*9th Low-Emittance Ring workshop*
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# *CSR impedance and the impact of quadrupolar wakes on beam dynamics*

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# 1. CSR impedance

## 1.1 What is CSR (Coherent Synchrotron Radiation)?

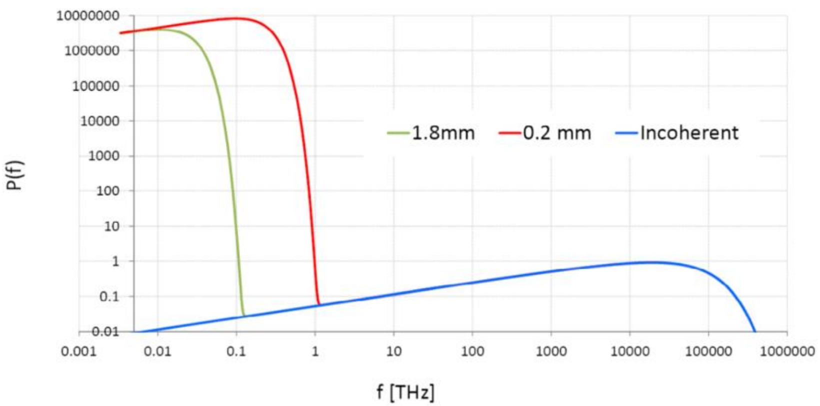
*(from J.B. Murphy, ICFA News Letter **35**, 2004 and excerpts/descriptions of McMillan, Schiff and Schwinger therein)*

- A bunch of electrons moving (under a dipolar magnetic field) on a circular orbit of radius $\rho$ radiate power via synchrotron radiation.
- Total power radiated by *N* particles: $$W_{tot} = \sum_n \left| \sum_{s=1}^{N} \exp(in\phi_s) \right|^2 W_n$$ ($W_n$: Radiated power by a single electron in the *n*-th harmonic. $\phi_s$: Instantaneous angular position of an electron)
- If all particles are bunched at the same azimuthal position ➔ $W_{tot} = N^2 \sum_n W_n$,
- If their positions were random and uncorrelated but characterized by a normalized distribution $f(\phi)$,

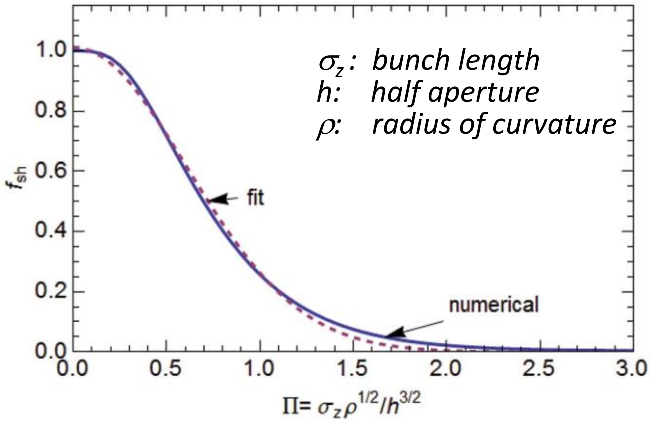
$$W_{tot} = \sum_n \left[ N + (N^2 - N) \left| \int f(\phi) e^{in\phi} d\phi \right|^2 \right] W_n$$

*( 1st term: ISR (Incoherent Synchrotron Radiation*
*2nd term: CSR (Coherent Synchrotron Radiation )*

- CSR is emitted at long wavelengths and may be effectively suppressed by metallic shielding




*Power spectrum of the synchrotron radiation emitted by a 0.2 mm bunch (red line) and by a 1.8 mm bunch (green line) including incoherent radiation of a single particle (blue line). From A. Novokhatsuki, SLAC-PUB-14893, 2012)*




*Ratio of the loss factor $\kappa_{sh}/\kappa_{free}$ versus shielding parameter for Gaussian beam moving between conducting planes (from K.L.F. Bane, workshop TWIICE, 2014).*

## 1.2 CSR wakes

*(from K.L.F. Bane et al., J. Synchrotron Rad. (2014). **21**, 937)*

- Longitudinal CSR wake in ***free space***

$$\frac{W_0(s)}{L} = -\frac{Z_0 c}{2\pi}\frac{H(-s)}{3^{4/3}\rho^{2/3}(-s)^{4/3}},$$

*(Y. Derbenev et al, Report TESLA FEL-1995–05. DESY; J.B. Murphy et al., Part. Accel. 57, 9, 1997)*

*(H(s) = 1 for s > 1 and H(s) = 0 for s < 0)*

➔ Unlike other wakes, in the case of free-space CSR the ***driving particle needs to be behind the test particle*** for the wake to be nonzero, and that the overall minus sign indicates energy gain by the test particle

- Longitudinal short-range CSR wake in the midplane between ***two perfectly conducting parallel plates***

*( J.B. Murphy et al., Part. Accel. 57, 9, 1997)*

$W_{tot}(s) = W_0(s) + W_1(s)$ with

$$\frac{W_1(s)}{L} = -\frac{Z_0 c}{8\pi^2\rho^{2/3}}\left(\frac{\Pi}{\sigma_{z0}}\right)^{4/3} G\left(\frac{\Pi s}{\sigma_{z0}}\right),$$

where $\Pi = \sigma_{z0}\rho^{1/2}/h^{3/2}$ : Shielding parameter

$$G(\zeta) = 8\pi\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2}\frac{Y_k(\zeta)[3 - Y_k(\zeta)]}{[1 + Y_k(\zeta)]^3},$$

where $Y_k$ are the solutions of $Y_k - \frac{3\zeta}{k^{3/2}}Y_k^{1/4} - 3 = 0.$

