Final Cooling Lattice Design

Elena Fol

C. Rogers, D. Schulte, B. Stechauner, A. Latina, A. Grudiev

IMCC 3rd Annual Meeting
CERN, 12-15 March 2024
• Final Cooling: Overview and Baseline
• Previous Steps and Current work
• Integrating Realistic RF-systems
• Optimization methods
• Start-to-end Lattice Simulation in RF-Track
• Conclusions and Next Steps
Ionisation cooling (the reduction of occupied phase-space by muons): the only technique compatible with muon’s lifetime (2.2 μs), demonstrated by MICE collaboration.

Final Cooling Channel: reduction of transverse emittance on the cost of longitudinal emittance growth.

<table>
<thead>
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<th>Parameter</th>
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Ionisation cooling: the only technique that works on the timescale of the muon lifetime

- Muons passing through a material → energy loss due to the interaction with absorber material
- Reduction of normalised beam emittance
- Re-accelerating the beam to restore the longitudinal momentum
Baseline Design and simulation tools

**Baseline: MAP study**

- Starting beam parameters:
  \[ \epsilon_\perp = 300 \mu m, \epsilon_\parallel = 1.5 mm, \sigma t = 50 mm, \sigma E = 3.2 MeV \]
- High-field magnets 25—32 T, beam momenta ranging from 135- 70 MeV/c
- Achieved in previous studies*: \( \epsilon_\perp = 55 \mu m \), with \( \epsilon_\parallel = 76 \) mm, \( \Delta N_\mu = 50\% \)
- Target is \( \epsilon_\perp = 25 \mu m \): using 40 T solenoid and further optimization
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First steps using ICOOL simulations:
- Python-wrapper to ease generation of input files and tracking results analysis
- Linear optics matching
- Transverse cooling using Liquid Hydrogen absorber
  - Studied transverse aspects only
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**Towards integrated Final Cooling design:**
- RF-Track (developed by A. Latina): [https://gitlab.cern.ch/rf-track/download](https://gitlab.cern.ch/rf-track/download)
- Includes collective effects, relevant lattice elements (absorbers, stating wave RF-cavities, solenoids), Python and Octave interface
  - easy to combine with advanced optimisation algorithms
- Specific ionisation cooling effects have been recently added (multiple scattering, muon decays)
  - *Further presented studies are focused on RF-Track simulations (thanks to A. Latina)*

See Andrea’s talk tomorrow: [https://indico.cern.ch/event/1325963/contributions/5828922/](https://indico.cern.ch/event/1325963/contributions/5828922/)
Design optimisation strategy

I. Estimate **optimal momenta and absorber lengths** in every cell, with objective $\epsilon_\perp = 25 \mu m$.

- Provides **starting momenta** and **absorber lengths** for all cells

II. **Optics control**, ensure low beta-function in absorber by **optimizing solenoid field and matching coils**

- **Mitigates emittance blow up** in the fridge fields and **controls the optics in absorber** region

III. **Optimize acceleration and rotation** of the bunch after absorber (simplified RF model)

- Provides **drifts** and **rotation** “kicks” initial estimates for RF-system design

Focus of today’s talk

IV. Integrated **end-to-end simulation** of the complete cooling channel using RF-Track

- **Optimize a realistic RF system**: frequencies, phases, gradients to **control the longitudinal dynamics**
- Current Limitations
- Developed tools and methods
Design optimisation strategy

I. **Estimate optimal momenta and absorber lengths** in every cell, with objective $\epsilon_{\perp} = 25\mu m$.

- Provides **starting momenta** and **absorber lengths** for all cells

\[
\frac{dE}{ds} = 4eN_0 \varepsilon_{\perp} \gamma m c^2 Z \left[ \frac{1}{\beta^2} \ln \left( \frac{2m c^2 \beta}{\Gamma(Z)} \right) - 1 - \frac{\delta}{2\beta} \right]
\]

\[
\frac{d\epsilon_{\perp}}{ds} = -\frac{\epsilon_{\perp}}{\beta^2} \frac{dE}{ds} + \frac{\beta_1 E_s^2}{2\beta^3 m c^2 L_R E}
\]

II. **Optics control**, ensure low beta-function in absorber by optimizing solenoid field and matching coils

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✓ Tracking simulations using **40T and optimised parameters** confirm the potential for **lower emittance**

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**Analytical:** $\epsilon_{\perp} = 24\mu m$

