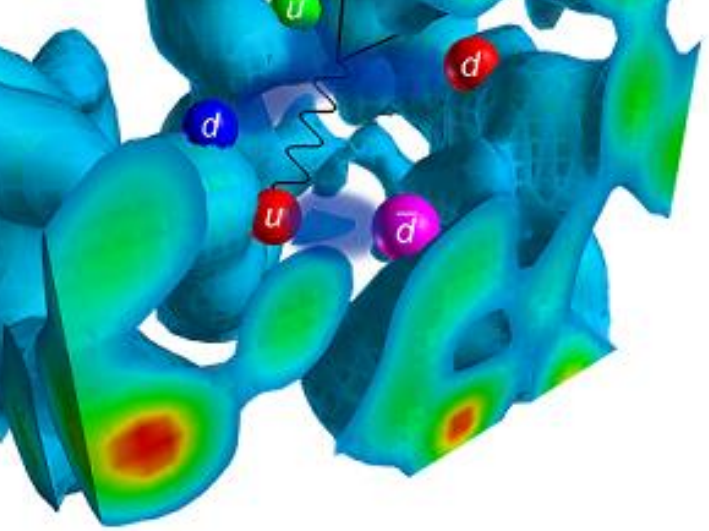


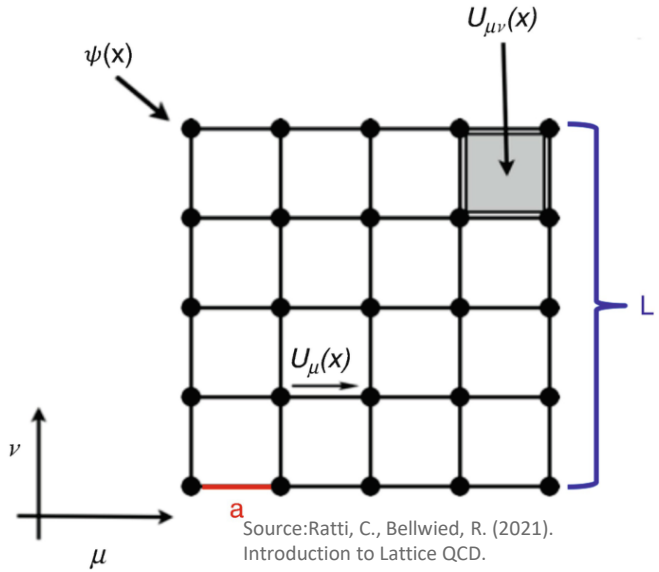
HSF-INDIA

Correlation Function Analysis

Adam Zacharia Anil

Indian Institute of Science Education and Research, Thiruvananthapuram

15th September



Source:Ratti, C., Bellwied, R. (2021). Introduction to Lattice QCD.

Mentors:
Archana Radhakrishnan (TIFR)
Brij Kishor Jashal (TIFR)

https://github.com/HSF-India/correlation_function_analysis

Adam Zacharia Anil

About me

I am an aspiring astrophysicist currently pursuing a BS-MS dual degree at the Indian Institute of Science Education and Research (IISER), Thiruvananthapuram. With a major in Physics and a minor in Data Science, I am passionate about unlocking the mysteries of the universe through the power of big data analysis and computational methods.

Where am I from

Kerala, India

I am from Kollam which is one of the 14 districts in Kerala

Research Interests

Astrophysics,Particle Physics

I am Interested Gravitational wave astrophysics, GRBs , Astro- particle -physics, Kilonoave

More Info

Website

https://www.adamzacharia.com/

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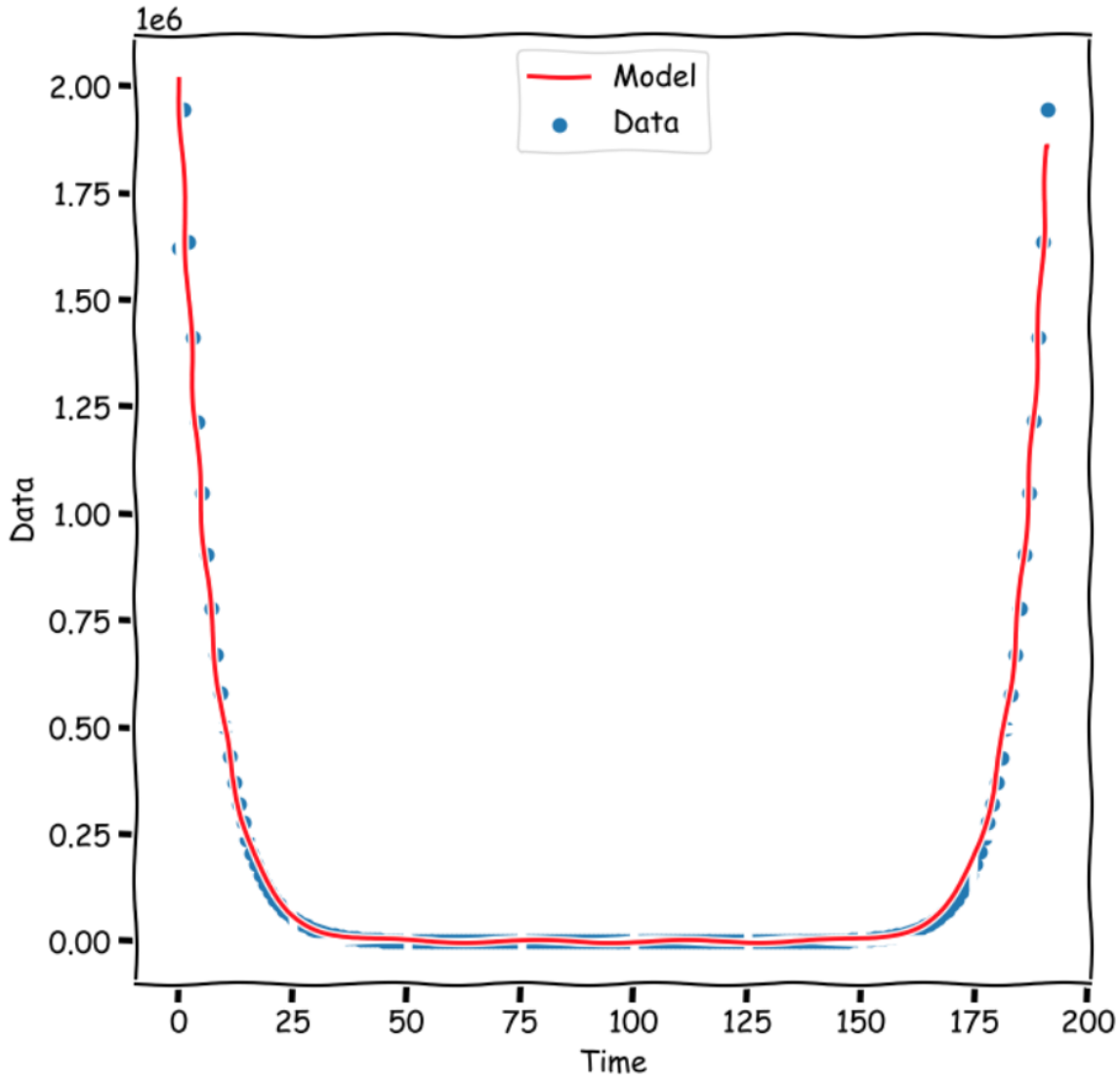
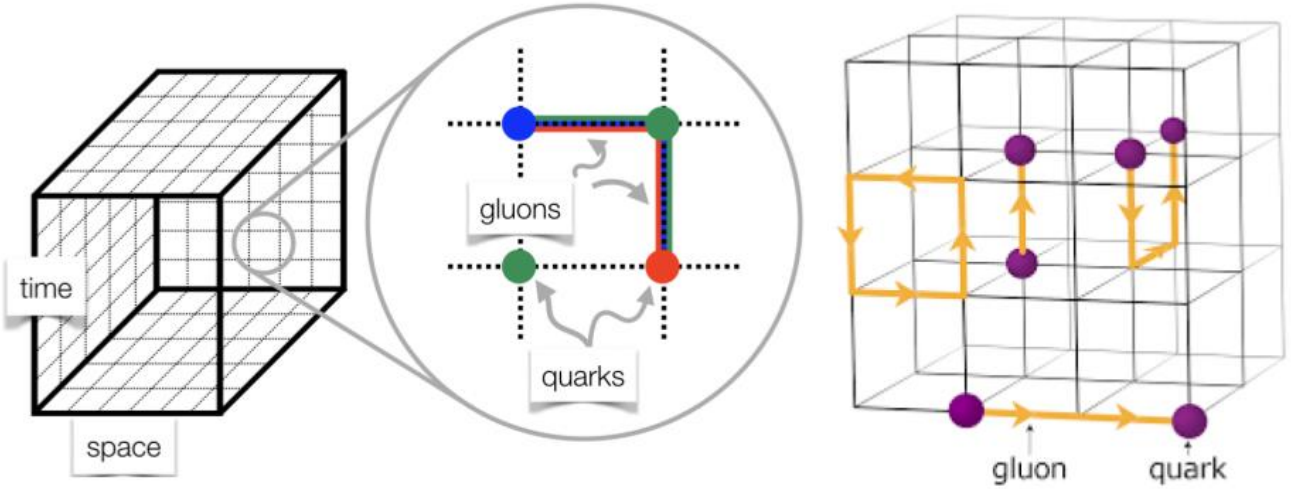


