PDFs at approximate N3LO accuracy

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Particle Physics Day 2023

Overview

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2. Theory Uncertainties
3. Cross-Sections
4. Evolution
5. NNPDF4.0 with QCD@aN3LO (preliminary)
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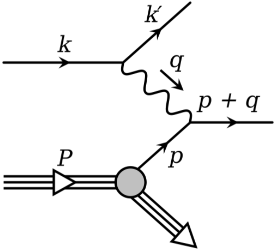
Introduction

Parton Distribution Functions



Parton Distribution Functions (PDF) $\mathbf{f}(x, \mu_F^2)$

- describe the fundamental constituents of the proton: quarks, gluons
- μ_F-dependence: DGLAP equations!
- x-dependence: fit!

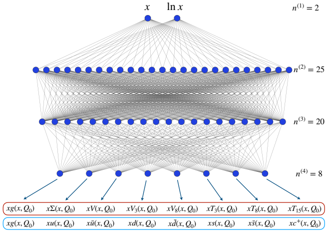
How to fit a PDF?

Experiment:

p_T [GeV]	Dimuon cross section (pb)	Dielectron cross section (pb)	Dilepton cross section (pb)
0 - 1.0	8.8945 ±0.23955	9.0042 ±0.29596	9.2821 ±0.29778
1.0 - 2.0	23.05 ±0.61831	23.482 ±0.75597	22.786 ±0.70765
2.0 - 3.0	31.799 ±0.84297	32.848 ±1.0437	32.062 ±0.96367
3.0 - 4.0	35.663 ±0.93277	37.025 ±1.1681	36.225 ±1.0621
4.0 - 5.0	36.455 ±0.94616	37.578 ±1.1785	36.882 ±1.0681
5.0 - 6.0	35.099 ±0.90895	36.201 ±1.1302	35.579 ±1.0246
6.0 - 7.0	33.122 ±0.85768	34.275 ±1.0673	33.547 ±0.96264
7.0 - 8.0	30.967 ±0.8023	32.2 ±1.0003	31.324 ±0.89554
8.0 - 9.0	28.702 ±0.74434	29.834 ±0.92491	29.089 ±0.82773
9.0 - 10.0	26.669 ±0.69291	27.369 ±0.84751	26.933 ±0.76278

taken from [JHEP12.061]

Methodology:



taken from [EPJC82.428]

Theory:

$$
\begin{aligned}
s'^2 \frac{d^2\sigma^{(1),H,fin}_{\vec{\kappa},bb',g}}{dt_1 du_1} = {}& \alpha\alpha_s g^{\kappa_1}_{b,Q} g^{\kappa_2}_{b',Q} K_{g\gamma} N_C C_F \Bigg[-\frac{2}{u_1} P^H_{\kappa_3,gg}(x_1) \\
& \left\{ B^{(0)}_{\vec{\kappa},\mathrm{QED}}(x_1 k_1) \left(\ln\left(\frac{s_4^2}{m^2(s_4+m^2)} \right) - \ln(\mu_F^2/m^2) \right) - 2B^{(1)}_{\vec{\kappa},\mathrm{QED}}(x_1 k_1) \right\} \\
& + C_A \frac{s_4}{2\pi(s_4+m^2)} \left(\int d\Omega_n \, R_{\vec{\kappa},\mathrm{OK}} \right)^{finite} \\
& + 2C_F \frac{s_4}{2\pi(s_4+m^2)} \int d\Omega_4 \, R_{\vec{\kappa},\mathrm{QED}} \Bigg] . \qquad (5.36)
\end{aligned}
$$

taken from [1910.01536]

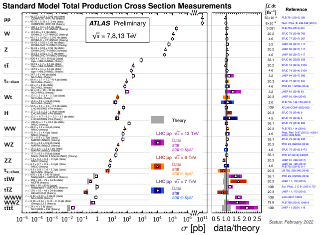
Strategy:

repeat until converged:

guess candidate PDF $f(Q_0^2) \to$ compute theory predictions $T \to$ compare to data D

Call for Precision

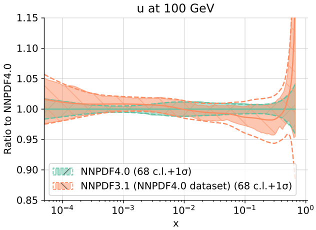
Experiment:



taken from

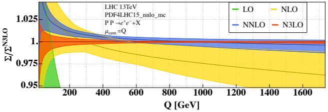
ATL-PHYS-PUB-2022-009

Methodology:



taken from [EPJC82.428]

Theory:



taken from [JHEP03.116]

but

which PDF to use?

