Inferring the Initial Condition for the Balitsky - Kovchegov Evolution Equation

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QCD theory group, University of Jyväskylä

Particle Physics Day, Oct 2023

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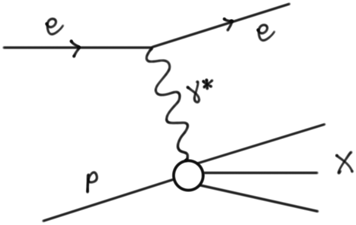
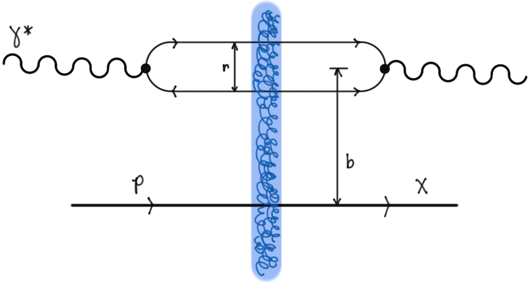
Deep Inelastic Scattering in the Dipole Picture



IMF picture



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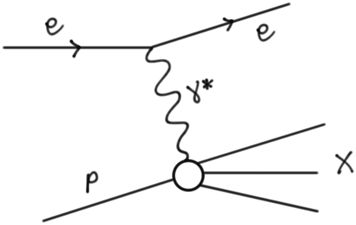

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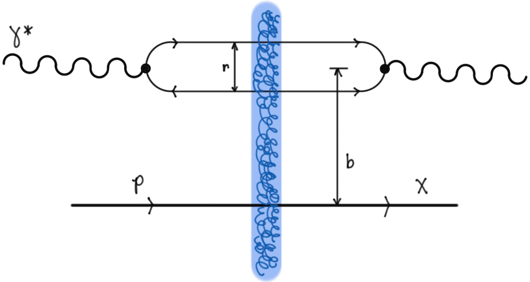

Dipole Picture

$$\sigma_{T,L}^{\gamma * p}(x, Q^2) \sim \frac{\sigma_0}{2} \times \mathcal{N}(r, y) \times \{\mathrm{LCWF}\}$$

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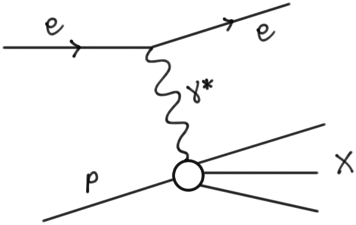

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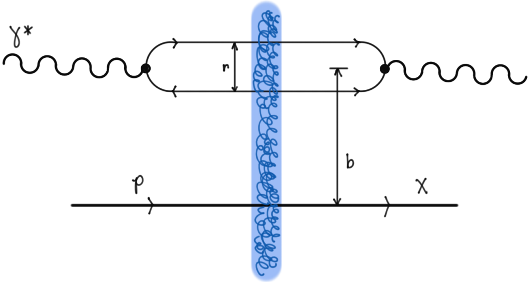

Dipole Picture

proton transverse size

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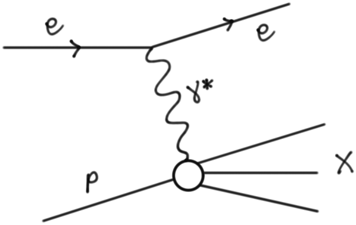

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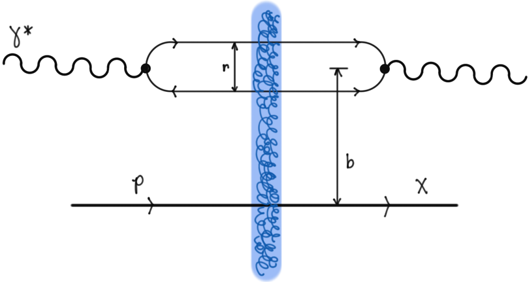