What is Lattice QCD ?

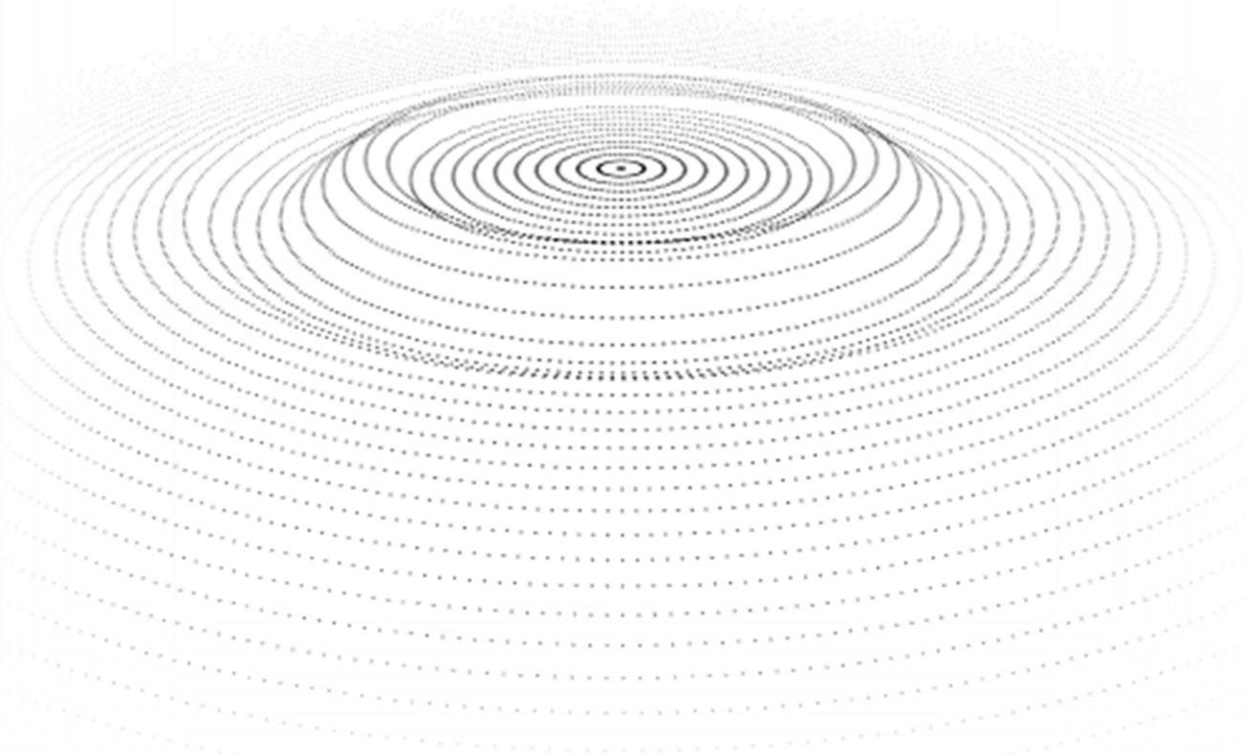
Lattice QCD is a computational method in quantum physics that discretizes spacetime onto a grid or lattice. By doing this, it enables numerical solutions for the non-perturbative regime of Quantum Chromodynamics, which describes the strong force interactions between quarks and gluons to form hadrons.



Source: edyrd

What is Two point Correlation function ?

A two-point correlation function measures the probability amplitude for a particle to propagate from one point in spacetime to another. Mathematically, it represents the expectation value of the product of two field operators, separated in time. By studying how this function decays with time separation, one can extract the mass (like a proton or a pion) associated with the quark fields.



Spectral Decomposition of Correlation Functions

We can look at the spectral decomposition to see the time dependence gives the energies. The two-point correlation function can thus be expressed as a sum of exponentials, where each exponential term corresponds to a specific energy state. This expression enables the isolation and identification of both the ground and excited states. By examining the behavior of the correlation function at different time separations, one can target specific states.

The spectral decomposition of a two-point correlation function is given by

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j(0)|0\rangle = \sum_p \frac{Z_i^{p*} Z_j^p}{2E_p} e^{-E_p t}$$

Source:Amparo, Elena, "Spectral Analysis of Lattice QCD Two-Point Correlation Functions" (2018). Undergraduate Honors Theses. Paper 1229.

