9th Low Emittance Rings Workshop 2024

Impact of beam-ion interactions in diffraction-limited storage rings

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Beam-ion instability

$$M_{\text{ions}} = \underbrace{\begin{pmatrix} \cos\phi & \beta\sin\phi \\ -\frac{1}{\beta}\sin\phi & \cos\phi \end{pmatrix}}_{\text{Focusing by electron bunch}} \underbrace{\begin{pmatrix} 1 & L_{\text{drift}} \\ 0 & 1 \end{pmatrix}}_{\text{Drift in bunch spacing}}$$

$$\beta = \beta c/\omega_I^b \qquad \phi = \omega_I^b L_{\text{bunch}}/c$$

$$\omega_I^b = \omega_I \sqrt{\frac{L_{\text{drift}}}{L_{\text{bunch}}}}$$

- Focusing force is linear!
- Trapping condition

$$|\text{Tr}M| = \left| 2 - \left(\frac{\omega_I L_{\text{drift}}}{c} \right)^2 \right| < 2 \Rightarrow \frac{\omega_I L_{\text{drift}}}{c} < 2$$

Trapping condition is for ions between two electron bunches!

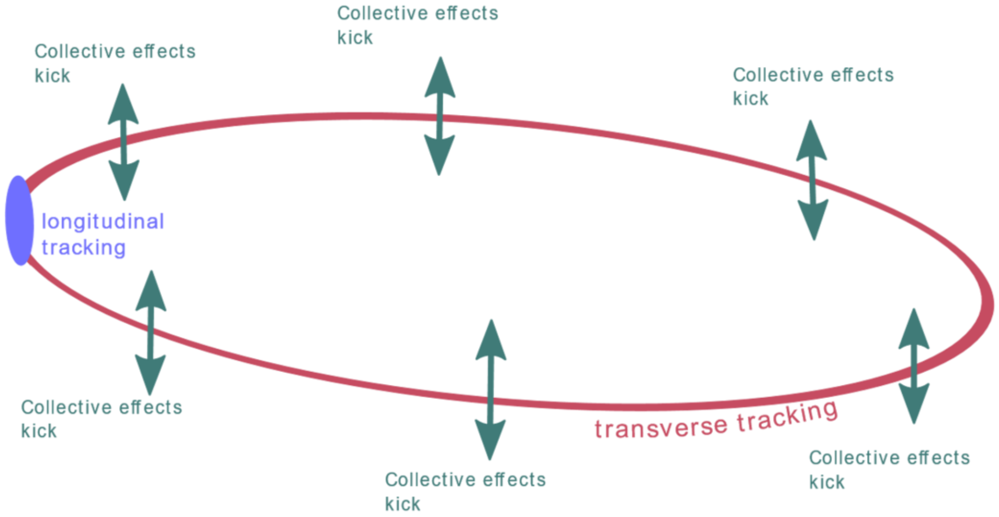
- Ion bounce frequency

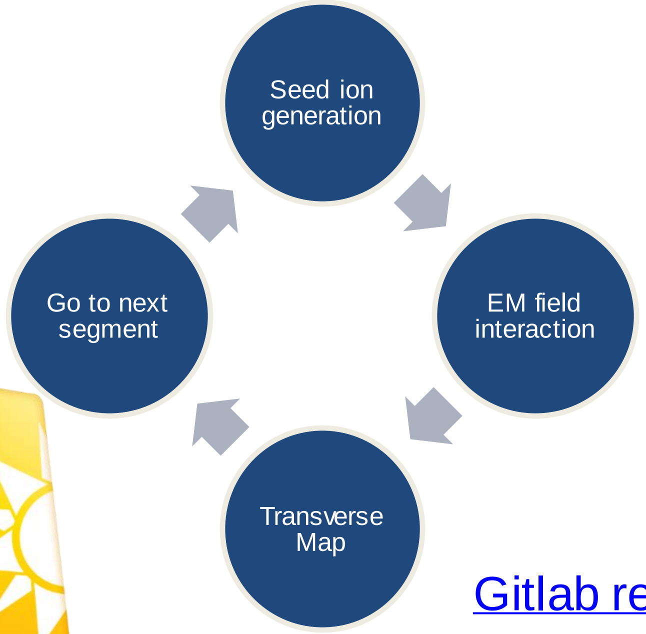
$$\omega_I = c\sqrt{\frac{4N_b r_p}{3L_{\text{drift}} A\sigma_{x,y}(s)[\sigma_x(s)+\sigma_y(s)]}}$$

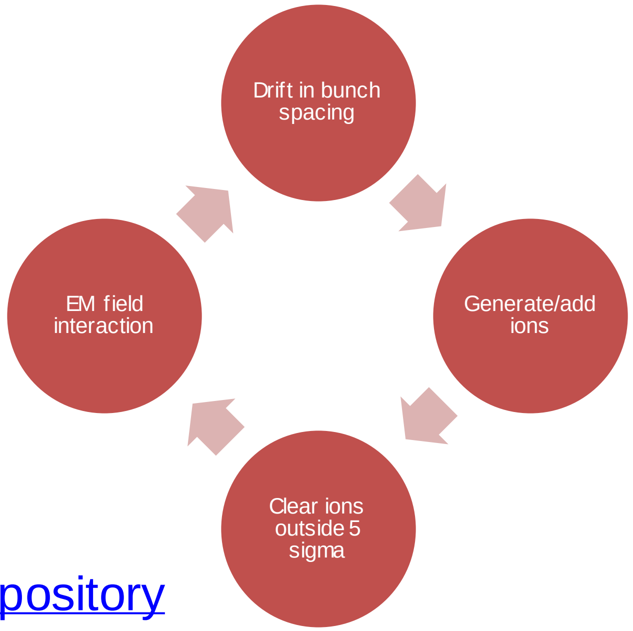
$$\frac{\partial^2 y_I(s,t|z')}{\partial t^2} + \underbrace{\omega_I^2(z)\left[y_I(s,t|z') - y_b(s,ct-s)\right]}_{\text{linear force}} = 0$$

$$\frac{\partial^2}{\partial s^2} y_e(s|z) + \frac{\omega_\beta^2}{v^2} y_e(s|z) + \overbrace{\frac{\omega_e^2 z}{v^2 l}\left[y_e(s|z) - \underbrace{\frac{1}{z}\int_0^z y_I\left(s, \frac{s+z'}{v}|z'\right)}_{\text{mean ion position at position } z} \right]}^{\text{beam-ion coupling force}} = 0$$

Our ion simulation approach







Gitlab repository

- Code developed in-house (as a branch of mbtrack2, to be integrated to the main branch)
- Ring is divided into segments
- Each segment consists of transverse tracking transformation
- Collective effects (wakefields, ions, etc.) can be placed in between segments
- Longitudinal tracking, rad. effects, etc. are done once per turn
- Each beam-ion interaction point can have different parameters:
  - Residual gas density
  - Ion species
  - Local Twiss parameters
- Electromagnetic interaction can be modelled as
  - Linear force (for comparisons with the theory)
  - Bassetti-Erskine formula (**this presentation**)
  - Particle-in-Cell Poisson solver (PIC solver)

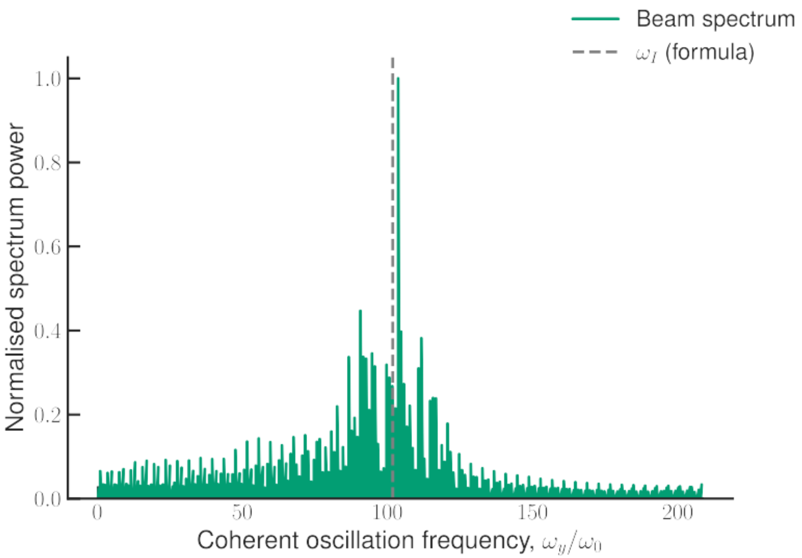
Beam-ion interaction components

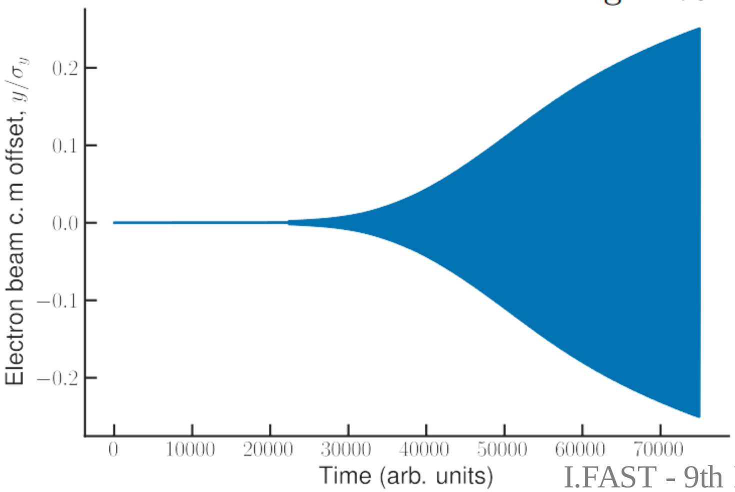
1. **Build-up**
   - Are ions accumulating bunch-to-bunch?
   - What is the transverse distribution of ions?
   - What determines equilibrium ion density?
2. **Instability**
   - How strong is the instability?
   - How to model it accurately?

