Final Cooling Lattice Design

Elena Fol

C. Rogers, D. Schulte, B. Stechauner, A. Latina, A. Grudiev

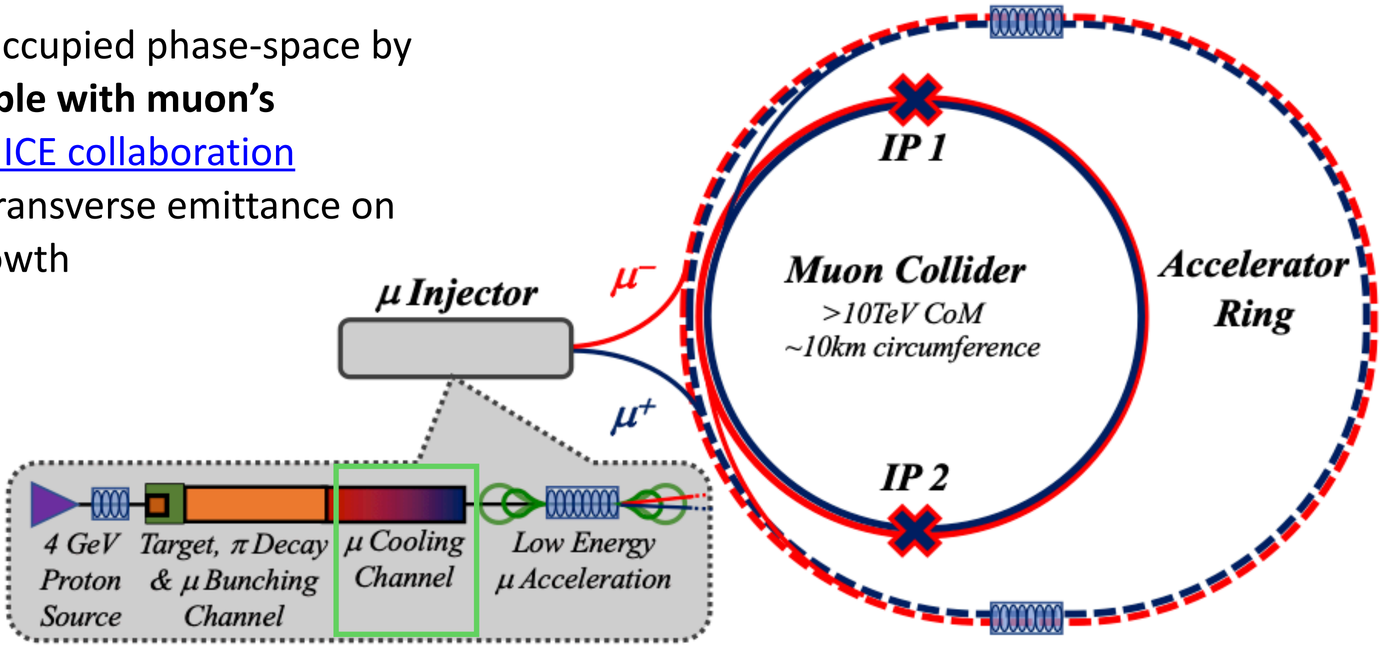
IMCC 3nd Annual Meeting
CERN, 12-15 March 2024

Outline

- Final Cooling: Overview and Baseline
- Previous Steps and Current work
- Integrating Realistic RF-systems
- Optimization methods
- Start-to-end Lattice Simulation in RF-Track
- Conclusions and Next Steps

Final Cooling for Muon Collider

- **Ionisation cooling** (the reduction of occupied phase-space by muons)**: the only technique compatible with muon's lifetime (2.2 μs)**, demonstrated by MICE collaboration
- **Final Cooling Channel:** reduction of transverse emittance on the cost of longitudinal emittance growth



Parameter	Unit	3 TeV	10 TeV	14 TeV
L	10^{34} cm^{-2}s^{-1}	1.8	20	40
N	10^{12}	2.2	1.8	1.8
f_r	Hz	5	5	5
P_{beam}	MW	5.3	14.4	20
C	km	4.5	10	14
<B>	T	7	10.5	10.5
ε_L	MeV m	7.5	7.5	7.5
σ_E / E	%	0.1	0.1	0.1
σ_z	mm	5	1.5	1.07
β	mm	5	1.5	1.07
ε	μm	25	25	25
$\sigma_{x,y}$	μm	3.0	0.9	0.63

$$\mathcal{L} \propto \gamma \langle B \rangle \sigma_\delta \frac{N_0}{\epsilon \epsilon_L} f_r N_0 \gamma$$

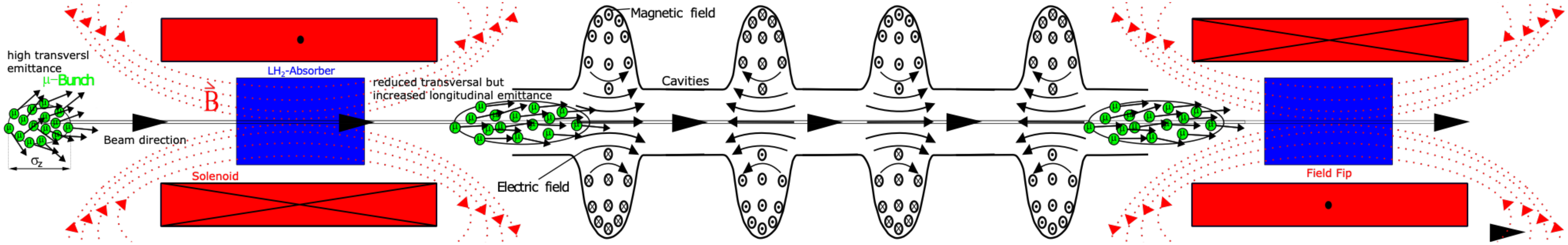
High energy
High field in collider ring
Large energy acceptance
Dense beam
High beam power

Technology and challenges of Final Cooling

Ionisation cooling: the only technique that works on the **timescale of the muon lifetime**

- Muons passing through a material —> energy loss due to the interaction with absorber material
- Reduction of normalised beam emittance
- Re-accelerating the beam to restore the longitudinal momentum

B. Stechauner



Momentum loss is opposite to motion, p, px, py, ΔE decrease

Momentum gain is purely longitudinal

$$\frac{d\varepsilon_T}{ds} = -\frac{1}{\beta^2 E}\frac{dE}{ds}\varepsilon_T + \frac{\beta\gamma\beta_T}{2}\frac{d\theta_0^2}{ds}$$

Energy loss term

Multiple scattering term

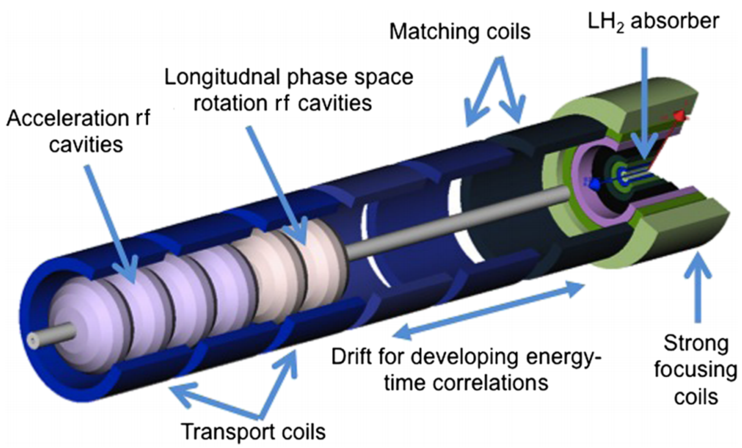
Baseline Design and simulation tools

Baseline: MAP study

- Starting beam parameters:
  $\epsilon_{\perp} = 300\mu m, \epsilon_{\parallel} = 1.5mm, \sigma t = 50mm, \sigma E = 3.2MeV$
- High-field magnets **25—32 T**, beam momenta ranging from **135- 70 MeV/c**

