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Beams
Department

Optimising Resonance Driving Terms Using MAD-NG Parametric Differential Maps.

I.FAST 9th Low Emittance Rings Workshop 2024

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CERN-BE/ABP

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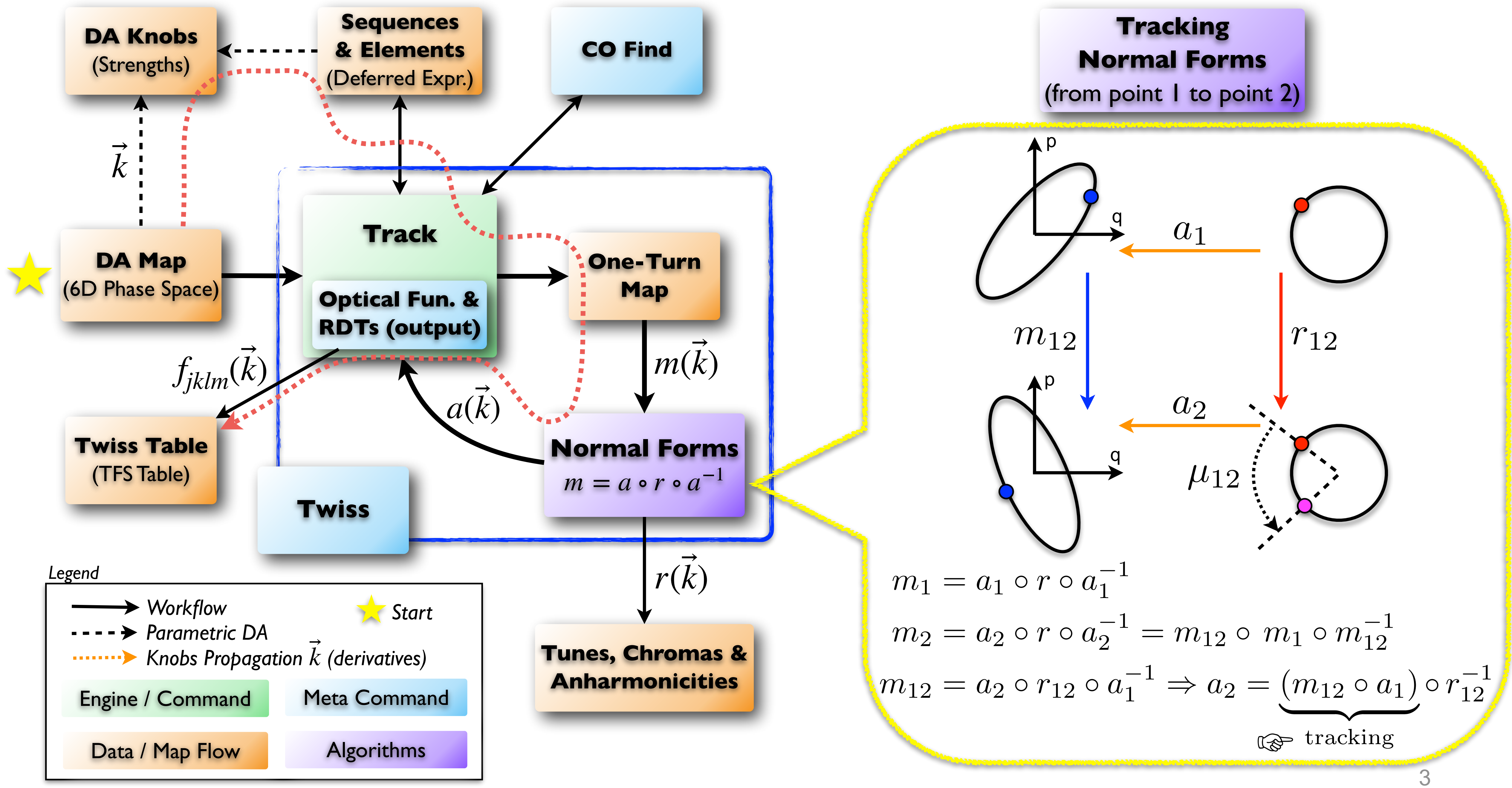
MAD-NG's Features