Spectral Decomposition of Correlation Functions

where $Z_i^p = \langle 0|\mathcal{O}_i|p\rangle$ gives the overlap factor of the eigenstate $|p\rangle$ and the interpolating operator $\mathcal{O}_i$ [4]. Let

$$v^m = \begin{bmatrix} v_1^m \\ \vdots \\ v_n^m \end{bmatrix}.$$

denote the generalized eigenvector corresponding to the eigenvalue λ_m. Then combining the spectral decomposition with the GEVP we obtain

$$C_{ij}(t)v_j^m = \sum_p \frac{1}{2E_p} Z_i^{p*} Z_j^p v_j^m e^{-E_p t} = \lambda_m(t) C_{ij}(t_0) v_j^m = \sum_p \frac{1}{2E_p} Z_i^{p*} Z_j^p v_j^m \lambda_m(t) e^{-E_p t_0}$$

for all $j \leq n$ and thus

$$\sum_p \frac{Z_i^{p*}}{2E_p} Z^p \cdot v^m \left[e^{-E_p t} - \lambda_m(t) e^{-E_p t_0}\right] = 0$$

$$= \frac{Z_i^{m*}}{2E_m} Z^m \cdot v^m \left[e^{-E_m t} - \lambda_m(t) e^{-E_m t_0}\right] + \sum_{p \neq m} \frac{Z_i^{p*}}{2E_p} Z^p \cdot v^m \left[e^{-E_p t} - \lambda_m(t) e^{-E_p t_0}\right.$$

for all m, i. This implies that the time dependence of the eigenvalues is given by

$$\lambda_m(t) = e^{-E_m(t-t_0)}$$

Source:Amparo, Elena, "Spectral Analysis of Lattice QCD Two-Point Correlation Functions" (2018). Undergraduate Honors Theses. Paper 1229.

Let's Talk about the Data

The data I have is of the pseudo -Two Correlation point Function

The simulation is performed on $N_f = 2 + 1 + 1$open source gauge ensembles from MILC collaboration

milc-qcd/**milc_qcd**

MILC collaboration code for lattice QCD calculations

8 Contributors | 15 Issues | 32 Stars | 27 Forks

0	1.63E+06
1	1.97E+06
2	1.68E+06
3	1.47E+06
4	1.28E+06
5	1.12E+06
6	9.64E+05
7	8.24E+05
8	7.03E+05
9	6.02E+05
10	5.17E+05
11	4.44E+05
12	3.80E+05
13	3.24E+05
14	2.78E+05
15	2.40E+05
16	2.08E+05
17	1.82E+05
18	1.60E+05
19	1.41E+05
20	1.23E+05
21	1.07E+05
22	9.33E+04
23	8.04E+04
24	6.90E+04

So how do we fit?

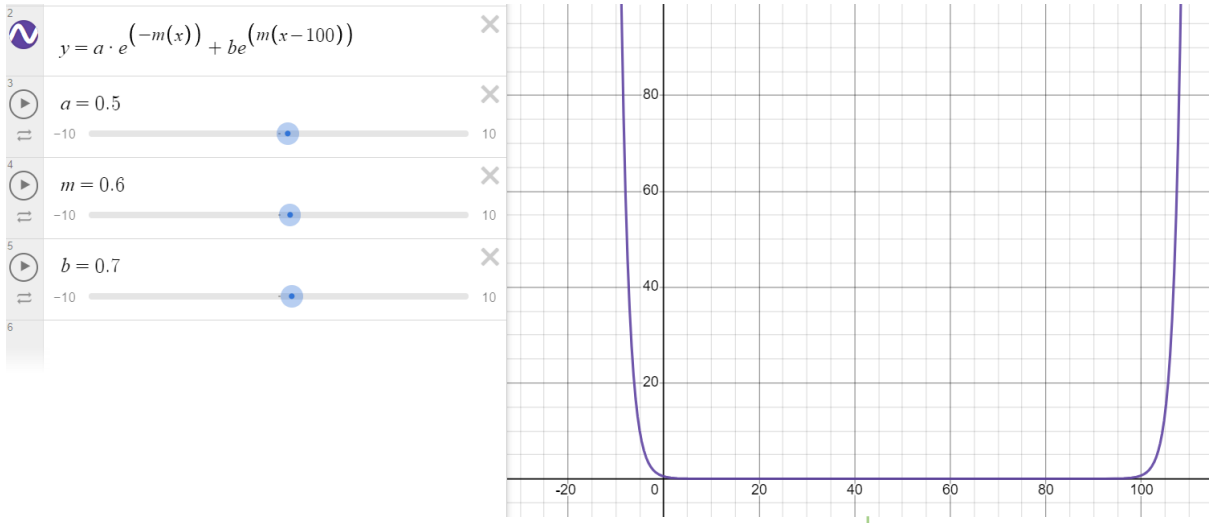
Step 1

Identify and derive the required mathematical equation that represents the behavior of the system or process to be modeled.

$$Y = Ae^{-m(x)} + Be^{m(x-n)}$$

Step 2

Using graphing calculators or platforms, like Desmos, to visualize the equation.



