Semi-analytical estimation of optics correction
+ an evaluation of vibration

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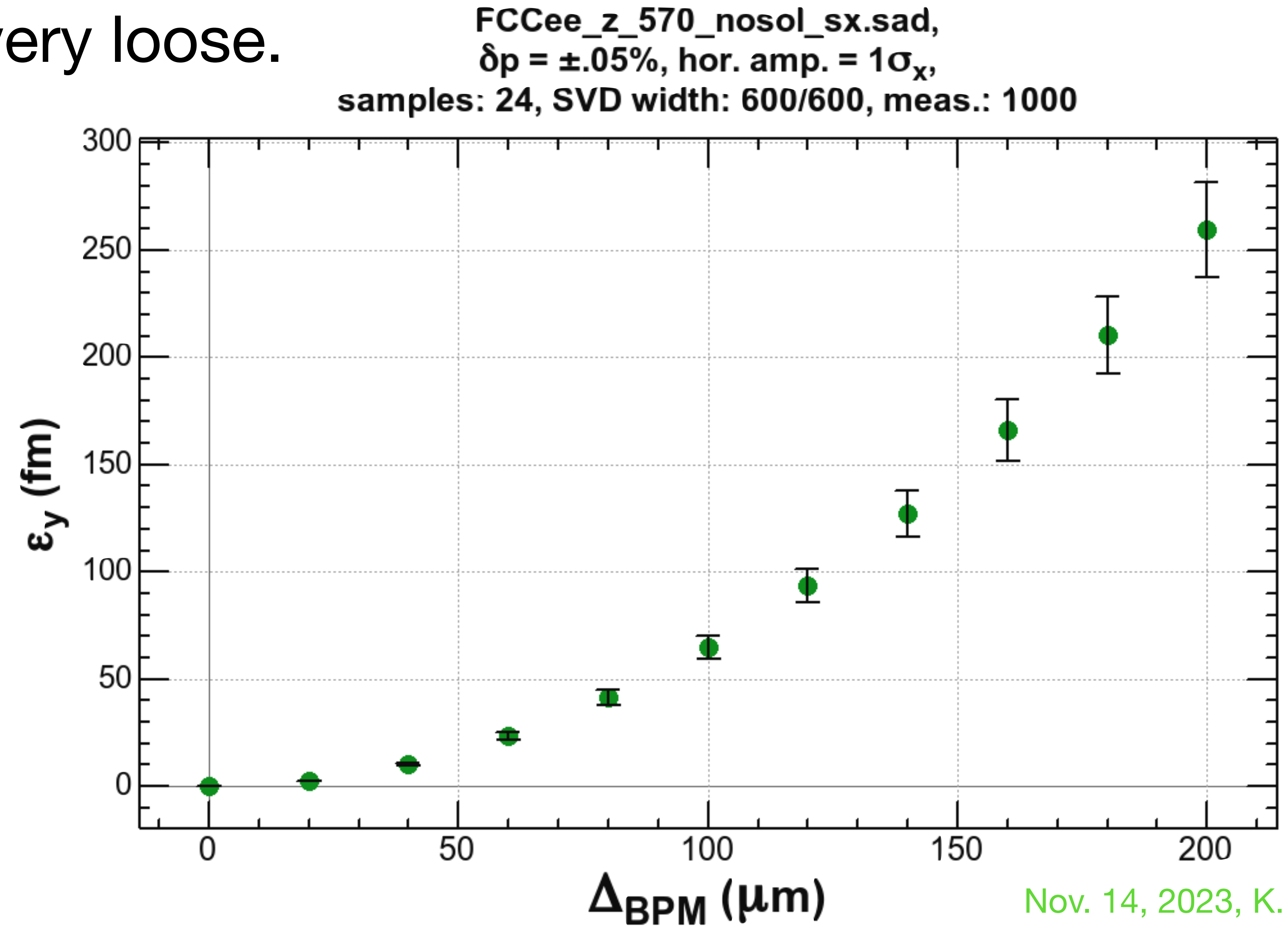
- In these studies, we use the Z-lattice, with all magnets including the IR, unless mentioned otherwise.

Optics correction with less effect on closed orbit distortion

- At the Tuning Workshop in June, it has been shown that the vertical misalignments of sextupoles can be simply corrected using their skew-quadrupole correction windings. after p. 12 of https://indico.cern.ch/event/1242395/contributions/5417968/attachments/2672866/4633889/Optics_Oide_230626.pdf
  - Using analytic response matrix from the skew winding to these parameters.
  - By turn-by-turn measurements of the x-y coupling parameter R_2 and the vertical dispersion. The requirements on the BPM resolution seem very loose.

$$\delta R_2 = \sqrt{\beta_x \beta_y \beta_{xk} \beta_{yk}} \frac{\sin \Delta\psi_x \sin(\mu_y - \Delta\psi_y) - \sin \Delta\psi_y \sin(\mu_x - \Delta\psi_x)}{2(\cos \mu_x - \cos \mu_y)} k_s + O[k_s^2] ,$$

$$\delta\eta_y = \sqrt{\beta_y \beta_{yk}} \frac{\cos(\Delta\psi_y - \mu_y/2)}{2 \sin(\mu_y/2)} \eta_{xk} k_s ,$$



Optics correction with less effect on closed orbit distortion (2)

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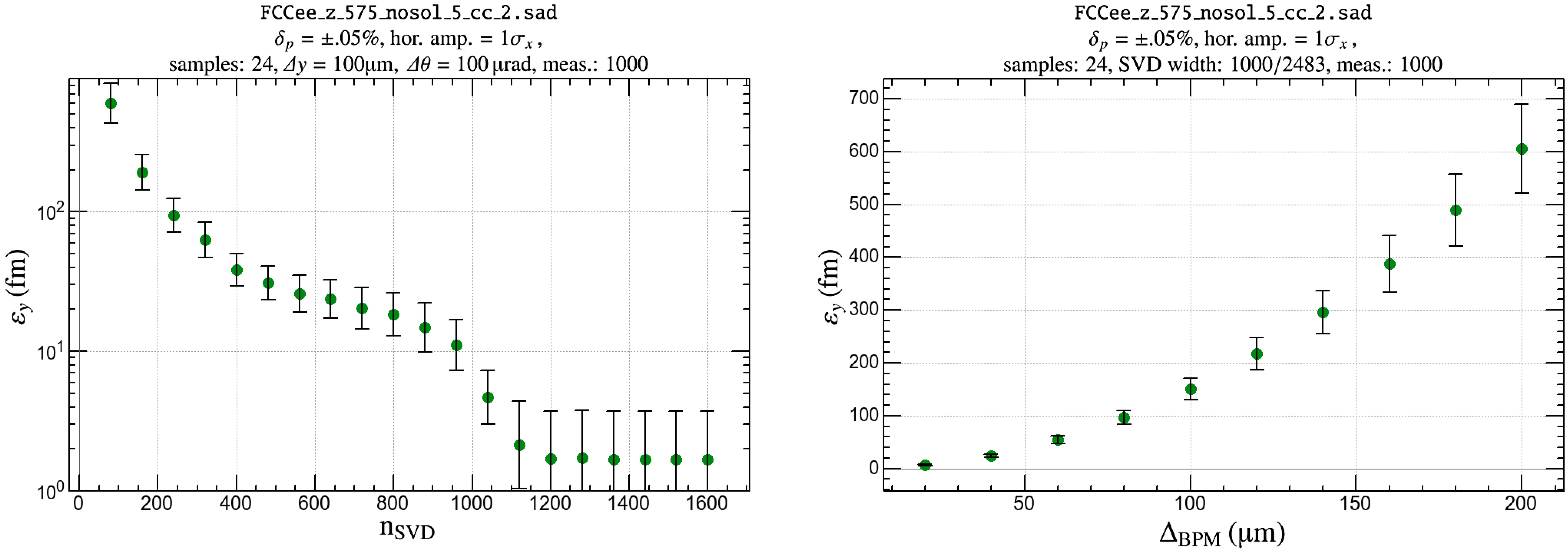
- This method can be extended to the correction of the roll error of quadrupoles:
  - By attaching skew quad corrector at every quadrupole.
  - The required single-shot BPM resolution is now 150 μm.

