CSR effects in low-emittance electron storage rings

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Acknowledgements

T. Agoh, K. Bane, M. Blaskiewicz, A. Blednykh, Y. Cai, Y.-C Chae, R. Lindberg, S. Di Mitri, E. Karantzoulis, S. Kramer, K. Ohmi, Y. Papaphilippou, G. Stupakov

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Outline

- Theories of CSR driven microwave instability threshold
- CSR impedance calculation using CSRZ code
- Prediction of CSR instability via simulations
- Interplay of CSR, CWR, RW and geometric wakes
- Summary

- For an overview of CSR, See R. Nagaoka's talk in this workshop.

Analytic theories of CSR driven MWI threshold

- Stupakov-Heifets (S-H) theory [1]
  - Beam becomes unstable when $(\pi R/(2h))^{3/2} \leq kR < 2\Lambda^{3/2}$.
  - For Gaussian bunch, the theory is valid when $k\sigma_z \gg 1$ (coasting-beam approximation).
  - The S-H theory was translated to bunch current threshold [2]:

$$I_b > \frac{\pi^{1/6}}{\sqrt{2}} \frac{ec}{r_0} \frac{\gamma}{\rho^{1/3}} \alpha_p \delta_0^2 \sigma_z \frac{1}{\lambda^{2/3}}$$

- Improvements on S-H theory
  - Simulation of MWI with steady-state parallel-plates model of CSR impedance [3].

$$I_{\text{th1}} = \frac{4\pi(E/e)\eta\sigma_\delta^2\sigma_z^{1/3}}{Z_0\rho^{1/3}} S_{\text{th1}}$$

  - Simulation of MWI with steady-state rectangular-chamber model of CSR impedance [4].

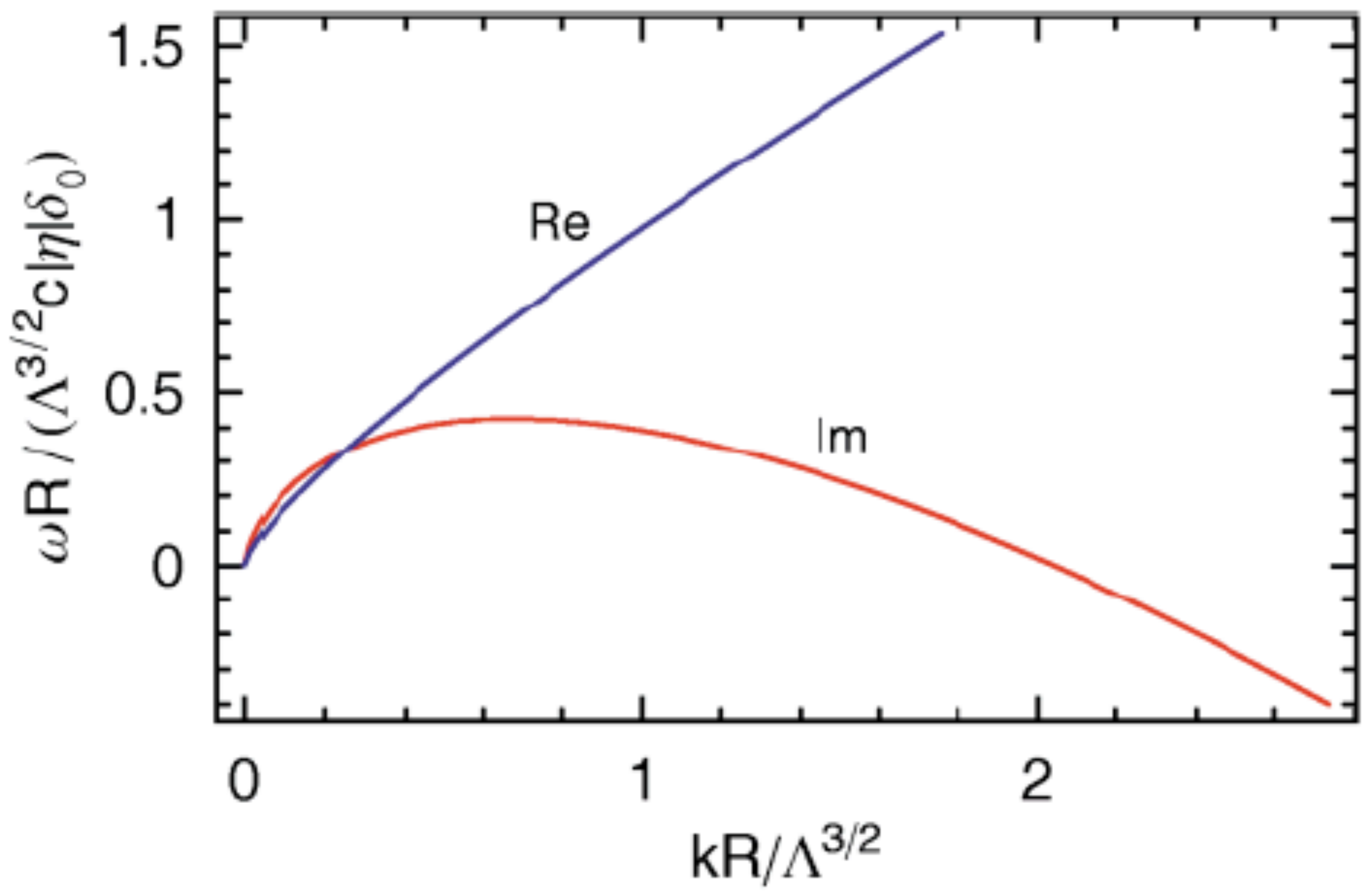


FIG. 1. (Color) The imaginary (Im) and real (Re) parts of the frequency ω as functions of $kR/\Lambda^{3/2}$, for a positive value of η. For negative values of k, the frequency can be found from the relation $\omega(-k) = -\omega^*(k)$ which follows from Eq. (9).

$$1 = \frac{ir_0 cZ(k)}{\gamma} \int \frac{d\delta\,(d\rho_0/d\delta)}{\omega + ck\eta\delta}$$

$$S_{\text{th1}} \approx 0.5 + 0.12\Pi$$

$$\Pi \equiv \sigma_z\sqrt{\rho/h^3}$$

[1] G. Stupakov and S. Heifets, PRST-AB 5, 054402 (2002).

[2] J. Byrd, et al., PRL 89, 22, Nov. 2002.

[3] K.L. F. Bane, Y. Cai, and G. Stupakov, Phys. Rev. ST Accel. Beams **13**, 104402 (2010).

[4] Y. Cai, Phys. Rev. ST Accel. Beams **17**, 020702 (2014).

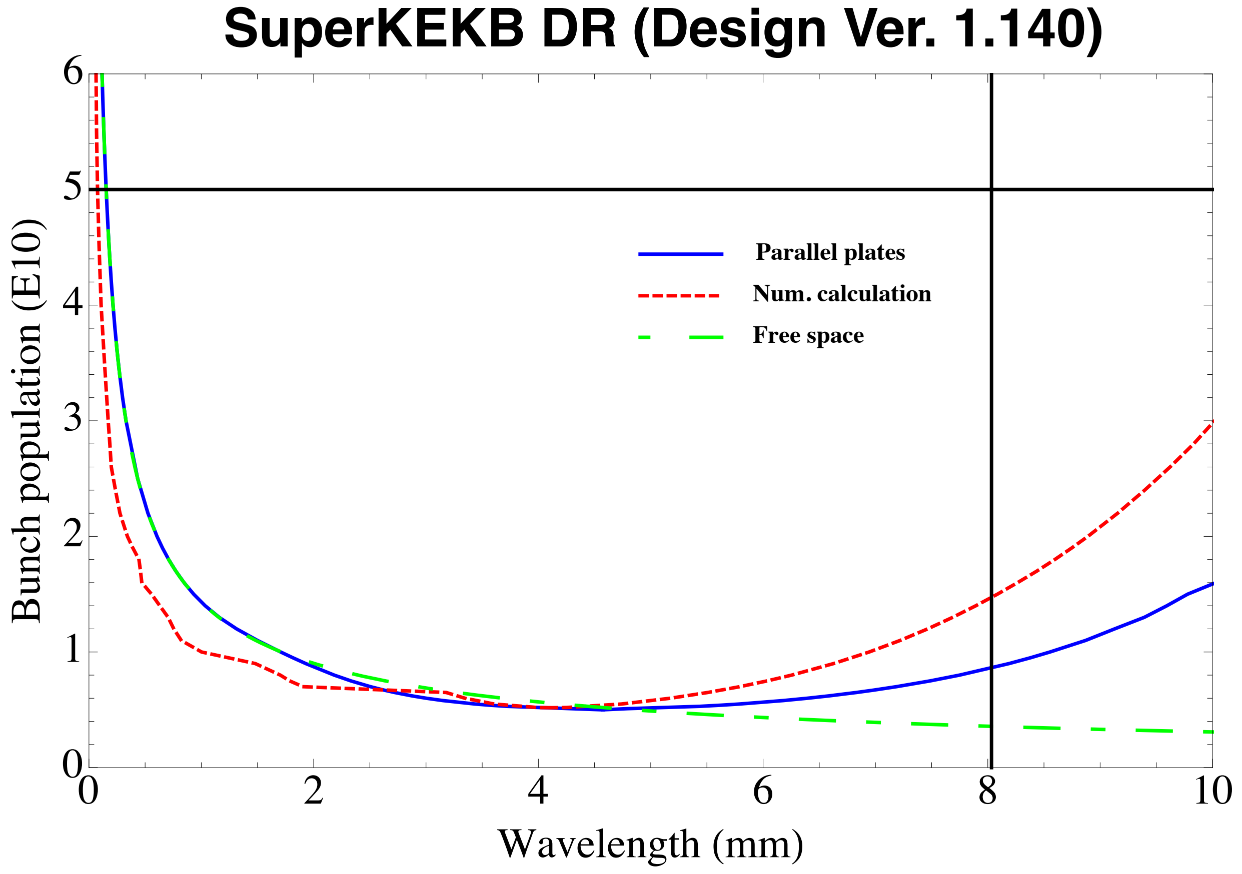
Analytic theories of CSR driven MWI threshold (cont'd)

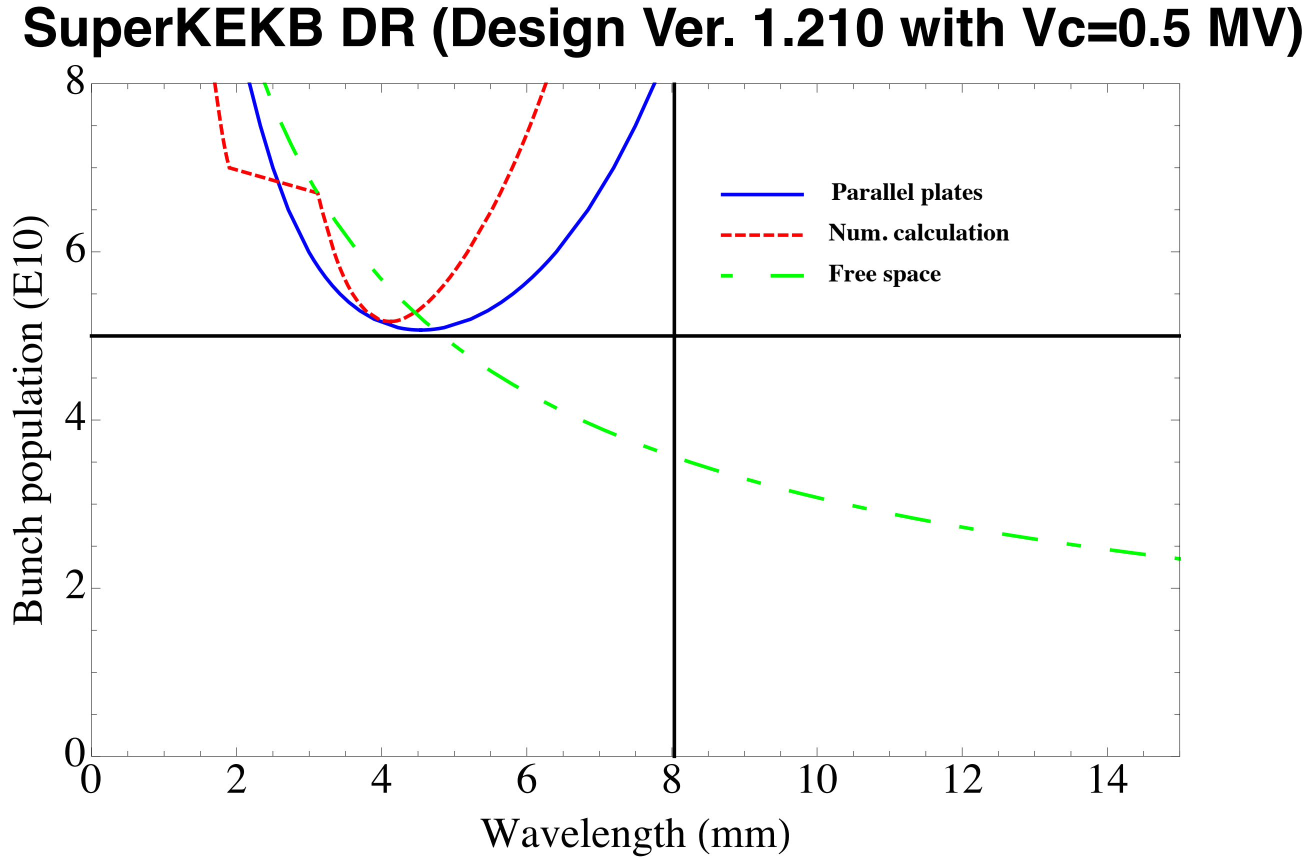
- The case of absolute impedance $Z(k)$
  - An alternative way of extending S-H theory is to solve the dispersion relation numerically with $A = \Omega/(ck\eta\sigma_p)$ [1]:

$$-if(I_b)\frac{Z_{\parallel}(k)}{k}G(A) = 1$$

$$G(A) = \int_{-\infty}^{\infty} dp \frac{pe^{-p^2/2}}{A+p} \qquad f(I_b) = \frac{I_b}{2\pi(E/e)\eta\sigma_p^2\sigma_z}$$

  - The impedance $Z(k)$ can be obtained by analytical or numerical methods, including transient effects and/or chamber shielding.
  - For the impedance $Z(k)$, it is good to use data as a smooth function of k (broadband impedance).
  - For CSR impedance in storage rings, the low-frequency part of $Z(k)$ is mainly determined by chamber shielding, the high-frequency part is mainly determined by transient effects.





[1] D. Zhou et al., "CSR in the SuperKEKB Damping Ring", in Proceedings of IPAC'10, Kyoto, Japan.

Analytic theories of CSR driven MWI threshold (cont'd)

- The case of absolute impedance $Z(k)$
  - Further simplification of the dispersion-relation problem with threshold condition Im[Ω]=0 [1]:

$$G(A) = \int_{-\infty}^{\infty} dp \frac{pe^{-p^2/2}}{A+p} = \sqrt{2\pi} + i\pi A e^{-\frac{A^2}{2}} \left[\text{sgn}[\text{Im}[A]] + i\,\text{erfi}\left[\frac{A}{\sqrt{2}}\right]\right]$$

$$G_r(A_r) = \sqrt{2\pi} - \pi A_r e^{-A_r^2/2} \text{erfi}[A_r/\sqrt{2}]$$

$$G_i(A_r) = \text{sgn}[\eta] \pi A_r e^{-A_r^2/2}$$

$$\boxed{\frac{G_\text{i}(A_\text{th})}{G_\text{r}(A_\text{th})} = \frac{Z_r(k)}{Z_i(k)}} \qquad \boxed{\frac{I_\text{th}}{2\pi(E/e)\eta\sigma_\delta^2\sigma_z} = \frac{kZ_r}{G_\text{i}(A_\text{th})(Z_r^2 + Z_i^2)}}$$

→ Roughly, $I_\text{th} \propto 1/(Z_\parallel/n)$

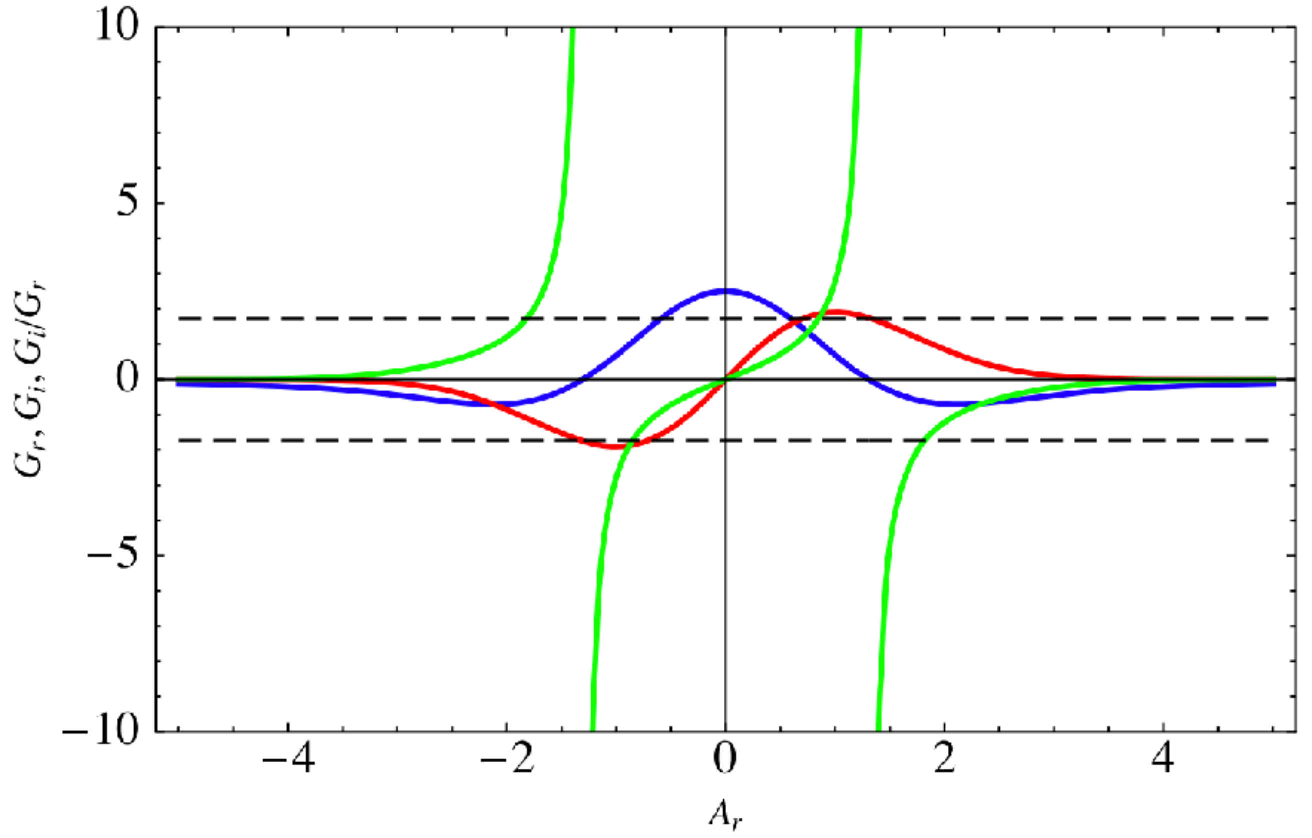


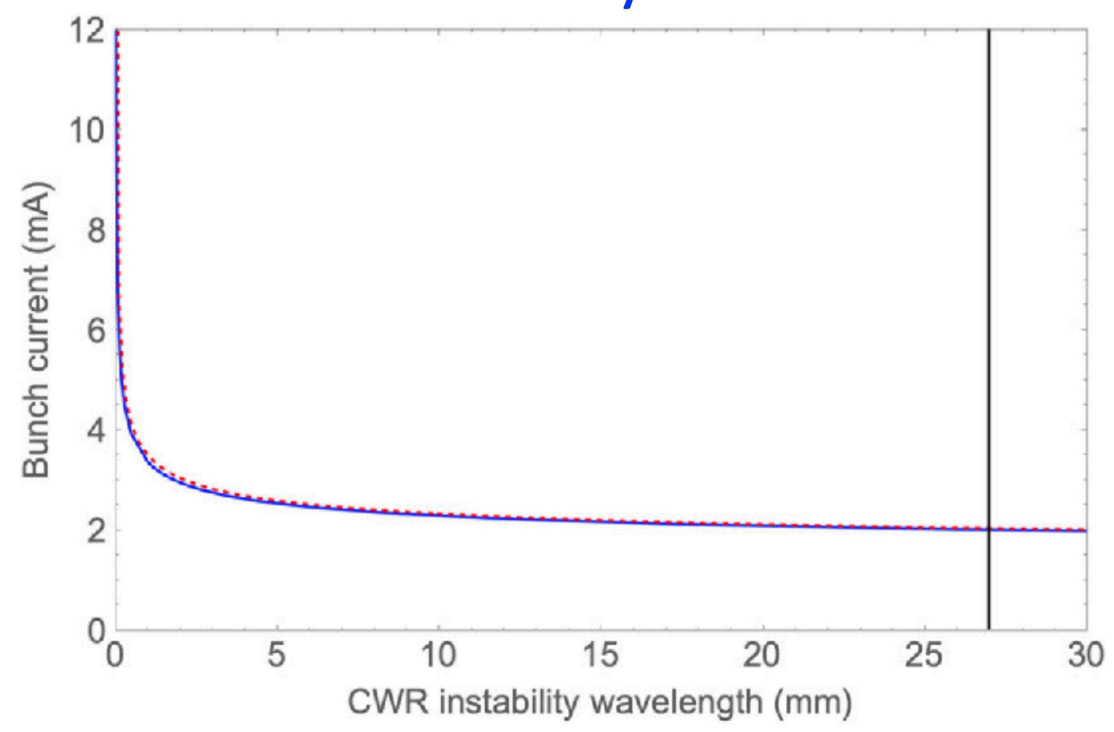
FIG. 13. The functions $G_r(A_r)$ (blue line), $G_i(A_r)$ (red line), and $G_i(A_r)/G_r(A_r)$ (green line) with real A_r and positive η. The horizontal dashed lines indicate $G_i/G_r = \pm\sqrt{3}$.