• Achieved in previous studies*: $\varepsilon_{\perp}$ = 55 µm, with $\varepsilon_{\parallel}$ = 76 mm, ΔN_{μ}= 50%

•Target is $\varepsilon_{\perp}$ = 25µm: using 40 T solenoid and further optimization



High field – low energy muon ionization cooling channel
Hisham Kamal Sayed, Robert B. Palmer, and David Neuffer
Phys. Rev. ST Accel. Beams **18**, 091001 – Published 4 September 2015

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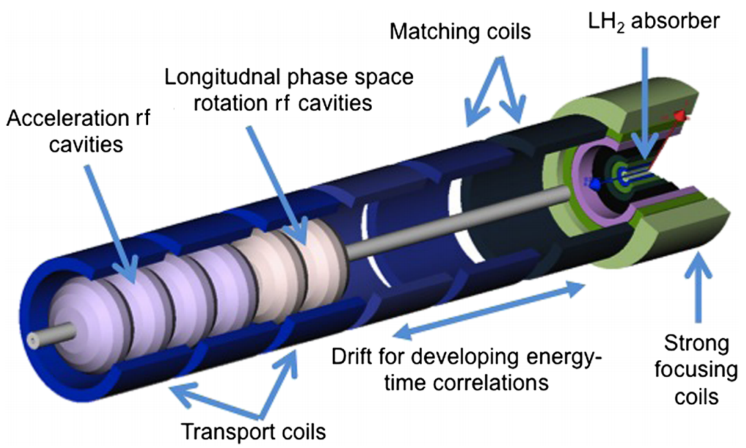
First steps using ICOOL simulations:

✓ Python-wrapper to ease generation of input files and tracking results analysis
✓ Linear optics matching
✓ Transverse cooling using Liquid Hydrogen absorber
▸ Studied transverse aspects only



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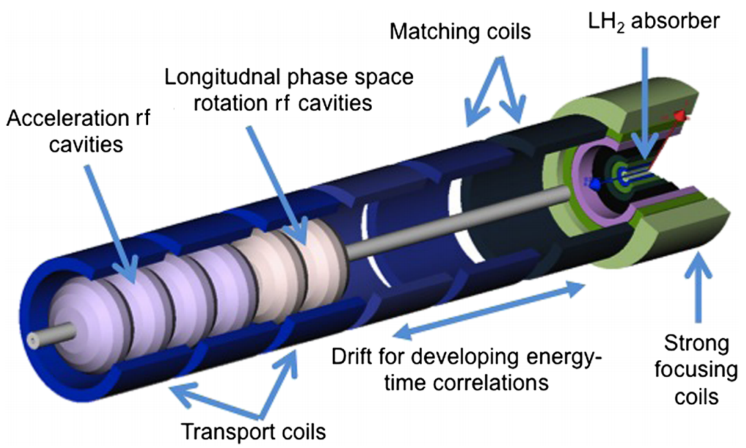

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Towards integrated Final Cooling design:

- RF-Track (developed by A. Latina): https://gitlab.cern.ch/rf-track/download
- Includes collective effects, relevant lattice elements (absorbers, stating wave RF-cavities, solenoids), Python and Octave interface
  ➡ easy to combine with advanced optimisation algorithms
- Specific ionisation cooling effects have been recently added (multiple scattering, muon decays)

➡ ***Further presented studies are focused on RF-Track simulations (thanks to A. Latina)***
See Andrea's talk tomorrow: https://indico.cern.ch/event/1325963/contributions/5828922/

International
MUON Collider
Collaboration

Design optimisation strategy

I. Estimate **optimal momenta and absorber lengths** in every cell, with objective $\epsilon_{\perp} = 25\mu m$.

➡ Provides **starting momenta** and **absorber lengths** for all cells

II. **Optics control**, ensure low beta-function in absorber by **optimizing solenoid field and matching coils**

➡ **Mitigates emittance blow up** in the fridge fields and **controls the optics in absorber** region

III. **Optimize acceleration and rotation** of the bunch after absorber (simplified RF model)

➡ Provides **drifts** and **rotation** “kicks” initial estimates for RF- system design

Focus of today’s talk

IV. Integrated **end-to-end simulation** of the complete cooling channel using RF-Track

➡ **Optimize a realistic RF system**: frequencies, phases, gradients to **control the longitudinal dynamics**
➡ Current Limitations
➡ Developed tools and methods

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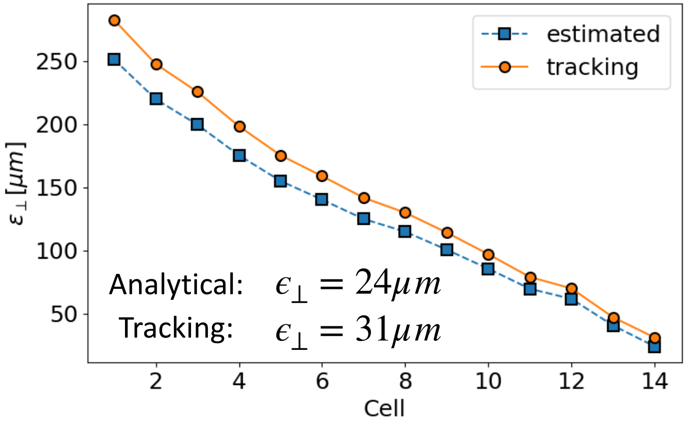
$$\frac{dE}{ds} = 4\pi N_A \rho r_e^2 m_e c^2 \frac{Z}{A}\left[\frac{1}{\beta^2}\ln\left(\frac{2m_e c^2\gamma^2\beta^2}{I(Z)}\right) - 1 - \frac{\delta}{2\beta^2}\right]$$

$$\frac{d\epsilon_\perp}{ds} = -\frac{\epsilon_\perp}{\beta^2 E}\frac{dE}{ds} + \frac{\beta_\perp E_s^2}{2\beta^3 m c^2 L_R E}$$



✓ Tracking simulations using **40T** and **optimised parameters** confirm the potential for **lower emittance**

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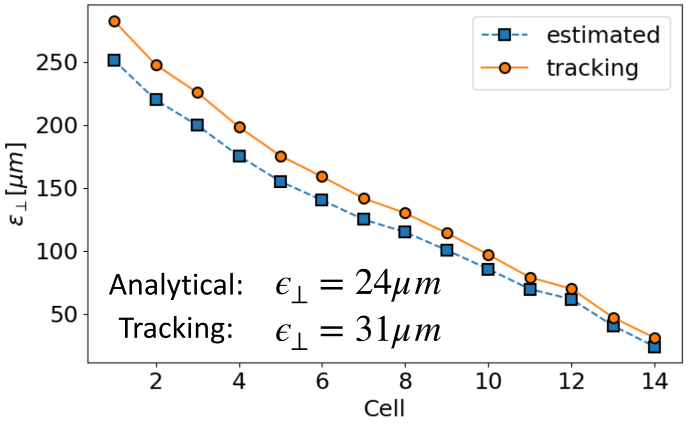