$W_1(s)$ : Contribution of the image charges generated by the metal plates. It is in general nonzero for both signs of argument

## 1.3 Specific CSR impedances *(from K.L.F. Bane et al., J. Synchrotron Rad. (2014). **21**, 937*

Solid lines: Shielded
Dashed lines: Free space

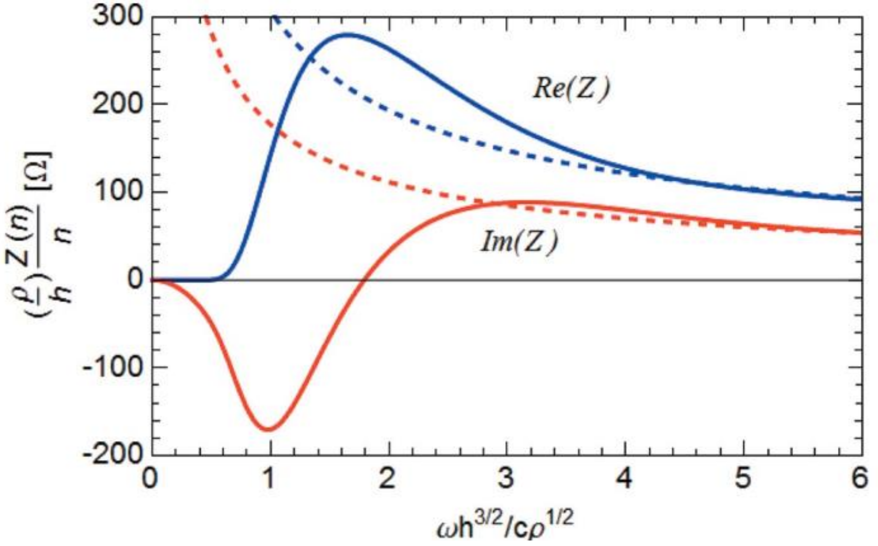
- Longitudinal CSR impedance in ***free space***

$$\frac{Z(\omega)}{L} = \frac{Z_0}{2\pi}\frac{\Gamma(2/3)}{3^{1/3}}\left[\exp\left(\frac{\pi i}{6}\right)\right]\left(\frac{\omega}{c\rho^2}\right)^{1/3},$$

- Longitudinal short-range CSR wake in the midplane between ***two perfectly conducting parallel plates***

$$(\frac{\rho}{h})(\frac{Z(n)}{n})_{csr\_p} = \frac{16\pi^3 2^{1/3}}{c}[n(\frac{h}{\rho})^{3/2}]^{-4/3}$$
$$\times \sum_{p>=1,3,\ldots}[Ai'(u)Ci'(u) + uAi(u)Ci(u)],$$

$A_i(u)$, $C_i(u)$ : Airy functions




*Y. Cai (IPAC2011) showed that in terms of scaled impedance, both free space and shielded (parallel-plates) impedances can be compared on the same plot:*

◊ At high frequencies, shielding has no effect (both converge to the same values )

◊ Shielding suppresses $ReZ$ for frequencies roughly below $\omega h^{3/2}/c\rho^{1/2}$

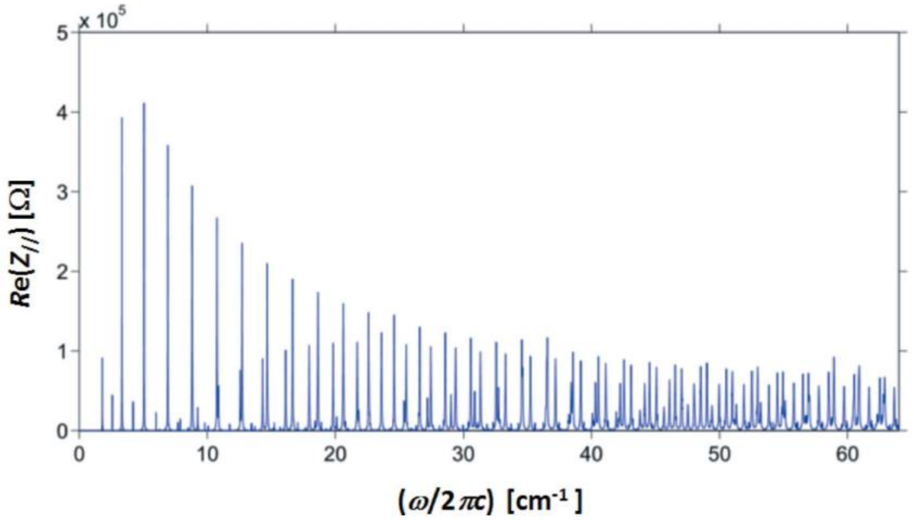
◊ A broadband peak on $(ReZ)_{shielded}$ at $\omega \approx c\rho^{1/2}/h^{3/2}$