Ingredients for an aN3LO PDF Fit

Factorization:

$$T(Q^2) = C(Q^2) \otimes f(Q^2) = C(Q^2) \otimes E(Q^2 \leftarrow Q_0^2) \otimes f(Q_0^2) \tag{1}$$

Upgrade perturbative elements:

$$X = X^{(0)} + a_s X^{(1)} + a_s^2 X^{(2)} + a_s^3 X^{(3)} + \dots \tag{2}$$

partonic cross-sections C:

- DIS
- LHC (and everything else)

evolution E:

- splitting functions
- transition matrix elements

Account for uncertainties!

Theory Uncertainties

Covariance Matrix Formalism [EPJC79.931]

To maximize the (Bayesian) probability $P(T|D)$ for a theory prediction T to describe a data point D we minimize

$$\chi^2 = (T - D)^T C^{-1} (T - D) \tag{3}$$

and assuming the experimental cov. matrix C is independent of a theoretical cov. matrix S we can minimize:

$$\chi^2 = (T - D)^T (C + S)^{-1} (T - D) \tag{4}$$

Both can contain:

- statistical uncertainties (e.g. QM vs. MC)
- systematic uncertainties (e.g. resolution vs. scale)

Strategy:

decompose $S^{\mathrm{tot}} = S^{\mathrm{MHOU}} + S^{\mathrm{IHOU}}$

MHOU Using Scale Variations [EPJC79.931]

$$T(Q^2, \mu_r^2, \mu_f^2) = C(Q^2, \mu_r^2) \otimes E(Q^2 \leftarrow Q_0^2, \mu_f^2) \otimes f(Q_0^2) \quad (5)$$

Factorization scale variations:

- finite knowledge of splitting functions
- correlated across datasets as PDFs are universal

Renormalization scale variations:

- finite knowledge of partonic cross-sections
- correlated for a given process (DIS NC, DIS CC, TOP, JETS, ...)

Choose:

$$\mu = \kappa \mu_0 \qquad \kappa \in \{1/2, 1, 2\} \quad (6)$$

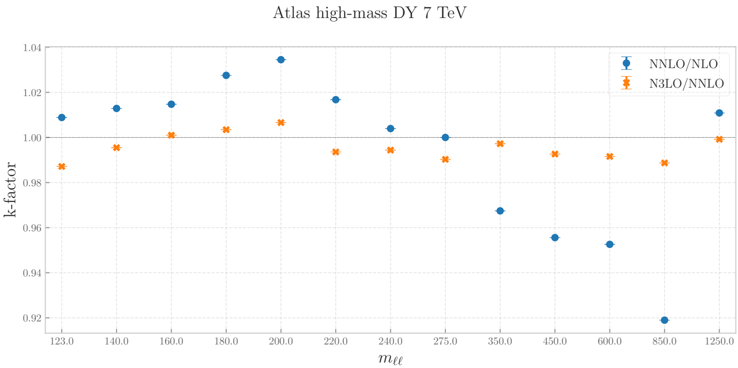
Cross-Sections

DIS with QCD@N3LO

- light coefficient functions [VVM05],[MVV05],[MV00],[MRV08],[MVV09] ✓
- massive coefficient functions $\rightarrow$ approximation in MSc thesis of N. Laurenti ✓
  i.e. combine in a suitable way:
  - threshold limit $s \rightarrow 4m^2$
  - high-energy limit $s \rightarrow \infty$
  - asymptotic limit $Q^2 \gg m^2$

DY with QCD@N3LO

- even if available, most codes are private
- use `n3loxs` [JHEP12.066] to obtain k-factor for inclusive distributions, e.g. [PLB725.223]



Evolution

Transition Matrix Elements

Use whatever is available in literature:

- I. Bierenbaum, J.Blümlein, and S. Klein. Mellin Moments of the $\mathcal{O}(\alpha_s^3)$ Heavy Flavor Contributions to unpolarized Deep-Inelastic Scattering at $Q^2 \gg m^2$ and Anomalous Dimensions. *Nucl. Phys. B*, 820:417–482, 2009. arXiv:0904.3563, doi:10.1016/j.nuclphysb.2009.06.005.
- J. Blümlein. Analytic continuation of mellin transforms up to two-loop order. *Computer Physics Communications*, 133(1):76–104, Dec 2000. URL: http://dx.doi.org/10.1016/S0010-4655(00)00156-9, doi:10.1016/s0010-4655(00)00156-9.
- J. Ablinger, A. Behring, J. Blümlein, A. De Freitas, A. Hasselhuhn, A. von Manteuffel, M. Round, C. Schneider, and F. Wißbrock. The 3-Loop Non-Singlet Heavy Flavor Contributions and Anomalous Dimensions for the Structure Function $F_2(x, Q^2)$ and Transversity. *Nucl. Phys. B*, 886:733–823, 2014. arXiv:1406.4654, doi:10.1016/j.nuclphysb.2014.07.010.
- J. Ablinger, A. Behring, J. Blümlein, A. De Freitas, A. von Manteuffel, and C. Schneider. The 3-loop pure singlet heavy flavor contributions to the structure function f2(x,q2) and the anomalous dimension. *Nuclear Physics B*, 890:48–151, Jan 2015. URL: http://dx.doi.org/10.1016/j.nuclphysb.2014.10.008, doi:10.1016/j.nuclphysb.2014.10.008.
- J. Ablinger, J. Blümlein, A. De Freitas, A. Hasselhuhn, A. von Manteuffel, M. Round, and C. Schneider. The $\mathcal{O}(\alpha_s^3 T_f^2)$ contributions to the Gluonic Operator Matrix Element. *Nucl. Phys. B*, 885:280–317, 2014. arXiv:1405.4259, doi:10.1016/j.nuclphysb.2014.05.028.
- J. Ablinger, J. Blümlein, A. De Freitas, A. Hasselhuhn, A. von Manteuffel, M. Round, C. Schneider, and F. Wißbrock. The transition matrix element a_gq(n) of the variable flavor number scheme at o(α_s^3). *Nuclear Physics B*, 882:263–288, May 2014. URL: http://dx.doi.org/10.1016/j.nuclphysb.2014.02.007, doi:10.1016/j.nuclphysb.2014.02.007.
- A. Behring, I. Bierenbaum, J. Blümlein, A. De Freitas, S. Klein, and F. Wißbrock. The logarithmic contributions to the $\mathcal{O}(\alpha_s^3)$ asymptotic massive Wilson coefficients and operator matrix elements in deeply inelastic scattering. *Eur. Phys. J. C*, 74(9):3033, 2014. arXiv:1403.6356, doi:10.1140/epjc/s10052-014-3033-x.
- J. Ablinger, J. Blümlein, S. Klein, C. Schneider, and F. Wissbrock. The $\mathcal{O}(\alpha_s^3)$ Massive Operator Matrix Elements of $\mathcal{O}(n_f)$ for the Structure Function $F_2(x, Q^2)$ and Transversity. *Nucl. Phys. B*, 844:26–54, 2011. arXiv:1008.3347, doi:10.1016/j.nuclphysb.2010.10.021.
- J. Blümlein, J. Ablinger, A. Behring, A.De Freitas, A. von Manteuffel, and C. Schneider. Heavy Flavor Wilson Coefficients in Deep-Inelastic Scattering: Recent Results. *PoS*, QCDEV2017:031, 2017. arXiv:1711.07957, doi:10.22323/1.308.0031.
- J. Ablinger, A. Behring, J. Blümlein, A. De Freitas, A. Goedicke, A. von Manteuffel, C. Schneider and K. Schonwald. The Unpolarized and Polarized Single-Mass Three-Loop Heavy Flavor Operator Matrix Elements $A_{gg,Q}$ and $\Delta A_{gg,Q}$ arXiv:2211.0546

Approximating Splitting Functions

Splitting functions are not known fully analytically, but some partial information:

