The Vertical FFA for Muon Acceleration

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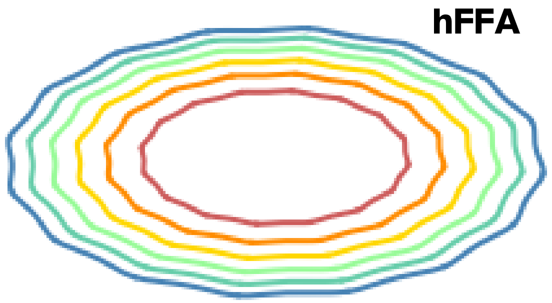
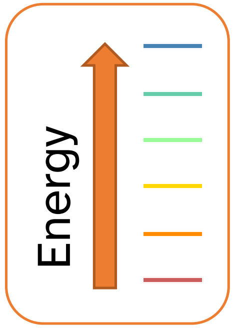
Why FFAs?



"Conventional" horizontal-excursion FFA (hFFA):

- Orbits move outwards with increasing energy
- Fields increase radially

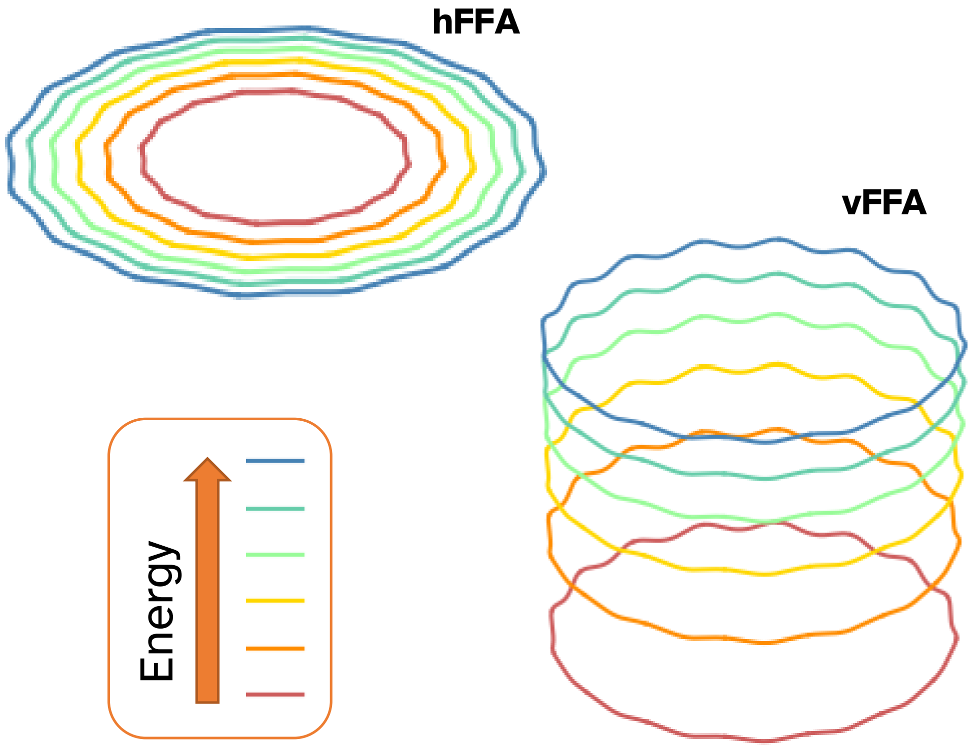


Time-independent magnetic fields means…

- No ramp times
  - Rate of acceleration limited only by RF
  - Mitigates engineering challenges of designing and powering fast-ramping dipoles
- All magnets can be superconducting

hFFA options for muon acceleration studied in detail by J. Scott Berg (see 2023 IMCC meeting)

Introduction to the vFFA



Vertical-excursion FFA (vFFA):

- Higher energy orbits are vertically translated copies of lower energy orbits
- Zero chromaticity if fields increase with vertical coordinate (Z) following scaling law

$$B = B_0 e^{mZ}$$

Zero path length difference means…

→ Zero momentum compaction factor α_c

→ Transitionless

→ Quasi-isochronous for relativistic particles

S. Brooks, "Vertical orbit excursion fixed field alternating gradient accelerators," Physical Review Special Topics - Accelerators and Beams, vol. 16, 08 2013.

vFFA difficulties

- Non-planar orbits

(identified first in tracking study!)