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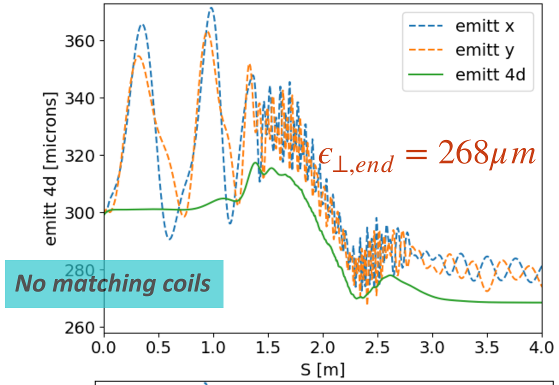

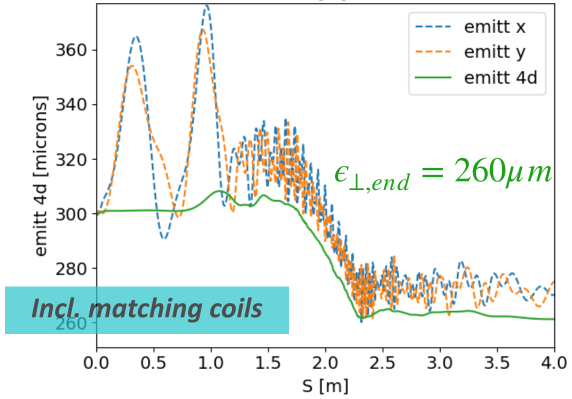

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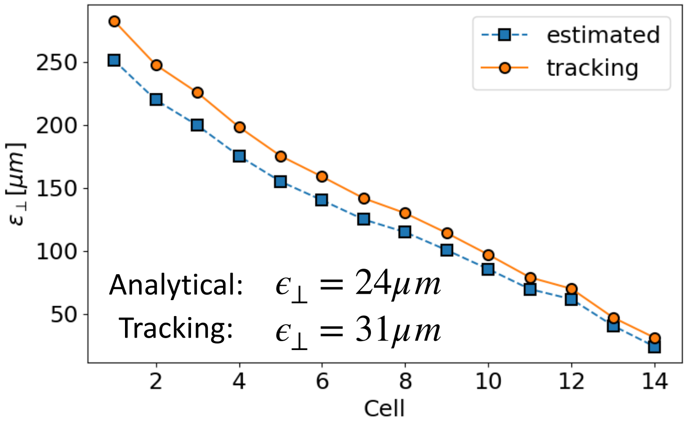

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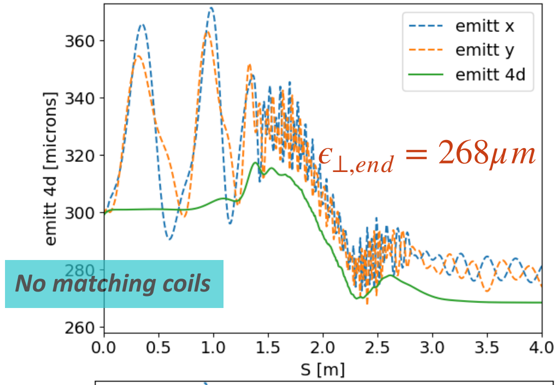

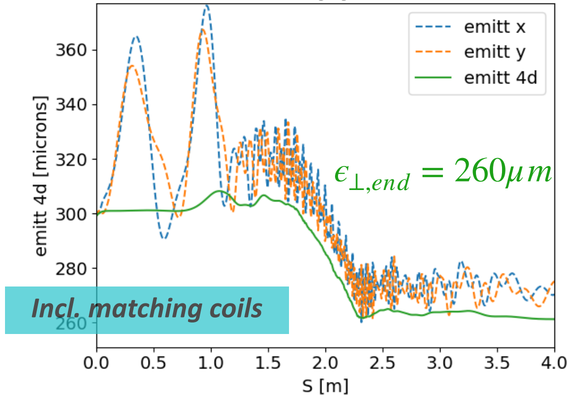

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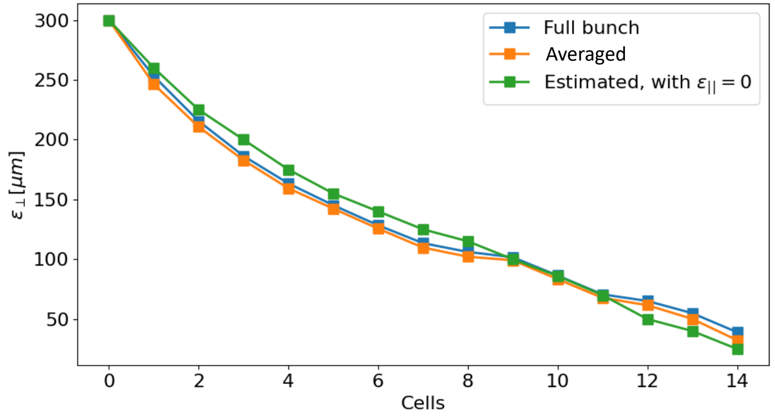


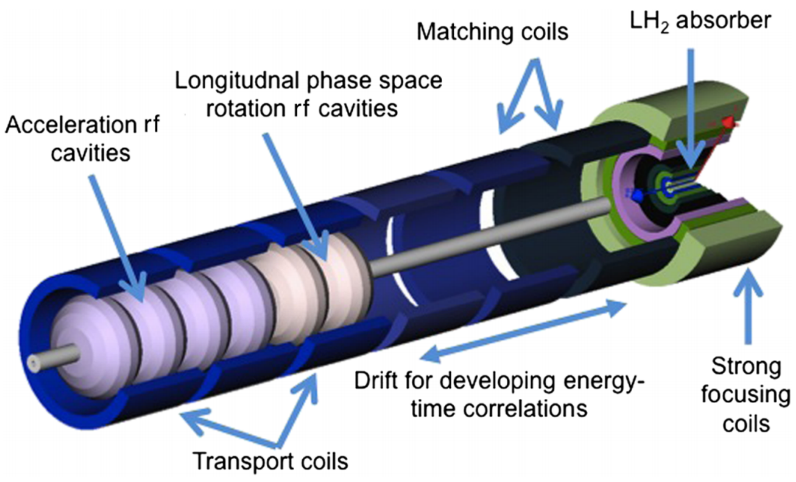
✓ *Transverse emittance = 32 micron, Longitudinal emittance = 77 mm*

✓ *Problem: Transmission (only ~29%) => more acceleration, higher momenta at the start of last cells?*

Integrated Lattice Optimization

Focus of today's talk

IV. Integrated **end-to-end simulation** of the complete cooling channel using RF-Track



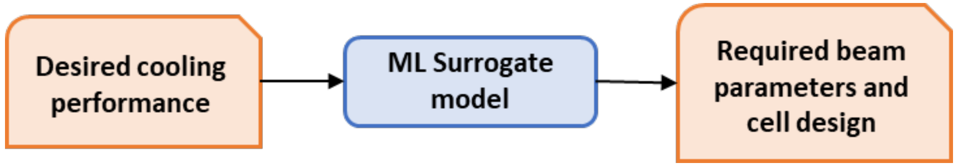
- Global optimization: would have **14 parameters** to optimize in **each cell**
- Expected to need **~14 cells in total**
- Cell-by-cell approach, testing different optimization algorithms

- **Objective function** : $\frac{\epsilon_\perp \epsilon_{||}}{N_\mu}$

- **Free parameters:**
  - Absorber (liquid hydrogen) thickness
  - Drift length
  - Number of accelerating RF cavities, rf phase
  - Number of rotating RF cavities, rf phase
  - B-field in RF region to match the field in the cooling cell and the change in momentum