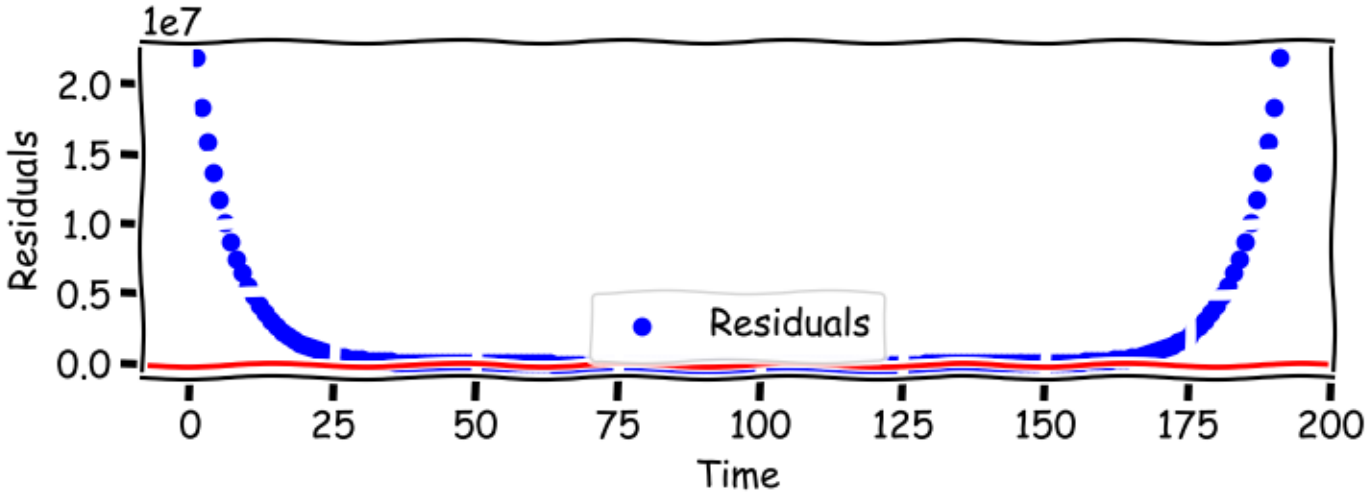
Step 3

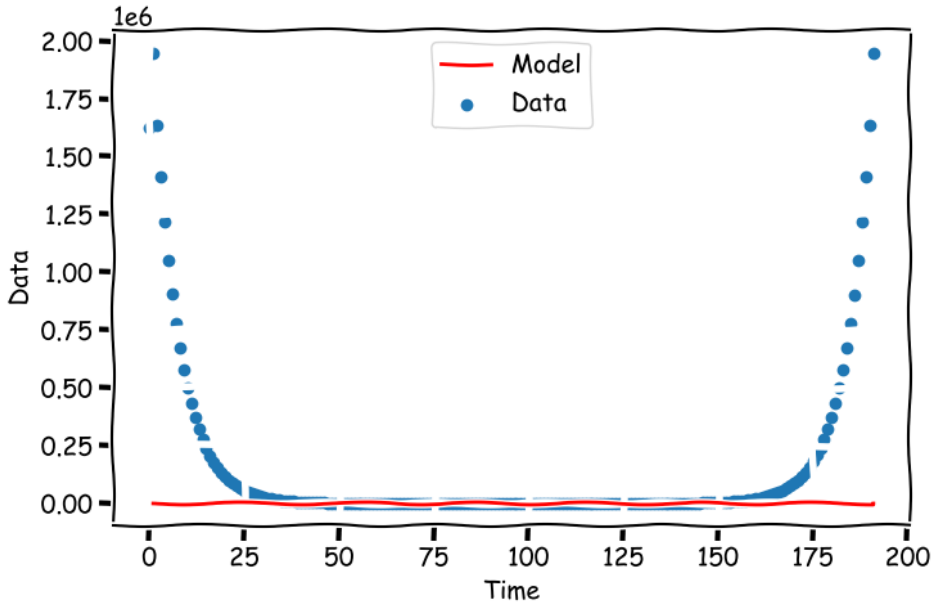
Translating the mathematical equation into Python code

```
def model_func(x, m, a, b):
    return a * np.exp(-m * x) + b * np.exp(m * (x-n))
```

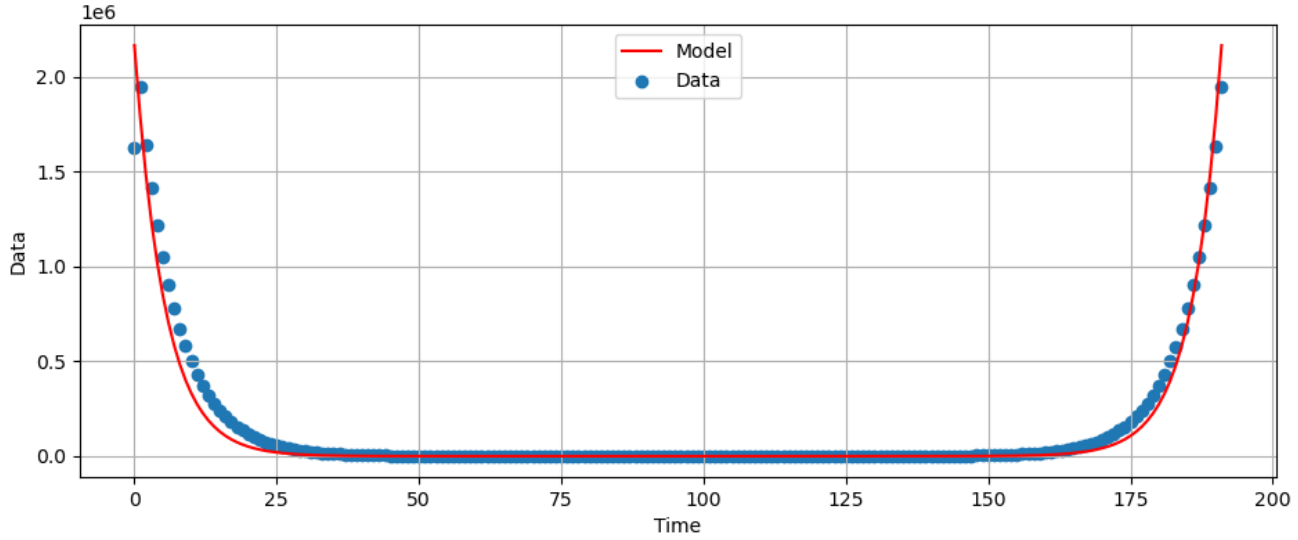
After Hundred Lines Of Code And A Thousand Errors Later.

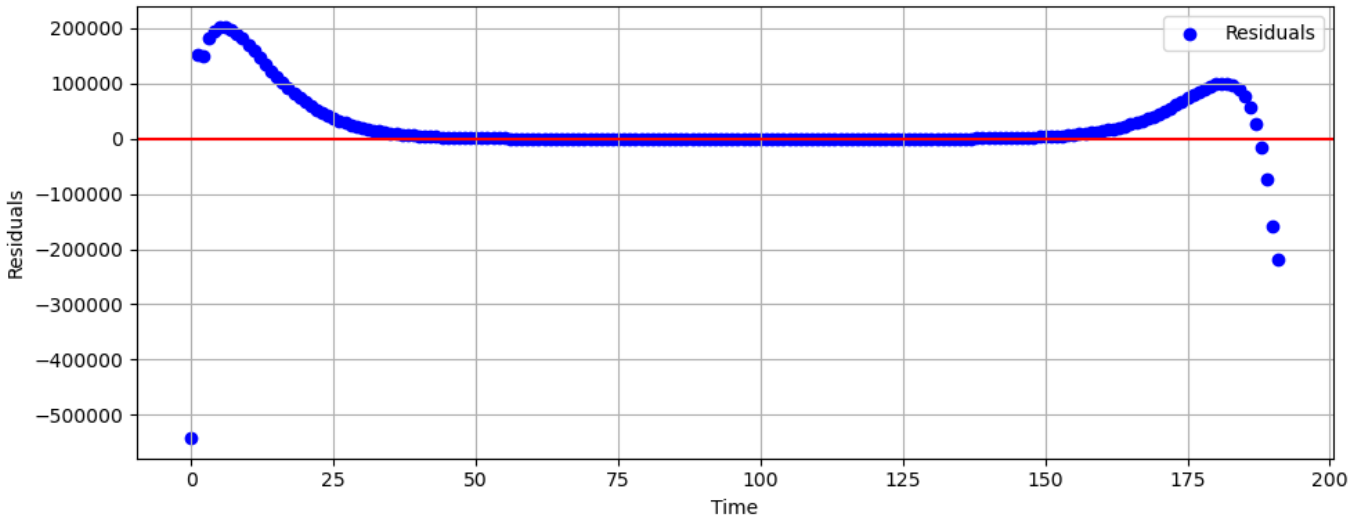
Translating abstract mathematical concepts into executable code can frequently result in small, yet frustrating mistakes. These seemingly trivial errors can consume an unexpectedly large amount of debugging time. I will give a clearer picture, here about some challenges and issues I've grappled with during such conversions:





Fit Result





Is this really the final result ??