Dipole Picture

dipole-target scattering amplitude

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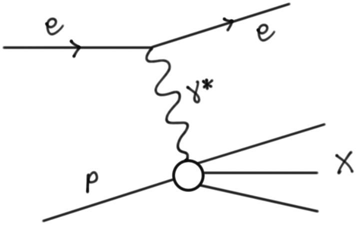

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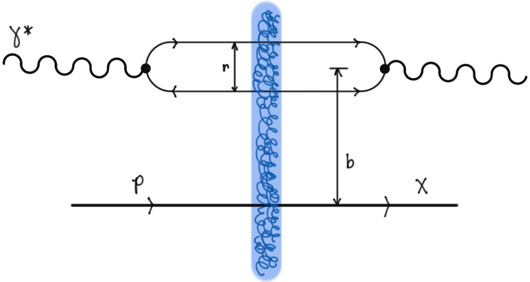

Dipole Picture

$$\sigma_{T,L}^{\gamma * p}(x, Q^2) \sim \frac{\sigma_0}{2} \times \mathcal{N}(r, y) \times \{\mathrm{LCWF}\}$$

$$\textbf{rcBK:}\, \mathcal{N}(r, x = x_0; Q_{s0}^2, \gamma, e_c) \xrightarrow{C^2} \mathcal{N}(r, y)$$

Objectives

- Constrain model parameters, $[Q_{s0}^2, \gamma, e_c, C^2, \sigma_0/2]$, that parametrize the BK IC ...

  ... via Bayesian inference using HERA structure function data.
- Account for correlated experimental uncertainties in HERA data.
- Provide predictions and uncertainties for other observables.

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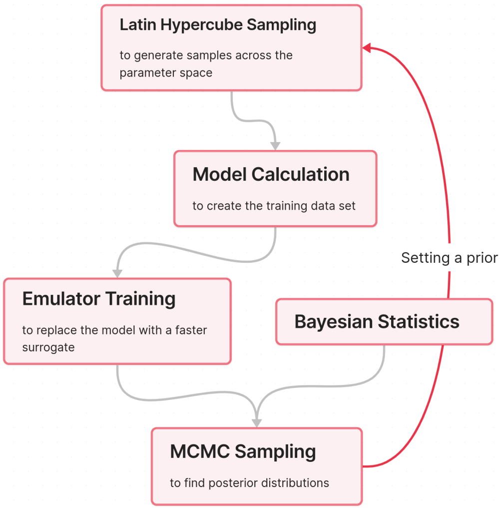
Typical Bayesian Workflow



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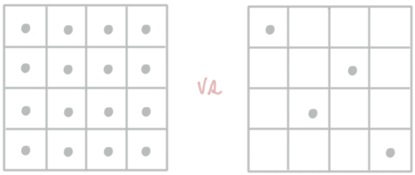
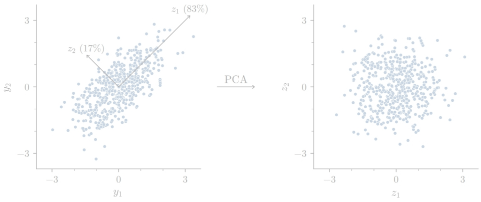
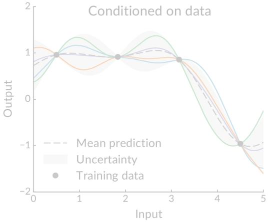


Principal Component Analysis

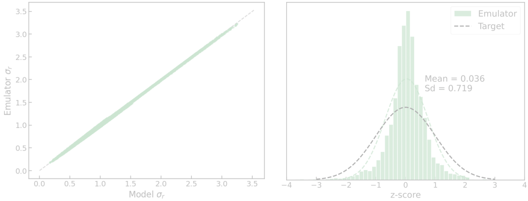


Gaussian Process Emulator

GPs learn the parameter dependence of the model!



