BEAM-BEAM AND IMPEDANCE INSTABILITY MODELLING WITH THE CIRCULANT MATRIX MODEL AND XSUITE

R. Soos (*Université Paris Saclay - CERN*), **X. Buffat** (*CERN*), **A. Faus-Golfe** (*IJCLab*),
with the collaboration of **M. Migliorati** (*Sapienza Universita e INFN*)

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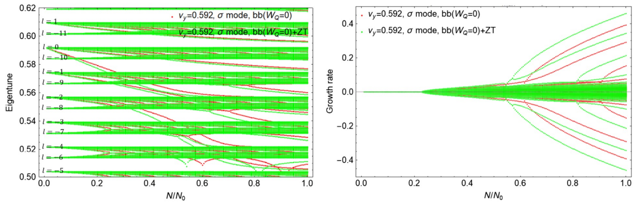
Introduction

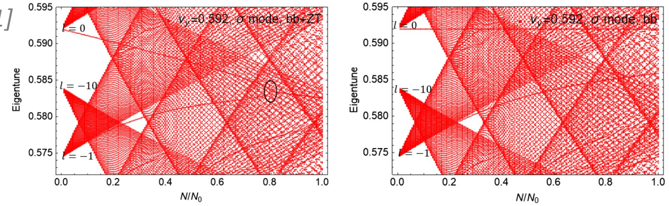
Study context for the FCC-ee

Context:

Interplay between impedance (wakefields) and beam-beam has a growing interest for building new accelerators [1]
CMM and Xsuite showed agreement with LHC and VEPP measurements [2], [3]



[1]



[1]

Goal: Test / Understand the tools independently for beam-beam and impedance.

Xsuite CMM

Next step: Study the interplay between beam-beam and impedance with Xsuite and CMM

[1] Y. Zhang, et al., Phys. Rev. Accel. Beams 26, 064401 (2023)
[2] S. White, et al., Phys. Rev. ST Accel. Beams 17, 041002 (2014)
[3] E. A. Perevedentsev and A. A. Valishev, Phys. Rev. ST Accel. Beams 4, 024403 (2001)

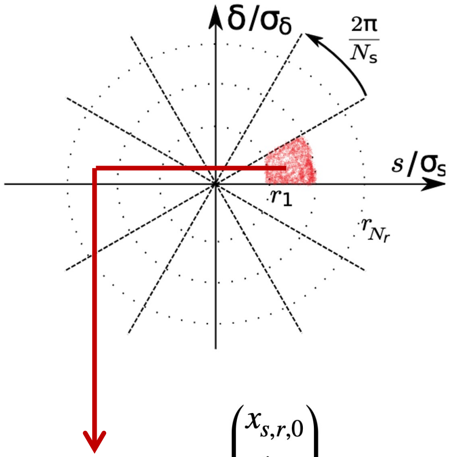
Introduction

Circulant Matrix Model (CMM)

Use linear algebra on a one turn matrix that represents a system, applied on discretized longitudinal phase space

Polar longitudinal phase space

Slicing structure



$$\vec{X}_{r,s,0} = \begin{pmatrix} x_{s,r,0} \\ x'_{s,r,0} \\ y_{s,r,0} \\ y'_{s,r,0} \end{pmatrix}$$

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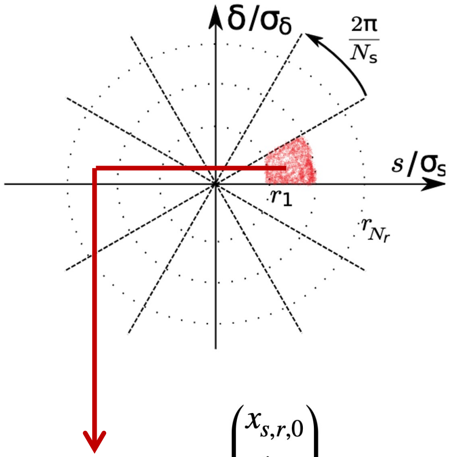

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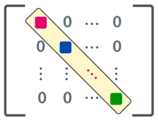
Slicing structure

1 turn system matrix

$$\overrightarrow{X}_t = M_{sys}^t \overrightarrow{X}_0$$

$$M_{lattice} M_w .. M_{BB}$$

Diagonalize to get eigenvalues



Rotation / Circulant matrix

Introduction

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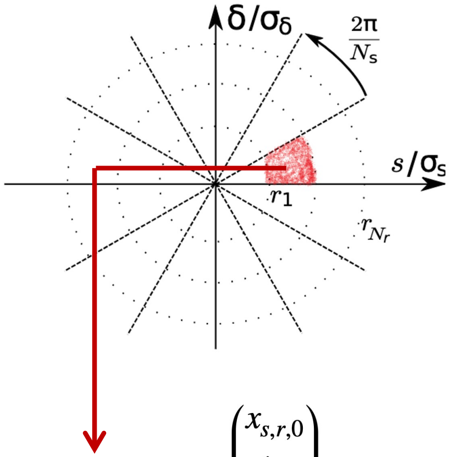

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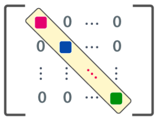

Rotation / Circulant matrix

Linearized coherent flat beam-beam kick (from Bassetti – Erskine formula)

Beam 1

$$M_{BB,0} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{\partial \Delta x'_{coh}}{\partial \delta_x} & 1 & -\frac{\partial \Delta x'_{coh}}{\partial \delta_y} & 0 & \frac{\partial \Delta x'_{coh}}{\partial \delta_x} & 0 & \frac{\partial \Delta x'_{coh}}{\partial \delta_y} & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-\frac{\partial \Delta y'_{coh}}{\partial \delta_x} & 0 & -\frac{\partial \Delta y'_{coh}}{\partial \delta_y} & 1 & \frac{\partial \Delta y'_{coh}}{\partial \delta_x} & 0 & \frac{\partial \Delta y'_{coh}}{\partial \delta_y} & 0 \\
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\frac{\partial \Delta y'_{coh}}{\partial \delta_x} & 0 & -\frac{\partial \Delta y'_{coh}}{\partial \delta_y} & 0 & -\frac{\partial \Delta y'_{coh}}{\partial \delta_x} & 0 & -\frac{\partial \Delta y'_{coh}}{\partial \delta_y} & 1
\end{pmatrix}$$

Beam 2

Matrix elements example

Introduction

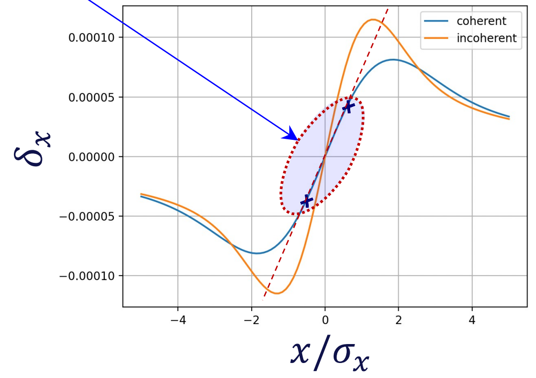
Circulant Matrix Model (CMM)

Use linear algebra on a one turn matrix that represents a system, applied on discretized longitudinal phase space

Linearized integrated coherent flat beam-beam kick
(from Bassetti – Erskine formula)

$\frac{\partial \Delta x'_{coh}}{\partial \delta_x}$

transverse beam-beam kicks,
main frequencies = coherent kick



Matrix elements example

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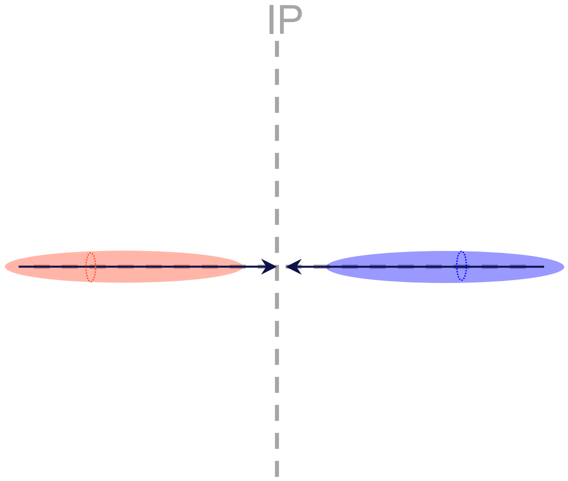
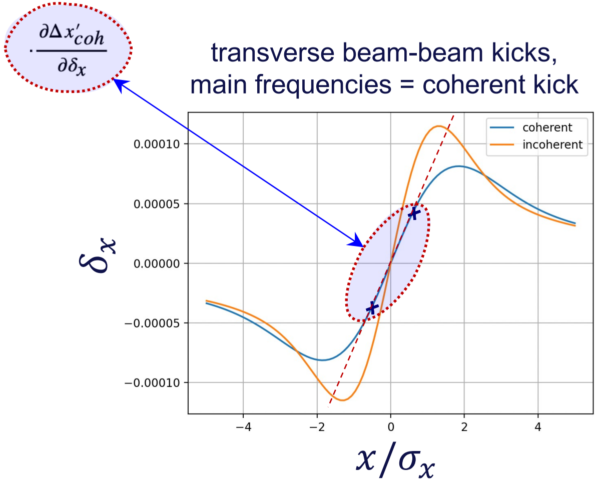


What is in the beam-beam model?