- **Integrated platform designed for Methodical Accelerators Design (optics).**
  - ➡ **Flexible language** ➠ **fast, simple, and general purpose scripting language.**
    - ~70% of the code is written in the Lua(JIT) scripting language, ~30% in C and C++.
  - ➡ **Flexible technologies** ➠ **self-contained, all-in-one and modular application.**
    - Single "copy & run" application, no dependencies (requires Gnuplot installed for plotting).
  - ➡ **Efficient & Portable technologies** ➠ **embeds a Tracing Just in Time compiler.**
    - Same results everywhere (LNX, OSX, WIN), extensive unit tests (>8000) and examples.
    - Extremely simple and fast Foreign Function Interface to external libraries in C, C++, Fortran, etc…
  - ➡ **Easy to extend & support** ➠ **embeds an online profiler and debugger.**
    - Adding new elements with new physics takes a day (assuming known explicit exact equations).
- **5D and 6D physics using high-order differential algebra and symplectic integrators.**
  - ➡ Combined physics, combined elements, misalignments & errors, local & global frames, all elements fringe fields, forward, backward and reverse tracking.
  - ➡ Physics & Maths in Lua and C/C++, **performance is x10-x70 faster than MADX-PTC**.
  - ➡ Support and development for new physics extensions is extremely easy…
- **Development open source.**
  - ➡ License GPL V3, Manual (~200p, covers <25%), Lua Manual (30p).
  - ➡ Sources https://github.com/MethodicalAcceleratorDesign/MAD
  - ➡ Releases & Manual https://cern.ch/mad/releases/madng/
  - ➡ Online Manual https://cern.ch/mad/releases/madng/html/
  - ➡ Learn Lua in 15 min: https://www.youtube.com/watch?v=REReUFgii5A

MAD-NG's Parametric Differential Maps Principle

Track a **high-order differential algebra** (DA) map on the closed orbit (optionally) equipped with **parameters** (knobs) to obtain the **one-turn map** m, then compute the **closed non-linear normal form** $m = a \circ r \circ a^{-1}$ and track the normalising map a to extract the **optical functions** (α, β, μ, etc.) and the **resonant driving terms (RDTs)** along the lattice.



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MAD-NG studies: RDTs for HL-LHC

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```
-- HL-LHC setup
MADX:load("hllhc_saved.seq", "hllhc_saved.mad")
MADX.lhcb1.beam = beam {particle="proton", energy=450}
MADX.lhcb2.beam = beam {particle="proton", energy=450}
MADX.lhcb2.dir  = -1 -- bv = -1

-- list of RDTs
local rdts = {"f4000", "f3100", "f2020", "f1120"}

-- loop over lhcb1 and lhcb2
for _,lhc in ipairs{MADX.lhcb1, MADX.lhcb2} do

  -- create phase-space damap at 4th order
  local X0 = damap {nv=6, mo=4}

  -- compute RDTs along HL-LHC
  local mtbl = twiss {sequence=lhc, X0=X0, trkrdt=rdts}

  -- plot RDTs along HL-LHC
  plot_rdt(mtbl, rdts)