**Tracking:** $\epsilon_{\perp} = 31\mu m$
Design optimisation strategy

I. Estimate optimal momenta and absorber lengths in every cell, with objective $\epsilon_\perp = 25\,\mu m$.

- Provides starting momenta and absorber lengths for all cells

\[
\frac{dE}{ds} = 4\pi N \frac{\rho^2 m c^2 Z}{2} \left[ \frac{1}{\beta^2} \ln \left( \frac{2 m c^2 \beta^2}{f(Z)} \right) - 1 - \frac{\delta}{2\beta^2} \right]
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\[
\frac{d\epsilon_\perp}{ds} = \frac{\epsilon_\perp}{\beta^2 E} \frac{dE}{ds} + \frac{\beta_2 E_s^2}{2\beta^3 m c^2 L_R E}
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\[
\epsilon_\perp, end = 268\,\mu m
\]

\[
\epsilon_\perp, end = 260\,\mu m
\]

Analytical: $\epsilon_\perp = 24\,\mu m$
Tracking: $\epsilon_\perp = 31\,\mu m$
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I. Estimate optimal momenta and absorber lengths in every cell, with objective \( \epsilon_\perp = 25 \mu m \).

⇒ Provides starting momenta and absorber lengths for all cells

\[
\frac{dE}{ds} = 4\pi \alpha r_s^2 m_e c^2 Z \left( \frac{1}{\beta^2} \left( \frac{2 m_e c^2 \gamma^2}{I(Z)} \right) - 1 - \frac{\delta}{2\beta} \right)
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✓ Tracking simulations using 40T and optimised parameters confirm the potential for lower emittance

✓ Transverse emittance = 32 micron, Longitudinal emittance = 77 mm

✓ Problem: Transmission (only ~29%) => more acceleration, higher momenta at the start of last cells?
IV. Integrated end-to-end simulation of the complete cooling channel using RF-Track

- **Global optimization:**
  - would have **14 parameters** to optimize in each cell
  - Expected to need ~**14 cells in total**
  - Cell-by-cell approach, testing different optimization algorithms

- **Objective function:**
  \[ \frac{\epsilon_\perp \epsilon_\parallel}{N_\mu} \]

- **Free parameters:**
  - Absorber (liquid hydrogen) thickness
  - Drift length
  - Number of accelerating RF cavities, rf phase
  - Number of rotating RF cavities, rf phase
  - B-field in RF region to match the field in the cooling cell and the change in momentum
Integrated Lattice Optimization: Methods

- **Optimization procedure:**
  - Run optimization for each cell, a few iterations
  - Create a surrogate model to estimate the initial parameters
  - Bayesian Optimization, BOBYQA

- **Input:** $\epsilon_{\perp,\text{start}}, P_{\perp,\text{start}}, \epsilon_{\perp}, \sigma_t, \sigma E, N_{\mu}$

- **Output:** $L_{\text{drift}}, N_{\text{rot}}, N_{\text{cav}}, \phi_{RF}, L_{\text{absorber}}, L_{\text{sol}}$

- **Fast design estimate**

- **Use as initial guess** for optimisation algorithms (optimal solution is found within fewer steps)

Peter I. Frazier: A Tutorial on Bayesian Optimization. arxiv:1807.02811

Py-BOBYQA: Derivative-Free Optimizer for Bound-Constrained Minimization
Integrated Lattice Optimization: Methods

- **Optimization procedure:**
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  ➡️ Use as initial guess for optimisation algorithms (optimal solution is found within fewer steps)

  - **Input:** \( \epsilon_{\perp,\text{start}}, P_{z,\text{start}}, \epsilon_{\perp}, \sigma_t, \sigma_E, N_{\mu} \)
  - **Output:** \( L_{\text{drift}}, N_{\text{rot}}, N_{\text{cav}}, \phi_{\text{RF}}, L_{\text{absorber}}, L_{\text{sol}} \)

  ➡️ Fast design estimate

**Example, cell 4:** \( \epsilon_{\perp,\text{start}} = 170 \mu m \)

**Target:**

\( \epsilon_{\perp} = 150 \mu m, \sigma_t = 400 mm, \sigma_E = 2.0 MeV, N_{\mu} = 75\% \)

Simulated with parameters predicted by ML-model:

\( \epsilon_{\perp} = 149 \mu m, \sigma_t = 404 mm, \sigma_E = 3.5 MeV, N_{\mu} = 69\% \)

Optimiser, 150 steps, starting with predicted parameters:

\( \epsilon_{\perp} = 150 \mu m, \sigma_t = 280 mm, \sigma_E = 2.1 MeV, N_{\mu} = 71\% \)
Integrated Lattice Optimization: Methods

• How to **speed up** simulations-based design optimization?
  ▶ Surrogate models to replace slow-executing simulations
    (used for optics matching in ICOOL simulations)