S. Machida, D.J. Kelliher, J-B. Lagrange, and C.T. Rogers. Optics design of vertical excursion fixed-field alternating gradient accelerators Phys. Rev. Accel. Beams (2021)

- Coupled optics
  - Focussing is dominated by skew quadrupole components

- Design for vFFA muon accelerator based on FODO cells has been done as an exercise in 2019 – derived from numerical simulation

S. Machida, J-B. Lagrange, M. Topp-Mugglestone. Application of the FFA concept to a muon collider complex, Proc. IPAC 12 (2021)

  - Design is unoptimized (large circumference, large required B-field)
  - No analytic model, hard to understand system and means of optimisation…

(until now)

Approaching the vFFA

Building an analytic model:

1. **Determine closed orbit**
   There must exist a continuum of closed orbits for all energies
   Closed orbits must be consistent with the fields required by the scaling law

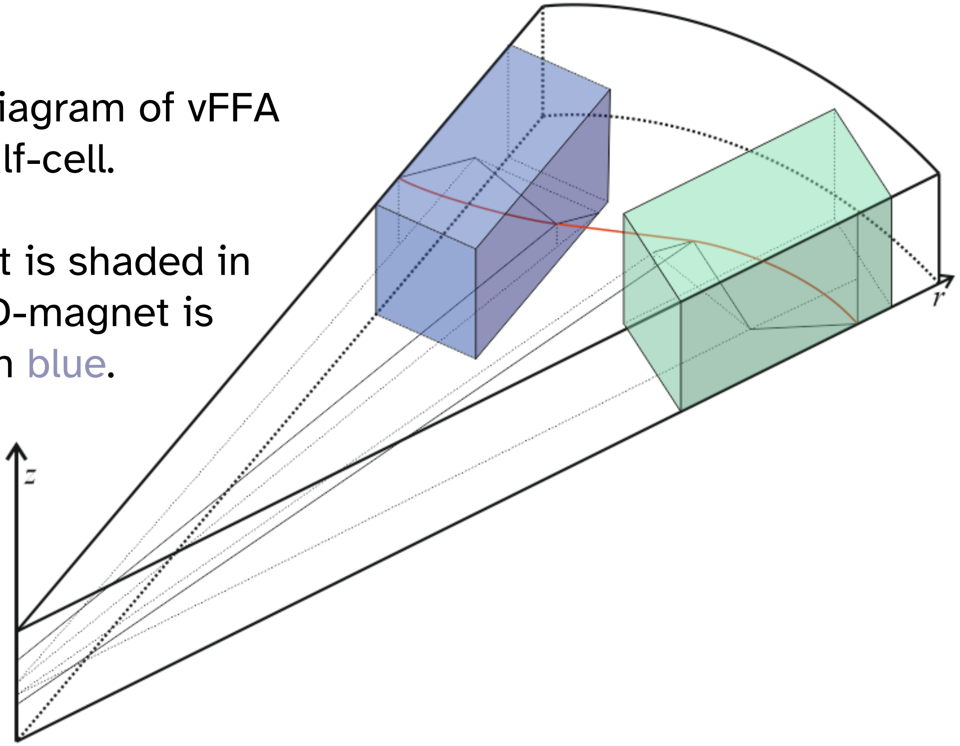
2. **Expand optics around closed orbit**
   Magnetic fields are all predetermined by the scaling law