  - **Application-1**: Scaling law of Coherent Wiggler Radiation (CWR) instability in damping rings (only valid for positive η):

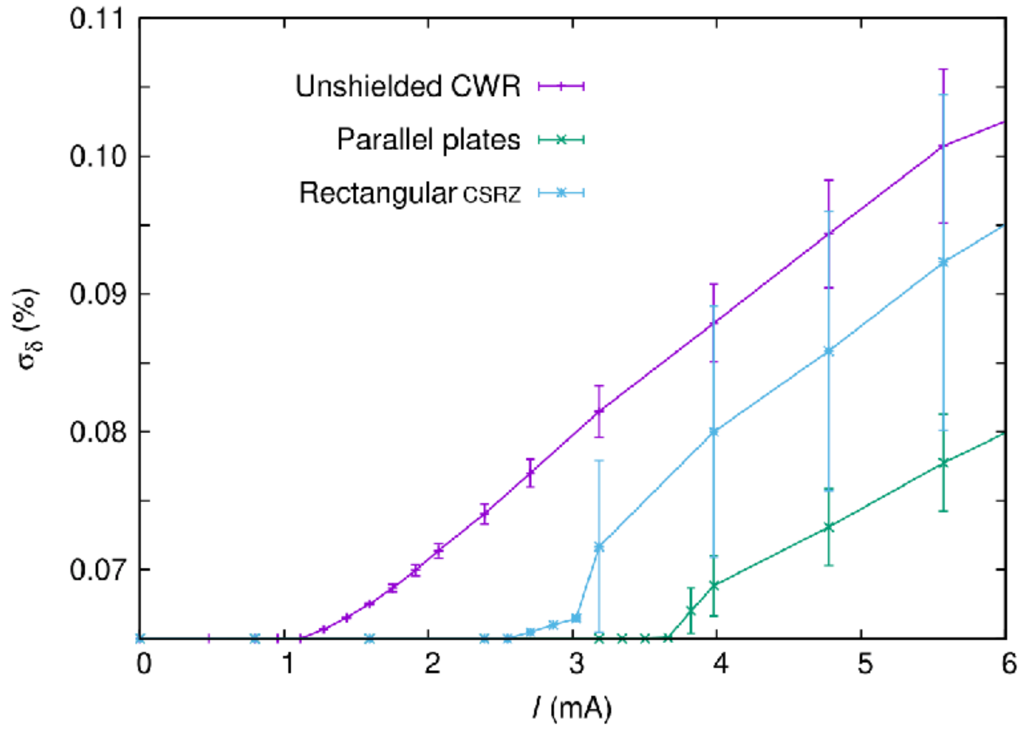
$$I_b^{th}(\lambda) \approx \frac{8\pi\sqrt{2\pi}(E/e)\eta\sigma_p^2\sigma_z}{LZ_0\theta_0^2 \ln \frac{2k_w\lambda}{\pi\theta_0^2}}$$

  - This scaling law well explains the simulated CWR instability in the ring cooler for EIC.

Theory



VFP simulation (R. Lindberg)



$\sigma_{z0} = 48$ mm

[1] A. Blednykh et al., Phys. Rev. Accel. Beams **26**, 051002 (2023).

Analytic theories of CSR driven MWI threshold (cont'd)

- The case of absolute impedance $Z(k)$
  - **Application-2**: Scaling law of CSR instability with parallel-plates steady-state model [1]:

$$I_{\rm th2} = \frac{4\pi(E/e)\eta\sigma_\delta^2\sigma_z^{1/3}}{Z_0\rho^{1/3}}S_{\rm th2} \qquad S_{\rm th2} \approx 0.384\Pi^{2/3} \qquad \Pi \equiv \sigma_z\sqrt{\rho/h^3}$$

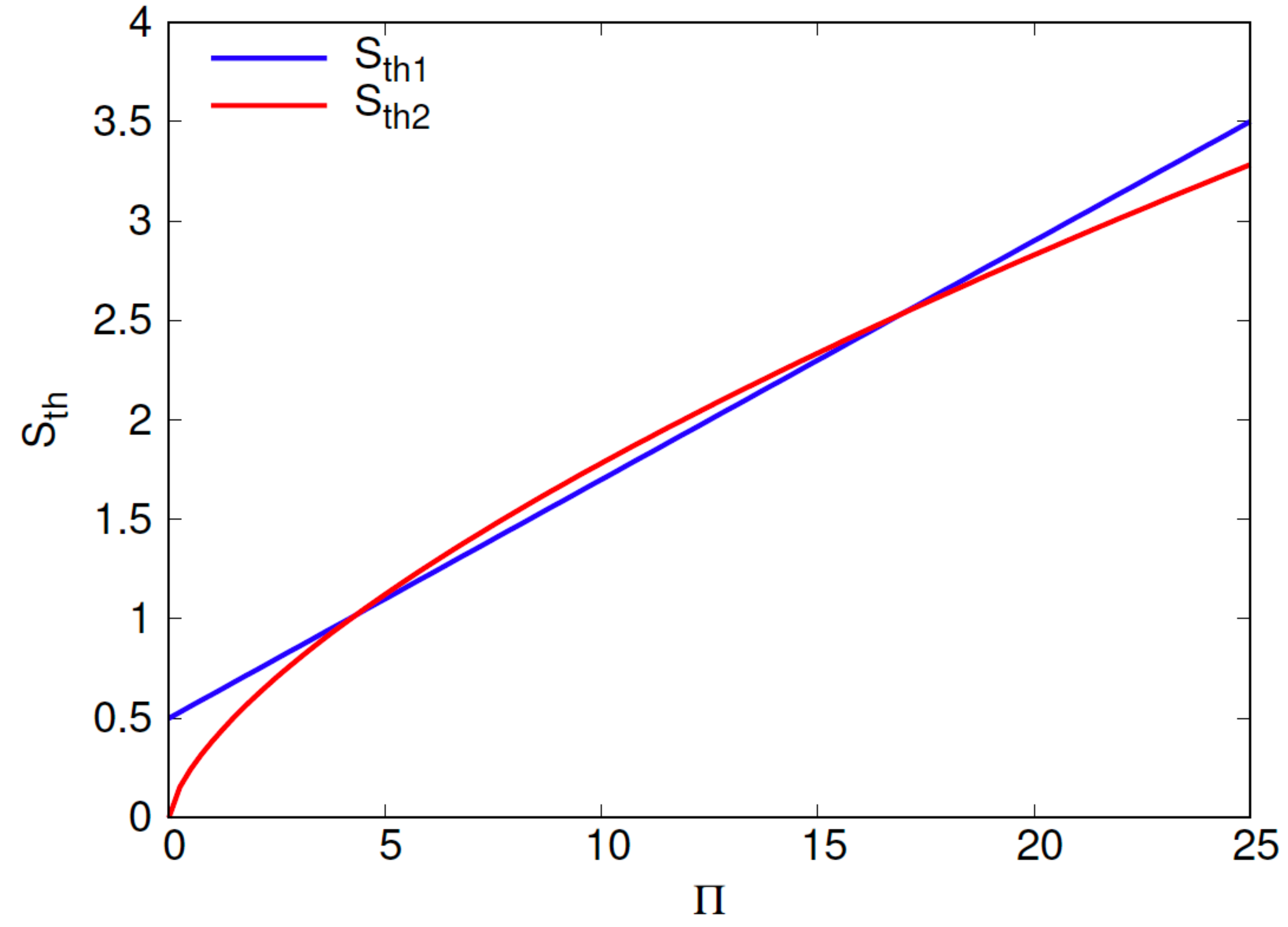
  - This scaling law (first found by Y. Cai [2]) is valid when $\Pi \gg 0.5$. It suggests CSR threshold is proportional to $\gamma\eta\sigma_\delta^2\sigma_z^{4/3}$, but independent of ρ [2].
  - The linear scaling law of $S_{\rm th1}$ is an approximation of $S_{\rm th2}$.
  - **Application-3**: Scaling law of Resistive Wall (RW) instability [1]:

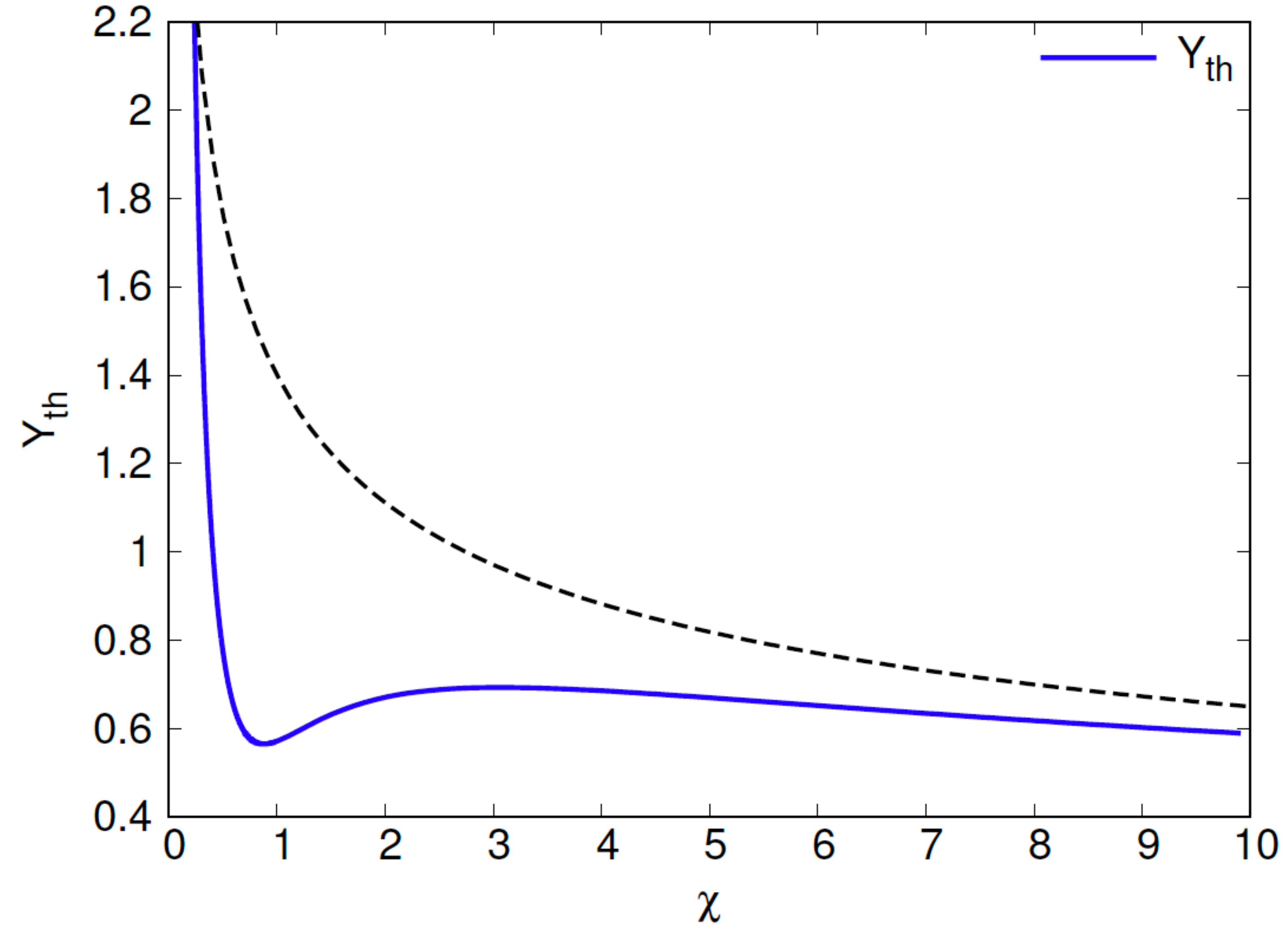
$$Z_\parallel^{\rm RW}(k) = \frac{f_Y Z_0 L_{\rm RW}}{\pi h\left(2\sqrt{\frac{iZ_0\sigma_c}{k}} - ihk\right)} \qquad \chi = \frac{\sqrt{2Z_0\sigma_c}}{hk^{3/2}}$$

$$I_{\rm th} = \frac{2\pi^2(E/e)\eta\sigma_\delta^2\sigma_z(2Z_0\sigma_c h)^{2/3}}{f_Y Z_0 L_{\rm RW}}{\rm Min}[Y_{\rm th}(\chi)] \qquad Y_{\rm th}(\chi) = \frac{1}{G_{\rm i}(A_{\rm th})\chi^{1/3}}$$

  - ${\rm Min}[Y_{\rm th}(\chi)] \approx 0.566$. The scaling law is valid when

$$\Pi_{\rm RW} \equiv \sigma_z\left(\frac{Z_0\sigma_c}{h^2}\right)^{1/3} \gg 0.73$$





[1] S. Dastan et al., to be published. [2] Y. Cai, IPAC2011, FRXAA01.

CSR impedance calculation

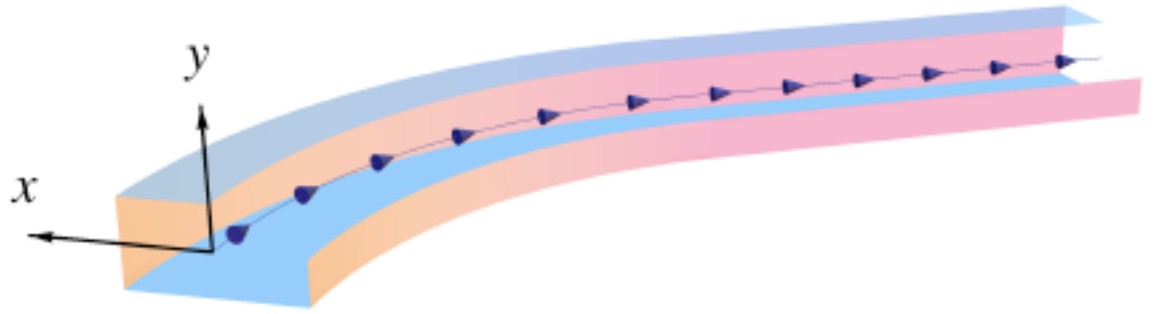
- Impedance calculation based on parabolic equation
  - G. Stupakov, T. Agoh et al. developed the method of calculating CSR impedance using parabolic equation (PE).
  - The CSRZ code following this line to solve PE:

$$\frac{\partial \vec{E}_\perp}{\partial s} = \frac{i}{2k}\left[\nabla_\perp^2 \vec{E}_\perp - \frac{1}{\epsilon_0}\nabla_\perp \rho_0 + 2k^2\left(\frac{x}{R(s)} - \frac{1}{2\gamma^2}\right)\vec{E}_\perp\right]$$

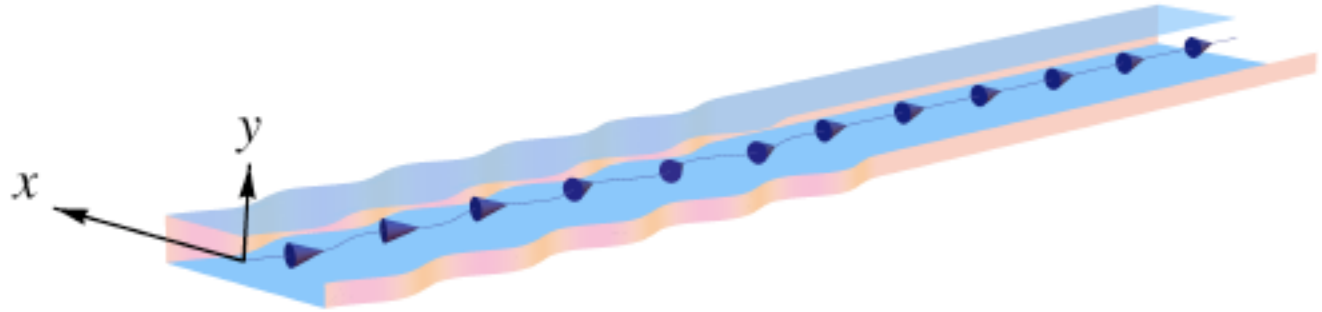
$$E_s = \frac{i}{k}\left(\nabla_\perp \cdot \vec{E}_\perp - \mu_0 c J_s\right) \qquad Z(k) = -\frac{1}{q}\int_0^\infty E_s(x_c, y_c)ds$$

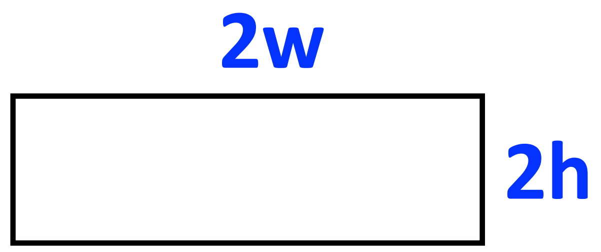
  - CSRZ takes into account: Arbitrary curvature of beam orbit $R(s)$ (CSR), finite beam energy γ (space charge effects, SC), and resistive wall (RW). The total impedance is not a simple sum of $Z_{CSR} + Z_{SC} + Z_{RW}$, but includes their interaction.
  - CSRZ uses Gaussian charge distribution in x-y plane, assuming $\sigma_x > \sigma_y$. Self-field is calculated by Bassetti-Erskine formulae.
  - Currently, CSRZ assumes uniform rectangular chamber referring to the beam orbit.
  - See [1] for an overview and [2] for details of CSRZ code.
  - See [3,4,5] for recent applications.

A single bend



A wiggler with "wiggling" chamber





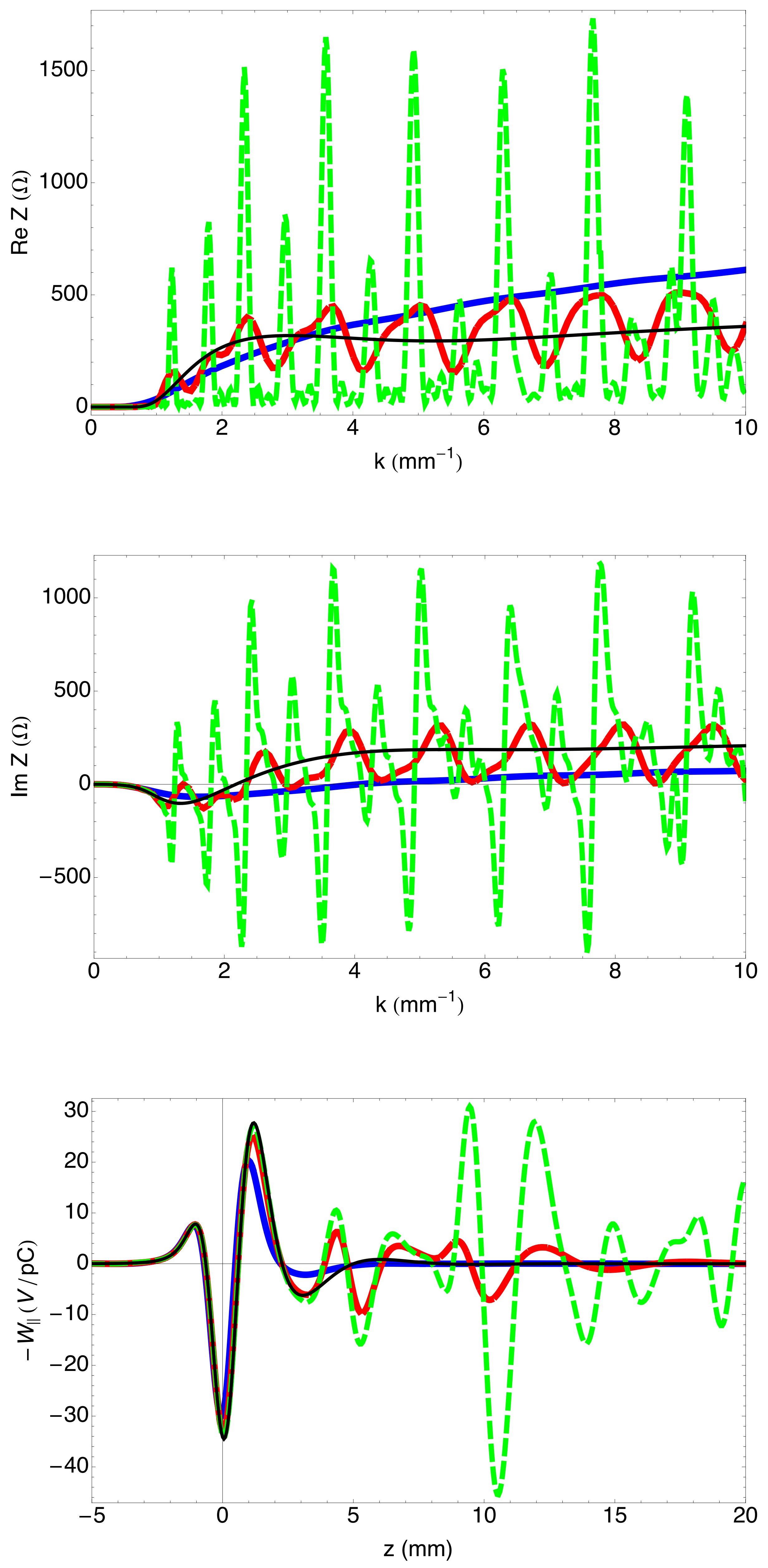
[1] D. Zhou et al., "An Alternative 1D Model for CSR with Chamber Shielding", in Proceedings of IPAC'12, New Orleans, Louisiana, USA.

