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# Nonlinear dynamics analysis and optimization in low-emittance rings by minimizing the fluctuations of resonance driving terms along the ring

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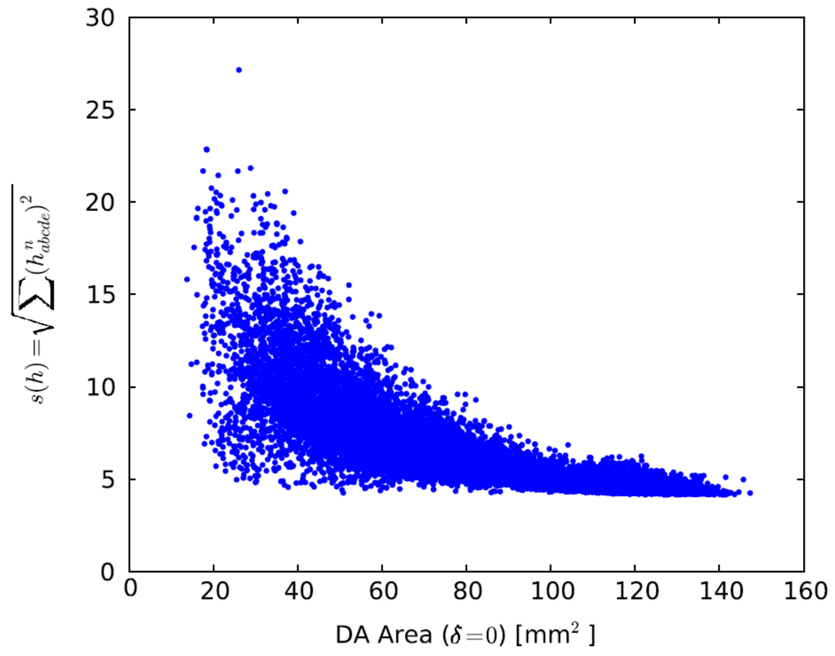
# Outline

- Introduction
- Analysis of on-momentum dynamics
- Analysis of off-momentum dynamics
- Preliminary optimization
- Conclusion

# Dynamic aperture optimization

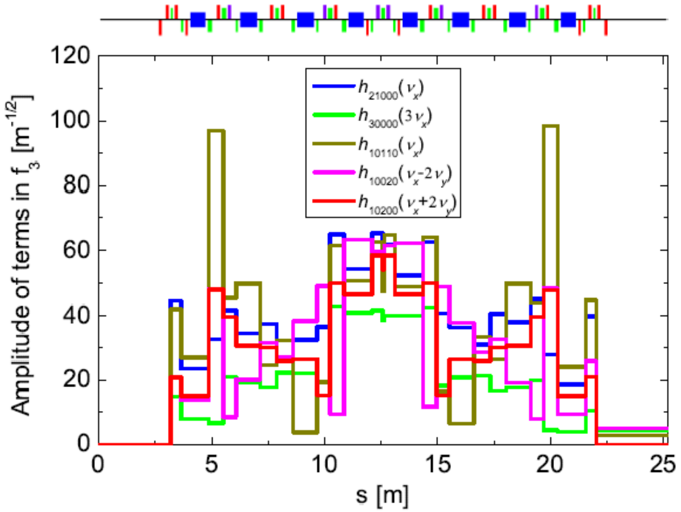
- ➢ Two widely-used approaches for dynamic aperture (DA) optimization
  - ① Minimization of resonance driving terms (RDTs)
  - ② Tracking based direct optimization with evolutionary algorithms
- ➢ The 1st analytical approach:
  - Small RDTs is a necessary but not sufficient condition for large DAs (see the figure);
  - Fast optimization, physical feedback.
- ➢ The 2nd numerical approach:
  - In principle, the largest DA can be found;
  - Very time-consuming, no physical feedback.
- ➢ For better DA optimization:
  - Powerful analytical approach (to be further developed) + powerful numerical approach.



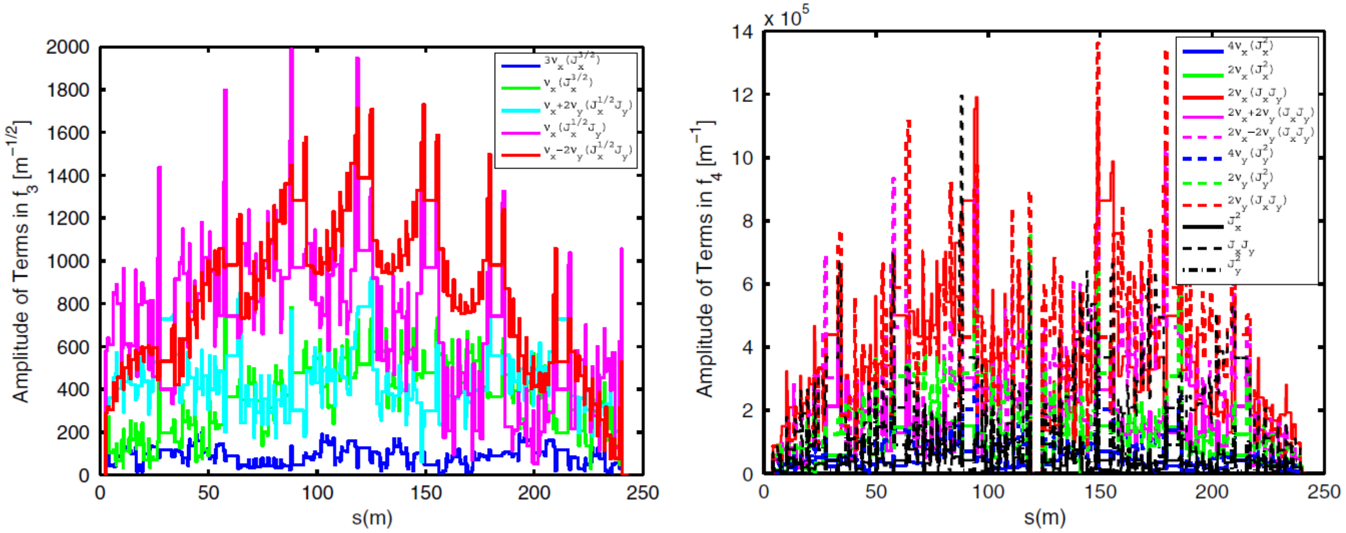

Correlation between DA area and RDTs.
L. Yang, et al., PRST-AB, 14, 054001 (2011).

# Control of the variation / fluctuation of RDTs along the longitudinal position

- Nonlinear dynamics cancellation
  - The nonlinear cancellation within a lattice cell (left figure) is more effective than the cancellation over some cells (right figure) in enlarging the DA.
  - The variation / fluctuation of RDTs along the longitudinal position in the former is smaller than that in the latter, which inspires us that reducing the RDT fluctuations could be very effective in enlarging the DA.



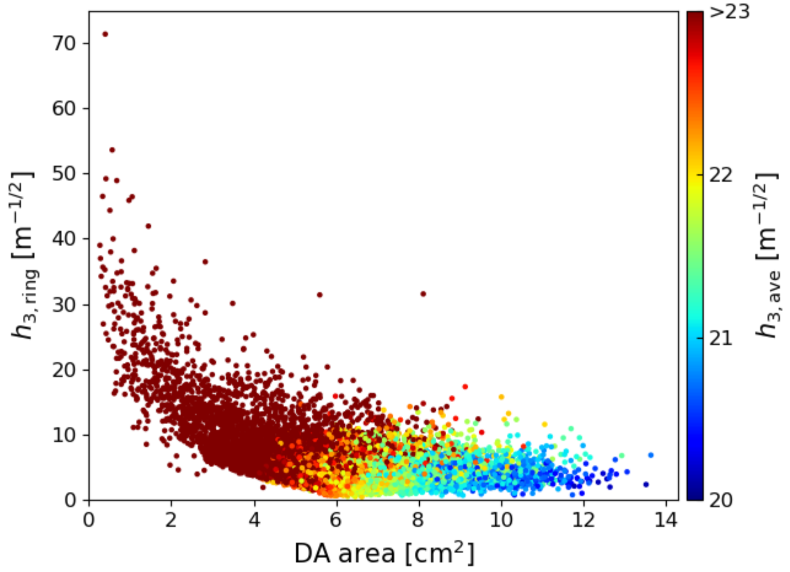

The variation of 3rd-order RDTs along a lattice cell.