◊ There is a frequency range in which $(ReZ)_{shielded} > (ReZ)_{freespace}$

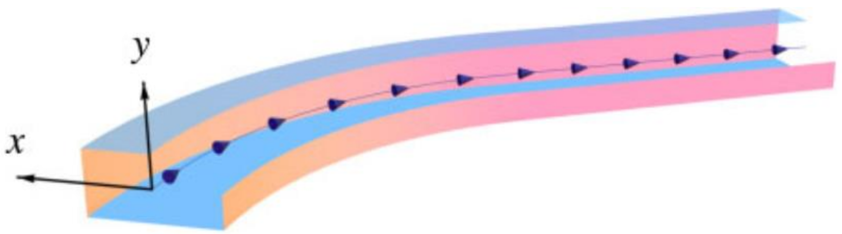
- $Z_{CSR}$ of ***closed toroidal chambers / Numerical calculations*** *of $Z_{CSR}$ of a ring composed of dipole and drift chambers*

Warnock and Morton (Part. Accel. **25**, 113, 1990) analytically obtained $Z_{CSR}$ of a torus with a rectangular cross section by field matching. They found ***whispering-gallerylike modes*** from a forest of high-frequency spikes in $ReZ_{CSR}$, which generate resonances and possibly coupled-bunch oscillations.

Triggered by the work of Stupakov and Kotelnikov (PRSTAB **12**, 104401, 2009), algorithms to numerical compute $Z_{CSR}$ of a ring composed of dipole chambers of arbitrary cross sections and drifts in between were developed.



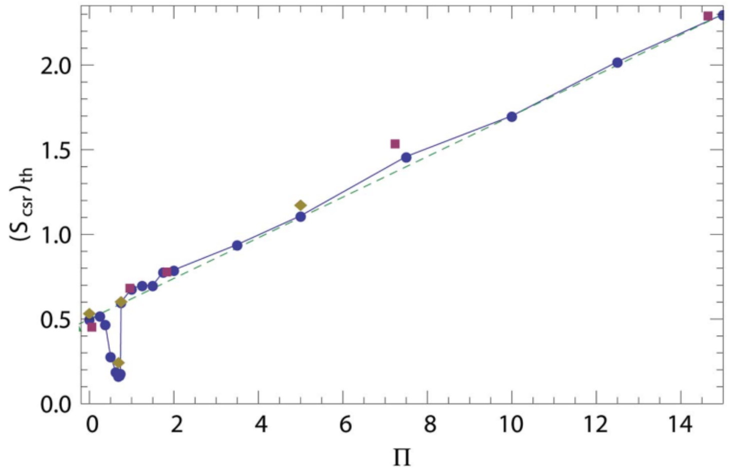

*ReZ calculated for closed toroidal chambers having a rectangular cross section of the ANKA ring (Warnock, R. et al., PAC2013)*




*D. Zhou et al. (Jpn. J. Appl. Phys. **51**, 016401, 2011) developed a code CSRZ using the finite difference method to compute $Z_{CSR}$ of a ring composed of a series of dipole chambers*

- Studies of the impact of obtained $Z_{CSR}$ on the beam dynamics

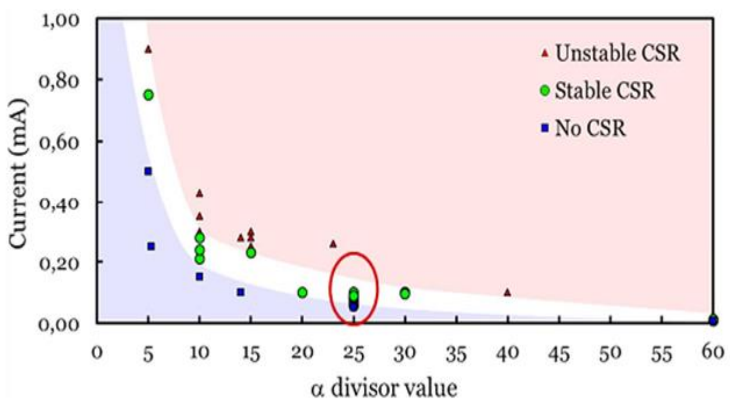
Since the early 2010s, many theoretical and numerical studies were made to understand the microwave instabilities driven by $Z_{CSR}$s, in parallel to extensive experimental studies in several rings that can alter (lower) momentum compaction factor $\alpha$.



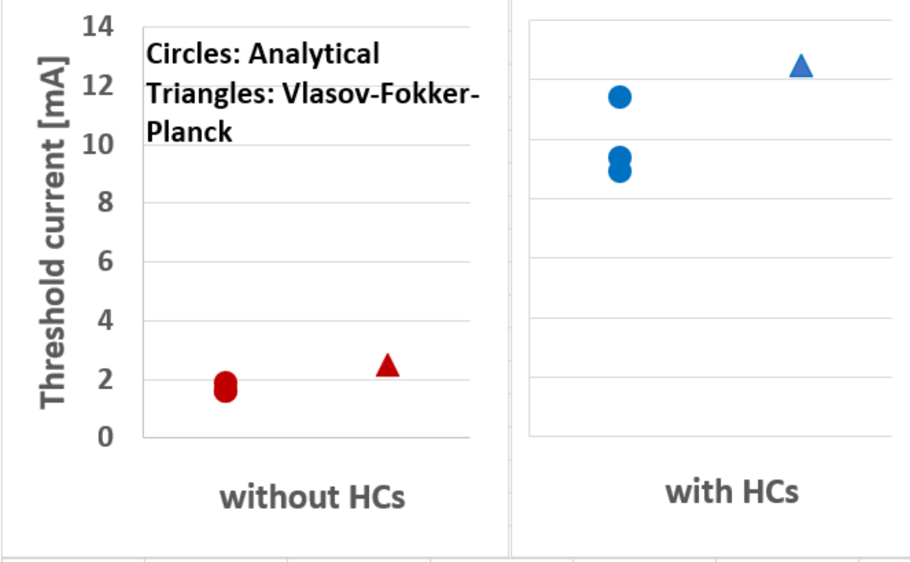

Normalized strength at threshold versus shielding parameter calculated using the $Z_{shielded}$ (*K.L.F. Bane et al., PRSTAB **13**, 104402*). VFP (Vlasov-Fokker-Planck: blue circles) and LV (Linearized Vlasov: red squares) solvers are compared