[2] D. Zhou, Coherent Synchrotron Radiation and Microwave Instability in Electron Storage Rings, Ph.D. thesis, SOKENDAI and KEK, 2011.

[3] G. Stupakov and D. Zhou, PRAB 19, 044402 (2016). [4] A. Gamelin, et al., NIM-A 999 (2021): 165191. [5] L. Carver et al., PRAB 26, 044402 (2023).

CSR impedance calculation (cont'd)

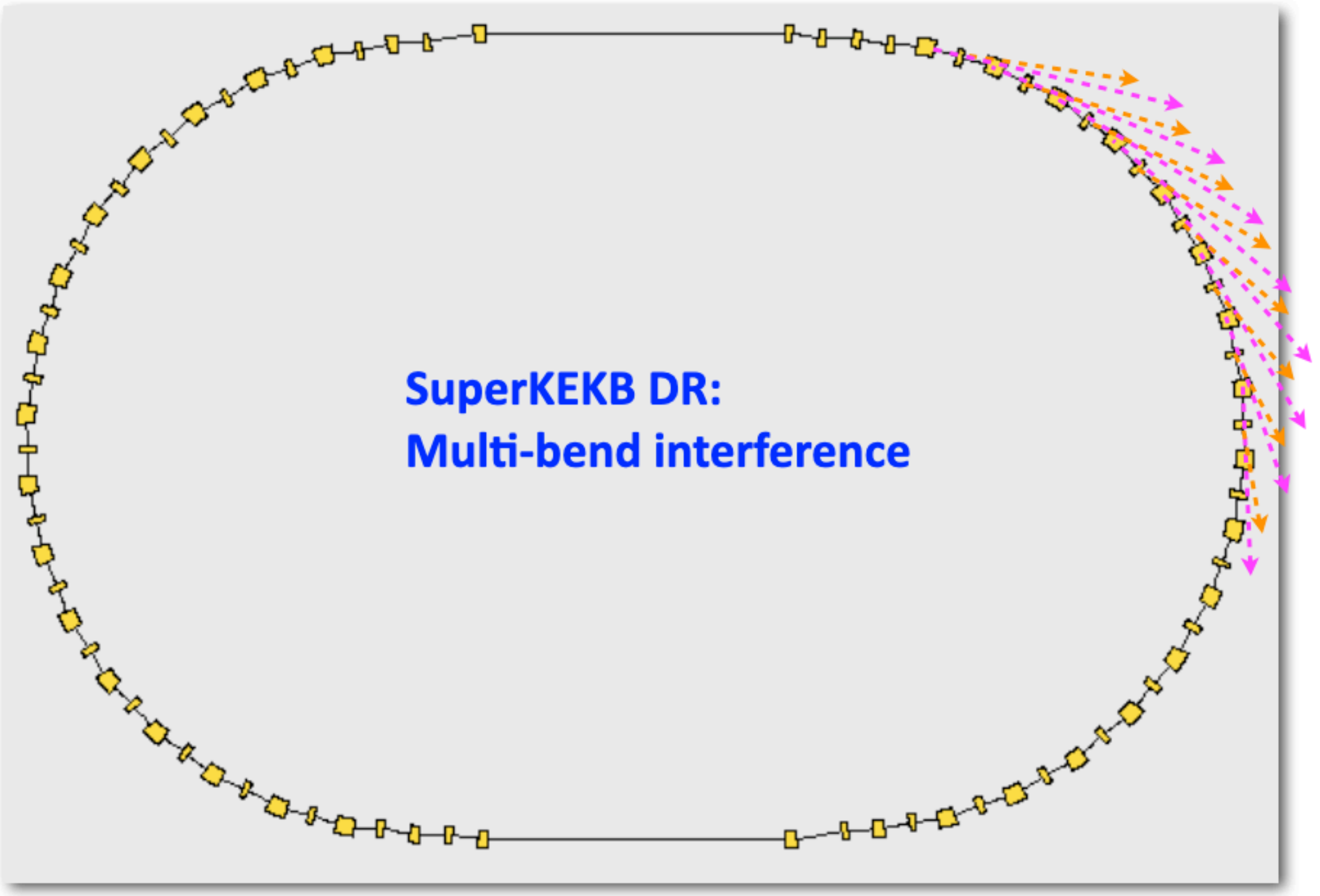
- Examples of CSR impedance by CSRZ
  - A single bend with varied length: w/h=30/15 mm, R=5 m, L_{bend}=0.5/2/8 m.
  - Black/Blue/Red/Green lines: Steady-state parallel-plates/L=0.5/L=2/L=8 m. For convenience of comparison, the impedance amplitude is scaled to L=1 m.
  - "Short bend": Transient effect at the entrance and exit is important.
  - "Long bend": Excited eigenmodes of a toroidal chamber (or "whispering gallery modes" by R. Warnock [1]).
  - "Overtaking field": Short-range wake fields, space charge like.
  - "Trailing field": Long-range wake fields, relevant to excited eigenmodes.

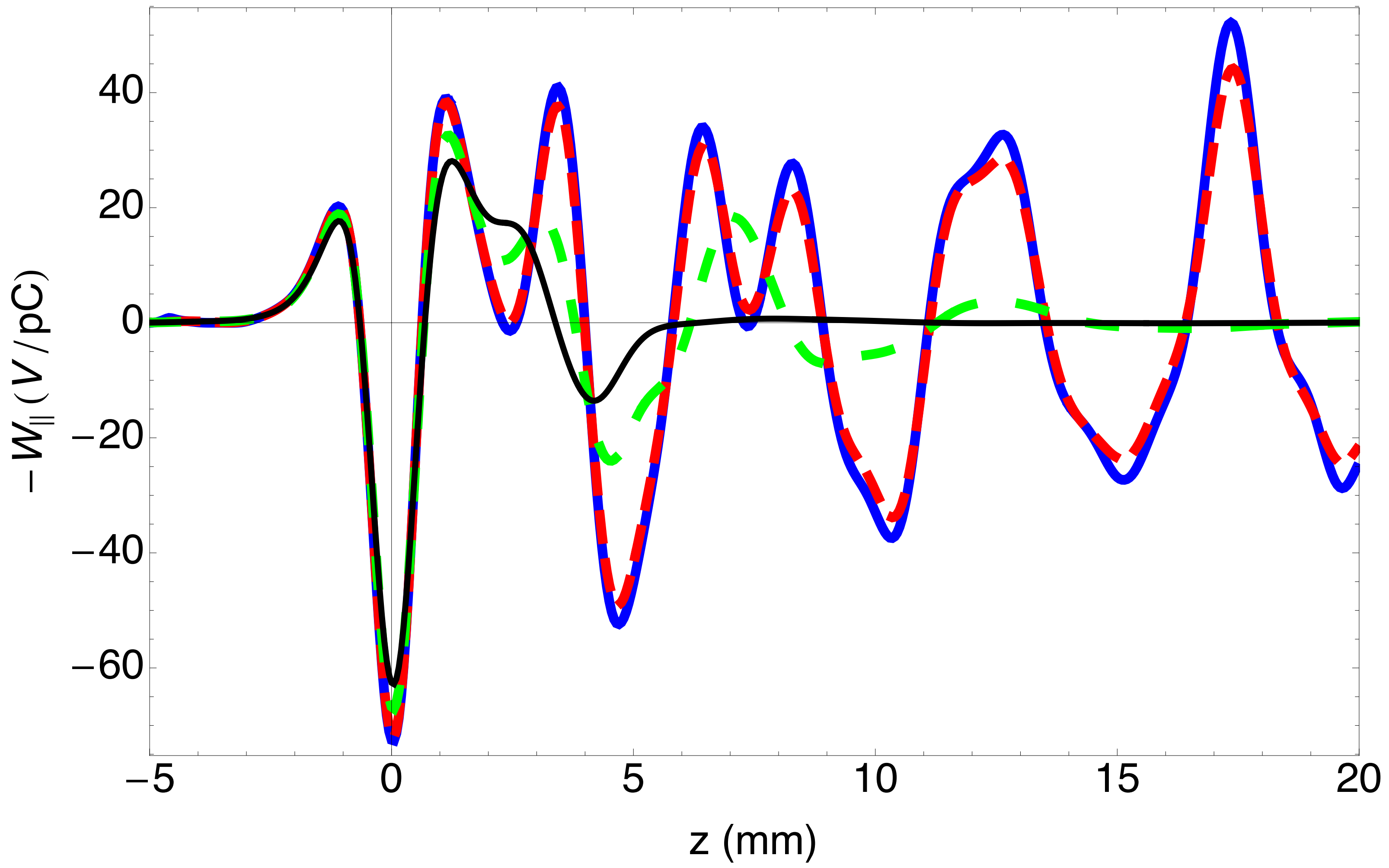


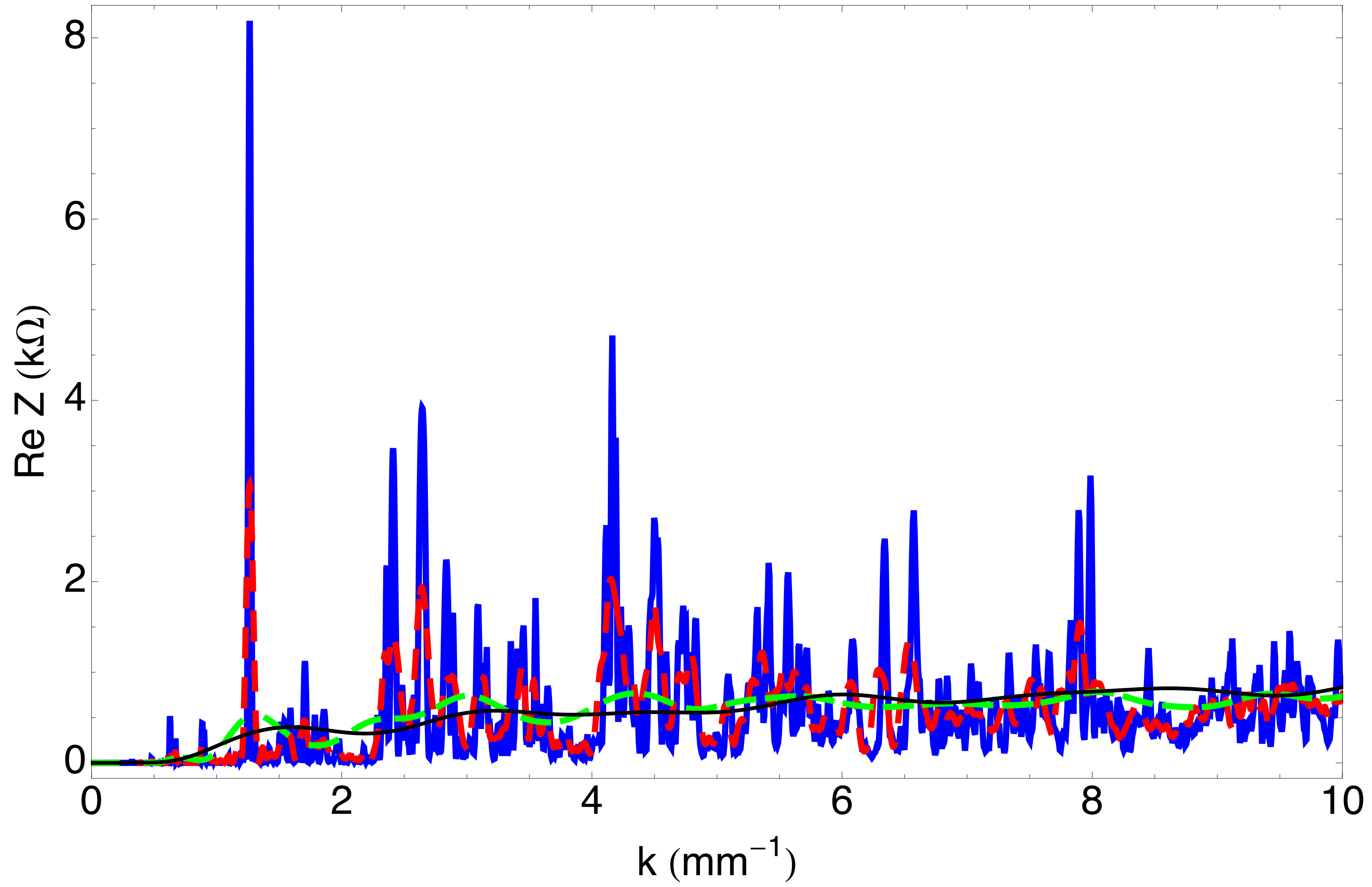
[1] R. Warnock, arXiv:1708.05500.

CSR impedance calculation (cont'd)

- Examples of CSR impedance by CSRZ
  - A realistic ring with multiple-bends: assuming smooth chamber.
  - SuperKEKB DR as an example [1]: a/b=34/34 mm, L_{bend}=0.74/0.29 m, R=2.7/-3 m (reverse bends), L_{drift}=0.9 m, N_{cell}=1/6/16.
  - Multi-bend interference: CSR fields generated by multiple bends propagate along the chamber together with the beam. The fields interfere to produce a pattern of "narrow-band spikes".
  - The real part of CSR impedance should correspond to SR spectrum in measurement.







[1] D. Zhou, et al., Jpn. J. Appl. Phys. 51 (2012) 016401.

Prediction of CSR instability via simulations

- **Check list** before running simulations
  - Scaling law with PP-SS CSR model:

$$I_{\text{th2}} = \frac{4\pi(E/e)\eta\sigma_\delta^2\sigma_z^{1/3}}{Z_0\rho^{1/3}}S_{\text{th2}}$$

→ Global picture

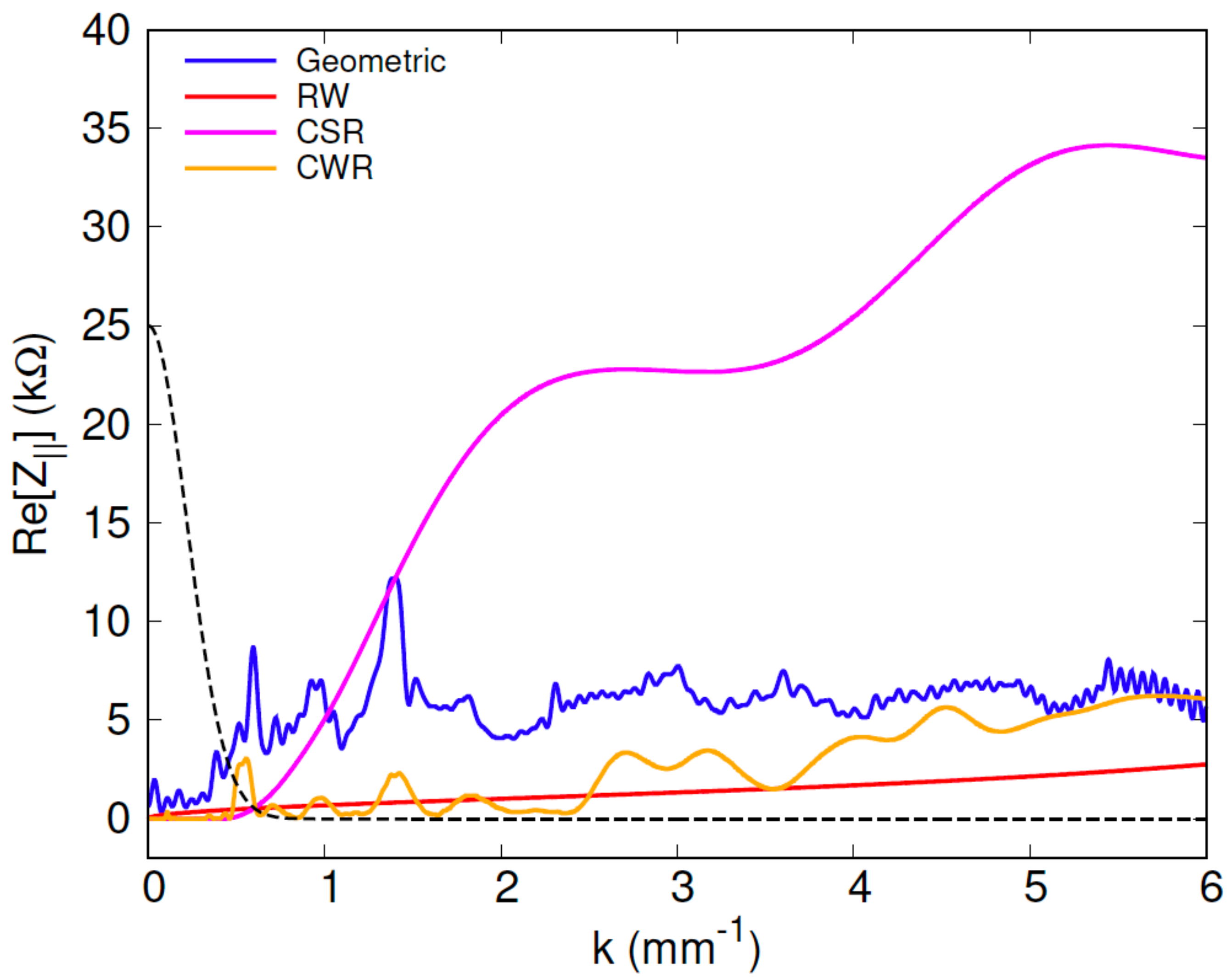
  - Critical wavenumber: $k_c = 3\gamma^3/(2\rho)$. Interaction distance of CSR: $z \gg 1/k_c$.
  - Wall shielding threshold: $k_w = \pi\sqrt{\rho/(2h)^3}$. When $\sigma_z \ll 1/k_w$, wall shielding is not crucial; when $\sigma_z \gg 1/k_w$, wall shielding becomes important (according to $S_{\text{th1}} \approx 0.5 + 0.11\sigma_z k_w$).
  - NOTE: $\sigma_z \gg 1/k_w$ does not mean CSR is negligible. In theory, there always is a finite I_{th} for any $\sigma_z k_w$.
  - Critical CSR wavenumber: $k_{\text{th}} = 2\sqrt{\rho/h^3} \sim 2k_w$. CSR around k_{th} determines the threshold current.
  - Radiation formation length: $l_f = (24\rho^2\sigma_z)^{1/3}$. For long magnet $l_b \gtrsim l_f$, transient effects are negligible; for short magnet $l_b < l_f$, transient effects become significant.
  - Catch-up distance: $l_c = 2\sqrt{2\rho w}$ with w the distance from the beam orbit to the side wales and path difference $\Delta s = \frac{4}{3}\sqrt{\frac{2w^3}{\rho}}$. When $\Delta s \lesssim \sigma_z$, reflected CSR plays a role.
  - Slippage length: $l_s = \eta\sigma_\delta C$. Lumping the CSR impedance of distributed bends into one point is valid only when $l_s \ll \lambda_{\text{CSR}}$.

