TMCI Theory of Flat Chambers Revisited

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Outline

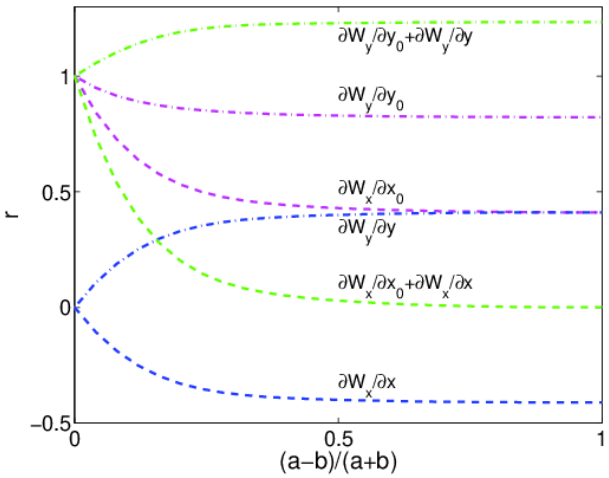
- Deficiencies with the computation of the transverse single bunch mode evolution
- Introduction into Lindberg's theory (PRAB 19 120402, 2016) and its new aspects
- Application of Lindberg's theory on the 2-mode case: vertical and horizontal & single resonator and composed resonator model
- Consideration of an increasing number of non-zero radial modes for each azimuthal mode: vertical and horizontal
- Application of the many radial mode approach to realistic impedance models

  we will often use HEADTAIL (G.Rumolo et al.) for comparison studies

- Conclusions

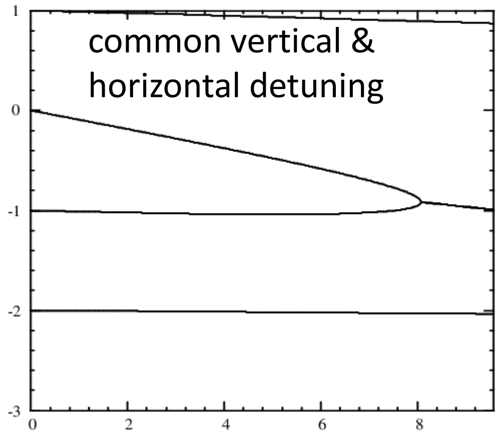
This work is not closely related with the ALBA-II design since it applies above all to flat vacuum chambers

TMCI: from radial to flat chambers

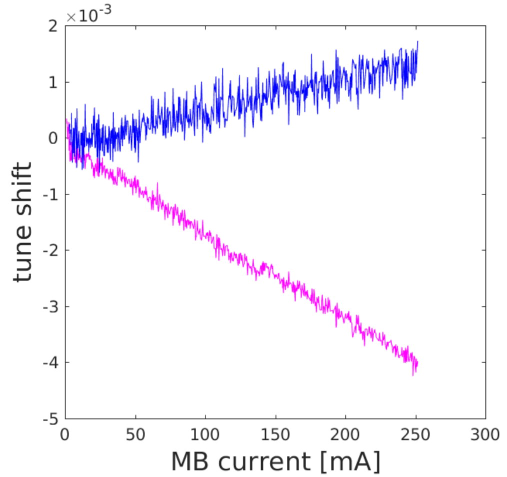


according to Yokoya's theory of resistive wall impedance of general cross section

circular vacuum chambers

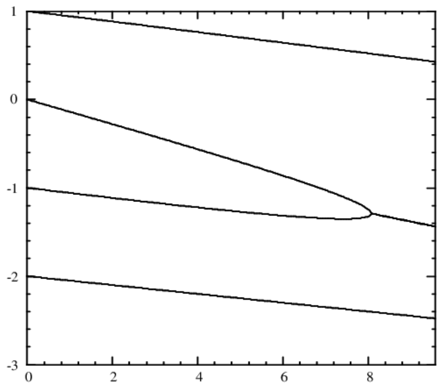


addition of quadrupolar detuning generated by flat chambers



inspired by quadrupolar detuning in multi-bunch

flat vacuum chambers

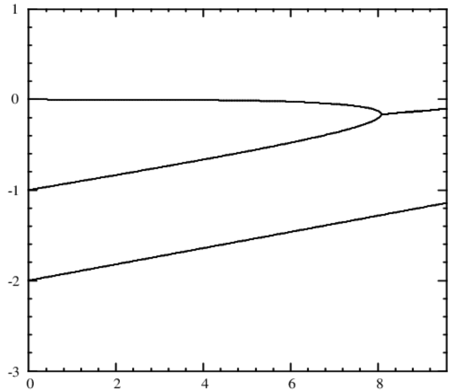


total vertical detuning

the single bunch threshold does not change, the slopes of all modes change by the same amount

called **quadrupolar tune shift theory**

flat vacuum chambers



horizontal detuning

qualitatively ok, but quantitatively unsatisfactory
e.g. prediction for the vertical threshold is 40-50% larger

Why do we need a new approach ?