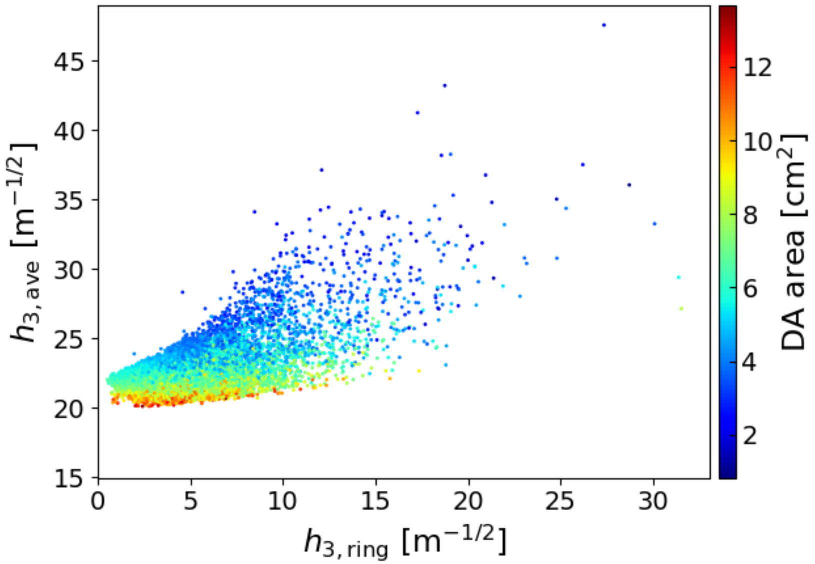



The variation of 3rd- and 4th-order RDTs along 8 lattice cells.
Y. Cai, et al., PRST-AB, 15, 054002 (2012).

# Correlation between RDT fluctuations and DA

- The RDT fluctuations can be quantitatively represented by the average RDTs.
- Lattice example: the SSRF storage ring lattice
- Minimizing RDT fluctuations is much more effective than minimizing the commonly used one-turn RDTs in enlarging the DA. (Each dot is a nonlinear solution in the figures.)

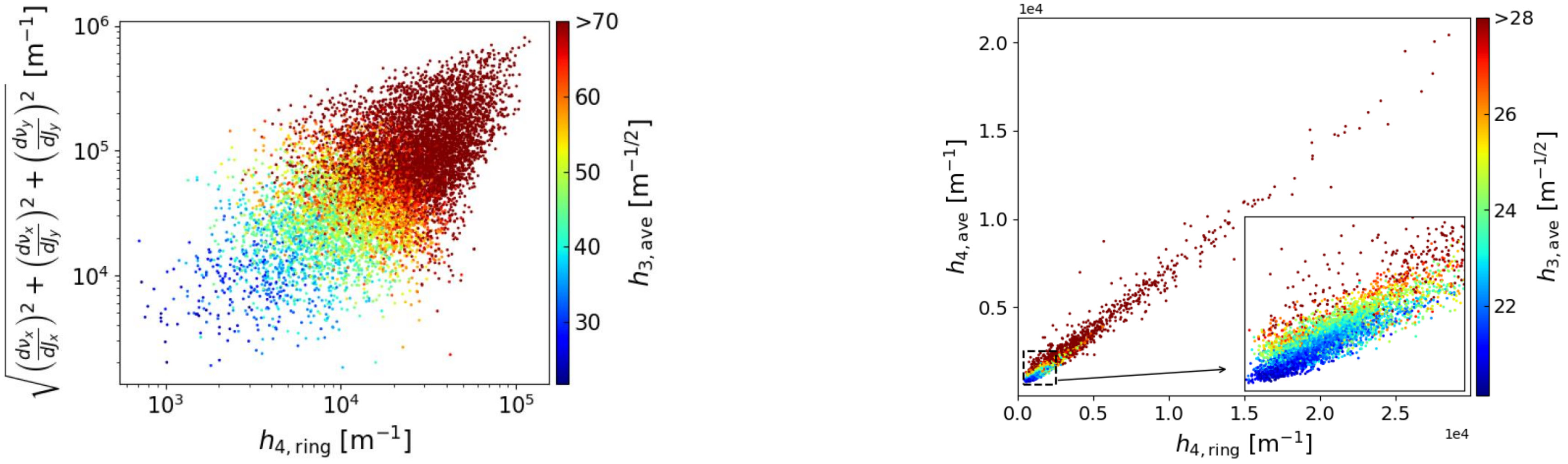



Correlation between DA area, 3rd-order one-turn RDTs ($h_{3,\ ring}$) and 3rd-order RDT fluctuations ($h_{3,\ ave}$) .
B. Wei, et al., PRAB, 26, 084001 (2023).

# Correlation between low- and higher-order RDTs

- As we know, for the commonly used one-turn RDTs, if the 3rd-order one-turn RDTs are smaller, the 4th-order one-turn RDTs can be larger.
- Reducing 3rd-order RDT fluctuations helps to reduce amplitude-dependent tune shifts (ADTS), 4th-order one-turn RDTs and 4th-order RDT fluctuations.

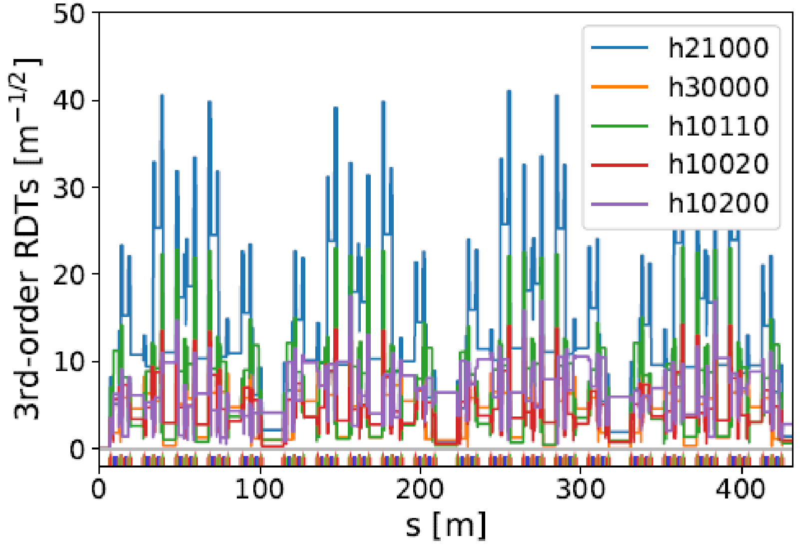


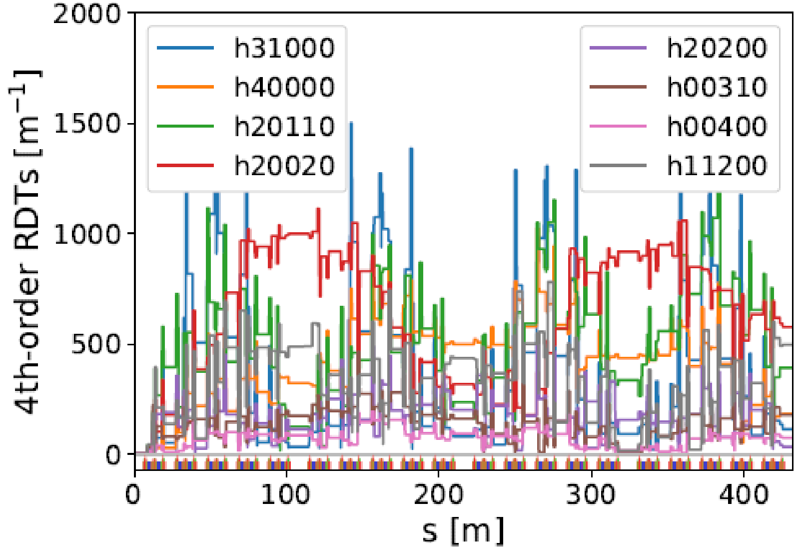

Correlation between 3rd-order RDT fluctuations and ADTS terms & 4th-order RDTs.
B. Wei, et al., WEPL078, IPAC2023. & B. Wei, et al., PRAB, 26, 084001 (2023).