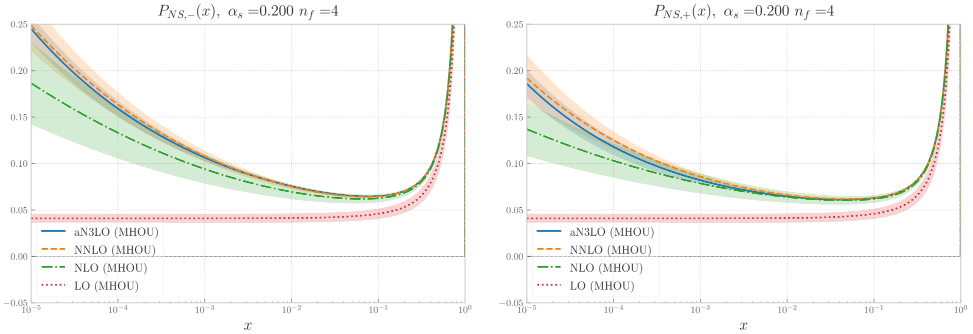
- large n_f contributions [NPB915.335], [JHEP10.041], [2310.01245]
- small x limit (from BFKL) [JHEP06.145], [JHEP08.135]
- large x limit (from soft) [NPB832.152], [JHEP04.018], [JHEP09.155]
- some (low) moments [JHEP10.041], [PLB825.136853], [PLB842.137944], [2307.04158], [2310.05744]

Strategy:

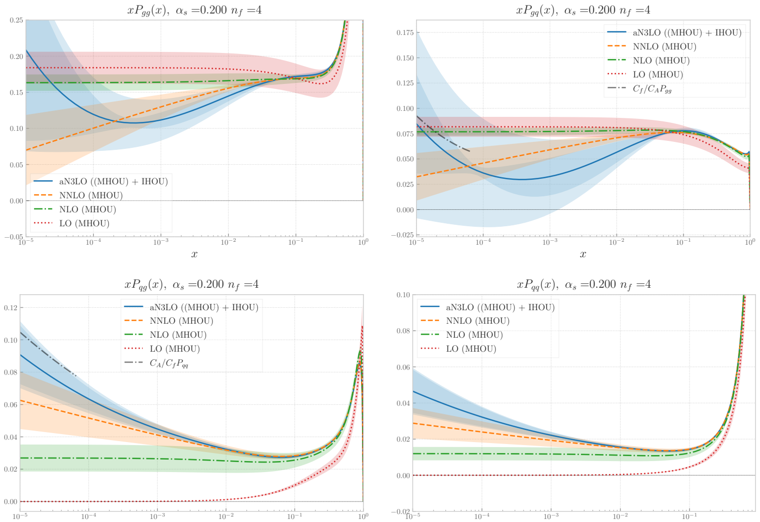
combine known limits and add sub-leading functions to ensure moments $\Rightarrow$ IHOU