Integrated Lattice Optimization: Methods

- **Optimization procedure:**
  - Run optimization for each cell, a few iterations
  - Create a surrogate model to estimate the initial parameters
  - Bayesian Optimization, BOBYQA



- Input: $\epsilon_{\perp start}, P_{z,start}, \epsilon_{\perp}, \sigma_t, \sigma E, N_\mu$
- Output: $L_{drift}, N_{rot}, N_{cav}, \phi_{RF}, L_{absorber}, L_{sol}$

➡ Fast design estimate

➡ **Use as initial guess** for optimisation algorithms (optimal solution is found within fewer steps)

Peter I. Frazier: *A Tutorial on Bayesian Optimization*. arxiv:1807.02811
Py-BOBYQA: Derivative-Free Optimizer for Bound-Constrained Minimization

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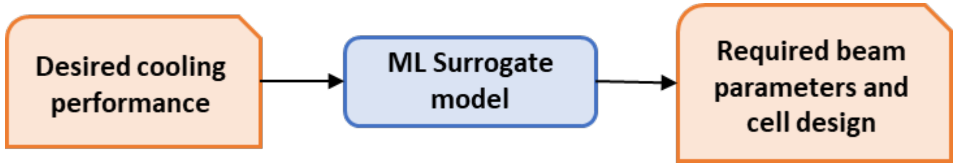

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Example, cell 4: $\epsilon_{\perp,start} = 170\mu m$

Target:

$\epsilon_{\perp} = 150\mu m, \sigma_t = 400mm, \sigma E = 2.0MeV, N_\mu = 75\,\%$

Simulated with parameters predicted by ML-model:

$\epsilon_{\perp} = 149\mu m, \sigma_t = 404mm, \sigma E = 3.5MeV, N_\mu = 69\,\%$

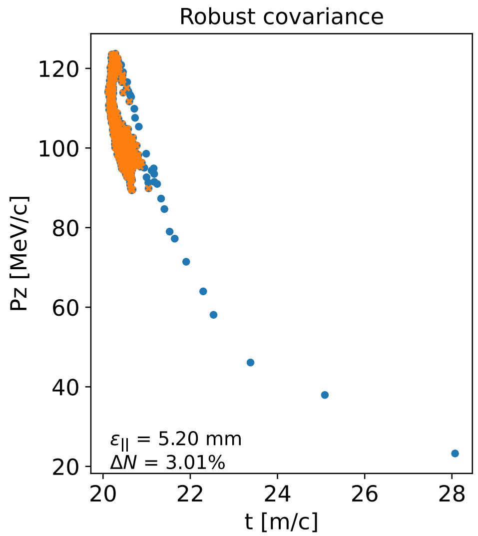
Optimiser, 150 steps, starting with predicted parameters:

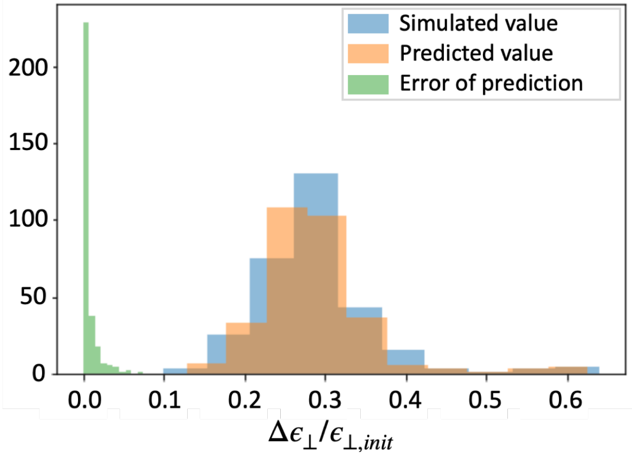
$\epsilon_{\perp} = 150\mu m, \sigma_t = 280mm, \sigma E = 2.1MeV, N_\mu = 71\,\%$

Integrated Lattice Optimization: Methods

- How to **speed up** simulations-based design optimization?
  - Surrogate models to replace slow-executing simulations (used for optics matching in ICOOL simulations)
- How to **estimate initial optimization parameters**?
  - Surrogate models to provide optimizers with "warm start"
  - Bayesian Optimization
- Robust **emittance estimation** during optimization?
  - Clustering for detection of tails biasing the emittance calculation

More details were in presented at 4th ICFA Beam Dynamics Mini-Workshop on Machine Learning for Particle Accelerators: [*"ML-assisted design of Final Cooling System for a Muon Collider"*]

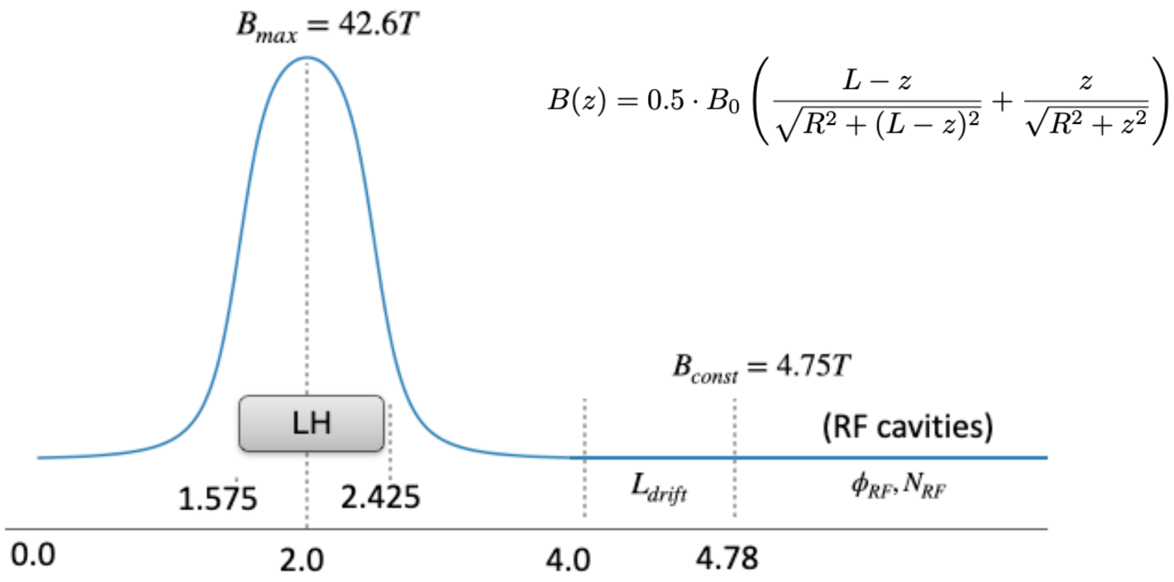




Details on simulations setup

- **General layout**

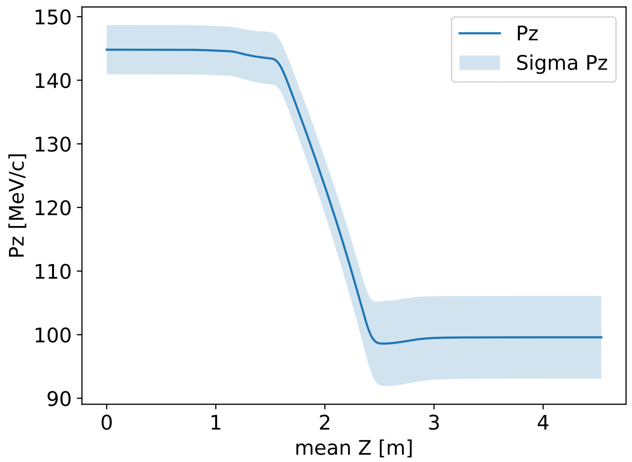
Example for cell 1:
Absorber thickness: 0.85
Solenoid length = 1.48 m