end -- end of loop
```

4th-order DA map

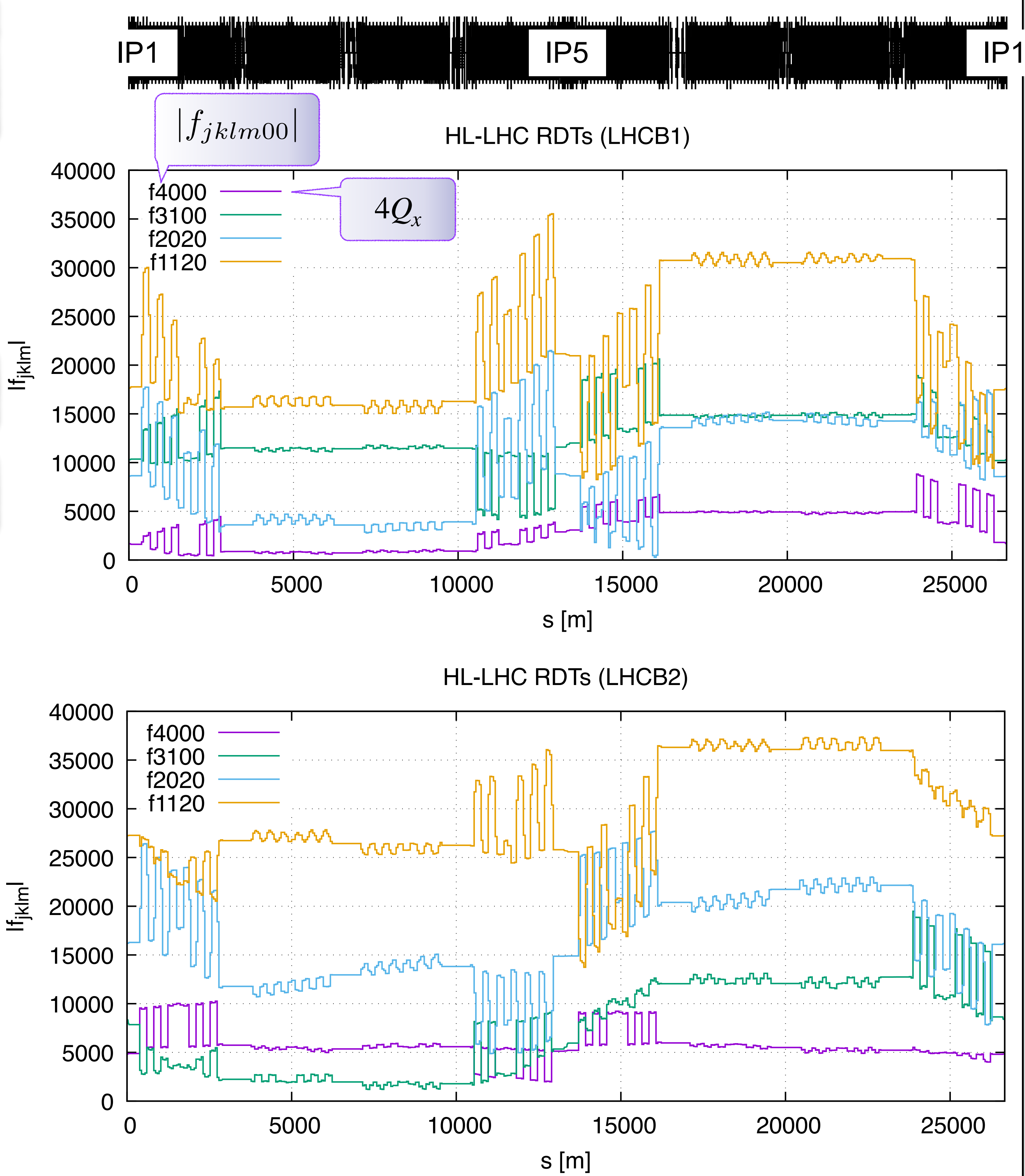
$$f_{jklm} = \frac{h_{jklm}}{1 - e^{2\pi i[(j-k)\nu_x + (l-m)\nu_y]}}$$

Resonances: $N = (j-k)Q_x + (l-m)Q_y$

Spectral lines: $H(1+k-j, m-l)$

$V(k-j, 1+m-l)$

These 2 RDTs' plots take 32 sec in MAD-NG 40 min in MADX-PTC



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MAD-NG studies: Parametric RDTs for HL-LHC

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```
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MADX:load("hllhc_saved.seq", "hllhc_saved.mad")
MADX.lhcb1.beam = beam {particle="proton", energy=450}
MADX.lhcb2.beam = beam {particle="proton", energy=450}
MADX.lhcb2.dir  = -1 -- bv = -1

-- list of knobs for both sequences
local knbs = { LHCB1 = {'ksf1.a45b1','ksf2.a45b1'},
               LHCB2 = {'ksf1.a45b2','ksf2.a45b2'} }

-- list of RDTs: 4Qx w 1st and 2nd derivatives vs knobs
local rdts = {"f40000000", "f40000010", "f40000020",
              "f40000011", "f40000001", "f40000002",}

-- loop over lhcb1 and lhcb2
for _,lhc in ipairs{MADX.lhcb1, MADX.lhcb2} do

  -- select knobs
  local knb = knbs[lhc.name]

  -- create phase-space damap at 6th order (mo=4th+po)
  local X0 = damap {nv=6, mo=6, np=#knb, po=2, pn=knb}

  -- set knobs: scalar + TPSA -> TPSA
  for _,k in ipairs(knb) do MADX[k] = MADX[k]+X0[k] end

  -- compute RDTs along HL-LHC
  local mtbl = twiss {sequence=lhc, X0=X0, trkrdt=rdts }

  -- plot RDTs along HL-LHC
  plot_rdt(mtbl, rdts)

end -- end of loop
```
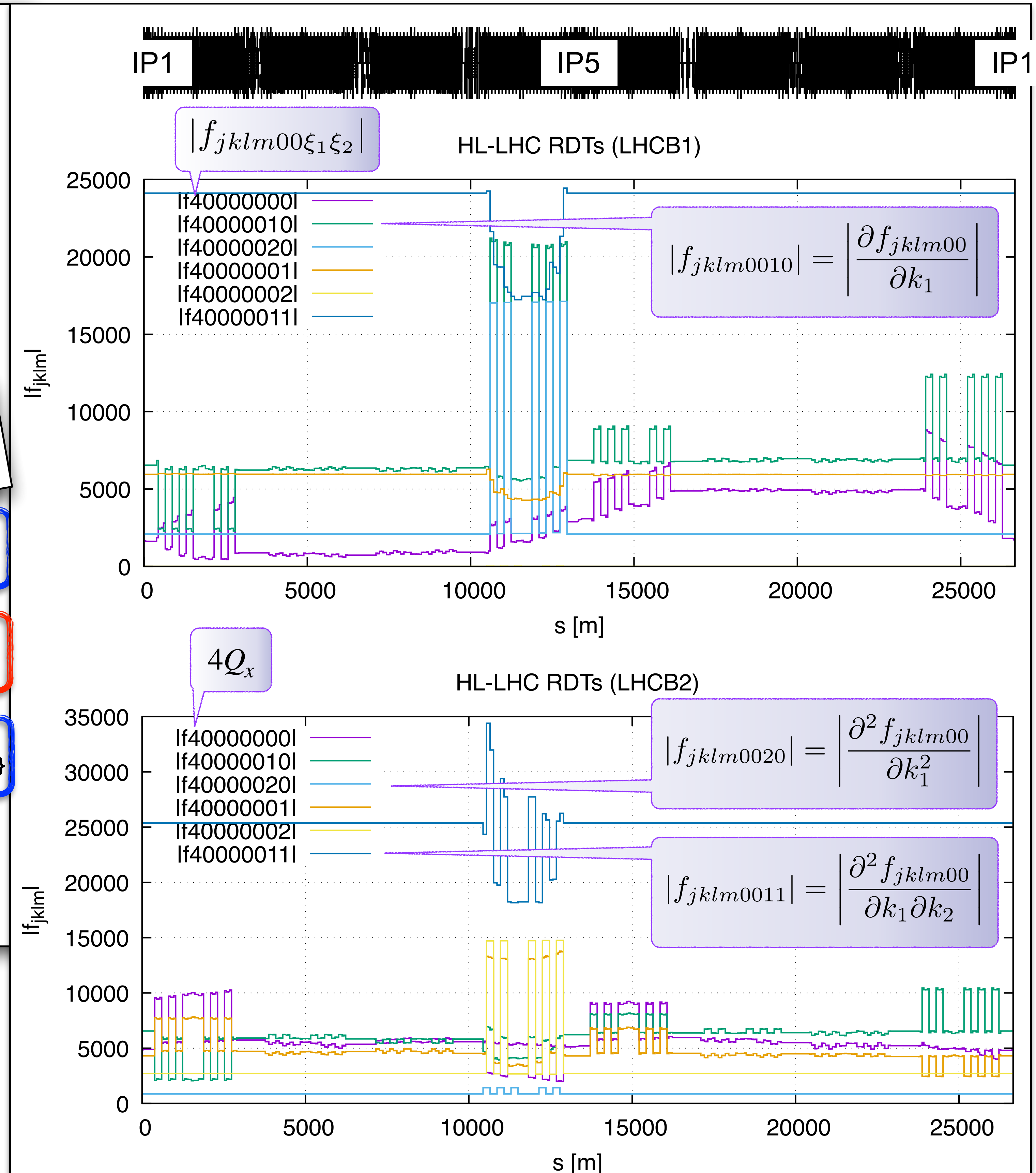

