Recent advancements in perturbative QCD at high density

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Kaapo Seppänen

In collaboration with: T. Gorda, R. Paatelainen, and S. Säppi

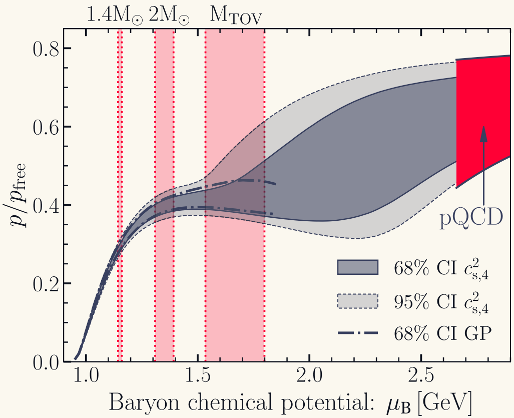
Based on: K.S. et al. JHEP '23 and K.S. et al. PRL '23

HELSINGIN YLIOPISTO
HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

HELSINKI INSTITUTE OF PHYSICS

QCD thermodynamics at high density

- **Thermodynamics of cold and dense** ($T = 0$, $\mu_B \neq 0$) **QCD matter** is largely unknown due to the Sign Problem of lattice field theory
- Perturbation theory viable at high baryon chemical potential μ_B and zero temperature T as QCD coupling $\alpha_s \ll 1$
- **pQCD constrains the neutron-star equation of state** (EOS) (see the fig.)
- High-order corrections to pQCD pressure should decrease the width of the uncertainty band



(adapted from Annala et al. [2303.11356])

Framework for calculating dense pQCD pressure

1. Generate Feynman diagrams from partition function:

$$p(\mu) \sim \ln Z = \ln \int \mathcal{D}\overline{\psi}\psi\overline{c}cAe^{-S_{\mathrm{QCD}}}$$

$$\overset{\mathrm{pQCD}}{=} \text{sum of connected vacuum diagrams (no ext. legs)}$$

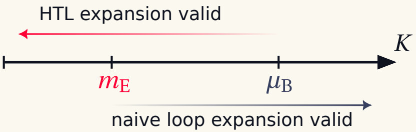
- Fermionic 4-momenta at finite density: $P^\alpha = (p^0 - i\mu, \mathbf{p})$

2. Calculate multi-loop integrals in dimensional regularization

Resummations required due to IR divergences associated with in-medium screening!

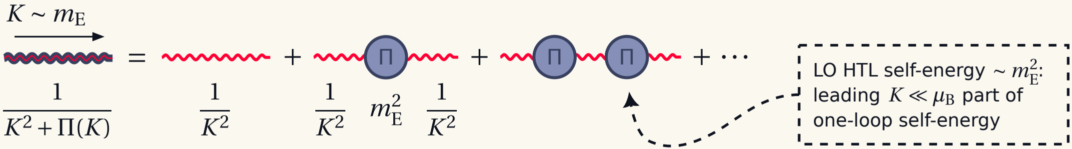
Momentum scales

- Only two scales at finite μ_B and zero T:
  - **Hard** scale $\sim \mu_\mathrm{B}$: full QCD
  - **Soft** scale $m_\mathrm{E} \sim \sqrt{\alpha_s}\mu_\mathrm{B}$: hard thermal loop (HTL) effective theory
- Soft, **dynamically screened gluons** HTL-resummed to handle infrared divergences of full QCD

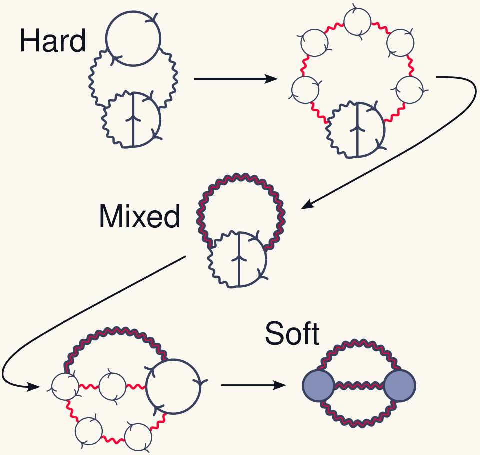


HTL-resummed vertices:

HTL-resummed propagator:



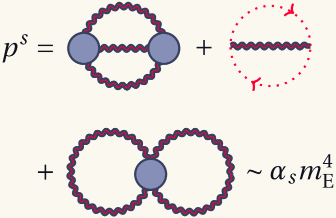
HTL resummation at α_s^3 (N^{3}LO i.e. the next unknown order)



Three kinematic sectors based on the number of soft loop momenta:

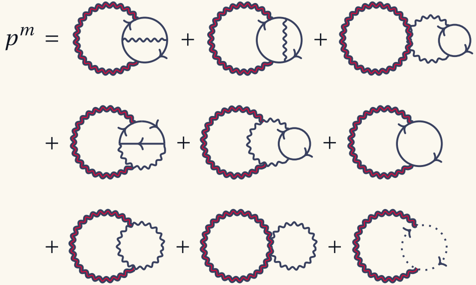
- **Hard**: No soft momenta, four-loop diagrams in full QCD
- **Mixed**: One soft momentum, a mix of HTL theory and full QCD
- **Soft**: Two soft momenta, two-loop diagrams in HTL theory

Soft sector



- Self-interacting soft gluons ⇒ integrals in HTL theory
- Determines the leading logarithm $\alpha_s^3 \ln^2 \alpha_s$
- Complete computation recently by our group: R. Paatelainen et al. PRL ’21 & PRD ’21

Mixed sector (our latest work, see K.S. et al. PRL '23)

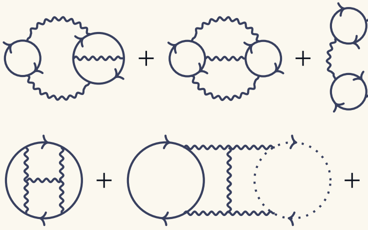


$$\sim \int_K \mathrm{Tr}\left\{G_{\mathrm{LO}}(K)\left[\Pi^{2,\mathrm{HTL}}(K) + \Pi^{1,\mathrm{Pow}}(K)\right]\right\} \sim \alpha_s m_{\mathrm{E}}^4$$

- Soft gluons interact with hard quarks and gluons $\Rightarrow$ mix of HTL theory and full QCD
- Contains **NLO** corrections to LO HTL **gluon self-energy**:
  - Two-loop HTL correction $\Pi^{2,\mathrm{HTL}} \sim \alpha_s m_{\mathrm{E}}^2$
  - One-loop power correction $\Pi^{1,\mathrm{Pow}} \sim \alpha_s K^2$
- $\Pi^{2,\mathrm{HTL}}$ and $\Pi^{1,\mathrm{Pow}}$ recently computed at finite T and μ_{B} in general covariant gauge by our group: K.S. et al. JHEP '23
- Together with p_s determines the **next-to-leading logarithm** $\alpha_s^3 \ln \alpha_s$

Hard sector

$p^h =$



$+$ 46 diagrams

$$\sim \left(\text{IR div.}\right) + c_2 \ln^2 \frac{3\overline{\Lambda}}{2\mu_\mathrm{B}} + c_1 \ln \frac{3\overline{\Lambda}}{2\mu_\mathrm{B}} + c_0 \sim \alpha_s^3 \mu_\mathrm{B}^4$$

- Interactions between hard modes only $\Rightarrow$ integrals in full QCD
- c_2 and c_1 known from RG invariance: $\mathrm{d}p/\mathrm{d}\overline{\Lambda} = 0$
- Value of c_0 from 51 four-loop diagrams $\Rightarrow$ project for the coming years

Results for N^{3}LO pressure up to $O(\alpha_s^3 \ln \alpha_s)$

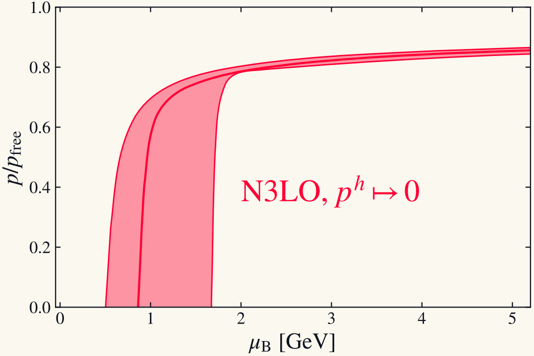
$$\frac{p}{p_{\text{free}}} = 1 - 2\left(\frac{\alpha_s}{\pi}\right) - N_f\left(\frac{\alpha_s}{\pi}\right)^2\left[\ln\left(N_f\frac{\alpha_s}{\pi}\right) + 3\ln\frac{3\bar{\Lambda}}{2\mu_{\text{B}}} + 5.0021\right]$$

$$+N_f^2\left(\frac{\alpha_s}{\pi}\right)^3\left[c_{3,2}\ln^2\left(N_f\frac{\alpha_s}{\pi}\right) + c_{3,1}(\bar{\Lambda})\ln\left(N_f\frac{\alpha_s}{\pi}\right) + c_{3,0}(\bar{\Lambda})\right] + O(\alpha_s^4)$$