- **Flat beams***

** new*

transverse beam-beam kicks,
main frequencies = coherent kick



Matrix elements example

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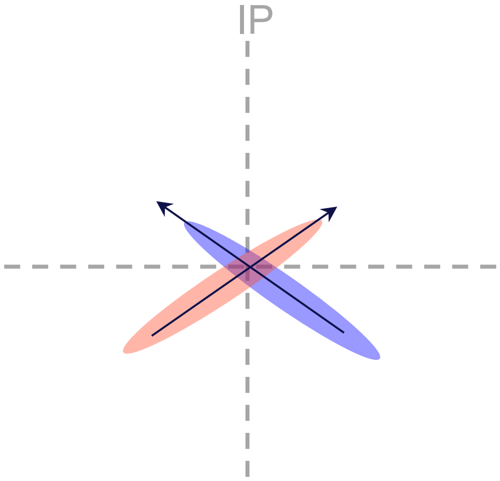
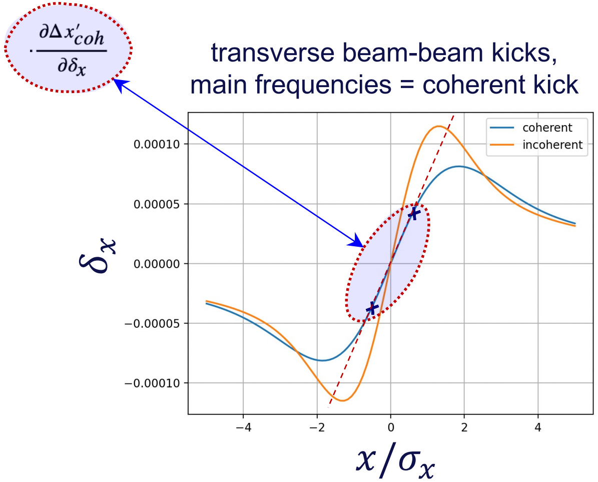


What is in the beam-beam model?

- Flat beams*
- **Crossing angle**

* *new*



Matrix elements example

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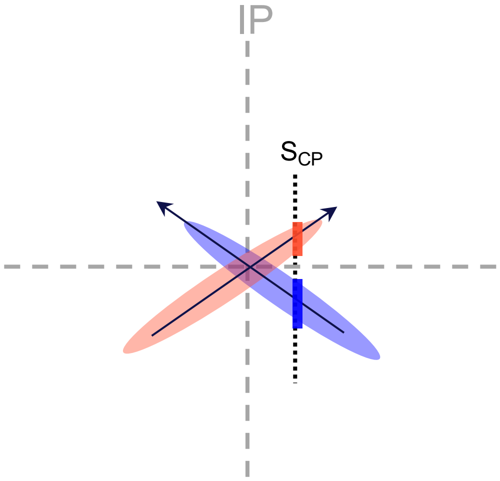
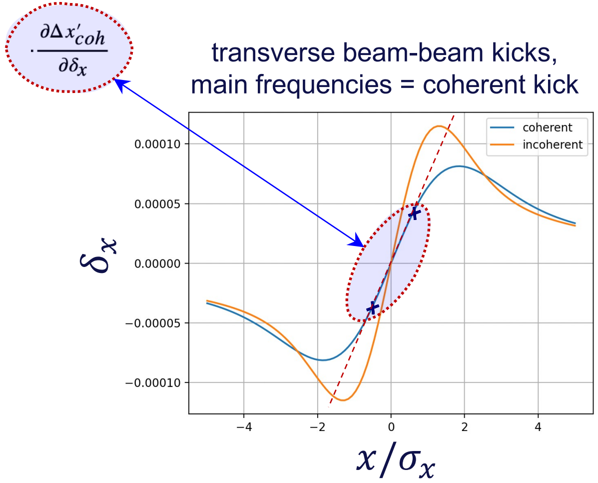


What is in the beam-beam model?

- Flat beams*
- Crossing angle
- **Drifts (IP -> CP -> IP)**

* *new*



Matrix elements example

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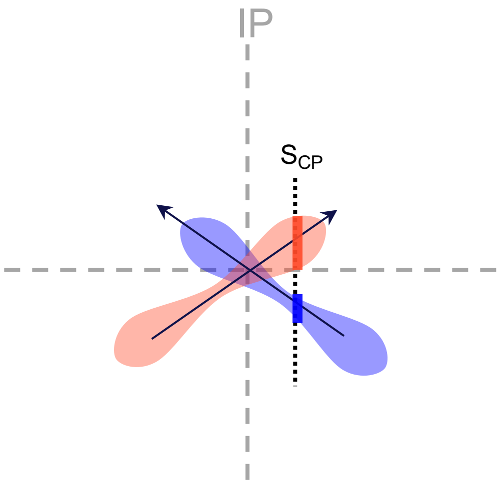
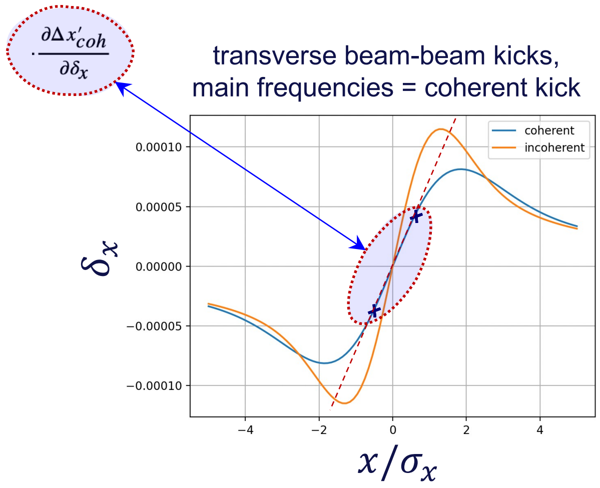


What is in the beam-beam model?

- Flat beams*
- Crossing angle
- Drifts (IP -> CP -> IP)
- **Hourglass**

** new*

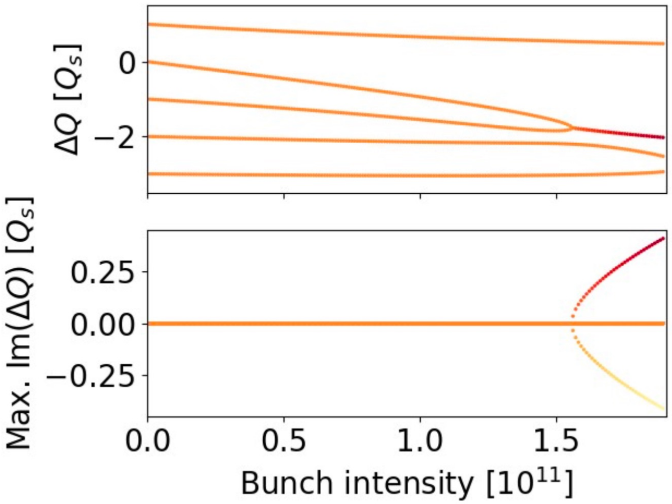


Introduction

Circulant Matrix Model (CMM)

Output: Oscillation modes and growth rates

Example of a transverse mode coupling instability (TMCI) with the circulant matrix model (CMM)



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Circulant Matrix Model (CMM)