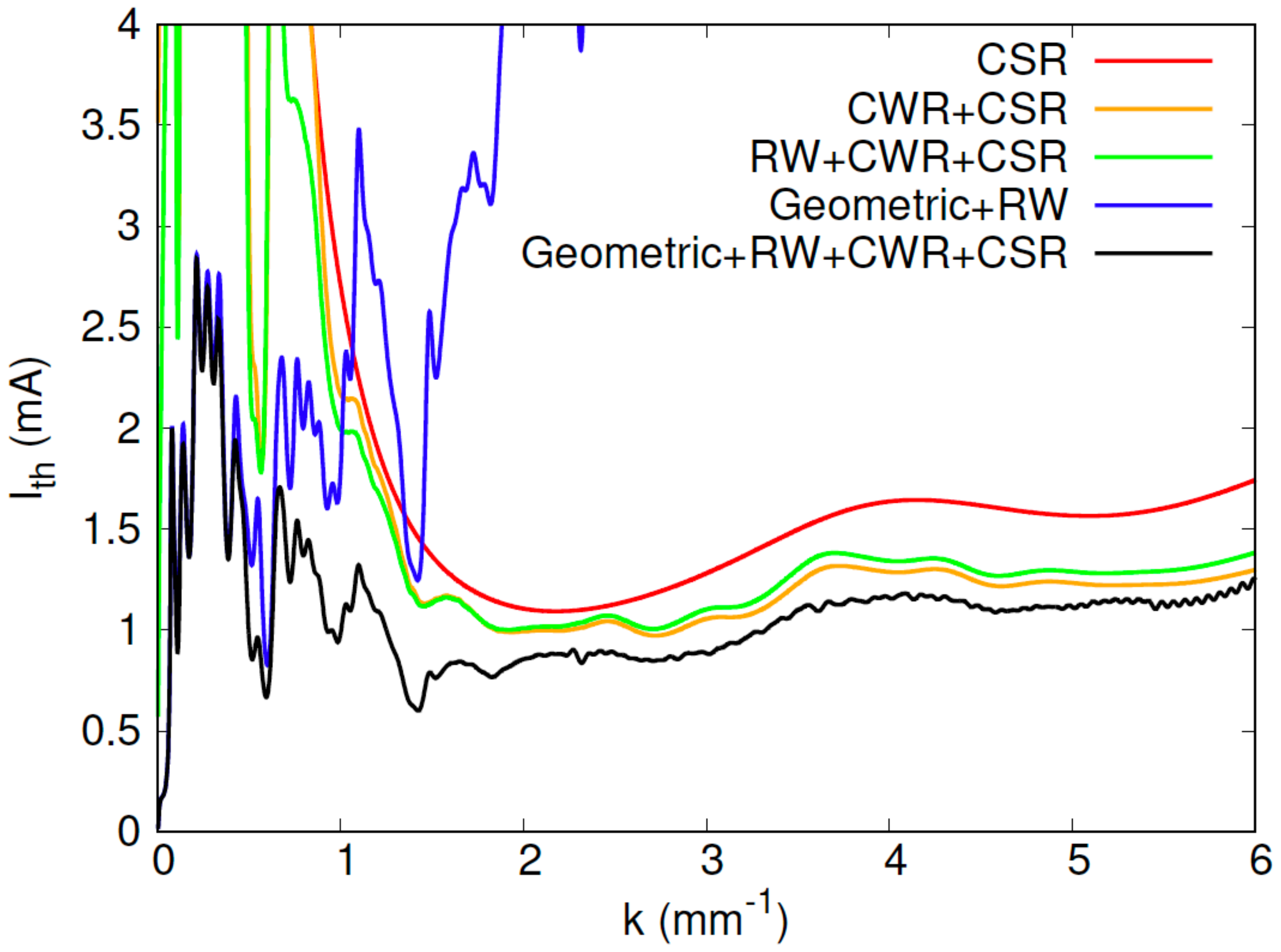
→ Proper choices of Impedance models
→ Proper setup of simulations

Prediction of CSR instability via simulations (cont'd)

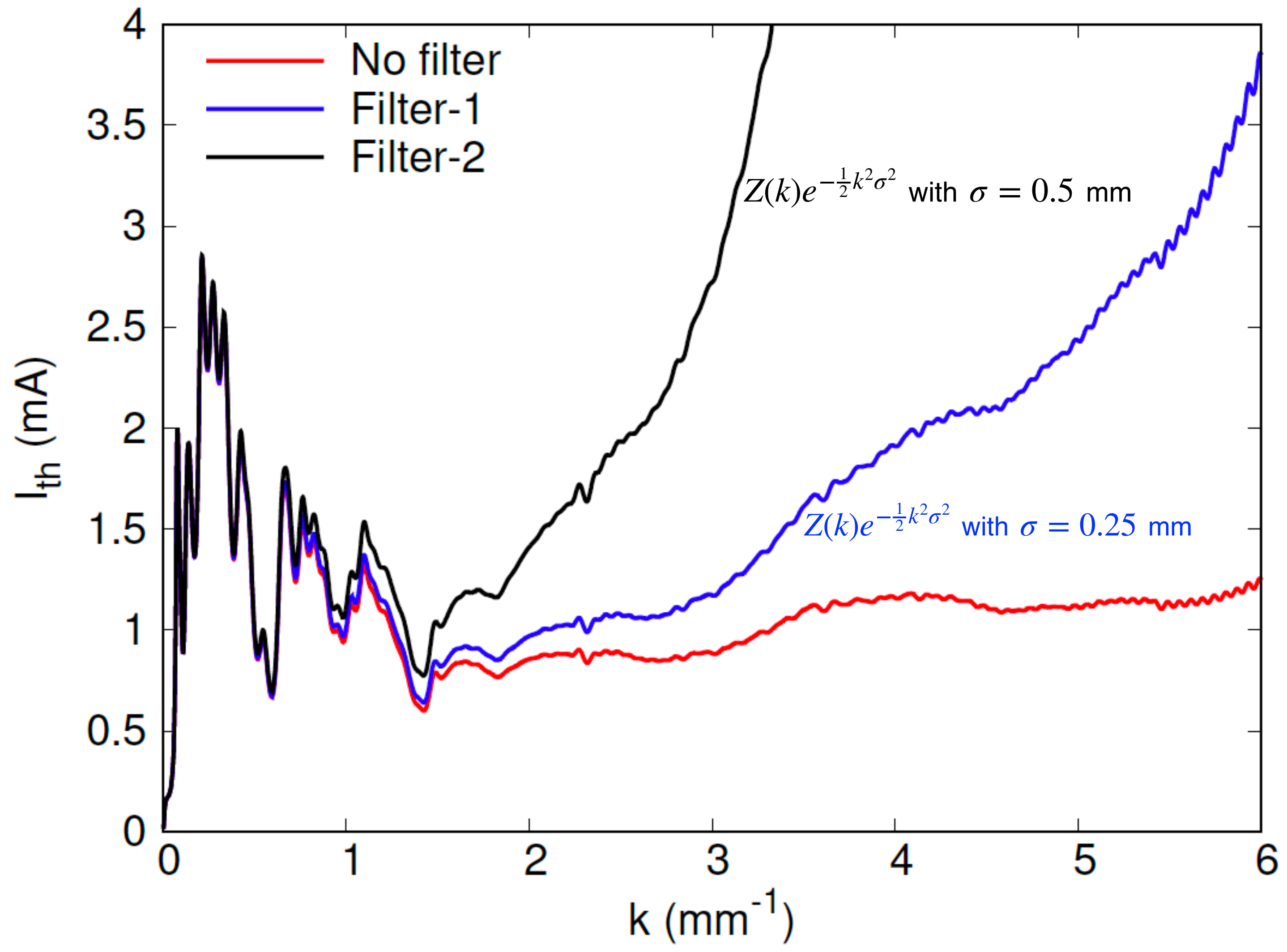
$h = 45$ mm, $\rho = 74.7$ m

- Example-1: SuperKEKB LER
  - Step-1: Impedance modeling of CSR/CWR by CSRZ, RW by IW2D, and geometric wakes by GdfidL, CST, and ECHO3D [1].
  - Step-2: Instability analysis to determine $I_{th}(k)$.
  - Step-3: Choosing important parameters: maximum k_{max} for impedance model, minimum mesh size $\Delta z \ll 2\pi/k_{max}$.
    - k_{max}=6 mm^{-1} $\rightarrow$ f_{max}=286 GHz.
  - Note: Be careful in choosing filtering function to damp high-frequency impedances.

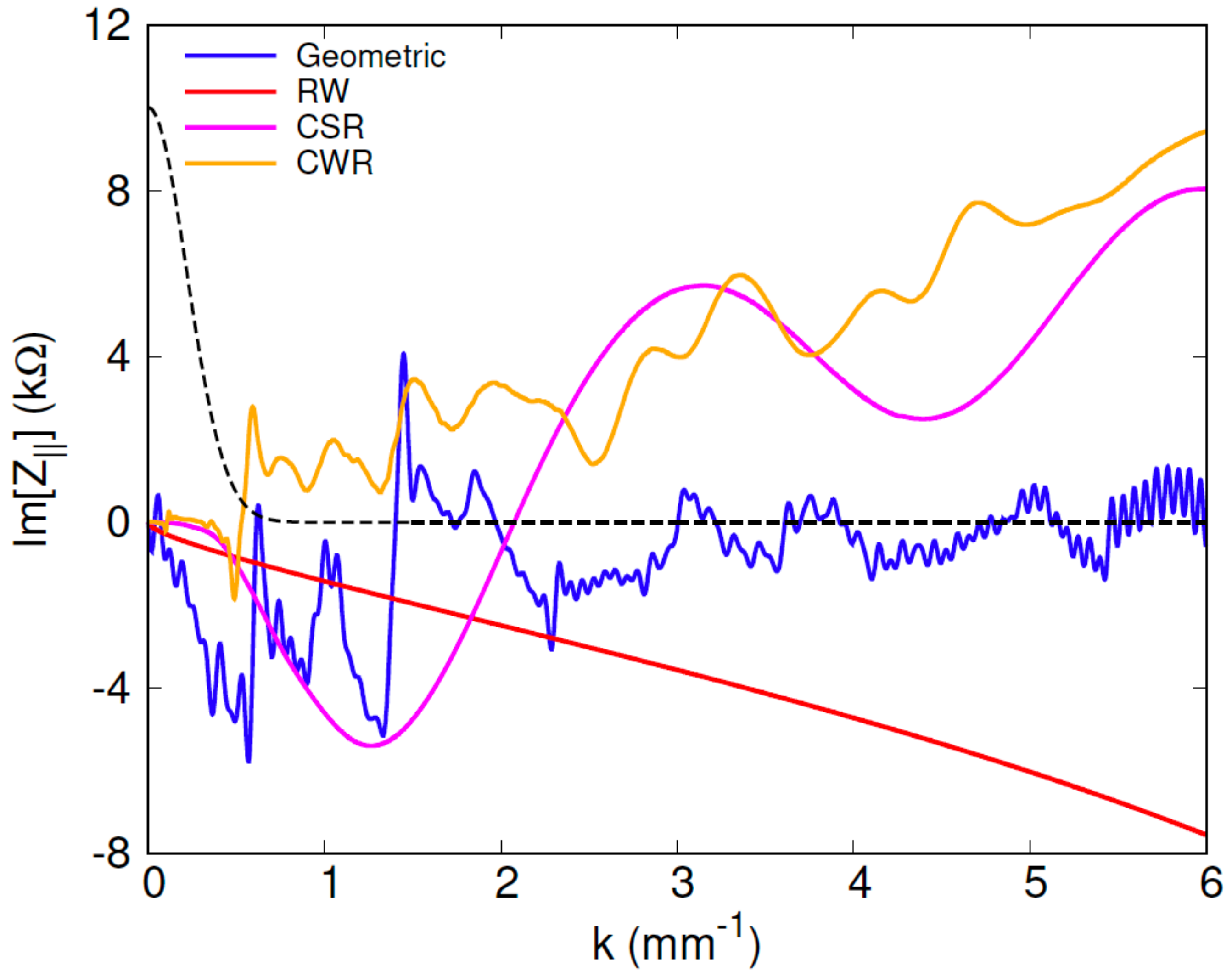




CSR is important source for MWI in SuperKEKB LER



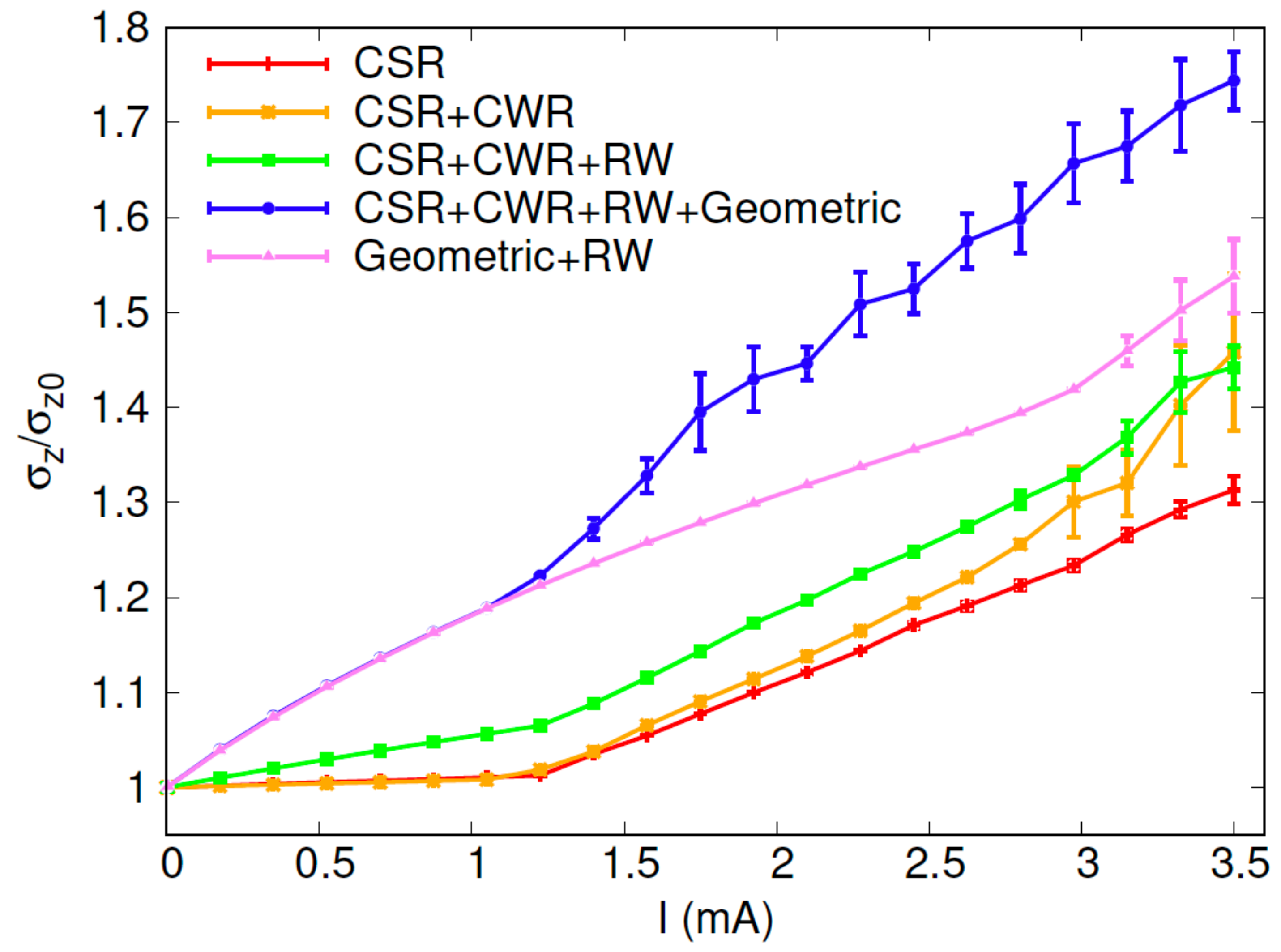
Filtered impedance models underestimate MWI threshold

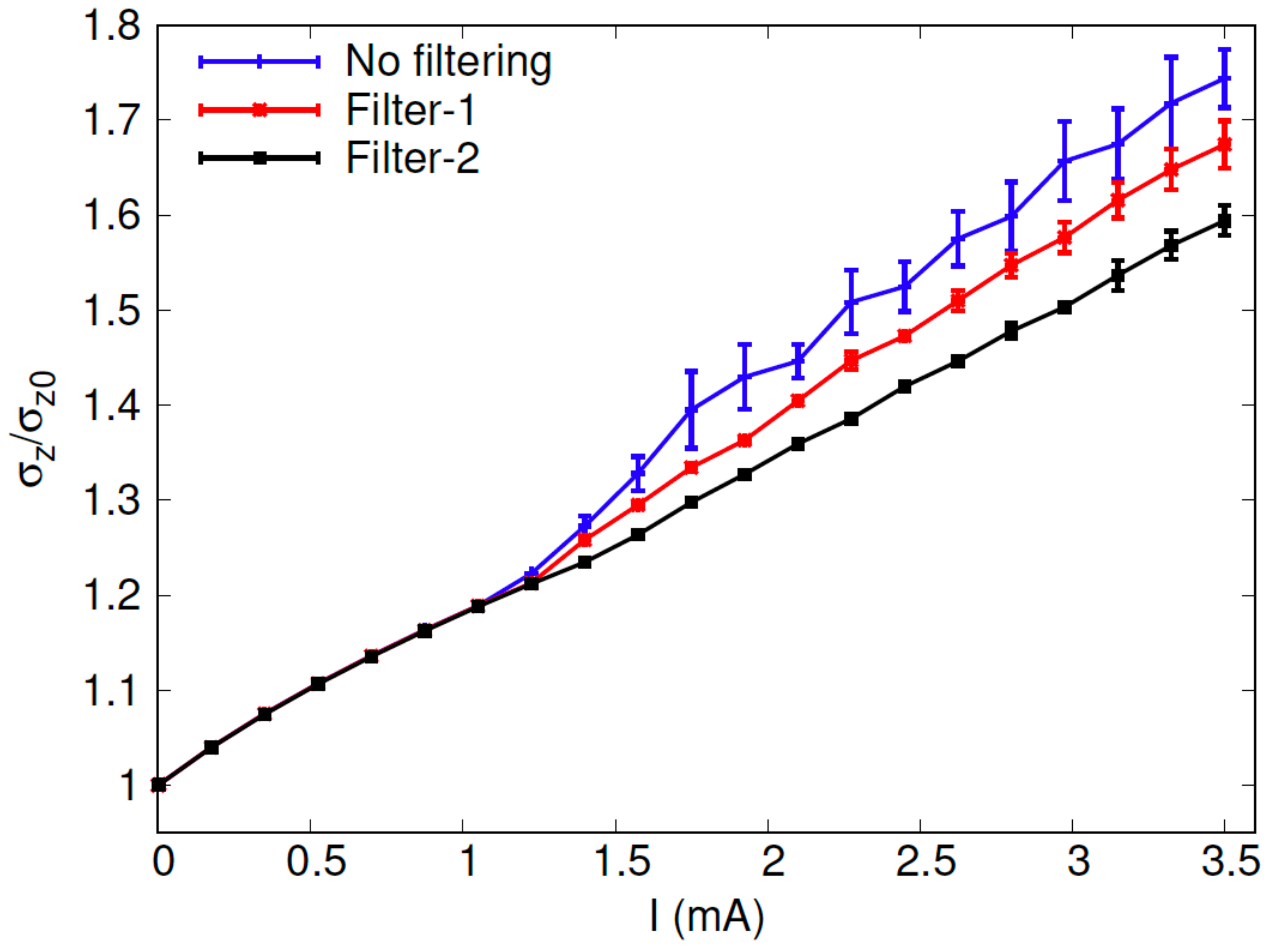


[1] T. Ishibashi et al 2024 JINST 19 P02013.