Until today we cannot well predict the TMCI-thresholds on vertical and horizontal plane with existing mode-coupling programs.

in particular when using quadrupolar tune shift theory :

$$(\Delta\Omega - m\omega_s)a_p^m + \mathrm{I}\cdot \sum_{n,q}(D_{p,q}^{m,n}a_q^n) + q\cdot a_p^m = 0$$

however Lindberg's theory is different :

$$(\Delta\Omega - m\omega_s)a_p^m + \mathrm{I}\cdot \sum_{n,q}(D_{p,q}^{m,n} + Q_{p,q}^{m,n})a_q^n = 0$$

Could Lindberg's theory fix the large discrepancy between theory and measurement of TMCI ?

How Lindberg's theory will be applied including simplifications

- In this approach the Planck-Fokker terms are neglected (TMCI is very strong).
- Zero chromaticity is assumed.
- We focus almost exclusively on the resonator model (f_{res}, Q, R_s) (Resistive wall impedance (RW) was already treated in Lindberg's own publication)
- Bunch length 15.4ps and for the ALBA impedance model 22ps
- Both dipolar and quadrupolar impedance are considered.
- in the following "mode crossing" comprises pure mode crossing but also mode coupling
- The eigenvalues of the linearized Vlasov equation provide the mode frequencies as a function of current

Actually, Lindberg's theory agrees with MOSES for pure dipolar impedance

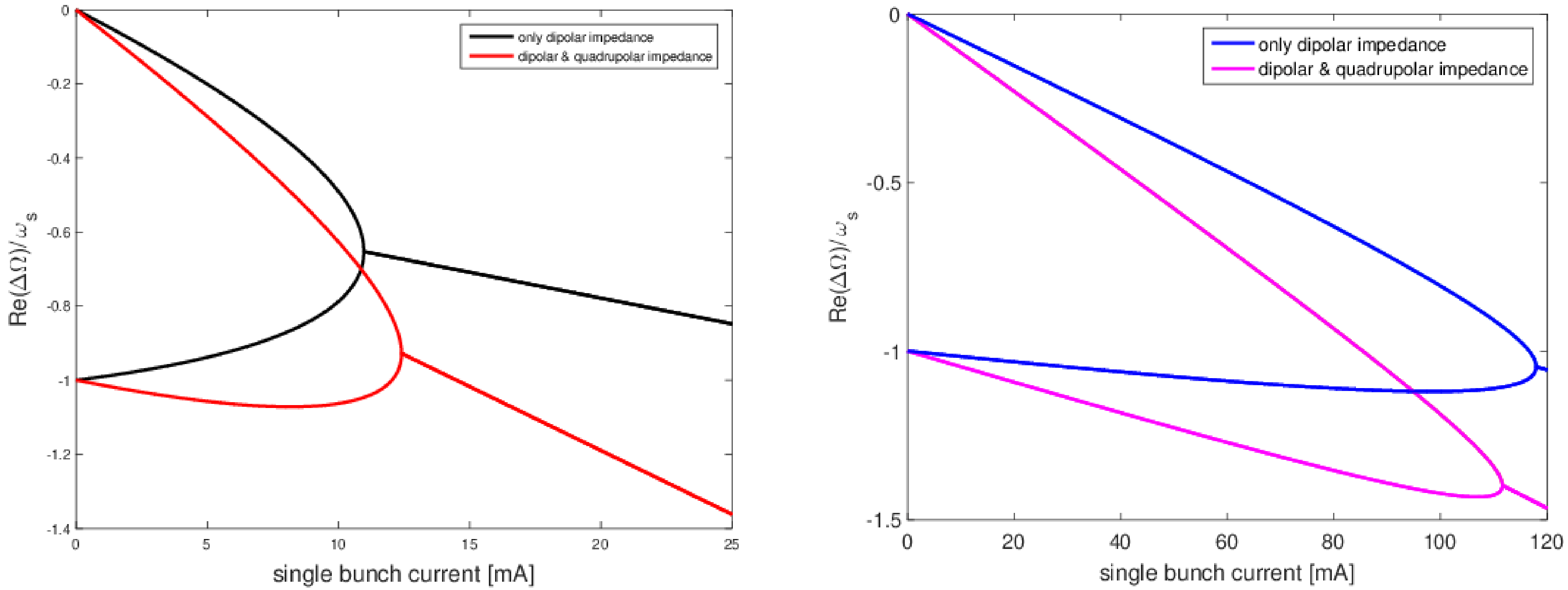
2-mode theory (vertical & fixed ρ)

$$\rho := \frac{Z_Q(\omega)}{Z_D(\omega)} = 0.5$$

according to K. Yokoya

consideration of only 2 azimuthal modes and fixed ρ

single resonator impedance model (dipol+quad) is used.



according to Lindberg's theory we find that including the quadrupolar impedance the vert. TMCI changes in contrast to quadrupolar tune shift theory

However, the effect is not very large and is only sensible if specific resonator models (parameters) are chosen. also in case of resistive wall impedance the effect is rather small.