- How about 5th-order RDTs if 3rd-order RDT fluctuations are reduced?
  - The formula for calculating 5th-order RDTs is very complicated.
  - Can be demonstrated using frequency map analysis (FMA).

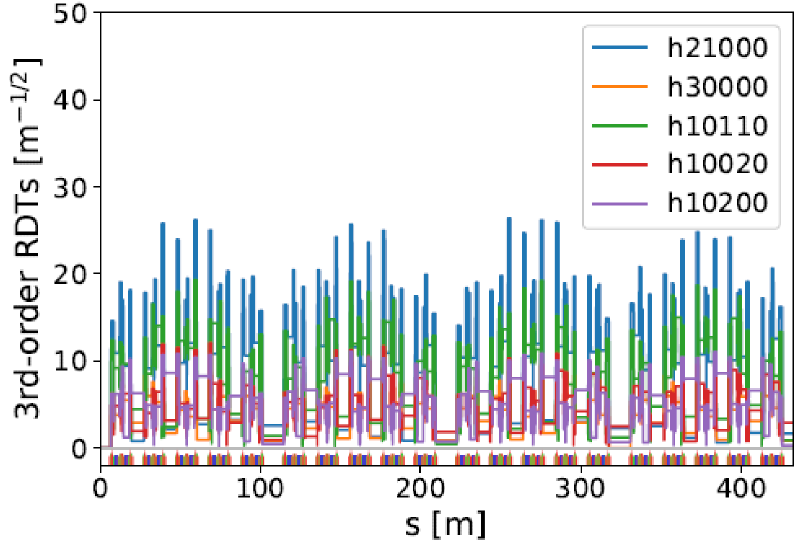
# Correlation between low- and higher-order RDTs

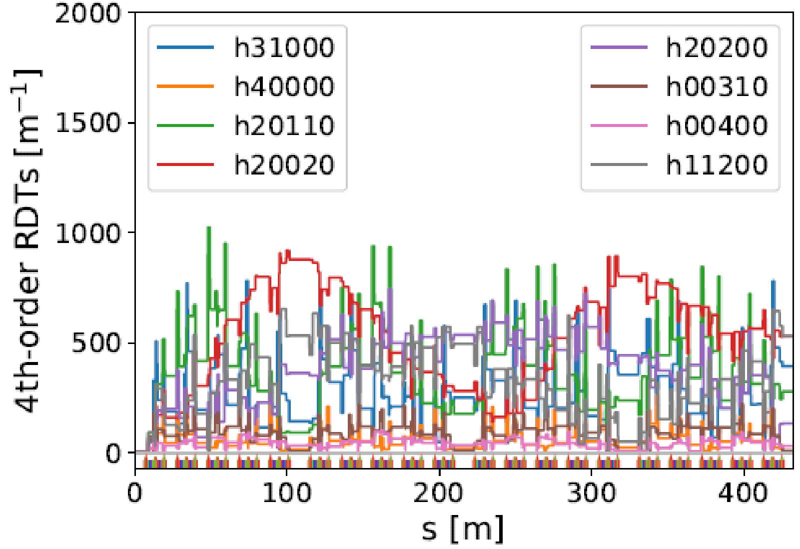
## nonlinear solution with larger RDT fluctuations



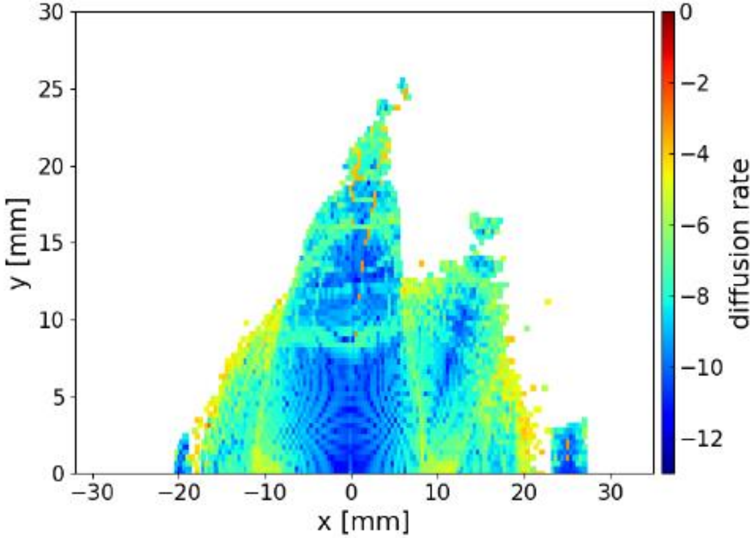

## nonlinear solution with smaller RDT fluctuations

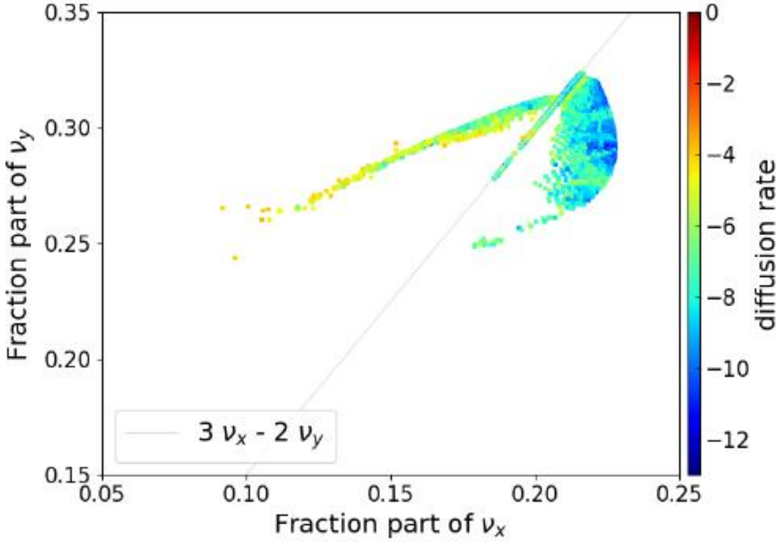


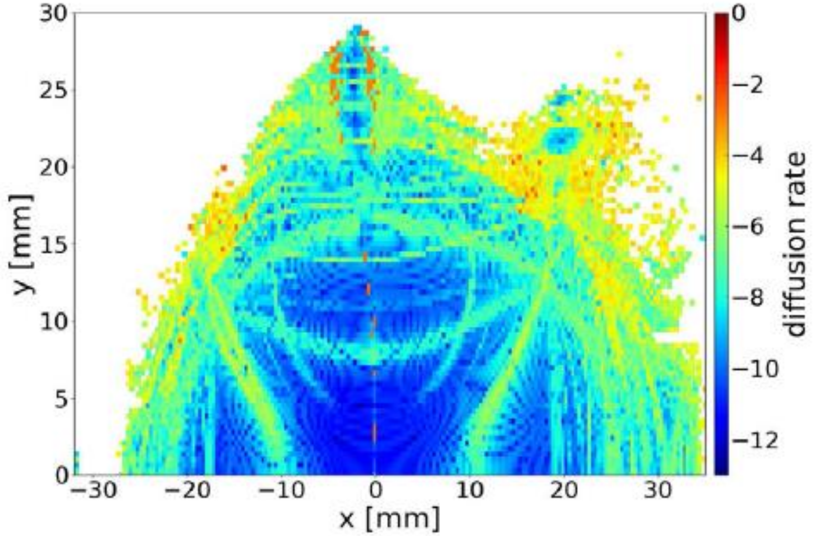
# Correlation between low- and higher-order RDTs