k_1 k_2

6th-order parametric DA map

$$f_{jklmno\xi_1\xi_2} = \frac{\partial^{\xi_1+\xi_2} f_{jklmno}}{\partial k_1^{\xi_1} \partial k_2^{\xi_2}}$$

$$\text{DA map order} = j+k+l+m+n+o+\sum_i \xi_i$$



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MAD-NG studies: lower RDTs for LHC@injection

Loading LHC Sequences & Optics (1)

```
MADX:load'lhc_seq.madx'
MADX:load'inj_optics.madx'
MADX.lhcb1.beam = beam {particle='proton', energy=450}
MADX.lhcb2.beam = beam {particle='proton', energy=450}
MADX.lhcb2.dir  = -1 -- set LHCB2 as reversed
```

Building Parametric DA Map (2)

```
local prms = { -- param./knob names (strings)
  -- 16 strengths of trim quadrupoles families
  'kqtf.a12b1', 'kqtf.a23b1', ···, 'kqtf.a81b1',
  'kqtd.a12b1', 'kqtd.a23b1', ···, 'kqtd.a81b1',
  -- 16 strengths of octupoles families
  'kof.a12b1' , 'kof.a23b1' , ···, 'kof.a81b1',
  'kod.a12b1' , 'kod.a23b1' , ···, 'kod.a81b1',
}
```

32 Circuit Knobs 16 MQT + 16 MO

```
-- DA map representing parametric phase-space
local X0 = damap {nv=6, mo=5, np=#prms, po=1, pn=prms}
```

5th-order parametric DA map

```
-- convert scalars to GTPSAs within MADX env.
for _,knb in pairs(prms) do
  MADX[knb] = MADX[knb] + X0[knb]
end
```

Use strengths as DA map parameters

Parametric Normal Forms & Setup (3)

```
-- function to compute non-linear normal forms
local function get_nf (lhc, X0)
  local _, mflw = track {sequence=lhc, X0=X0}
  return normal(mflw[1]):analyse("all")
end
```

Twiss-like RDTs @ IP1 (faster for single point)

```
-- save reference values
local nf = get_nf(X0, MADX.lhcb1)
local q1ref  = nf:q1{1}
local q2ref  = nf:q2{1}
local q1jref = nf:anhx{1,0}
local q2jref = nf:anhy{0,1}
```

A solution is found by: MAD-NG in 3 min MADX-PTC in 45 min (using finite difference approx.)