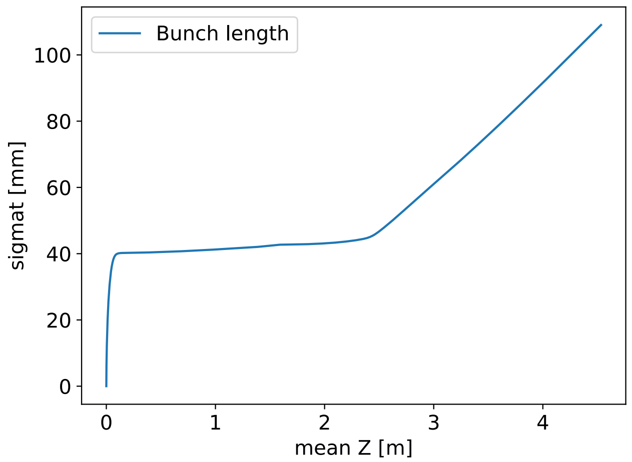


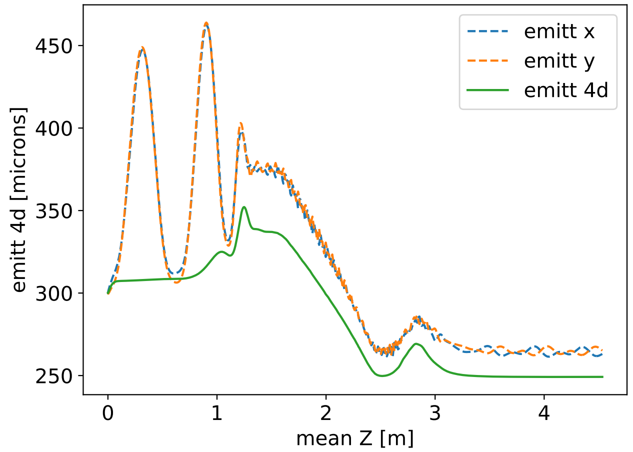
- **Simulating RF-systems:**
  - SW cavity - model: pillbox, fixed length = 0.25 m
  - Rotating cavities: rf phase to be optimised to provide the energy spread minimising rotation (and partially acceleration)
  - Accelerating cavities: RF-Track routine to find the phase providing maximum acceleration
- f_{RF} according to $\lambda = \sigma_t/20, G = 1.88 * \sqrt{(f_R F)}$ (optimistic assumption for gradients, see IMCC report)

Beam parameters evolution inside a cooling cell

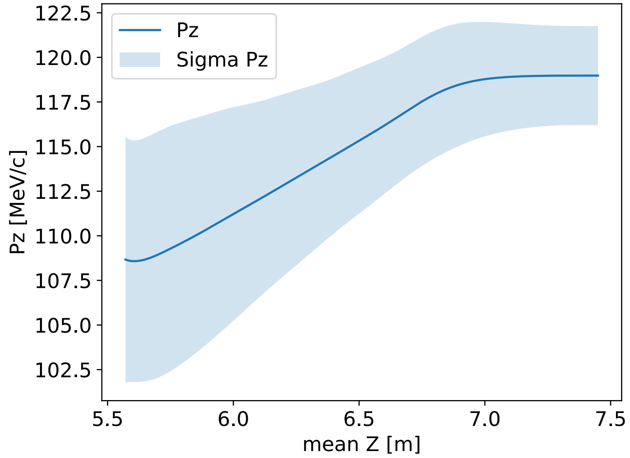
Cell 1, passing through liquid hydrogen absorber

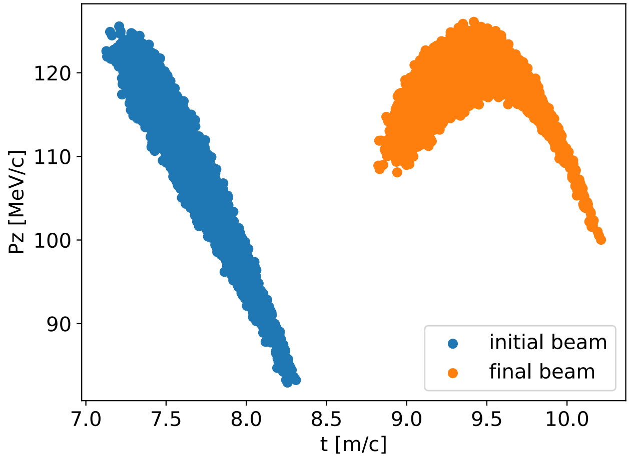






Rotation and acceleration





Integrated Lattice Optimization: Current results

Cell	LH_2 [m]	Drift [m]	N_{RF} rot.	N_{RF} accel.	f_{RF} [MHz]	G [MV/m]	$\phi_{RF,rot.}$ degrees
1	0.85						
2	0.466	0.3238	5	5	111.06	19.81	-180
3	0.46958	1.363	10	7	56.85	14.17	90
4	0.4	2.5	9	8	40.13	11.9	51
5	0.3	1.8358	7	2	34.91	11.11	-10
6	0.25	2.0	5	10	30.61	10.4	-54
7	0.3	0.984	5	14	11.637	6.823	-82
8	0.1	3.6464	2	7	16.17	8.04	67
9	0.17	3.64	2	11	13.38	7.32	67
10	0.08	2.555	11	2	8.226	5.39	-6
11	0.0541	2.895	11	4	5.676	4.48	-96

$P_{z,start}$ [MeV/m]	σE_{start} [MeV]	σt_{start} [mm]	$P_{z,end}$ [MeV/m]	σE_{end} [MeV]	σt_{end} [mm]	$\epsilon_{\|\|}$ [mm]	$\epsilon_{\perp}$ [μm]	N [%]
145.0	3.1	49.8	99.8	4.3	129.8	2.3	239.2	98
119.1	2.1	209.2	89.1	2.6	201.2	4.8	190.2	95
118.5	4.0	284.8	88.5	4.0	394.9	6.4	157.3	90
113.1	5.7	819.5	87.5	3.7	362.8	12.5	133.3	83
93.9	3.7	357.6	62.7	5.5	738.1	19.1	103.6	76
83.0	6.8	4606.2	58.0	2.7	1209.7	23.6	86.1	63
89.5	2.2	1378.5	55.3	3.0	1271.0	31.3	64.0	55
71.0	2.7	1785.7	56.4	3.1	1617.2	41.4	54.9	49
75.7	3.1	2120.8	52.3	3.5	1967.6	49.1	44.0	40
61.2	2.1	3199.0	43.5	2.8	2740.0	68.8	35.3	35
60.7	2.3	3456.5	49.5	2.9	3143.8	86.2	31.4	31