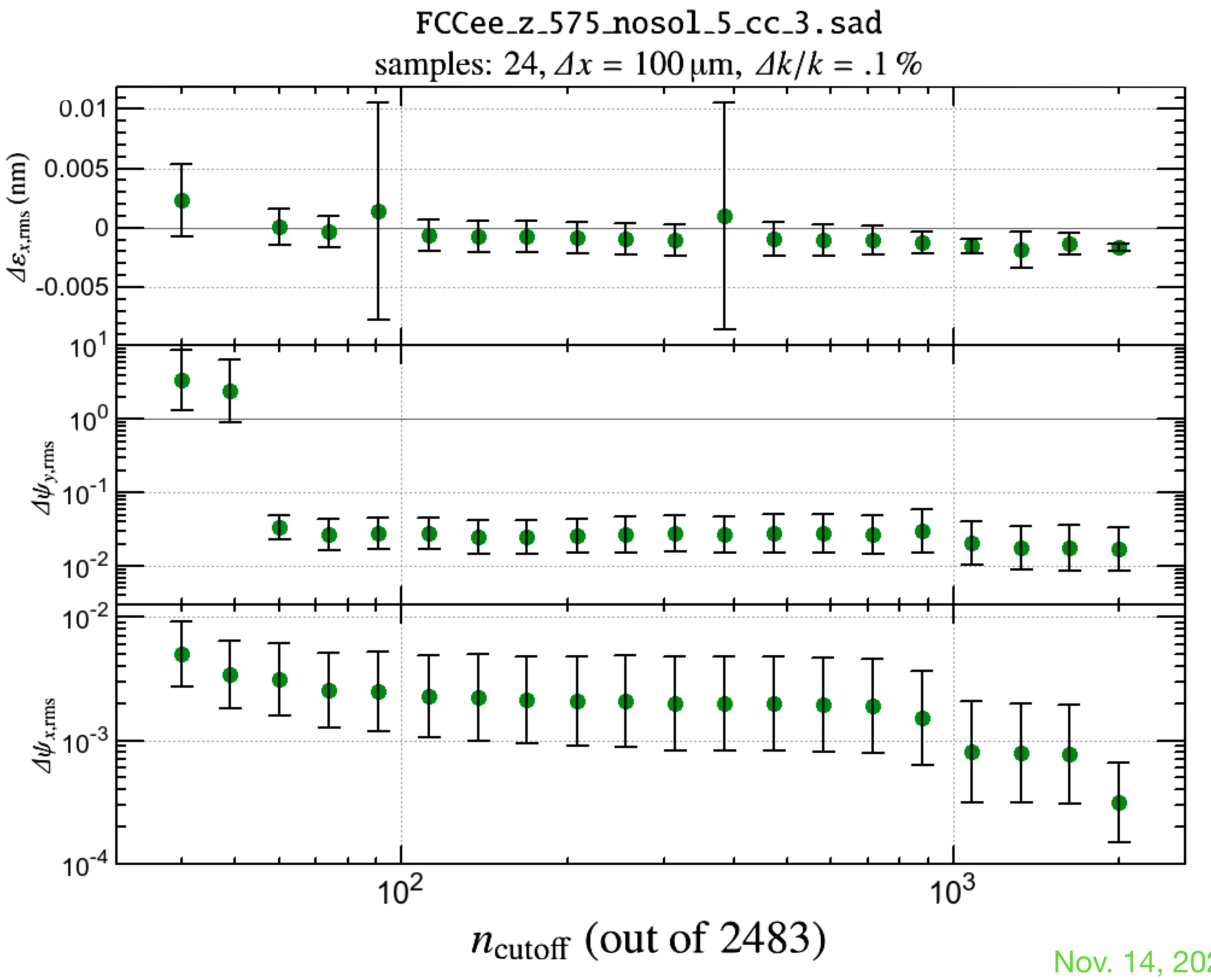


Optics correction Optics correction with less effect on closed orbit distortion (3)

- The same method is applicable to horizontal/vertical betatron phases and the horizontal dispersion correction.
  - The phase correction is nearly equivalent to β-correction, as $\delta\psi \approx \delta\beta/\beta$.
  - The phase error can be directly measured by TbT BPMs.
  - Again, the result (right) seems promising.

$$\delta\psi_{x,y} = \pm\frac{\cos(\Delta\psi_{x,y} - \mu_{x,y})\sin\Delta\psi_{x,y}}{2\sin\mu_{x,y}}\beta_{x,y}\delta k_n$$

$$\delta\eta_x = \sqrt{\beta_x\beta_{xk}}\frac{\cos(\Delta\psi_x - \mu_y/2)}{2\sin(mu_x/2)}\eta_x\delta k_n$$



