

DETERMINISTIC APPROACH TO MBA LATTICE DESIGN

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iFAST – 9th Low Emittance Workshop, CERN, Switzerland,
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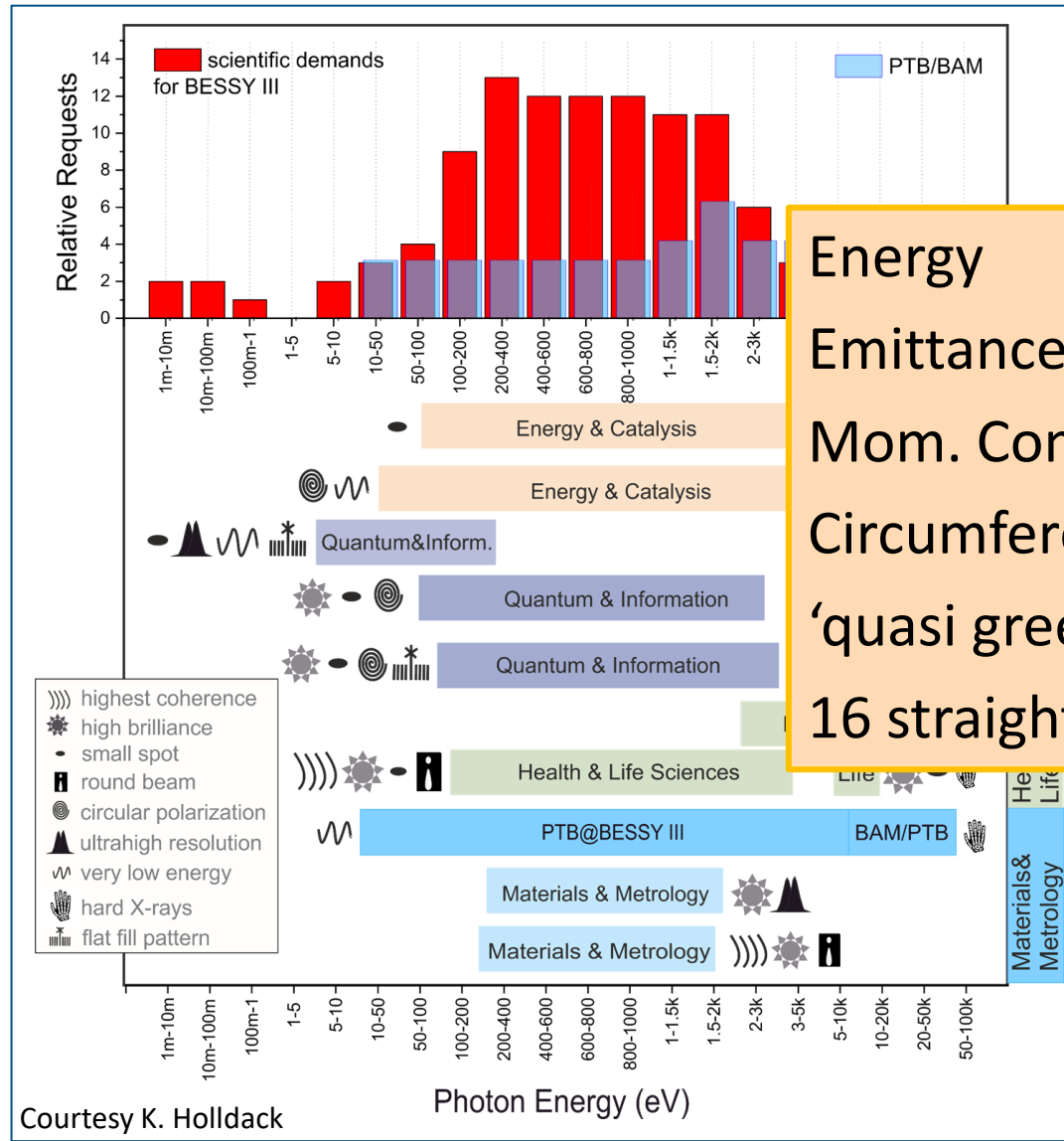
Find the
optimal
solution

Deterministic Approach to MBA Lattice Design

- Introductory remarks
- Layout of the unit cell
- Current 'working horse' lattice
- Validation of results
- Non-linear optimization
- Conclusion

BESSY III Requirements & Objectives

Courtesy Paul Goslawski



FACILITY PARAMETERS

Energy 2.5 GeV

Emittance 100 pm

Mom. Compaction $\alpha > 1e-4$

Circumference ~350m

'quasi green field' ~350m

16 straights à 5.6 m

RING PARAMETERS

1. Ring Energy **2.5 GeV**
(1.7 GeV)
2. Emittance **100 pm rad**
(5 nm rad)
3. Circumference **350 m**
16 straights @ 5.6 m
(240 m @ 4.5 m)
4. Low beta straights & maybe round beams
5. **Metrology source**
Homogenous bends
Measuring the field at the source point with a $100 \mu\text{m}$ probe in a volume of $10 \times 10 \times 10 \text{ mm}$
1-2 bends per arc
6. Momentum compaction factor **> 1.0e-4**

applications

Already at BESSY II, a 3rd generation **without** combined function bends

Courtesy K. Holldack

Lattice design is a many-parameter optimization in a high-dimensional space.

4th generation light sources

Bessy III

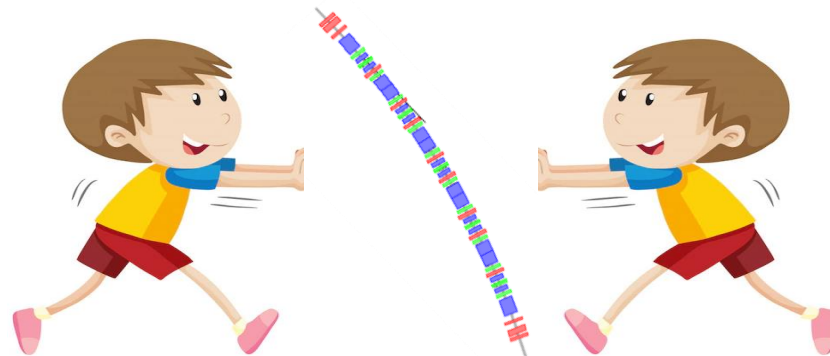
- 6 dipoles
- 10 reverse bends
- 24 quadrupoles
- 19 sextupoles, octupoles?
- drifts

3rd generation light sources

Bessy II

- dipole
- 7 quadrupoles
- 6 sextupoles
- drifts
- => many parameters to optimize

=> too many parameters to handle easily



OPTION:

Take existing lattice and push towards own needs and demands.

<https://www.shutterstock.com/image-vector/boys-pulling-pushing-boxes-illustration-376336384>

OPTION: Use multi-objective Genetic Algorithms (MOGA) or machine learning



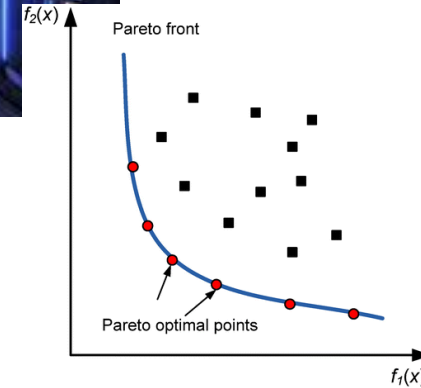
<https://www.pngaaa.com/detail/4902300>

Drawbacks:

- Excessive computer resources
- No learning curve
- Many equivalent solutions
- 'a' solution – not necessarily the optimum

