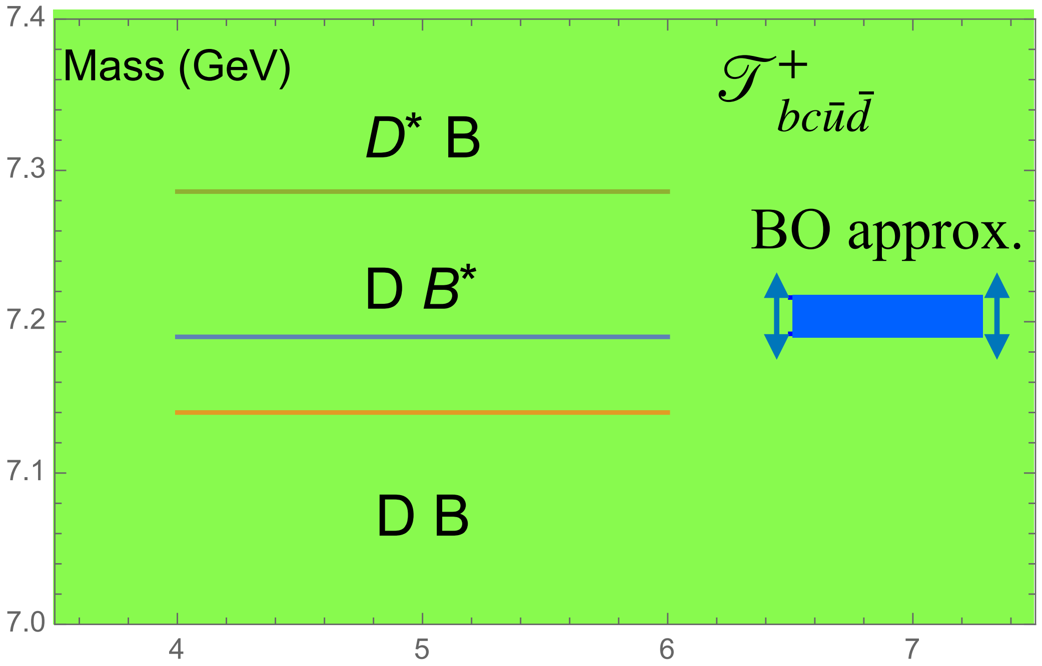
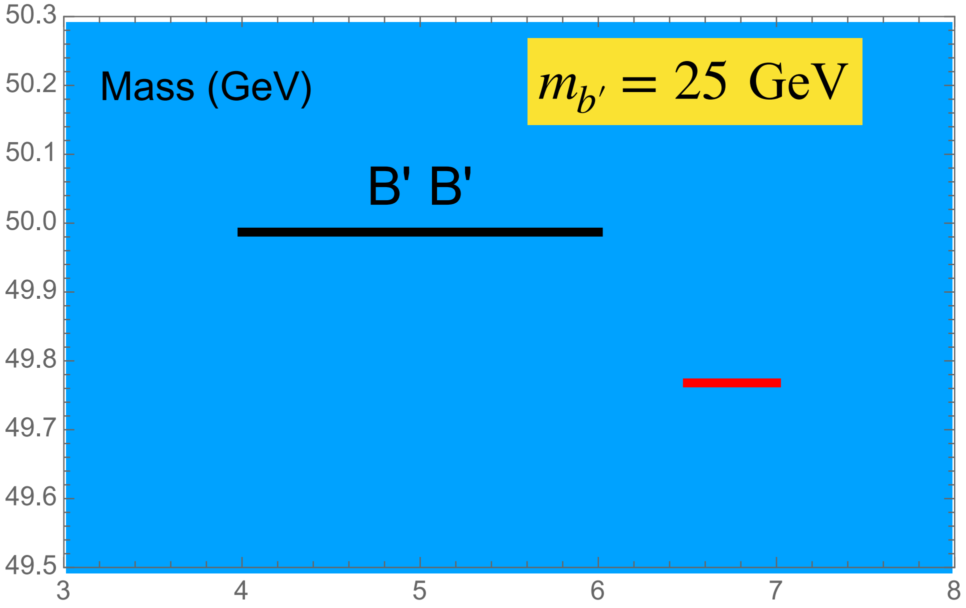
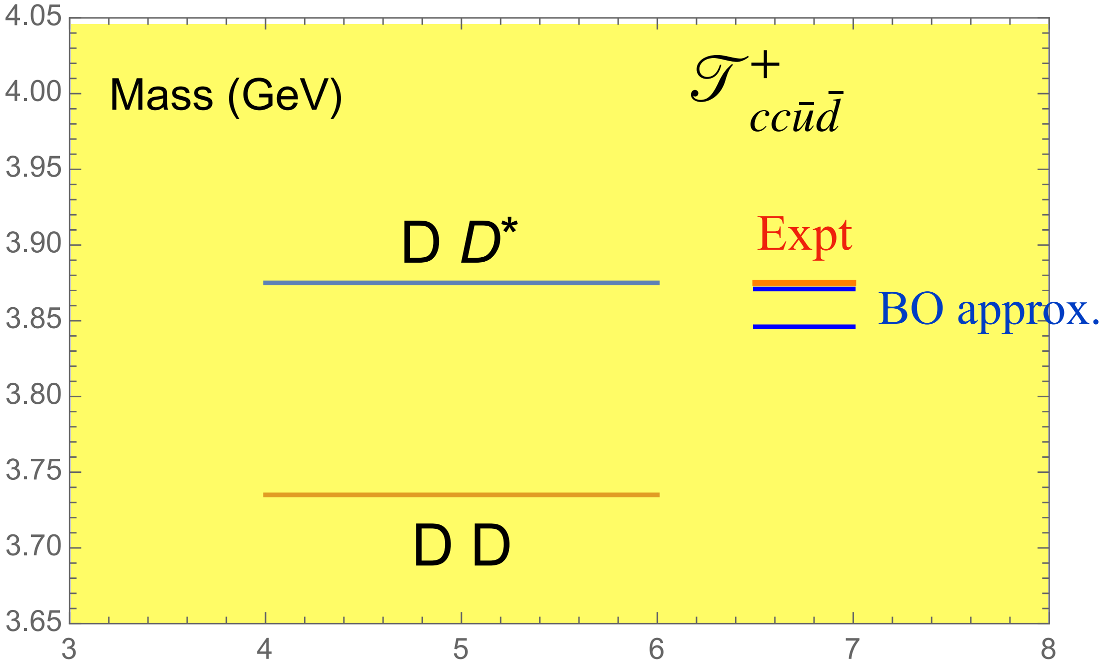