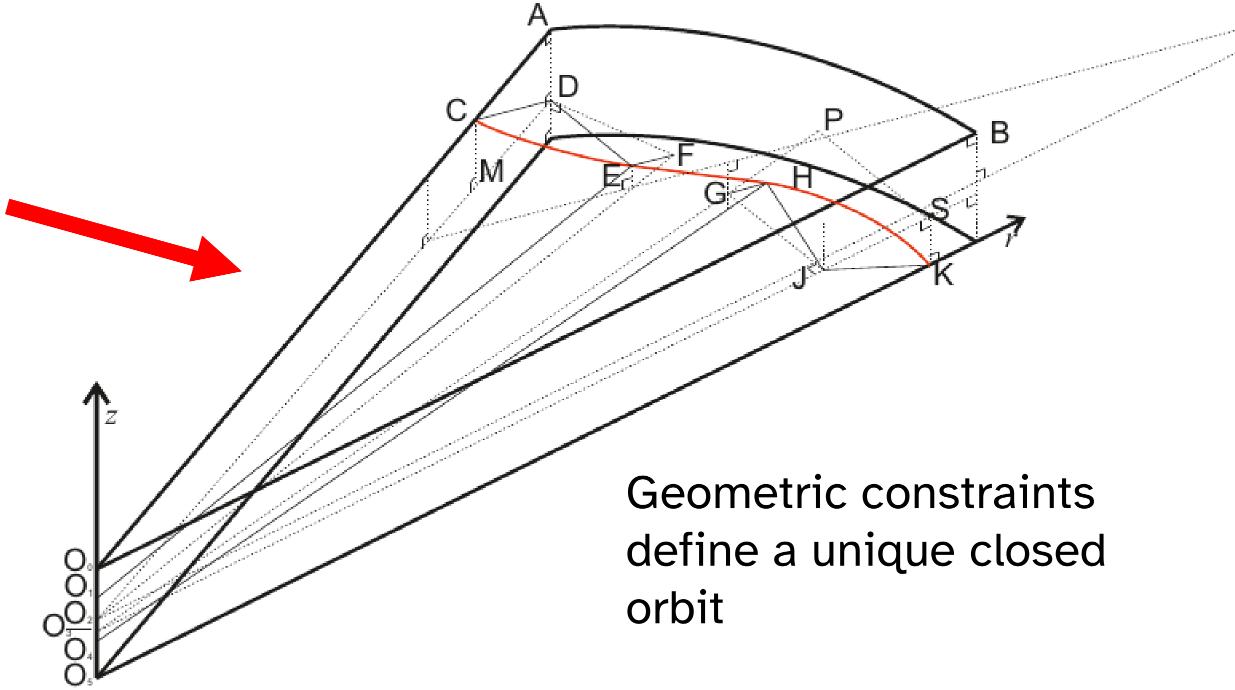
Approaching the vFFA

1. **Determine closed orbit**

Figure: diagram of vFFA FODO half-cell.

F-magnet is shaded in green; D-magnet is shaded in blue.





Geometric constraints define a unique closed orbit

Approaching the vFFA

2. Expand optics around closed orbit

vFFA magnet body Hamiltonian:

$$\mathcal{H} \simeq \frac{p_x^2}{2} + \frac{p_z^2}{2} - \frac{1}{\rho + \frac{\sin\gamma}{m}} \left[\cos\gamma \left(m + \frac{2\sin\gamma}{\rho} \right) xz - \frac{1}{2} m \left(x^2 - z^2\right) \sin\gamma \right] + \frac{1}{\rho\left(\rho + \frac{\sin\gamma}{m}\right)} \left(x^2 \cos^2\gamma + z^2 \sin^2\gamma\right)$$

x, z : horizontal and vertical transverse coordinates

m : normalised field gradient

ρ: radius of curvature (in 3D!)

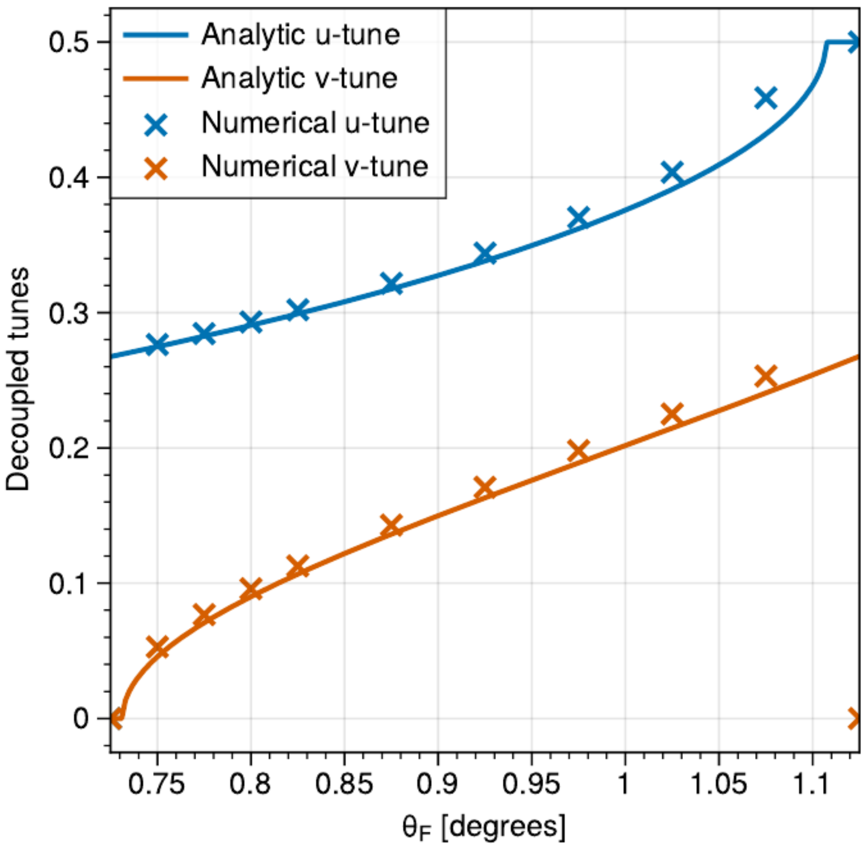
γ : a new parameter called 'inclination'
(angle of the plane of curvature in a vFFA magnet)