TABLE I. Relevant accelerator and beam parameters in the uniform filling pattern.

Parameter	SOLEIL	SOLEIL II	Units
Circumference, C	354.10	353.74	m
Energy, E	2.75	2.75	GeV
Beam current, I	500	500	mA
Bunch spacing, L_{sep}	0.85	0.85	m
Horizontal emittance, ε_x	4000	83	pm rad
Vertical emittance[a], ε_y	83	25	pm rad
Average pressure, p	1×10^{-9}	1×10^{-9}	mbar
Horizontal tune, Q_x	18.16	54.2	
Vertical tune, Q_y	10.22	18.3	
Bunch number, n_b	416	416	

[a] assuming 30 % emittance coupling via white noise excitation





Ion trapping in SOLEIL and SOLEIL II (30% coupling)

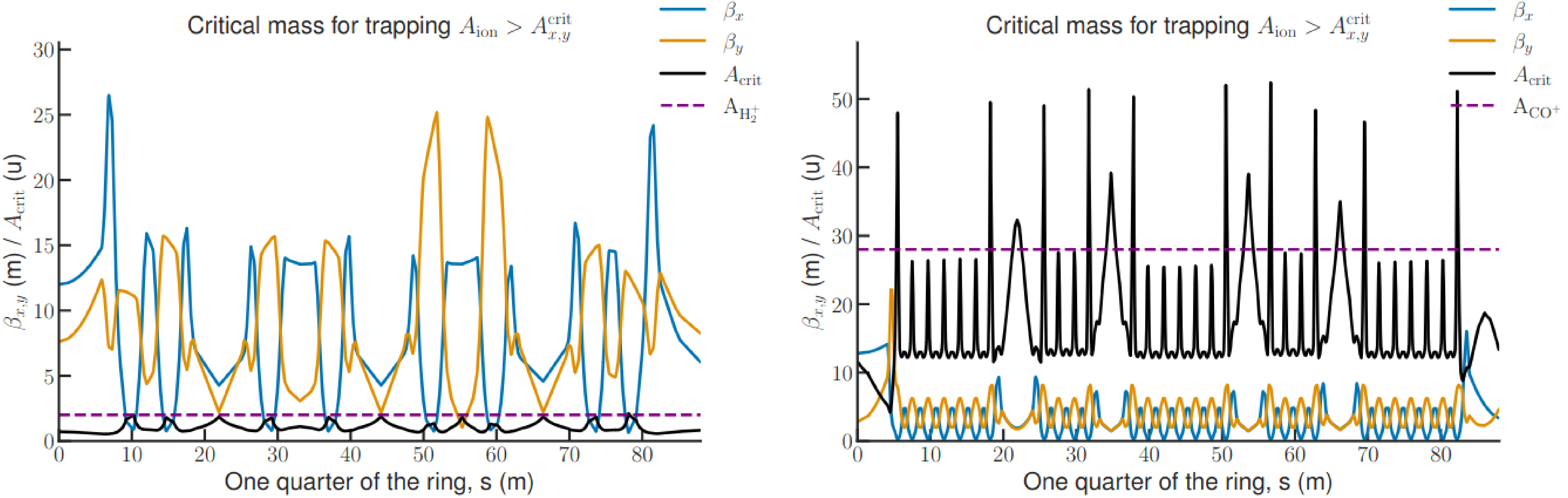


FIG. 1. Beta-functions and critical ion trapping masses along the ring for SOLEIL (left) and SOLEIL II (right).

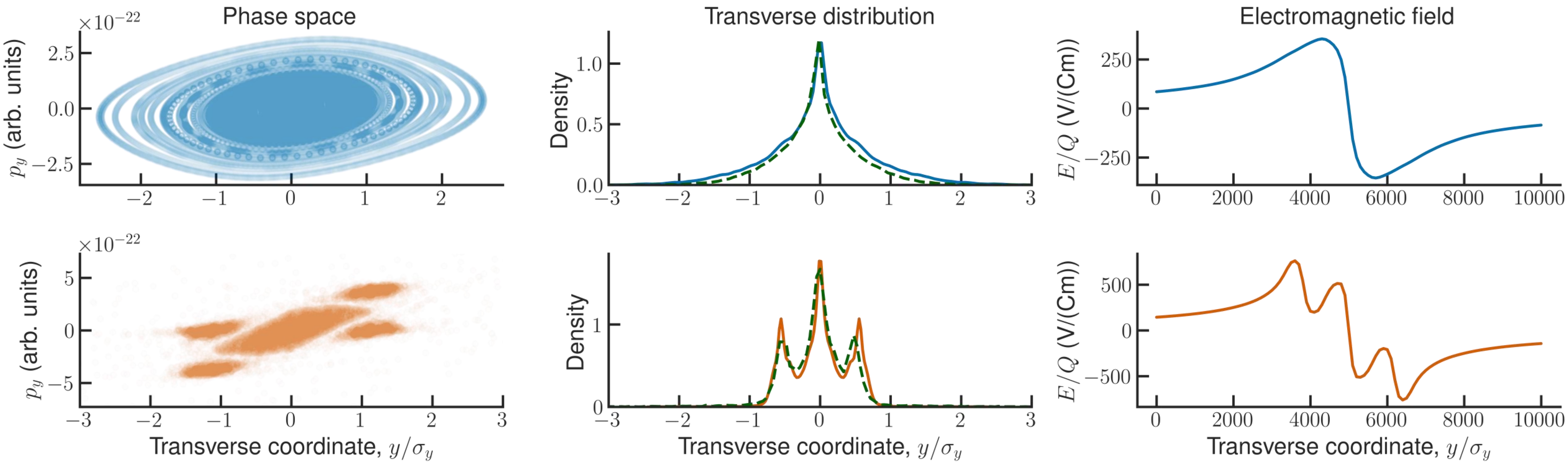
$$M_{\rm ions} = \underbrace{\begin{pmatrix} \cos\phi & \beta\sin\phi \\ -\frac{1}{\beta}\sin\phi & \cos\phi \end{pmatrix}}_{\text{Focusing by electron bunch}} \underbrace{\begin{pmatrix} 1 & L_{\rm drift} \\ 0 & 1 \end{pmatrix}}_{\text{Drift in bunch spacing}}$$

$$|{\rm Tr} M| = \left| 2 - \left(\frac{\omega_I L_{\rm drift}}{c} \right)^2 \right| < 2 \Rightarrow \frac{\omega_I L_{\rm drift}}{c} < 2$$

$$A^{\rm crit}_{x,y}(s) = \frac{N_e L_{\rm sep} r_p}{2\sigma_{x,y}(s)[\sigma_x(s) + \sigma_y(s)]}$$

Transverse distribution of ions

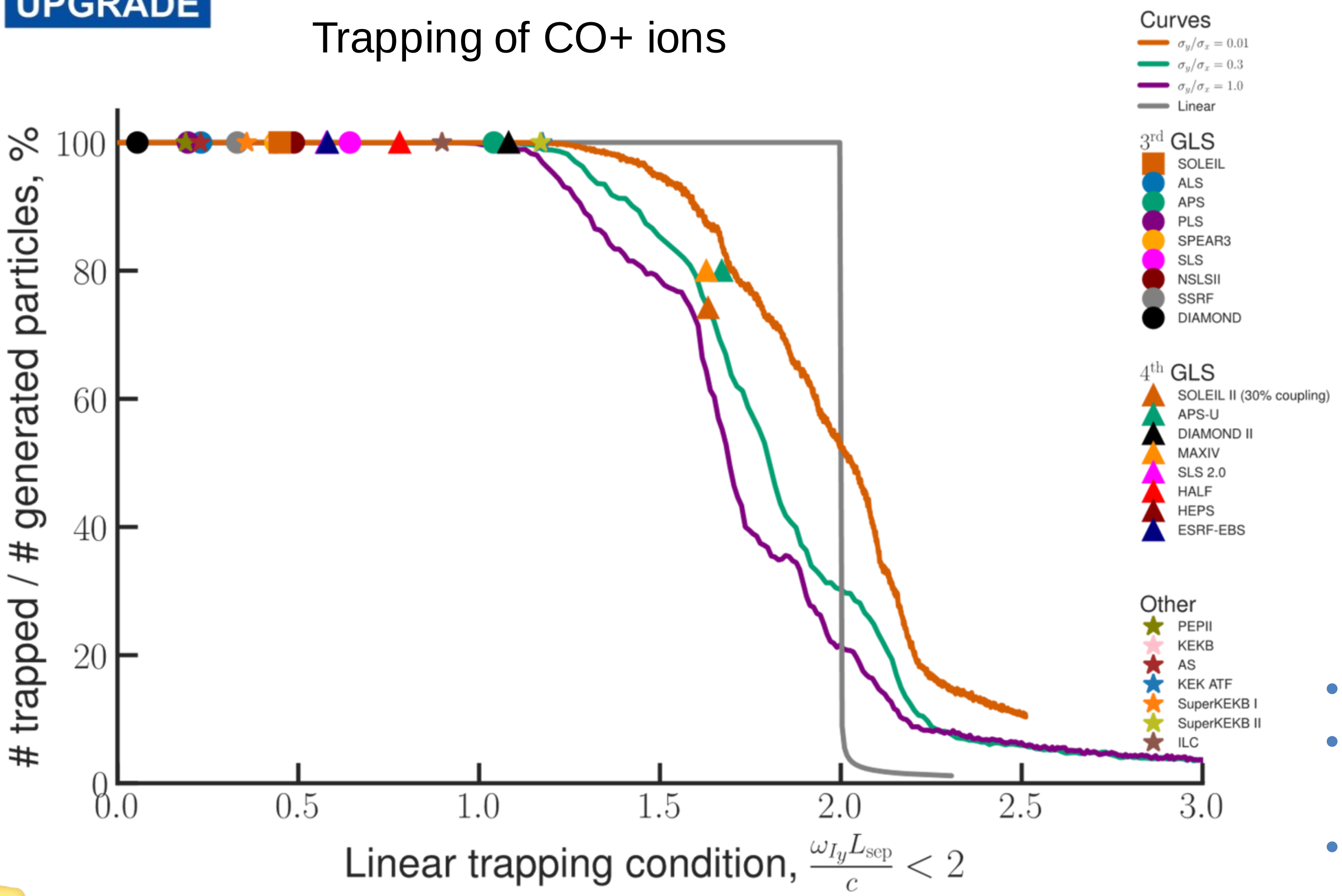
Top plots – SOLEIL, Bottom plots – SOLEIL II