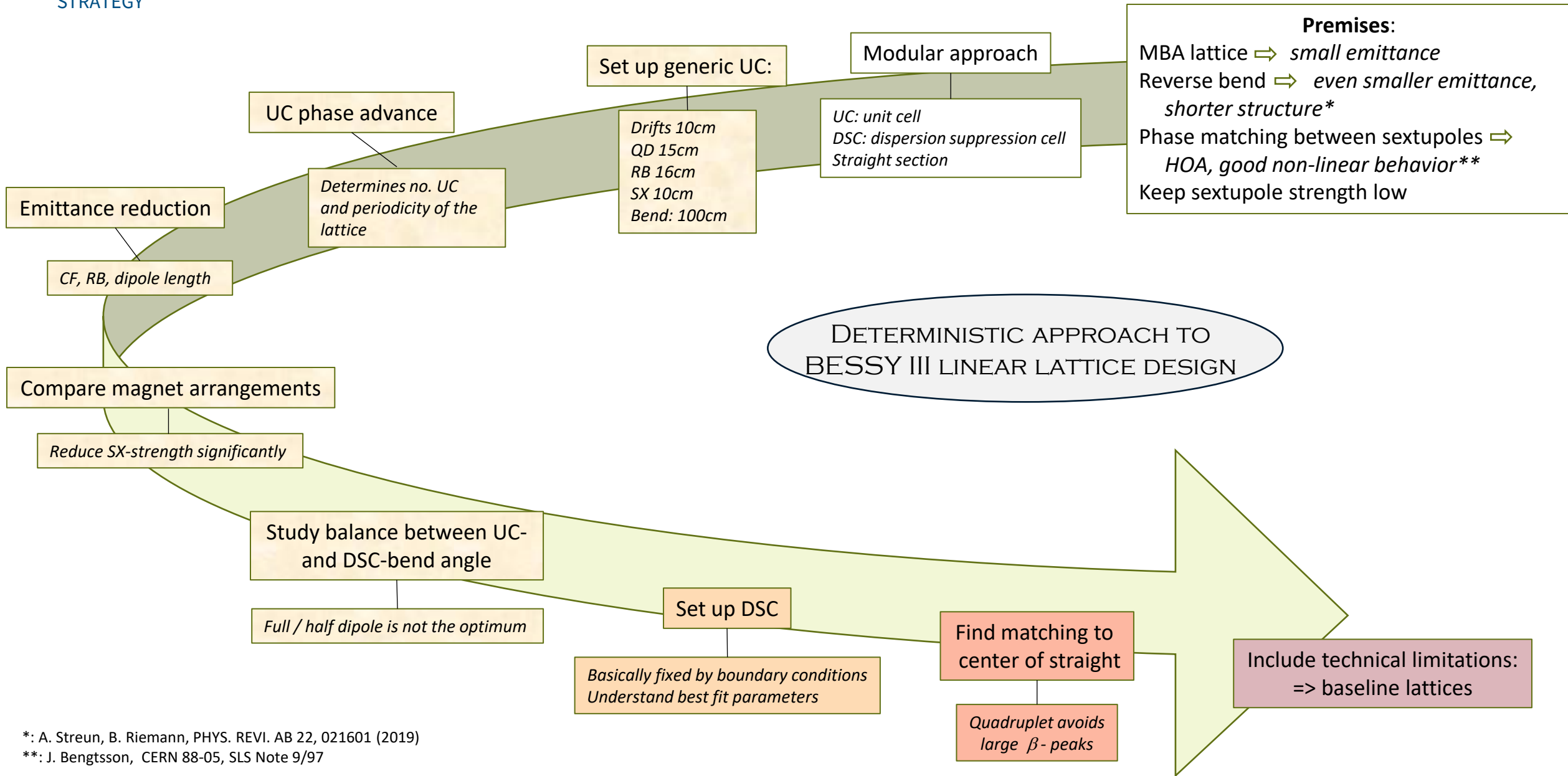


<https://www.shutterstock.com/de/search/server-room>



HZB approach: Deterministic Lattice Design (regular MBA)

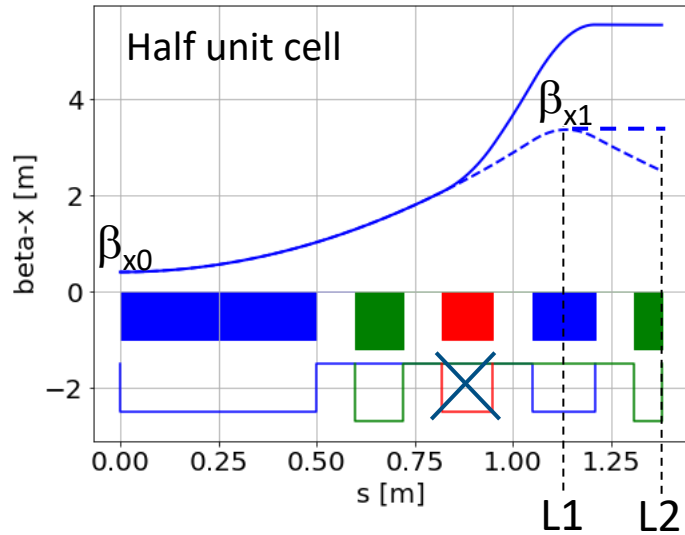
- Divide lattice into small, generic subsections
Cuts down on the number of parameters per section
- Optimize subsection
Understand the functionality of each element
Why a reverse bend? Combined function or separate function magnets? Which magnet order?
- Compose baseline lattice
- Injection straights, super bends ...
all regarded as perturbations that do not alter the basic design choices



*: A. Streun, B. Riemann, PHYS. REVI. AB 22, 021601 (2019)

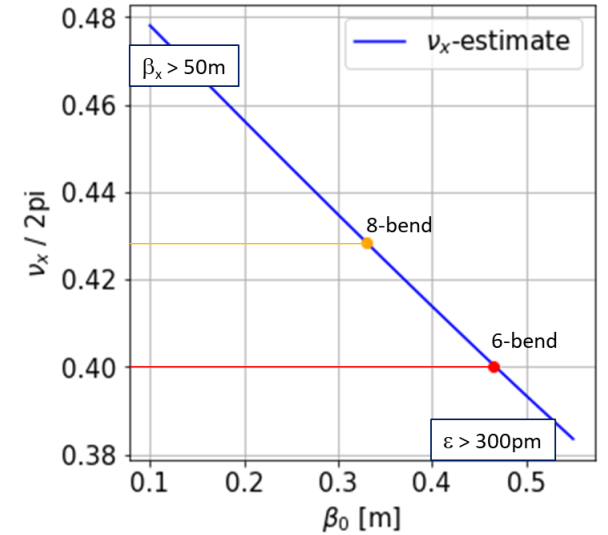
** : J. Bengtsson, CERN 88-05, SLS Note 9/97

HORIZONTAL PHASE ADVANCE



Phase matching for HOA:

- *unit cell*: $v_{x,y} * (n+1) = N$,
n: no UCs, N: integers
- *super period*: $\varphi_{x,y} * p = M$
p: no. super periods, M: integer



An upper estimate for the horizontal phase advance can be calculated for QD = 0

$$2\pi v_x < \int_0^{L1} \frac{\beta_{x0}}{\beta_{x0}^2 + s^2} ds + \int_0^{L2} \frac{1}{\beta_{x1}} ds = \arctan(L1 / \beta_{x0}) + L2 / \beta_{x1}$$

$v_x = f(\beta_{x0})$ - 90% of phase advance accumulates before QD

UC		L(arc)* [m]	16 periods
3	4*0.5 = 2 4*0.25 = 1		
4	5*0.4 = 2	23.2	371
5	6*0.5 = 3 6*0.33 = 2		
6	7*0.4286 = 3	28.8	460

*: straight = 5.6+2.8m, DSC = 1.8m, UC = 2.8m

HOA condition fixes no. of UCs and periodicity ($v_y = 0.1$)

- 3 free parameters left in UC: dipole-angle and -length, RB angle

Smaller emittance by increasing J_x : reverse bend or gradient bend?

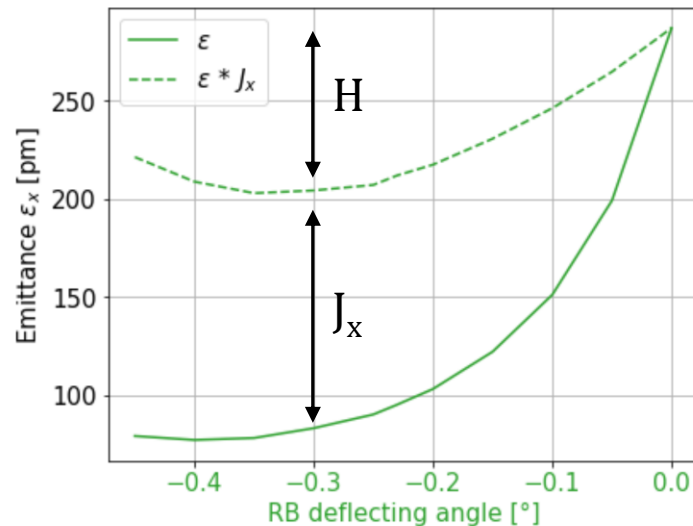
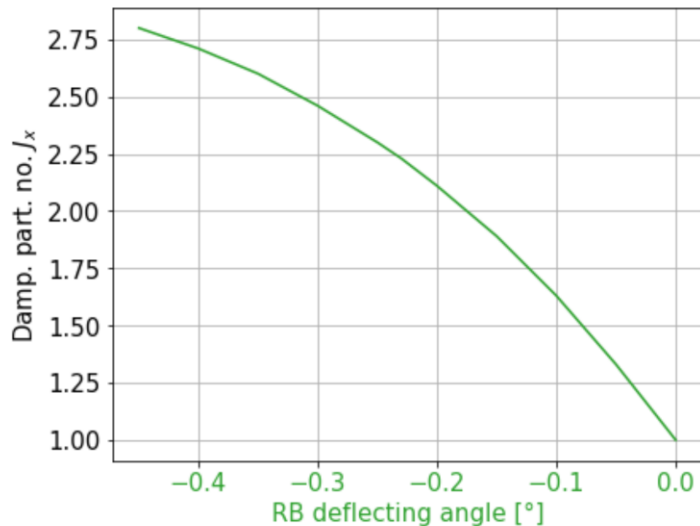
$$\varepsilon = \frac{C_a \gamma^2}{J_x} \int_0^C \frac{H(s)}{|\rho(s)|} ds$$

$$J_x = 1 - \frac{I_{4x}}{I_{2x}}, \quad I_{4x} = \int_0^L \frac{\eta_x}{\rho^3} + \frac{2K\eta_x}{\rho} ds, \quad I_{2x} = \int_0^L \frac{1}{\rho^2} ds$$

- 1) $K < 0 \Rightarrow$ gradient bend
- 2) $\rho < 0, K > 0 \Rightarrow$ RB

1) Reverse bends (RB): focusing quadrupoles, shifted from the central trajectory to create a small dipole field \Rightarrow combined function dipole

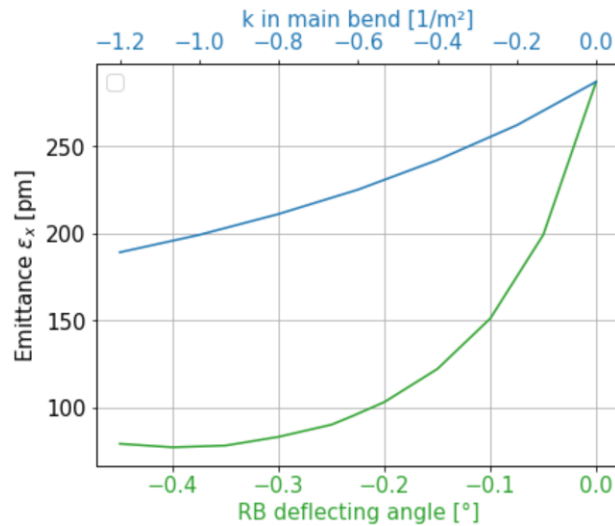
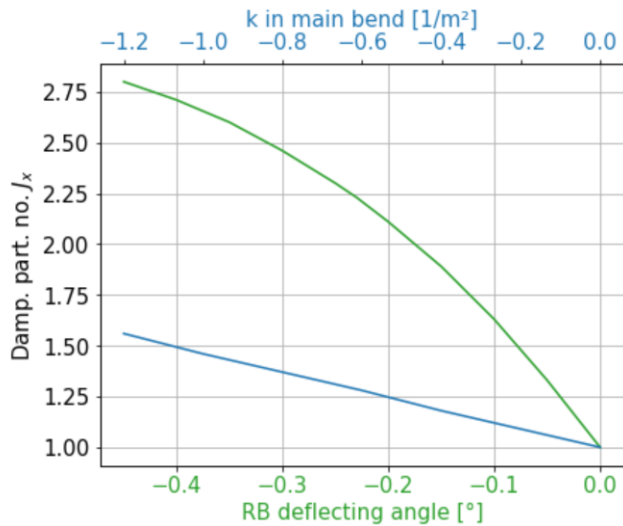
- Deflection angle and length \sim 5-10% of main bend
- ρ same order of magnitude as the main bend, $\rho < 0$