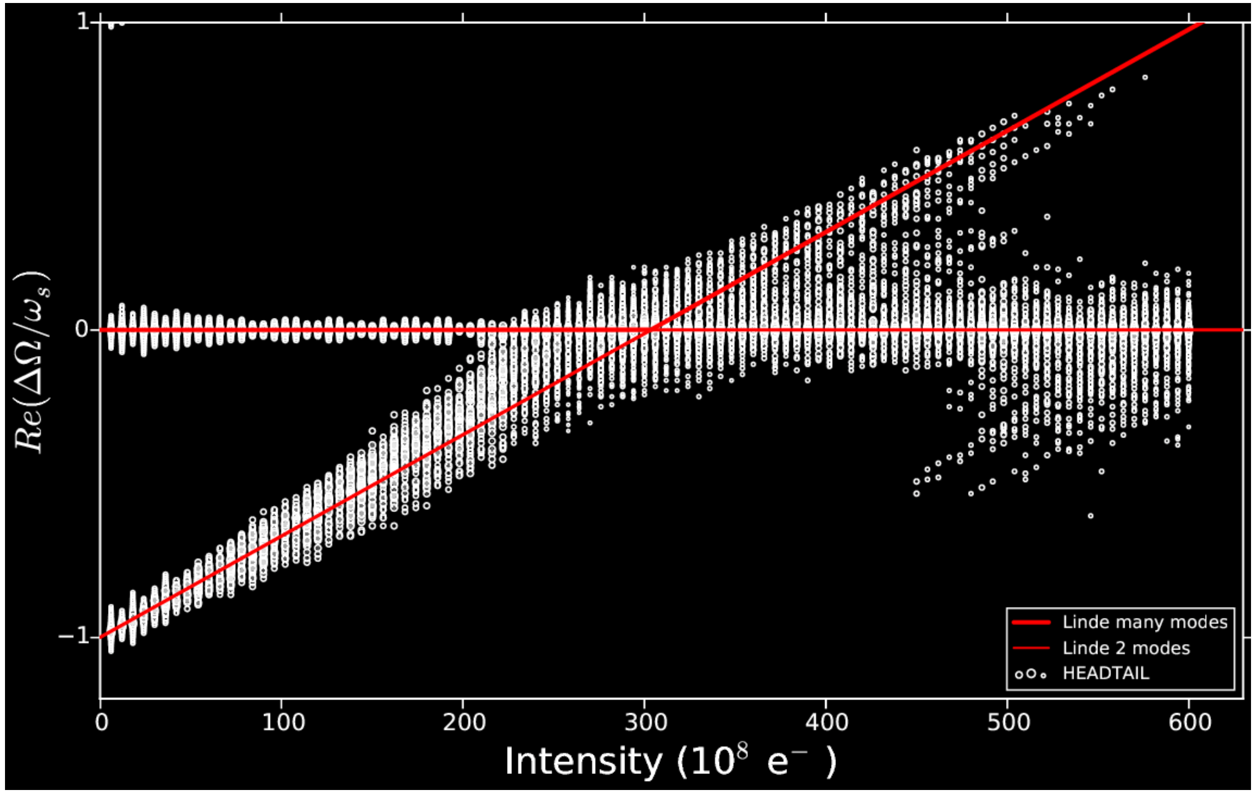
2-mode theory (horizontal & fixed ρ)

consideration of only 2 azimuthal modes and fixed ρ

single resonator impedance model (dipol+quad) is used.

$$\rho := \frac{Z_Q(\omega)}{Z_D(\omega)} = -1$$

according to K. Yokoya



overlay of Lindberg's mode theory with Headtail (G. Rumolo et al.).

The change of mode crossing current (threshold) comes from the different slope of mode m=-1.