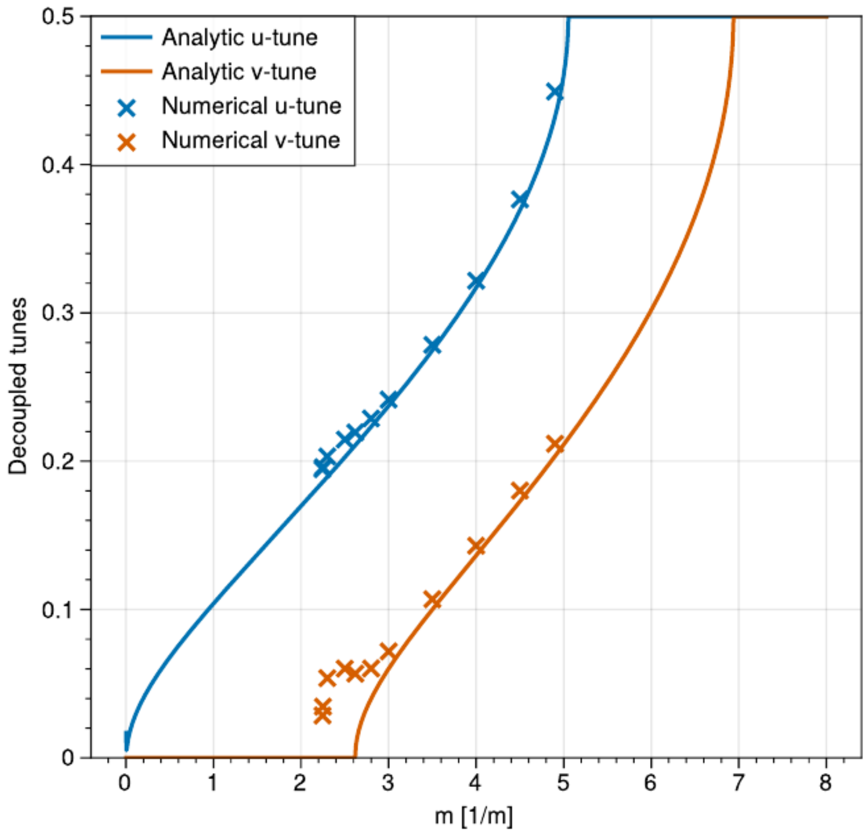
Hamiltonian has normal quad + skew quad + geometric terms!

vFFA linear optics recipe

- Choose set of input parameters
- Compute closed orbit geometry for lattice type (FODO, triplet, straight cell…)
- Input geometric parameters into transfer matrices derived from Hamiltonian
- Compute transfer matrix for each element of the cell

Testing the model





- Agrees with simulation!
  - Tested with 2019 muon accelerator lattice across different input parameters
- Divergence at low *m*-values
  - Expected behaviour due to fringe field effects
  - *m* should be large in a muon accelerator to minimise excursion

What can we do with a vFFA?

“One ring to rule them all” design from 2019

One accelerator on LHC-scale to take the role of RCS1+2+3

	2019 Lattice
Circumference [m]	28350
Number of Cells	810
Injection Energy [TeV]	0.05
Extraction Energy [TeV]	1.5
F-magnet length [m]	12.0
D-magnet length [m]	12.0
Drift length [m]	5.5
Peak Dipole Field [T]	8.7
m-value [1/m]	6.8
Excursion [m]	0.50
Tune	(0.40,0.086)

- Large circumference (> LHC)
- Large peak field (8.7 T)
- Large excursion (50 cm)
- Derived from numerical simulation

Can this design be optimised?