## 1.4 Two conventional ways of treating the CSR in light source rings

- CSR is considered negative as a source of microwave instability that blows up the energy spread of the beam
- A number of light source rings (2nd and 3rd generations) such as SuperACO, UVSOR, NSLS, SOLEIL, MLS, … tried to make use of CSR as a useful THz source and sought for ways to attain steady states of CSR by lowering the momentum compaction $\alpha$
- In particular, the group from the Univ. of Lille has made a series of experimental and numerical studies on the SOLEIL ring to control and enhance the CSR in storage rings with RF feedback actions → *A talk by C. Evain in this workshop on their latest progress*
- In ultra-low-emittance storage rings, chamber apertures are generally much smaller and dipole fields lower, so microwave instability may be expected not important → Evaluations for SOLEIL II find low $I_{threshold}$s and bunch lengthening with HCs effective in increase them (*C. Evain et al., I-FAST Beam dynamics/diagnostics workshop 2022*) → *More to hear from Demin Zhou in this workshop*
- Magnet lattice of an ultra-low emittance ring generally appears to have much less flexibility to operate the ring in low $\alpha$ mode




*Single bunch CSR threshold versus momentum compaction factor $\alpha$ measured at SOLEIL (from M.-A. Tordeux et al., ESLS2012)*

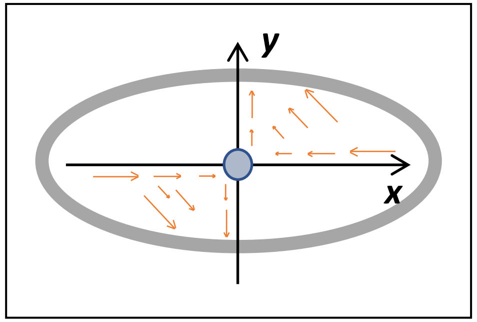



*Calculated CSR threshold w/ and w/o bunch lengthening with HCs for SOLEIL II (from C. Evain et al., I-FAST Beam dynamics/diagnostics workshop 2022)*

| Machine energy $E_0$ [GeV] | 2.75 |
|---|---|
| Horizontal emittance $\varepsilon_{H0}$ [pm·rad] | 80 |
| Revolution time $T_0$ [ms] | 1.18 |
| Longitudinal damping time $t_d$ [ms] | 11.9 |
| Incoherent synchrotron frequency $f_s$ [kHz] | 1.458 |
| Vacuum chamber height $h$ [mm] | 5.0 |
| Zero current bunch length $\sigma_{z0}$ | 2.78 |
| Zero current energy spread $\sigma_{E0}$ | 9×10$^{-4}$ |
| Main RF frequency $f_{RF}$ [MHz] | 354 |
| Main RF voltage $V_{RF}$ [MV] | 1.38 |
| Harmonic assumed for HCs | 3 |
| HC voltage [MV] | 0.32 |
| (Bunch lengthening factor)$_{max}$ assumed | 4.4 |

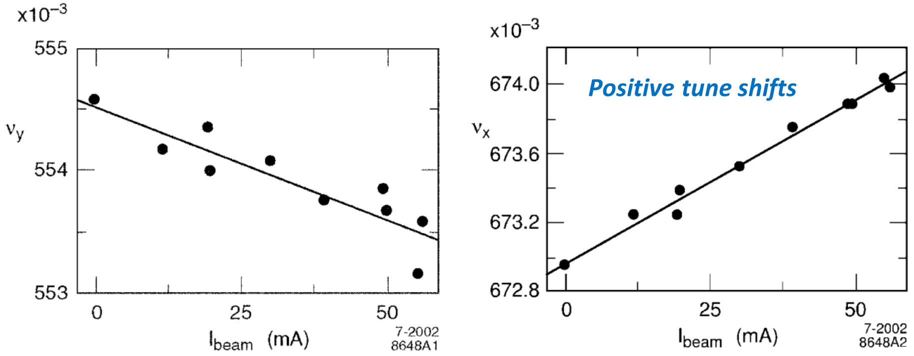
*SOLEIL II machine and beam parameters assumed in the simulation*

## 2. Impact of Quadrupolar Wakes on Beam Dynamics




◊ With a resistive-wall (RW) chamber having ***non-circular*** cross section**,**

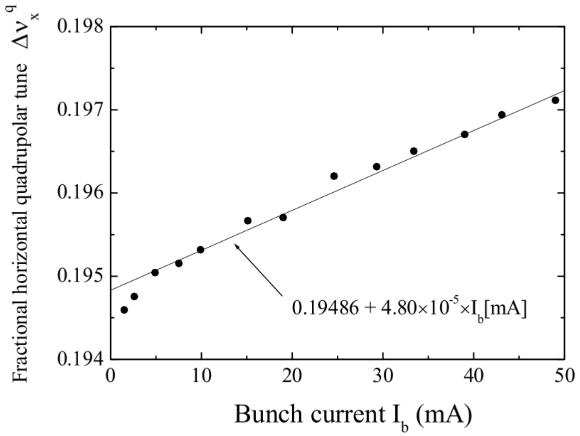
- A transverse wake is created even if the driving beam stays on axis.
- Non-oscillating wake fields add up to build an extremely long-range field.
- Its leading field component is "***quadrupolar***". Trailing particles are focused (defocused) "***incoherently***".
- Focusing strength depends linearly on the beam current and on the chamber cross section geometry.