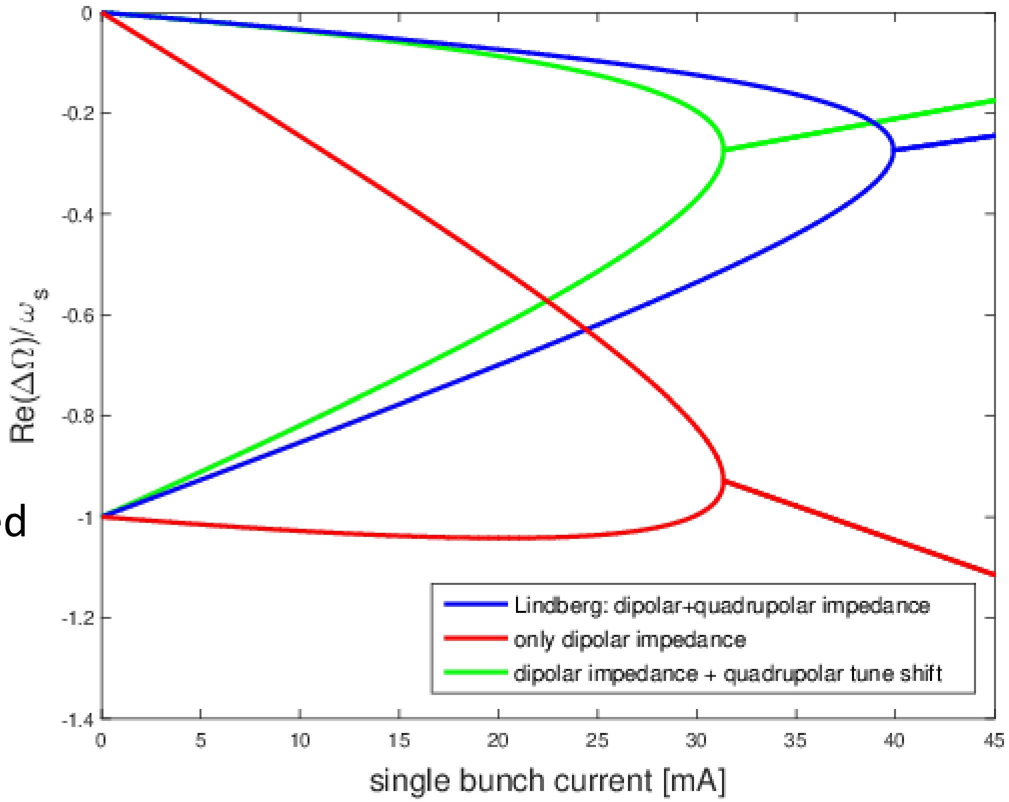
there is even no mode coupling anymore, it's just mode crossing and the crossing happens at a higher current than in case of mode coupling

so on the horizontal plane the mode evolution is very different from that on the vertical plane

Example of 2-mode theory
ALBA's horizontal impedance model

being a realistic impedance model spectral equality between dipolar & quadrupolar impedance is not longer required, so ρ is no longer properly defined.

appreciate the difference between the quadrupolar tune shift theory (green) and the result of the Lindberg's theory (blue)
Note that this result changes if many radial modes are included (next slides).



Unfortunately we could not check this experimentally due to the risk of heatload at high single bunch current

As $\rho = -1$, is no longer the case, mode coupling again occurs, but a higher current, so it's something intermediate between the original (pre-Lindberg) mode coupling and pure mode crossing

Many radial modes

treating the transversal mode evolution as 2-(azimuthal) mode case is a tremendous simplification (we solve an intego-differential (Vlasov) equation in an infinite function space)

approach: consider increasing number of radial modes in the mode evolution

this is done by an iterative procedure: increase #radial modes in each iteration step and check if the mode slope converges

Two scenarios are distinguished:

- spectrally equal dipolar and quadrupolar impedance ρ=0.5 (Vert) and ρ = -1 (Horz)

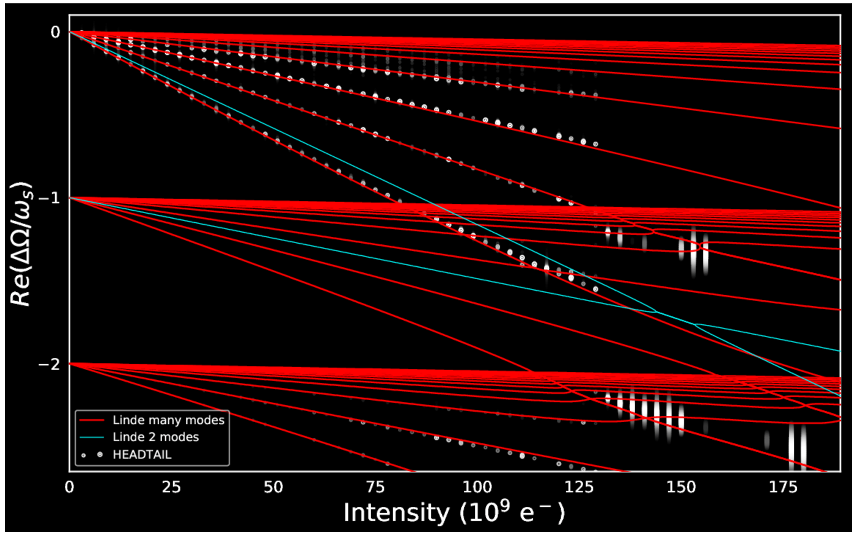
  - convergence criterion is Cauchy ε=0.01 applied on the lowest radial mode m=0(vertical) or m=-1 (horizontal)
  - in the iteration only 2 azimuthal modes are considered: vertical plane with m=0 or horizontal plane with m=-1.
  - resonator impedance models in the range of of $\omega_r\sigma_\tau \leq 5.2$ and $1 \leq Q \leq 800$ underwent the iterative procedure
  - RW-impedance only requires the check of one model
  - used bunch length length σ_τ=15.4ps and also σ_τ=30.8ps
  - a couple of selected models were compared to HEADTAIL (several azimuthal modes were admitted and no iteration was applied)

- spectrally unequal dipolar and quadrupolar impedance as part of realistic impedance models (Vert. and Horz.)