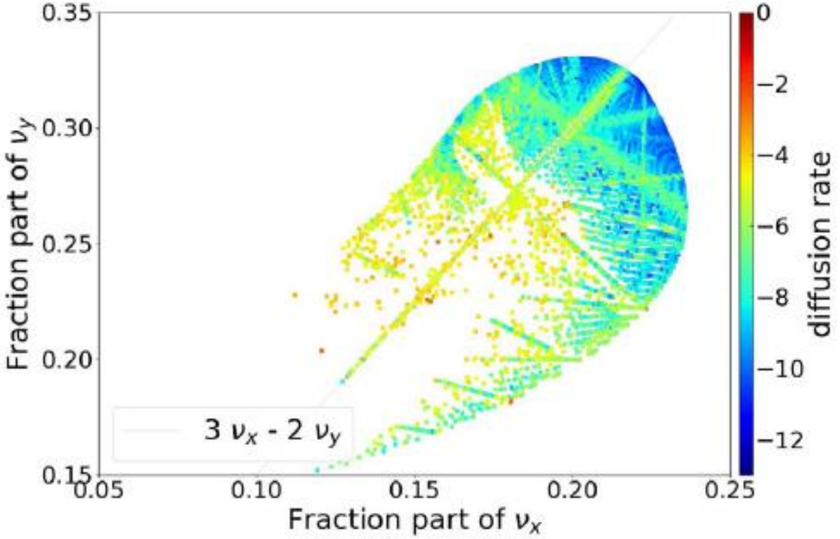
nonlinear solution with larger RDT fluctuations




nonlinear solution with smaller RDT fluctuations




Reducing 3rd-order RDT fluctuations can also help to reduce 5th-order RDTs.

# Physics behind minimizing RDT fluctuations

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Consider a lattice with *N* sextupole kicks, the nonlinear driving terms in the one-turn map can be computed as:

$$e^{:h:} = \Pi_{i=1}^{N} e^{:\hat{V}_i:} \equiv e^{:S_N:}$$

where $\hat{V}_i$ is the $i$-th normalized sextupole kick.

Considering only *t* sextupole kicks, we can obtain the cumulative nonlinear terms resulting from these *t* sextupole kicks: $e^{:S_t:} = \Pi_{i=1}^{t} e^{:\hat{V}_i:}$.

$$e^{:S_t:} = e^{:S_{t-1}:} e^{:\hat{V}_t:}$$

BCH formula:

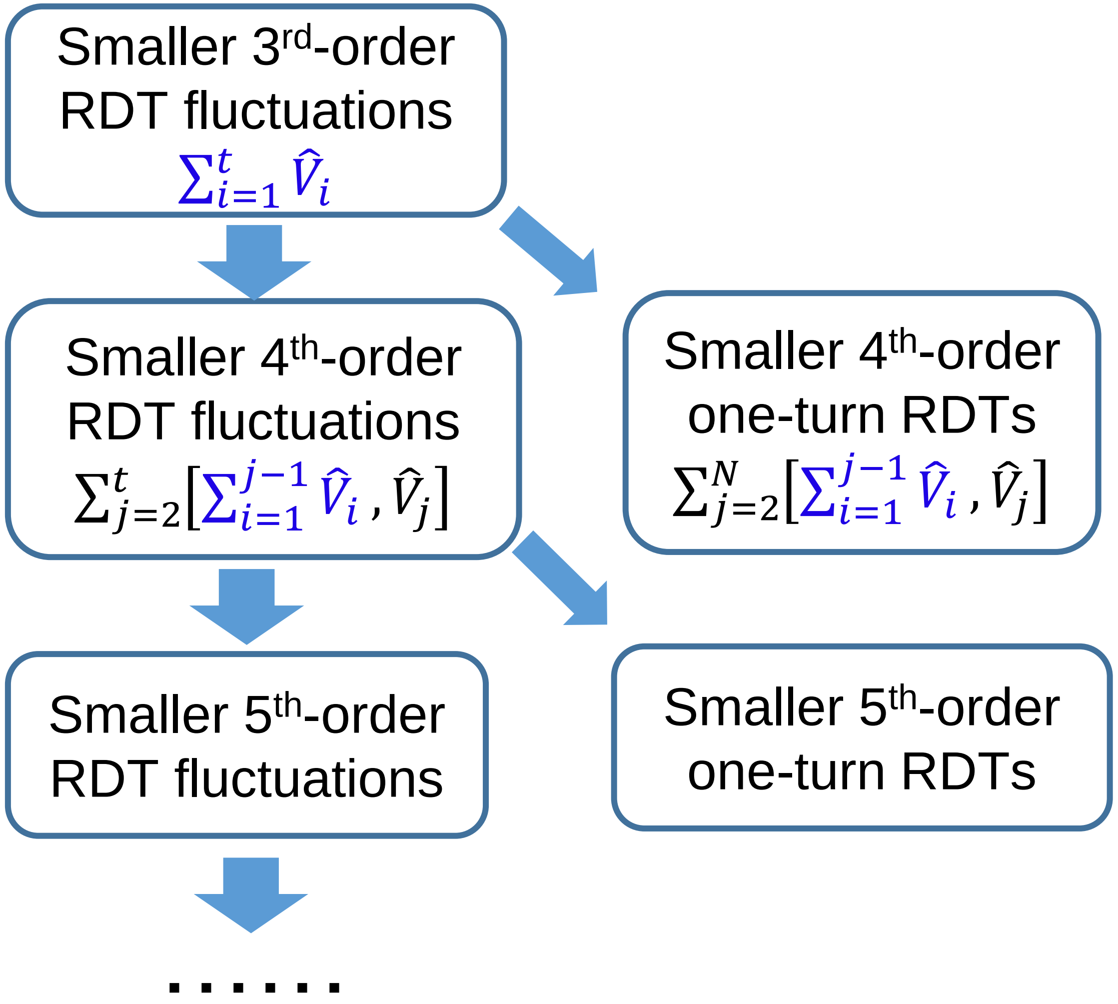
$$S_t = S_{t-1} + \hat{V}_t + \frac{1}{2} :S_{t-1}: \hat{V}_t + \frac{1}{12} :S_{t-1}:^2 \hat{V}_t + \frac{1}{12} :\hat{V}_t:^2 S_{t-1} + \cdots$$

The crossing terms of the cumulative nonlinear terms and the current nonlinear terms result in higher-order nonlinear terms.

# Physics behind minimizing RDT fluctuations

- Reducing low-order RDT fluctuations can help reduce higher-order and even higher-order RDTs.

- Based on minimizing RDT fluctuations, it would not be necessary to calculate higher-order RDTs in DA optimization.
  - For RDTs higher than 4th-order, they are not only more computationally complicated but also more numerous.




Schematic of the correlation between low- and higher-order RDTs.
Z. Bai, et al., TU1B4, FLS2023.

# Analysis of off-momentum dynamics

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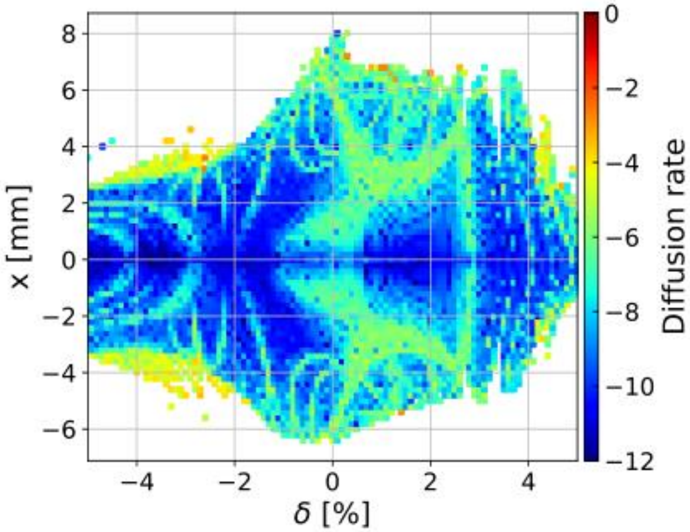
- To analyze off-momentum nonlinear dynamics, the natural idea is to reduce the chromatic RDT fluctuations.
  - We try to minimize the fluctuations of h20001 and h00201 that drive beta function variation with momentum, but it is not so effective.