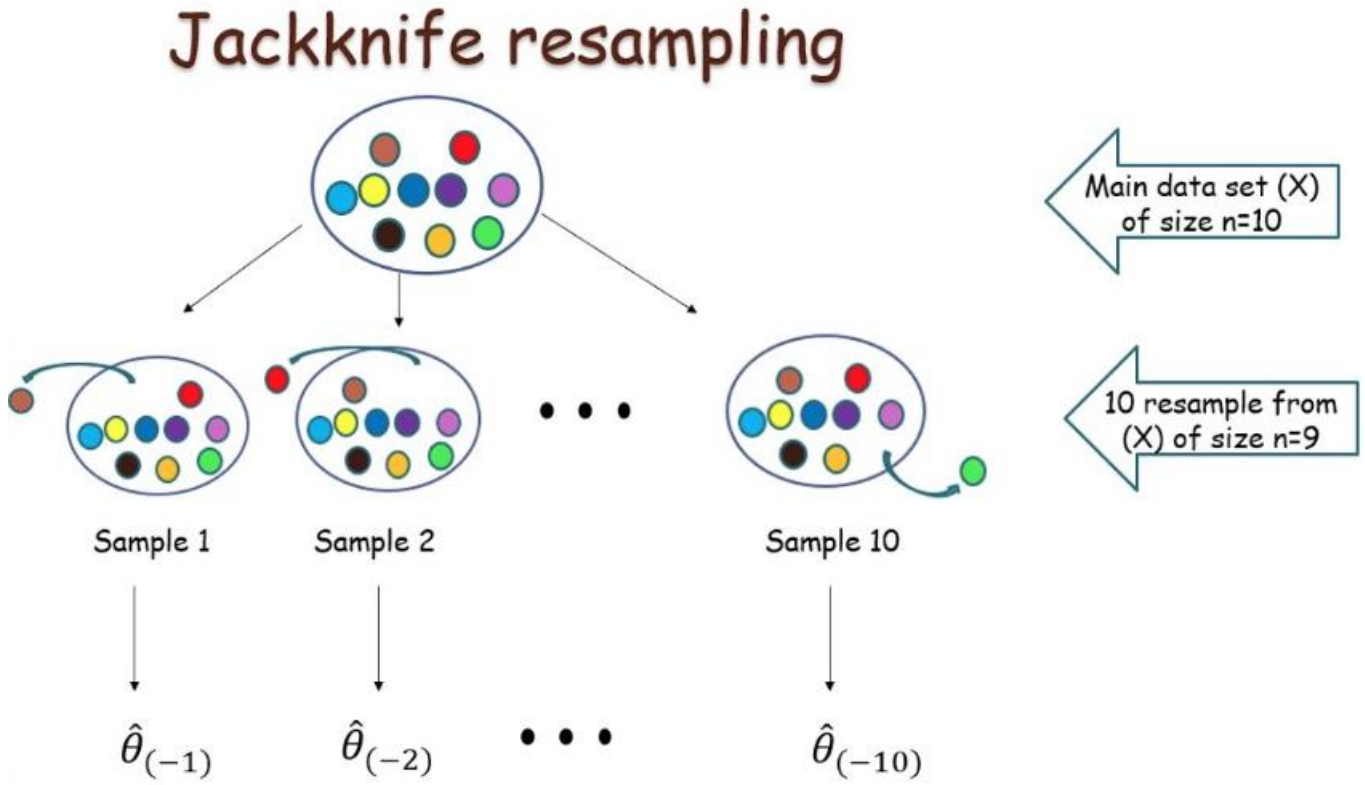
How can we improve it??

How do we remove bias and other errors??

Jackknife Resampling

The jackknife method involves systematically leaving out one observation at a time from the sample set and calculating the estimate over n-1 observations.

By doing this for all observations in the sample and examining the variation among the results, one can estimate the bias and variance of the estimator.