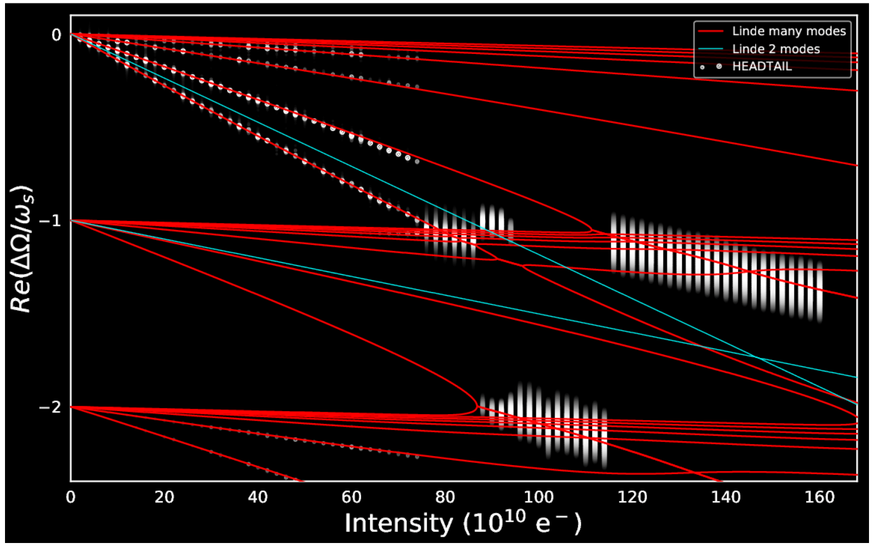
Many radial modes VERTICAL plane

3 examples: comparison of Lindberg's theory with HEADTAIL

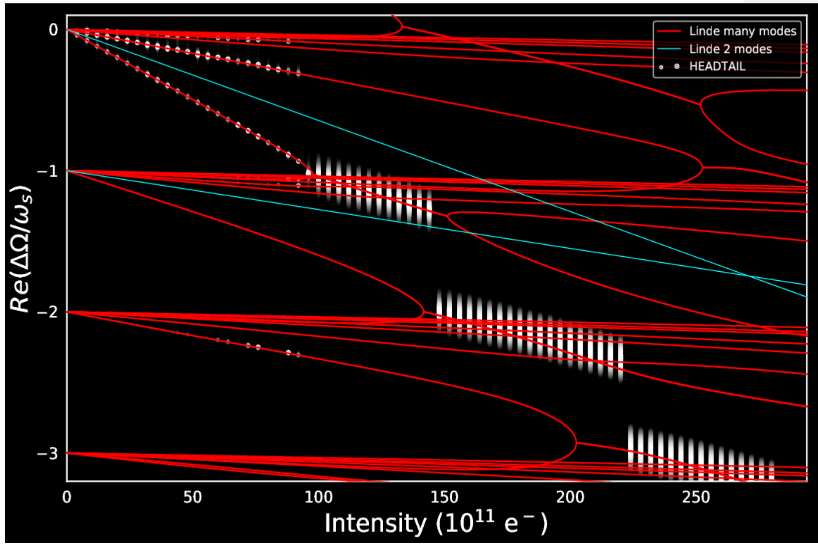
2-(azimuthal) mode description in cyan, the numerous radial modes in red



R_s=5MΩ, Q=5, f_{res}=51GHz



R_s=5MΩ, Q=50, f_{res}=39GHz



R_s=5MΩ, Q=800, f_{res}=24GHz

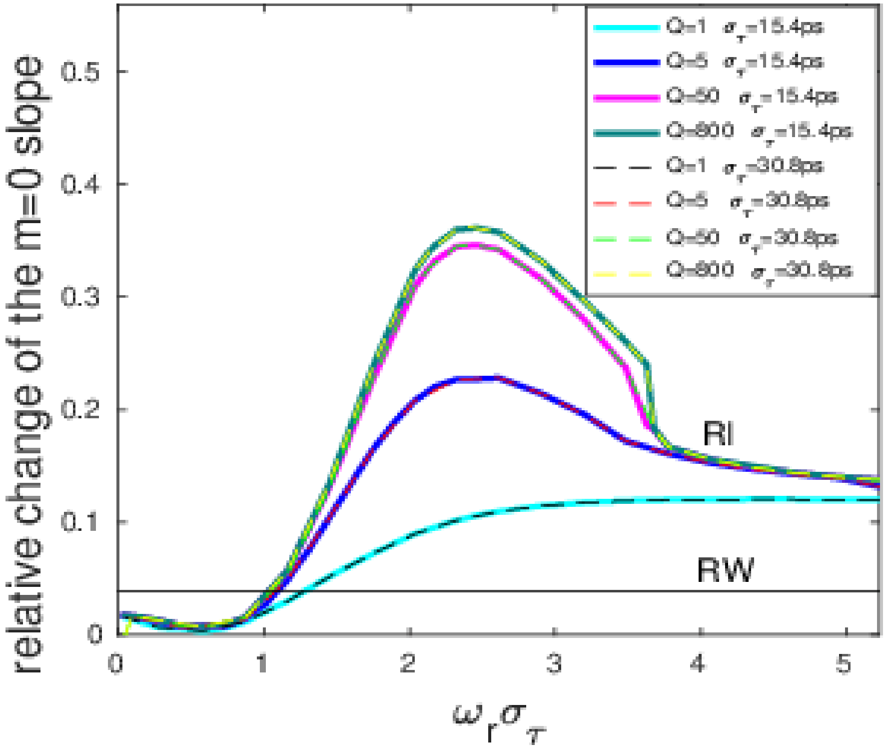
transverse mode evolution under the effect of different resonator impedances characterized by R_s, f_{res} and Q

