

Recent advancements in perturbative QCD at high density

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Based on: [K.S. et al. JHEP '23](#) and [K.S. et al. PRL '23](#)

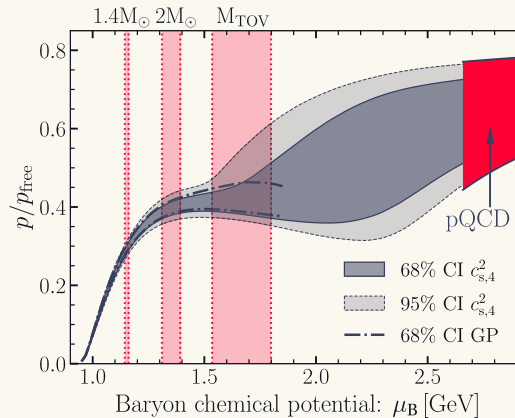


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QCD thermodynamics at high density

- **Thermodynamics of cold and dense** ($T = 0$, $\mu_B \neq 0$) **QCD matter** is largely unknown due to the Sign Problem of lattice field theory
- Perturbation theory viable at high baryon chemical potential μ_B and zero temperature T as QCD coupling $\alpha_s \ll 1$
- **pQCD constrains the neutron-star equation of state (EOS)** (see the fig.)
- High-order corrections to pQCD pressure should decrease the width of the uncertainty band



(adapted from [Annala et al. \[2303.11356\]](#))

Framework for calculating dense pQCD pressure

- 1 Generate Feynman diagrams from partition function:

$$p(\mu) \sim \ln Z = \ln \int \mathcal{D}\bar{\psi}\psi\bar{c}c A e^{-S_{\text{QCD}}}$$

$\stackrel{\text{pQCD}}{=} \text{sum of connected vacuum diagrams (no ext. legs)}$

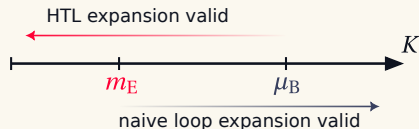
- Fermionic 4-momenta at finite density: $P^\alpha = (p^0 - i\mu, \mathbf{p})$

- 2 Calculate multi-loop integrals in dimensional regularization

Resummations required due to IR divergences associated with in-medium screening!

Momentum scales

- Only two scales at finite μ_B and zero T :
 - **Hard** scale $\sim \mu_B$: full QCD
 - **Soft** scale $m_E \sim \sqrt{\alpha_s} \mu_B$: hard thermal loop (HTL) effective theory
- Soft, **dynamically screened gluons**
HTL-resummed to handle infrared divergences of full QCD



HTL-resummed vertices:

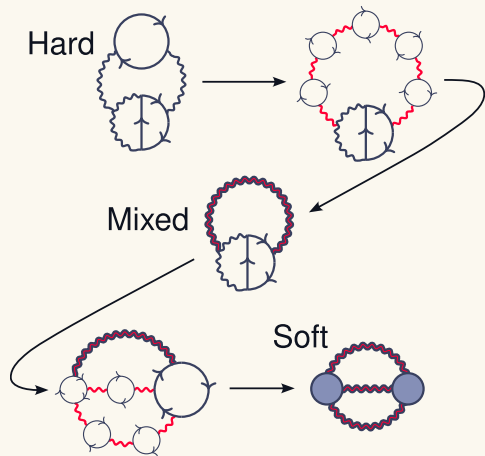


HTL-resummed propagator:

$$\begin{aligned}
 \frac{1}{K^2 + \Pi(K)} &= \frac{1}{K^2} + \frac{1}{K^2} \frac{m_E^2}{K^2} \frac{1}{K^2} + \dots \\
 \xrightarrow{K \sim m_E} &= \text{[Diagrammatic expansion: wavy line} + \text{wavy line} \cdot \text{blue circle} \cdot \text{wavy line} + \text{wavy line} \cdot \text{blue circle} \cdot \text{blue circle} \cdot \text{wavy line} + \dots]
 \end{aligned}$$

LO HTL self-energy $\sim m_E^2$:
leading $K \ll \mu_B$ part of
one-loop self-energy

HTL resummation at α_s^3 (N³LO i.e. the next unknown order)



Three kinematic sectors based on the number of soft loop momenta:

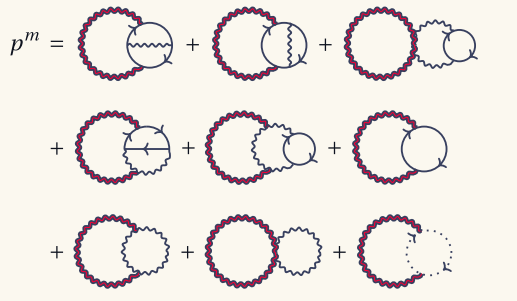
- **Hard:** No soft momenta, four-loop diagrams in full QCD
- **Mixed:** One soft momentum, a mix of HTL theory and full QCD
- **Soft:** Two soft momenta, two-loop diagrams in HTL theory

$$p^s = \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} \sim \alpha_s m_E^4$$

The diagram shows the soft sector contribution p^s as a sum of three terms. The first term is a self-energy loop with two blue vertices and a wavy gluon line. The second term is a tadpole diagram with a wavy gluon line and a red dashed loop. The third term is a sunset diagram with two wavy gluon lines and a central blue vertex. The entire sum is proportional to $\alpha_s m_E^4$.

- Self-interacting soft gluons \Rightarrow integrals in HTL theory
- Determines the leading logarithm $\alpha_s^3 \ln^2 \alpha_s$
- Complete computation recently by our group: R. Paatelainen et al. PRL '21 & PRD '21

Mixed sector (our latest work, see K.S. et al. PRL '23)



The image shows a series of Feynman diagrams representing the calculation of p^m . The diagrams are arranged in three rows, each containing three diagrams separated by plus signs. The first row shows diagrams with a red wavy line (gluon) and a white circle (quark). The second row shows diagrams with a red wavy line and a white circle, with a double-headed arrow indicating a specific interaction. The third row shows diagrams with a red wavy line and a white circle, with a dashed line indicating a specific interaction. Below the diagrams is the equation:

$$\sim \int_K \text{Tr} \left\{ G_{\text{LO}}(K) \left[\Pi^{2,\text{HTL}}(K) + \Pi^{1,\text{Pow}}(K) \right] \right\} \sim \alpha_s m_E^4$$

- Soft gluons interact with hard quarks and gluons \Rightarrow mix of HTL theory and full QCD
- Contains **NLO** corrections to LO HTL **gluon self-energy**:
 - Two-loop HTL correction $\Pi^{2,\text{HTL}} \sim \alpha_s m_E^2$
 - One-loop power correction $\Pi^{1,\text{Pow}} \sim \alpha_s K^2$
- $\Pi^{2,\text{HTL}}$ and $\Pi^{1,\text{Pow}}$ recently computed at finite T and μ_B in general covariant gauge by our group: [K.S. et al. JHEP '23](#)
- Together with p_s determines the **next-to-leading logarithm** $\alpha_s^3 \ln \alpha_s$

Hard sector

$$\begin{aligned}
 p^h = & \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} \\
 & + \text{[Diagram 4]} + \text{[Diagram 5]} + 46 \text{ diagrams} \\
 \sim & (\text{IR div.}) + c_2 \ln^2 \frac{3\bar{\Lambda}}{2\mu_B} + c_1 \ln \frac{3\bar{\Lambda}}{2\mu_B} + c_0 \sim \alpha_s^3 \mu_B^4
 \end{aligned}$$

- Interactions between hard modes only
 \Rightarrow integrals in full QCD
- c_2 and c_1 known from RG invariance:
 $d\rho/d\bar{\Lambda} = 0$
- Value of c_0 from 51 four-loop diagrams \Rightarrow project for the coming years

Results for N³LO pressure up to $O(\alpha_s^3 \ln \alpha_s)$

$$\frac{p}{p_{\text{free}}} = 1 - 2\left(\frac{\alpha_s}{\pi}\right) - N_f \left(\frac{\alpha_s}{\pi}\right)^2 \left[\ln\left(N_f \frac{\alpha_s}{\pi}\right) + 3 \ln \frac{3\bar{\Lambda}}{2\mu_B} + 5.0021 \right] \\ + N_f^2 \left(\frac{\alpha_s}{\pi}\right)^3 \left[c_{3,2} \ln^2\left(N_f \frac{\alpha_s}{\pi}\right) + c_{3,1}(\bar{\Lambda}) \ln\left(N_f \frac{\alpha_s}{\pi}\right) + c_{3,0}(\bar{\Lambda}) \right] + O(\alpha_s^4)$$