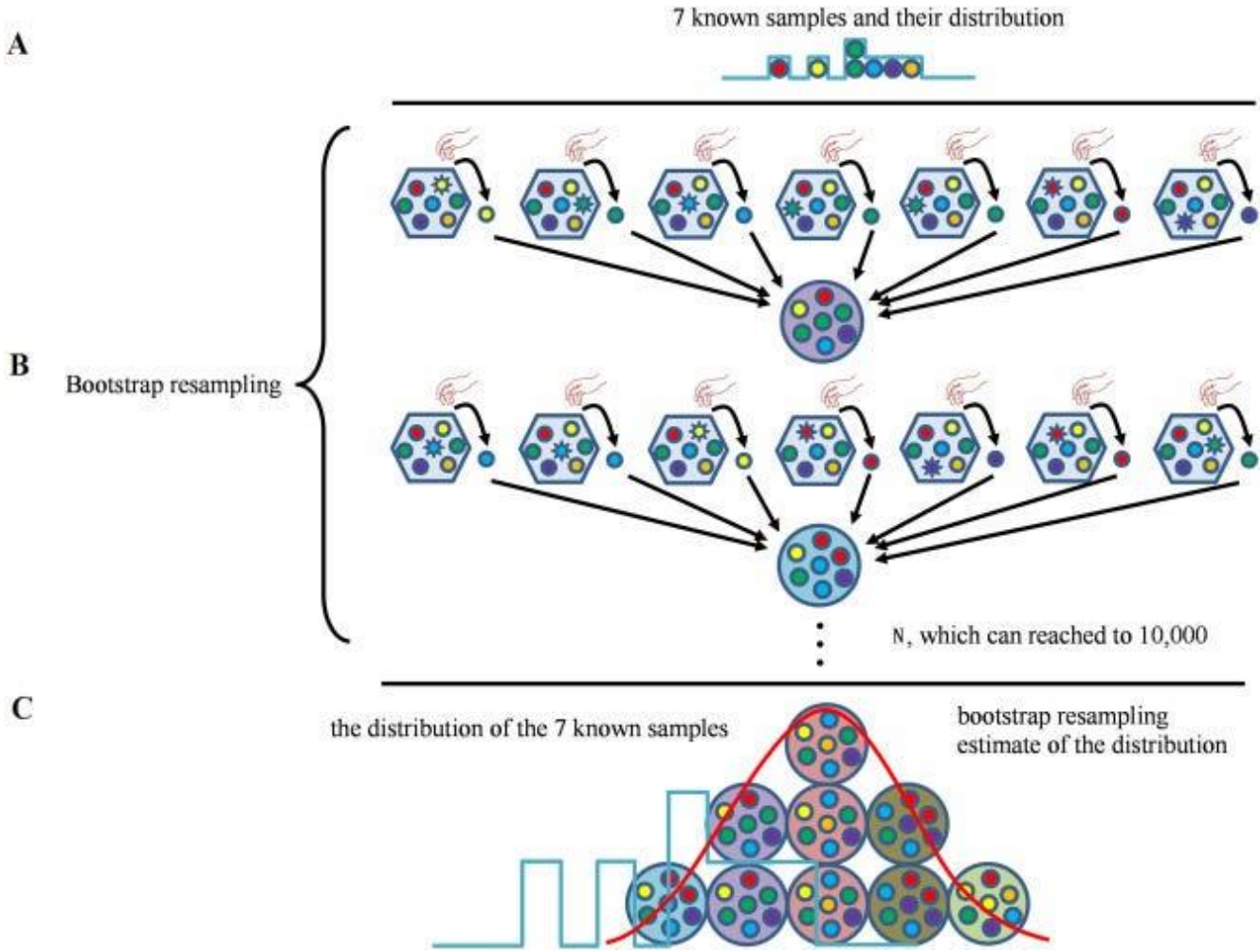


Source: Statistics Time

Bootstrap Resampling

Bootstrap estimation is a powerful resampling technique used to infer the distribution of a statistic (like the mean or variance) by repeatedly sampling subsets of the observed data with replacement.

The key idea is to simulate the process of obtaining new sample sets by drawing from the original dataset. By doing this many times (say thousands or even millions), you can build an empirical distribution of the statistic and estimate its variance, confidence intervals, and more.



Source: Ming Cui et al (2017)

Jackknife VS Bootstrap

Jackknife

- Simpler formulation
- Deterministic results
- Bias correction

Bootstrap

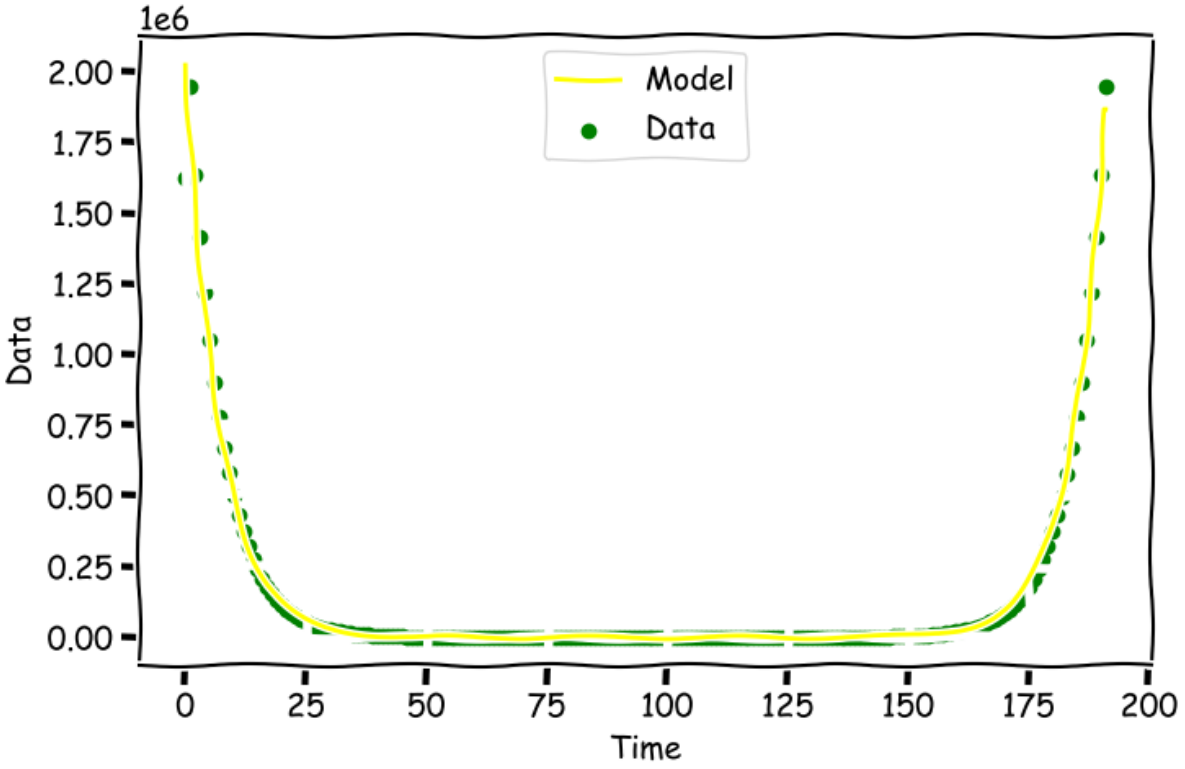
- Greater versatility
- Accurate error estimates
- Useful for complex statistics

Time	Jackknife Estimations									
0	1623842	1623840	1623296	1624126	1623908	1622867	1623973	1624254	1624086	16
1	1947857	1948103	1947611	1948318	1948285	1947173	1948178	1948359	1948165	19
2	1636704	1637156	1636742	1637337	1637366	1636337	1637187	1637328	1637037	16
3	1414201	1414835	1414456	1415012	1415017	1414131	1414772	1414993	1414539	14
4	1216760	1217548	1217162	1217703	1217657	1216913	1217343	1217658	1217071	12
5	1048755	1049599	1049218	1049729	1049642	1049029	1049260	1049642	1048981	10
6	903880.3	904666.9	904300.6	904760.1	904627.4	904147.2	904249.6	904658.2	903968.7	904
7	779574.3	780209.2	779873.9	780287.7	780123.8	779747.9	779829.2	780191.1	779498.6	7
8	672825.6	673300.7	672991.3	673386.4	673215.9	672895.2	673036.2	673286.7	672613.1	672
9	581001.3	581384.9	581073	581472.2	581300.2	581012.3	581222.8	581358.1	580747	580
10	501960.8	502299.6	501954.2	502369.1	502200.6	501918.2	502206.9	502224.1	501720.7	501
11	433476.9	433778.8	433409.1	433830.5	433669.6	433371.6	433742.1	433668.1	433271	433
12	373966.6	374208.5	373851.2	374255.3	374112.7	373802.6	374221.6	374094.9	373779.1	373
13	322773.8	322946.6	322619.3	322989.5	322876.7	322564.1	323002.7	322836.3	322581.9	322
14	278776.1	278904.5	278618.5	278937.3	278852	278555.1	278997.4	278795.1	278584.7	278
15	240891.5	241001.4	240753.6	241025.6	240957	240695.5	241114.9	240888.2	240724.5	240
16	208226.7	208328.7	208111.7	208357.6	208294.1	208065.3	208448.4	208216.5	208107.7	2
17	180072.5	180180.4	179982.7	180208.4	180145.9	179944.8	180281.7	180073.8	180003.5	179
18	155751.6	155880.2	155694.2	155895.5	155834.5	155658.7	155942.3	155779.2	155721.3	155
19	134661.9	134810	134639.8	134814.6	134750	134595	134832.9	134712.5	134660.4	134