the lowest radial mode's slope is steeper in case of many radial modes than it were if only 1 radial mode were considered

this is actually what we are looking for

parametrization of the vertical slope change (m=0)

change of slope (sign not considered) for different resonator impedance parameters under increase of the number of radial modes



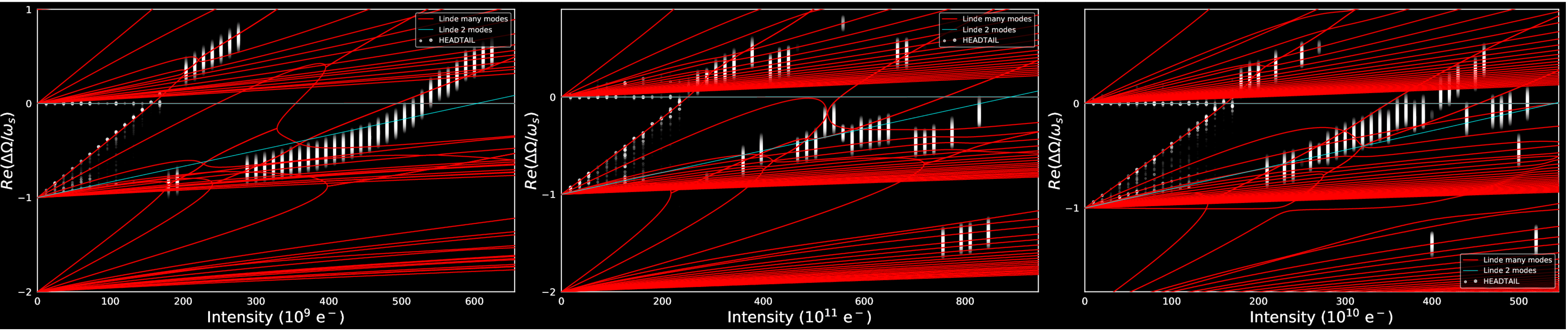
the graph covers the parameter space where the iterative procedure led to convergence.

for certain **R**(esonator) **I**(mpedance) models the change in slope is more than 35%, however, for **R**(esistive)**W**(all) impedance it is only 3.4%.

Many radial modes HORIZONTAL plane

3 examples of mode evolution due to Lindberg's theory compared to HEADTAIL

2-(azimuthal) mode description in cyan, the numerous radial modes in red



R_s=5MΩ, f_{res}=24GHz, Q=800 | R_s=5MΩ, f_{res}=39GHz, Q=50 | R_s=5MΩ, f_{res}=51GHz, Q=5

- to achieve convergence up to 29 radial modes had to included, therefore the computation effort is very large.

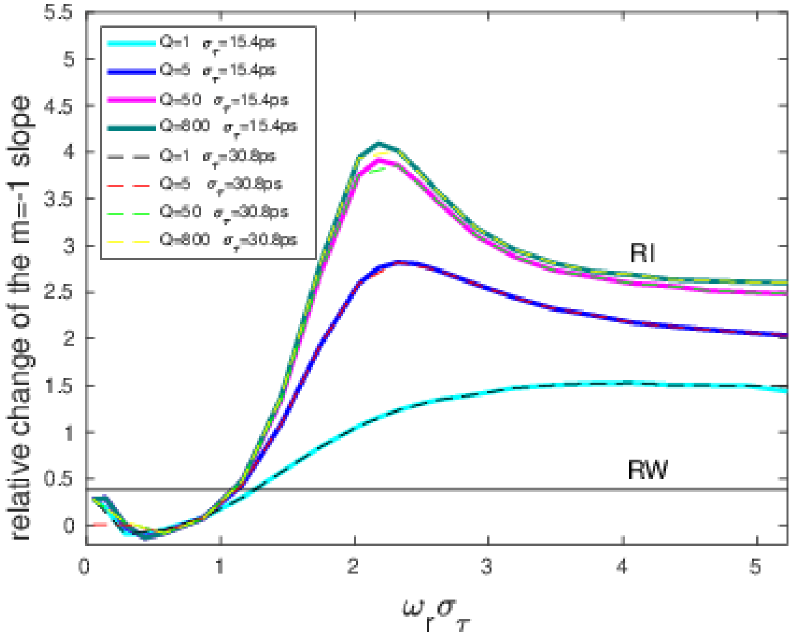
  increasing number of radial modes is limited by computing precision