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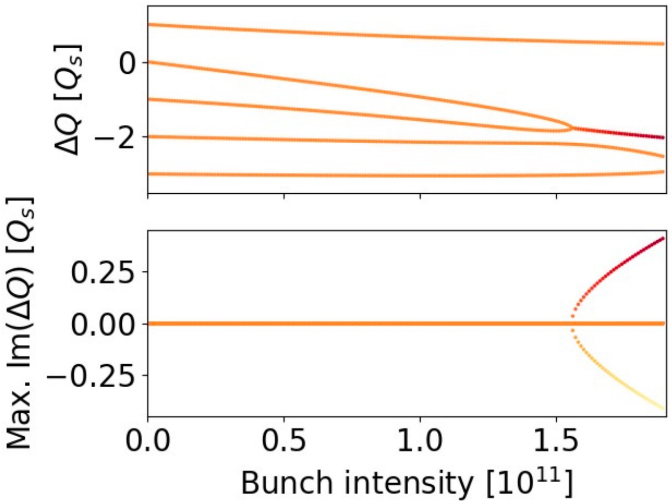
Example of a transverse mode coupling instability (TMCI) with the circulant matrix model (CMM)

Eigenvalues / Eigenvectors



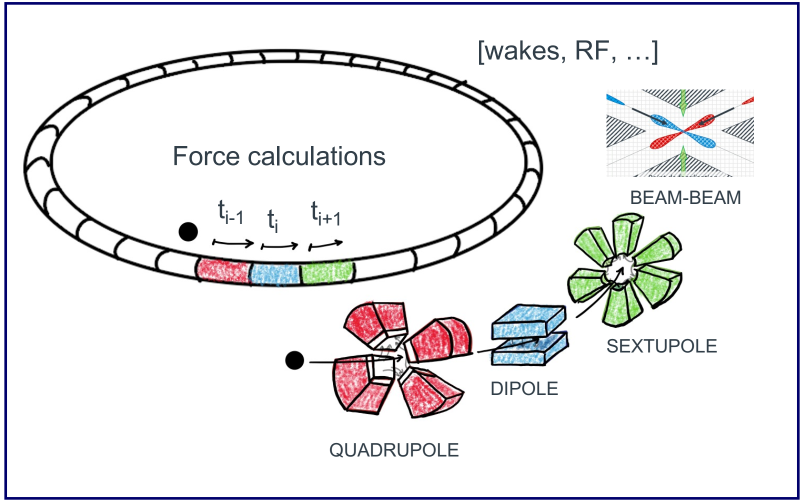
Real parts give oscillation modes

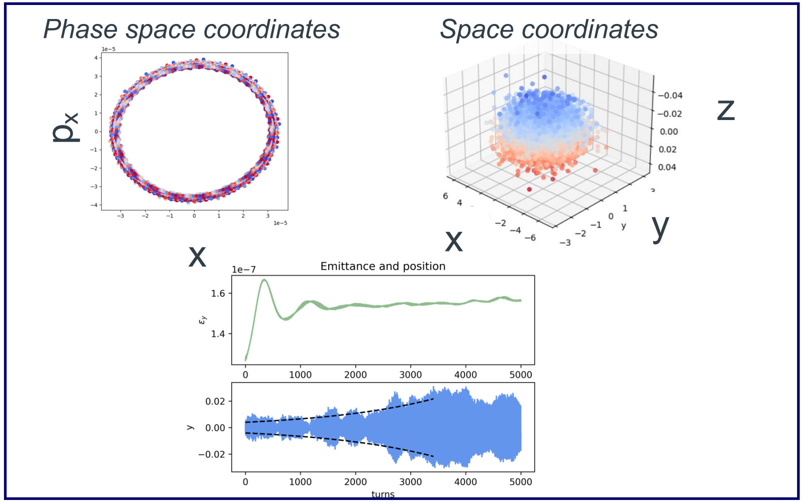
Imaginary parts give growth rates

Introduction

Xsuite

Tracking: follows particles through specified elements in the accelerator for a given a number of turns.





Output: Several outputs linked to the particle parameters can be retrieved.

Introduction

Xsuite

Tracking: follows particles through specified elements in the accelerator for a given a number of turns.

Can be coupled with PyHEADTAIL

- *Wakefields simulations*
- *Active feedbacks*

Detailed beam-beam model (including strong-strong)
(Peter Kicsiny 15/11 at 9:00)

Possibility to improve longitudinal model (e.g. multiple RF stations)

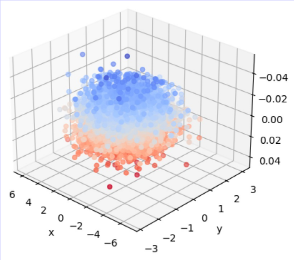
Output: Several outputs linked to the particle parameters can be retrieved.

Introduction

Complementarity

Xsuite

Output:
Turn by turn parameters of the particles in the beam



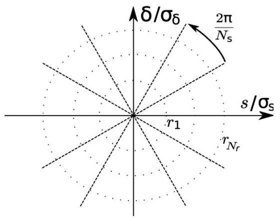
Advantages:
Closer to reality, non-linear models.

Drawbacks:
Difficult to interpret results, slower

Study of BOTH wakefields and beam-beam interactions possible

CMM*

Output:
Eigenvalues ⇔ tunes and growth rates



Advantages:
We can see all oscillation modes and the growth rates quickly

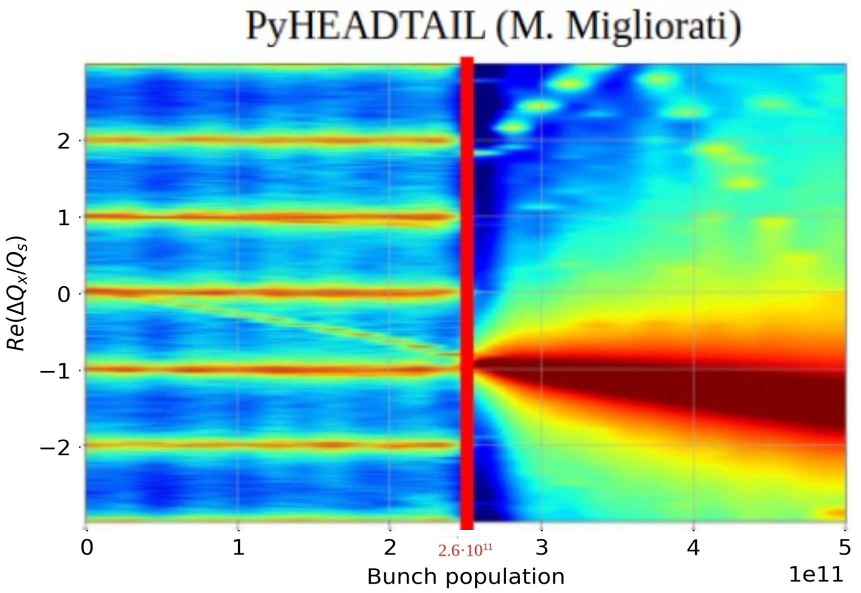
Drawbacks:
Linear model, cannot show non-linear effects

Circulant Matrix Model

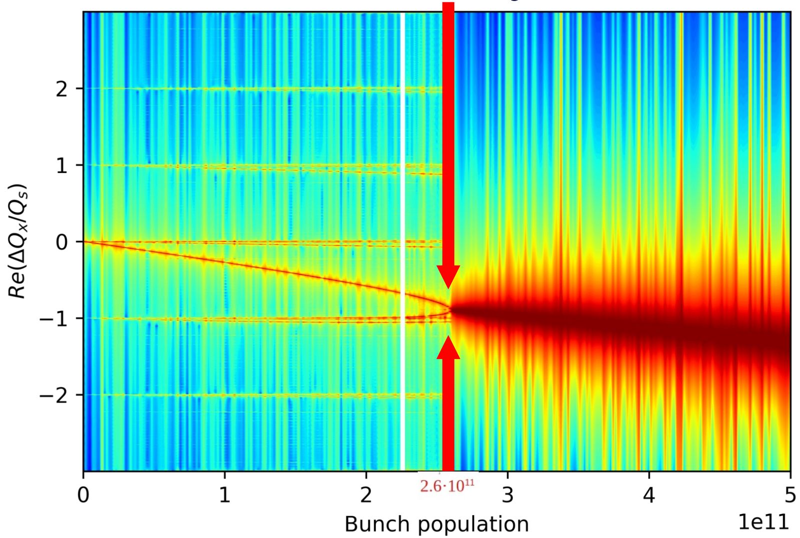
Simulation of impedance

Wakes benchmarked with PyHEADTAIL

Transverse wakefields for FCC-ee at Z energy; **good agreement!**



Xsuite - PyHT



Bunch length without beamstrahlung (5.2mm), L = 90.66 km , linearized RF, no longitudinal wakes.