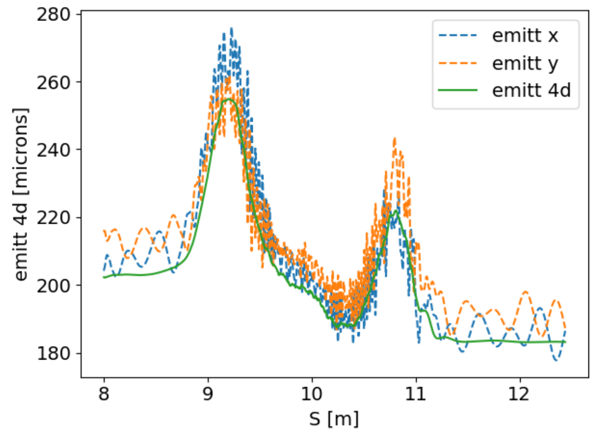
- Already cell 8 achieves better performance compared to the baseline:
  8 cells, $\epsilon_{\perp} = 55\mu m, \epsilon_{||} = 41mm$ vs. 16 cells,$\epsilon_{\perp} = 55\mu m, \epsilon_{|||} = 76mm$
- Potential to improve the transmission by minimising the relative energy spread
- Potential to combine with other cooling techniques
- Current results of 6D cooling could allow to start final cooling at < 300 micron

Executing start-to-end simulations and optimisation

- RF-Track, pre-compiled version, download: https://gitlab.cern.ch/rf-track/download
- Cells are described in JSON format
- Python script to read the cells description and to set-up and run RF-Track simulation
- Optimisation script with defined objective function executing the base lattice
- Post-processing, displaying results
- Simulation data management and Surrogate models training

```
{ "cell_n": 2,
  "pz": 118.64,
  "abs_len": 0.466,
  "entr_coil_bz": 3.10,
  "entr_coil_r": 0.4,
  "entr_coil_offset": 0.615,
  "exit_coil_bz": 2.46,
  "exit_coil_r": 0.7,
  "exit_coil_offset": 0.583,
  "sol_len": 1.75,
  "low_bz_cool": 4.629,
  "low_bz_rf": 4.79,

  "freq_accel": 111.06,
  "grad_accel": 19.81,
  "drift_len": 0.3238,
  "nrot": 5,
  "naccel": 5,
  "cell_len": 0.25,
  "phase_rot": -180
```

```
json_data = read_json_file(filename)

channel_params = cells_from_json(json_data)

for cell_params in channel_params:
   cooling_cell = CoolingCell(**cooling_cell_data)
   cooled_beam = cooling_cell.cool_in_cell(beam_to_track)
   utils.plot_results(cooled_beam)


beam_to_track.load(“./optimized_beam_{}”.format(cell_n-1))
beam_end_cell.save("./optimized_beam_{}".format(cell_n))
```



https://github.com/MuonCollider-WG4/muon_final_cooling

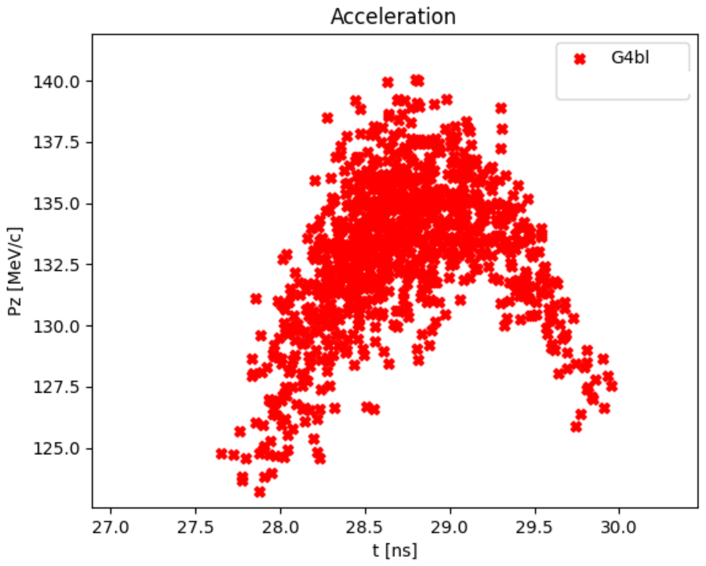
RF cavity in RF-Track vs. G4Beamline

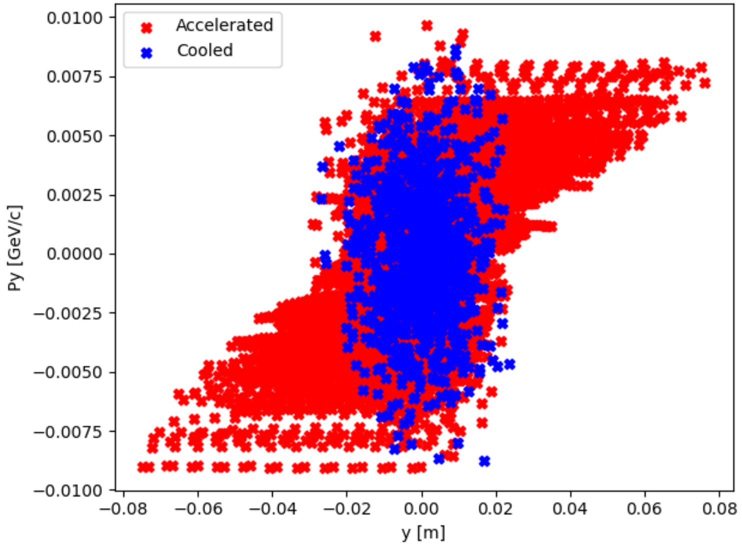
RFTrack
- TM011
- E = E sin(kz) sin(ωt)
- open cavity, no windows

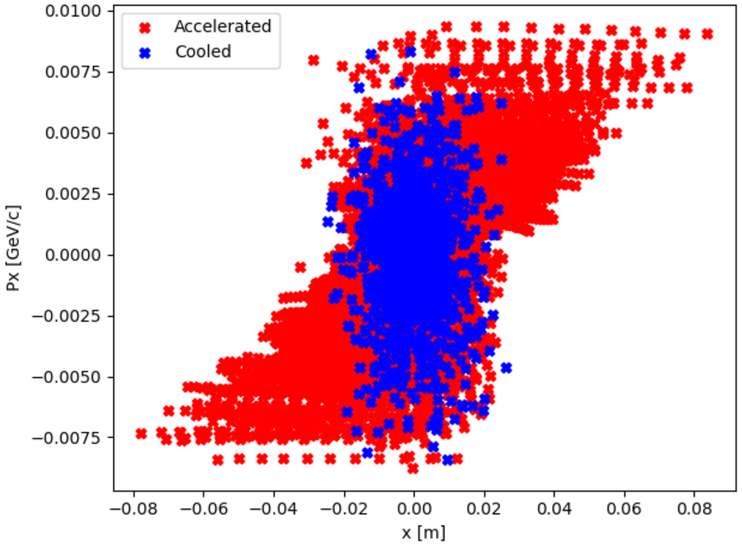
G4BL
- TM010
- E = E sin(ωt)
- Implements windows

Acceleration with G4bl

- cavity length = 0.25 m
- Accelerate from 100 MeV/c to 135 MeV/c, reduce energy spread from 4.2 MeV to 2.3 MeV
- Emittances: transverse 232 micron, longitudinal 3.7 mm