- Dashed green lines indicate distributions obtained in elegant for the same case

Nonlinear trapping criterion

Trapping of CO+ ions



$$|\mathrm{Tr}M| = \left|2 - \left(\frac{\omega_I L_{\mathrm{drift}}}{c}\right)^2\right| < 2 \Rightarrow \frac{\omega_I L_{\mathrm{drift}}}{c} < 2 \quad M_{\mathrm{ions}} = \underbrace{\begin{pmatrix} \cos\phi & \beta\sin\phi \\ -\frac{1}{\beta}\sin\phi & \cos\phi \end{pmatrix}}_{\text{Focusing by electron bunch}} \underbrace{\begin{pmatrix} 1 & L_{\mathrm{drift}} \\ 0 & 1 \end{pmatrix}}_{\text{Drift in bunch spacing}}$$

Nonlinear trapping criteria $x = \frac{\omega_i L_{\mathrm{sep}}}{c}$

- X < 1 Linear regime
- 1 < x < 1.5 Quasilinear regime
- 1.5 < x < 2 Nonlinear regime
- x > 2-2.5 weakly trapped

- Simulations for SOLEIL cases
- Back-of-the-envelope calculation for other accelerators
- 80% of generated ions are trapped after 1000 bunch passes

Equilibrium ion density

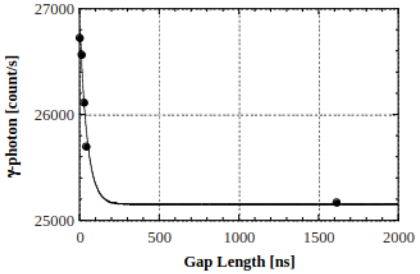


Figure 2: Counts of the γ-photons at a stored current of 100 mA as a function of the gap length.

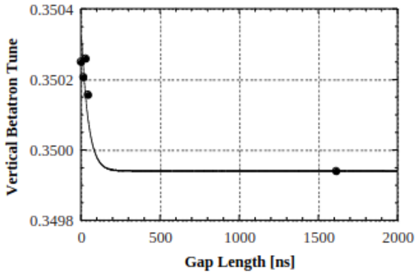
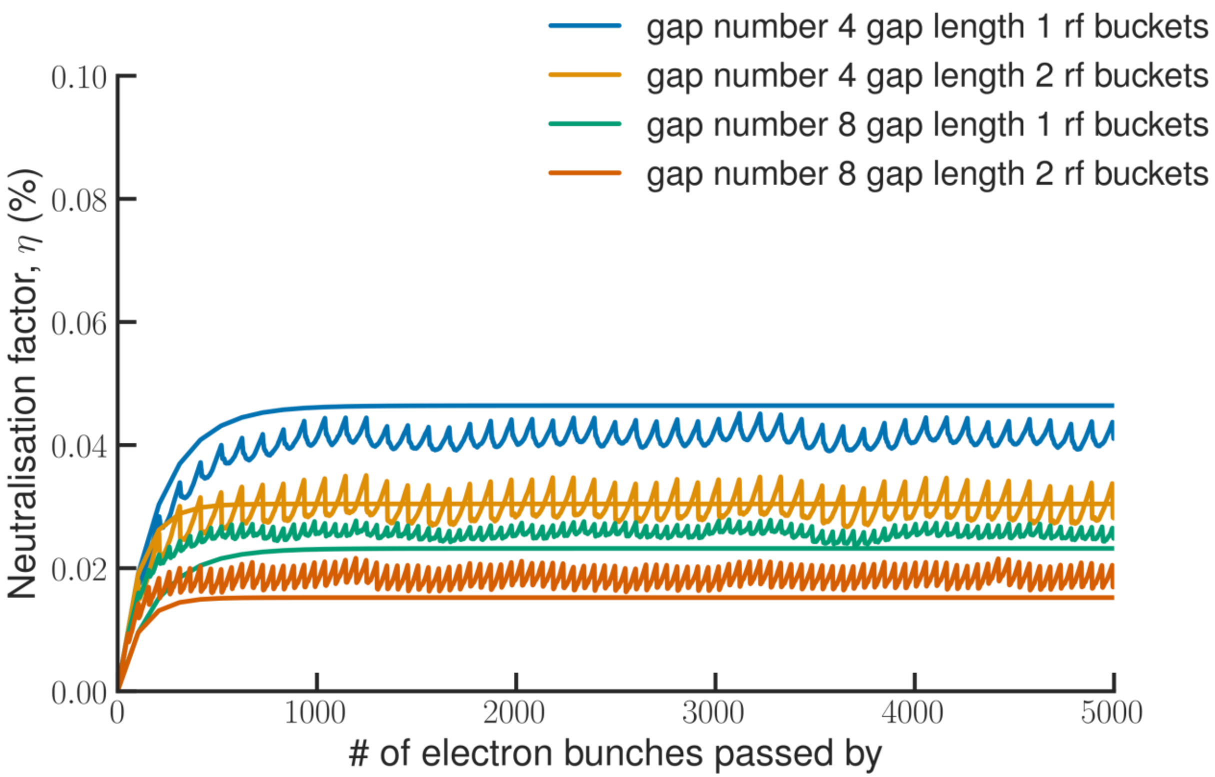


Figure 6: Vertical betatron tunes at the stored current of 100 mA as a function of the gap length.

$$f_{\text{ions}}(n) = N_{\text{ions}} M_{\text{train}} \sum_{k=0}^{n} \left[\exp\left(-n_{\text{empty}} L_{\text{sep}} / L_{\text{diffusion}}\right)\right]^k$$

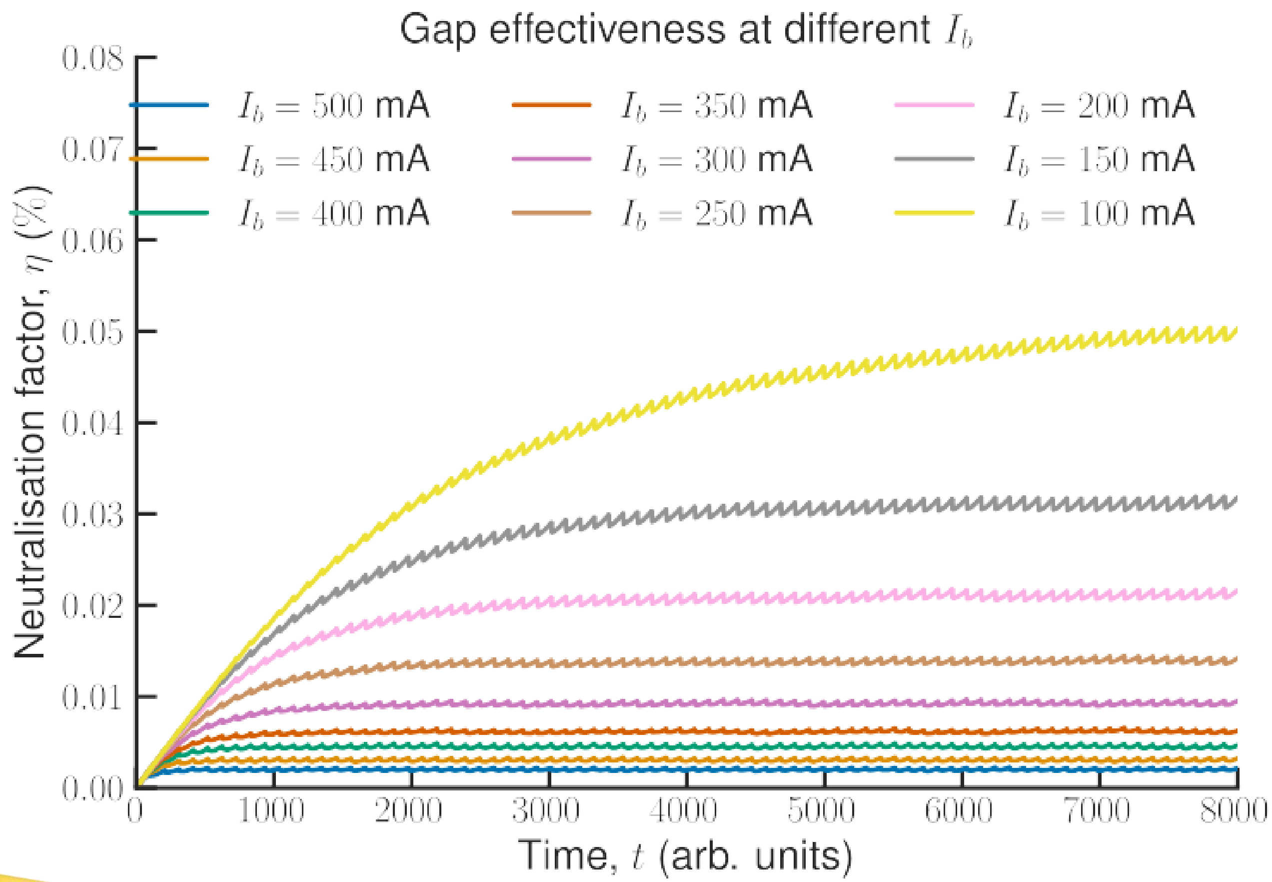
$$L_{\text{diffusion}} = f(\omega_I, \sigma_x, \sigma_y, I)$$