Non-Singlet Splitting Functions

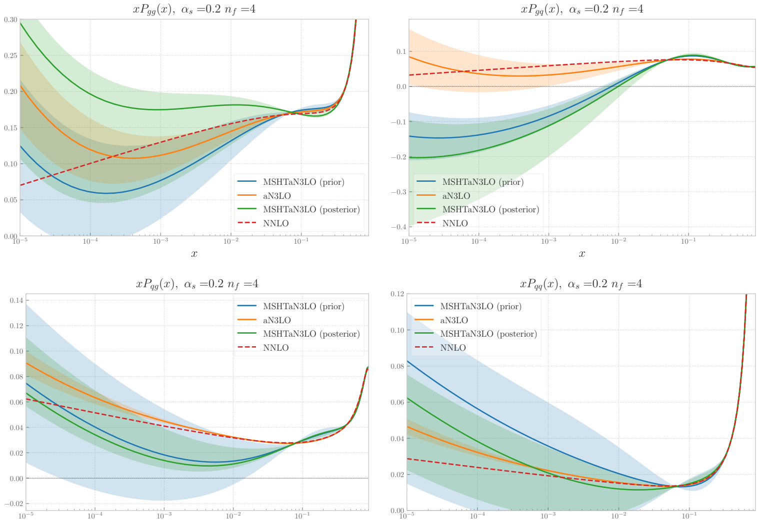
8 known moments ✓



Singlet Splitting Functions

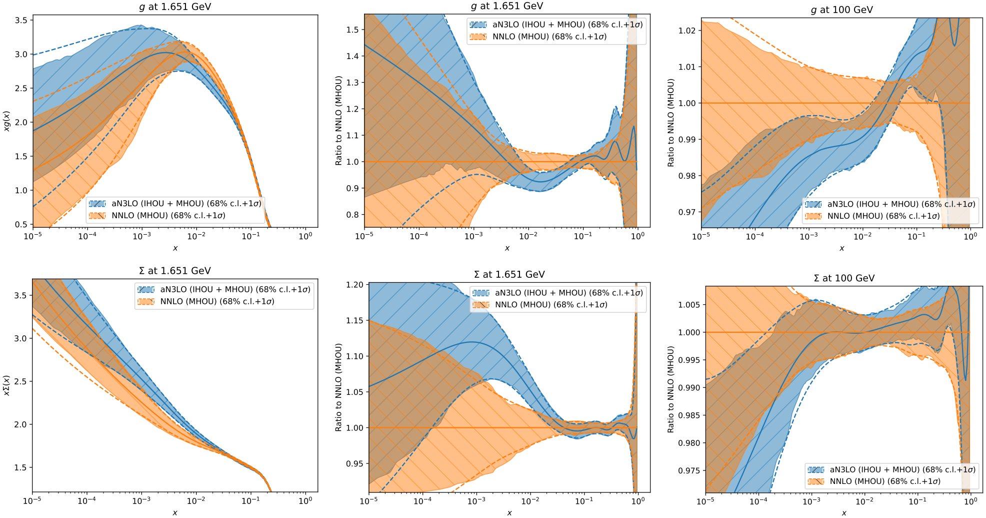


Comparison with MSHT [EPJC83.185]



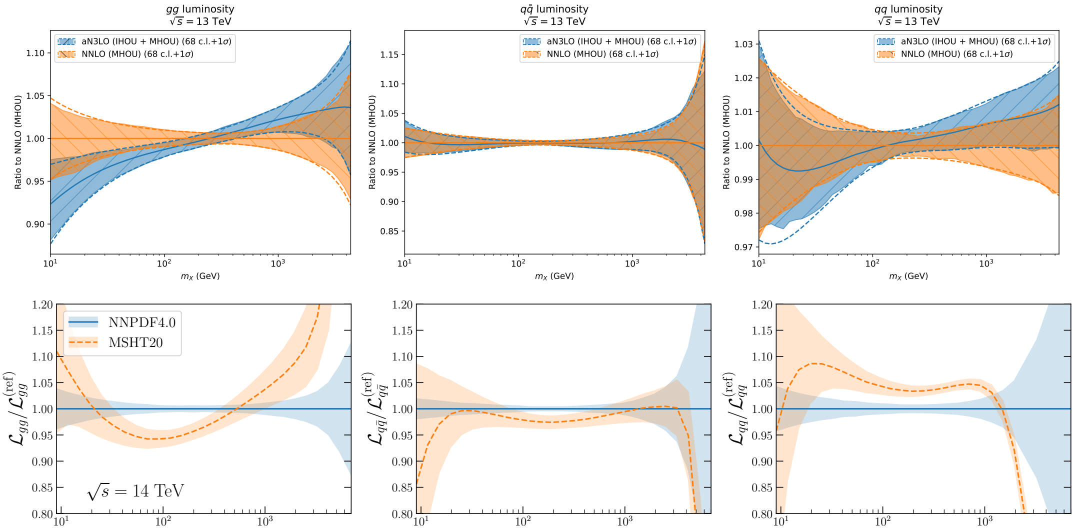
NNPDF4.0 with QCD@aN3LO (preliminary)

PDFs

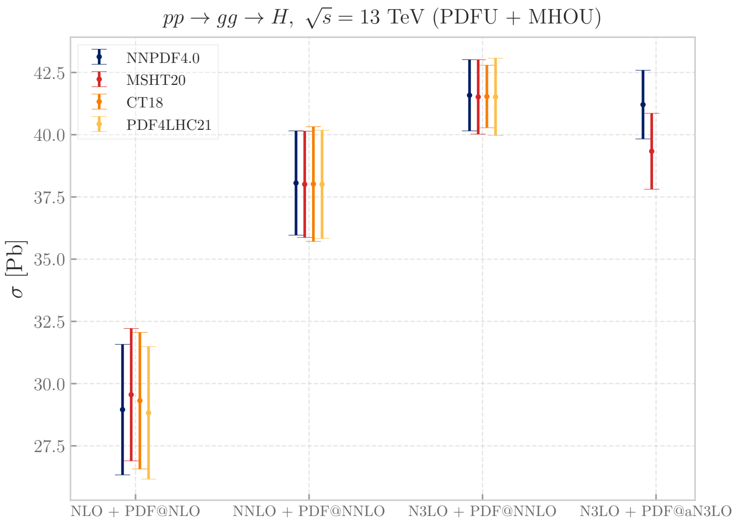


g at 1.651 GeV
$xg(x)$
aN3LO (IHOU + MHOU) (68% c.l.+1σ)
NNLO (MHOU) (68% c.l.+1σ)
x
g at 1.651 GeV
Ratio to NNLO (MHOU)
g at 100 GeV
Σ at 1.651 GeV
$x\Sigma(x)$
Σ at 100 GeV

Luminosities



Some First Pheno . . .