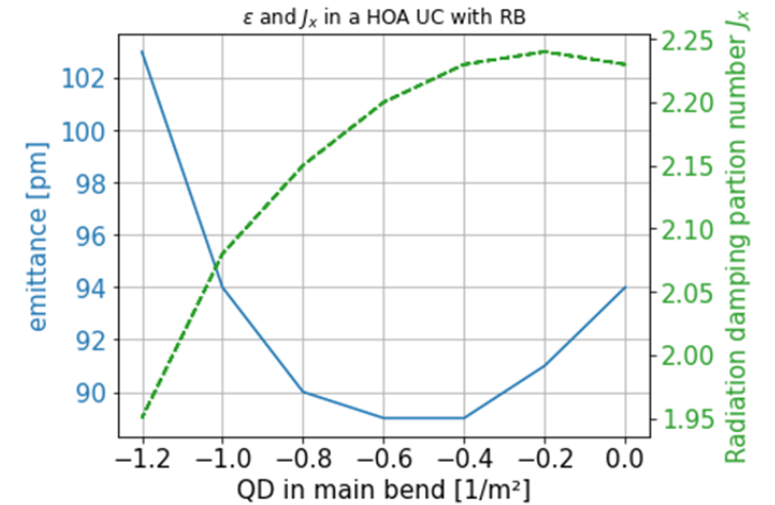
RB: Emittance reduction without stronger focusing!

2) Combined function main bend

J_x and emittance for: a) reverse bend, homogeneous main bend
 b) no reverse bend, gradient main bend



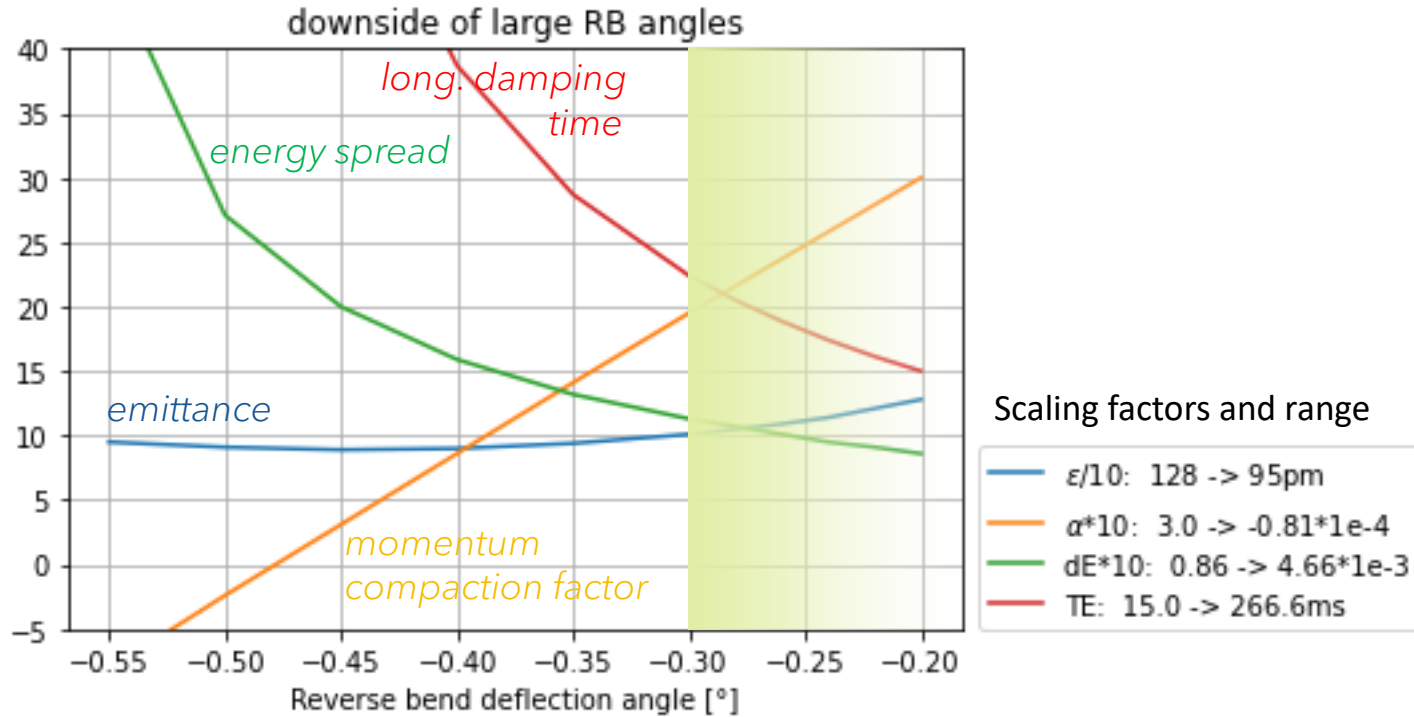
⇒ RB larger effect on emittance than gradient bend



RB (-0.23°) and gradient in bend - not beneficial

⇒ RB completely overtakes the role of the combined function dipole and is more efficient!

THERE IS NO SUCH THING AS A FREE LUNCH...

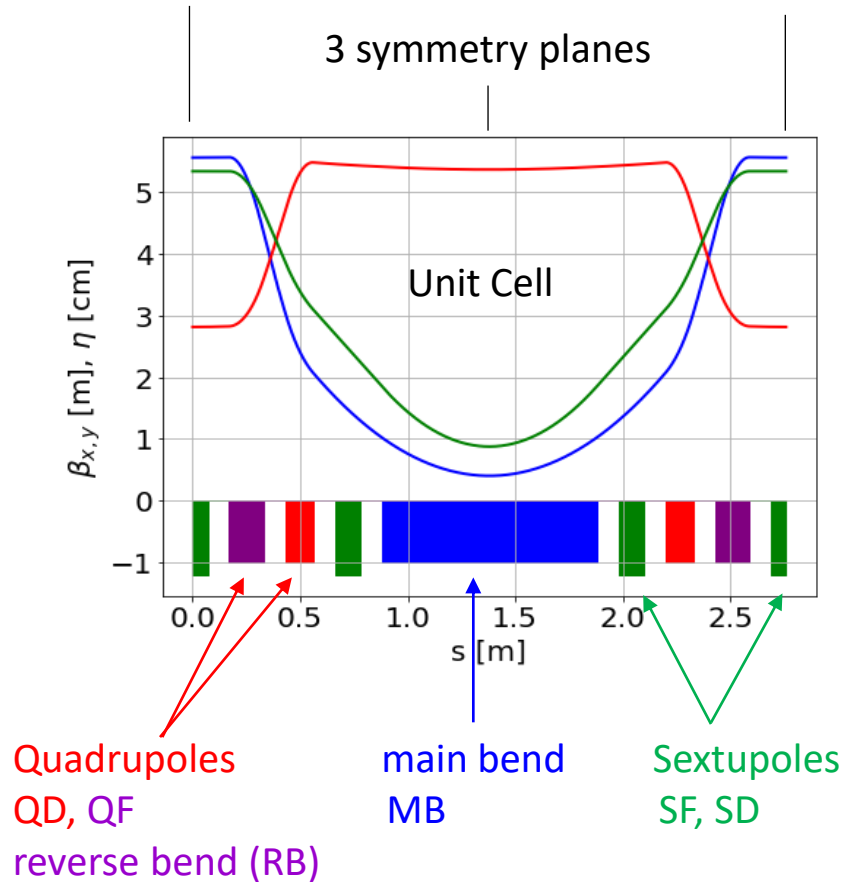


$$\alpha_c = \frac{1}{C} \int_0^c \frac{\eta(s)}{\rho(s)} ds$$

$\alpha_c > 1 * 10^{-4}$ translates to

$\alpha_{c,UC} > 2 * 10^{-4}$ in UC

Strong limitation of RB angle by longitudinal plane



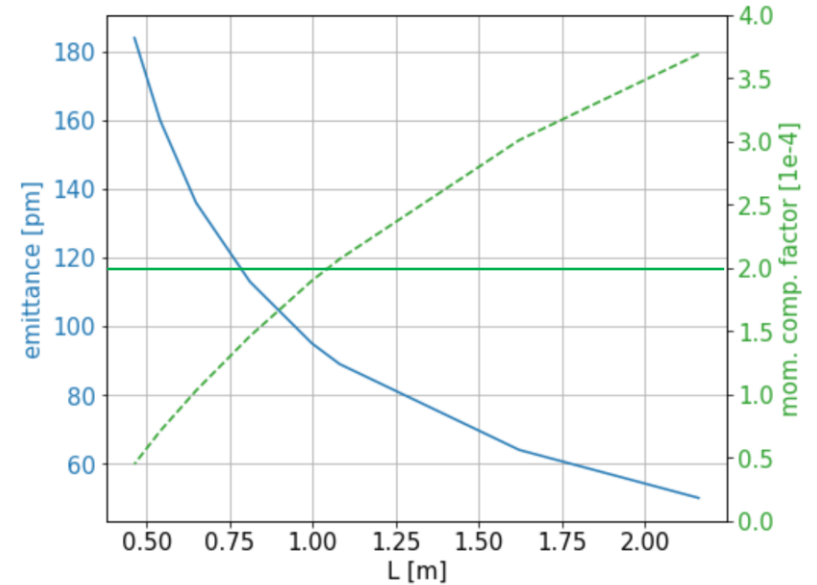
QD, QF – set by phase advance

RB angle – as large as possible, limited by α_c , τ_z , Δ_E

MB angle – approx. given by no. UC and super-period

MB length

Emittance reduction by MB length



Longer bends decrease emittance *and* increase α_c , but limited by circumference (factor 64, 4UC * 16p)