Validation



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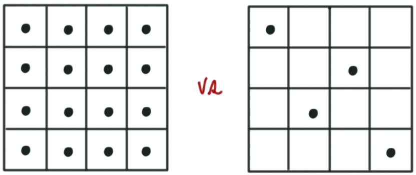

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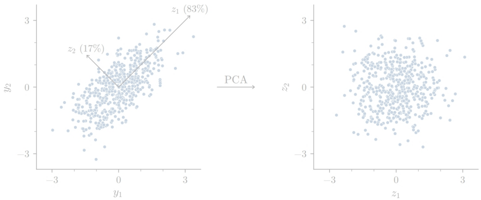

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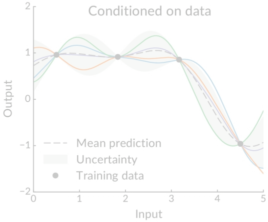

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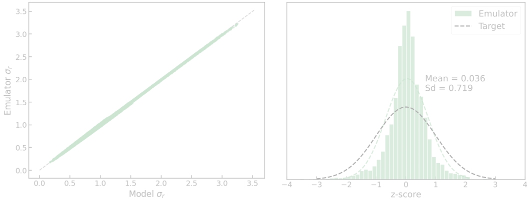

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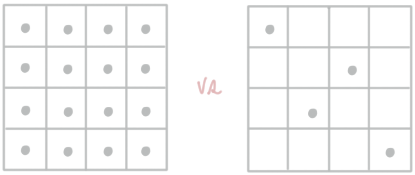

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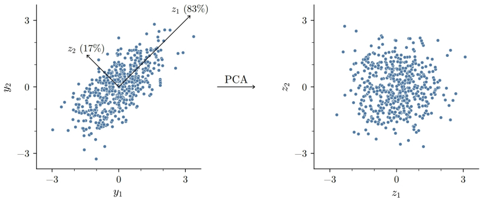

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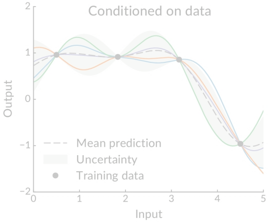

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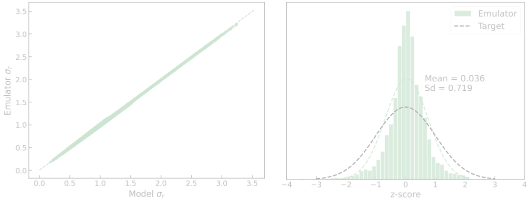

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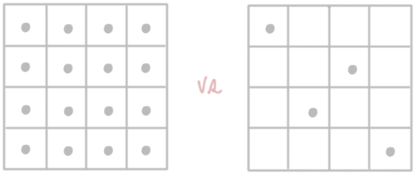

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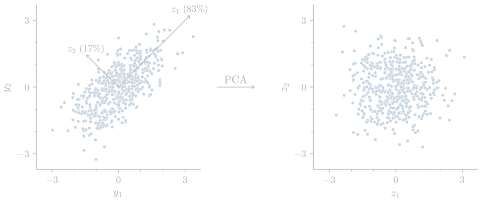

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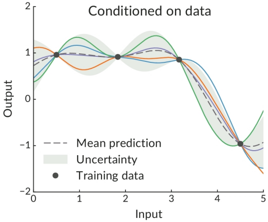

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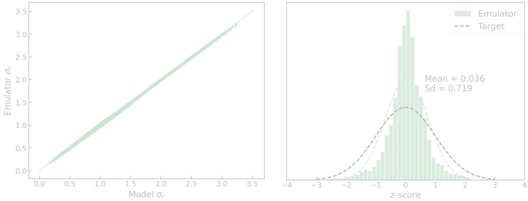

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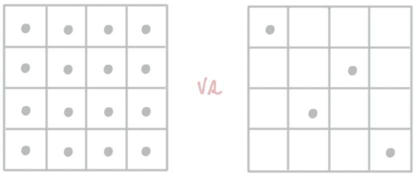

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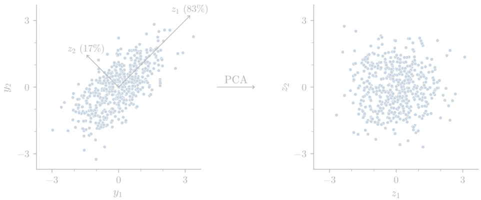