- the inclusion of radial modes changes the steepest slope of m= -1 tremendously on the horizontal plane.

parametrization of the horizontal slope change(m=-1)

change of slope for different horizontal resonator impedance parameters
under increase of the number of radial modes

The effect is 10x larger than on the vertical plane, so the scale on the y-axis is 10x larger.

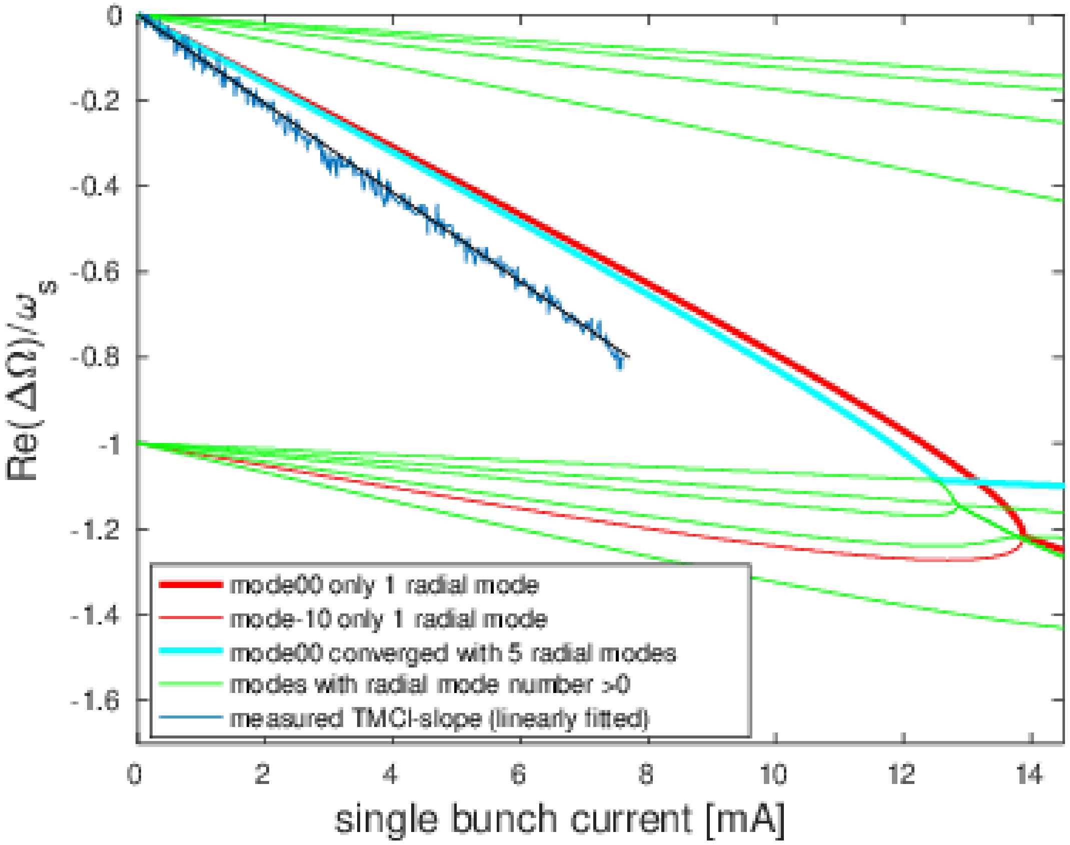


the graph covers the parameter space where convergence could be achieved.

for certain **R**(esonator) **I**(mpedance) models the change in slope is reaches 400%, however, for **R**(esistive) **W**(all) impedance it is only 39%.

ALBA's vertical impedance model as multi-mode case

iterative procedure applied on spectrally unequal dipolar and quadrupolar impedance
as part of ALBA's vertical impedance model



no check-up
with Headtail

inspite reinforced convergence criterion iteration stops early and the slope increases only by 4%.

the desired substantial slope increase in order to overcome
the discrepancy of theory and measured TMCI slope does not happen (well only a very little bit).