Correction with misalignments of quadrupoles and BPMs

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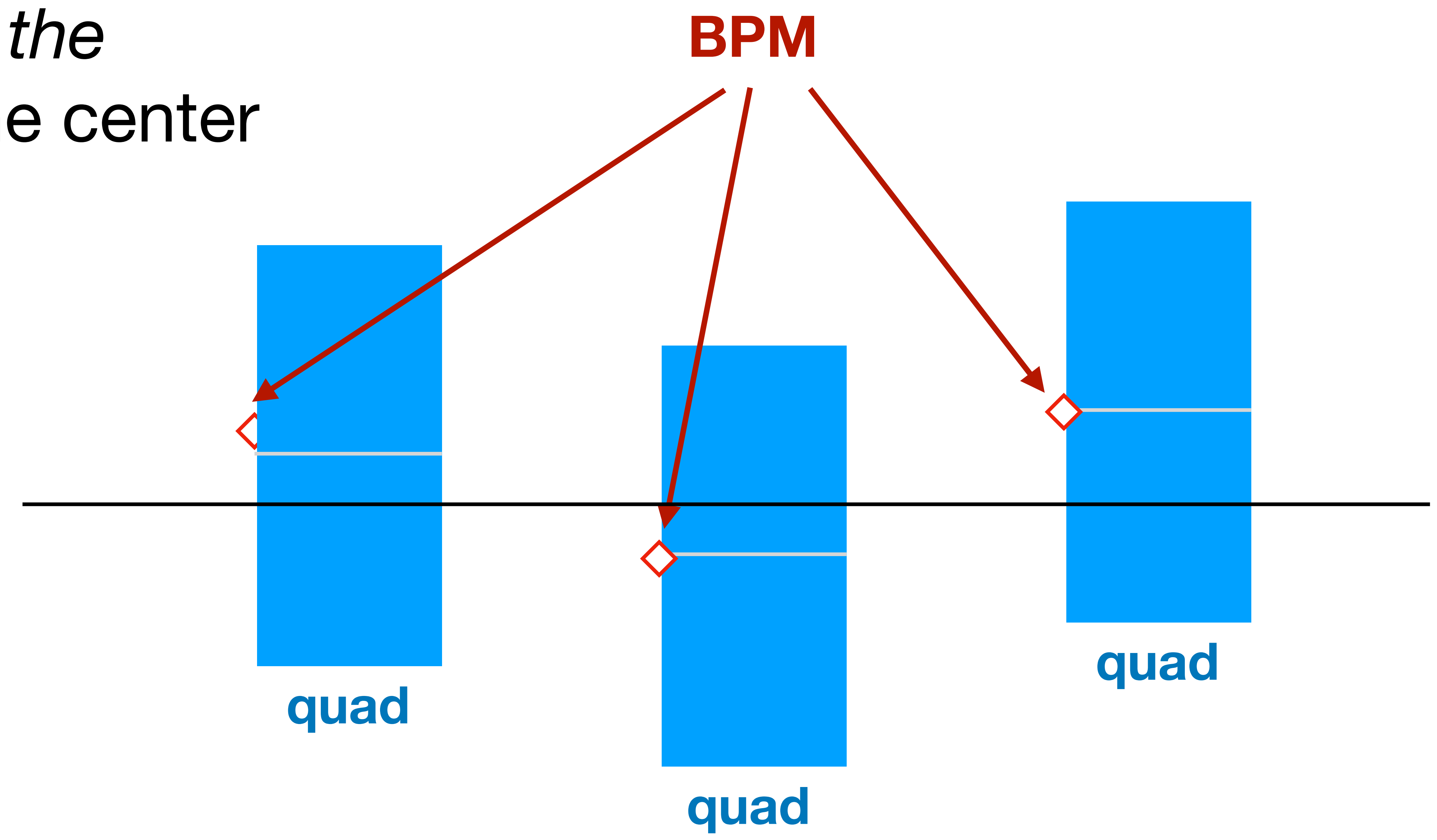
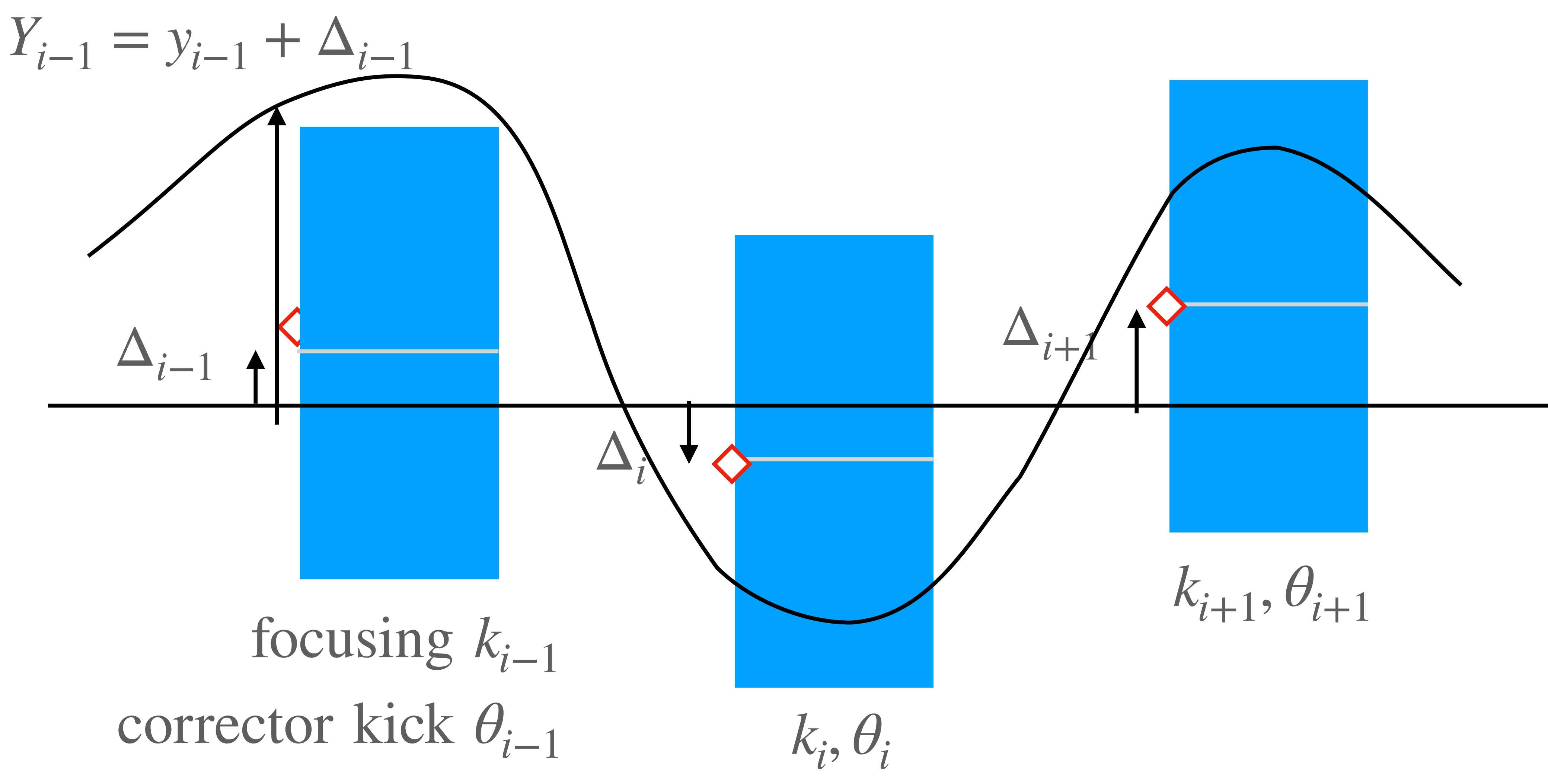
- The estimation above has not included the misalignments of quadrupoles and BPMs.
  - The coupling/dispersion measurements does not depend on the origin of the BPMs.
- However, once misalignments of quadrupoles are introduced, the situation complicates.
- Usually, we can calibrate the offset of a BPM *relative to the center of the quadrupole* attached.
- However, the misalignments of quadrupoles *relative to the design plane* are much larger than the BPM offset to the center of the quad.
- Then the question is how can we estimate the misalignment of the quad itself?



Estimation of quadrupole misalignment

- Hereafter let us assume that each quadrupole has a dipole corrector completely nested, which is indeed possible by a superconducting quad.
- Each BPM is exactly placed on the axis of the quad.
- Each quad is misaligned from the design plane by Δ_i.
- If the orbit at the i-th quad wrt the design plane is Y_i, the BPM reading becomes $y_i = Y_i - \Delta_i$.
- The i-th quad has additional dipole kick $\theta_i - k_i\Delta_i$, where θ_i is the kick applied by the corrector, and $-k_i\Delta_i$ is the kick generated by the offset of the quad.



Then, for a set of three successive quads/BPMs, there are relations:

$$\begin{pmatrix} y_i + \Delta_i \\ p_{y,i} \end{pmatrix} = \mathrm{M}_{i,i-1} \begin{pmatrix} y_{i-1} + \Delta_{i-1} \\ p_{y,i-1} \end{pmatrix} + \mathrm{K}_{i,i-1} \begin{pmatrix} 0 \\ \theta_{i-1} - k_{i-1}\Delta_{i-1} \end{pmatrix} ,$$

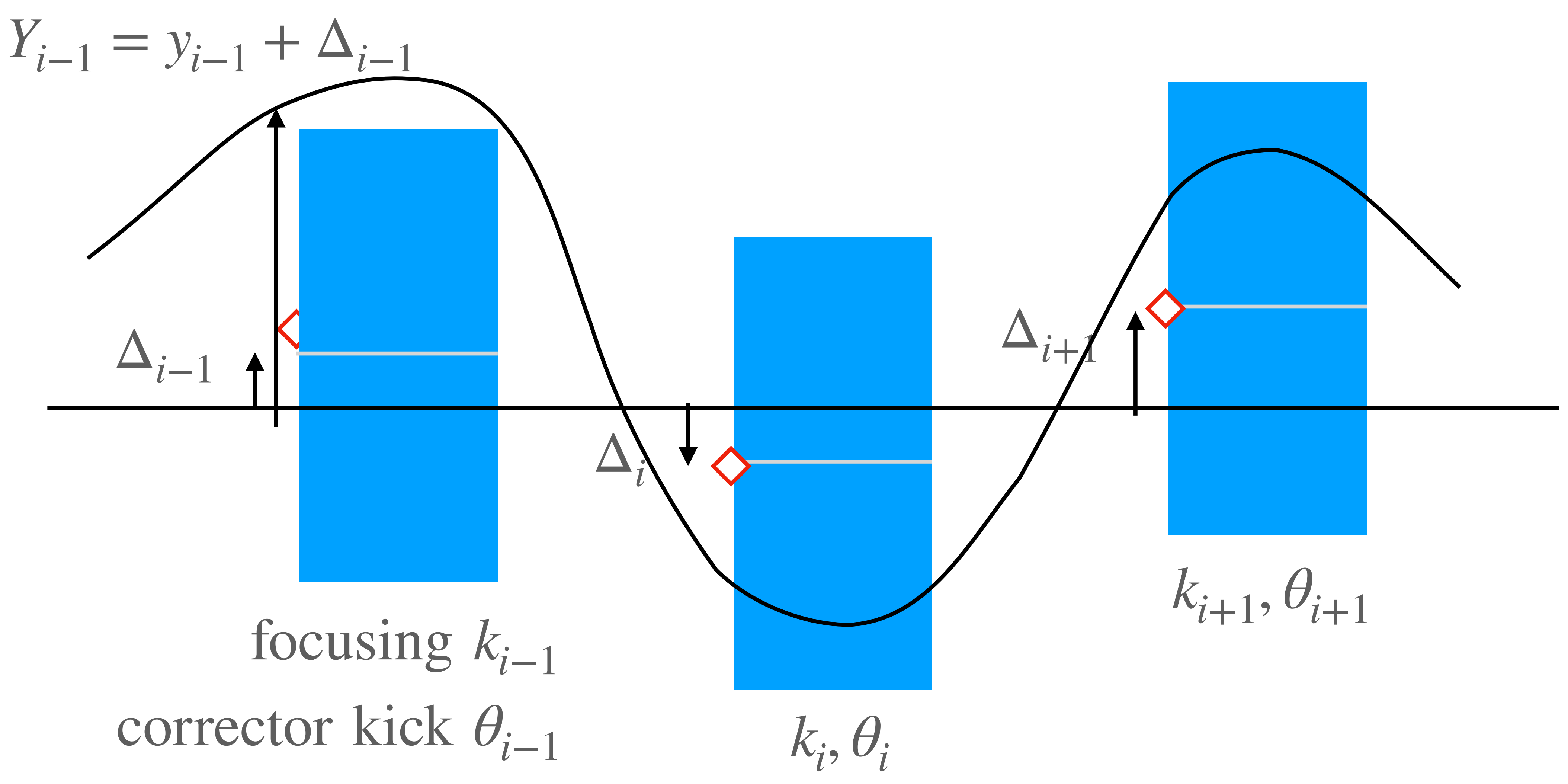
where $\mathrm{M}_{i,i-1}$ and $\mathrm{K}_{i,i-1}$ are the transfer matrix and the kick-response matrix from $(i-1)$th BPM and quad to the i-th BPM, respectively.