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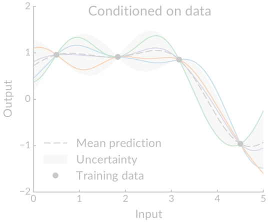

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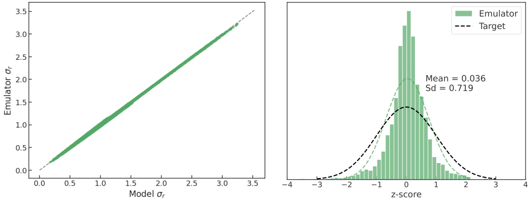
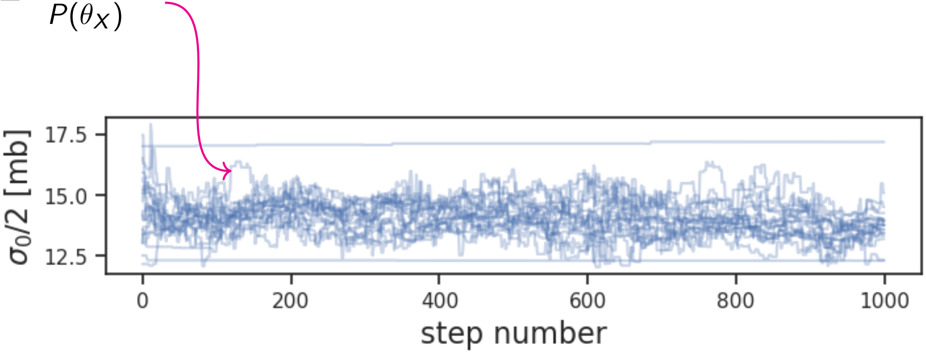


Acceptance probability:

$$\alpha = \frac{P(\theta_{X+1})}{P(\theta_X)}$$



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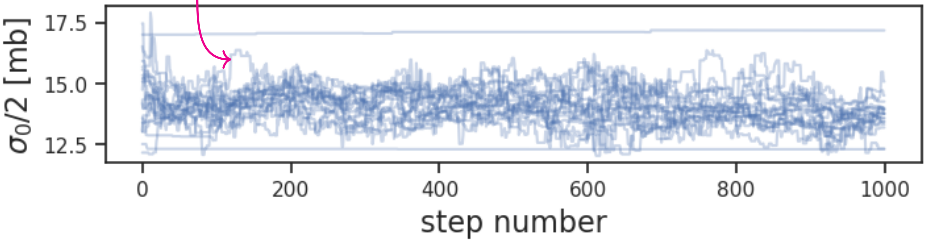
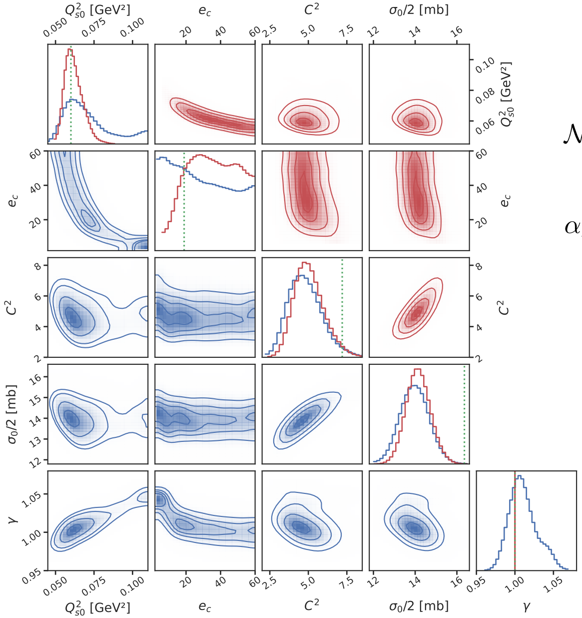
Bayesian Statistics

$$P(\theta) = \text{posterior} = \text{likelihood} \times \text{prior}$$

- Likelihood: how well data matches the model at θ
- Prior: bounds of the parameter space