BONUS slide

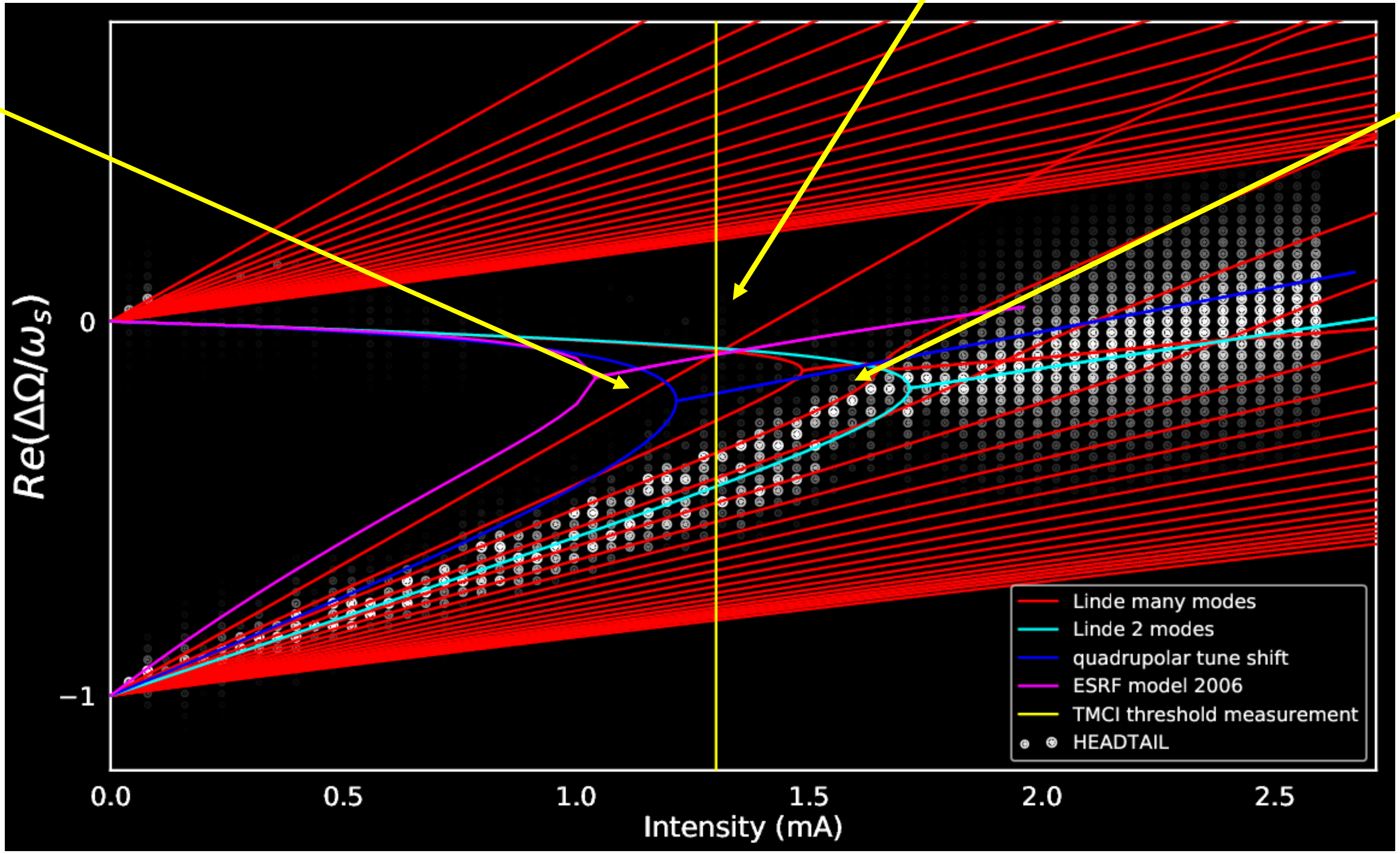
horizontal impedance model of ESRF(2004) as multi-mode case

The OLD ESRF model was used since the iterative procedure applied on the ALBA horizontal impedance model doesn't converge well (contains resonator impedances outside the tested parameter space)

TMCI measurement

predictions 2006
quad tune shift theory

predictions 2024 (Lindberg)



the OLD ESRF horizontal model does converge upon increasing the number of radial modes

now a larger horizontal TMCI is predicted which is much more natural since the vertical TMCI's prediction is also much larger than the measurement (grosso modo we expect the same deficiencies on both planes)

agreement with the Headtail simulation is quite good.

Exploration of Lindberg's theory (2016) leads to following conclusions

- Different treatment of quadrupolar impedance according to Lindberg compared to the quadrupolar tune shift theory leads to slightly (mostly) smaller TMCI-threshold on the vertical plane & sensibly (mostly) larger TMCI-threshold on the horizontal plane.
- While the slope of m=0 hardly changes, the slope of m=-1 indeed changes sensibly which explains the threshold changes.
- Applied on 2 realistic impedance models – ALBA and ESRF -- the improvement was only 4% for ALBA (vertical), but larger for the OLD ESRF (horizontal).
- Therefore on the horizontal plane Lindberg's theory can produce the most interesting results.
- The approach is limited by numerical stability of the computation of the quadrupolar matrix elements.
- In the parameter space of application of Lindberg's theory good agreement with HEADTAIL could be achieved.
- It turns out that in this type of study HEADTAIL is very useful and could also be applied beyond the tested parameter space.

Acknowledgements & References

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