Estimation of quadrupole misalignment (2)

If we combine this with the equation for $(i+1, i)$, we can derive the relation between three successive offsets as:

$$f_{a,i}\Delta_{i-1} + \Delta_i + f_{b,i}\Delta_{i+1} = c_{a,i}y_{i-1} + c_{0,i}y_i + c_{b,i}y_{i+1} + v_{a,i}\theta_{i-1} + v_{0,i}\theta_i \,,$$

where coefficients $f_{a,i}$, $f_{b,i}$, $c_{a,i}$, $c_{0,i}$, $c_{b,i}$, $v_{a,i}$, $v_{0,i}$ are all expressed in terms of the matrices M and K. The rhs of above is determined by the measured orbit y_i and the corrector setting θ_i. So we can solve the above equation for $\Delta_i, i = (1..n)$, once an orbit is measured. Actually this equation is expressed by solving a nearly bi-diagonal matrix.



Estimation of quadrupole misalignment (3)

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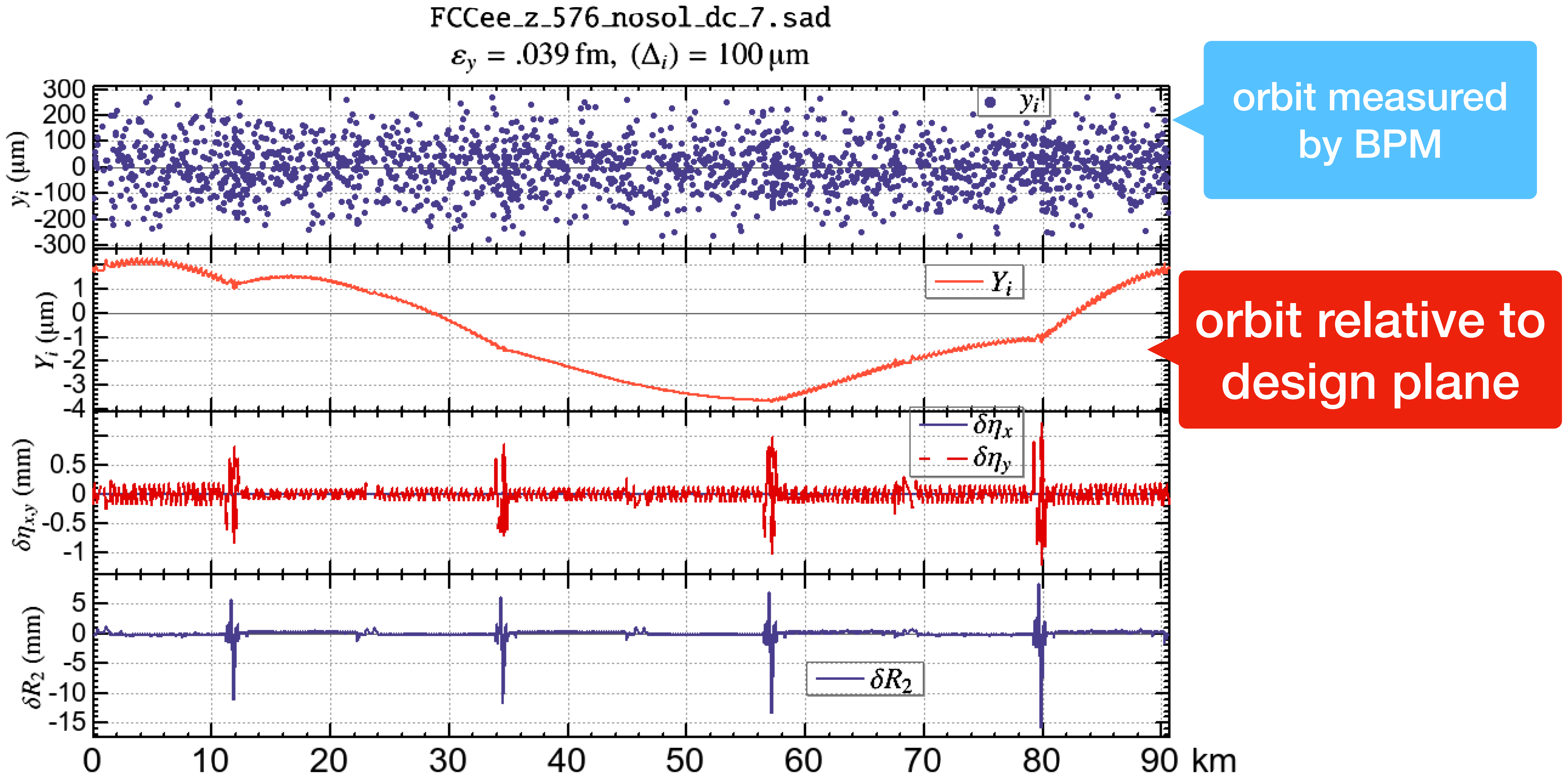
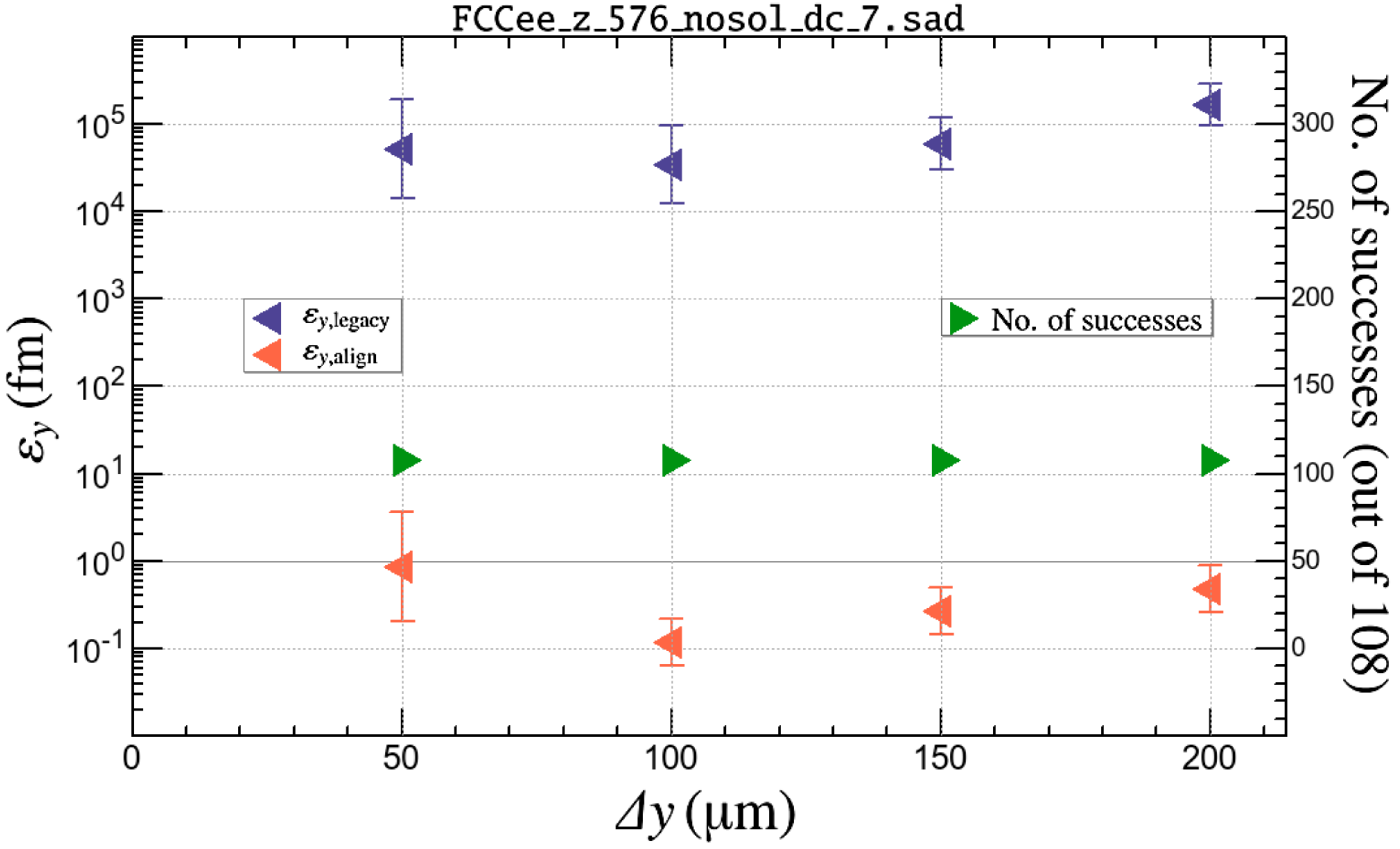
- If we solve the equation above, we can determine all misalignments of quadrupoles.
- For the lattice at Z, the solution looks nearly perfect; it is possible to reach vertical emittance below 1 fm, under up to 200 μm misalignments of quadrupoles (together with BPMs).
- Here all sextupoles are nested on nearby quads, except for crab/YCCS ones (SY*), which are set to zero here.