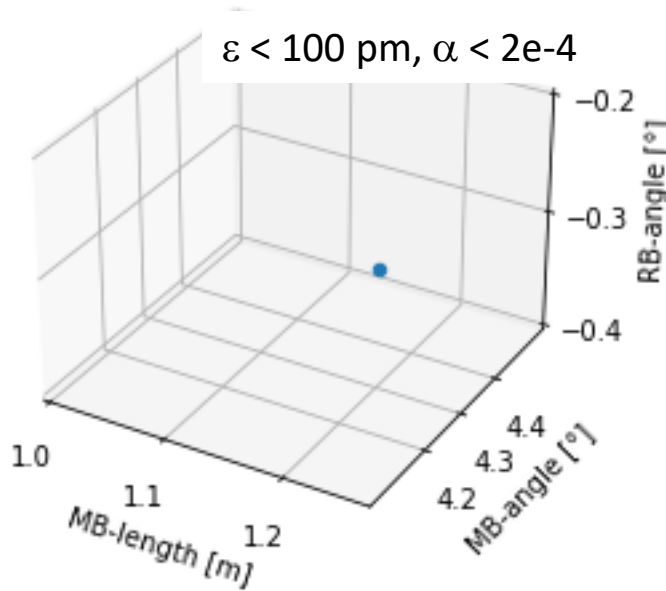
ϵ , α - functions of MB-length and -angle and RB-angle

Crude grid 3x3:

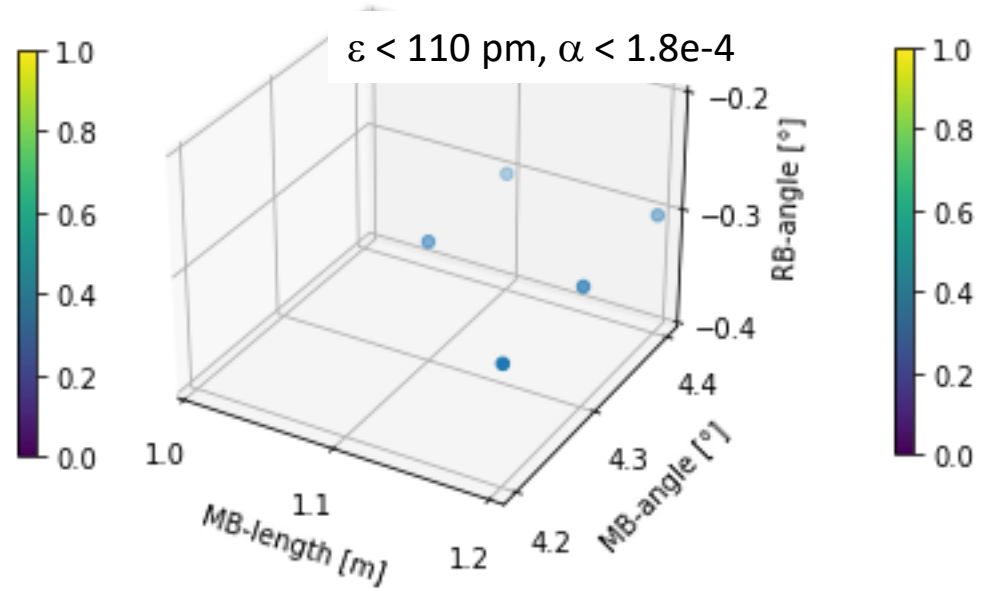
MB-length = [1.0m, 1.1m, 1.2m]

MB-angle = [4.2°, 4.3°, 4.4°]

RB-angle = [-0.2°, -0.3°, -0.4°]



M-L [m]	MB-a [°]	RB-a [°]	ϵ [pm]	α [1e-4]
1.2	4.3	-0.3	94	2.2



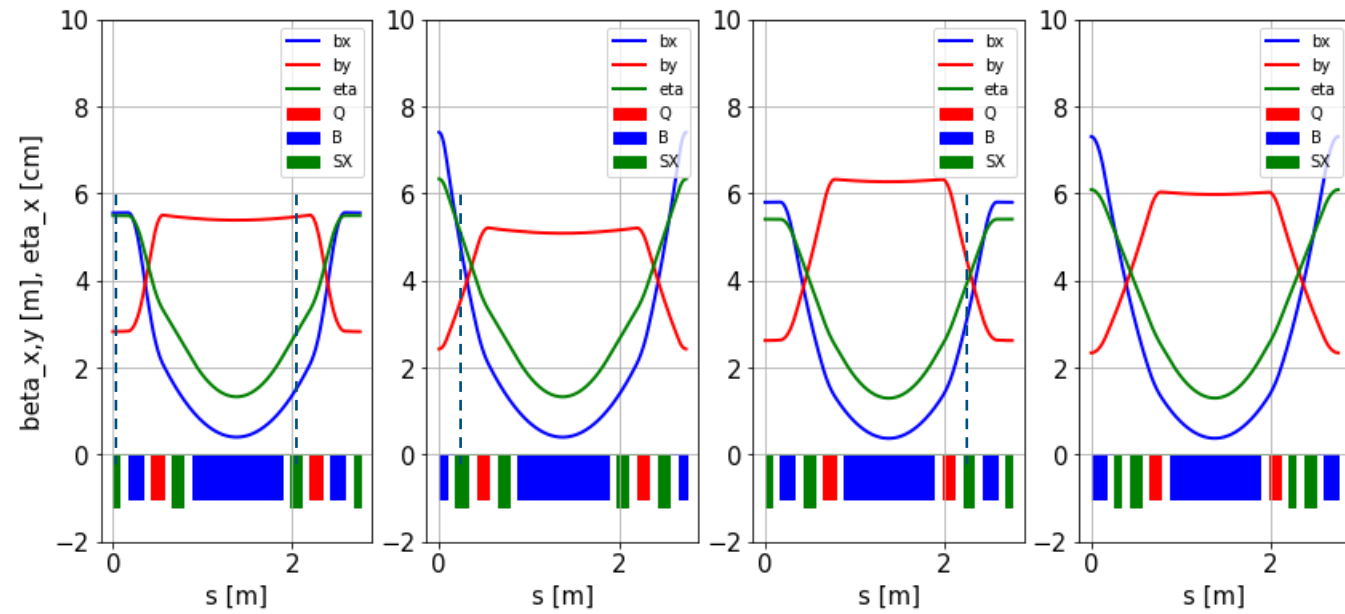
M-L [m]	MB-a [°]	RB-a [°]	ϵ [pm]	α [1e-4]
1.1	4.3	-0.3	101	1.9
1.1	4.4	-0.3	109	2.1
1.2	4.2	-0.3	86	2.0
1.2	4.3	-0.3	94	2.2
1.2	4.4	-0.3	102	2.4

Standard angle distribution:
 $360^\circ / 16 / 5 = 4.5^\circ \Rightarrow \text{MB} = 4.5^\circ, \text{DSB} = 2.25^\circ$
 Better: MB = 4.3°, RB = -0.3°, DSB = 2.65°

Magnetic arrangement of the SF-Unit Cell:

4 magnet permutations

- a) place the RB or SF at the outside
 - b) place QD or SD next to the central dipole
- Drifts remain 0.1m, $v_x = .4$, $v_y = .1$



	setup	ϵ [pm]	ξ_x	ξ_y	SD [1/m ²]	SF [1/m ²]
1	SF last – SD central	95	-0.75	-0.28	-17.8	10.1
2	RB last – SD central	98	-0.83	-0.24	-32.1	18.9
3	SF last – QD central	106	-0.74	-0.29	-18.3	15.2
4	RB last – QD central	103	-0.82	-0.25	-30.3	26.2
		+/- 5%	+/- 6%	+/- 9%	+/- 30%	+/- 44%

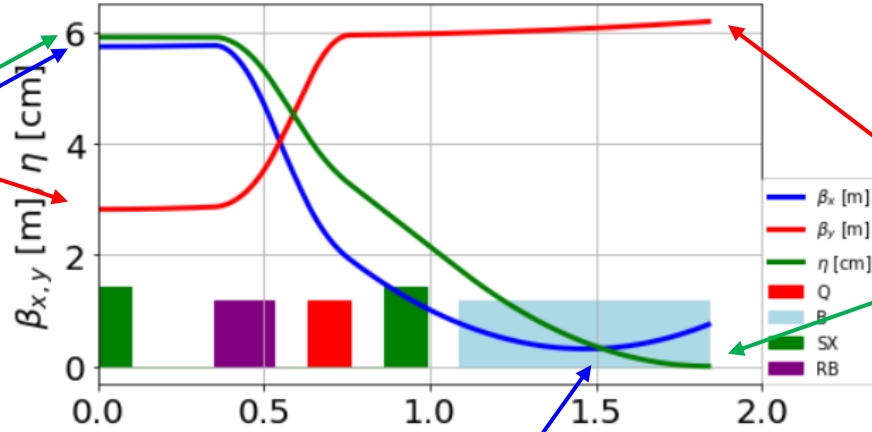
UC completely analyzed => 'unique' solution!!!

Magnet permutations have a moderate impact on emittance and chromaticity, but significant impact on the sextupole strength!