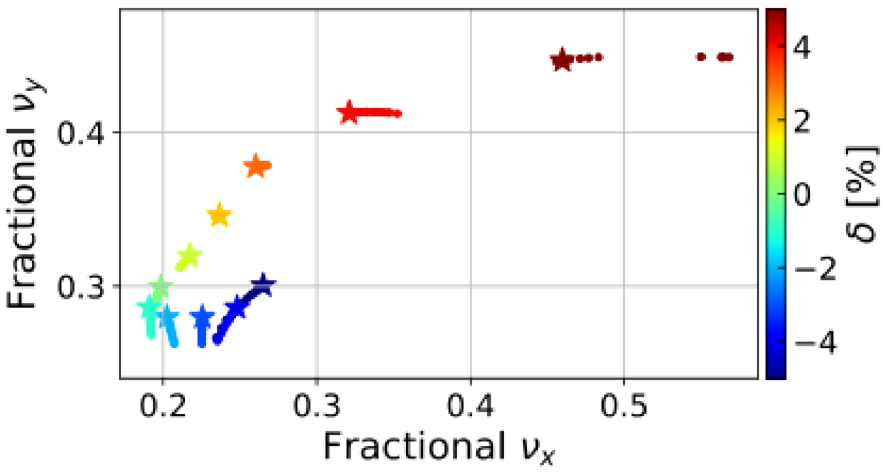
- Instead of using chromatic RDTs, recently we found that using off-momentum nonlinear driving terms (NDTs, including off-momentum RDTs and off-momentum ADTS terms is more direct and more effective in enlarging off-momentum DAs .
  - Calculate off-momentum linear optics;
  - Calculate off-momentum nonlinear driving terms based on off-momentum linear optics.

# Off-momentum ADTS

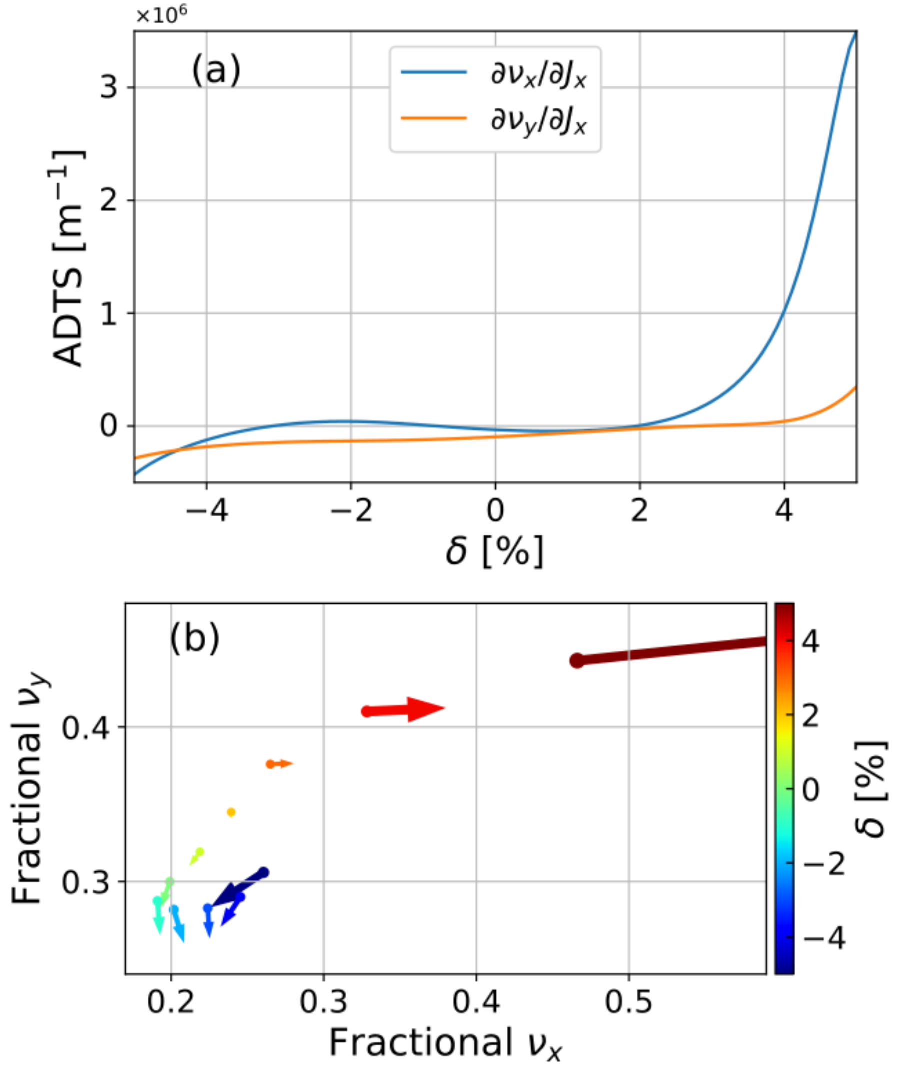
Lattice: SOLEIL II TDR lattice




Off-momentum DAs of the reference solution



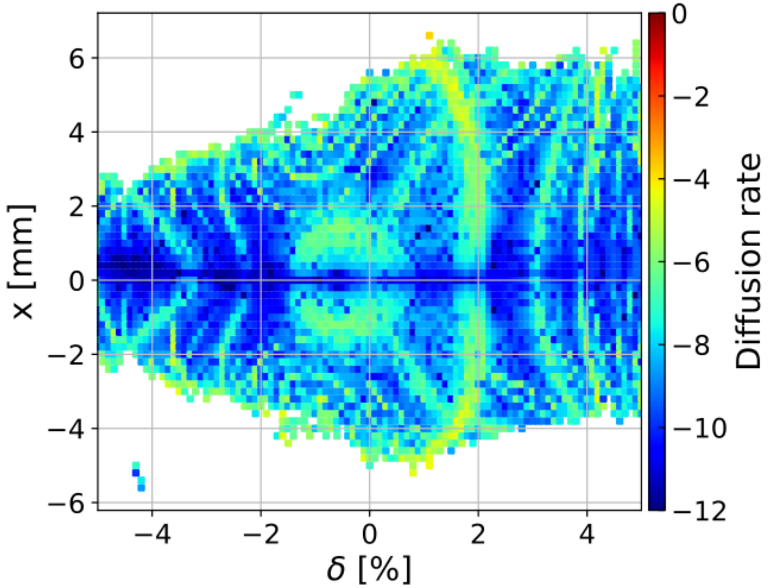

Off-momentum ADTS tracked with FMA
(Stars are tunes of off-momentum particles with x=0, and dots for |x| <= 2 mm.)