Optimizing RDTs (4 & 5)

```
match {
  -- compute non-linear normal forms
  command := get_nf(), -- returns nf used below
```

Jacobian [10x32] filled derivatives vs. knobs used by optimiser

```
  -- compute Jacobian from parametric maps
  jacobian = \nf,_,J =>
    for k=1,32 do -- fill [10x32] J matrix
      J:set(1,k, nf:q1{1,k} or 0)
      J:set(2,k, nf:q2{1,k} or 0)
      J:set(3,k, nf:anhx{1,0,0,k})
      J:set(4,k, nf:anhy{0,1,0,k})
      J:set(5,k, nf:gnfu{"2002",k}.re)
      J:set(6,k, nf:gnfu{"2002",k}.im)
      J:set(7,k, nf:gnfu{"4000",k}.re)
      J:set(8,k, nf:gnfu{"4000",k}.im)
      J:set(9,k, nf:gnfu{"0040",k}.re)
      J:set(10,k,nf:gnfu{"0040",k}.im)
    end
  end,
```

$$\frac{\partial Q_x}{\partial k_i}, \frac{\partial Q_y}{\partial k_i}, \frac{\partial^2 Q_x}{\partial J_x \partial k_i}, \frac{\partial^2 Q_y}{\partial J_y \partial k_i}$$

$$\frac{\partial f_{2002}}{\partial k_i}, \frac{\partial f_{4000}}{\partial k_i}, \frac{\partial f_{0040}}{\partial k_i}$$

```
  -- variables in MADX env. to use as knobs
  variables = {
    {name=prms[1] , var='MADX[prms[1]]' },
    ···,
    {name=prms[32], var='MADX[prms[32]]'},
  },
```

32 knobs to vary

10 constraints to satisfy

```
  -- target constraints as equalities to zero
  equalities = {
    {name = 'q1'    , expr = \nf -> nf:q1{1} - q1ref},
    {name = 'q2'    , expr = \nf -> nf:q2{1} - q2ref},
    {name = 'q1j1'  , expr = \nf -> nf:anhx{1,0} - q1jre
    {name = 'q2j2'  , expr = \nf -> nf:anhy{0,1} - q2jre
    {name = 'f2002r', expr = \nf -> nf:gnfu{"2002"}.re  - 0},
    {name = 'f2002i', expr = \nf -> nf:gnfu{"2002"}.im) - 0},
    {name = 'f4000r', expr = \nf -> nf:gnfu{"4000"}.re) - 0},
    {name = 'f4000i', expr = \nf -> nf:gnfu{"4000"}.im) - 0},
    {name = 'f0040r', expr = \nf -> nf:gnfu{"0040"}.re) - 0},
    {name = 'f0040i', expr = \nf -> nf:gnfu{"0040"}.im) - 0},
  },
} -- close match
```

Invariant

$$Q_x, Q_y, \frac{\partial Q_x}{\partial J_x}, \frac{\partial Q_y}{\partial J_y}$$

Minimise

f_{2002}

f_{4000}

f_{0040}

Match has the same structure as in MAD-X with “use_macro”

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MAD-NG studies: DA increase for LHC@injection

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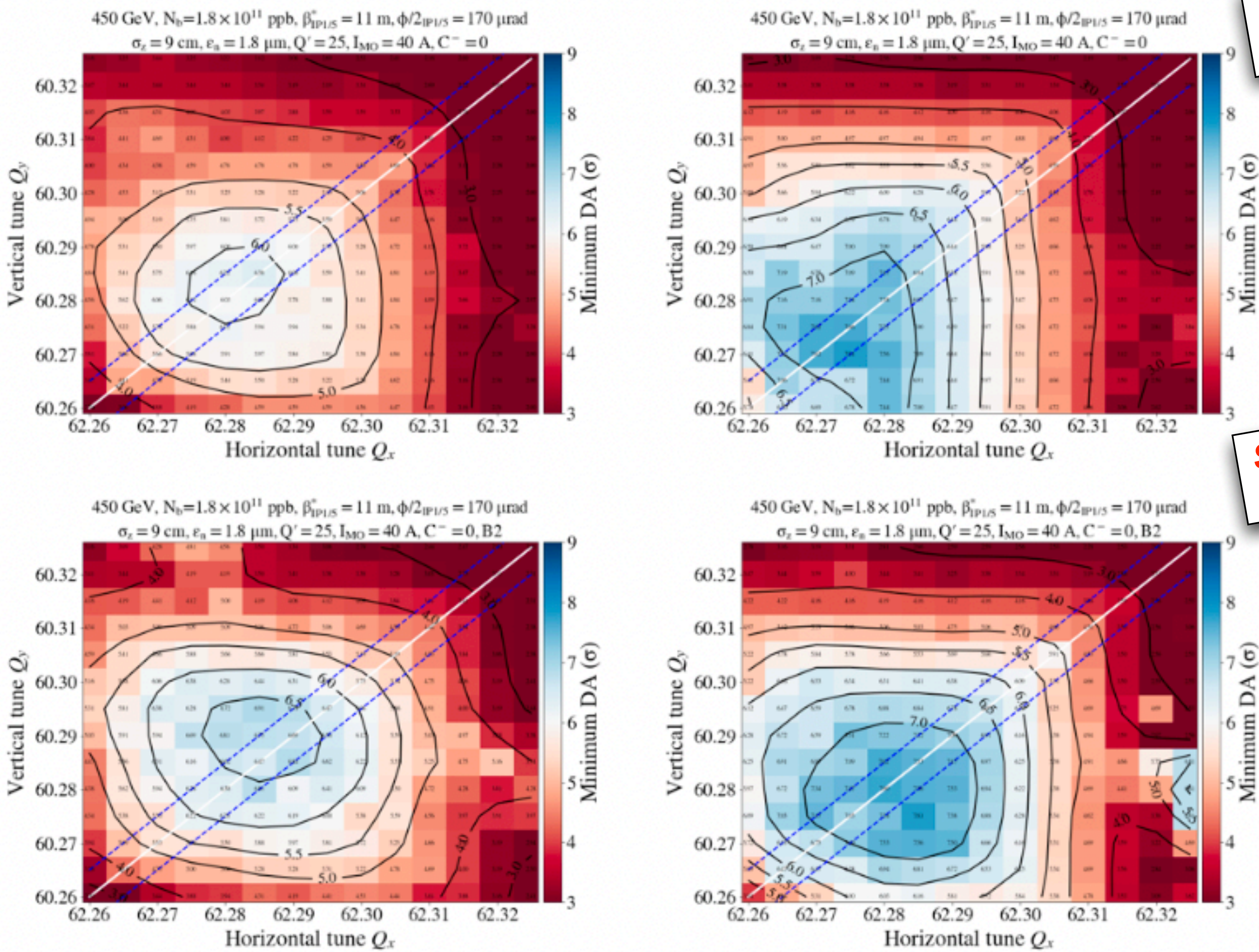
Dynamic Aperture Improvements