$$f_{a,i}\Delta_{i-1} + \Delta_i + f_{b,i}\Delta_{i+1}$$
$$= c_{a,i}y_{i-1} + c_{0,i}y_i + c_{b,i}y_{i+1} + v_{a,i}\theta_{i-1} + v_{0,i}\theta_i \,,$$



Results do not depend on the magnitude of misalignments.

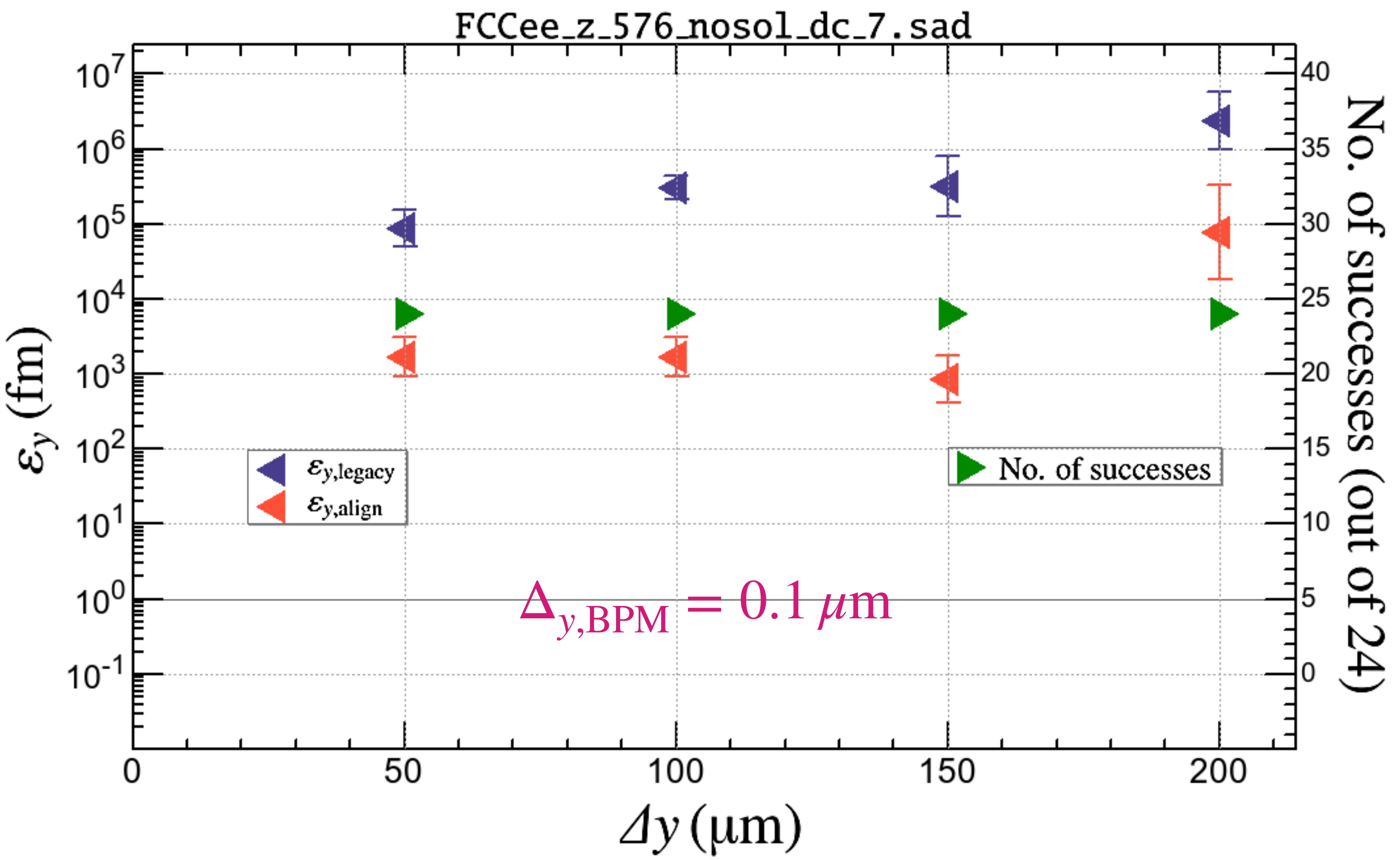


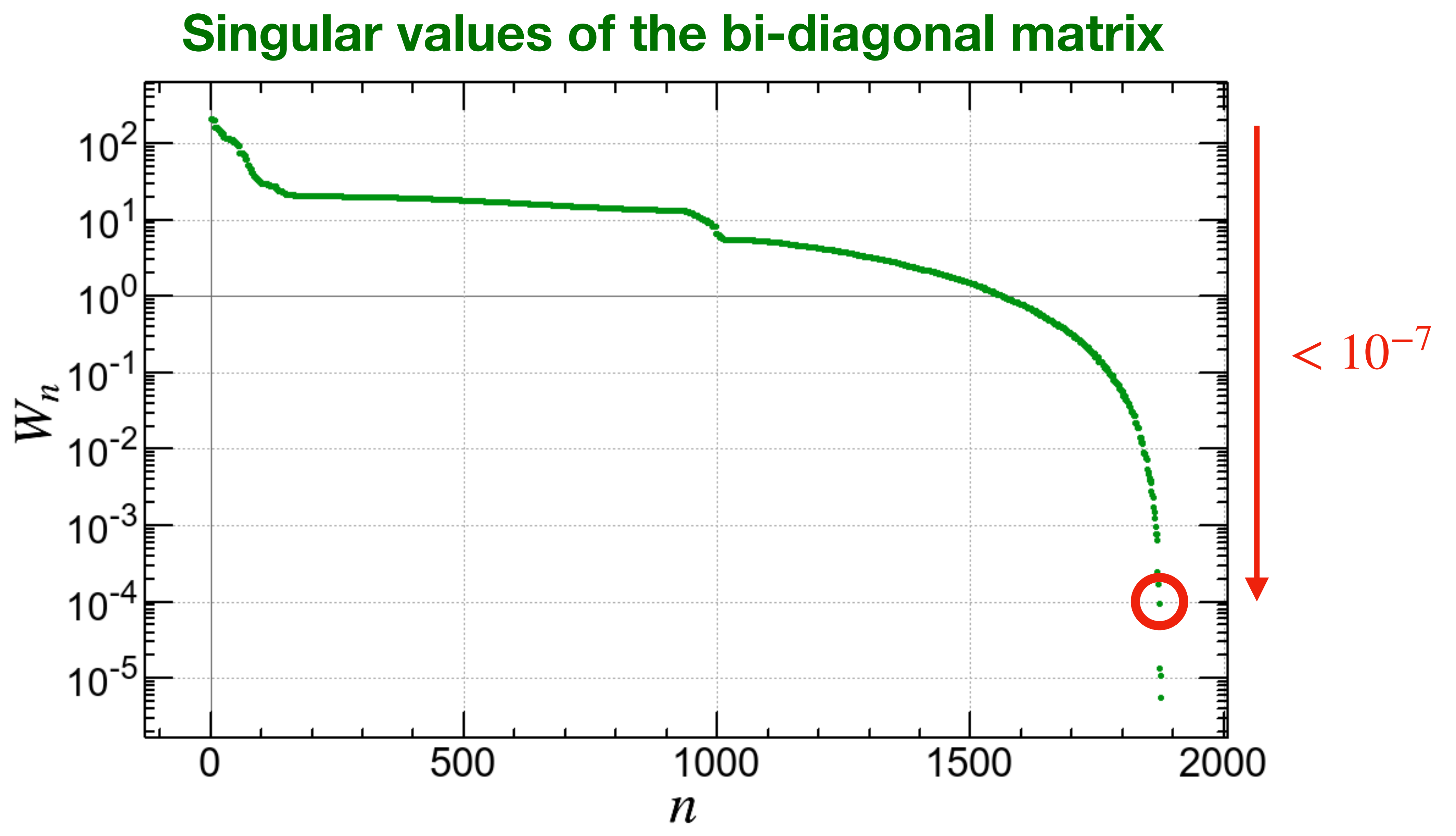
Estimation of quadrupole misalignment (4)

- However, the result above is only true with zero BPM measurement/alignment errors.
- If we add BPM errors, the estimation results in huge vertical emittance.
- One reason it that this solution needs to use almost all singular values of the bi-diagonal matrix up to (maximum)-3.

$$f_{a,i}\Delta_{i-1} + \Delta_i + f_{b,i}\Delta_{i+1}$$
$$= c_{a,i}y_{i-1} + c_{0,i}y_i + c_{b,i}y_{i+1} + v_{a,i}\theta_{i-1} + v_{0,i}\theta_i \,,$$

To reach 1 nm accuracy by a BPM with 100 μm single-shot resolution BPM, we need 10^6 turns (= 5 minutes) with 10000 bunches in the ring. Besides, the BPM offset to the quad center must have the same accuracy.





Vibration of quadrupoles

https://indico.cern.ch/event/1186798/contributions/5062667/attachments/2533360/4362351/Vib_Oide_221025.pdf

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1.3 Non-resonant vibration

Next let us look at the off-resonant contribution of Eq. (7). If we roughly approximate the tune-dependent term by 1, the integrated power spectrum in a range $\omega \geq \omega_c$ is given by