Simulation of impedance

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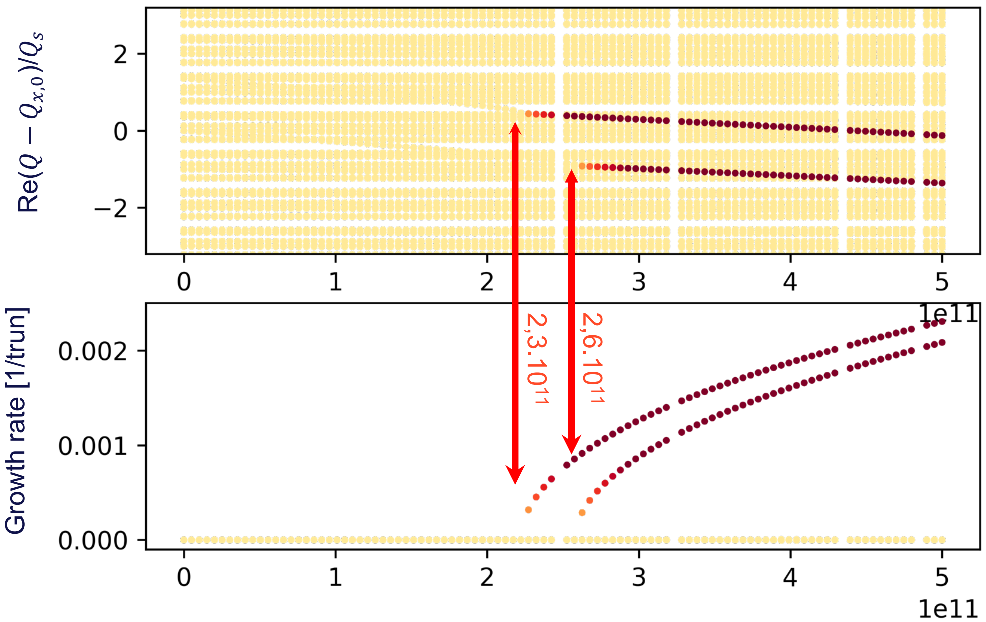
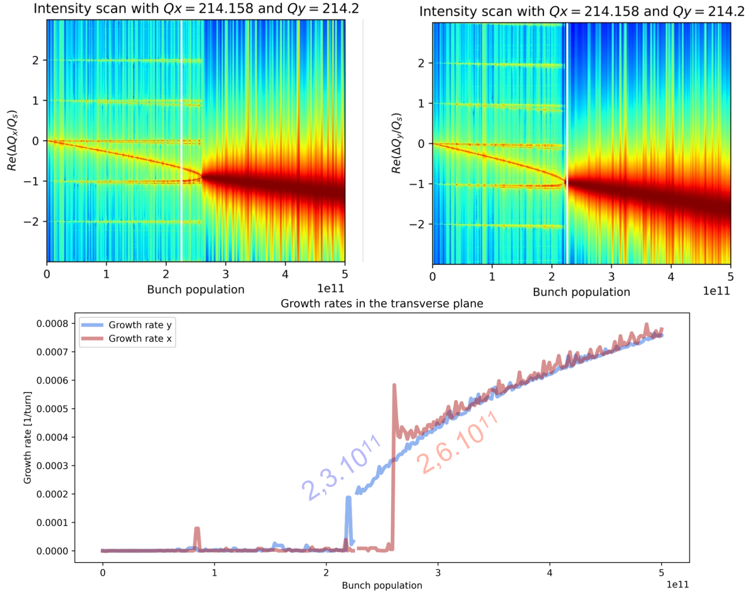
Transverse wakefields for FCC-ee at Z energy; **good agreement!**

Intensity scan with CMM



Intensity scan with Xsuite - PyHT



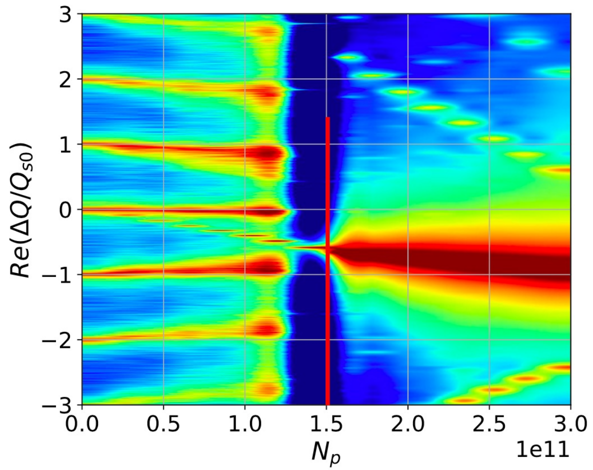
Bunch length without beamstrahlung (5.2mm), L = 90.66 km , linearized RF, no longitudinal wakes.

Simulation of impedance

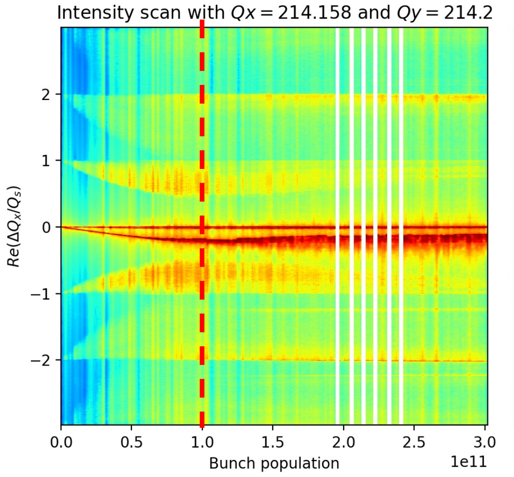
Wakes benchmarked with PyHEADTAIL

Transverse **+ longitudinal** wakefields for FCC-ee at Z energy; **ongoing benchmark!**

PyHEADTAIL (M. Migliorati)



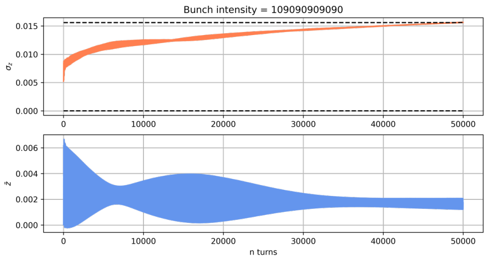
Xsuite - PyHT



Same behaviour until 1.10^{11}

Differences may arise from:

- *Not properly set longitudinal **slicer** (longitudinal bunch size goes over the slicer limit)*

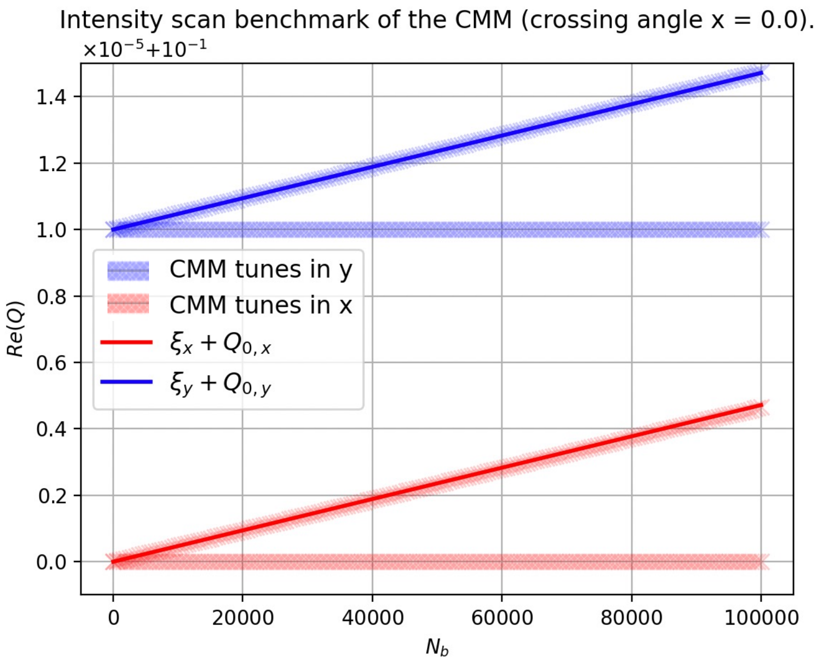


- *Use **non-linear** longitudinal map instead for **linear** map.*

Bunch length without beamstrahlung (5.2mm), L = 90.66 km , linearized RF.