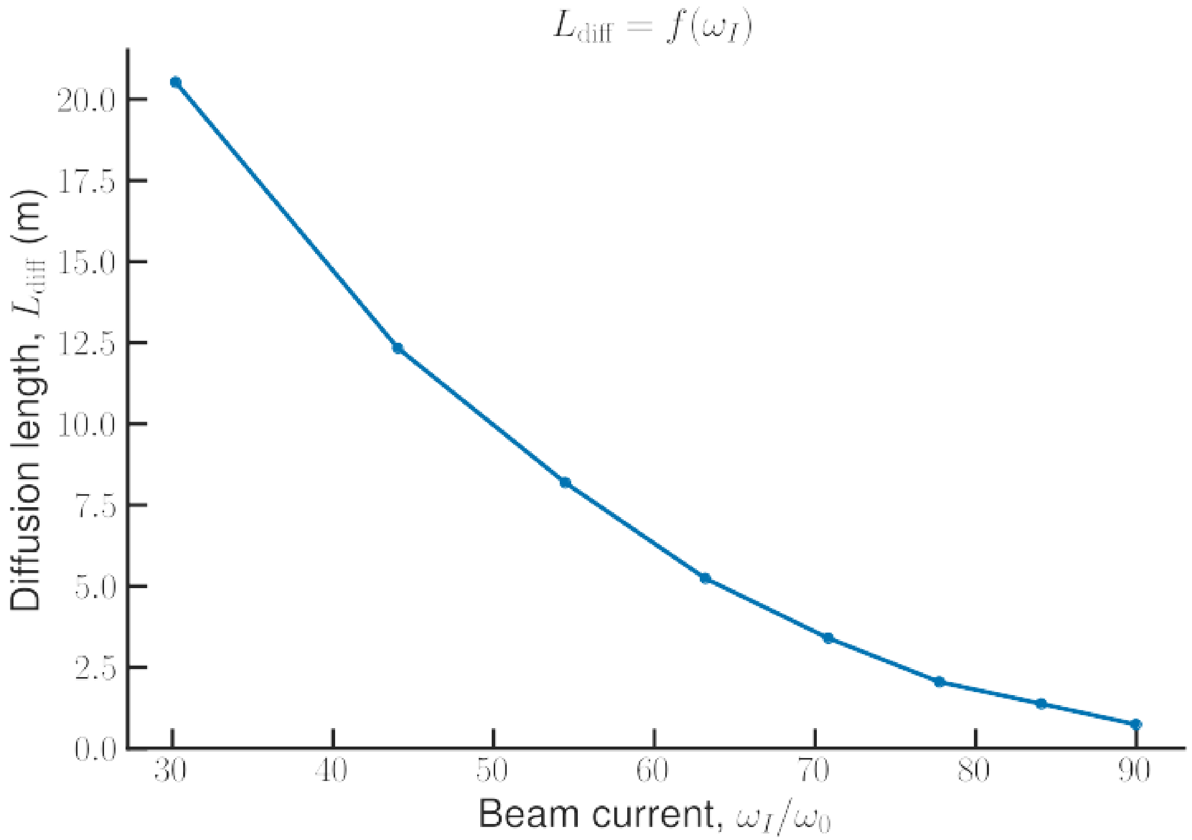
- Linear with train length
- Exponential with gap length
- Possible to extend for arbitrary filling/beam current pattern



[1] Wang, L., Cai, Y., Raubenheimer, T. O., Fukuma, H. (2011). PRSTAB, 14(8), 084401. https://doi.org/10.1103/PhysRevSTAB.14.084401
[2] Bosch, R. A. (2000). NIMA, *450*(2–3), 223–230. https://doi.org/10.1016/S0168-9002(00)00298-9
[3] M. Takao, H. Ego, Y. Kawashima, Y. Ohashi, T. Ohshima, and H. Saeki, https://jacow.org/e02/papers/WEPRI037.pdf

Diffusion length



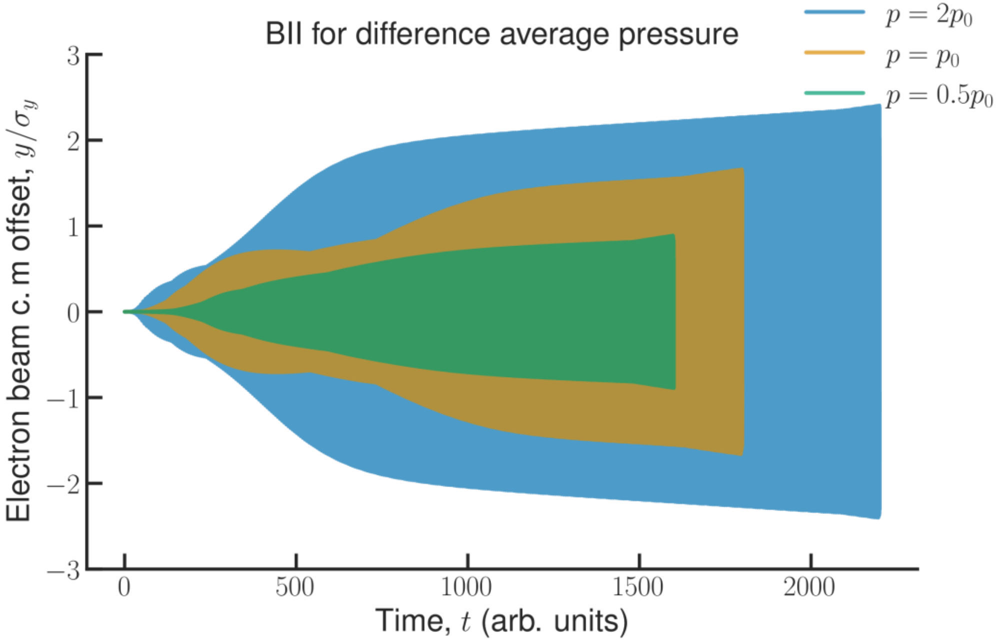


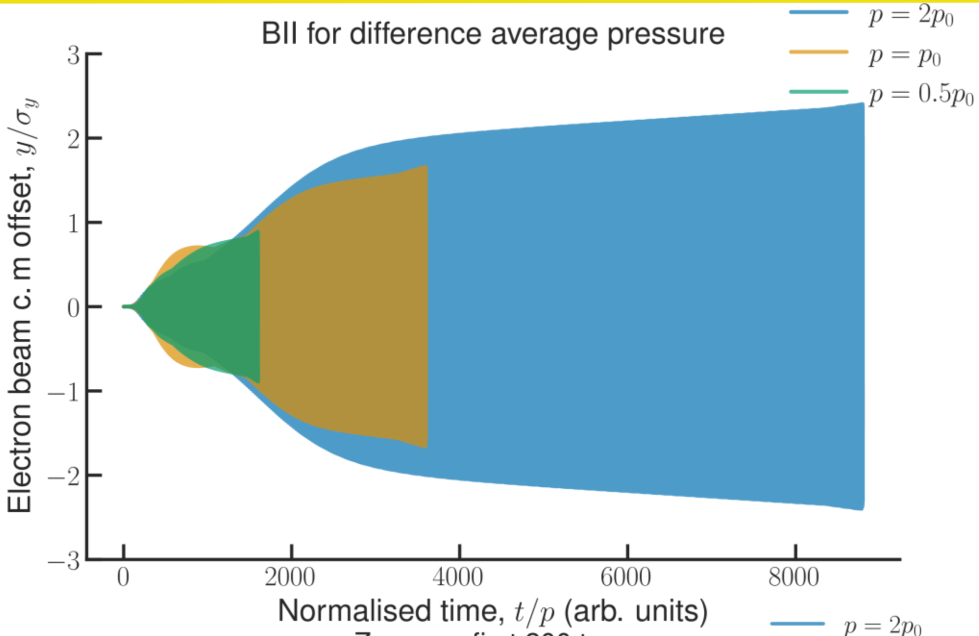
- Diffusion length could be obtained as a function of ω and L
- Diffusion length alone determines gap effectiveness
- With semianalytical formula we can predict the optimal gap length/number of gaps