Let’s see what the analytic model can tell us…

2019 lattice optimisation

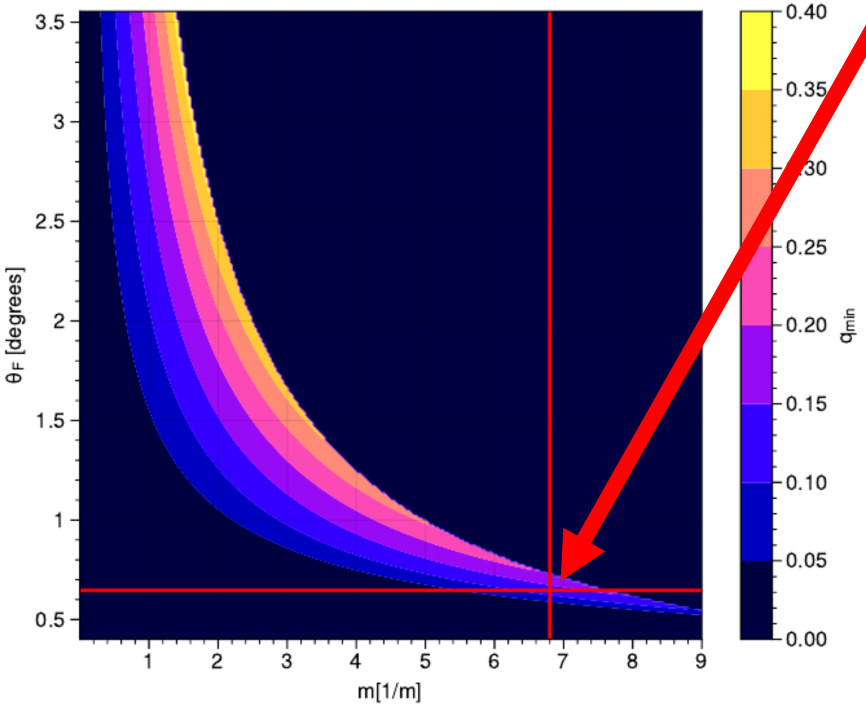
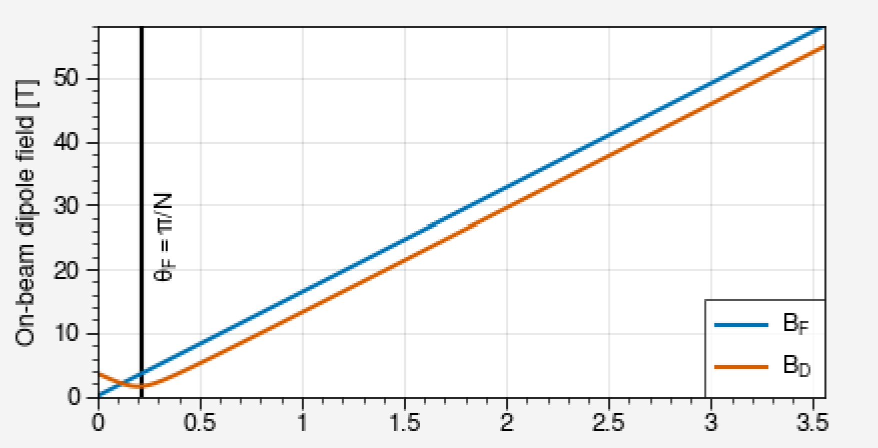


Figure: stability region as a function of m, θ_F
(normalised field gradient, bending angle in F-magnet)

Original lattice configuration



- Peak fields can be decreased by decreasing θ_F whilst increasing m to maintain stability

2019 lattice optimisation

- End result:

	2019 Lattice	Optimisation
Circumference [m]	28350	25200
Number of Cells	810	720
Injection Energy [TeV]	0.05	0.05
Extraction Energy [TeV]	1.5	1.5
F-magnet length [m]	12.0	14.5
D-magnet length [m]	12.0	9.5
Drift length [m]	5.5	5.5
Peak Dipole Field [T]	8.7	7.1
m-value [1/m]	6.8	7.57
Excursion [m]	0.50	0.45
Tune	(0.40,0.086)	(0.44, 0.098)

- Smaller circumference by 3.15 km
- 1.6 T smaller maximum dipole field
- 5cm smaller excursion
- Same drift length

However, excursion is still large, and circumference is greater than RCS stages

→ What if we target the same circumferences and energy scales as the RCS chain?