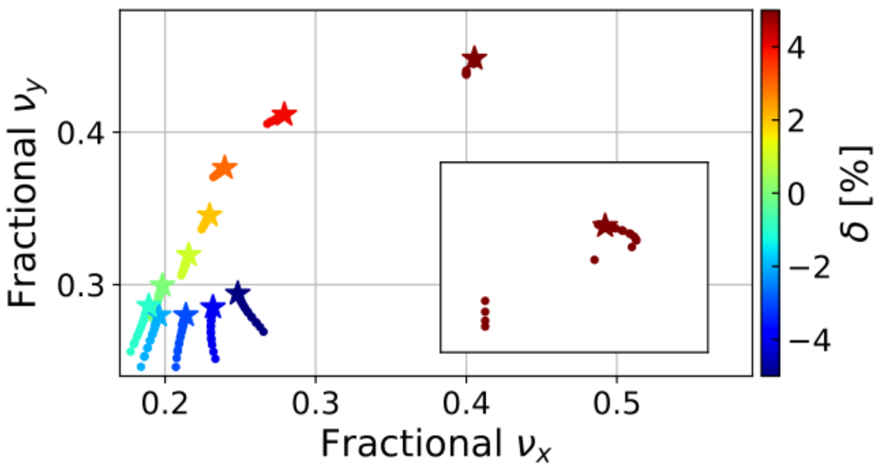


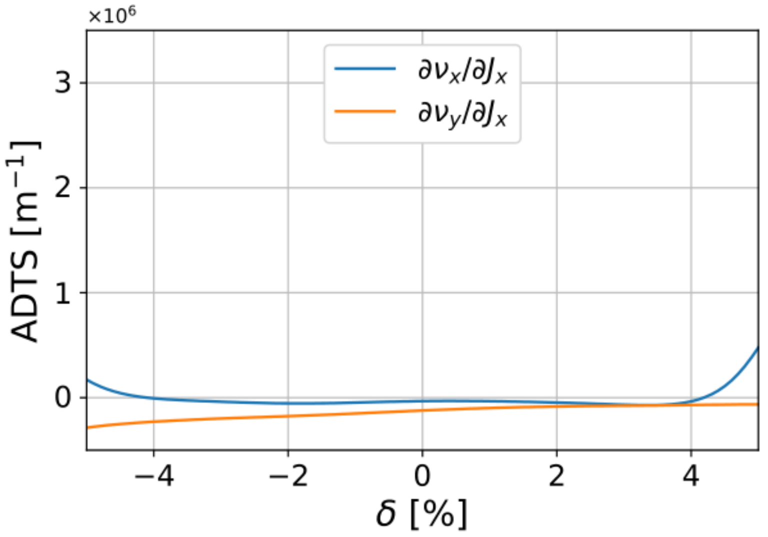

Off-momentum ADTS calculated using NDTs
(Dots are tunes of off-momentum particles on off-momentum closed-orbits (x≠0 when δ ≠0).)

The analyzed ADTS terms are consistent with the FMA result.

# Control of off-momentum ADTS

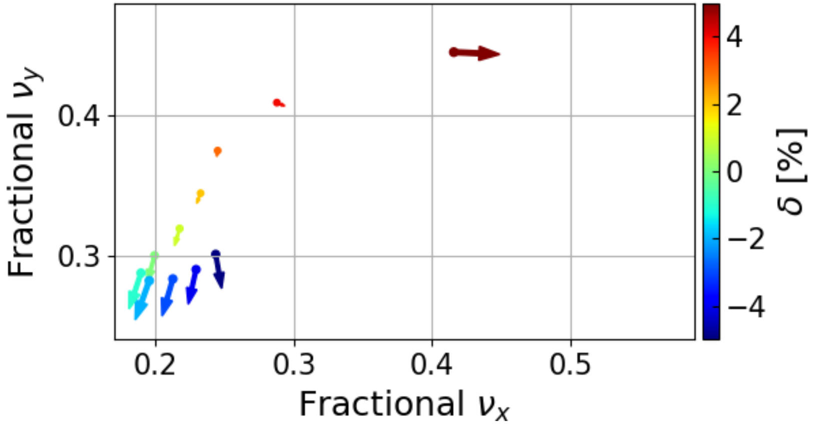



A solution obtained by controlling both off-momentum ADTS at $\delta = 5\%$ and tune shifts with momentum

Off-momentum ADTS tracked with FMA (Stars are tunes of off-momentum particles with x=0, and dots for |x| <= 2 mm.)



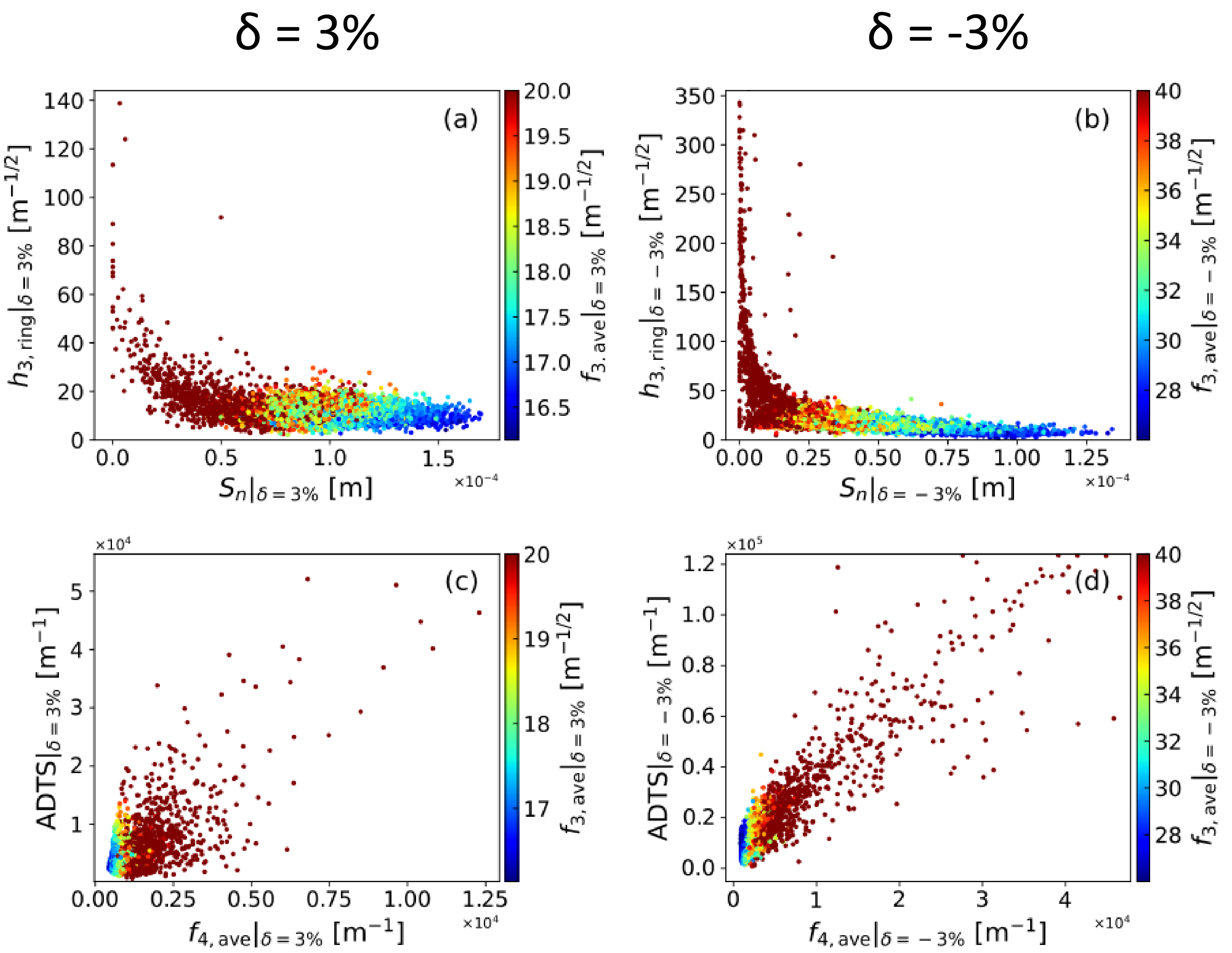

Off-momentum DA at $\delta = 5\%$ is much enlarged.

Off-momentum ADTS calculated using NDTs (Dots are tunes of off-momentum particles on off-momentum closed-orbits (x≠0 when δ ≠0).)