Dispersion Suppression Cell:

- Guideline: As close as possible to half unit cell – to keep phase matching, ε
- Fitting: use RB, QD, RB-angle, B-length, drift
=> Deviation from 'generic' setup
- Mismatch in dispersion limits angle distribution, $\sim 4.2\text{-}4.3^\circ / 2.85\text{-}2.65^\circ$
- Gradient bend helpful for fitting

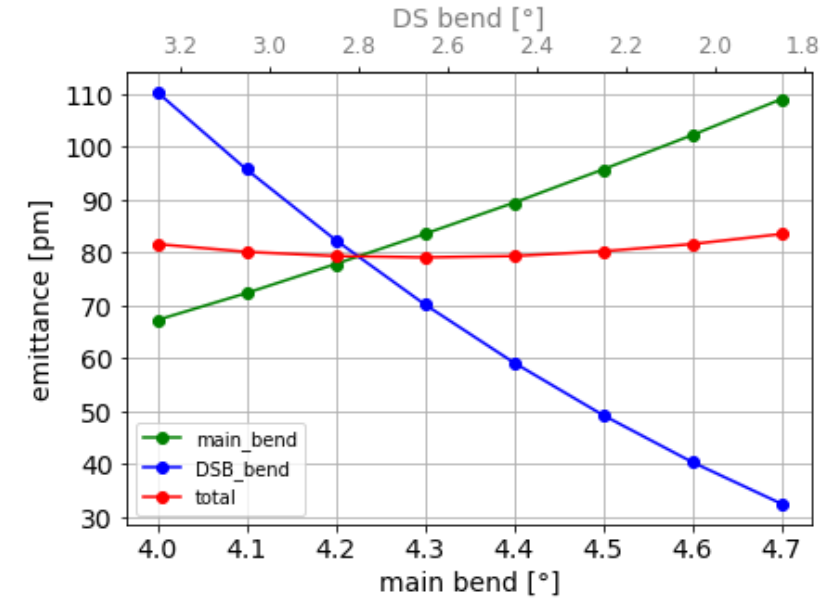
$\beta_{x,y}, \eta$, set by UC



β_{min} 40% inside dipole for minimal emittance

α_y close to zero

η, η' zero



Emittance of MB and DSB for different angle distributions. TME in main bend, 0.8T, and optimal positioning of $\beta_x = 0.1$ in DSB.

Phase matching for HOA:

- *unit cell* : $v_{x,y} * (n+1) = N$,
n: no UCs, N: integers
- *super period*: $\varphi_{x,y} * p = M$
p: no. super periods, M: integer

Quadrupoles in straight need to fit:

- $\beta_{x,y} = 3.0\text{m}$
- $\alpha_{x,y} = 0.0\text{m}$
- phase advance

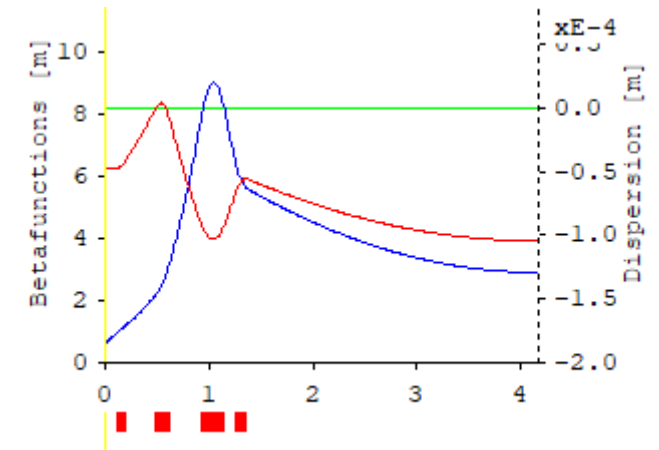
Phase advance closely related to $\beta_{x,y}$
=> 4 quadrupoles

φ_x	M		φ_y	p_y
2.69	43		0.75	12
2.75	44		0.81	13
2.81	45		0.87	14
2.87	46		0.94	15

Matching to the straight section:

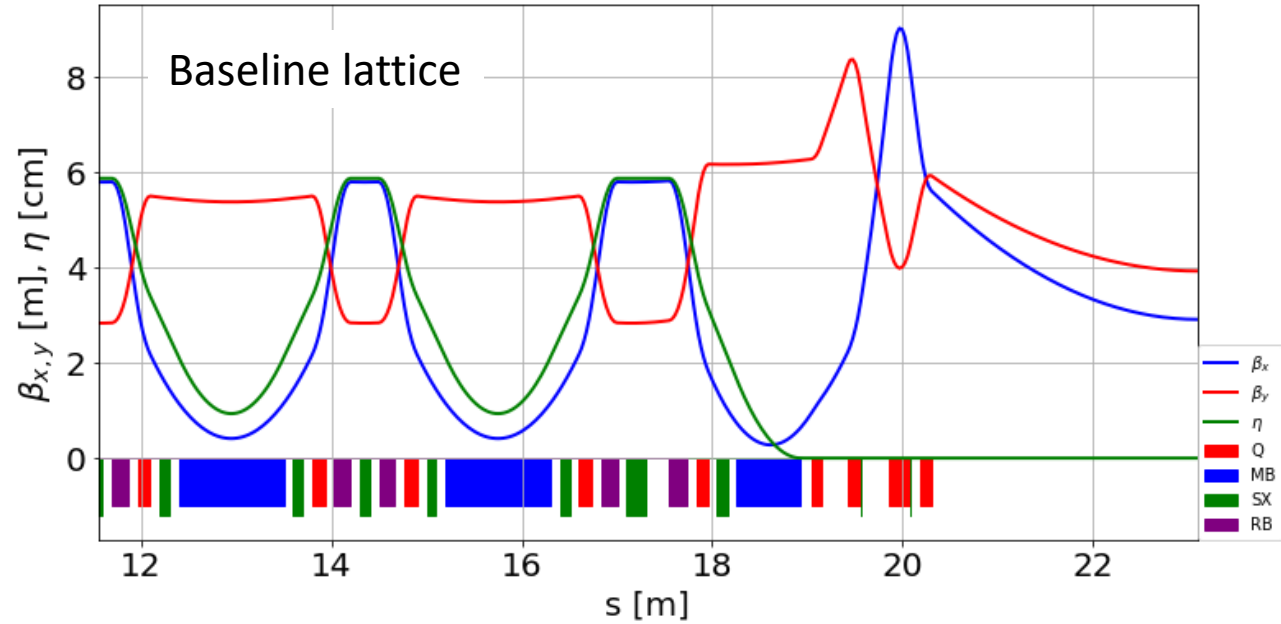
- Offline script: scan 4 gradients and 4 drifts and select the appropriate solution
- Chose tune close to HOA-condition 43.72, 12.78
- fit $\alpha_{x,y} = 0$, lattice tune => small mismatch in $\beta_{x,y}$

=> lattice complete
incorporate technical limitations



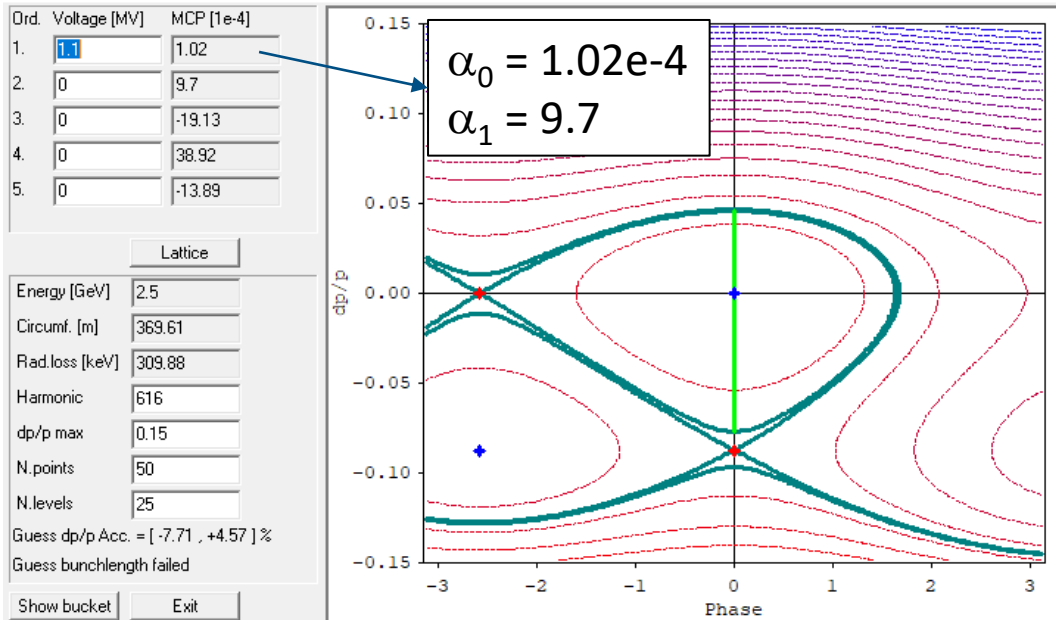
Technical design limits (conservative)

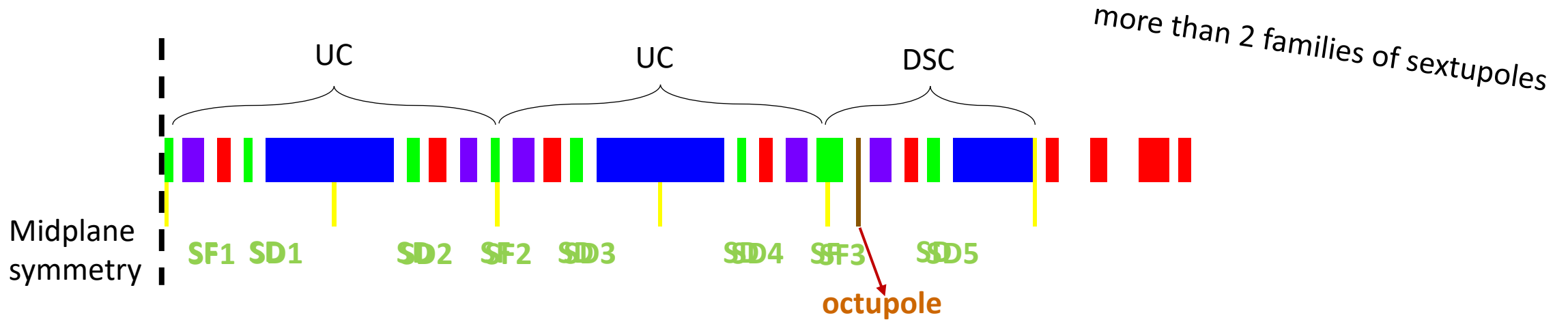
Dipoles	1.4T
Com. func. dipoles	0.8T, 15T/m & 30T/m
Quadrupoles	80T/m (less for RB)
Sextupoles	4000T/m ²
Drifts	> 100mm
Bore diameter	25mm (18mm inner vacuum)