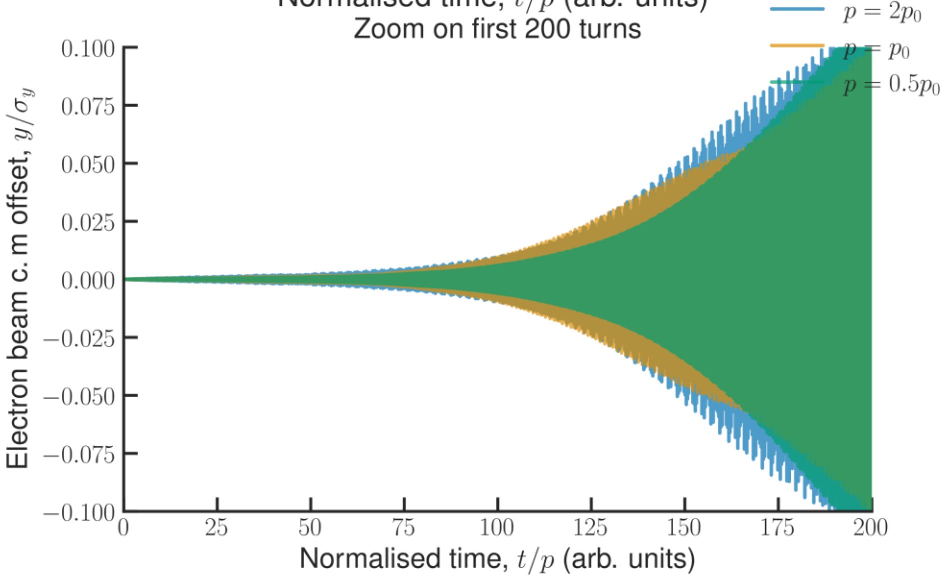
Instability scaling with vacuum pressure

- According to various G. Stupakov derivations

$$\tau^{-1} \propto p$$







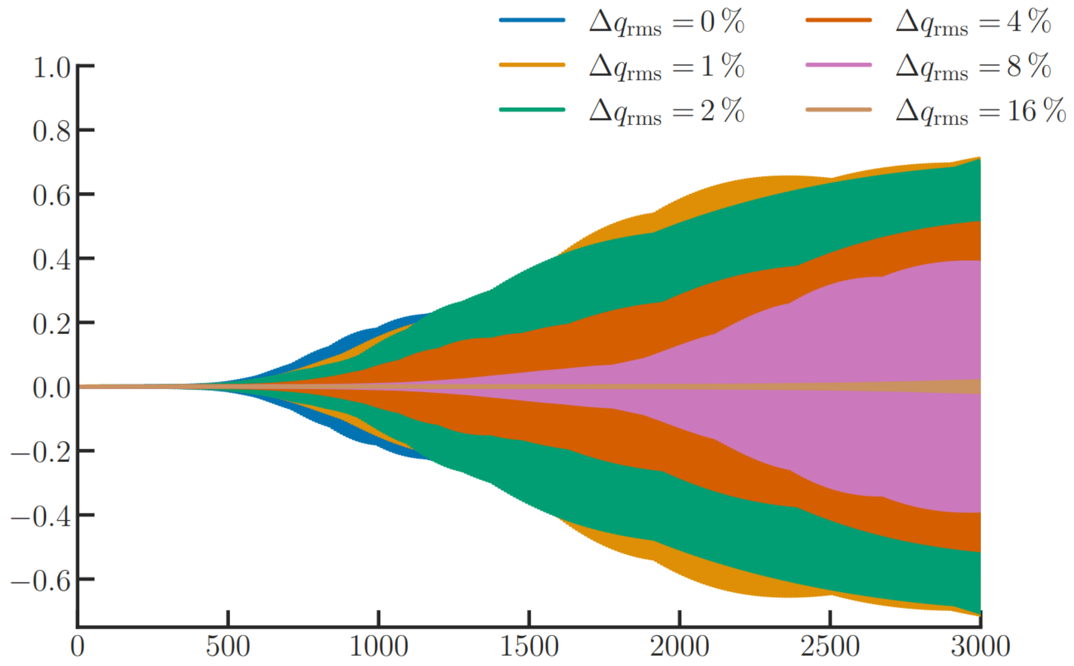
Conclusion:

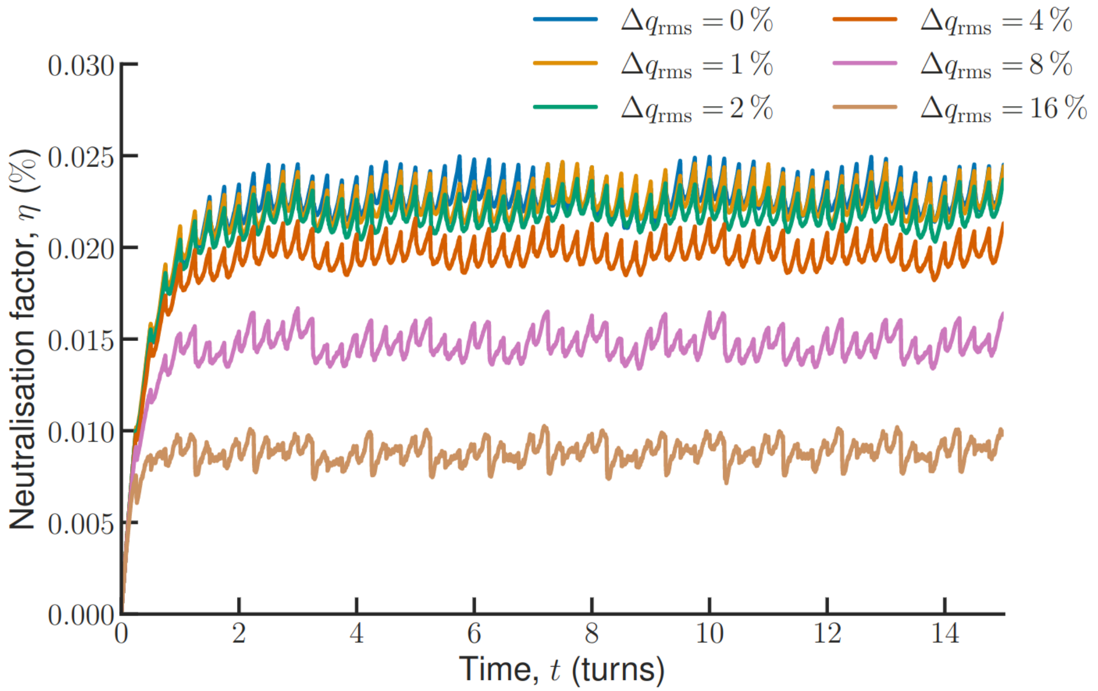
- Stupakov's results confirmed with tracking

- twice the pressure mean twice as fast BII

Bunch-to-bunch charge variation

(4 gaps, each 1 empty rf bucker long)





- Modelled as a Gaussian noise
- Result is qualitatively similar to [1]
- 4 gaps with 1 empty rf bucket each

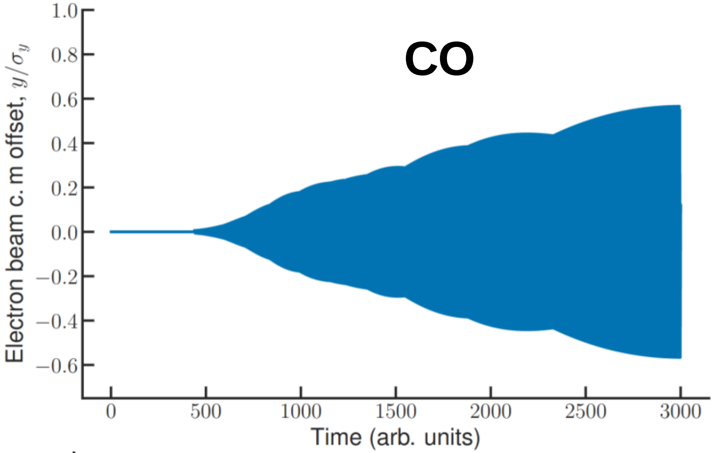
Interpretations:

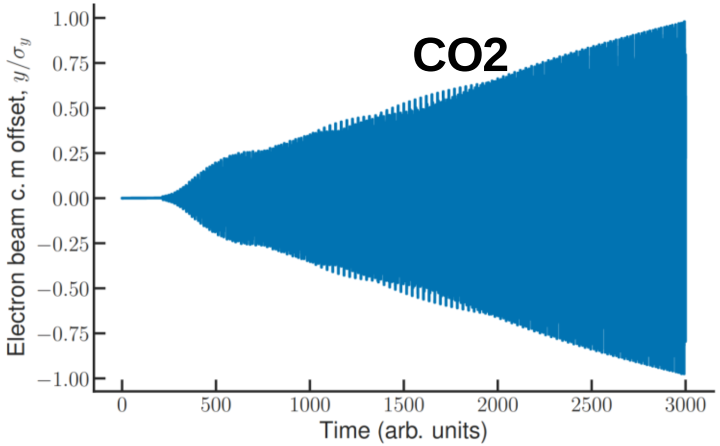
- Ion frequency variation -> decoherence
- "Gradient Error" in the "lattice for ions" [2] -> affects ion accumulation (confirmed)