Muon vFFAs as an alternative to RCS

Proof-of-concept designs

	Stage 1 vFFA	Stage 4 vFFA
Circumference [m]	5990	35000
Number of Cells	810	1000
Injection Energy [TeV]	0.06	1.5
Extraction Energy [TeV]	0.3	5
F-magnet length [m]	6.36	18.76
D-magnet length [m]	4.24	13.12
Drift length [m]	1.05	1.56
Peak Dipole Field [T]	7.14	14.8
m-value [1/m]	30	10.44
Excursion [m]	0.048	0.12
Tune	(0.39, 0.13)	(0.41, 0.070)

- Using analytic model, we show that parameters similar to RCS1 and RCS4 can be achieved with a vFFA
  - Same footprint
  - Same energy ranges
  - Smaller peak dipole field than RCS4
  - 5cm excursion for stage 1
  - 12cm excursion for stage 4

An isochronous, fixed-field, zero-chromatic muon accelerator is possible based on the vFFA technology!

Unsolved problems…

- What is the optimum configuration for a Muon Collider complex containing a vFFA?
  - Energy ranges were optimised for RCS designs – could we reoptimize to include one or more vFFAs?
- Large-aperture magnets and RF
  - Significant engineering challenge
- Injection and extraction systems…
  - Coupled optics make kicking/bumping orbit difficult

However…

Decoupled insertions

- Coupling effects can be (locally) negated within a straight vFFA insertion
  - Requires inclination = 90 degrees

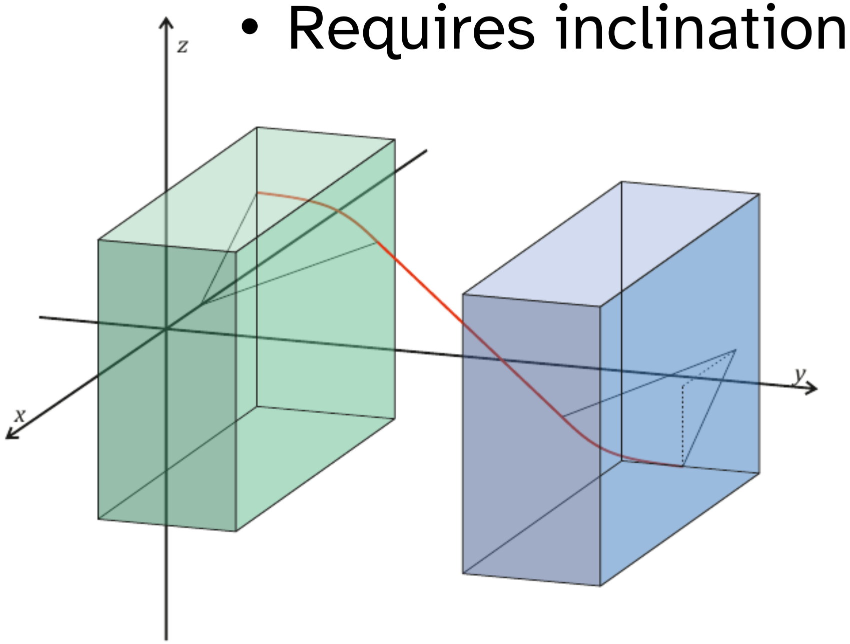


Figure: 3D geometry of a straight cell in a vFFA

The inclination angles are given by $\widehat{HDE}$ and $\widehat{AFB}$ for each magnet.

In a straight vFFA cell in a FODO or triplet configuration, the inclination for magnets of opposite polarity must be equal; in the special case of a 90 degree inclination this leads to a complete negation of all coupling effects within the cell.