Simulation of beam-beam

Benchmark CMM



beam-beam parameter

=

tune shift of pi-mode far from resonances.

$$\xi_x \approx \Delta Q_{\pi x}$$

beam-beam parameter with crossing angle:

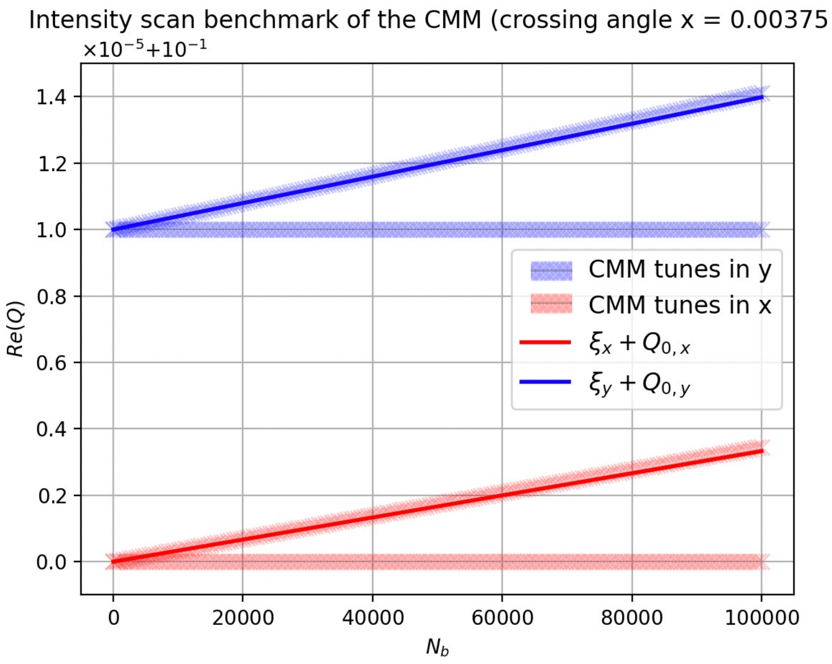
$$\xi_{x,y} = \frac{N r_0 \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

$$\sigma_{x,\theta} = \sqrt{\sigma_z^2 \tan(\theta/2) + \sigma_{x,0}^2}$$

Crossing angle

Simulation of beam-beam

Benchmark CMM



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Simulation of beam-beam

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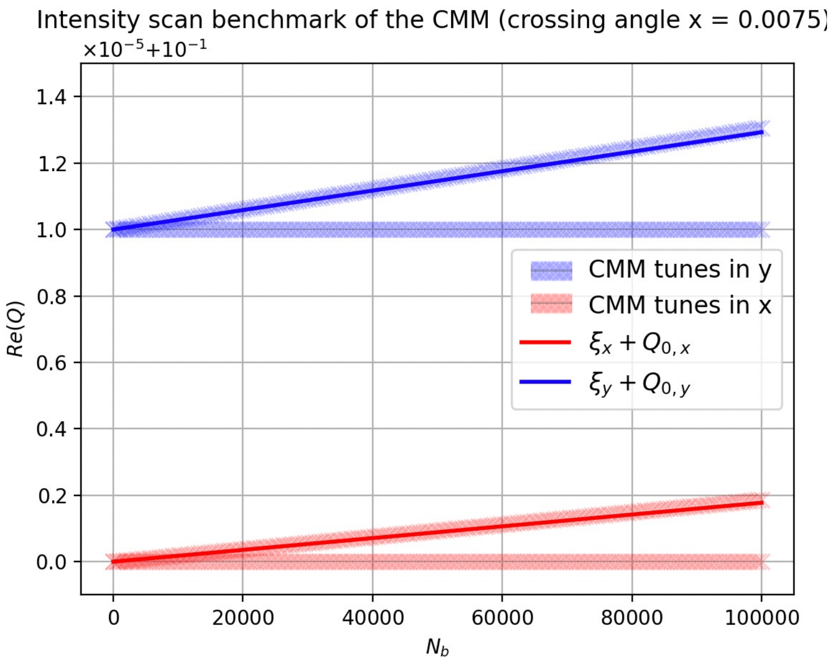

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Simulation of beam-beam

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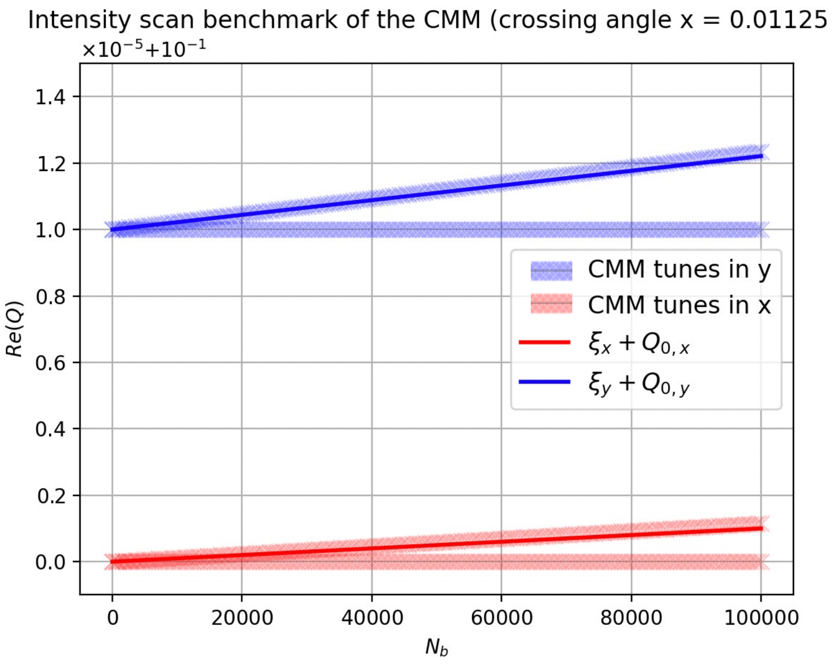

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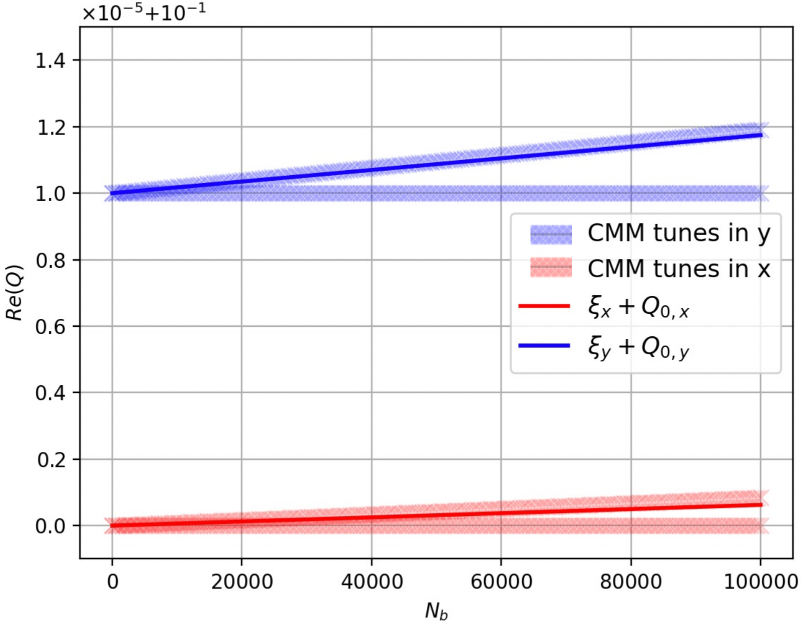
Crossing angle

Simulation of beam-beam

Benchmark CMM

Intensity scan benchmark of the CMM (crossing angle x = 0.015).