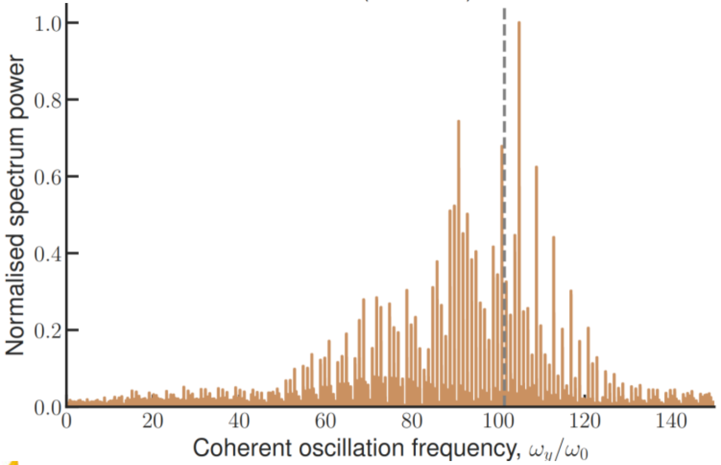
[1] J. Calvey and M. Borland, "Simulation of incoherent ion effects in electron storage rings," https://doi.org/10.1103/PhysRevAccelBeams.24.124401
[2] Workshop on PP in the SPS, SPS-PP-1, 1980

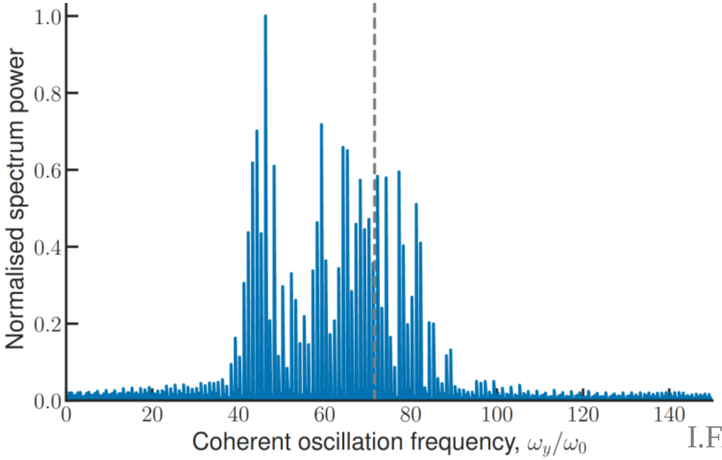
Considering gas mixture (CO and CO2)

Species	H_2	CH_4	CO	CO_2	Units
SOLEIL II					
Cross-section[a], Σ_I	0.36	2.09	1.78	2.79	Mbarn
Partial pressure, p	0.67	0.02	0.16	0.12	pbar
$\langle \omega_{I_x} \rangle / \omega_0$	267.7	94.6	71.5	57.1	
$\langle \omega_{I_y} \rangle / \omega_0$	386.2	136.5	103.2	71.6	