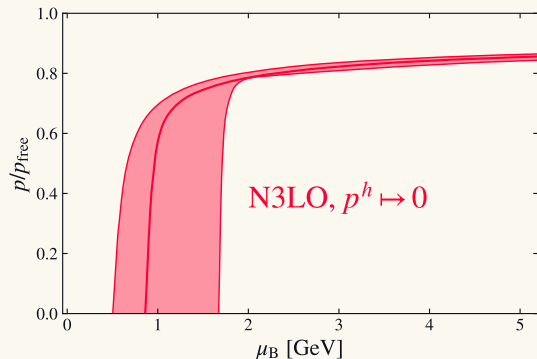
$$c_{3,2} \quad \frac{11}{12}$$


$$c_{3,1} \quad -6.5968(12) - 3 \ln \frac{3\bar{\Lambda}}{2\mu_B}$$

$$c_{3,0} \quad 5.1342(48) + \frac{2}{3}c_0 - 18.284 \ln \frac{3\bar{\Lambda}}{2\mu_B} - \frac{9}{2} \ln^2 \frac{3\bar{\Lambda}}{2\mu_B}$$

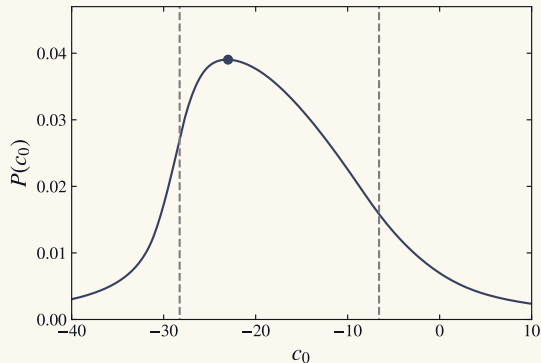
- The sum $p = p_s + p_m + p_h$ involves logarithms of the ratio of the soft and hard scales $\ln(m_E/\mu_B) \sim \ln \alpha_s$
- **Next-to-leading logarithm**
 $\alpha_s^3 \ln \alpha_s$ now determined \Rightarrow brings the result on par with its high-temperature counterpart by [Kajantie et al. PRD '03](#)
- Full N³LO result still sensitive to unknown hard constant c_0

Results for N³LO pressure with only soft and mixed sectors

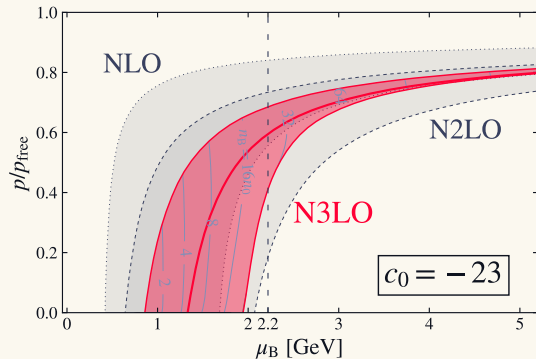


- N³LO result including only screened gluonic sectors (soft+mixed) incredibly well-behaved with nearly vanishing renormalization-scale dependence
- Conclusion: hard sector main source for uncertainty
- Stark contrast to the high- T case

Full N³LO pressure with fixed c_0



- Full N³LO with $c_0 = -23$ most consistent with lower-order results according to Bayesian analysis



- Actual computation of c_0 may lead to a significantly improved EOS usable even at $\mu_B = 2.2$ GeV ($n = 27n_0$), cf. N²LO at $\mu_B = 2.7$ GeV ($n = 40n_0$)

Summary and outlook

- pQCD essential for constraining the neutron-star EOS
- New state-of-the-art result for cold and dense pQCD EOS computed to $O(\alpha_s^3 \ln \alpha_s)$ level
- Our NLO results for the gluon self-energy supersede LO HTL results from the 1980s (Braaten & Pisarski)

Future projects:

- ① Computation of hard and dense 4-loop QCD diagrams to determine c_0
 - Probably takes many years and requires the development of new techniques
 - Promising steps: Pablo's talk, Säppi et al. PRD '22 and Österman et al. JHEP '23
- ② Extend the HTL framework to full NLO level
- ③ Next-to-leading logarithmic resummation of the cold and dense EOS (leading logarithm done: Fernandez & Kneur PRL '22)

Thank you! Questions?