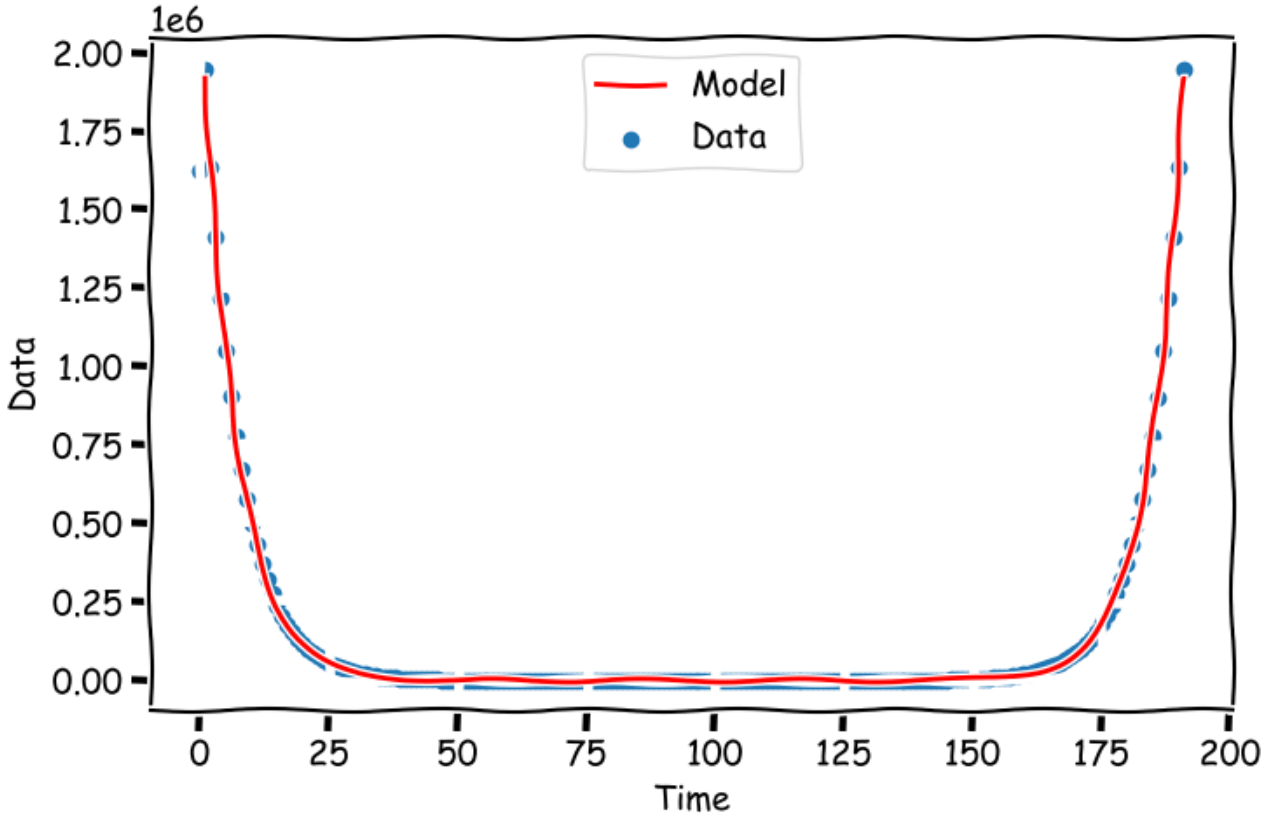
Time	Estimation	Estimation	Estimation	Estimation	Estimation	Estimation	Estimation	Estimation	Estir
0	1619676	1630143	1617932	1617544	1632964	1630956	1630914	1626608	162
1	1950107	1952227	1950410	1954378	1956771	1944159	1953996	1943836	194
2	1641579	1644449	1629687	1639445	1640120	1634298	1637616	1633531	162
3	1414394	1424314	1423285	1413894	1414102	1412782	1417851	1408051	142
4	1219065	1228074	1218104	1222654	1222823	1213096	1218979	1220537	122
5	1060552	1042274	1051748	1045090	1046036	1058623	1049298	1048279	105
6	903770.2	908493.1	907885.7	901771.7	900423.5	903327.2	895913.8	909208.9	890
7	775709.3	766864.6	783863.6	782894.3	783045.1	779593.4	781874.5	781679.9	769
8	674687.1	675161.9	666153	675124.6	677482.2	677530.2	676345.4	671201.4	674
9	578136.6	585851.9	584925.2	581374.4	581230.4	579458	585168.6	581992.5	580
10	498316.3	499343.5	499868.1	502306.5	506055.5	501878.1	502742.1	509045.5	507
11	434454.7	433196.2	434734.5	430767.2	425307.9	436923.1	436174.3	431670.3	429
12	375690.6	370319.7	373874.9	370410.2	376875	367512.4	369072.5	371513.8	374
13	326639.6	318454.8	322332.8	321674.5	322341.5	321120.1	322130.7	320008	322
14	278969.1	278016.4	279073.3	279273.3	279537	278774.1	280548.2	277163.3	278
15	237653	241176.5	242559.8	240398	240115.3	240178	242043	242030.9	238
16	207628.1	205857.7	211077.5	205251.6	206181	208966.8	207753.6	208691.7	210
17	181233.1	180354.3	181076.5	179516.1	179457.5	179326.7	180472.2	181758.5	179

Newer Result

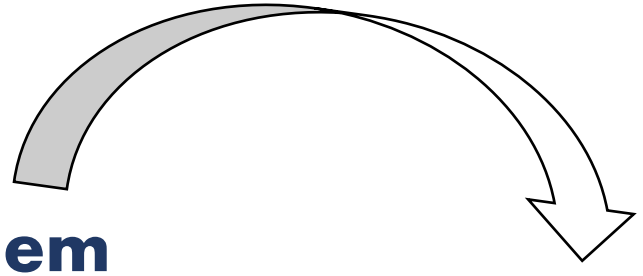
Bootstrap



Jackknife



Future plans



Generalized Eigenvalue Problem

The Generalized Eigenvalue Problem (GEVP)is a mathematical approach used to derive energy levels of hadronic states from the numerical data. By analyzing matrices of correlation functions with various operators over different time separations, the GEVP produces the best linear combination of the operators that project to a particular state. These eigenvalues represent the exponential decay rates of the correlation functions, which are directly linked to the energies of the particles under study.