Decoupled insertions

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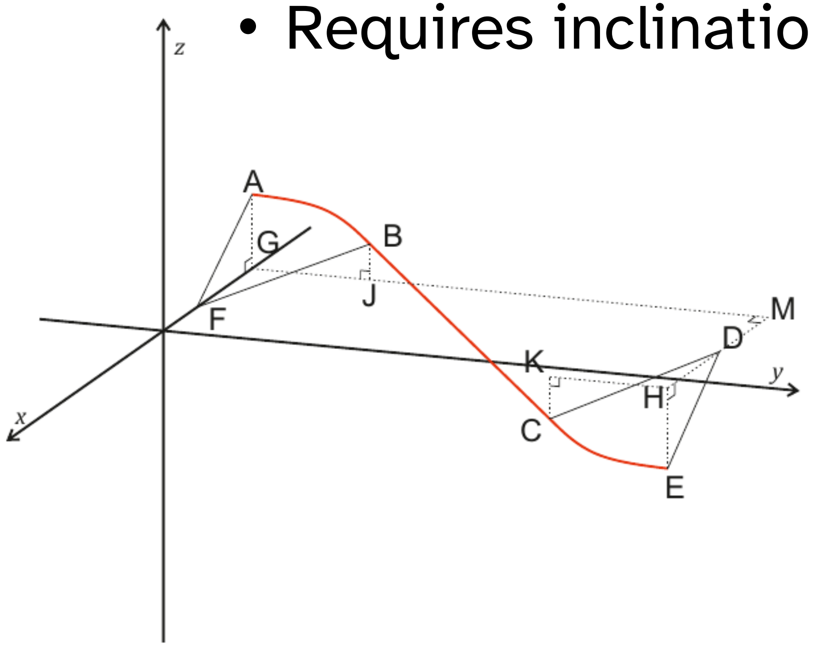

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Decoupled insertions

- Coupling effects can be (locally) negated within a straight vFFA insertion
  - Requires inclination = 90 degrees

→ Easy *(easier)* design of injection and extraction systems

→ Dispersion suppression is possible in a decoupled straight using existing technology

→ RF acceleration could be done in dispersion-suppressed straights

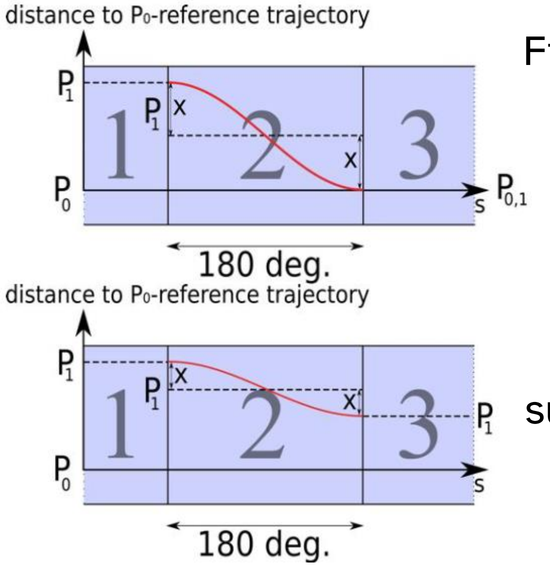

Figure:
Complete dispersion suppression scheme for an FFA

Partial dispersion suppression scheme in an FFA

Source: J-B. Lagrange

Conclusions

- **We now have analytic tools to understand vFFAs**
  - Straight cell, FODO, and triplet geometries have been studied
- **vFFA FODO rings can be built with**
  - Equivalent footprint to RCS designs · Achievable dipole fields · Fully superconducting magnets · Small orbit excursion

  …for an energy-efficient, zero-chromatic, isochronous accelerator
- **Coupling effects can be negated in straights**
  - Could help solve injection/extraction issues + dispersion suppression
- **More research needed to realise the vFFA alternative to RCS in context of a full facility design!**

Further reading

- **vFFA concept**
  - Original rediscovery by S. Brooks

    S. Brooks, "Vertical orbit excursion fixed field alternating gradient accelerators," Physical Review Special Topics - Accelerators and Beams, vol. 16, 08 2013.
- **vFFA modelling**
  - My PhD thesis (forthcoming!!)
- **vFFA numerical studies**
  - Numerical studies of linear optics and DA by M. Vanwelde

    M. Vanwelde, C. Hernalsteens, and N. Pauly, "Linear and nonlinear beam dynamics of vertical fixed-field accelerators," Phys. Rev. Accel. Beams, vol. 27, p. 024003, Feb 2024.
  - Machine Learning approach by A. Oeftiger

    A. Oeftiger, A. Santamaría García, J.-B. Lagrange, and S. Hirlander, "Active deep learning for nonlinear optics design of a vertical ffa accelerator," IPAC2023, WEPA026