# Control of off-momentum RDTs and their fluctuations

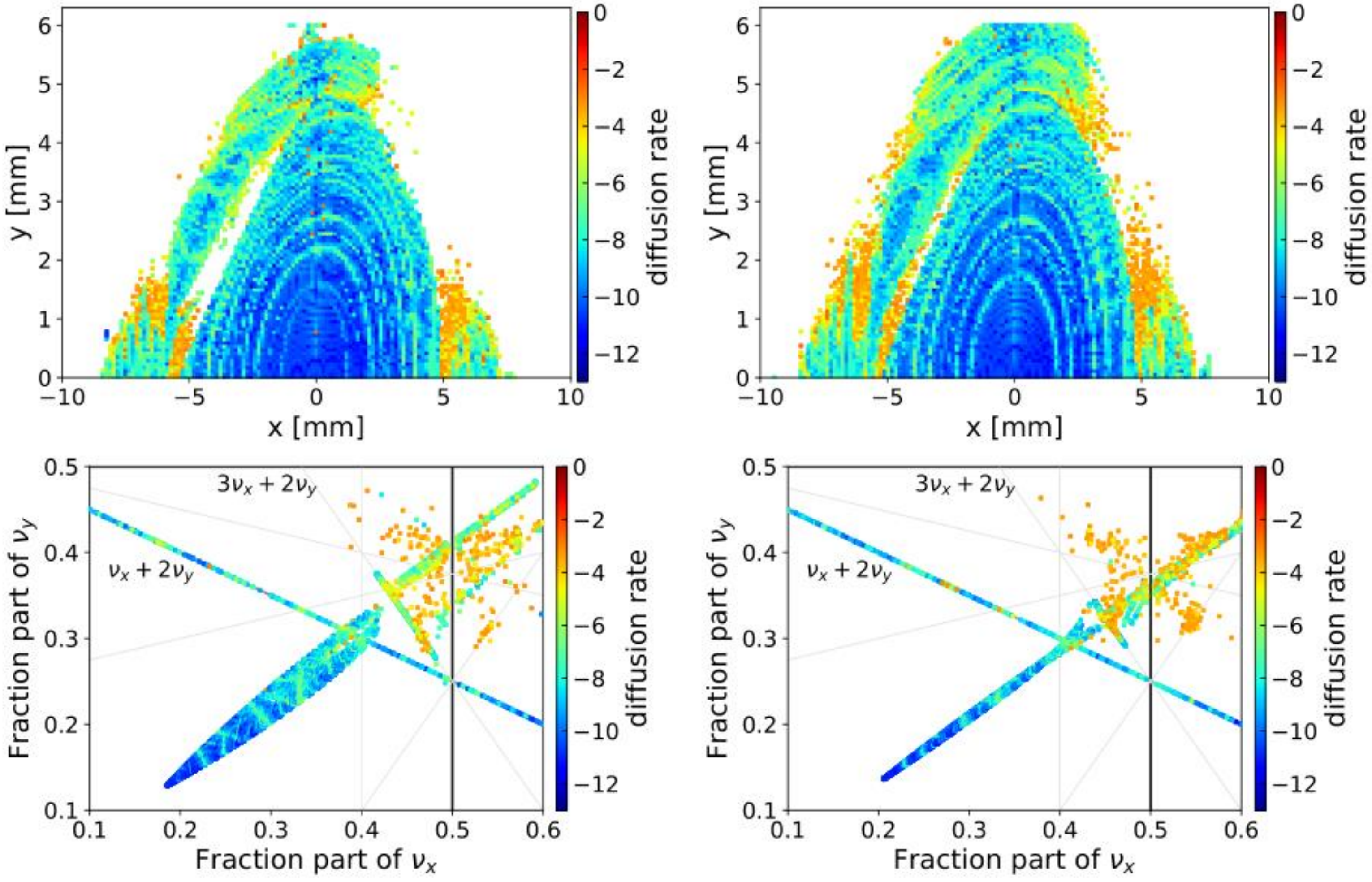
- SSRF lattice: the same solution set as in the on-momentum case
  - $S_n = S_{DA}/\sqrt{\beta_x \beta_y}$ is the normalized DA area.

- The same results were got as in the on-momentum case:
  - Minimizing off-momentum RDT fluctuations is much more effective in enlarging off-momentum DAs.
  - Reducing off-momentum low-order RDT fluctuations can help to reduce off-momentum ADTS and off-momentum higher-order RDTs.



# Off-momentum analysis for the HALF lattice

- Lattice: HALF storage ring
- The hole in the off-momentum DA (δ=-2%) involves multiple resonance lines, including the 3rd-order resonance $\nu_x + 2\nu_y$ and 5th-order resonance $3\nu_x + 2\nu_y$.
- Reducing the 3rd and 4th-order off-momentum RDT fluctuations can repair the hole.



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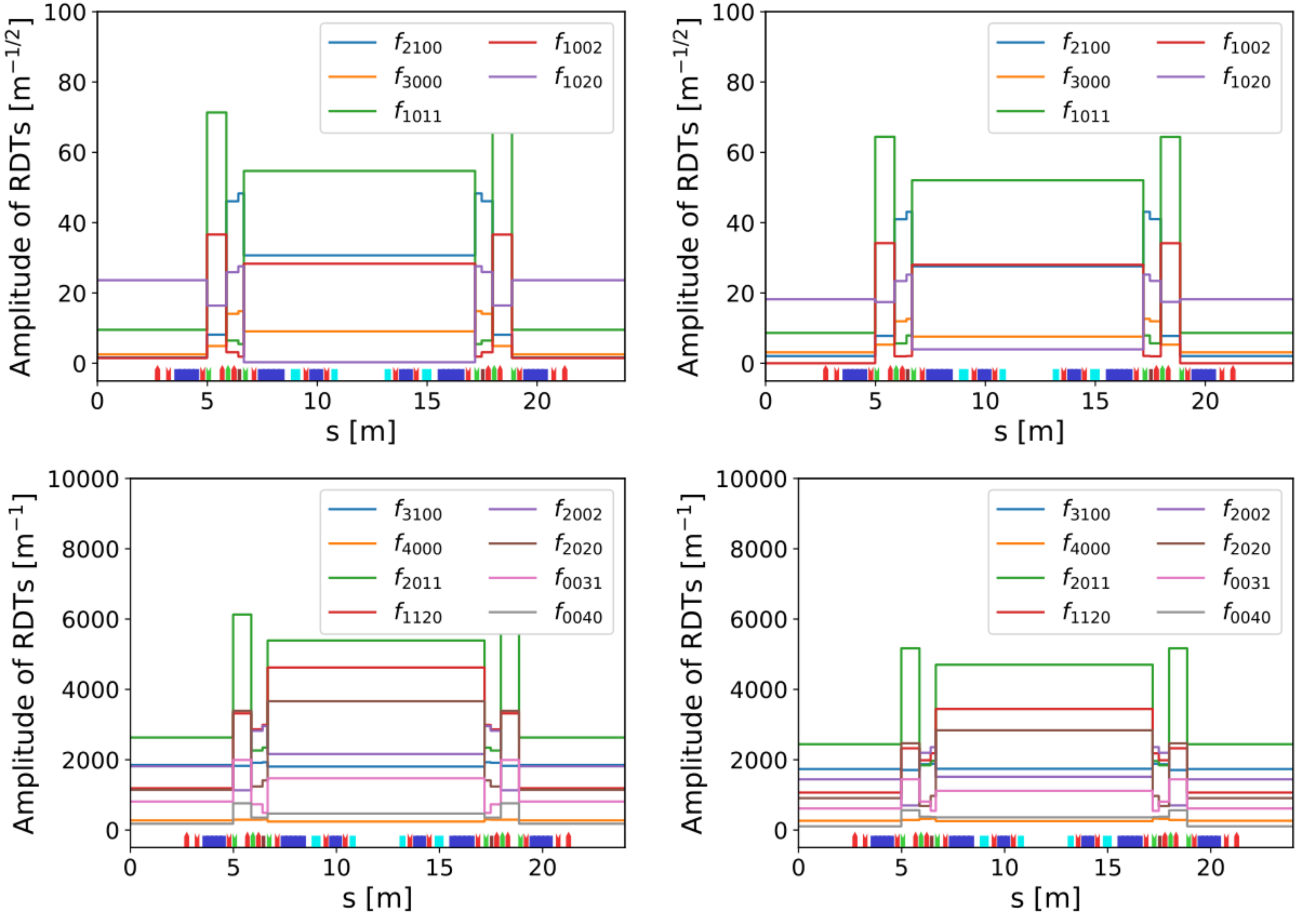