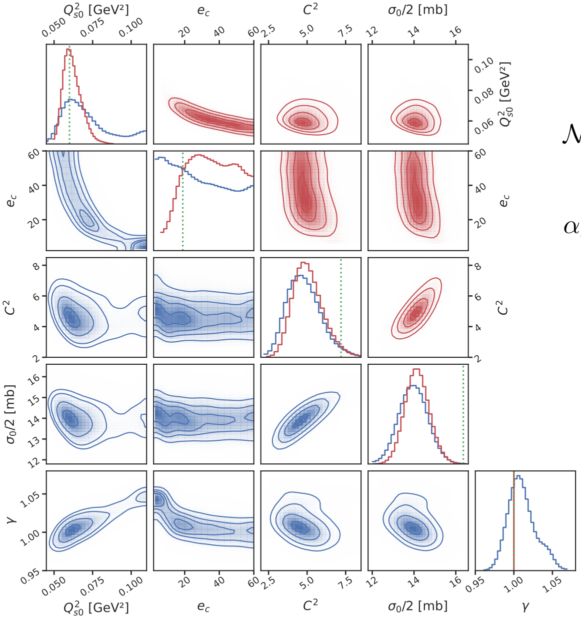


Results: Posterior Distribution

$$\mathcal{N}(\mathbf{r}, x_0) = 1 - \exp\left[-\frac{(\mathbf{r}^2 Q_{s0}^2)^\gamma}{4} \ln\left(\frac{1}{|\mathbf{r}|\Lambda_{\mathrm{QCD}}} + e_c \cdot e\right)\right]$$

$$\alpha_s(\mathbf{r}) = \frac{12\pi}{(33 - N_f)\log\left(\frac{4C^2}{\mathbf{r}^2\Lambda_{QCD}^2}\right)}$$

1. HERA data prefers a $\gamma \approx 1$
2. With γ as a free parameter, we allow a wider posterior distribution
3. Off-diagonal plots = correlations



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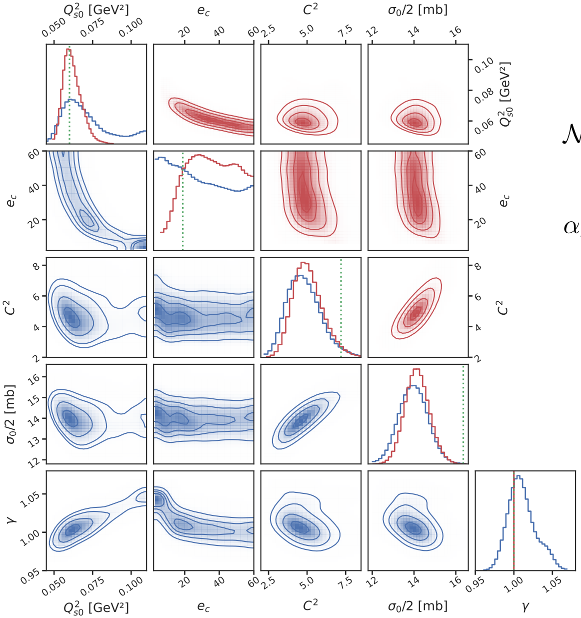

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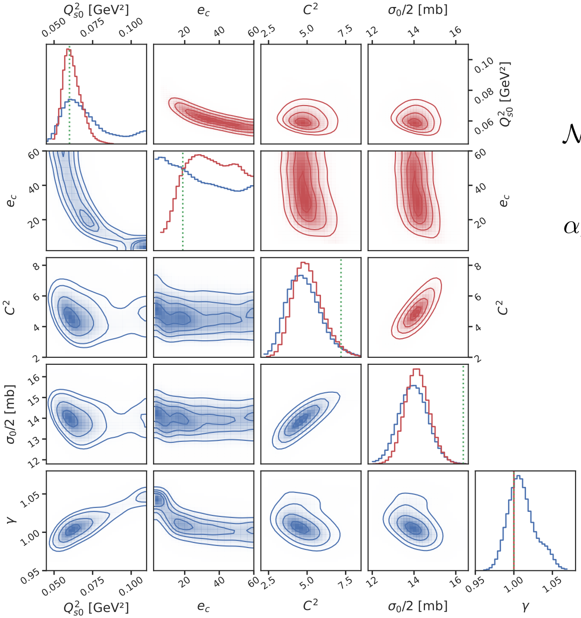