$$\begin{aligned}\langle \Delta y^{*2} \rangle &= \frac{N_q \beta^* \beta_q k_q^2}{4} \int_{\omega_c}^{\infty} S(\omega) \frac{d\omega}{2\pi} \\ &= \frac{N_q \beta^* \beta_q k_q^2 \sigma}{24\pi\omega_c^3} .\end{aligned} \tag{15}$$

In the case for the previous measurement, we estimate $\sigma \sim 1.6 \times 10^{-12}\,\mathrm{m}^2/\mathrm{Hz}$, then

$$\sqrt{\Delta y^{*2}} \sim 32.9\,\mathrm{nm} \tag{16}$$

for $\omega_c = 2\pi \times 1\,\mathrm{Hz}$. The assumption here is that below the critical frequency ω_c, an orbit feedback suppresses the beam oscillation perfectly. Thus the expected vibration reaches to the vertical beam size at the IP. Among the vibration, the dominant contribution comes from the final quads "QC{12}*". If we exclude them, the expected vibration becomes

$$\sqrt{\Delta y^{*2}}_{\text{excl. QC\{12\}*}} \sim 5.8\,\mathrm{nm} . \tag{17}$$

This value means that the contribution from other quads is small, but still not negligible. Suppressing the vibration of the final quads as well as an orbit feedback system working beyond 1 Hz will be crucial.

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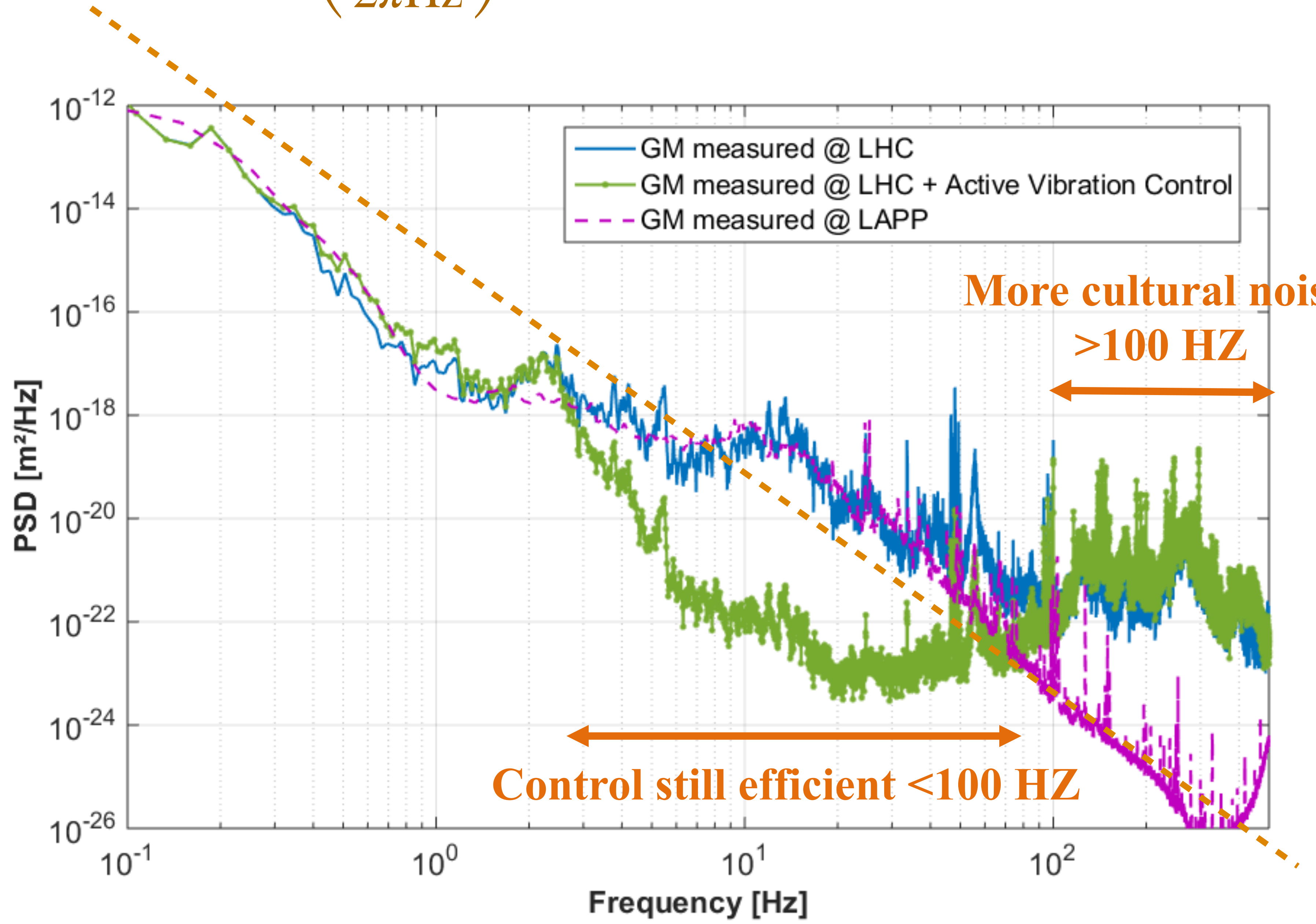
$$S(\omega) \sim 10^{-15} \left(\frac{\omega}{2\pi\mathrm{Hz}}\right)^{-4} \mathrm{m}^2/\mathrm{Hz}$$