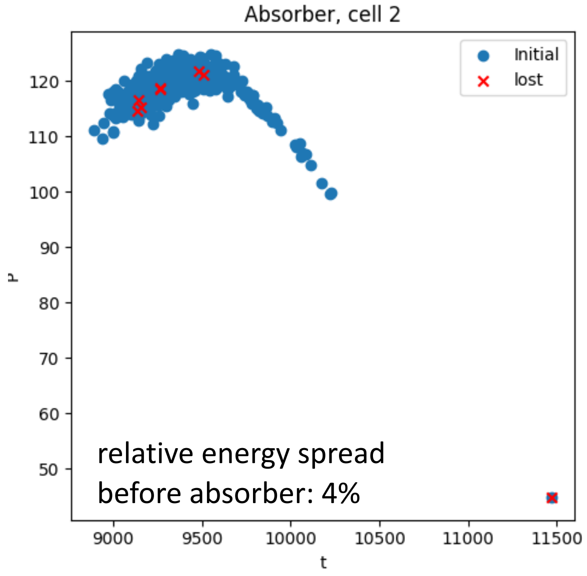


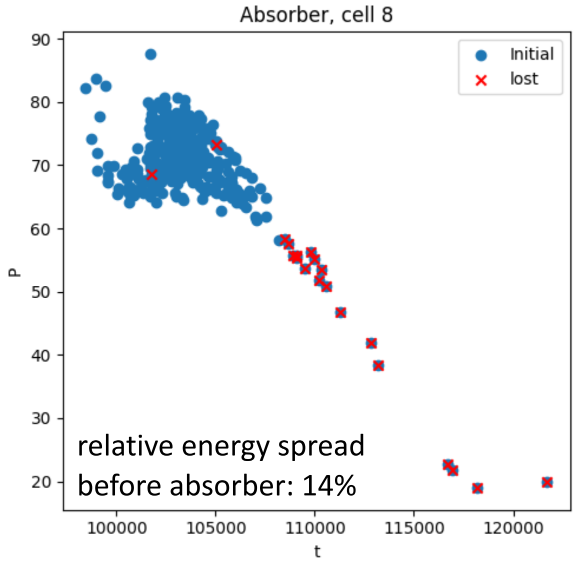
Challenges and potential improvements

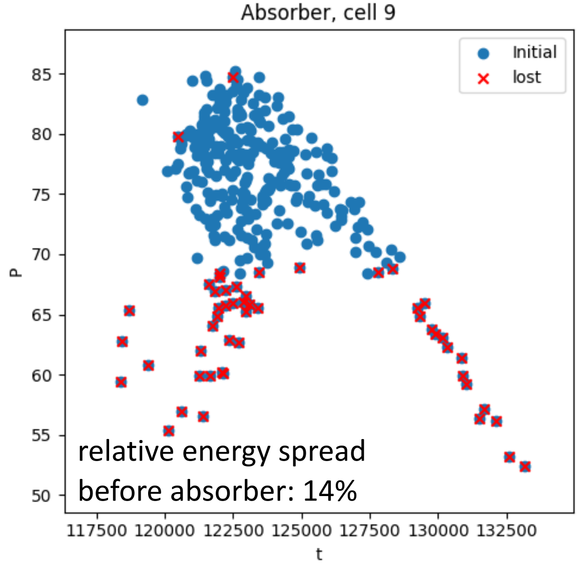
- Energy spread
- Transmission losses
- Large bunch length towards the end of the channel

- Improvement by using RF phase such that cavities combine acceleration and rotation?
- Better control over RF bucket size to avoid the transition losses?

After cooling	After cooling, cut	After drift and RF	$\sigma E_{accel}[\%]$
99.42	97.12	96.88	5.8
96.11	89.84	88.98	4.4
88.2	84.8	83.88	5.8
83.17	76.9	76.43	7
75.66	72.71	71.81	14.4
65.87	62.62	61.84	7.6
60.13	57.14	56.5	8.4
55.49	52.79	52	14
46.47	44	43	13.9
40.56	40	39	16.7
38	37	36	18
35	34		21







Summary and outlook

- ✓ Demonstrated a **strategy for the optimisation** of final cooling design
- ✓ Flexible **optimisation & simulation framework** for evolving design
- ✓ **Integrated lattice design** including all relevant elements
- ✓ **Shorter channel** achieving **better performance** compared to the baseline:

  8 cells, $\epsilon_{\perp} = 55\mu m, \epsilon_{||} = 41mm$ vs. 16 cells, $\epsilon_{\perp} = 55\mu m, \epsilon_{||} = 76mm$

- ✓ Currently achieved best performance: $\epsilon_{\perp} = 35\mu m, \epsilon_{||} = 68mm$

- Improvements of **longitudinal dynamics** control and transmission losses
- Consideration of **feasible RF-design options**: e.g. multi-harmonics RF (allows the use of higher frequencies, shorter acceleration path is possible.)
- Start-to-end simulations in G4Beamline

$P_{z,start}$ [MeV/m]	σE_{start} [MeV]	σt_{start} [mm]	$P_{z,end}$ [MeV/m]	σE_{end} [MeV]	σt_{end} [mm]	$\epsilon_{||}$ [mm]	$\epsilon_{\perp}$ [μm]	N [%]
145.0	3.1	49.8	99.8	4.3	129.8	2.3	239.2	98
119.1	2.1	209.2	89.1	2.6	201.2	4.8	190.2	95
118.5	4.0	284.8	88.5	4.0	394.9	6.4	157.3	90
113.1	5.7	819.5	87.5	3.7	362.8	12.5	133.3	83
93.9	3.7	357.6	62.7	5.5	738.1	19.1	103.6	76
83.0	6.8	4606.2	58.0	2.7	1209.7	23.6	86.1	63
89.5	2.2	1378.5	55.3	3.0	1271.0	31.3	64.0	55
71.0	2.7	1785.7	56.4	3.1	1617.2	41.4	54.9	49
75.7	3.1	2120.8	52.3	3.5	1967.6	49.1	44.0	40
61.2	2.1	3199.0	43.5	2.8	2740.0	68.8	35.3	35
60.7	2.3	3456.5	49.5	2.9	3143.8	86.2	31.4	31

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Thanks a lot for your attention!

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Back-up slides

Solenoid field parameters

Cell	Bz peak [T]	Solenoid Length [m]	Bz low [T]
1	43	1.48	4.75
2	43	1.75	4.75
3	43	1.0	4.7
4	43	1.0	4.7
5	43	1.0	4.7
6	43	1.11	4.7
7	41	1.33	2.1
8	41	1.0	2.0
9	41	1.4	1.1
10	39	1.0	0.86
11	39	1.0	0.86