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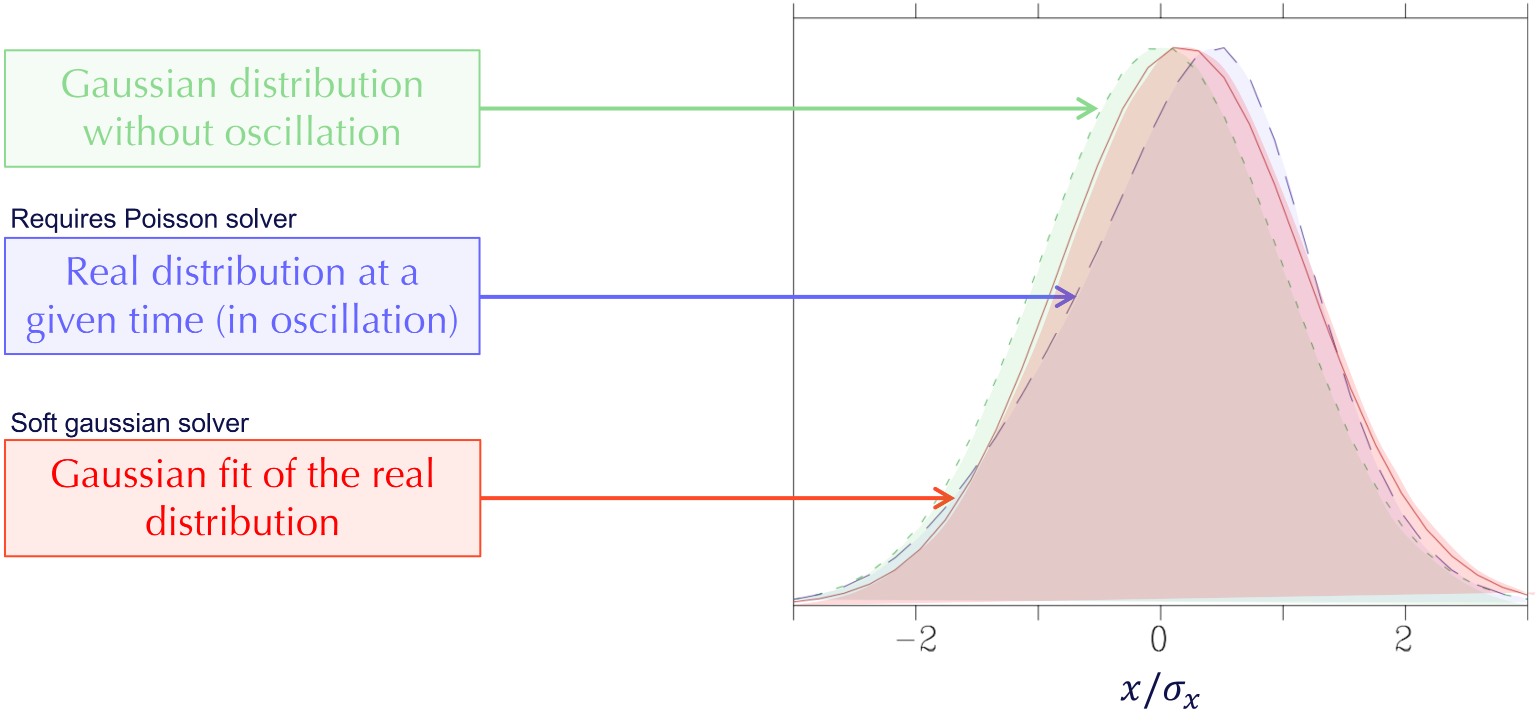
Crossing angle

Simulation of beam-beam

Benchmark Xsuite

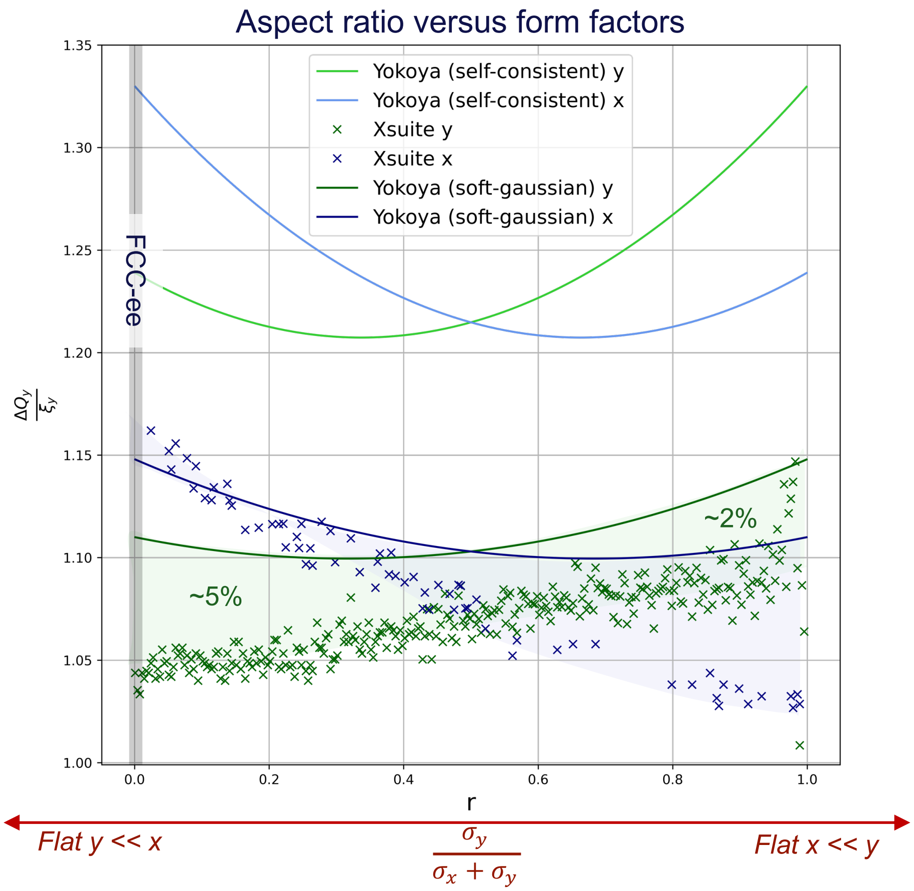


Limitation of the Gaussian approximation in beam-beam simulations, *Kaoru Yokoya - 2000*

Simulation of beam-beam

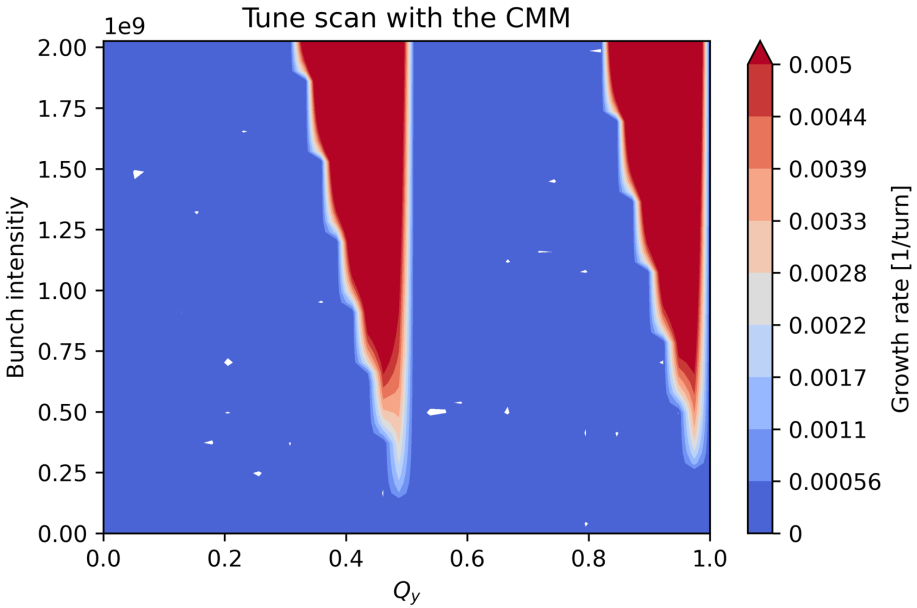
Benchmark Xsuite

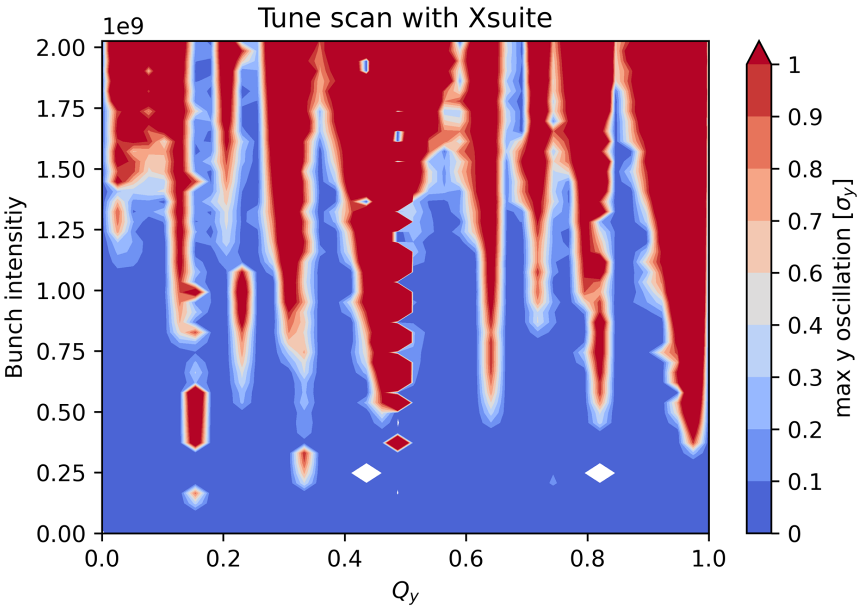
- Arbitrary tunes used for simulation to reduce numerical noise.
- Xsuite results show a 20% error with respect to Yokoya (self-consistent)
- Errors improve to only 5% when compared to Yokoya (soft-gaussian)
- Arbitrary transverse tunes close to zero (resonances?)