Dynamic aperture for beam 1 (top) and beam 2 (bottom) with old (left) and new (right) injection optics for LHC. Lowering the octupolar RDTs has significantly improved the dynamic aperture at injection.

Courtesy Sofia Kostoglou



See Rogelio's paper for details

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References

- HB2023 papers
  - L. Deniau et al., "Optimising Resonance Driving Terms Using MAD-NG Parametric Differential Maps".
  - R. Tomás et al., "Optics for Landau Damping with Minimised Octupolar Resonances in the LHC".
  - K. Paraschou et al.,"Emittance Growth from Electron Clouds Forming in the LHC Arc Quadrupoles".

- Papers and books on Differential Algebra, Nonlinear Normal Forms and RDTs.

  - **E. Forest**, M. Berz and J. Irwin, "***Normal Form Methods for Complicated Periodic Systems using Differential Algebra and Lie Operators***", Particle Accelerators, Vol. 24, pp. 91-107, 1989.
  - E. Forest, "*From Tracking Code to Analysis, Generalised Courant-Snyder Theory for Any Accelerator Model*", Springer, 2016.
  - A. Franchi, "*Studies and Measurements of Linear Coupling and Nonlinearities in Hadron Circular Accelerators*", PhD Thesis, 2006.
  - R. Bartolini and F. Schmidt, "*Normal Form via Tracking or Beam Data*", Particle Accelerators, Vol. 59, pp. 93-106, 1998.
  - T. Pugnat, R. Tomás, A. Franchi and B. Dalena, "*Non-linear Variation of the Beta-beating Measured from Amplitude*", 12th Int. Particle Acc. Conf., 2021.
  - L. Deniau and C.I. Tomoiaga, "*Generalised Truncated Power Series Algebra for Fast Particle Accelerator Transport Maps*", 6th Int. Particle Acc. Conf., 2015.

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Extra Slides

MAD-NG's Physics I

- **5D and 6D physics using differential algebra and symplectic integrators.**
  - **combined physics & elements, slicing & frames, easy to extend, etc…**
  - **x10-30 faster than MADX-PTC for TPSA tracking (Multivariate Taylor Series).**
- **Survey**: geometrical tracking (*global frame, linear algebra*)
  - Survey supports **multi-turns**, **ranged** and step-by-step **forward, backward** and **reverse** tracking. Return a Survey table *and* a Survey map flow (tracked context).
  - fully compatible with Track for superposition and observable points (e.g. table output, smooth plots, slicing, actions, sub-elements, **local in global frame**, etc…)
  - support **exact** misalignments, **permanent** misalignments, and patches.
- **Track**: dynamical tracking (*local mobile frame, differential algebra*)
  - Track supports **multi-particles** or **multi-damaps**, **multi-turns**, **ranged** and step-by-step **forward**, **backward** and **reverse** tracking of **charged** particles to **arbitrary** DA **order** with an arbitrary number of **parameters** (few hundreds). Return a Track table *and* a Track map flow (tracked context).
  - fully compatible with Survey for superposition and observable points.
  - support **exact** misalignments, **permanent** misalignments, **multipoles** & field errors **for all elements.** Can be combined freely with **patches**.
  - **symplectic tracking with integrators up to 8th order** on 5D (delta-p) and 6D (delta-rf) phase space *(exact=true, time=true, totalpath e.g. for thick RF)*.
  - provides true **thick lens** and thin lens tracking model, **radiation with photons tracking** (disabled in twiss), **fringe fields** (hard edge for all elements, including quads, solenoid, RFs), **mutable particles** (multiple beams), **exact patches** (translations, rotations & time-energy), 4D weak-strong beam-beam (sixtracklib), apertures (all kinds).
  - can search for the closed orbit to support relative initial coordinates.