• How to **estimate initial optimization parameters**?
  ▶ Surrogate models to provide optimizers with “warm start”
  ▶ Bayesian Optimization

• Robust **emittance estimation** during optimization?
  ▶ Clustering for detection of tails biasing the emittance calculation

More details were in presented at 4th ICFA Beam Dynamics Mini-Workshop on Machine Learning for Particle Accelerators:
"ML-assisted design of Final Cooling System for a Muon Collider"
Details on simulations setup

- **General layout**

  Example for cell 1:
  - Absorber thickness: 0.85 m
  - Solenoid length = 1.48 m

  ![Diagram](image)

- **Simulating RF-systems:**
  - SW cavity - model: pillbox, fixed length = 0.25 m
  - Rotating cavities: rf phase to be optimised to provide the energy spread minimising rotation (and partially acceleration)
  - Accelerating cavities: RF-Track routine to find the phase providing maximum acceleration

  \[ f_{RF} \text{ according to } \lambda = \sigma_t/20, \quad G = 1.88 \sqrt{f_R F} \] (optimistic assumption for gradients, see IMCC report)
Beam parameters evolution inside a cooling cell

**Cell 1, passing through liquid hydrogen absorber**

- **Rotation and acceleration**

![Graphs showing beam parameters evolution](image-url)
### Integrated Lattice Optimization: Current results

| Cell | \( LH_2 \) [m] | Drift [m] rot. | \( N_{RF} \) accel. | \( f_{RF} \) [MHz] | \( G \) [MV/m] | \( \phi_{RF,rot.} \) degrees | \( P_{z,start} \) [MeV/m] | \( \sigma E_{start} \) [MeV] | \( \sigma t_{start} \) [mm] | \( P_{z,end} \) [MeV/m] | \( \sigma E_{end} \) [MeV] | \( \sigma t_{end} \) [mm] | \( \epsilon_{||} \) [μm] | \( \epsilon_{\perp} \) [μm] | \( N \) [%] |
|------|----------------|----------------|-----------------|----------------|----------------|-----------------|-----------------|-----------------|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|----------------|
| 1    | 0.85           | 0.3238         | 5               | 111.06         | 19.81          | -180            | 145.0           | 3.1             | 49.8           | 99.8           | 4.3             | 129.8           | 2.3             | 239.2           | 98             |
| 2    | 0.466          | 1.363          | 10              | 56.85          | 14.17          | 90              | 119.1           | 2.1             | 209.2          | 89.1           | 2.6             | 201.2           | 4.8             | 190.2           | 95             |
| 3    | 0.46958        | 1.363          | 10              | 56.85          | 14.17          | 90              | 118.5           | 4.0             | 284.8          | 88.5           | 4.0             | 394.9           | 6.4             | 157.3           | 90             |
| 4    | 0.4            | 2.5            | 9               | 40.13          | 11.9           | 51              | 113.1           | 5.7             | 819.5          | 87.5           | 3.7             | 362.8           | 12.5            | 133.3           | 83             |
| 5    | 0.3            | 1.8358         | 7               | 34.91          | 11.1           | -10             | 93.9            | 3.7             | 357.6          | 62.7           | 5.5             | 738.1           | 19.1            | 103.6           | 76             |
| 6    | 0.25           | 2.0            | 5               | 30.61          | 10.4           | -54             | 83.0            | 6.8             | 4606.2         | 58.0           | 2.7             | 1209.7          | 23.6            | 86.1            | 63             |
| 7    | 0.3            | 0.984          | 5               | 11.637         | 6.823          | -82             | 89.5            | 2.2             | 1378.5         | 55.3           | 3.0             | 1271.0          | 31.3            | 64.0            | 55             |
| 8    | 0.1            | 3.6464         | 2               | 16.17          | 8.04           | 67              | 71.0            | 2.7             | 1785.7         | 56.4           | 3.1             | 1617.2          | 41.4            | 54.9            | 49             |
| 9    | 0.17           | 3.64           | 2               | 13.38          | 7.32           | 67              | 75.7            | 3.1             | 2120.8         | 52.3           | 3.5             | 1967.6          | 49.1            | 44.0            | 40             |
| 10   | 0.08           | 2.555          | 11              | 8.226          | 5.39           | -6              | 61.2            | 2.1             | 3199.0         | 43.5           | 2.8             | 2740.0          | 68.8            | 35.3            | 35             |
| 11   | 0.0541         | 2.895          | 11              | 5.676          | 4.48           | -96             | 60.7            | 2.3             | 3456.5         | 49.5           | 2.9             | 3143.8          | 86.2            | 31.4            | 31             |