SF-lattice

	2 SX
MB, RB, DSB [°]	4.3, -0.3, 2.65
Circumference [m]	369.6
Emittance [μm]	98
Mom. Comp.	1.03e-4
Mom. Acceptance, RF @ 1.2MV [%]	3.8, 4.7
Dyn. Aperture x,y [mm]	2, 6
OPA-Touschek lifetime, 2% coupling, 300mA [h]	5





Splitting up sextupole families:

- Sextupoles in DSC most important
- Single octupole reduces 2nd order vertical chromaticity
- Optimization of sextupoles by opa -> non-linear panel
- No harmonic sextupoles

SF-lattice	2 SX	8 SX, 1 Oct.
Main bend, RB [°]	4.3, -0.3	4.3, -0.3
Circumference [m]	369.6	369.6
Emittance [pm]	98	98
Mom. Comp.	1.03e-4	1.03e-4
Mom. Acceptance, RF [%] @ 1.2MV	3.8, 4.7	4.8, 4.7
Dyn. Aperture x,y [mm]	2, 6	3.2, 4.5
OPA-Touschek lifetime [h], 2% coupling, 300mA	5	11

Normalized Emittance

	ϵ [μm]	E [GeV]	ϵ/E^2 [$\mu\text{m}/\text{GeV}^2$]
MAX 4	336	3.0	37.3
SLS 2	123	2.4	21.3
Soleil 2	81	2.75	10.7
BESSY III	98	2.5	15.8

Dynamic aperture ($\Delta Q_{x,y} < 0.1$)

	x, y [mm]	$\beta_{x,y}$ [m]	acceptance [mm mrad]	no. sx-families	$\Sigma (b_3 * L)^*$ [1/m ²]	no. octupoles
MAX 4	15.3, 6.2	9.3, 4.8	26.2, 8.0	5	5180	3
SLS 2	3.0, 4.4	3.1, 3.3	2.9, 5.8	9	8148	8
Soleil 2	4.6, 1.4	11.5, 3.2	1.8, 0.6	16	20278	12
BESSY III	3.2, 4.5	2.9, 3.9	3.5, 5.2	8	5761	1

*: only chromatic sextupoles counted

Momentum acceptance ($\Delta Q_{x,y} < 0.1$)

	$\Delta p/p$ [%]	RF [MHz]	RF-acc. [%]	Touschek (literature) [h]	opa 2% coupling, 300mA
MAX 4	3.6	100	7.35	30	15
SLS 2	5.5	500	5.7	4	6
Soleil 2	2.4	352.2	8.1	2.7	2
BESSY III	4.8	500	4.7		11

The concept is ok!

- Lattice compares well to other projects
- Fewer non-linear elements needed

Data taken from opa-runs of lattices supplied by colleagues and from publications during LEL 2022 - 3rd Workshop on Low Emittance Lattice Design 26-29 June 2022 , ALBA, Barcelona

0:Manual optimization minimizing driving terms with *opa***A:**

Andrea Santa-Maria Garcia, KIT: use **Bayesian Optimization** to find best setting of chromatic SX and optionally, add harmonic SX and octupoles.

Optimization criterium: maximization of 3D-volume (x_{\max} , y_{\max} , $\Delta p/p_{\max}$)

B:

Automized minimization of Resonant Driving Terms, RDTs

- Small RDTs => tune is confined for larger amplitudes and momentum offset => large apertures and long lifetime
- $\beta_{x,y}$, η_x and $\mu_{x,y}$ – linear lattice properties
- Optimize sextupoles without further lattice calculations
- Minimization of local/global driving terms?*

*Jiajie Tan et al., Minimizing the fluctuation of resonance driving terms in dynamic aperture optimization, PHYS. REV. AB 26, 084001 (2023)

Strategy to set the sextupoles?

5.1.1 First Order Chromatic Terms

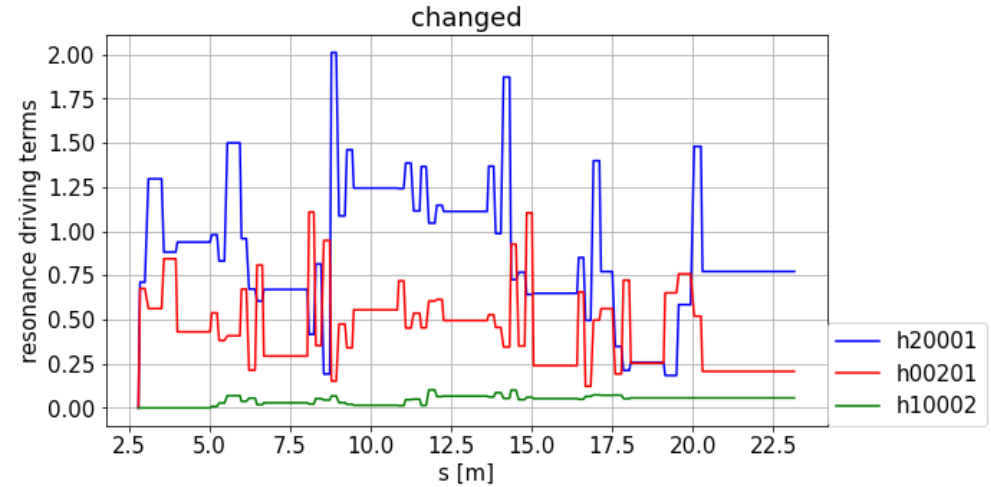
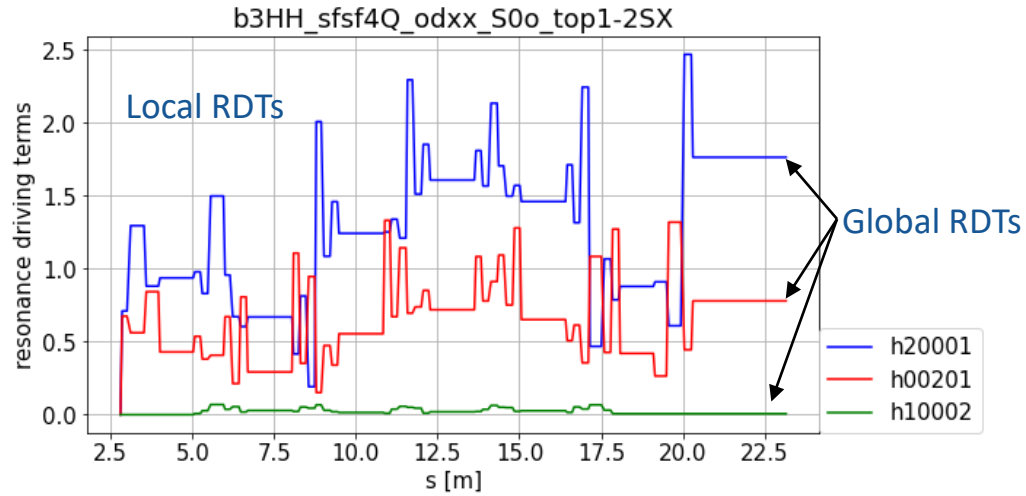
J. Bengtsson, SLS note 9/97

$$\begin{aligned}
 h_{20001} &= h_{02001}^* = \frac{1}{8} \sum_{i=1}^N [(b_2 L)_i - 2(b_3 L)_i \eta_{xi}^{(1)}] \beta_{xi} e^{i2\mu_{xi}} + O(\delta^2), \\
 h_{00201} &= h_{00021}^* = -\frac{1}{8} \sum_{i=1}^N [(b_2 L)_i - 2(b_3 L)_i \eta_{xi}^{(1)}] \beta_{yi} e^{i2\mu_{yi}} + O(\delta^2), \\
 h_{10002} &= h_{01002}^* = \frac{1}{2} \sum_{i=1}^N [(b_2 L)_i - (b_3 L)_i \eta_{xi}^{(1)}] \eta_{xi}^{(1)} \sqrt{\beta_{xi}} e^{i\mu_{xi}} + O(\delta^3) \quad (96)
 \end{aligned}$$

5.1.2 First Order Geometric Terms

$$\begin{aligned}
 h_{21000} &= h_{12000}^* = -\frac{1}{8} \sum_{i=1}^N (b_{3i} L) \beta_{xi}^{3/2} e^{i\mu_{xi}}, \\
 h_{30000} &= h_{03000}^* = -\frac{1}{24} \sum_{i=1}^N (b_{3i} L) \beta_{xi}^{3/2} e^{i3\mu_{xi}}, \\
 h_{10110} &= h_{01110}^* = \frac{1}{4} \sum_{i=1}^N (b_{3i} L) \beta_{xi}^{1/2} \beta_{yi} e^{i\mu_{xi}}, \\
 h_{10020} &= h_{01200}^* = \frac{1}{8} \sum_{i=1}^N (b_{3i} L) \beta_{xi}^{1/2} \beta_{yi} e^{i(\mu_{xi} - 2\mu_{yi})}, \\
 h_{10200} &= h_{01020}^* = \frac{1}{8} \sum_{i=1}^N (b_{3i} L) \beta_{xi}^{1/2} \beta_{yi} e^{i(\mu_{xi} + 2\mu_{yi})} \quad (97)
 \end{aligned}$$

Work in progress!