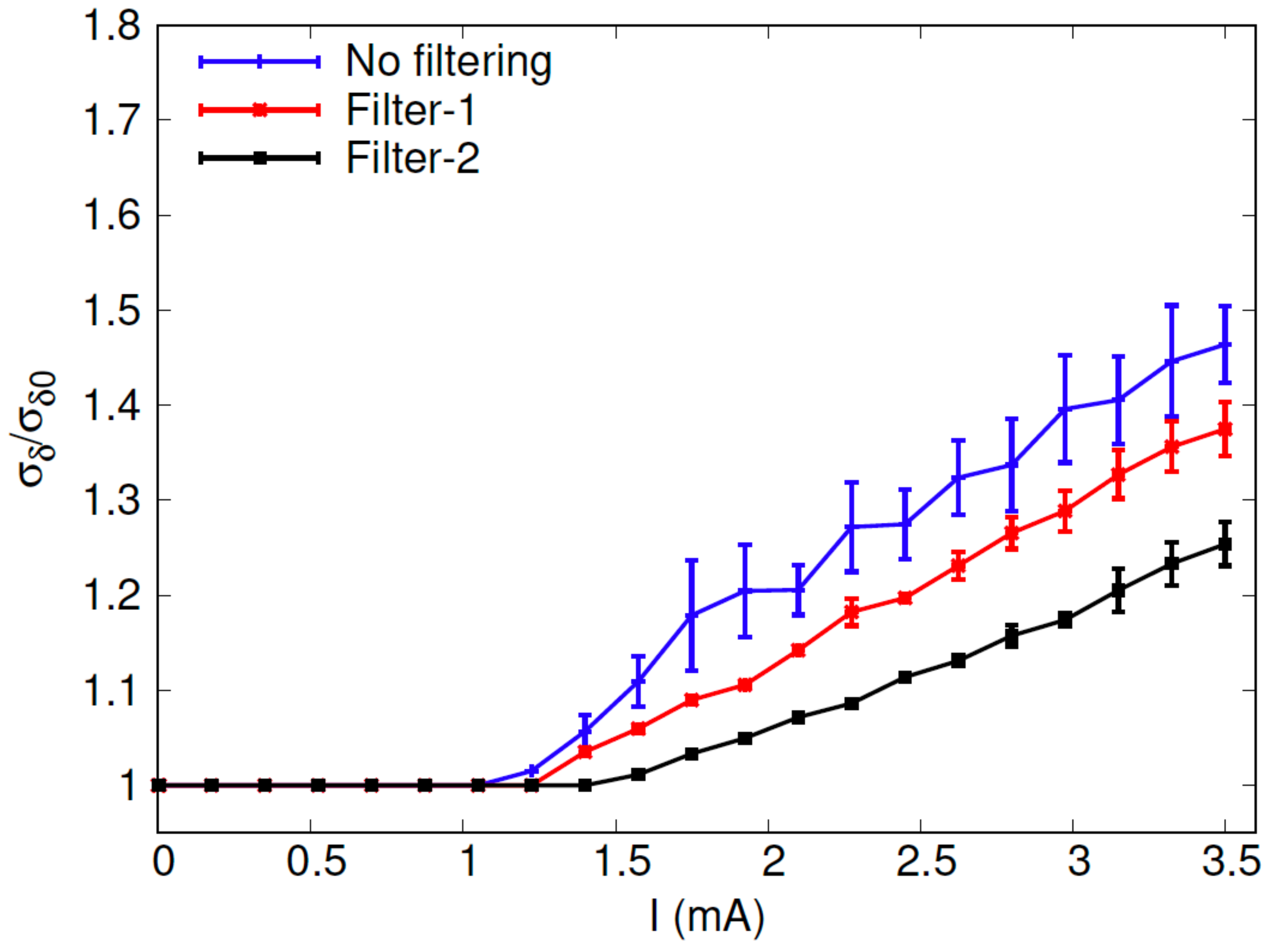
Prediction of CSR instability via simulations (cont'd)

- Example-1: SuperKEKB LER
  - Step-4: Run VFP simulations.
    - Different combinations of impedance sources: CSR sets MWI threshold
    - Different filtering functions for impedance model
  - Step-5: Check consistency between theories and simulations.
    - "Numerical arts": Interpolation, smoothing histogram, mesh size, number of wake kicks per turn, mesh boundaries, cutoff of impedance beyond $k_{\rm max}$, …
    - A good simulation should be well understood by a good theory

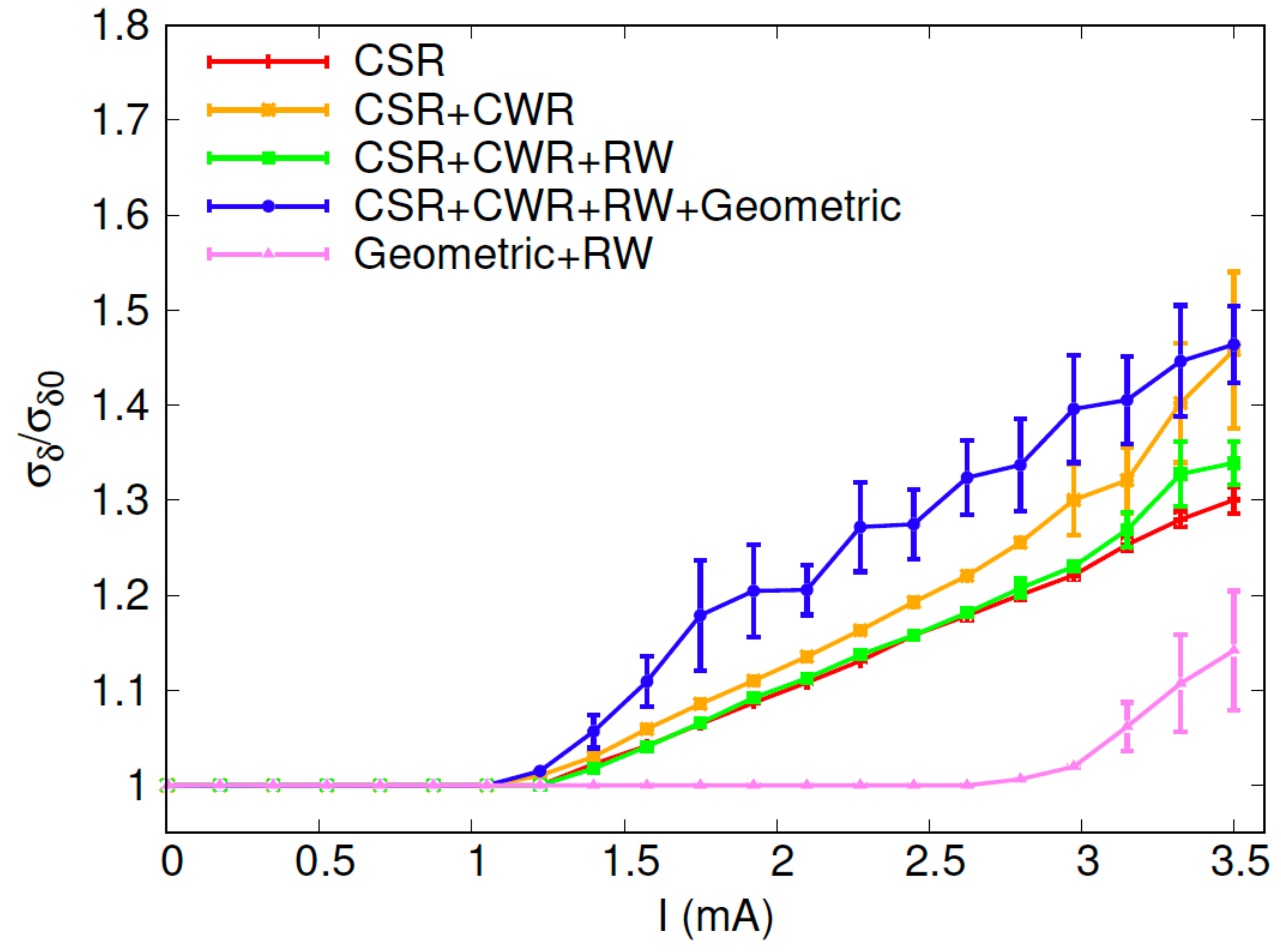




$\sigma_{z0} = 4.6$ mm

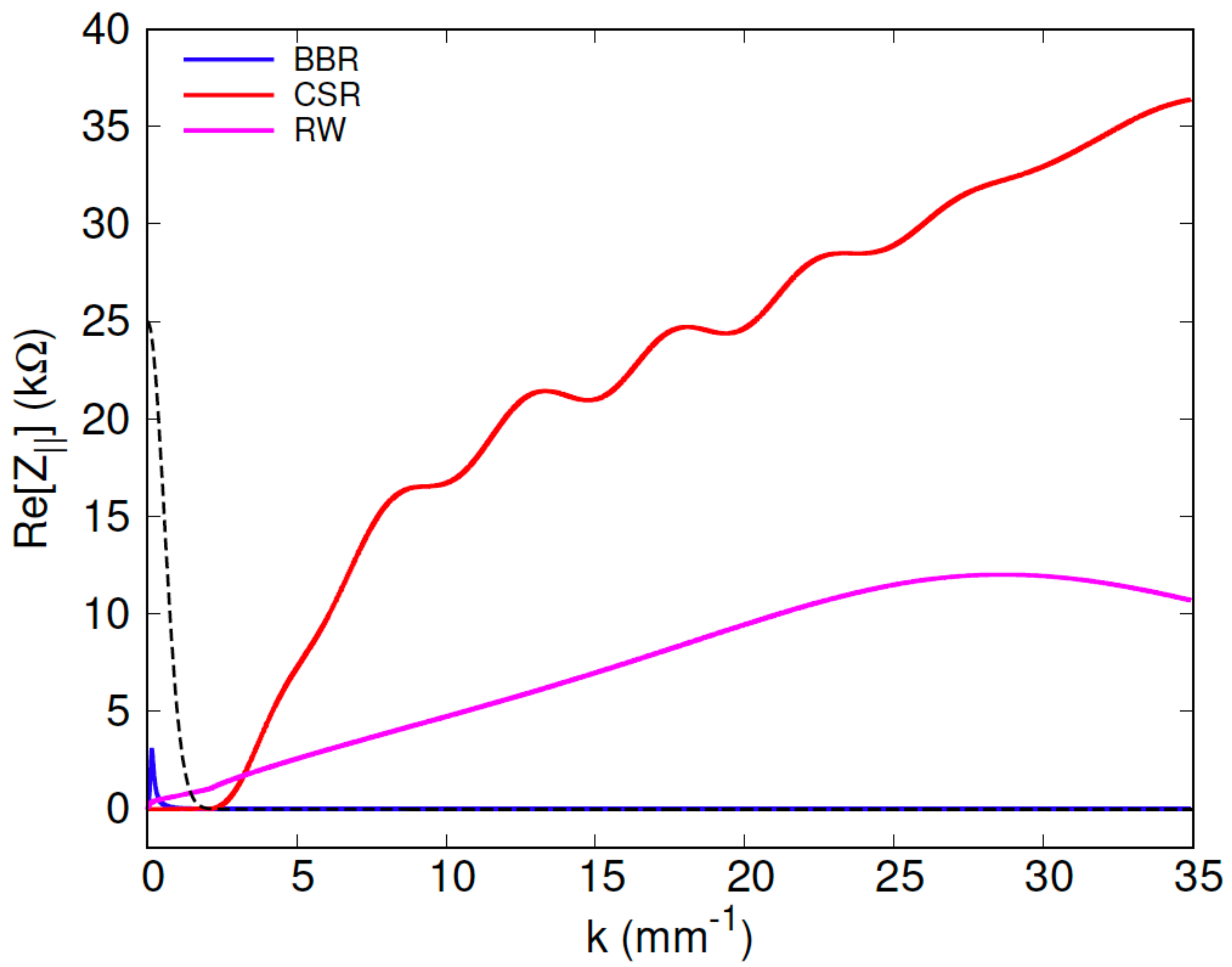


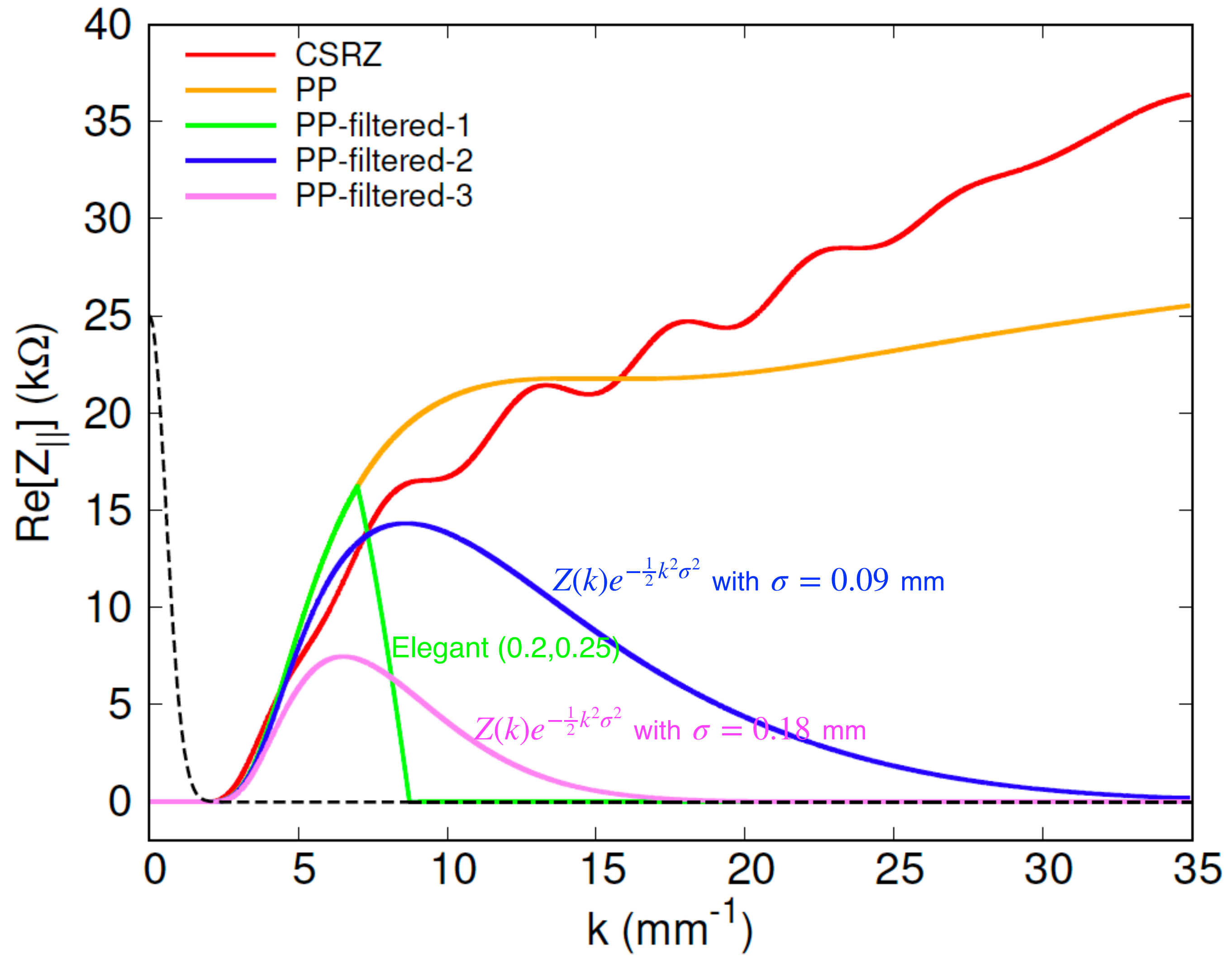
$\sigma_{\delta 0} = 7.53 \times 10^{-4}$

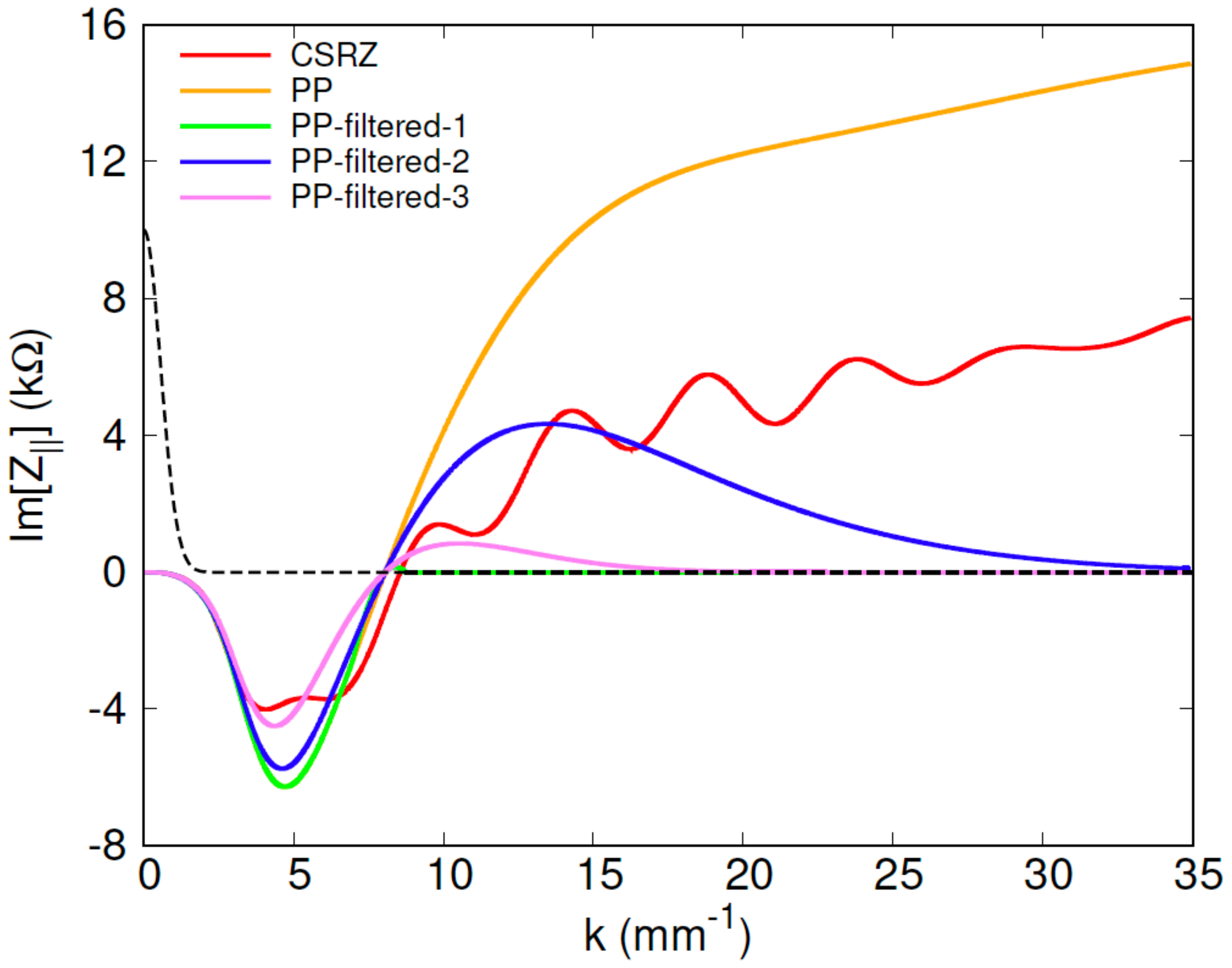


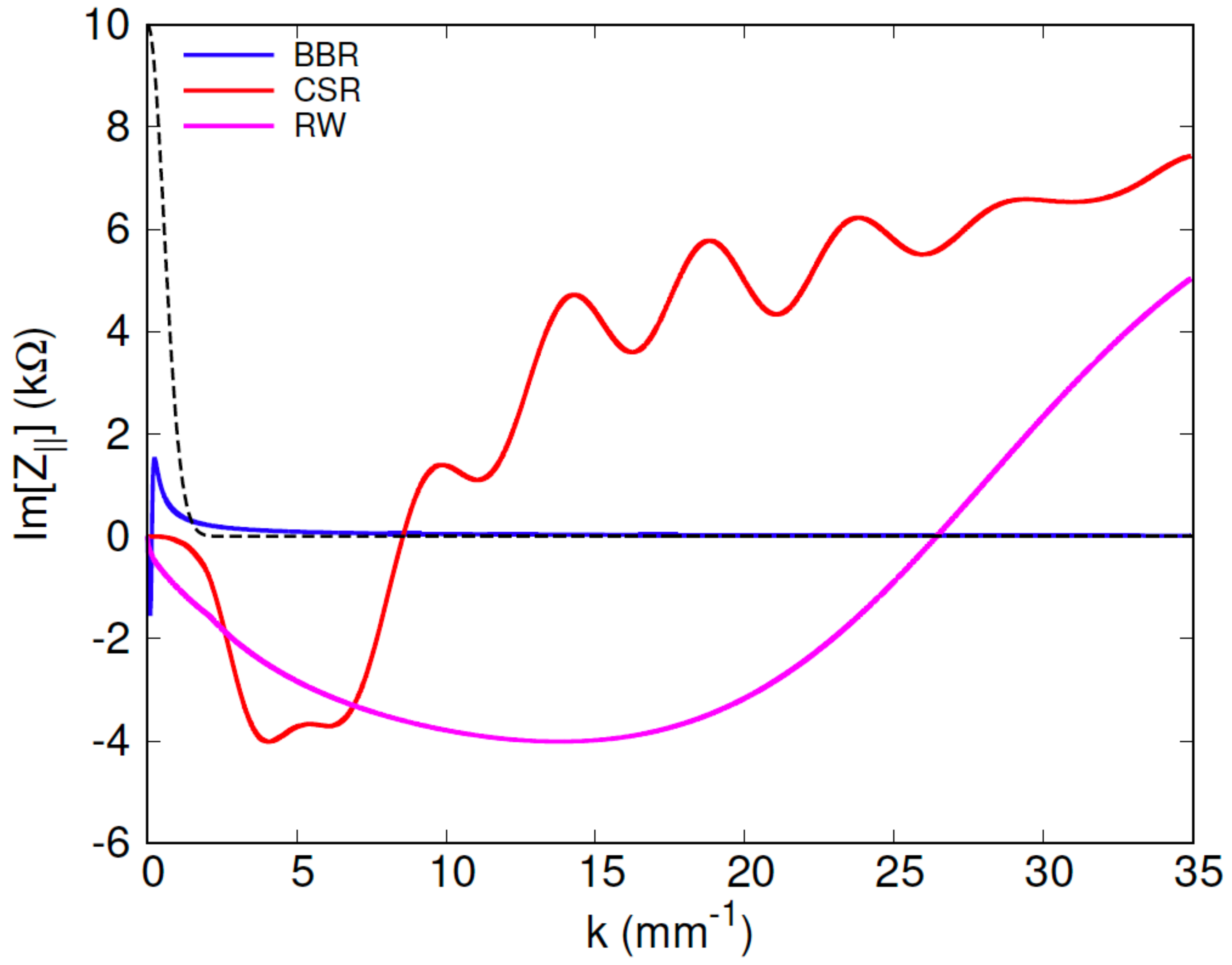
Prediction of CSR instability via simulations (cont'd)