MAD-NG's Physics II

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- **Cofind**: fix point search
  - Newton-based optimiser running **Track** with 1st order DA map or 7 particles (F-Diff).
  - extend Track with actions.
- **Twiss**: optics tracking
  - runs **Cofind** (closed orbit) - **Track** (one-turn map) - **Normal** - **Track** (optics, RDTs).
  - extend Track with actions to compute on-the-fly optics and fill twiss table (extended track table).
  - support strongly coupled optics, linear and non-linear dispersions, tunes, and chromaticities, RDTs, synchrotron integrals, compaction factor, phase slip factor, gamma transition, Montaigue functions, etc…
- **Match**: highly configurable optimiser
  - on the model of MAD-X `use_macro` approach, i.e. arbitrary user's setups & runs.
  - provides all kinds of local & global, linear & non-linear, optimiser (~20 algorithms).
  - very flexible, highly configurable with many physics-oriented setups (not just a penalty-function to minimise).
- **Correct**: orbit correction
  - provides few algorithms (e.g. SVD, Micado) to correct orbit using BPMs and Kickers. Supports many options.
- **Normal**: parametric normal forms & analysis
  - provides linear and non-linear parametric normal forms on DA map (used by twiss) to extract RDTs. Can also be applied at observable points in Track to track RDTs, either on-the-fly with actions or through post processing of DA maps saved in Track table.

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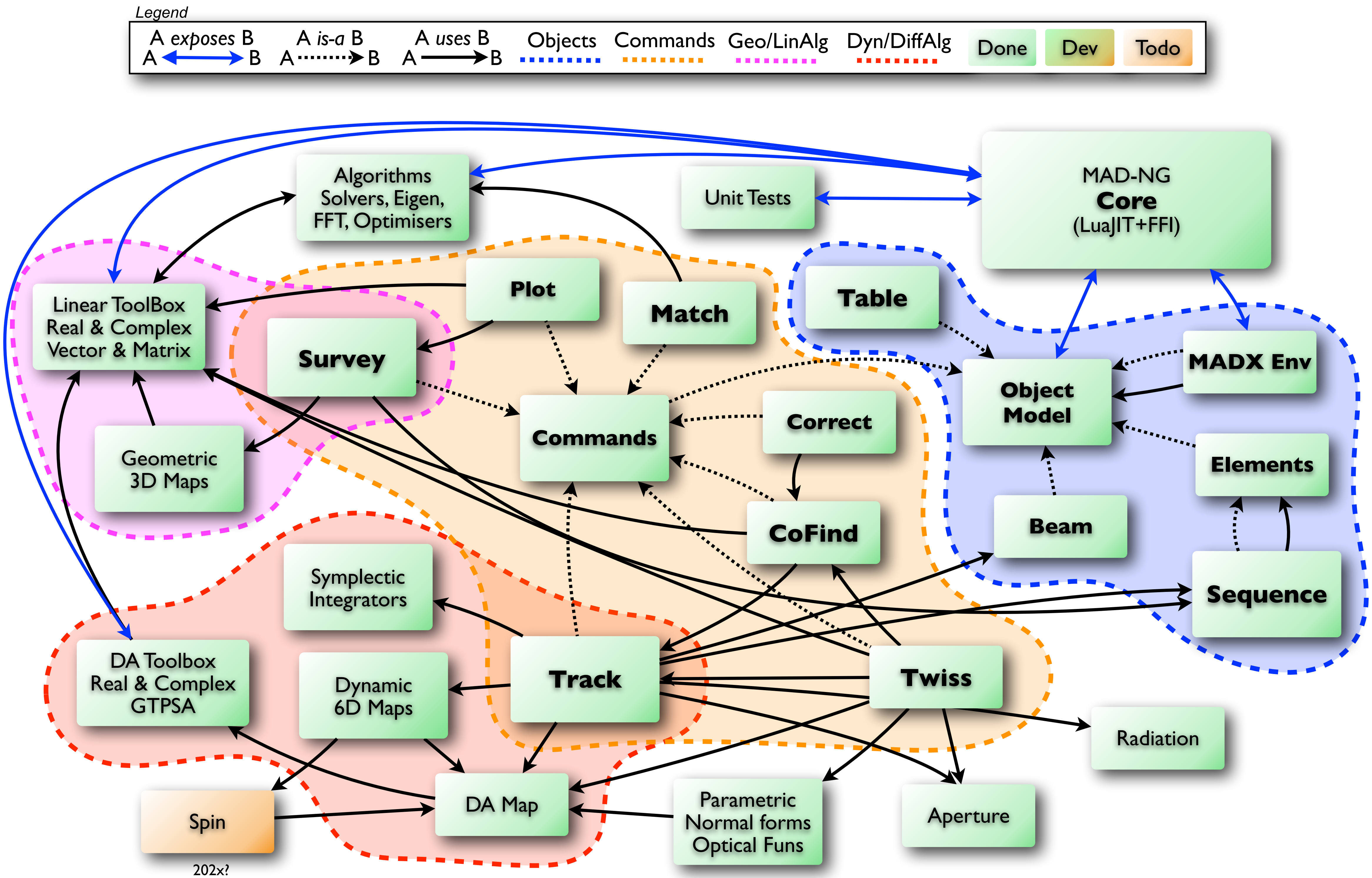
MAD-NG's Ecosystem



GTPSA in a nutshell

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- Generalised Truncated Power Series Algebra **IPAC 2015** **2017-2018** **Github MAD**
  - ➡ Multivariate Taylor polynomials of order n in ℝ & ℂ.
  - ➡ Powerful tool for solving differential equations (e.g. motion equations).