for very large b mass…
Eichten-Quigg's theorem
shows up more clearly





Is b quark mass large enough for a stable tetraquark?

Searching for $\mathcal{T}^0_{cb}$

- The state with $J^P = 1^+$ $(S_{\bar{q}\bar{q}} = 0,\ S_{cb} = 1)$ could be produced strongly in LHC collisions, in association with one $\bar{B}\bar{D}$ pair;
- Most likely, it decays strongly:

$$\mathcal{T}^0_{cb} \to D^0 B^{*0},\ D^+ B^{*-},\ \text{followed by } B^* \to B + \gamma$$

- Other spin states can be produced at the LHC, in particular $J^P = 0^+$, $(S_{\bar{q}\bar{q}} = 0,\ S_{cb} = 0)$
- $\mathcal{T}^0_{cb}(J^P = 0^+) \to D^0 B^0,\ D^+ B^-$ and no gamma ray in the final state: search for collisions: $p + p \to \bar{D}\bar{B} + DB + \ldots$

Searching for a stable $\mathcal{T}_{bb}^{-}(1^{+})$

- $\mathcal{T}_{bb}^{-}$ should be produced at LHCb, together with a $\bar{B}\bar{B}$ pair;
- if stable, $\mathcal{T}_{bb}^{-}$ should decay weakly with the b lifetime, at a ***detectable distance from the p-p interaction point***
- the expected weak decay $b \to c + \bar{c} + s \ (c + \bar{u} + d)$ gives rise to the chain decays

$$\mathcal{T}_{bb}^{-} \to \bar{D}_s(\pi^-) + \mathcal{T}_{bc}^{0} \to \bar{D}_s(\pi^-) + D^{+/0} + B^{*-/0} \to \bar{D}_s(\pi^-) + D + B + \gamma$$

in total

$$p + p \to \bar{B}\bar{B} + \ldots + (\bar{D}_s(\pi^-) + D + B + \gamma)_{(at\ a\ distance)}$$

- the weak decay could produce also the tetraquark $\mathcal{T}_{bc}^{0}$ $(J^P = 0^+)$, which would decay into $D + B$ without the gamma ray

Conclusions

- Quark model and Born-Oppenheimer give similar predictions for the $\mathcal{T}_{cc}^{+}$ mass, in agreement with the observed value;
- The BO approximation in QCD:
    - many similarities with the molecular picture, in particular for the feature of ***unconfined orbitals;***
    - the use of quark masses from meson spectrum is justified and crucial;
    - light quark hf interaction is much smaller than in the baryons: another effect of unconfined orbitals;
- A stable Double Beauty tetraquarks is still possible but not so clearcut;
- if $\mathcal{T}_{bb}^{-}$ is stable, it has a very interesting cascade weak decay:

$$\mathcal{T}_{bb}^{-} \to \bar{D}_s + \mathcal{T}_{cb}^{0}$$

Conclusions

- Quark model and Born-Oppenheimer give similar predictions for the $\mathscr{T}^{+}_{cc}$ mass, in agreement with the observed value;
- The BO approximation in QCD:
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  - light quark hf interaction is much smaller than in the baryons: another effect of unconfined orbitals;
- A stable Double Beauty tetraquarks is still possible but not so clearcut;
- if $\mathscr{T}^{-}_{bb}$ is stable, it has a very interesting cascade weak decay:

$$\mathscr{T}^{-}_{bb} \to \bar{D}_s + \mathscr{T}^{0}_{cb}$$

- The search for other spin and isotopic spin states is very important!!
- and so is the search of strange-double heavy tetraquarks: for each QQ' pair one predicts ***two*** $SU(3)_{flavour}$ ***multiplets***: $\mathbf{3}\ (S_{\bar{q}\bar{q}} = 0) \oplus \bar{\mathbf{6}}\ (S_{\bar{q}\bar{q}} = 1)$.