*Vertical (left) and horizontal (right) tune shifts with beam current in observed in the PEP-II high energy ring in 1998 (A. Chao et al., PRSTAB **5**, 111001 (2002))*

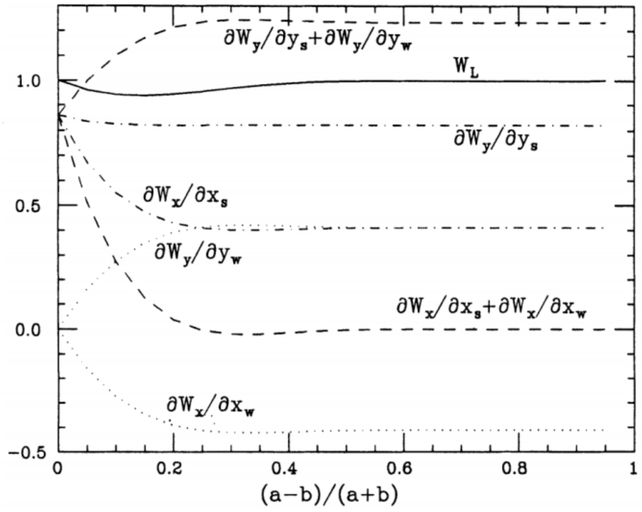
◊ Early studies addressed the ways to evaluate RW dipolar and quadrupolar wakes of non-circular chambers :

- *H. Henke, O. Napoly, EPAC90, 1046*
- *R.L. Gluckstern, J. van Zeijts, B. Zotter, Phys.Rev.* ***E47****, 1 (1993) 656*
- *K. Yokoya, Part. Acc.,* ***41****(1993) 221*
- *A. Burov, V. Lebedev "Transverse Resistive Wall Impedance for Multi-Layer Flat Chambers", EPAC 2002*




**Positive** *single bunch detuning* measured in the KEK-PF ring developing a *quadrupolar mode excitation (S. Sakanaka et al., PRSTAB* ***8****, 042801 (2005))*

*Form factors of asymptotic dipolar and quadrupolar RW wakes as a function of the cross section aspect ratio (a-b)/(a+b) for rectangular chambers, numerically obtained via the scheme developed by K. Yokoya, Part. Acc.,* ***41****(1993) 221 for arbitrary cross sections*



## 2.1 Long-range quadrupolar wakes in multibunch

- Early studies focused on the incoherent focusing strength felt by a "witness" particle trailing a "source" particle at a distance $z$ behind

$$x''_w = \frac{e}{E/e}\frac{\partial W_x}{\partial x_w}\cdot x_w = -k^e_{eff}(z)\cdot x_w \qquad k^e_{eff}(z) = D_\perp \cdot \frac{e}{E/e}\frac{c\sqrt{Z_0\,\rho_R}}{\pi^{3/2}b^3}\frac{1}{z^{1/2}}, \qquad D_\perp = \frac{\pi^2}{24}$$

for a flat chamber

- However, with the $z^{-1/2}$ dependence that comes from assuming an infinitely thick RW, the summation of fields over previous turns "diverges"
  - ➔ Infinite tune shifts!
  - ➔ Two groups studied ways to avoid this divergence by considering a RW of ***finite thickness*** across which the field penetrates:

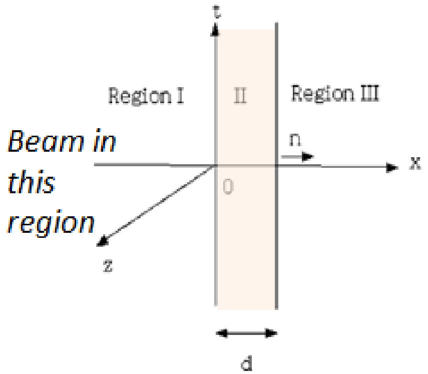
- Model of Chao, Heifets and Zotter: (*PRSTAB* ***5****, 111001 (2002)*)
  - Made use of the exponential decay known for a circular cross section chamber by assuming that the time-dependence should not be much different for flat chambers.

$$\frac{d\nu_{x,y}}{dI} = \pm\left(\frac{\pi r}{48\nu_{x,y}}\right)\left(\frac{Z_0}{E/e}\right)\left(\frac{R}{b}\right)^2\left(\frac{L}{C}\right), \qquad r = 1+\frac{b^2}{D^2}$$

→ Resultant tune shifts do not depend on the resistivity $\rho_R$ nor wall thickness *d*

- Model of Shobuda and Yokoya: (*Phys. Rev. E****66****, 056501 (2002)*)
  - Solved EM fields in a one-dimensional system assuming application to flat chambers