$c_{3,2}$	$\frac{11}{12}$
→ $c_{3,1}$	$-6.5968(12) - 3\ln\frac{3\bar{\Lambda}}{2\mu_{\text{B}}}$
$c_{3,0}$	$5.1342(48) + \frac{2}{3}c_0 - 18.284\ln\frac{3\bar{\Lambda}}{2\mu_{\text{B}}} - \frac{9}{2}\ln^2\frac{3\bar{\Lambda}}{2\mu_{\text{B}}}$

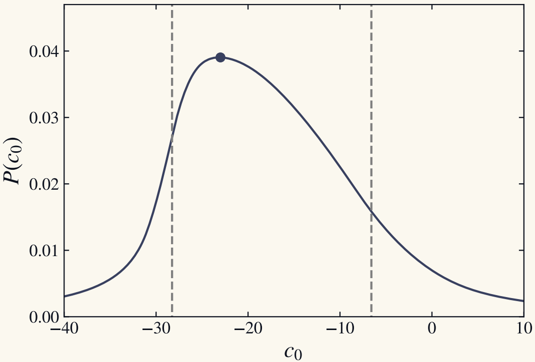
- The sum $p = p_s + p_m + p_h$ involves logarithms of the ratio of the soft and hard scales $\ln(m_{\text{E}}/\mu_{\text{B}}) \sim \ln \alpha_s$
- **Next-to-leading logarithm** $\alpha_s^3 \ln \alpha_s$ now determined $\Rightarrow$ brings the result on par with its high-temperature counterpart by Kajantie et al. PRD '03
- Full N^{3}LO result still sensitive to unknown hard constant c_0

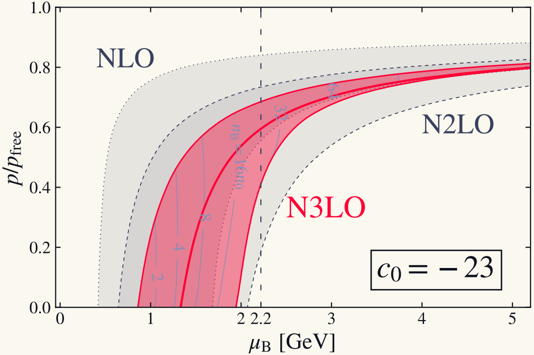
Results for N^{3}LO pressure with only soft and mixed sectors



- N^{3}LO result including only screened gluonic sectors (soft+mixed) incredibly well-behaved with nearly vanishing renormalization-scale dependence
- Conclusion: hard sector main source for uncertainty
- Stark contrast to the high-T case

Full N^{3}LO pressure with fixed c_0





- Full N^{3}LO with $c_0 = -23$ most consistent with lower-order results according to Bayesian analysis

- Actual computation of c_0 may lead to a significantly improved EOS usable even at $\mu_B = 2.2$GeV ($n = 27n_0$), cf. N^{2}LO at $\mu_B = 2.7$GeV ($n = 40n_0$)

Summary and outlook

- pQCD essential for constraining the neutron-star EOS
- New state-of-the-art result for cold and dense pQCD EOS computed to $O(\alpha_s^3 \ln \alpha_s)$ level
- Our NLO results for the gluon self-energy supersede LO HTL results from the 1980s (Braaten & Pisarski)

Future projects:

1. Computation of hard and dense 4-loop QCD diagrams to determine c_0
   - Probably takes many years and requires the development of new techniques
   - Promising steps: Pablo's talk, Säppi et al. PRD '22 and Österman et al. JHEP '23
2. Extend the HTL framework to full NLO level
3. Next-to-leading logarithmic resummation of the cold and dense EOS (leading logarithm done: Fernandez & Kneur PRL '22)

Thank you! Questions?