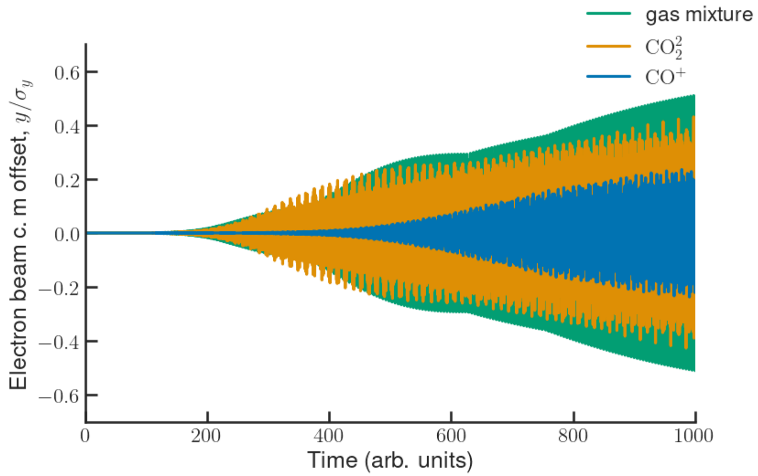


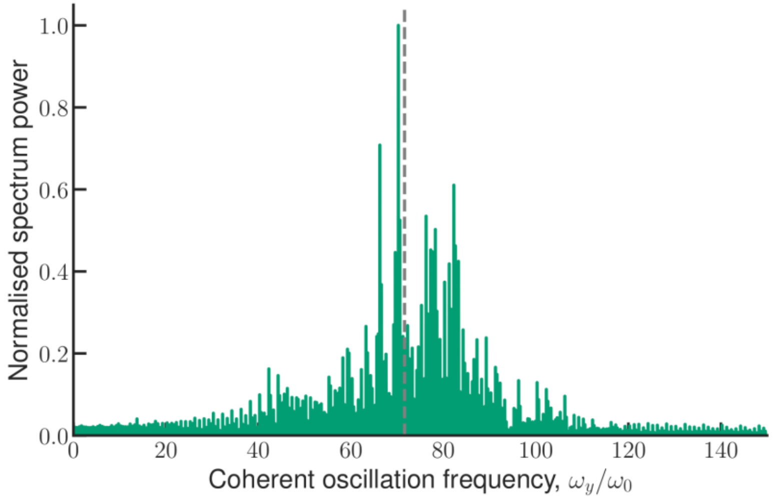






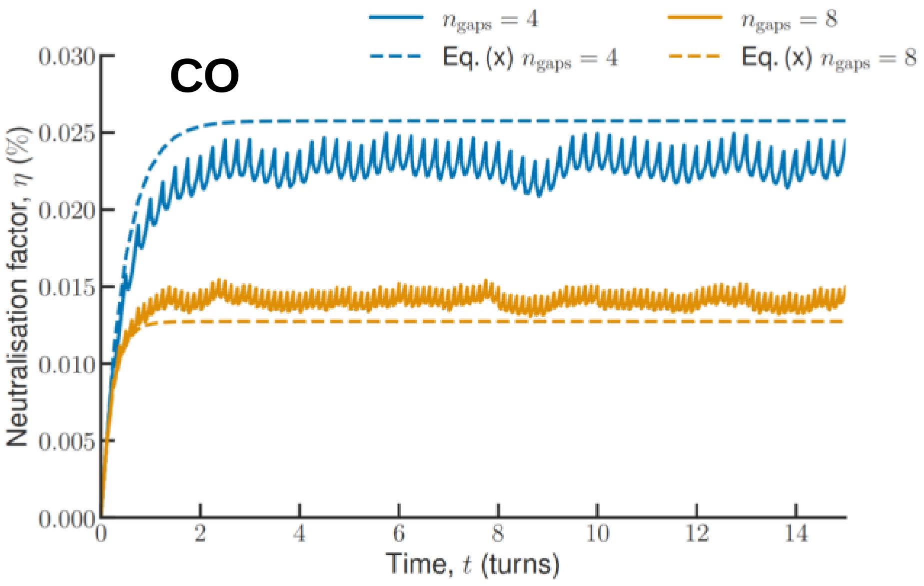
Mix of gases

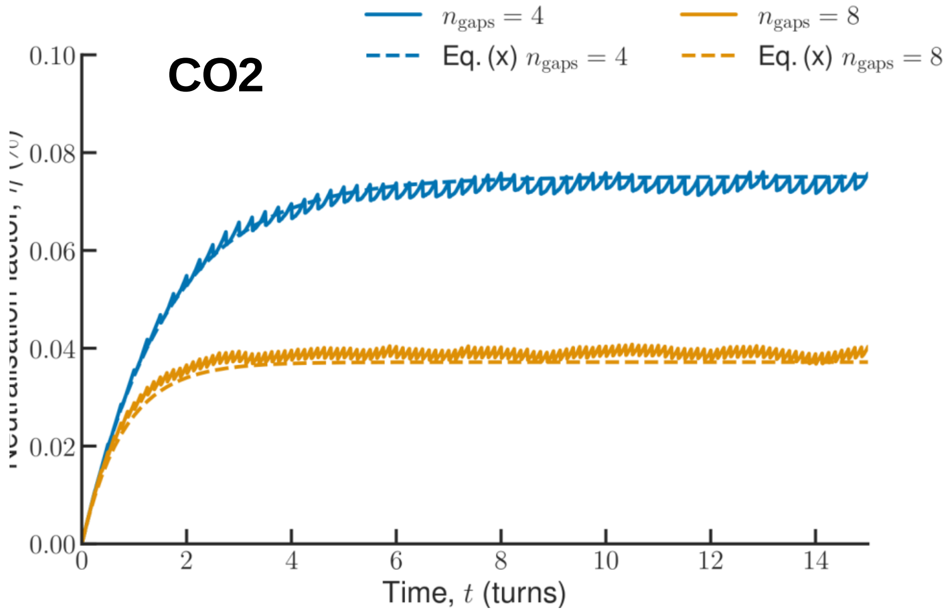




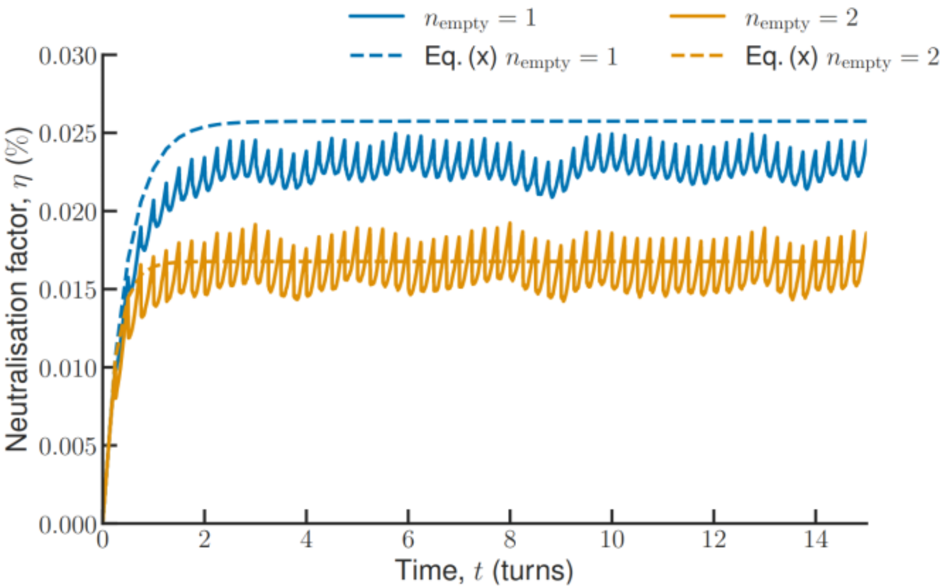
Gap effectiveness (CO vs CO2)

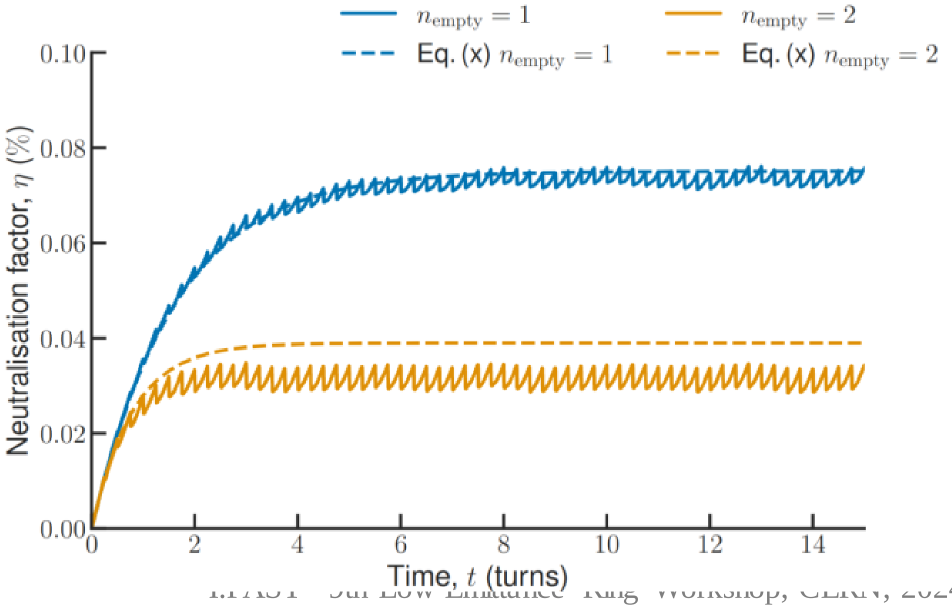
Gaps length of 1 varying number of gaps





4 gaps of varying gap length

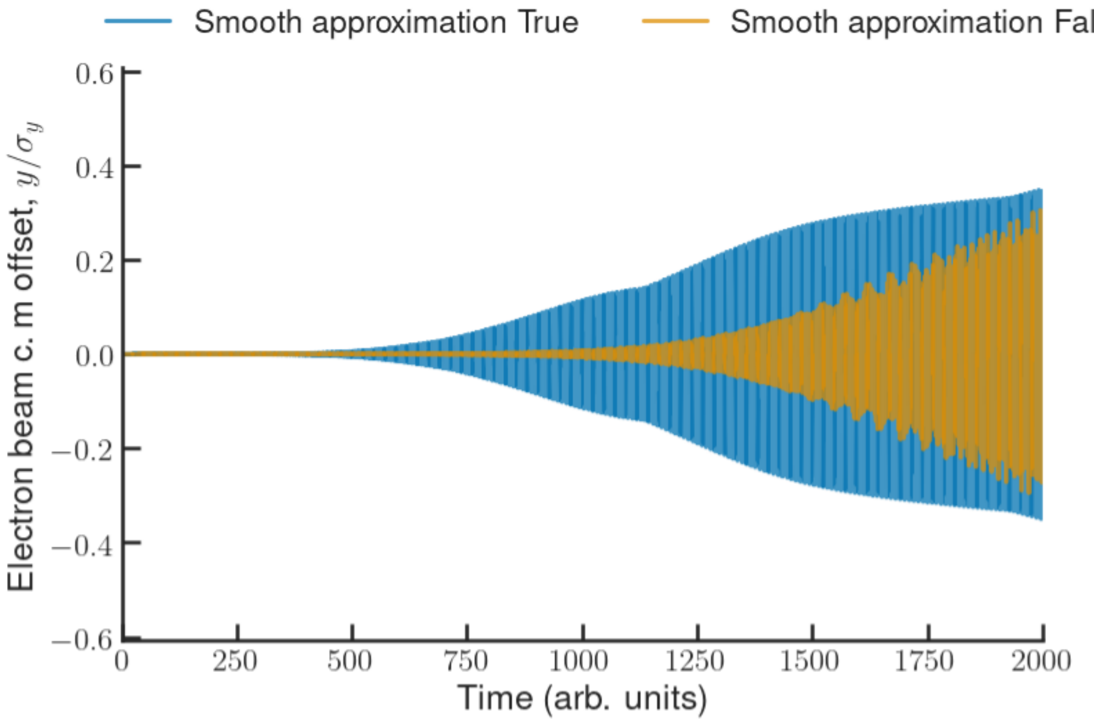




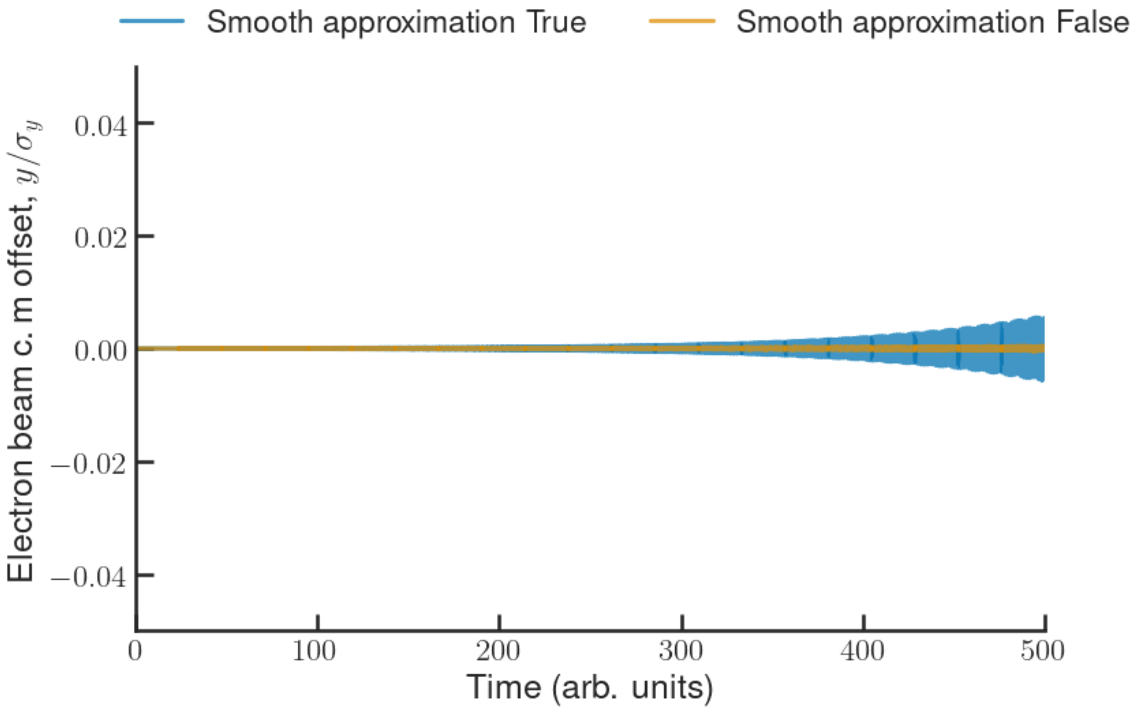
Conclusions

- mbtrack2 module for beam-ion instability to be released soon
- CO ions could still be trapped in 4-th generation light sources (4GSL)
- Trapping of ions in 4GSL is in a nonlinear regime:
    - Some ion are trapped
    - Others are lost due to nonlinear dynamics
- Transverse ion distribution can diverge from the analytical one in 4GSL
- Obtained nonlinear trapping criterion
- Gaps are very effective in 4GSL
- Optimal gap length is ~1-2 empty rf buckets
- Optimal gap length could be found via diffusion length
- Increasing the number of gaps is more effective
- Scaling of BII with vacuum pressure confirmed
- Bunch-to-bunch charge variation is not too important in 4GSL
- Transverse feedback is necessary to suppress the instability