- Example-2: Elettra 2.0
  - Step-1: Impedance modeling of CSR by CSRZ, RW by IW2D, and geometric wakes by broadband resonator (BBR).
    - Alternative CSR models: CSRZ, PP, filtered PP
    - $h = 7.5$ mm, $\bar{\rho} = 7.8$ m
    - BBR: Q=1, f_r=7 GHz, $R_s \approx 3000\ \Omega$ ($|\mathrm{Im}[Z_{\|}]/n| = 0.5\ \Omega$ [1])





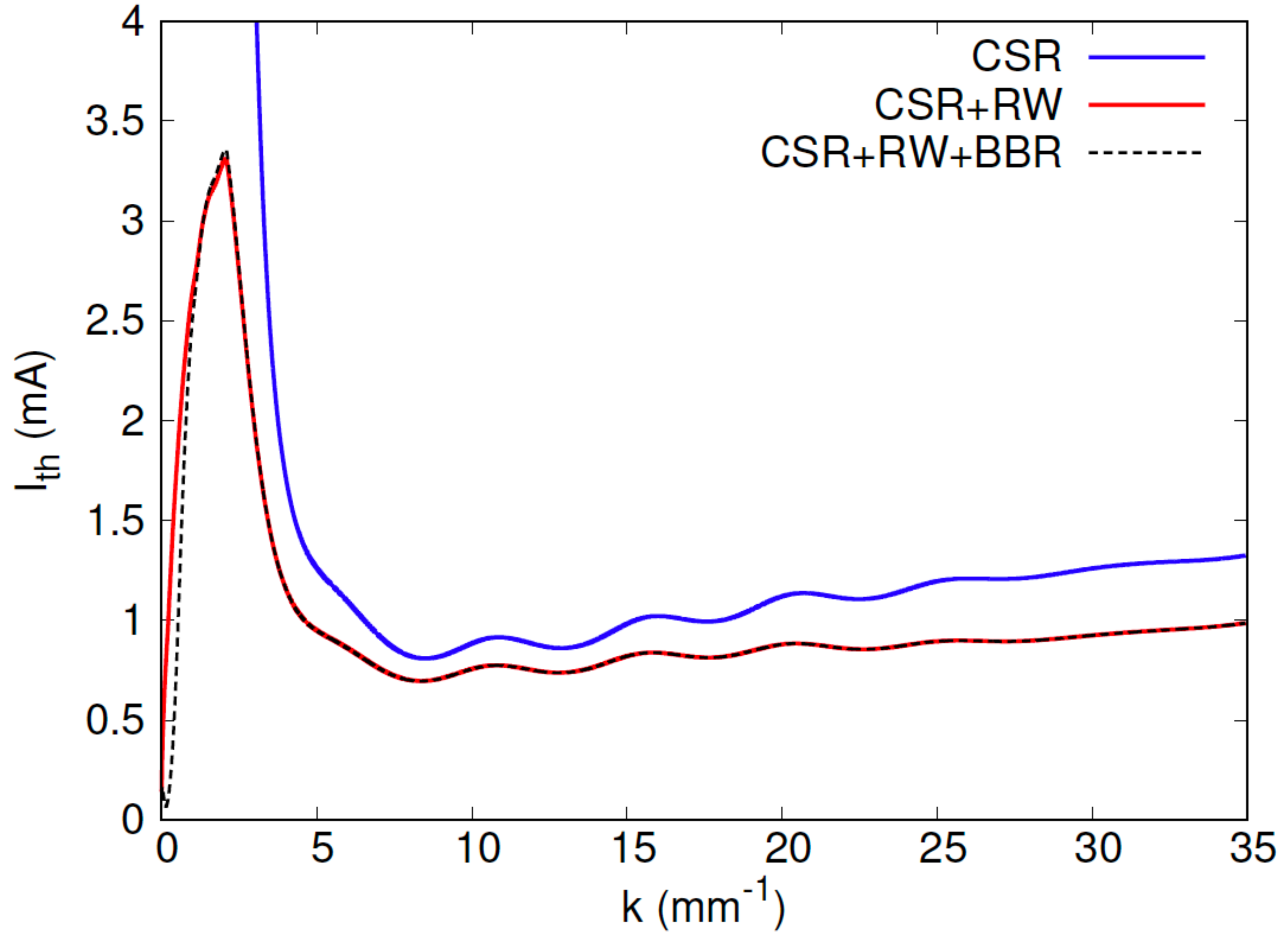




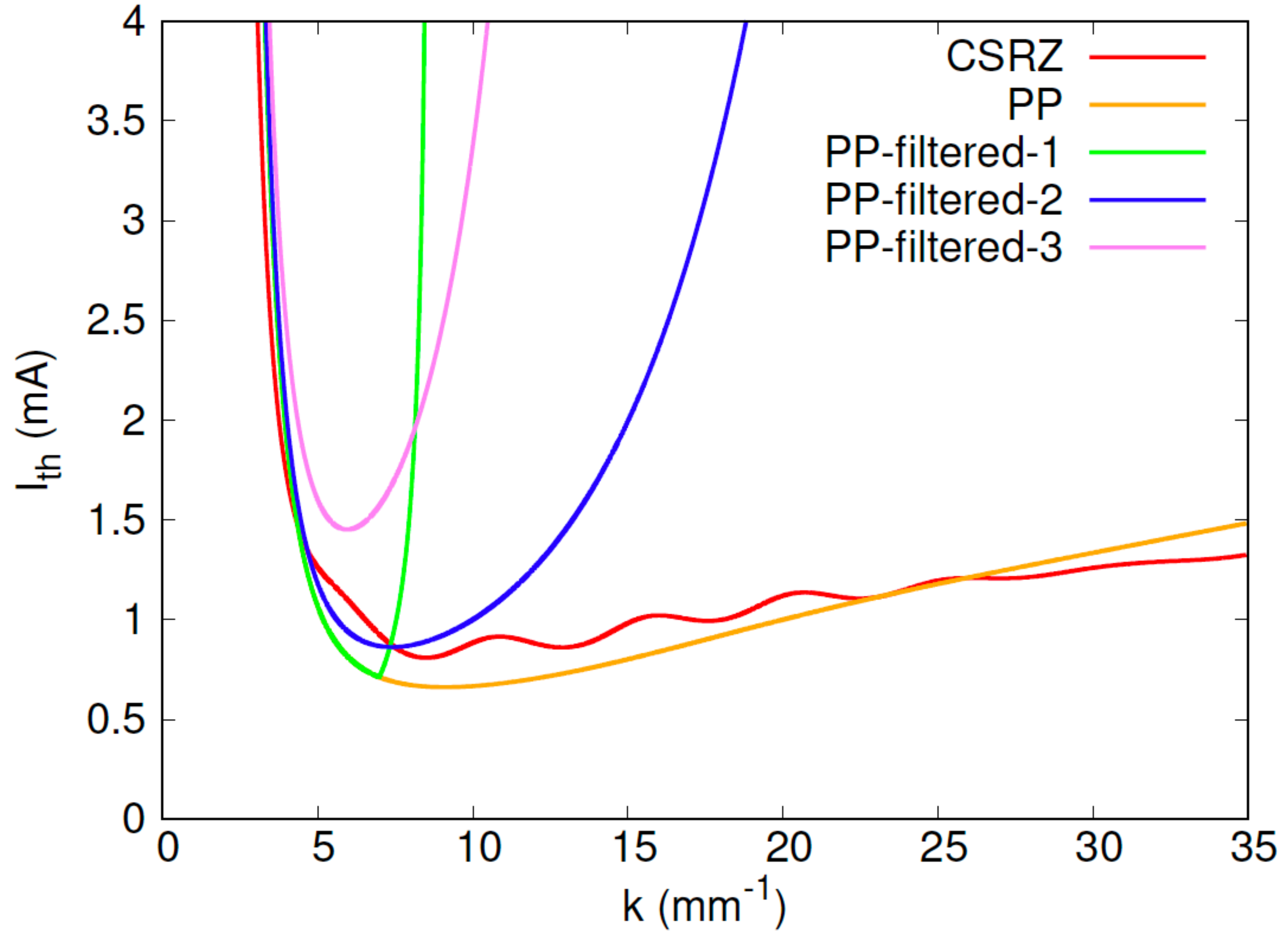
[1] Elettra 2.0 TDR, Part three: Machine and Infrastructure.

Prediction of CSR instability via simulations (cont'd)

- Example-2: Elettra 2.0
  - Step-2: Instability analysis to determine $I_{\text{th}}(k)$.
    - σ_{z0}=1.8 mm without harmonic cavity (3HC)
  - Step-3: Choosing important parameters: maximum k_{max} for impedance model, minimum mesh size $\Delta z \ll 2\pi/k_{\text{max}}$.
    - k_{max}=35 mm^{-1} $\rightarrow$ f_{max}=1.67 THz.



CSR and RW re important source for MWI in Elettra 2.0

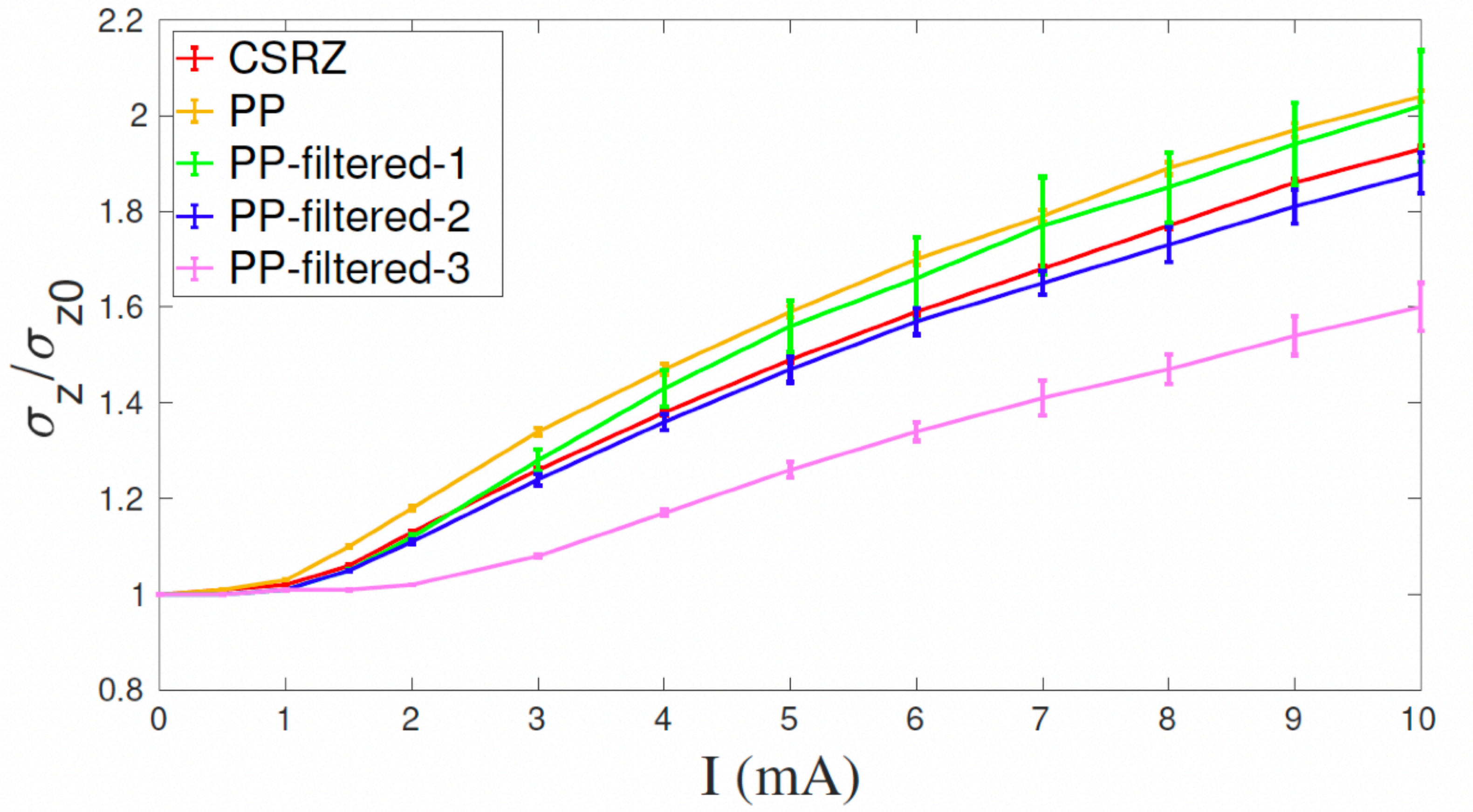


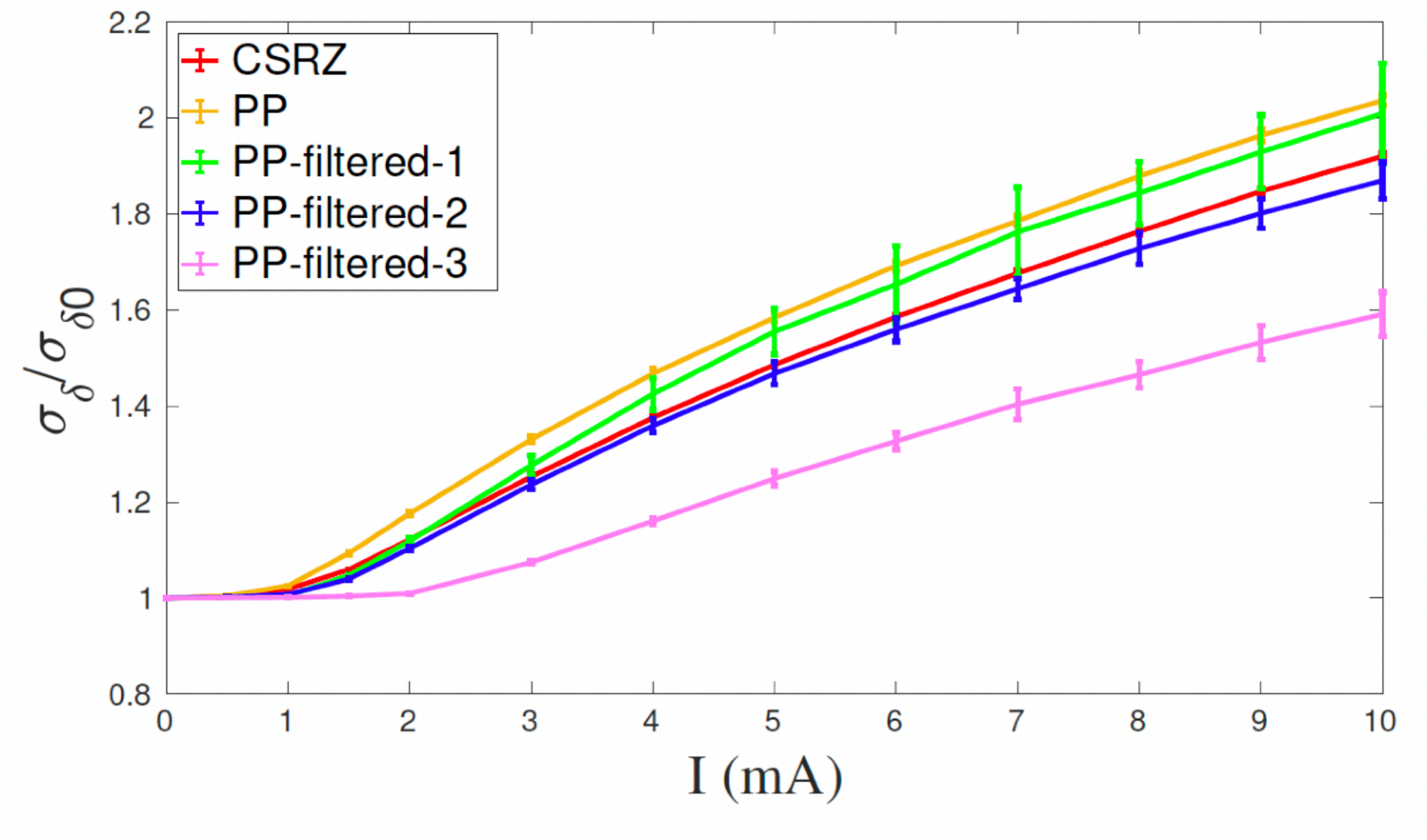
Filtered impedance models predicts higher MWI threshold

Prediction of CSR instability via simulations (cont'd)

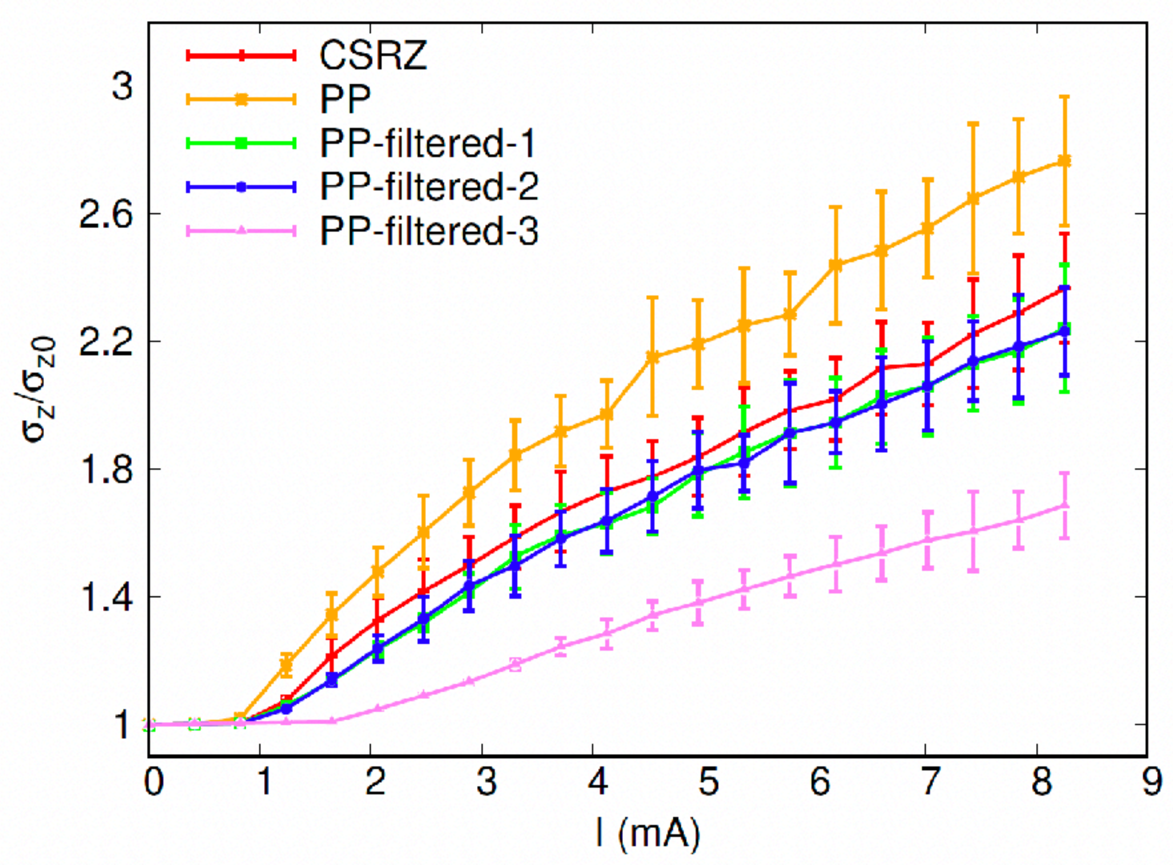
- Example-2: Elettra 2.0
  - Step-4: Run tracking simulations (Elegant) and VFP simulations (Only consider CSR)
    - General consistency between theory, Elegant and VFP simulations.
    - 3HC increases MWI threshold through reducing charge density as expected.