Simulation of beam-beam

Xsuite and CMM result comparison

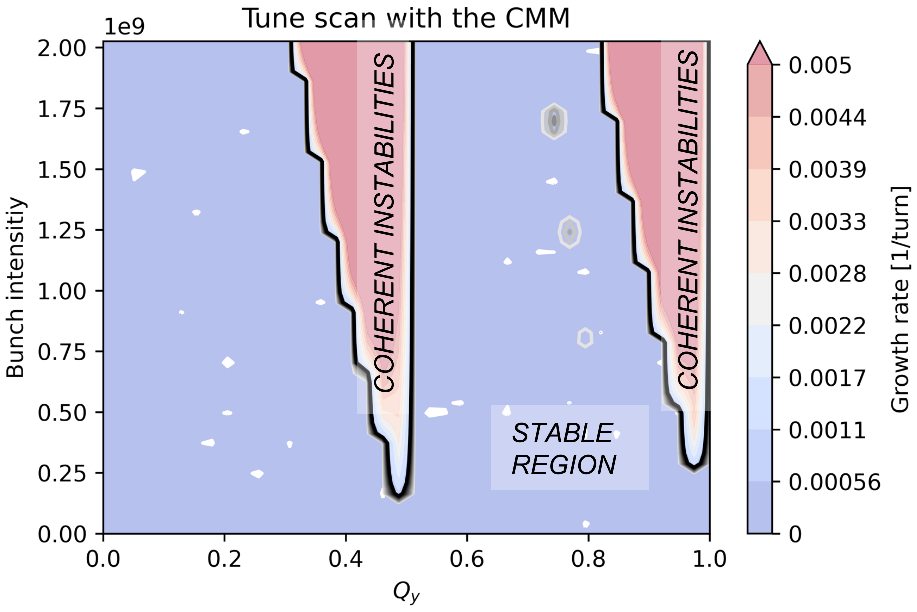


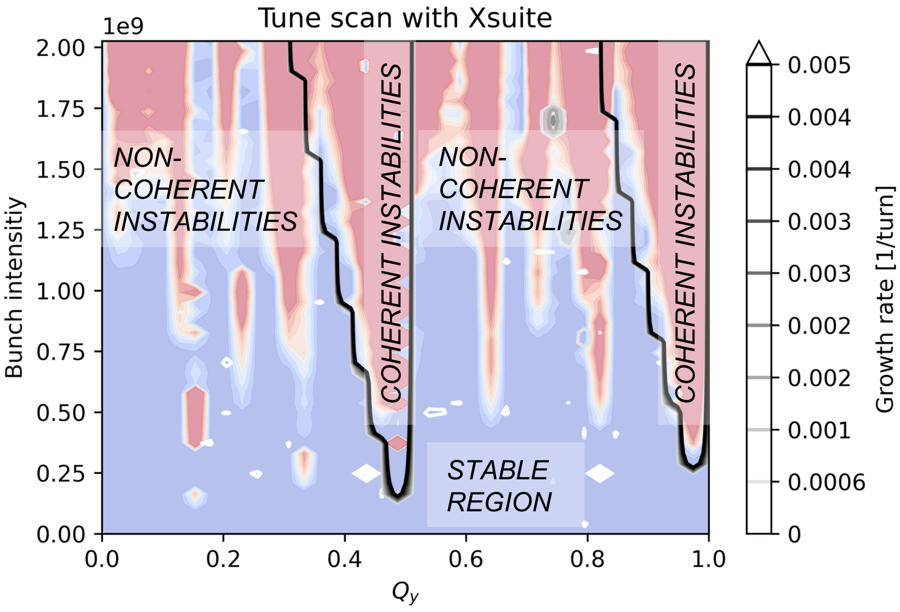


Non-FCC values, small beam-beam, parameter, no hourglass, no crossing angle.

Simulation of beam-beam

Xsuite and CMM result comparison





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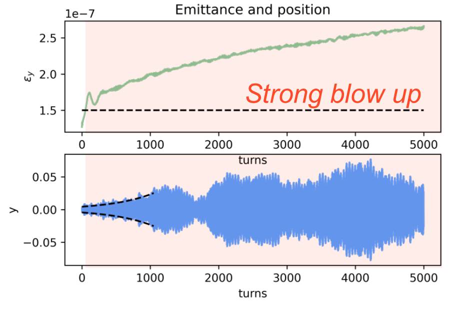
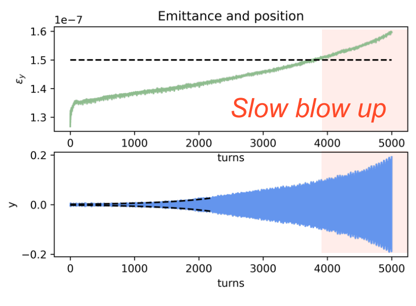
Simulation of beam-beam

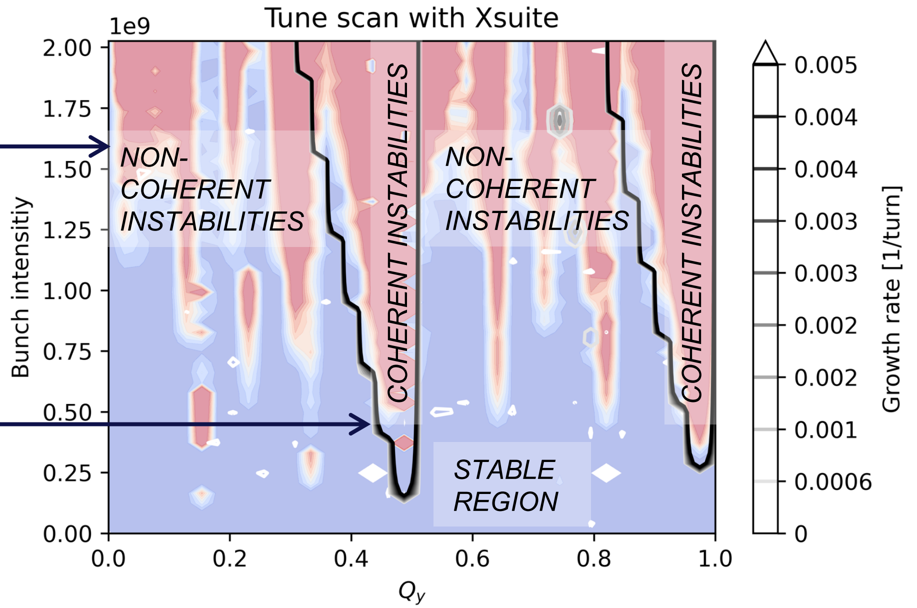
Xsuite and CMM result comparison

NON-COHERENT INSTABILITIES



COHERENT INSTABILITIES





Non-FCC values, small beam-beam, parameter, no hourglass, no crossing angle.

Summary and outlook

- **CMM adapted for FCC-ee requirements on beam-beam interactions.**
- **CMM and Xsuite benchmarked for transverse wake studies, ongoing work for longitudinal wakes.**
- **CMM and Xsuite benchmarked for beam-beam interaction studies.**

Next steps:

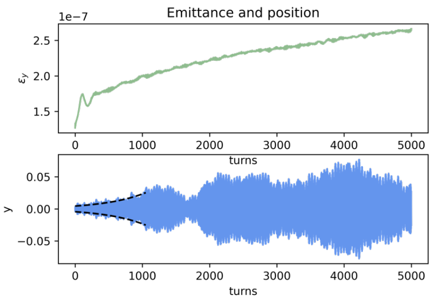
- *Put together wakefields and beam-beam interactions to;*
  - *Reproduce by simulation the known instabilities*
  - *Study the new instabilities that may arise*
- *Validate models and simulations with measures from existing lepton colliders?*

Thank you for your attention.