Minimization in local and global RDTs
insufficient to enlarge mom. acceptance
=> dominated by second order

PhD work @ HZB: Michael Arlandoo:

$$a_{xx} = -4/\pi \left(-6 \int_0^C ds \operatorname{Im}[3G_{0300}(s)h_{3000}(s) + G_{1200}(s)h_{2100}(s)] |2200\rangle + 6 \operatorname{Im}(3h_{3000}f_{0300} + h_{2100}f_{1200}) |2200\rangle \right)$$

↙ ↘
↙ ↘

local driving term
global driving terms

Gjklm : individual sextupole contribution to driving terms

$$f_{jklm} : h_{jklm} / (e^{i\Gamma} - 1), \Gamma = 2\pi[(j-k)Q_x + (l-m)Q_y]$$

To-do list:

- Incorporate engineering demands into generic lattice without losing performance
- Develop a *strategy* to optimize sextupoles, octupoles if necessary, hopefully analytically
- Special injection insert necessary?
- Error, lifetime, commissioning studies for first lattice candidate

} Lattice candidate

Summary:

The deterministic approach makes much sense:

- Insight into the functionality of lattice elements
- RBs more beneficial than combined function magnets
- Different distribution of bending angles helps
- Minimization of sextupole strength eases non-linear compensation
- All design parameters can be met with a minimum of non-linear elements
- Based on thorough investigations, the BESSY III lattice will be unsophisticated, but competitive in the community

Thank you for your attention!

Lattice design team: *Paul Goslawski, Bettina Kuske*: lattice development (tweak OPA*)
Michael Abo-Bakr: elegant, error studies, injection, special topics
Johan Bengtsson: non-linear optimization (HOA), *Thor-scsi/TRACY*, digital twin
Michael Arlandoo (PhD): Tribs@BESSY III, non-linear optimization, *Xsuite*

*: OPA, Lattice Design Code by A. Streun, PSI, <https://ados.web.psi.ch/opa/>

** : Xsuite, CERN initiative, <https://xsuite.readthedocs.io/en/latest/>

IPAC 2021

M. Arlandoo et al., "A first attempt at implementing TRIBs in BESSY III's design lattice", IPAC21, THPOPT003

J. Bengtsson et al., "Robust Design and Control of the Nonlinear Dynamics for BESSY-III", IPAC21, MOPAB048

P. Goslawski et al., "BESSY III & MLS II - Status of the Development of the New Photon Science Facility in Berlin", IPAC21, MOPAB126

B. Kuske, "Towards Deterministic Design of MBA-Lattices", IPAC21, MOPAB220

IPAC 2022

J. Bengtsson et al., "Robust design of modern Chasman-Green lattices - a geometric control theory approach", IPAC2023, WEPL037

P. Goslawski et al., "BESSY III Status Report and Lattice Design Process", IPAC22, TIPOMS010

B. Kuske et al., "Basic Design Choices for the BESSY III MBA Lattice", IPAC22, MOPOTK009

LEL-workshop 2022, ALBA, Barcelona, Spain

P. Goslawski, et al. talk, ""

B.C. Kuske, talk, "Deterministic design of multi bend HOA lattices"

IPAC 2023

M. Arlandoo et al., "Further investigations of TRIBs in BESSY III design MBA lattices", IPAC2023, WEPL109

P. Goslawski et al., "Update on the lattice design process of BESSY III: towards a baseline lattice", IPAC23, WEPL036

P. Goslawski et al., "BESSY III - status and overview", IPAC23, MOPA174

B.C. Kuske et al., "Further aspects of the deterministic lattice design app. for BESSY III", IPAC23, WEPL039

FLS-workshop 2023, Luzern, Switzerland

P. Goslawski et al., talk, "The BESSY III Lattice"

B.C. Kuske, P. Goslawski, "Deterministic Lattice Design approach for BESSY III", FLS2023, WE4P31

iFAST-Low Emittance Workshop 2024, CERN, Switzerland

P. Goslawski et al., talk, "The BESSY III Lattice"

B.C. Kuske, P. Goslawski, "Deterministic Approach to MBA Lattice Design"

Emittance is given by:
$$\varepsilon = \frac{C_q \gamma^2}{J_x} \int_0^C \frac{H(s)}{|\rho(s)|} ds$$

$$H(s) = \beta_x \eta'^2 + 2 \alpha_x \eta' \eta + \gamma_x \eta^2$$

γ Lorentz factor
 $Cq = 3.83e-13$ m quantum excitation number
 J_x damping partition number
 ρ dipole bending radius

For equal dipoles

$$\varepsilon \propto |1/\rho| * \langle H \rangle \propto |\theta| * \langle H \rangle$$

Many dipoles to reduce θ /dipole

For a homogeneous dipole and symmetric β -functions
 ($\alpha_x = \eta' = 0$ at dipole center)

$$\frac{1}{L\theta^2} \langle H \rangle = \frac{L}{\beta_0} \left[\left(\frac{\eta_0}{L\theta} \right)^2 - \frac{1}{3} \left(\frac{\eta_0}{L\theta} \right) + \frac{1}{20} \right] + \frac{1}{3} \frac{\beta_0}{L}$$

A. Streun, B. Riemann, PHYS. REV. AB 22, 021601 (2019)

$$\beta_0 = \frac{L}{\sqrt{15}}, \quad \eta_0 = \theta \frac{L}{6}, \quad \varepsilon \propto \theta^3 \frac{2}{3\sqrt{15}}$$

Theoretical Minimum Emittance - conditions (TME) (single dipole)

$\theta =$ half angle, $L =$ half length

Dipoles will also be long to manage Twiss parameters

Accumulated dipole length:
 BESSY II: 27.36m, 11%
 BESSY III: ~ 100m, 29%

MBAs				Hybrid MBAs			
Ring	Energy [GeV]	lattice	Circum. [m]	Ring	Energy [GeV]	lattice	Circum. [m]
Max4	3.00	7-BA	527.76	ESRF-EBS	6.00	7-H-BA	844
Alba	3.00	6-BA	269.0	Petra IV	6.00	6-H-BA	2300
Soleil	2.75	4-BA/7-BA	354.0	APS	6.00	7-H-BA	1100
Sirius	3.00	5-BA	518.25	HEPS	6.00	7-H-BA	1360
SLS 2	2.40	7-BA	288.0	ALS-U	2.00	9-H-BA	196.5
Elettra	2.40	6-BA	259.2				
Diamond	3.50	2*TBA	560.574				
Taiwan	3.00	5-4-4-5BA	518.4				

Low-energy rings tend to have problems with momentum acceptance/Touschek lifetime with Hybrid MBAs
=> start with a **regular MBA lattice**.

Thorsten Hellert, ALS:

Commissioning tool, based on MML, used for PETRA IV and ALS-U

LEL 2022 - 3rd Workshop on Low Emittance Lattice Design, Th. Hellert,
 "Toolkit for simulated commissioning"

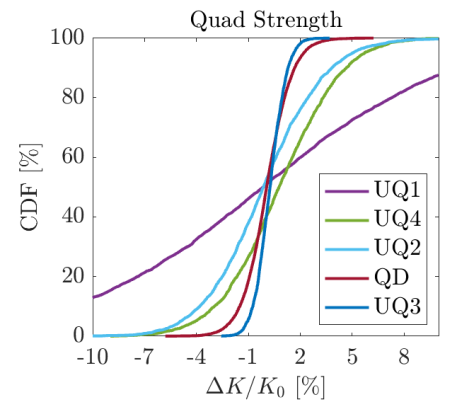
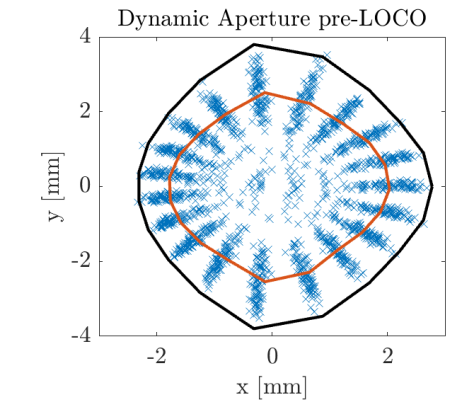
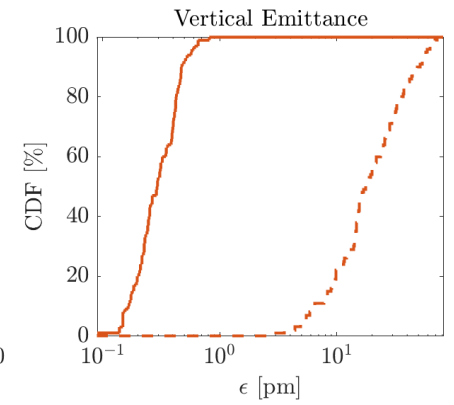
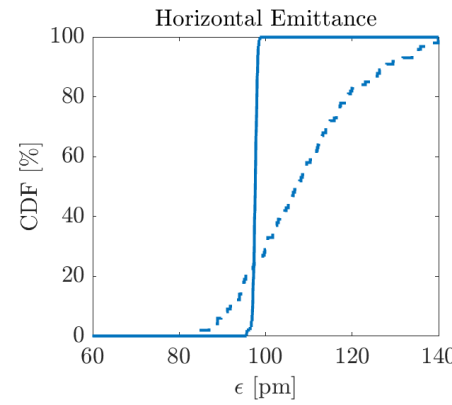
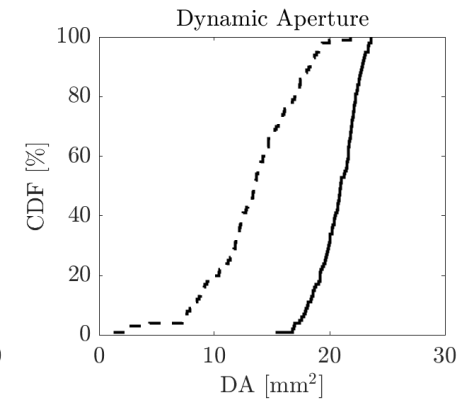
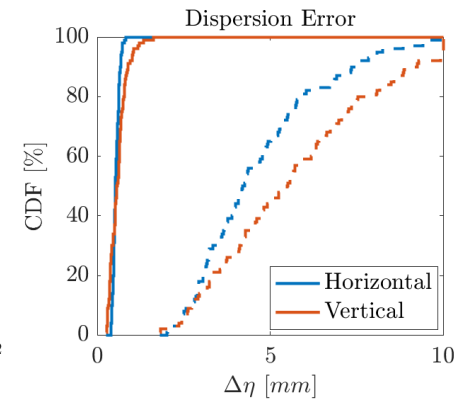
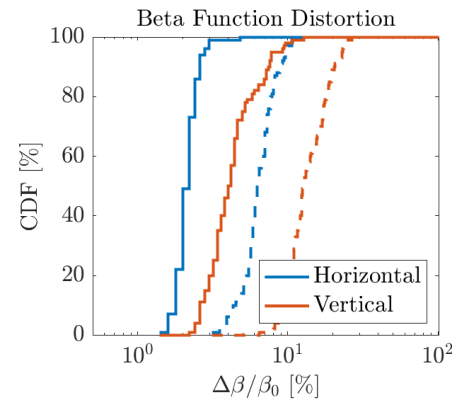
Very preliminary results!