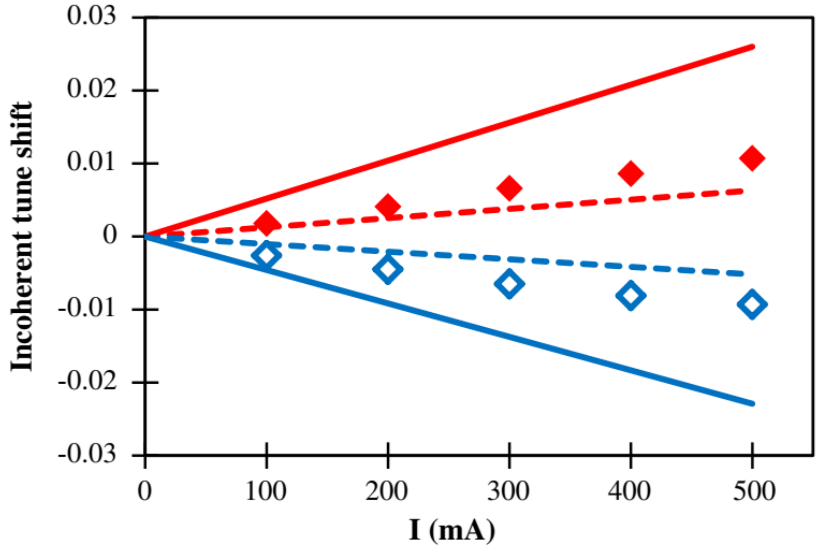
$$\frac{d\nu_{x,y}}{dI} = \pm D_{2xy}\frac{C\langle\beta_{x,y}\rangle}{4\pi}\frac{Z_0}{E/e}\cdot\frac{d}{\pi b^3},$$




→ Resultant tune shifts do not depend on the resistivity, but depends linearly on the wall thickness *d*

## 2.2 Quadrupolar wakes in multibunch fillings measured at SOLEIL and comparison with theory:

(*P. Brunelle, R. Nagaoka, R. Sreedharan, PRAB* ***19****, 044401 (2016)*)




*Incoherent betatron tune shifts vs current in uniform filling in the SOLEIL ring: Red: H, Blue V. Measured (diamond); Chao et al. (solid line), Shobuda and Yokoya (dashed line). The precision on measured tune values is 1 × $10^{-4}$.*

- A numerical program developed at SOLEIL to compute the overall incoherent focusing felt by a particle circulating in the ring in an arbitrary beam filling by evaluating the effective wake field of vacuum chambers piecewise
- Ring database used was composed of :
  - Realistic chamber configuration around the ring with information on $\partial W_x/\partial x_w$
  - Wall resistivity and thickness
  - Local integrated beta functions at the chamber location

| Model | Horizontal | Vertical |
|---|---|---|
| Chao et al. | +0.0260 (+0.0100) | −0.0229 (−0.0177) |
| Shobuda-Yokoya | +0.0080 (+0.0080) | −0.0067 (−0.0081) |

*Comparison between the derived analytical tune shift formula and numerical piecewise summation*

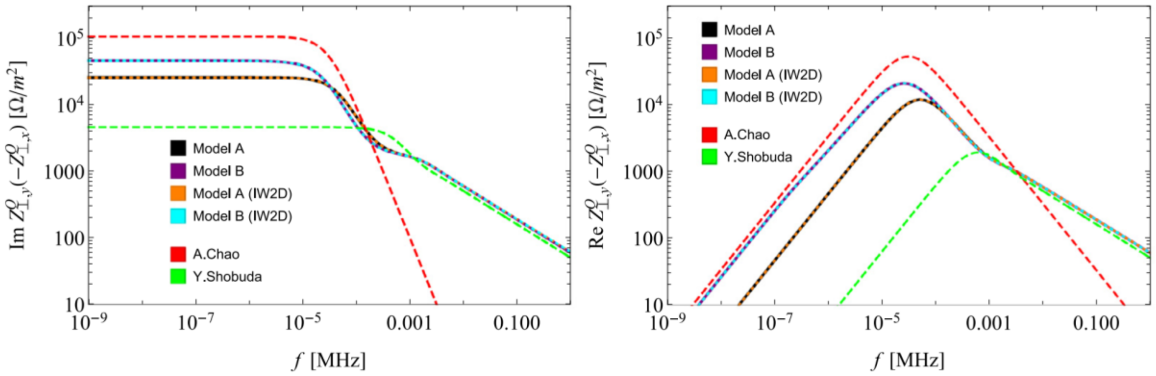
- Induced linear optics distortion was analyzed using LOCO by introducing virtual quadrupoles in the ring and subtracting off magnetic contributions when in-vacuum insertions were closed.
- $\Delta\beta/\beta$ of 1 ~2% level was deduced at 500 mA

| Contributor | Measured gradient | Calculated gradient |
|---|---|---|
| Long SS | +0.0009 | +0.00101 |
| Medium SS | +0.0018 | +0.00197 |
| In-vacuum IDs closed at minimum gap | +0.0043 | +0.00789 |