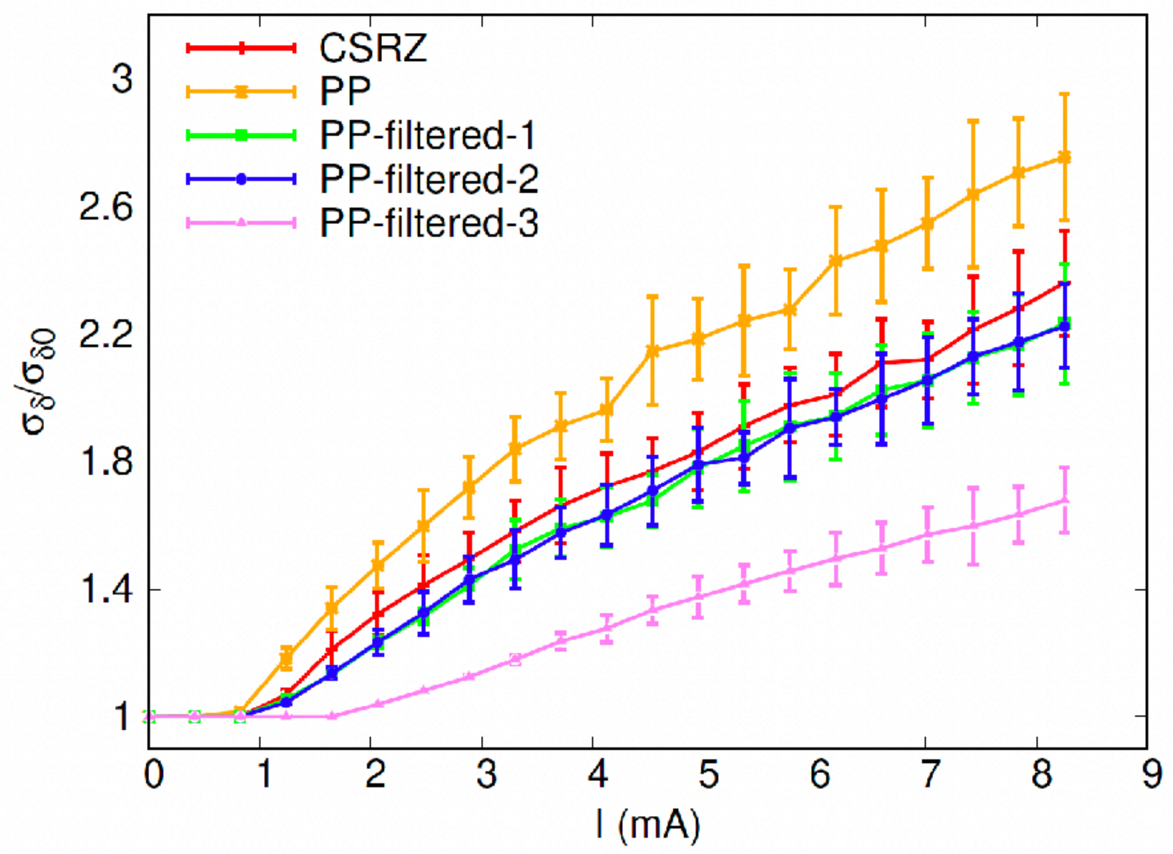
Elegant (10^6 particles, Δt=2x10^{-13} s, without 3HC)



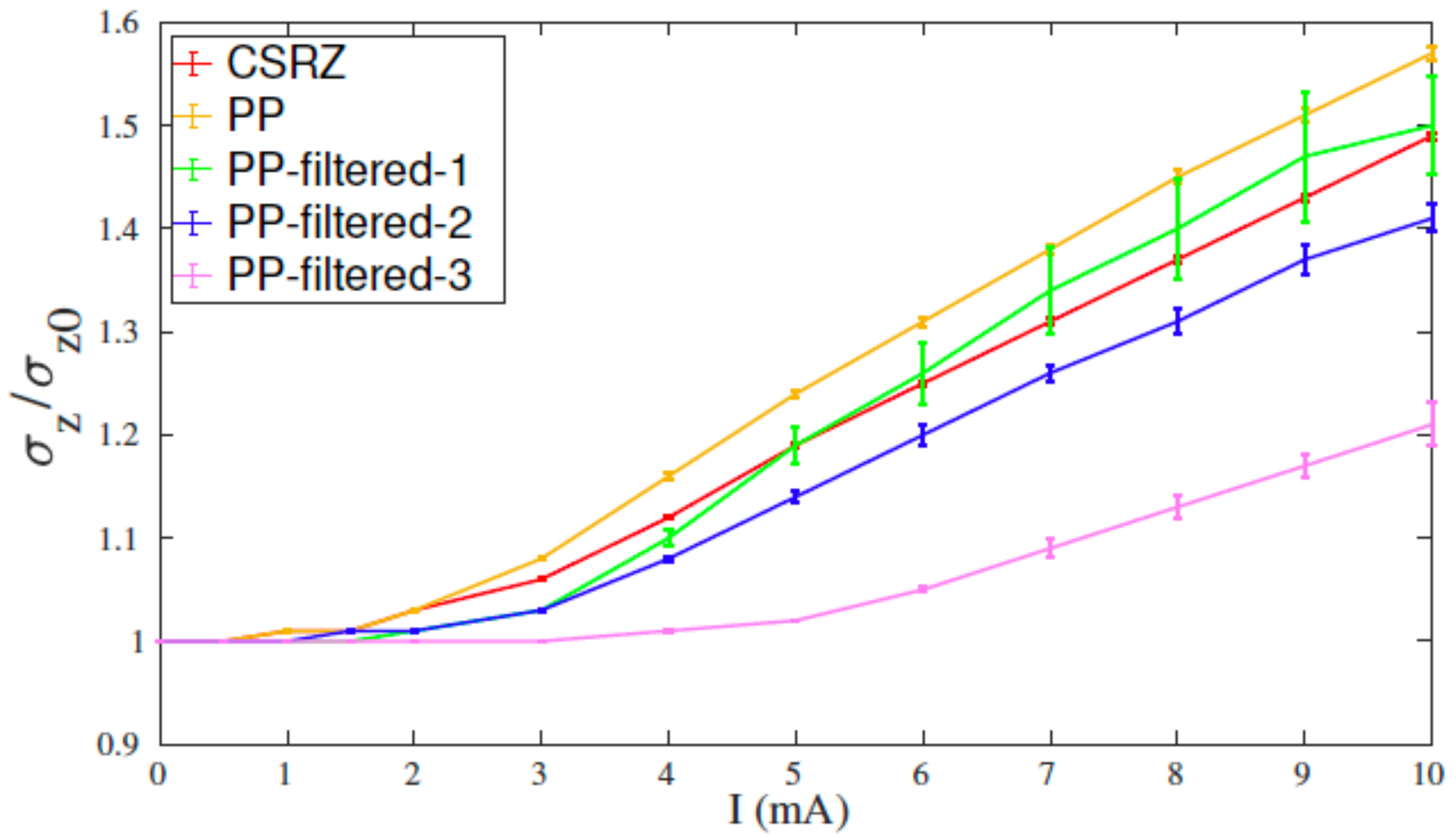


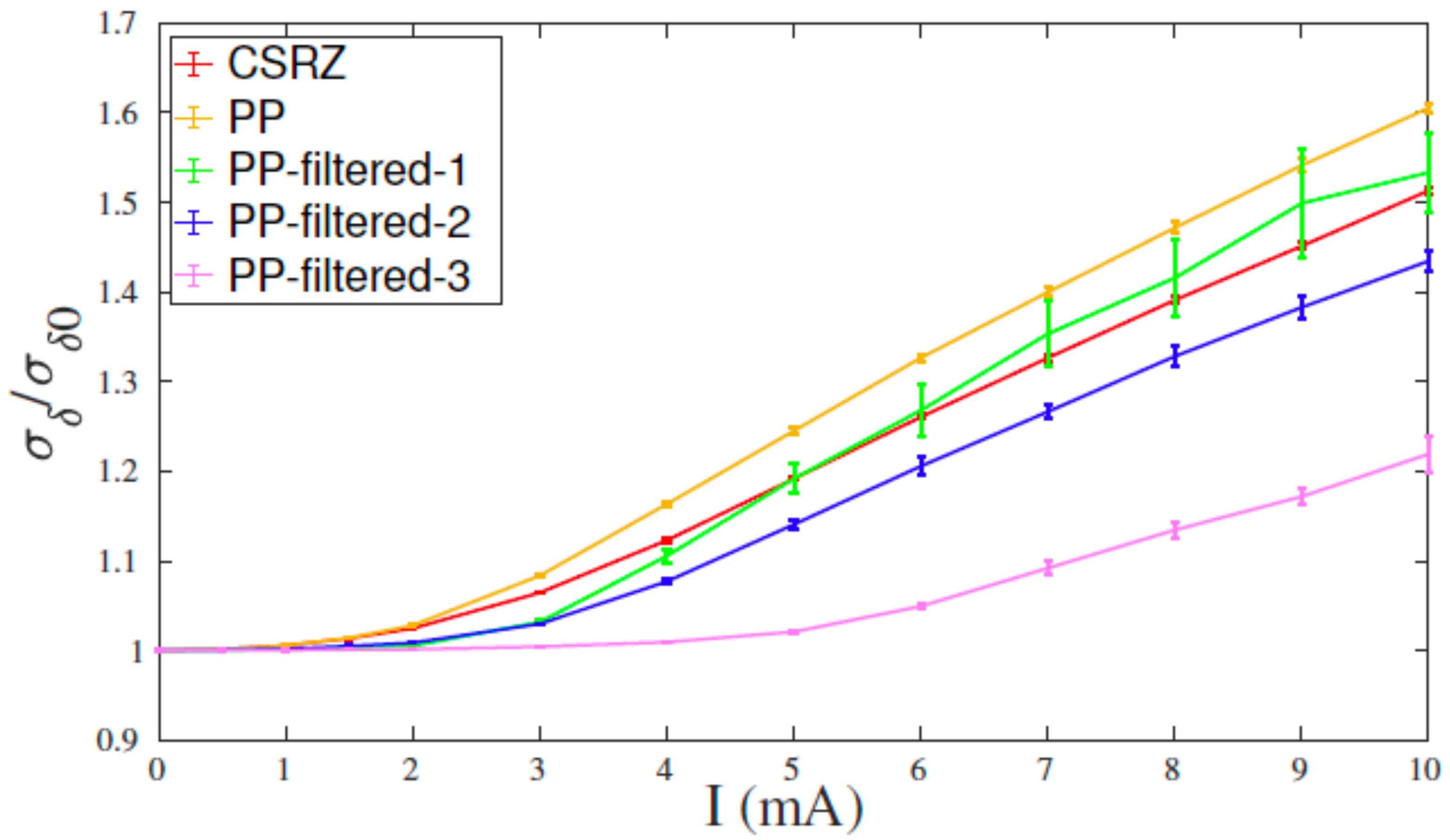
VFP solver (without 3HC)





Elegant (10^6 particles, Δt=2x10^{-13} s, with 3HC)

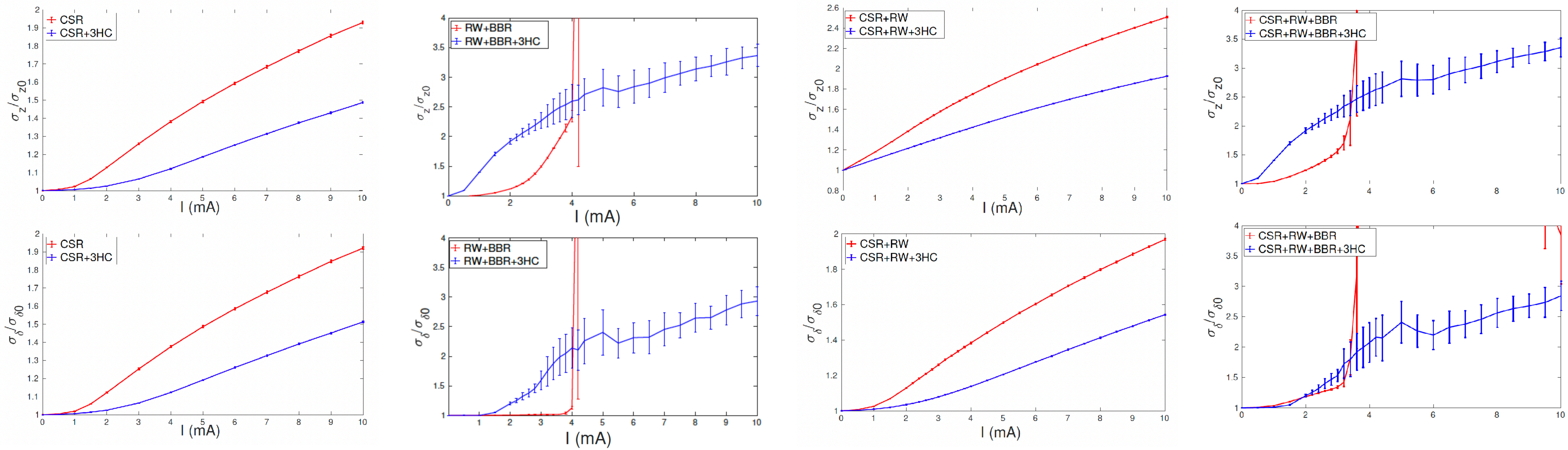




Prediction of CSR instability via simulations (cont'd)

- Example-2: Elettra 2.0
  - Step-4: Run tracking simulations (Elegant) simulations (Interplay of CSR, RW and BBR)
    - The (preliminary) results with 3HC seems to suggest that CSR is not a threat at Elettra 2.0.
  - Step-5: Check consistency between theories and simulations.
    - The results with BBR seems plausible. Further investigations (detailed calculations of geometric impedances and benchmarks with VFP solver) are planned. Realistic geometric impedance model is preferred.

Elegant (Only CSR) | Elegant (RW+BBR) | Elegant (CSR+RW) | Elegant (CSR+RW+BBR)



Comment from R. Lindberg: Elegant has difficulty when $k_r\sigma_z \lesssim 1$ (k_r: resonant frequency of BBR). Also true for VFP solver.

Summary

- Analytic theories (S-H theory and its extensions) are useful for determining the significance of CSR in low-emittance electron storage rings.
- Calculated high-frequency ($k\sigma_z \gg 1$) impedances (CSR, RW, and geometric impedances) can be used to estimate MWI threshold.
- Care should be taken in simulations of MWI with high-frequency ($k\sigma_z \gg 1$) impedances.
  - Play with "numerical arts".

- Not covered in this talk:
  - Narrow-band CSR impedance and its impact on MWI [1]
  - Accurate prediction of beam dynamics in the region well above MWI threshold

[1] Y. Cai, Phys. Rev. ST Accel. Beams **17**, 020702 (2014).

Backup

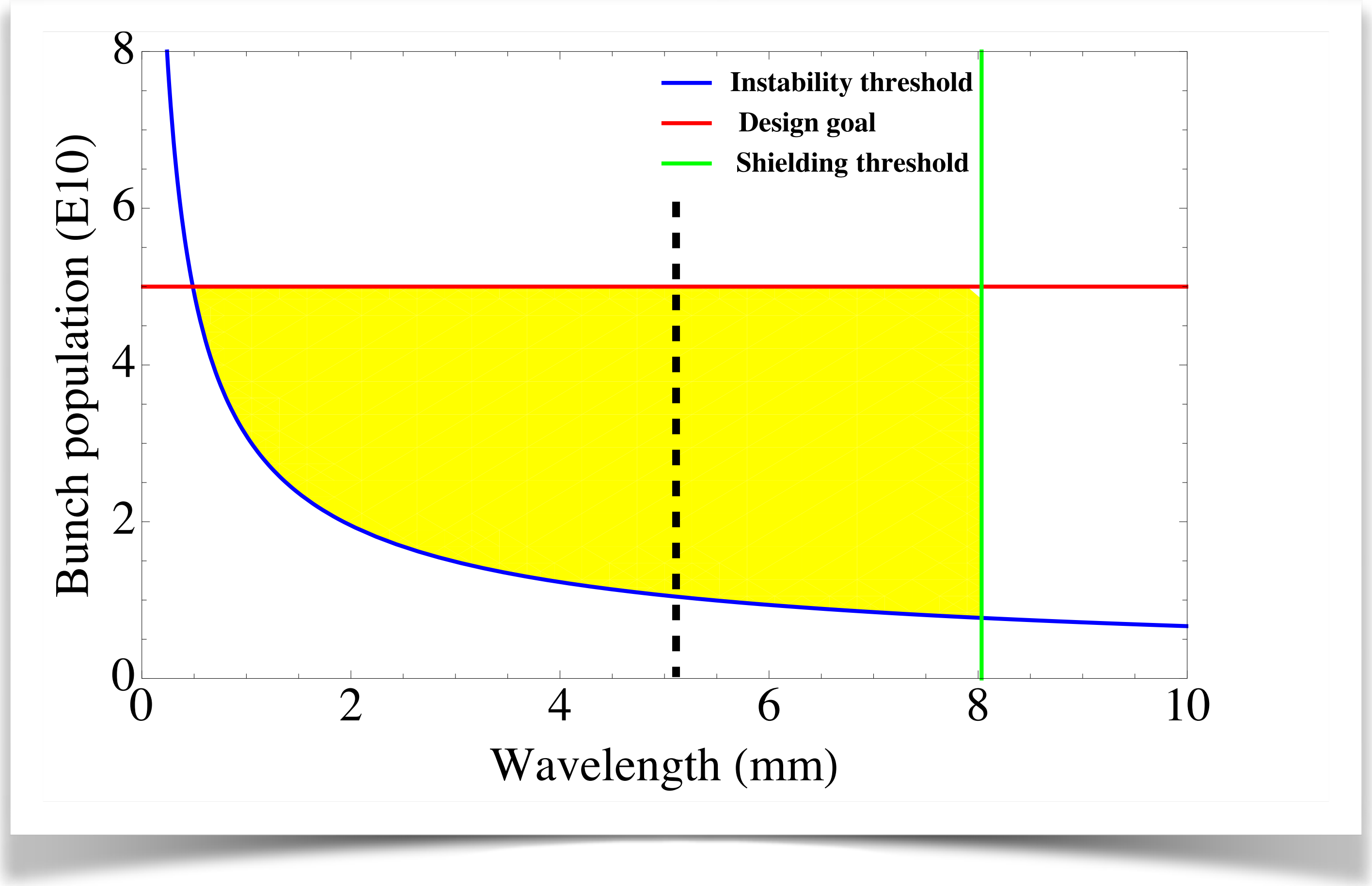
Analytic theories of CSR driven MWI threshold (cont'd)

- Apply S-H theory to electron storage rings
  - Quick estimate of CSR instability.
  - Very useful in the design stage of a storage ring.
  - "Yellow region" indicates "severity of instability".
  - For rings where CSR is marginally of concern, MWI simulations are required.