# Preliminary optimization (on-momentum case)

Comparison of three plans:

| Plans | Objective |
|---|---|
| Plan A | Minimizing 3rd-order RDT fluctuations $h_{3,\,\mathrm{ave}}$ |
| Plan B | Minimizing 3rd-order one-turn RDTs $h_{3,\,\mathrm{ring}}$ |
| Plan C | Maximizing DA area (tracked by ELEGANT*) |

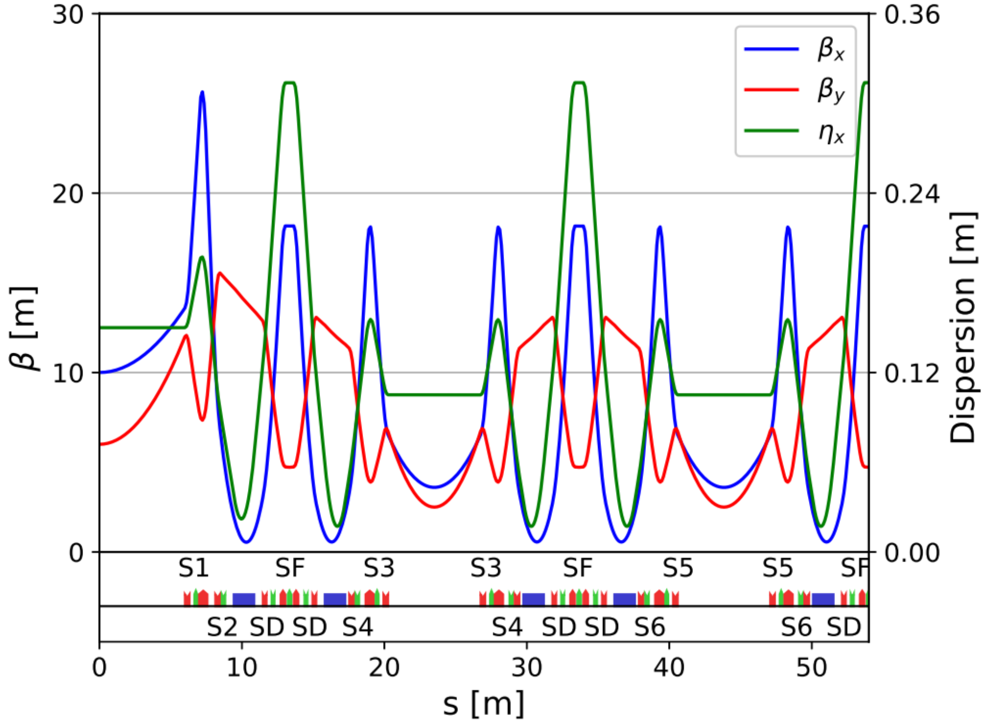
The same optimization settings in genetic algorithms:

- Sextupole variables: S1-S6;
- Constraints: $\xi^{(1)}_{x,y} = (1,\ \ 1),\ \ \xi^{(2)}_{x,y} < 50,$

$$\sqrt{\left(\frac{\mathrm{d}\nu_x}{\mathrm{d}J_x}\right)^2 + \left(\frac{\mathrm{d}\nu_x}{\mathrm{d}J_y}\right)^2 + \left(\frac{\mathrm{d}\nu_y}{\mathrm{d}J_y}\right)^2} < 10000\ \mathrm{m}^{-1}\ ;$$

- Population size = 300, 50 generations.

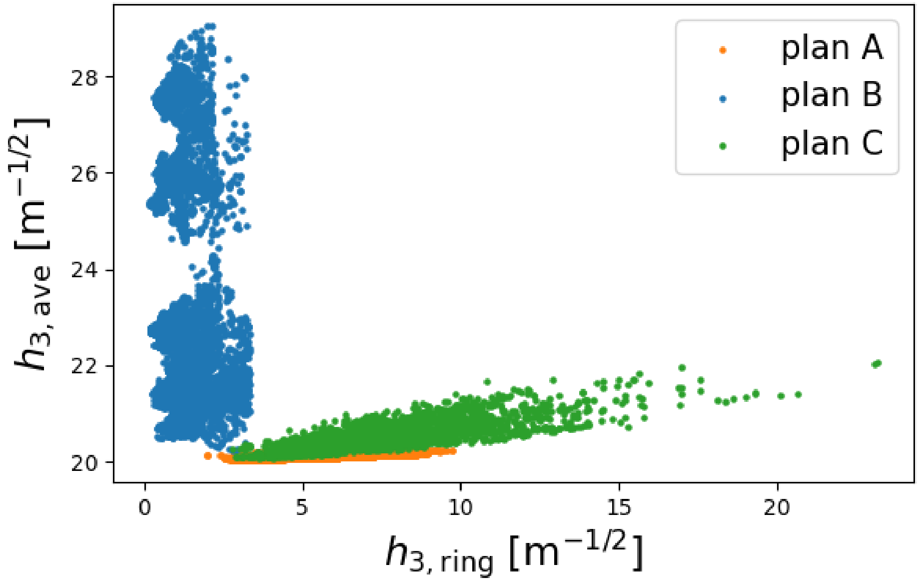
Lattice: SSRF lattice



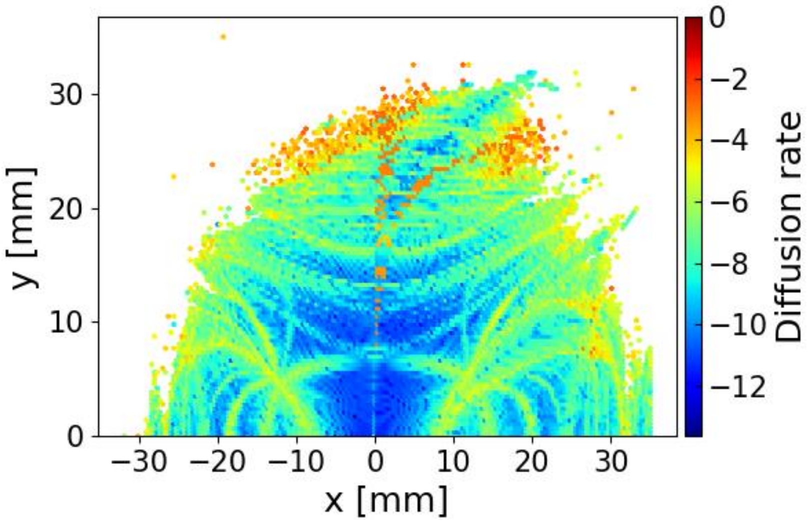

*n-lines: 9 lines, nx=36, 256 turns, xmax=ymax=0.035, n_split=1, split_fraction=0.2

# Preliminary optimization (on-momentum case)

| Plans | Objective | Time |
|---|---|---|
| Plan A | Minimizing 3rd-order RDTs fluctuation $h_{3,\,\mathrm{ave}}$ | 12 s |
| Plan B | Minimizing 3rd-order one-turn RDTs $h_{3,\,\mathrm{ring}}$ | 11 s |
| Plan C | Maxmizing DA area (tracked by ELEGANT) | 36 h |




The distribution of the optimized solutions.




One optimized DA in Plan A.

- Minimizing RDT fluctuations can quickly identify the region where good solutions are located.
- We can further combine Plan A and Plan C to achieve better nonlinear optimization.
- Optimizing both on- and off-momentum DAs and tune shifts with momentum is on-going.

# Conclusion

- Minimizing the fluctuations of RDTs along the longitudinal position is much more effective than minimizing the commonly-used one-turn RDTs in enlarging the DA.
- Reducing low-order RDT fluctuations can help to reduce both higher-order RDT fluctuations and higher-order one-turn RDTs.
- Instead of controlling chromatic RDT fluctuations, controlling off-momentum RDT fluctuations can be much better used for enlarging off-momentum DAs; and the two conclusions above are also valid for the off-momentum case.
- Based on minimizing RDT fluctuations using genetic algorithms, large DA solutions can be found very fast.

**Thank you for your attention!**