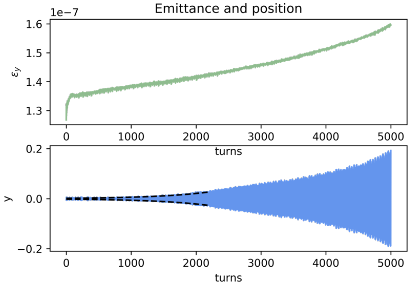
Simulation of beam-beam

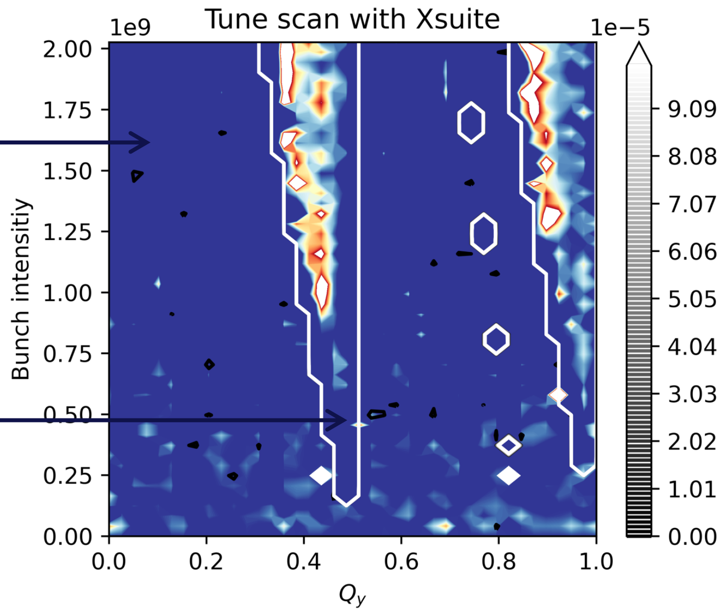
Xsuite and CMM result comparison

NON-COHERENT INSTABILITIES



COHERENT INSTABILITIES

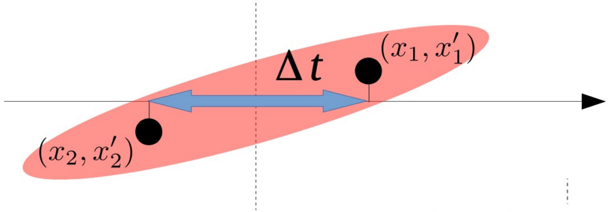




Non-FCC values, close to linear regime, no hourglass.

From Xavier Buffat's slides

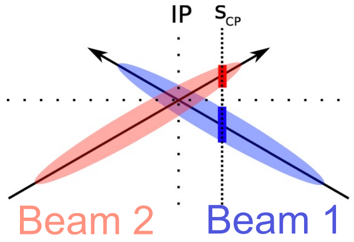
Single bunch wake in the circulant matrix model



$$\Delta x_2' = W_{dip}(\Delta t)\, x_1 + W_{quad}(\Delta t)\, x_2$$

$$\begin{pmatrix} x_1 \\ x_1' \\ x_2 \\ x_2' \end{pmatrix}_{t+1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ W_{dip}(\Delta t) & 0 & W_{quad}(\Delta t) & 1 \end{pmatrix} \cdot M \begin{pmatrix} x_1 \\ x_1' \\ x_2 \\ x_2' \end{pmatrix}_{t}$$

Other effects



Beam 1

$$M_{drift} = \begin{pmatrix} 1 & s_{CP} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -s_{CP} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Beam 2

3. FORCE CALCULATION

Electric field induced by beam-beam interaction for flat beams. ($\sigma_x \neq \sigma_y$)

Bassetti – Erskine formula 1980:

$$E = K \times \left(\mathcal{W}\left(\frac{(x-\delta_x) + i(y-\delta_y)}{\sqrt{2\pi(\sigma_x^2-\sigma_y^2)}} \right) - e^{\left(\frac{(y-\delta_y)^2}{2\sigma_y^2} - \frac{(x-\delta_x)^2}{2\sigma_x^2} \right)} \mathcal{W}\left(\frac{(x-\delta_x)\frac{\sigma_y}{\sigma_x} + i(y-\delta_y)\frac{\sigma_x}{\sigma_y}}{\sqrt{2\pi(\sigma_x^2-\sigma_y^2)}} \right) \right)$$

Construction of the beam-beam matrix.

$$\begin{aligned}
\frac{\partial}{\partial\delta_x}\Delta'_{x\,coh} &= \frac{\partial}{\partial\delta_x}\int\int_{\mathbb{R}^2} E_x(x-\delta_x, y-\delta_y)\psi(x,y)dxdy \\
\frac{\partial}{\partial\delta_y}\Delta'_{y\,coh} &= \frac{\partial}{\partial\delta_y}\int\int_{\mathbb{R}^2} E_y(x-\delta_x, y-\delta_y)\psi(x,y)dxdy \\
\frac{\partial}{\partial\delta_y}\Delta'_{x\,coh} &= \frac{\partial}{\partial\delta_y}\int\int_{\mathbb{R}^2} E_x(x-\delta_x, y-\delta_y)\psi(x,y)dxdy \\
\frac{\partial}{\partial\delta_x}\Delta'_{y\,coh} &= \frac{\partial}{\partial\delta_x}\int\int_{\mathbb{R}^2} E_y(x-\delta_x, y-\delta_y)\psi(x,y)dxdy
\end{aligned}$$

$$\psi(x,y) = \frac{-q}{E}\frac{e^{-\frac{x^2}{2\sigma_x}-\frac{y^2}{2\sigma_y}}}{2\pi\sigma_x\sigma_y}$$

$E_x = \Im[E]$ and $E_y = \Re[E]$

$$\begin{aligned}
\mathcal{W}(x) &= e^{-x^2}\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^x e^{z^2}dz\right) \\
K &= \frac{ne}{2\epsilon_0\sqrt{2\pi(\sigma_x^2-\sigma_y^2)}}
\end{aligned}$$

Numerical force calculation

$$\frac{\partial}{\partial\delta_x}\Delta'_{x\,coh} = \frac{1}{2\epsilon_x}\left(\int\int_{\mathbb{R}^2} E_x(x-\delta_x-\epsilon_x, y-\delta_y)\psi(x,y)dxdy - \int\int_{\mathbb{R}^2} E_x(x-\delta_x+\epsilon_x, y-\delta_y)\psi(x,y)dxdy\right)$$

Semi - analytical force calculation.

$$\frac{\partial}{\partial\delta_x}\Delta_{coh} = \int\int_{\mathbb{R}^2} \frac{\partial}{\partial\delta_x}E(x-\delta_x, y-\delta_y)\psi(x,y)dxdy$$

Analytical formula

$$\frac{\partial}{\partial X}E(X,Y) = K\left(\frac{2i}{S\sqrt{\pi}} - \frac{(X+iY)}{S^2}\mathcal{W}\left(\frac{X+iY}{S}\right) - \frac{2i}{S\sqrt{\pi}}\frac{\sigma_y}{\sigma_x}e^{\frac{Y^2}{2\sigma_y^2}-\frac{X^2}{2\sigma_x^2}}\right.$$

$$\left. - W\left(\frac{X\frac{\sigma_y}{\sigma_x}+iY\frac{\sigma_x}{\sigma_y}}{S}\right)e^{\frac{Y^2}{2\sigma_y^2}-\frac{X^2}{2\sigma_x^2}}\left(\frac{X}{\sigma_x^2} + 2\frac{X\frac{\sigma_y}{\sigma_x}+iY\frac{\sigma_x}{\sigma_y}}{S^2}\frac{\sigma_y}{\sigma_x}\right)\right)$$

$$\begin{aligned}
S &= \sqrt{2\pi(\sigma_x^2-\sigma_y^2)} \\
X &= x - \delta_x \\
Y &= y - \delta_y
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial x}\Delta'_{x\,coh} &= \Im\left[\frac{\partial}{\partial x}\Delta_{coh}\right] \\
\frac{\partial}{\partial y}\Delta'_{y\,coh} &= \Re\left[\frac{\partial}{\partial y}\Delta_{coh}\right] \\
\frac{\partial}{\partial y}\Delta'_{x\,coh} &= \Im\left[\frac{\partial}{\partial y}\Delta_{coh}\right] \\
\frac{\partial}{\partial x}\Delta'_{y\,coh} &= \Re\left[\frac{\partial}{\partial x}\Delta_{coh}\right]
\end{aligned}$$