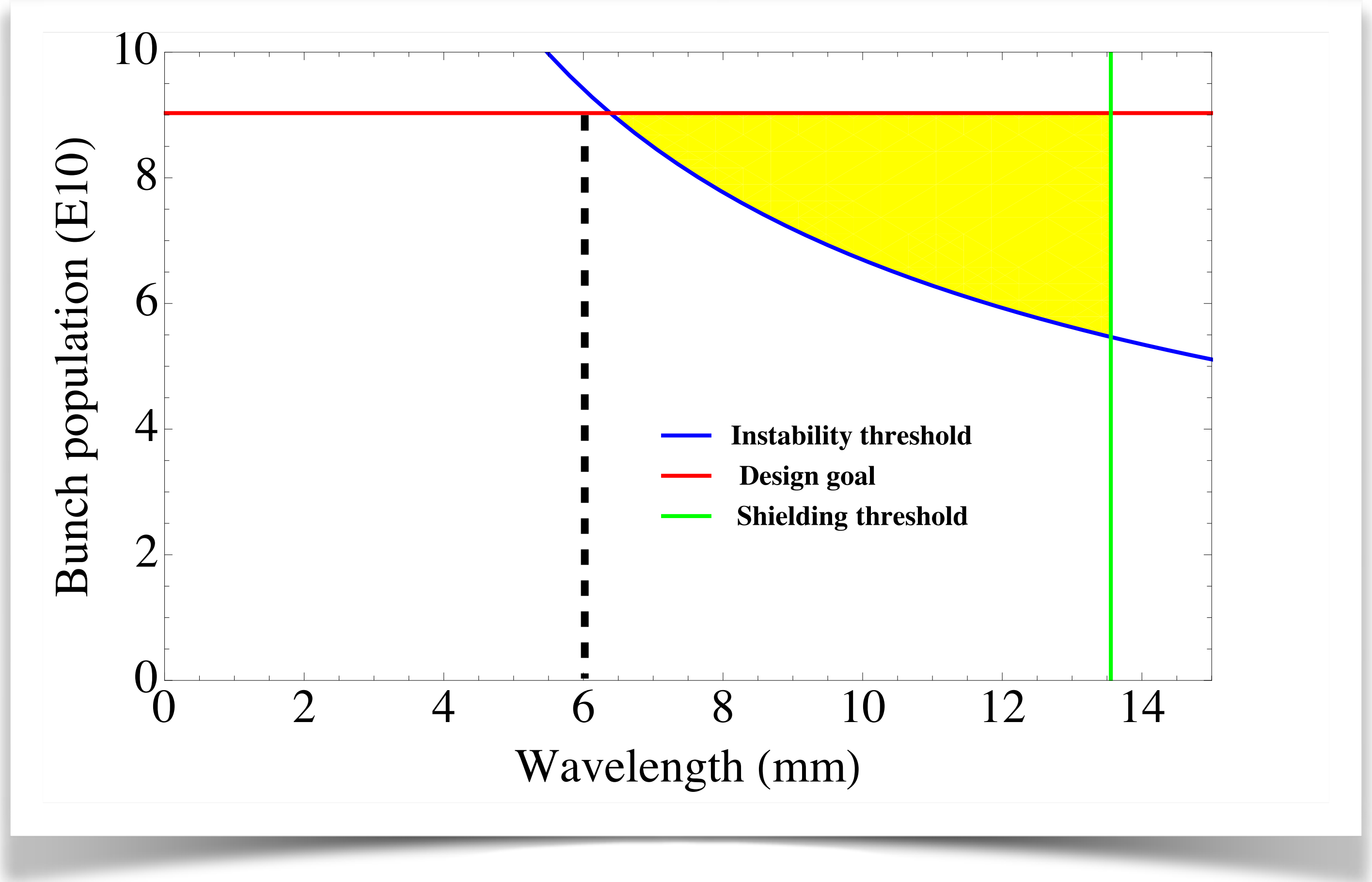
Parameters	SuperKEKB DR[1]	SLC DR[2]	ATF[3]	SuperKEKB LER[4]	SuperKEKB LER[5]	PEP-II LER[6]	ALS[6]	KEKB LER[7]
Circumference (m)	135.5	35.27	138.6	3016	3016	2200	196	3016
Energy (GeV)	1	1.21	1.54	4	3.5	3.1	1.5	3.5
Bending radius	2.43623	2.0372	5.73	15.87	15.87	13.7	4	15.87
Mom. compaction	3.43E-03	0.01814	2.17E-03	2.74E-04	2.74E-04	1.31E-03	1.41E-03	3.31E-04
Energy spread(10^{-4})	5.44	7.3	5.56	8.14	7.13	8.1	7.1	7.27
Bunch length (mm)	5.1	5.9	5	6	3	10	7	4.58
Bunch population (10^{10})	5	5	2	9.03	11.7	9.16	12.3	6.47
Pipe height@bends (mm)	34	15.6	24	90	90	50	40	94
Total bend. radius(2π)[8]	1	1	1	1	1	1	1	1

1) Design Version 1.140, Apr. 2010
2) SLC design handbook, Dec. 1984
3) ATF design and study report, KEK Internal 95-4
4) Nano-beam option design, Feb. 2008
5) High-current option design
6) G. Stupakov and S. Heifets, PRST-AB 5, 054402 (2002)
7) Machine operating parameters, Jun.17, 2009
8) Assumed

SuperKEKB DR (Design Ver. 1.140)

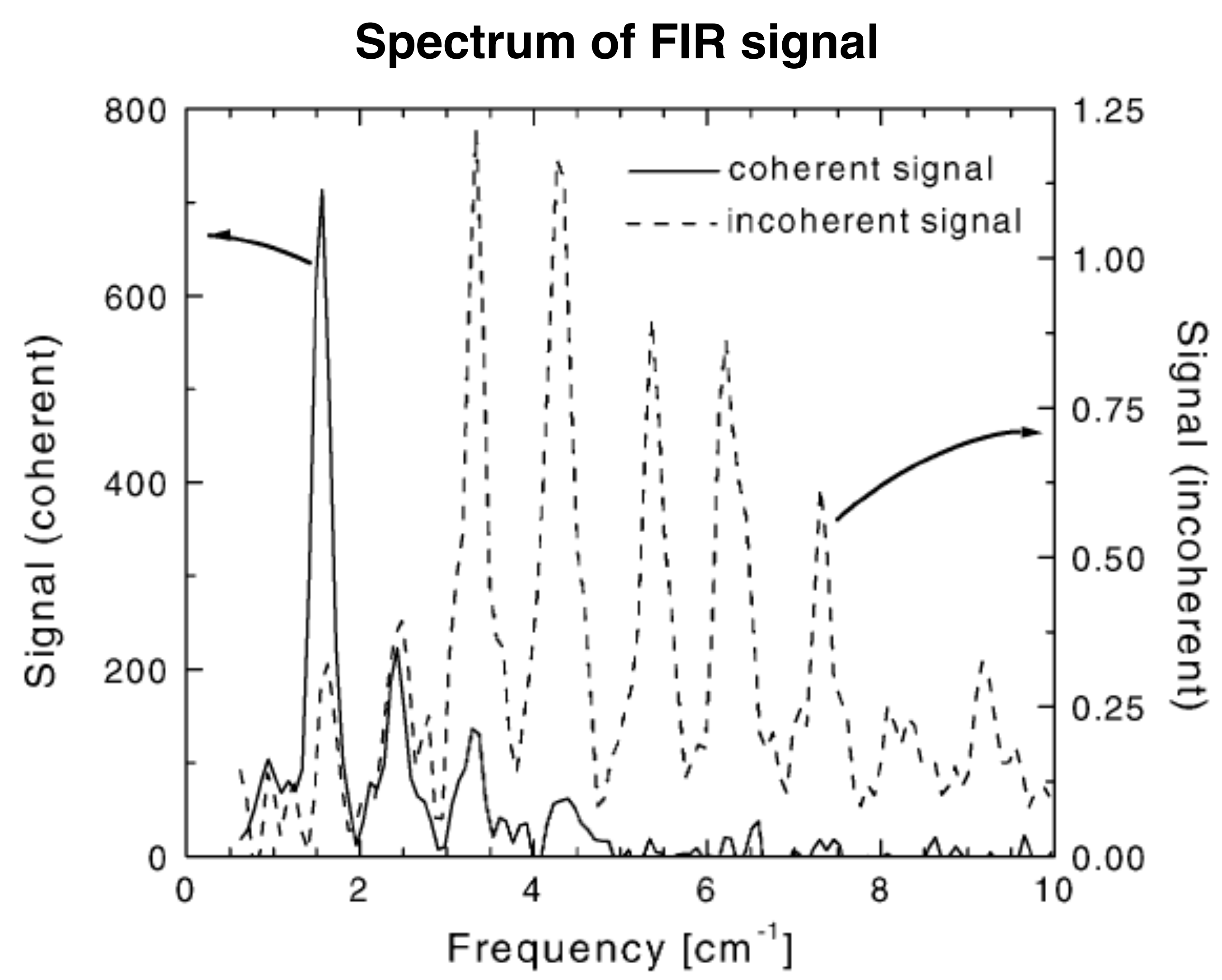


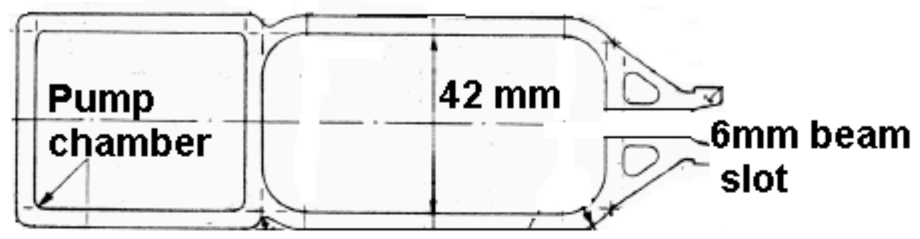
SuperKEKB LER (Design nano-beam option)

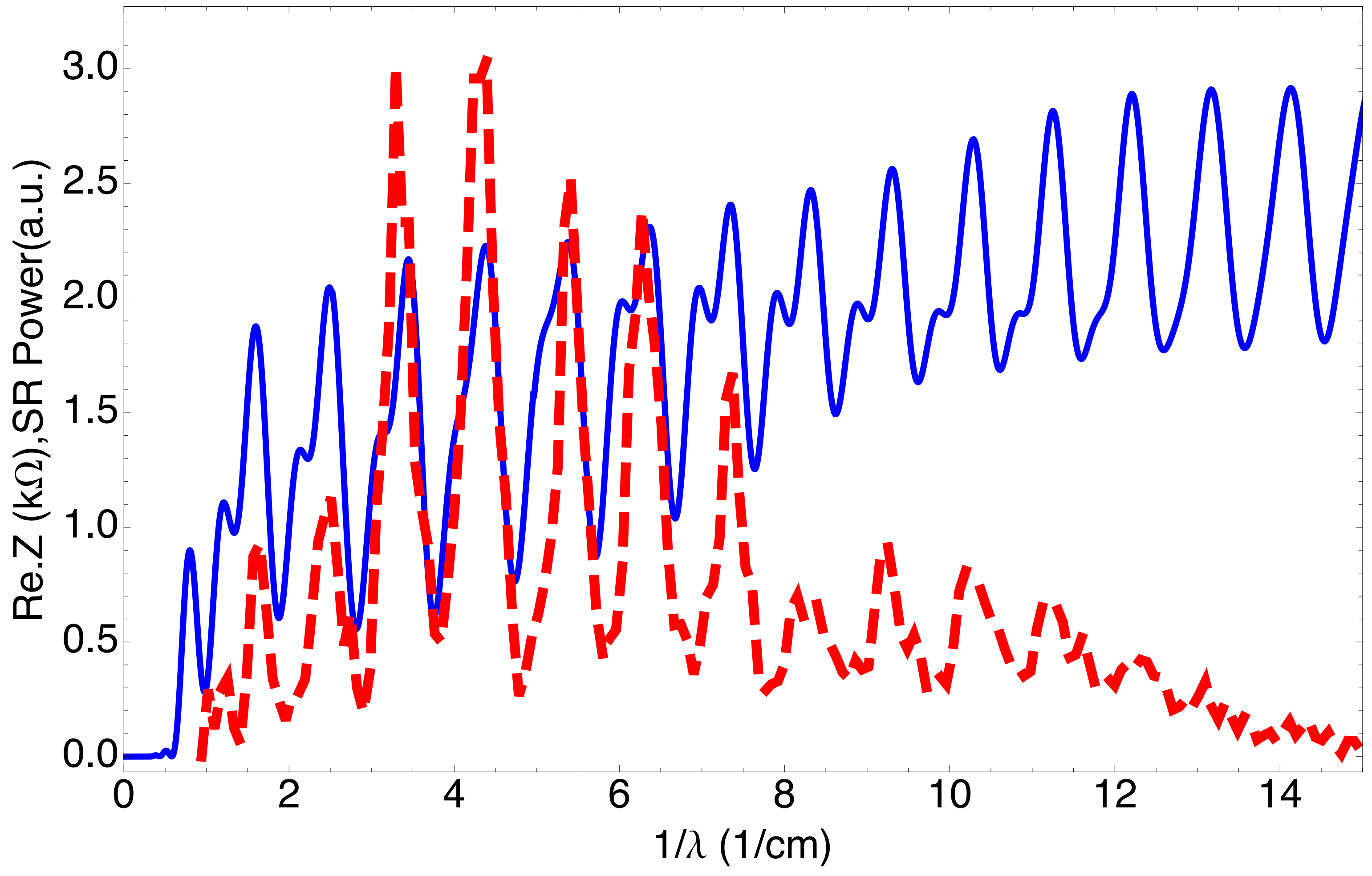


CSR impedance calculation (cont'd)

- Examples of CSR impedance by CSRZ
  - NSLS VUV as an example: a/b=80/42 mm, Lbend=1.5 m, R=1.91 m (Collaboration with S. Kramer)
  - Measured SR spectrum showed similar pattern of CSR impedance [6,8,10]. This is an evidence of multi-bend interference of CSR, or CSR in "whispering gallery modes".







[1] D. Zhou, IPAC'12, MOOBB03. [2] R. Warnock, arXiv:1708.05500. [3] G. Carr et al., NIM-A 463, 387 (2001).

CSR impedance calculation (cont'd)

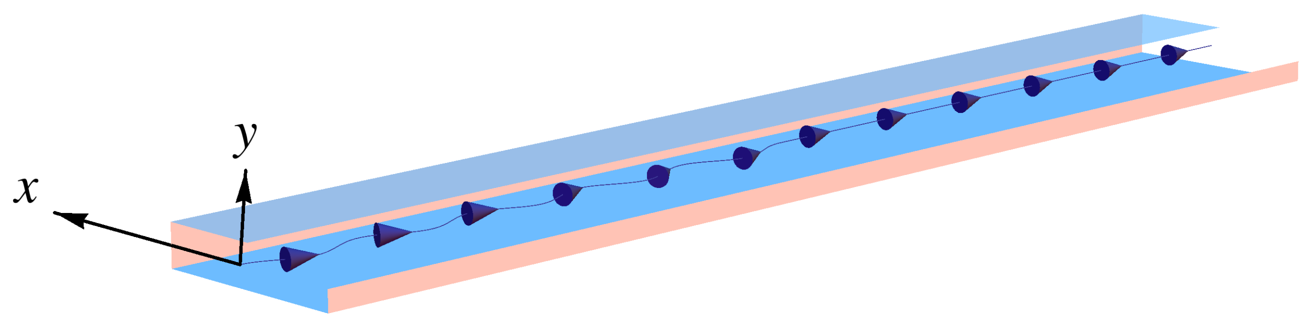
- Examples of CSR impedance by CSRZ
  - CSR in a wiggler/undulator: Coherent wiggler/undulator radiation (CWR or CUR).
  - The CWR spectrum can be calculated analytically (for example, see Refs.[1,2]):

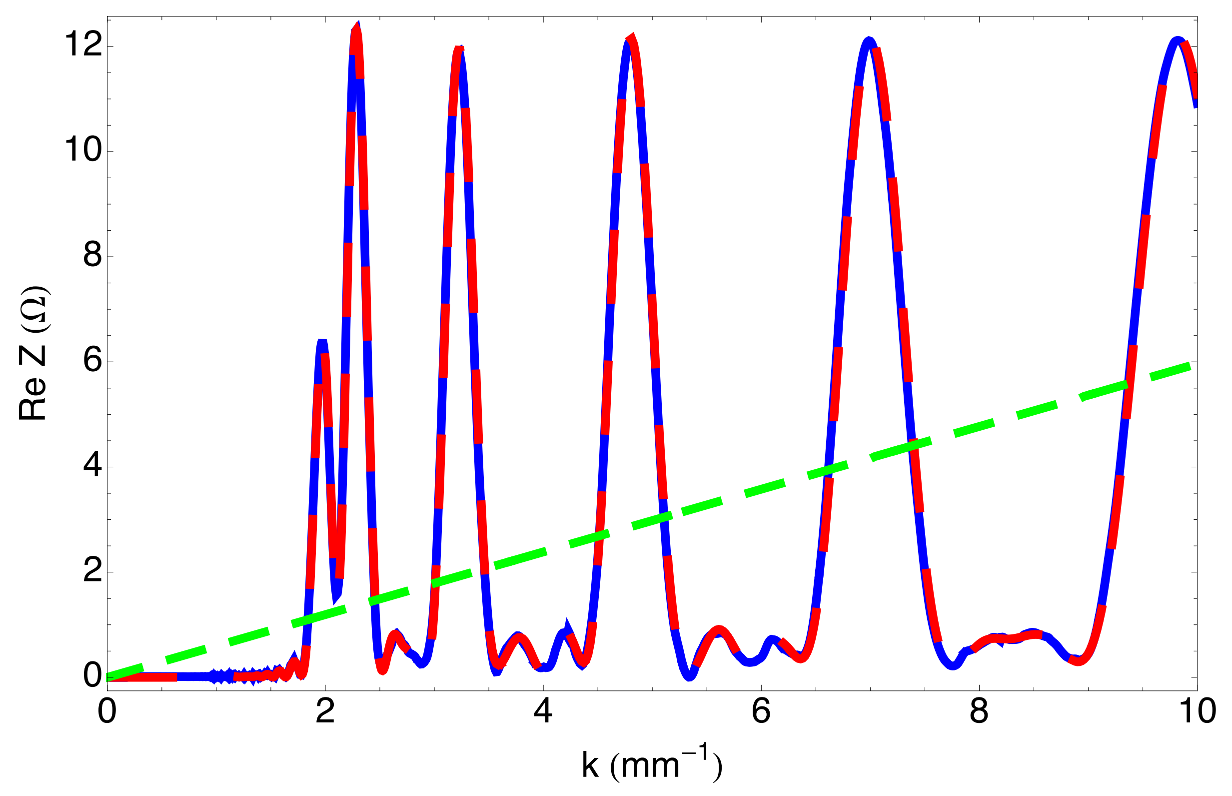
$$\mathrm{Re}\ Z(k) = \frac{4Z_0}{abR_0^2}\sum_{m=0}^{\infty}\sum_{p=1}^{\infty}\frac{k}{(1+\delta_{m0})k_z}\frac{\sin^2\left((k-k_z-k_w)L_w/2\right)}{(k-k_z)^2-k_w^2}$$

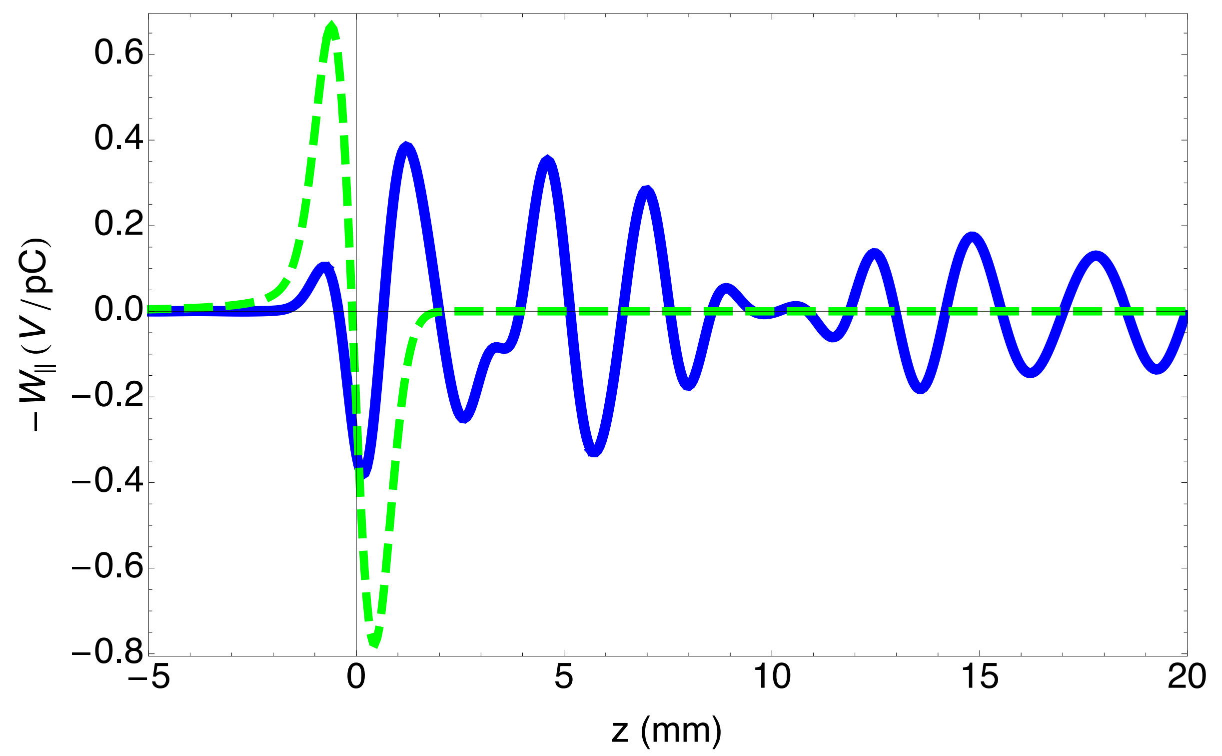
  - A weak wiggler: a/b=100/20 mm, λw=1 m, R0=100 m, N_{period}=10
  - Blue line by CSRZ; Red line by analytic theory with rectangular chamber; Green line by analytic theory in free space [3].

$$Z(k) = \frac{1}{4}Z_0 L_w k \frac{k_w}{k_0}\left(1-\frac{2i}{\pi}\left(\log\frac{4k}{k_0}+\gamma_E\right)\right)$$

  - For storage-ring light sources or THz FELs, it might be interesting to look at the interference of CUR + SC + RW.







[1] Y. Chin, LBL-29981, 1990.
[2] G. Stupakov and D. Zhou, KEK Preprint 2010-43.
[3] J. Wu et al., Phys. Rev. ST Accel. Beams **6**, 040701 (2003).