- Already cell 8 achieves better performance compared to the baseline:
  - 8 cells, \( \epsilon_{\perp} = 55\mu m, \epsilon_{||} = 41\text{mm} \) vs. 16 cells, \( \epsilon_{\perp} = 55\mu m, \epsilon_{||} = 76\text{mm} \)
- Potential to improve the transmission by minimising the relative energy spread
- Potential to combine with other cooling techniques
- Current results of 6D cooling could allow to start final cooling at < 300 micron
Executing start-to-end simulations and optimisation

- RF-Track, pre-compiled version, download: [https://gitlab.cern.ch/rf-track/download](https://gitlab.cern.ch/rf-track/download)
- Cells are described in JSON format
- Python script to read the cells description and to set-up and run RF-Track simulation
- Optimisation script with defined objective function executing the base lattice
- Post-processing, displaying results
- Simulation data management and Surrogate models training

```python
json_data = read_json_file(filename)
channel_params = cells_from_json(json_data)

for cell_params in channel_params:
    cooling_cell = CoolingCell(**cooling_cell_data)
    cooled_beam = cooling_cell.cool_in_cell(beam_to_track)
    utils.plot_results(cooled_beam)

beam_to_track.load("./optimized_beam_{}.format(cell_n-1))
beam_end_cell.save("./optimized_beam_{}.format(cell_n))
```

[https://github.com/MuonCollider-WG4/muon_final_cooling](https://github.com/MuonCollider-WG4/muon_final_cooling)
RF cavity in RF-Track vs. G4Beamline

**RFTrack**
- TM011
- $E = E \sin(kz) \sin(\omega t)$
- open cavity, no windows

**G4BL**
- TM010
- $E = E \sin(\omega t)$
- Implements windows

**Acceleration with G4bl**
- cavity length = 0.25 m
- Accelerate from 100 MeV/c to 135 MeV/c, reduce energy spread from 4.2 MeV to 2.3 MeV
- Emittances: transverse 232 micron, longitudinal 3.7 mm
Challenges and potential improvements

- Energy spread
- Transmission losses
- Large bunch length towards the end of the channel

- Improvement by using RF phase such that cavities combine acceleration and rotation?
- Better control over RF bucket size to avoid the transition losses?

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<th></th>
<th>After cooling</th>
<th>After cooling, cut</th>
<th>After drift and RF</th>
<th>$\sigma_{E_{\text{acc}}} [%]$</th>
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<td>Spread before</td>
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<td>absorber: 4%</td>
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Summary and outlook

- Demonstrated a **strategy for the optimisation** of final cooling design
- Flexible **optimisation & simulation framework** for evolving design
- **Integrated lattice design** including all relevant elements
- **Shorter channel** achieving **better performance** compared to the baseline:
  - 8 cells, $\epsilon_\perp = 55\mu m$, $\epsilon_\parallel = 41mm$ vs. 16 cells, $\epsilon_\perp = 55\mu m$, $\epsilon_\parallel = 76mm$

Currently achieved best performance: $\epsilon_\perp = 35\mu m$, $\epsilon_\parallel = 68mm$

- **Improvements of longitudinal dynamics control** and transmission losses
- **Consideration of feasible RF-design options**: e.g. multi-harmonics RF (allows the use of higher frequencies, shorter acceleration path is possible.)
- **Start-to-end simulations in G4Beamline**

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<th>$P_{z,\text{start}}$ [MeV/m]</th>
<th>$\sigma E_{\text{start}}$ [MeV]</th>
<th>$\sigma \tau_{\text{start}}$ [mm]</th>
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Thanks a lot for your attention!
Back-up slides
# Solenoid field parameters

<table>
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<tr>
<th>Cell</th>
<th>Bz peak [T]</th>
<th>Solenoid Length [m]</th>
<th>Bz low [T]</th>
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<td>11</td>
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Solenoid field in RF-Track:

\[
B(z) = 0.5 \cdot B_0 \left( \frac{L - z}{\sqrt{R^2 + (L - z)^2}} + \frac{z}{\sqrt{R^2 + z^2}} \right)
\]

<table>
<thead>
<tr>
<th>Cell</th>
<th>Aperture [mm]</th>
<th>LH [m]</th>
<th>(E_{\text{kin}, \text{start}}) [MeV]</th>
<th>(E_{\text{kin}, \text{exit}}) [MeV]</th>
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Initial phase space location of lost particles

- Cell 2: relative energy spread before absorber: 4%
Initial phase space location of lost particles

Cell 8: relative energy spread before absorber 14 %

Cell 9: relative energy spread before absorber 14 %