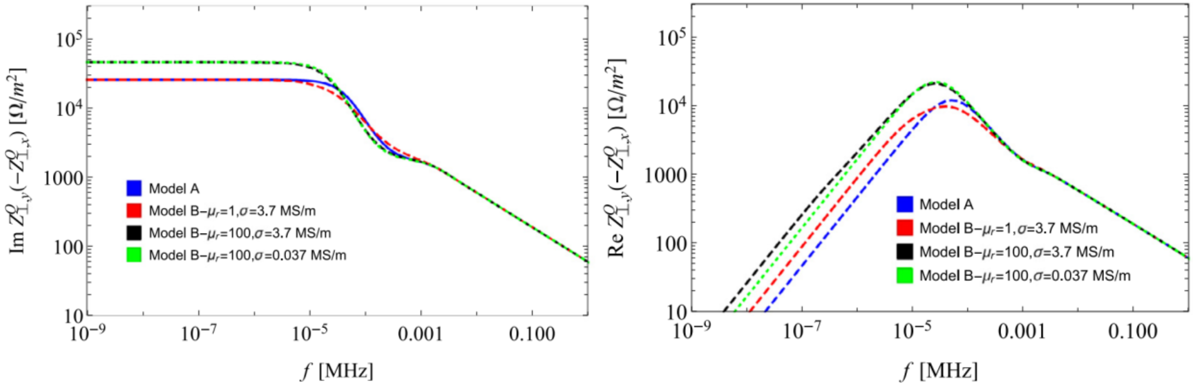
*Comparison of gradient [T/m] between measured and theory (Chao et al.)*

## 2.3 Dependence on conductivity, thickness and permeability of the magnets outside the vacuum chambers:

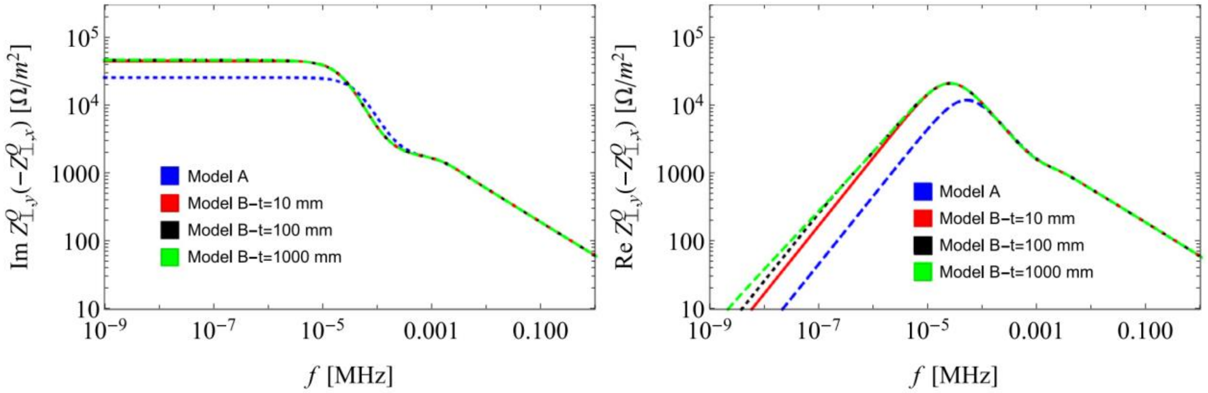
- Studies recently made by Y. T. Wang et al. exploring the contributions of magnets outside the vacuum chambers on the quadrupolar wakes (*Y. T. Wang et al., NIM,* ***A1029*** *(2022) 166414*)




*Imaginary (left) and real (right) part of quadrupolar impedance at extremely low frequencies. Model A: Copper chamber with air outside. Model B: Copper chamber first surrounded by magnets*




*Dependence of $Z_{quad}$ on the permeability $\mu_r$ and conductivity $\sigma$ of the magnets*



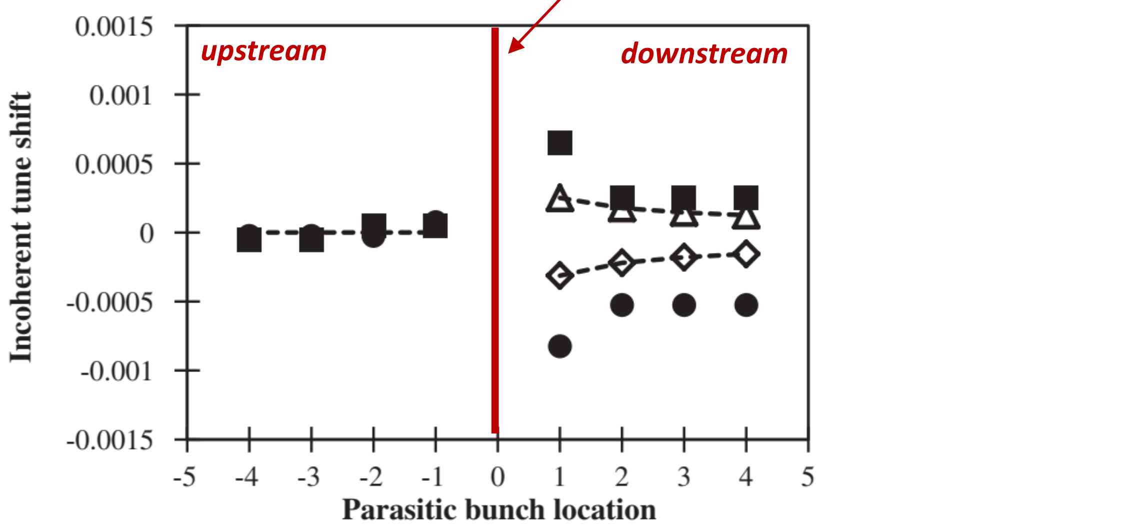

*Dependence of $Z_{quad}$ on the thickness of magnets*

- To be able to treat multilayers of different materials outside the vacuum chamber, quadrupolar wakes were studies in terms of their Fourier transforms $Z_{quad}(\omega)$ at extremely low frequencies (DC component) and compared with the results of the code *IW2D*
- While no dependence on the conductivity of the magnets was confirmed, which is consistent with the models of Chao et al. and Shobuda-Yokoya, a substantial dependence on the permeability $\mu_r$ was noticed on $ImZ_{quad}(\omega=0)$
- Predicted tune shifts for CEPC lied between those obtained by Chao et al. and Shobuda-Yokoya