Summary

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For PDFs at % accurarcy we need:

- include QED and EW effects $\rightarrow$ NNPDF4.0QED
- account for theory uncertainties $\rightarrow$ NNPDF4.0MHOU
- use N3LO accuracy $\rightarrow$ NNPDF4.0aN3LO

For PDFs with QCD@aN3LO we need:

- approximate splitting functions
- upgrade as many processes as possible
- account for theory uncertainties

Summary

For PDFs at % accurarcy we need:

- include QED and EW effects $\rightarrow$ NNPDF4.0QED
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For PDFs with QCD@aN3LO we need:

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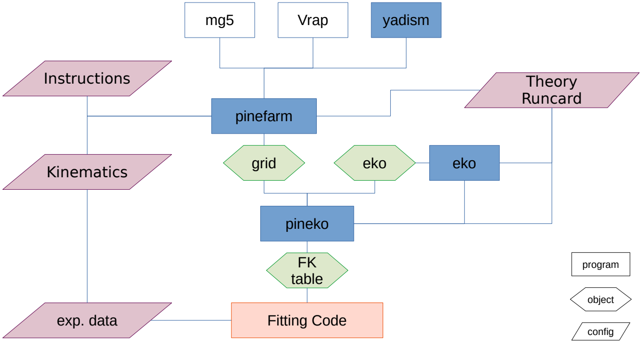
Danke! Thanks! Kiitos!

Backup slides

New Theory Prediction ~~Pipeline~~ Pineline [2302.12124]

Produce FastKernel (FK) tables!



The workhorse in the background: PineAPPL

Usage of PDFs with Theory Uncertainties [EPJC79.931]

- PDFs are universal
- we can assume uncertainties for a given process independent of the PDF

In the end we can just do:

$$(\delta\sigma^{tot})^2 = (\delta\sigma^{MHOU})^2 + (\delta\sigma^{PDF})^2 \tag{7}$$

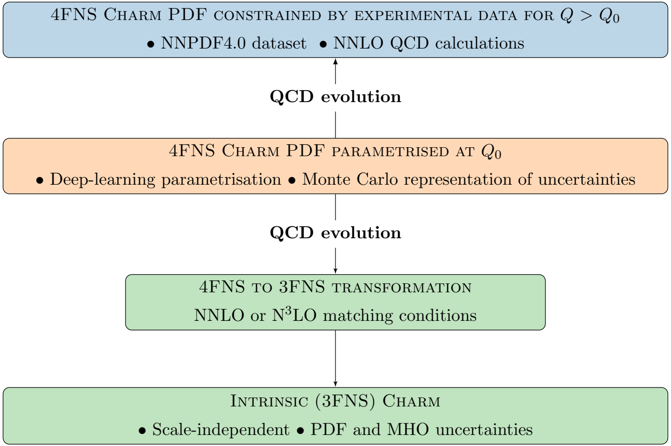
with (as usual):

$$(\delta\sigma^{MHOU})^2 = \langle (T_\sigma[\mu] - T_\sigma[\mu_0])^2 \rangle_{\mu \in V_\mu} \tag{8}$$

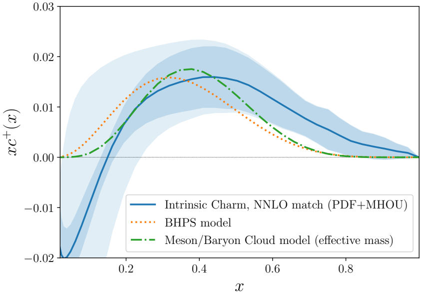
$$(\delta\sigma^{PDF})^2 = \langle (T_\sigma[f^{(k)}] - T_\sigma[f^{(0)}])^2 \rangle_{k=1 \dots N_{rep}} \tag{9}$$

Intrinsic Charm: Strategy [Nature608.483]

based on NNPDF4.0 [EPJC82.428]



Intrinsic Charm: PDF plot [Nature608.483]



[BHPS] or [Meson/Baryon Cloud Model]

- in **3FNS** a valence-like peak is present
- for $x \leq 0.2$ the perturbative uncertainties are quite large
- the carried momentum fraction is within **1%**