Beam size variation along the ring



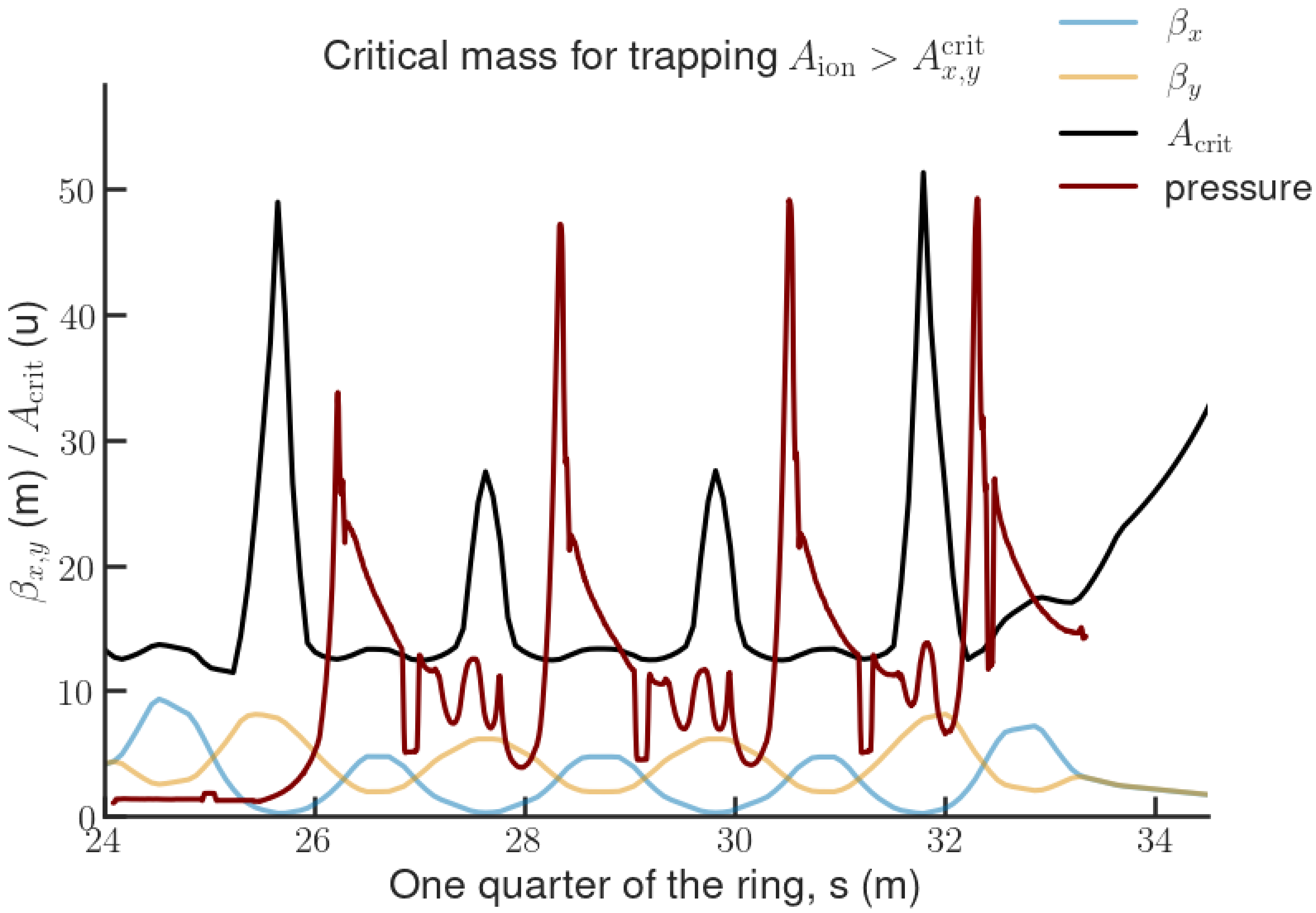
Zoom in on first 500 turns



- The growth rate should be decreased by orders of magnitude
- The nonlinear regime for large oscillations seems to be unaffected by it

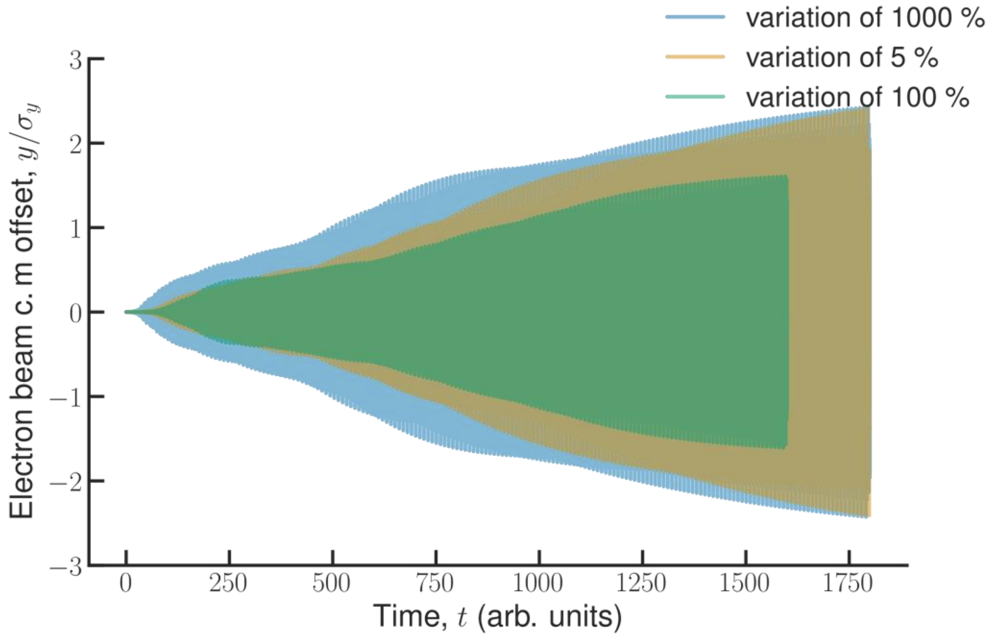
Vacuum pressure profile in 4BA arc

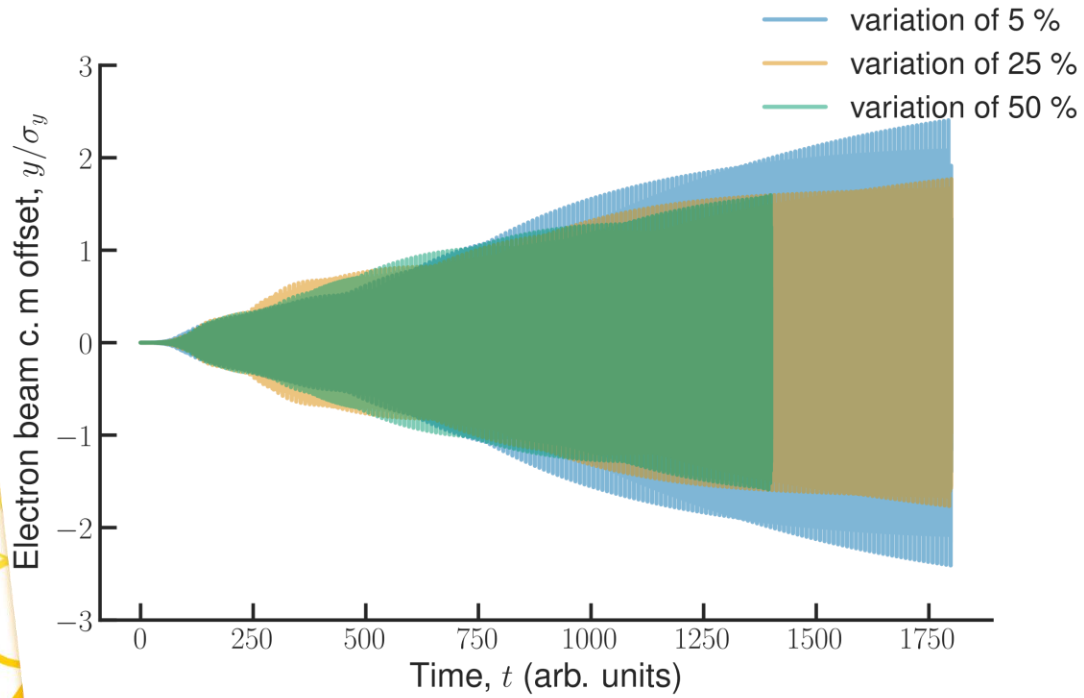
- Regular peaks in vacuum pressure
- Ion frequency variation due to beam size is not sampled because vacuum pressure is too low
- It's a good approximation to lump ion kicks to the locations of the peaks



Vacuum pressure variation along the ring

- Modelled as Gaussian noise
- Negligible effect for random variation
- Should be stronger for systematic variation if beam size variation is also included





Conclusion:

- random pressure variation along the ring is a weak effect
- Systematic variation might be a stronger if combined with beam-size variation