Source:Amparo, Elena, "Spectral Analysis of Lattice QCD Two-Point Correlation Functions" (2018). Undergraduate Honors Theses. Paper 1229.

Given a matrix of correlation functions, we would like to extract the energies of the eigenstates. Using a variational approach to find the linear combinations of operators which best approximate the eigenstates leads to a generalized eigenvalue problem.

Consider a diagonal Correlator $\langle 0|\Omega(t)\Omega^{\dagger}(0)|0\rangle$,where $\Omega = \sum_i v_i^* \mathcal{O}_i$ Is a linear combination of operators.

Then it can be decomposed as

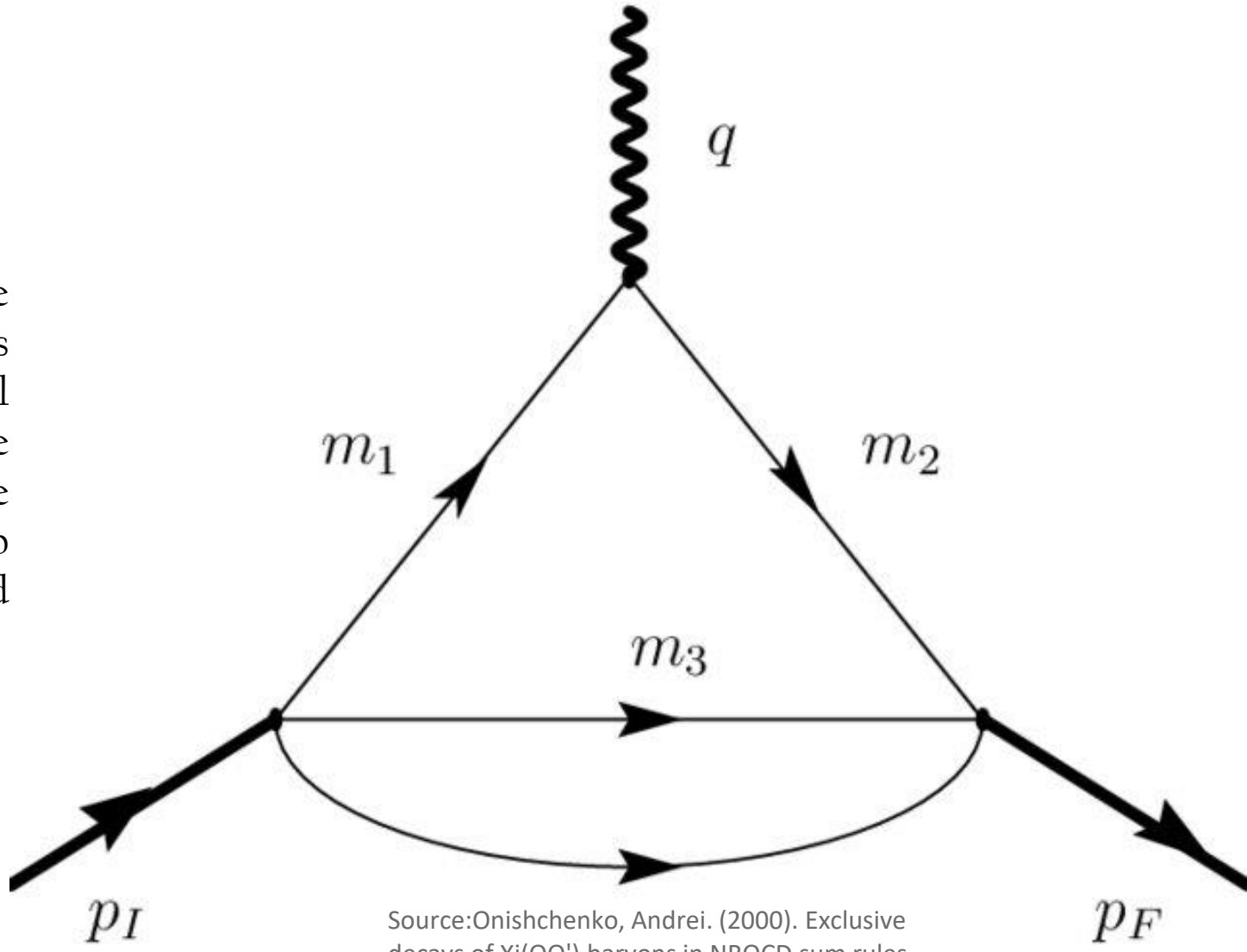
$$\langle 0|\Omega(t)\Omega^{\dagger}(0)|0\rangle = \sum_n \langle 0|e^{\hat{H}t}\Omega(0)e^{-\hat{H}t}|n\rangle\langle n|\Omega^{\dagger}(0)|0\rangle$$

$$= \sum_n \langle 0|\Omega(0)|n\rangle\langle n|\Omega^{\dagger}(0)|0\rangle e^{-E_n t} = \sum_n |\langle n|\Omega^{\dagger}(0)|0\rangle|^2 e^{-E_n t}$$

Future plans

Three Point Correlation Function

A three-point correlation function evaluates the interaction amplitude when a particle transitions from one state to another, mediated by an external current or operator at an intermediate spacetime point. Mathematically, it is the expectation value of the product of three field operators, where two are typically separated in time and the third represents the transition operator.



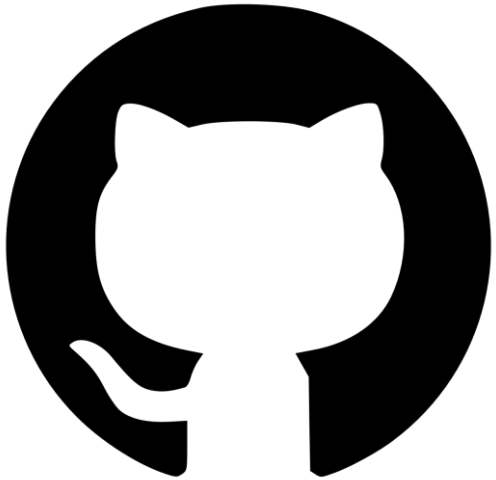
Source:Onishchenko, Andrei. (2000). Exclusive decays of Xi(QQ') baryons in NRQCD sum rules.

Future plans

One Ring To Rule Them All

The idea of this project is to design and implement a versatile software layer that can effectively perform fits and solve linear systems of equations. By building upon existing frameworks in lattice QCD, this software layer will enable researchers to extract multiple form-factors with ease and efficiency.

Source: The Lord of the Rings: The Fellowship of the Ring



THANK YOU

https://github.com/HSF-India/correlation_function_analysis