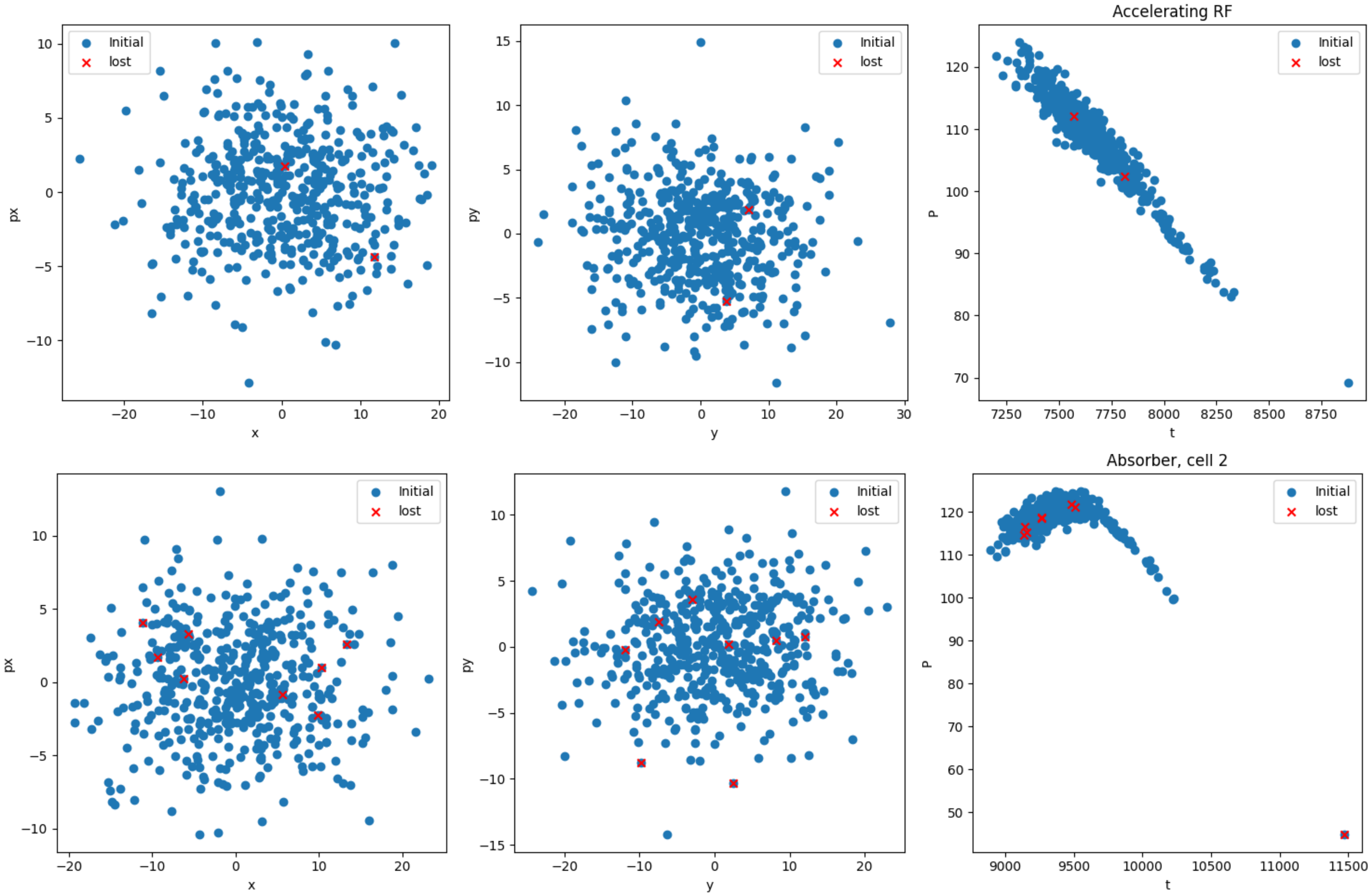
Solenoid field in RF-Track:

$$B(z) = 0.5 \cdot B_0 \left(\frac{L - z}{\sqrt{R^2 + (L - z)^2}} + \frac{z}{\sqrt{R^2 + z^2}} \right)$$

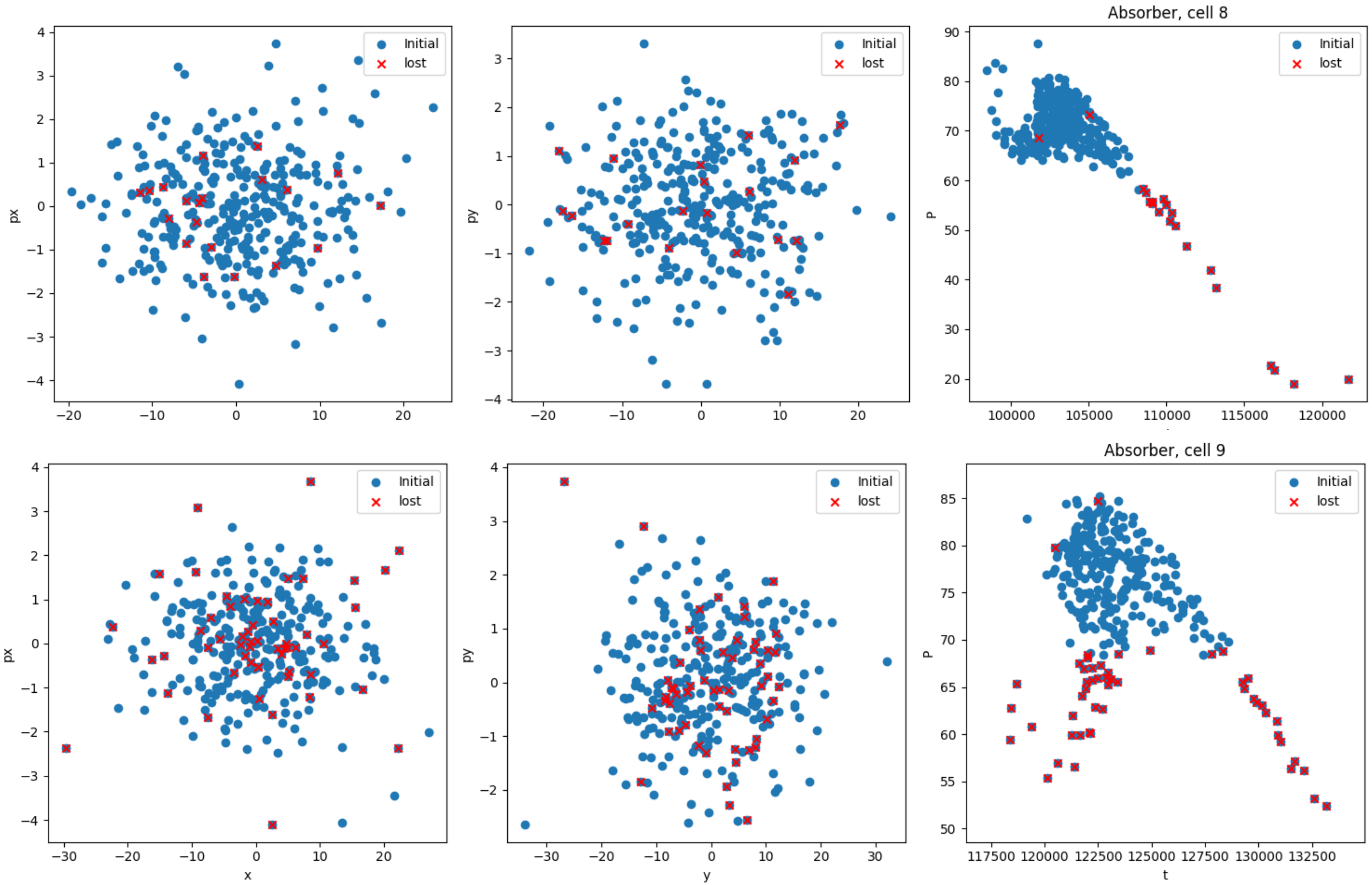
Cell	Aperture [mm]	LH [m]	E_{kin} , start [MeV]	E_{kin} , exit [MeV]
1	15.32	0.85	73.75	39.81
2	10.33	0.47	53.53	32.75
3	8.40	0.47	53.64	32.81
4	7.72	0.40	50.06	31.44
5	8.47	0.30	35.01	16.95
6	5.73	0.25	29.83	14.54
7	5.00	0.30	32.93	13.60
8	4.20	0.10	22.08	14.69
9	4.32	0.17	25.18	12.92
10	4.06	0.08	16.73	9.02
11	2.89	0.05	16.25	11.16
12	3.17	0.10	18.88	9.93

Initial phase space location of lost particles

- Cell 2: relative energy spread before absorber: 4%



Initial phase space location of lost particles



Cell 8: relative energy spread before absorber 14 %

Cell 9: relative energy spread before absorber 14 %