## 2.4 Short-range quadrupolar wakes felt by particles in a high intensity bunch: Measurement made at SOLEIL

- Unlike the multibunch fillings, particles in a high intensity bunch are under the direct short-range RW quadrupolar wakes, whose amplitude increases as their distance to the bunch center diminishes.

- Tunes of very low-intensity *parasitic* bunches, placed in the vicinity of a high intensity one, were measured with a bunch-by-bunch feedback system
  - ➔ An increase of the tune shifts observed as expected as the distance to the intense bunch decreases
  - ➔ Measured tune shifts > calculated (→ due to NEG which is missing in the calculations?)

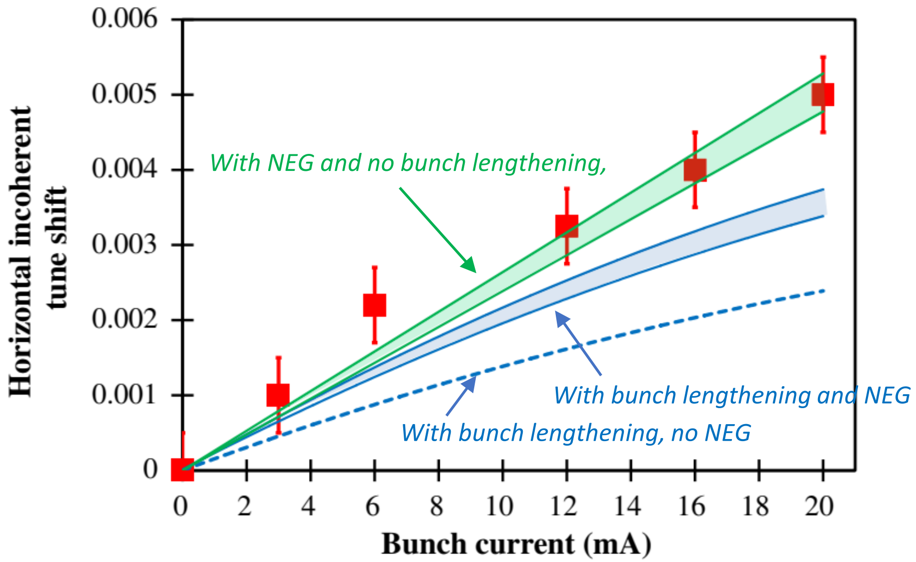



*Measured hor (dark squares), ver (dark circles); Calculated hor (white triangles), ver (white diamonds)*

- Incoherent tunes of an intense bunch were measured driving the beam on a coupling resonance. They were compared with theory using the quadrupole focusing given by the formula

$$\langle k_{\text{eff}}^{\text{bunch}} \rangle = \frac{4\pi/Q}{E/e} \int_0^\infty \text{Im} Z_x^{RW}(\omega) \cdot |\tilde{\rho}(\omega)|^2 d\omega \, (\text{m}^{-2})$$

- It was experimentally confirmed that a particle is focused nearly ***20 times*** in an intense bunch of 20 mA than in a high multibunch beam of 500 mA
- With inclusion of NEG, agreement with the measurement improves significantly



## 3. Summary

- CSR is understood to excite both single and multibunch longitudinal coherent instabilities. Numerical codes developed today would allow us to compute effectively the CSR impedance of many complex chamber structures and explore corresponding instabilities. However, the parallel-plate model has shown to describe well many observed CSR-induced instabilities.
- Maximizing the use of CSR in the bursting mode above the threshold is a developing area that requires both accelerator physics and technological studies (such as RF feedback). Possibilities for ultra-low emittance rings, particularly in terms of momentum compaction tuning are yet to be explored.
- Despite the general trend of smaller aperture $h$ and longer bending radius $\rho$ for ultra-low emittance rings which would enhance the shielding of CSR, the microwave threshold is found low for SOLEIL II and bunch lengthening with HCs effective in raising the threshold. Whether this is a general trend or not must be clarified with further studies.

- With the general trend of reducing the vacuum chamber half aperture $b$ for ultra-low emittance ring designs, the RW quadrupolar wakes tend to be stronger (as $\propto b^{-3}$) if we adopt non-circular chamber cross sections.
- For multibunch operations where the long-range RW wakes are important, the field penetration across the finite thickness walls plays a critical role in understanding the net equilibrium field reached in the vacuum chambers. Recent studies from the Beijing group have shown quantitatively the importance of permeability of the magnets sitting outside the chambers.
- On the other hand, particles in an intense bunch are under the influence of direct strong short-range RW fields, where the resistivity of the chamber materials plays a role. In particular, the NEG coating seems to enhance the incoherent tune shifts according to what measured at SOLEIL. The net impact on the ultra-low emittance tuning needs be evaluated in this case.

*Thank You for Your Attention*