Input

- 2 families of sextupoles
- Error model of ALS-U (100 seeds)
- Beam threading, RF commissioning, trajectory correction, BBA
- 6D tracking

Results

- No show-stoppers found – easy start-up
- Lattice distortions can be well-recovered
- Minimal dyn. Aperture degradation due to errors

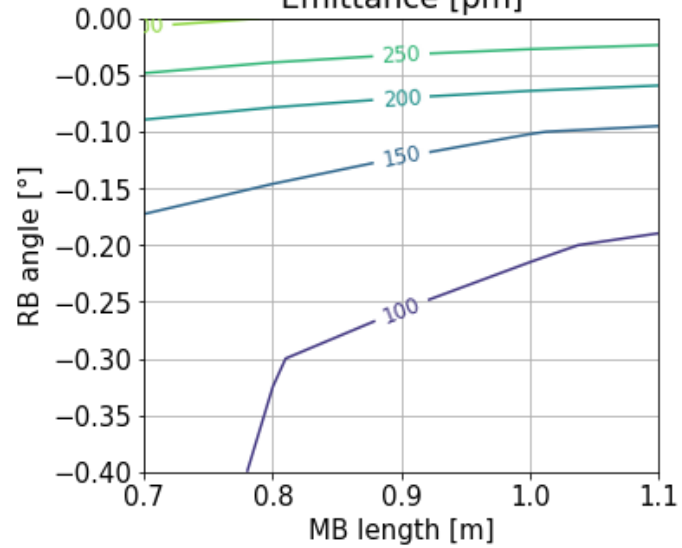


CDF: cumulated distribution function

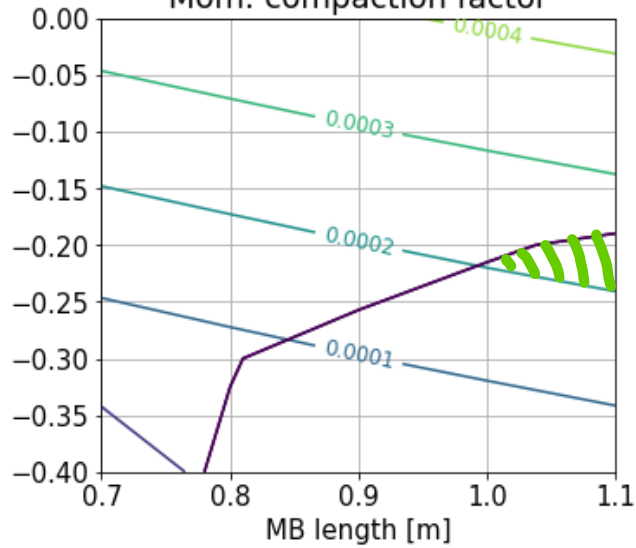
Optimal MB length, MB angle and RB angle depend on each other

MB angle = 4.3°

Emittance [pm]

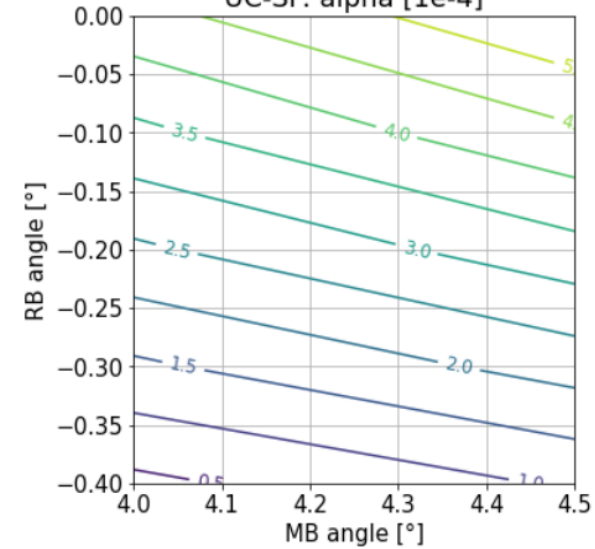


Mom. compaction factor

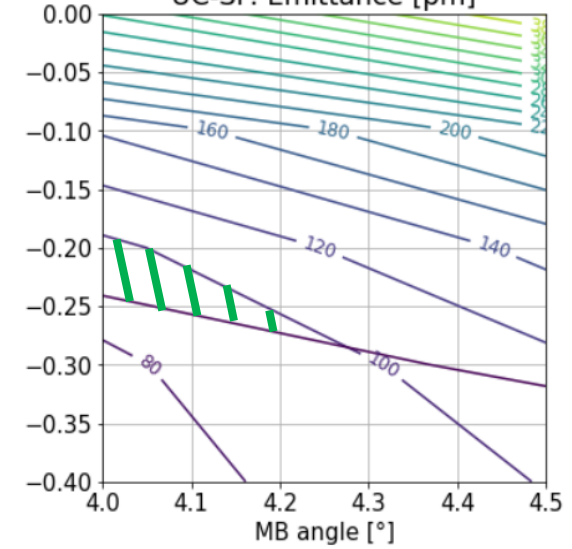


MB length = 1.1m

UC-SF: alpha [1e-4]



UC-SF: Emittance [pm]



$360^\circ / 16 / 5 = 4.5^\circ \Rightarrow \text{MB} = 4.5^\circ, \text{DSB} = 2.25^\circ$
 Even angle division not necessarily optimal, would need longer dipoles
 $\Rightarrow \text{MB} = 4.3^\circ, \text{RB} = -0.3^\circ, \text{DSB} = 2.65^\circ$

Dipole sources:

BESSY II: 0.855m, ~ 1.3 T => critical photon energy of **2.5 keV**

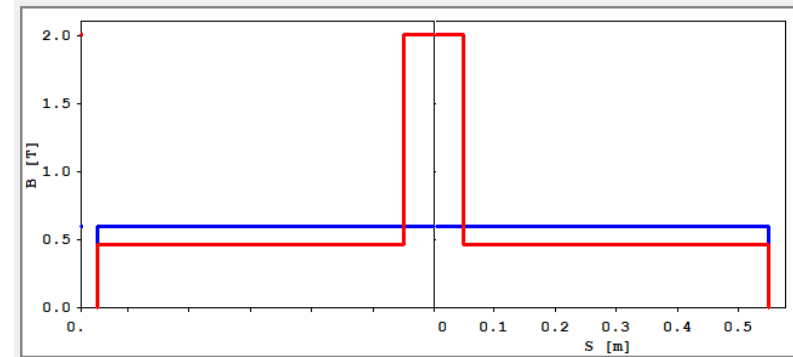
BESSY III: 1.1m, ~ 0.669 T => critical photon energy of **2.78 keV**, 1.3 T => **5.5 keV**, 2.0 T => **8.3 keV**

2.0T bend might replace superconducting WLS?

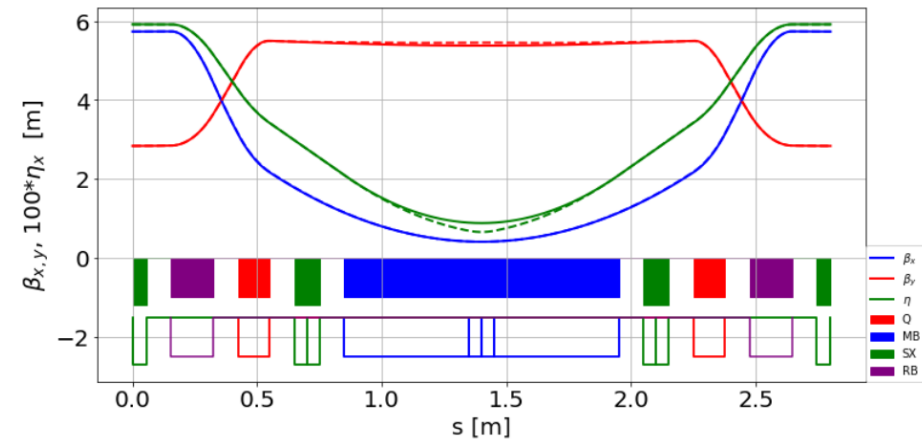
Solution: longitudinal gradient bend

- Different field strength in ‘one’ dipole
- Short 2T insert at center, 10cm
- Outer field 0.462T (0.669T)
- Same bending angle and length as main bend
- Fit Twiss functions at exit
- Fit $\beta_{x,y}$ using QD, RB-gradient, η_x using RB-angle
 - “plug-in solution”

2T dipoles	0	1 in ring	1 per arc
emittance	97	97	101
alpha	0.95e-4	0.95e-4	0.88e-4



Field profile of longitudinal gradient bend



Longitudinal gradient bends are often used to lower the emittance => $\beta_{x0}, \eta_{x0} \propto L$ => strong disturbance of Twiss parameters