TPSA coefficients

1 variable x at order n in the *neighbourhood* of the point a in the domain of the function f:

$$T_f^n(x;a) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n = \sum_{k=0}^{n} \boxed{\frac{f_a^{(k)}}{k!}}(x-a)^k$$

convergence of the remainder (i.e. truncation error):

$$\lim_{n\to\infty} R_f^n(x;a) = \lim_{n\to\infty} f(x) - T_f^n(x;a) = 0$$

f(x) is an analytic function, $T_f^n(x;a)$ *is a polynomial approximation nearby* a *with radius of convergence h:* $\min_{h>0} \lim_{n\to\infty} R_f^n(a \pm h; a) \neq 0$.

2 variables *(x,y)* at order 2 nearby *(a,b)*:

$$= f^{(1)}_{(a,b)}(x-a, y-b)$$

$$T_f^2(x,y;a,b) = f(a,b) + \boxed{\left.\frac{\partial f}{\partial x}\right|_{(a,b)}(x-a) + \left.\frac{\partial f}{\partial y}\right|_{(a,b)}(y-b)} + \ldots$$

homogeneous polynomials

f must not depend on the integration path, i.e. must derive from a potential!

$$+\frac{1}{2!}\left(\boxed{\left.\frac{\partial^2 f}{\partial x^2}\right|_{(a,b)}(x-a)^2 + \boxed{2\frac{\partial^2 f}{\partial x \partial y}}\Bigg|_{(a,b)}(x-a)(y-b) + \left.\frac{\partial^2 f}{\partial y^2}\right|_{(a,b)}(y-b)^2}\right)$$

$$= f^{(2)}_{(a,b)}(x-a, y-b)$$

v variables X at order n nearby A: **TPSA coefficients**

$$T_f^n(X;A) = \sum_{k=0}^{n} \frac{f_A^{(k)}}{k!}(X;A)^k = \sum_{k=0}^{n} \boxed{\frac{1}{k!}} \sum_{\boxed{|\vec{m}|=k}} \boxed{\binom{k}{\vec{m}} \frac{\partial^k f}{\partial X^{\vec{m}}}}\Bigg|_A (X;A)^{\vec{m}} \quad \text{with} \quad \boxed{\binom{k}{\vec{m}}} = \frac{k!}{c_1!\,c_2!\ldots c_v!}$$

monomials of order k **multinomial**

Accuracy of TPSA (myths and legends)

- GTPSA are **exact** to machine precision, **no** approximation for orders 0..n
  - ➡ derivatives are computed using **automatic differentiation** (AD).

from Wikipedia

> AD exploits the fact that every computer program, no matter how complicated, executes a sequence of elementary arithmetic operations (addition, subtraction, multiplication, division, etc.) and elementary functions (exp, log, sin, cos, etc.). By applying the chain rule repeatedly to these operations, **derivatives of arbitrary order can be computed automatically, accurately to working precision**, and using at most a small constant factor more arithmetic operations than the original program.
>
> Symbolic differentiation can lead to **inefficient code** and faces the di[...] computer program into a single expression, while numerical differentiation can i[...] round-off errors in the discretization process and cancellation. **Both classical methods have problems with calculating higher derivatives, where complexity and errors increase.**

Functions of TPSAs ≠ TPSAs as functions
exact ≠ approximate

- MAD-NG includes a complete toolbox (i.e. module) to handle DA using AD…
  - ➡ users have full access to GTPSA and DAmaps from the scripting language.
  - ➡ users can manipulate DAmaps stored in the MTable or the MFlow returned by Track.
- *So when DAmap/TPSA introduce errors? (Something that we never do…)*
  - ➡ If they are used as *functions* (e.g. evaluated), instead of *DA* (e.g. track, twiss).
  - ➡ High orders of $T_f^n(x;a)$ are used to interpolate at the new position by substitution, e.g. MADX.

$$T_f^n(x;a+h) = \sum_{k=0}^{n} \frac{f_{a+h}^{(k)}}{k!}(x-a-h)^k \,; \quad f(a+h) \approx \underbrace{\sum_{k=0}^{n} \frac{f_a^{(k)}}{k!}h^k}_{T_f^n(a+h;a)} \,; \quad f_{a+h}^{(k)} \approx \frac{\mathrm{d}^k T_f^n(x;a)}{\mathrm{d}x^k}(a+h)$$

Matrix codes don't do better!

order n is constant
order n-1 is linear in h

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