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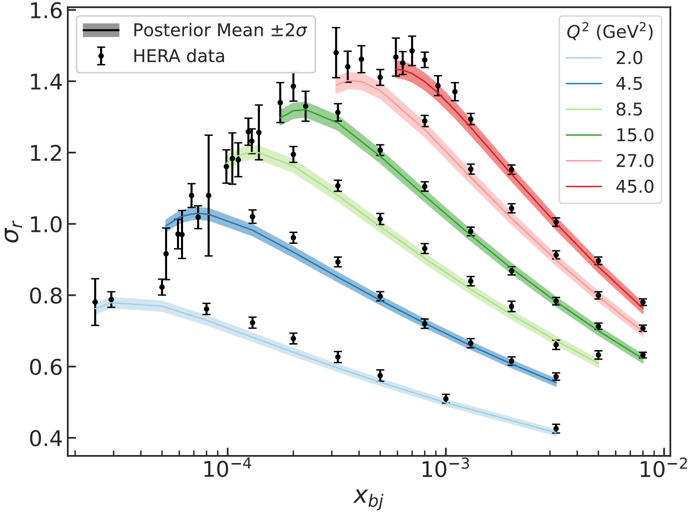
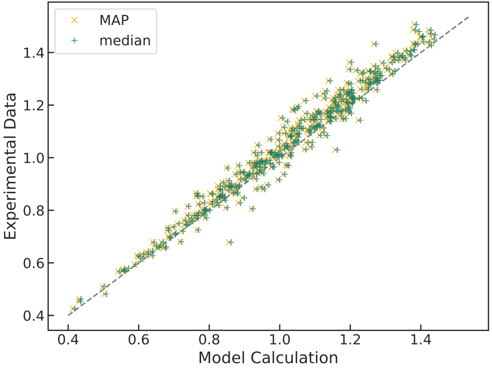
Posterior Samples, Median and MAP curves



"Best Fit" Values

5 - parameter	$Q_{s0}^2[GeV^2]$	γ	e_c	C^2	$\sigma_0/2[mb]$	χ^2/dof
median	0.067	1.01	27.5	4.72	14.0	1.63
MAP	0.076	1.01	15.6	4.47	13.9	2.06

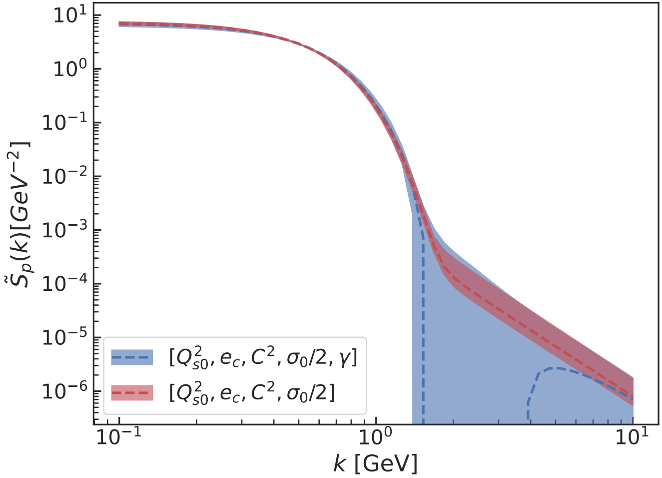


Applications to pA collisions

$$\mathrm{d}\sigma^{q+A\to q+X} = xg(x,\mathbf{k}^2)\tilde{S}_p(\mathbf{k})$$

$$\tilde{S}_p(\mathbf{k}) \to \text{2DFT of Dipole amplitude}$$
$$= \int \mathrm{d}^2\mathbf{r}\, e^{i\mathbf{k}\cdot\mathbf{r}}$$
$$\times \left[1 - \mathcal{N}(\mathbf{r}, x = x_0)\right]$$

- $\gamma > 1$ result to negative 2DFT values $\Rightarrow \sigma/k^2 < 0$
- Uncertainty estimates are now provided!



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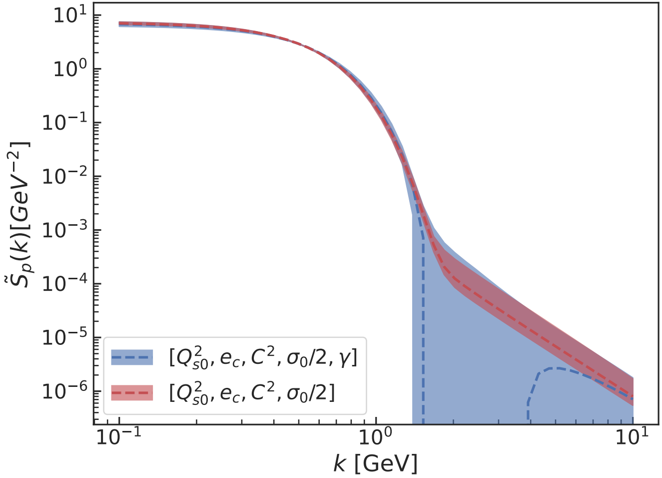

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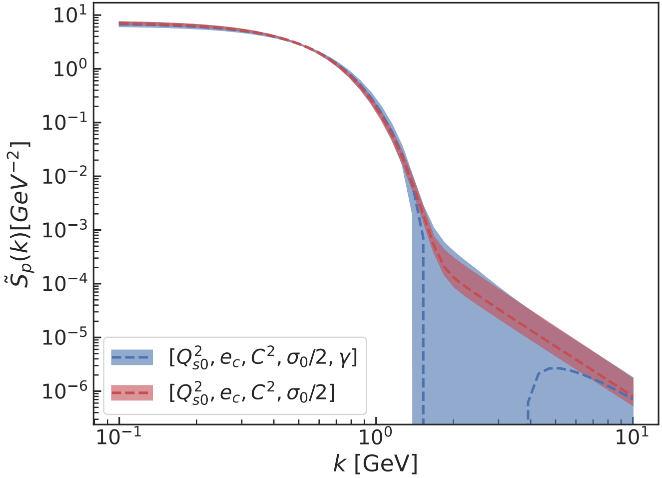

Summary

- Posterior distribution for $[Q_{s0}^2, \gamma, e_c, C^2, \sigma_0/2]$
- Correlations between parameters observed from the posteriors
- Uncertainty for the BK initial condition (novel!)
- necessary for calculations that propagate the uncertainties of non-perturbative BK IC and other observables

Further work

- Extension to other functional forms of the initial condition
- NLO fits to further probe saturation

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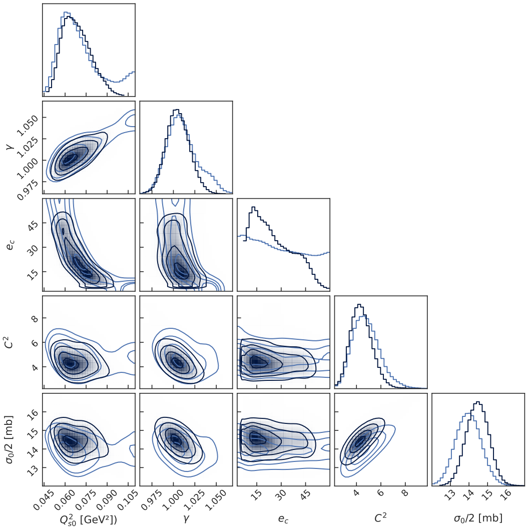
Back up Slides

Correlations



Initial and Evolved $\mathcal{N}(r, y)$