M. Serluca, et al.

https://indico.cern.ch/event/694811/contributions/2863859/attachments/1595533/2526938/2018_02_06_FCCee_MDI_workshop_Serluca.pdf

Correlation between $e^{\pm}$

Let us discuss the vertical displacement of a bunch at the IP, arriving at $t = 0$. The vertical displacement of a positron Δy^*_+ at the IP caused by a quadrupole for positrons, located at vertical phase advance ϕ_q from the IP, vibrating with an amplitude Δy_q and an angular frequency ω_q is written as:

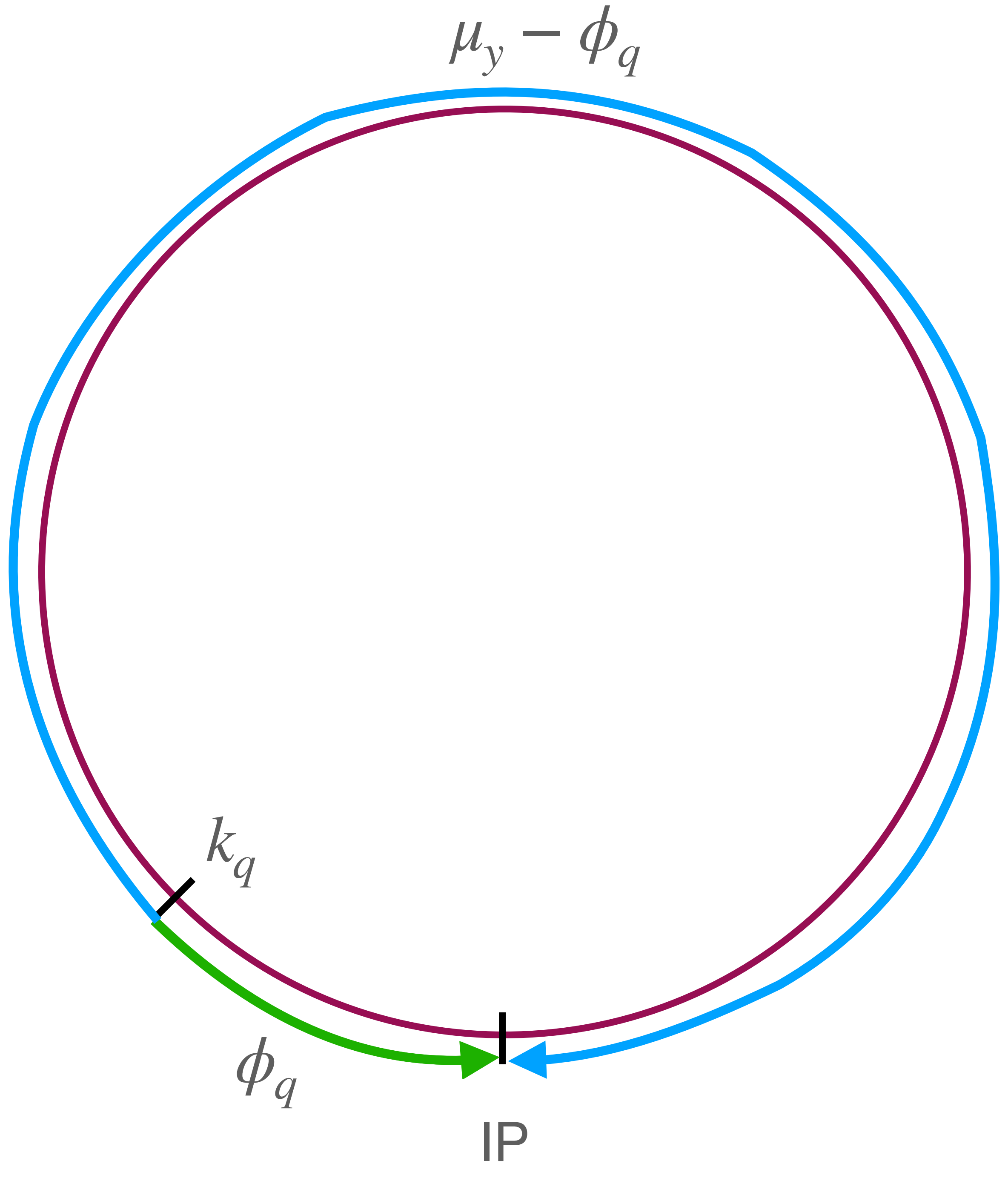
$$\begin{aligned}\Delta y^*_+ &= \sum_n \sqrt{\beta^*\beta_q}\exp(-nT_0/\tau_y - i\omega_q t_{q+})\sin(\phi_q + n\mu_y)k_q\Delta y_q \\ &= \sum_n \sqrt{\beta^*\beta_q}\exp(-n\alpha_y + i(n+n_q)\mu_q)\sin(\phi_q + n\mu_y)k_q\Delta y_q\,,\end{aligned} \tag{1}$$

where μ_y, T_0 & τ_y, are the vertical betatron angular tune, the revolution & damping times, and $\alpha_y \equiv T_0/\tau_y$, $\mu_q \equiv \omega_q T_0$, $n_q \equiv \phi_q/\mu_y$. β^*, β_q, k_q are the beta functions at the IP and the quadrupole, and the focusing strength of the quadrupole. We can assume that $t_{q+} \approx -(n+n_q)T_0$.

The current design of the collider optics employs a twin-aperture quadrupole magnet for arc quadrupoles. Although both quadrupoles for $e^{\pm}$ are mechanically tight coupled, they have opposit focusing/defocusing, having different *betas*. So we do not have to care about their interference on the vibration. Another source of the vibration is the final focusing quad QC1, having the same strength and polarity for two beams. The motion of the electron beam caused by the same quad is written as

$$\Delta y^*_- = \sum_n \sqrt{\beta^*\beta_q}\exp(-n\alpha_y + i(n+1-n_q)\mu_q)\sin(\phi_q + n\mu_y)k_q\Delta y_q\,, \tag{2}$$

by replacing $\phi_q \to \mu_y - \phi_q$ in Eq. (1), and the source time is given by $t_{q-} \approx -(n+1-n_q)T_0$.



$\beta^*_y = 0.7\,\text{mm}$, $\mu_y = 222.2 \times 2\pi$,
$\phi_q = \pi/2$, $T_0 = 0.3\,\text{ms}$, $\alpha = 4.28 \times 10^{-4}$,
$\beta_q = 12000\,\text{m}$, $k_q = 0.2\,/\text{m}$

Correlation between $e^{\pm}$ (2)

Then we sum up above over n from 0 to infinity, we obtain the difference between the positron and electron bunches as:

$$\Delta y^*(\infty) = A(f)\Delta y_q\,, \tag{3}$$

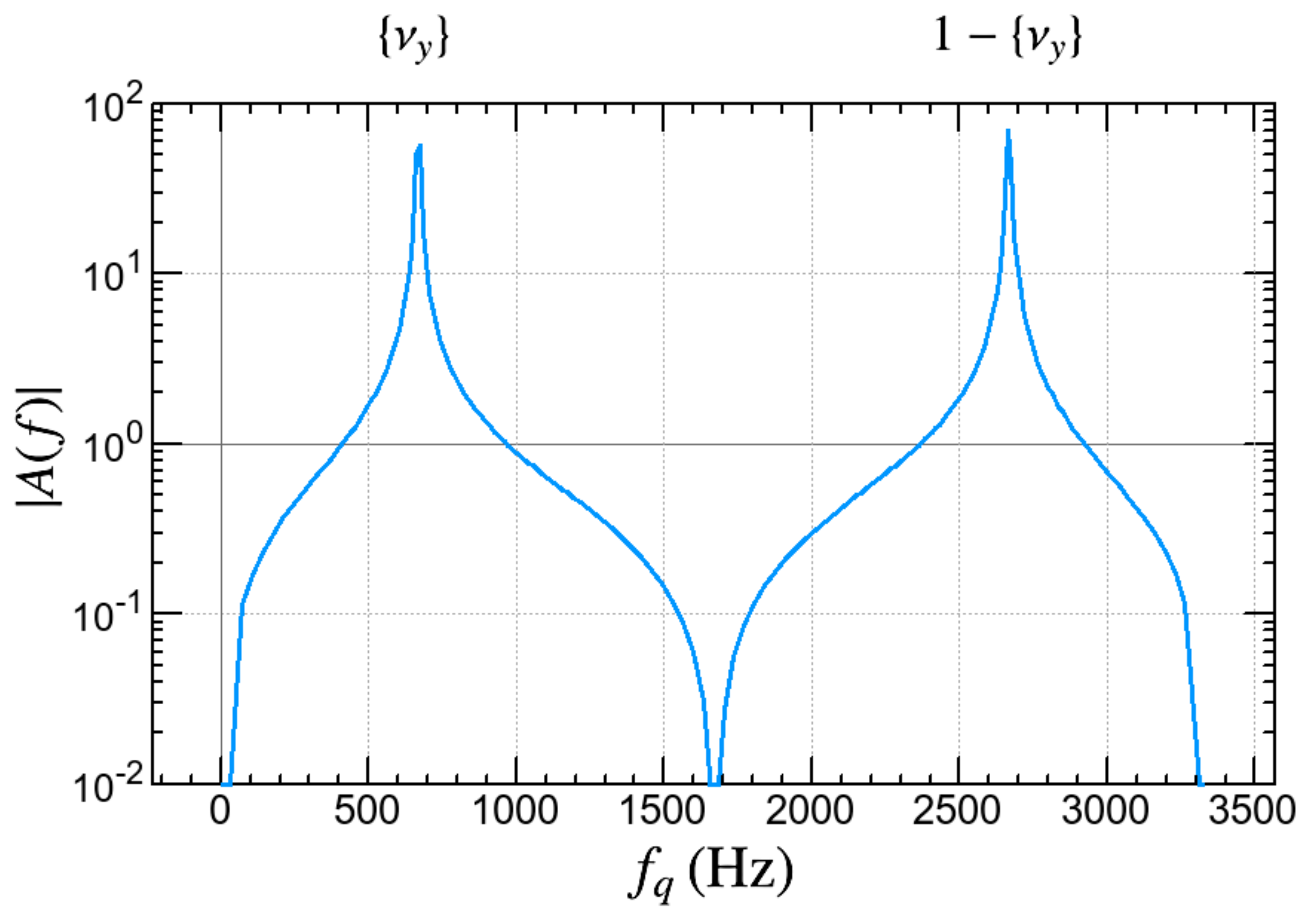
where f is the vibration frequency of the quadrupole $f = \omega_q/2\pi$, and $A(f)$ is the attenuation factor can be analytically obtained from above. We do not write $A(f)$ here as it is a little bit lengthy.

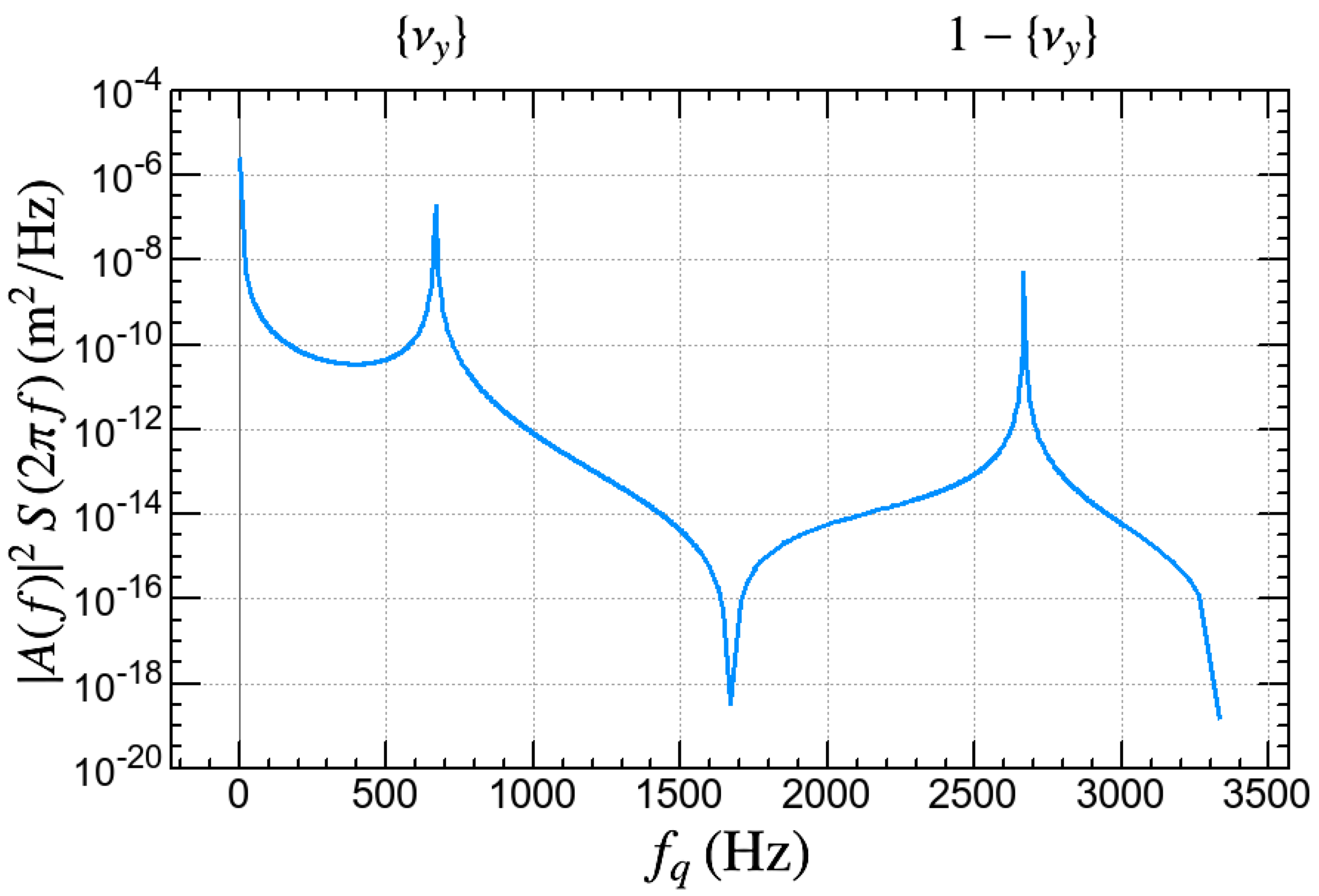
The right plot shows $A(f)$ for the final quad `QC1L1` with the optics for Z. Two peaks are seen at the fraction of tune, $\{\nu_y\}$ and $1-\{\nu_y\}$.

By integrating the product of $A(f)^2$ and the ground motion $S(\omega)$, from a critical frequency ω_c up to the revolution frequency. we obtain the magnitude of the vertical difference:

$$\begin{aligned}\langle \Delta y^{*2}\rangle &= \int_{f_c}^{1/T0} A(f)^2 S(2\pi f) df \\ &= (96\,\mathrm{pm})^2\,,\end{aligned} \tag{4}$$

for $fc = 1\,\mathrm{Hz}$. This number seems small even if we take all 8 similar quads into account.





Summary

- Several optics correction schemes have been tried for the FCC-ee Z lattice.
  - If errors not accompanying closed orbit distortion, ie., misalignments of sextupoles, rotation and strength errors of quadrupoles, corrections by skew/normal quad corrections work sufficiently.
    - hor./ver. emittances, betatron phase errors ($\approx \Delta\beta/\beta$), dispersions can be reduced below requirement, with a reasonable single-shot BPM error, $\sim 100\,\mu\mathrm{m}$.
  - The misalignment of quadrupoles & BPMs can be estimated by orbit measurement.
    - However, the required BPM measurement error seems extremely small, a nm level (for a long time/many bunch average).
      - An estimation with relaxed optics may help?

- Estimation of the correlation between $e^{\pm}$ by the final quad QC1L/R1 has been made.
  - The resulting relative offset at the IP will be small compared to the beam size, assuming the measured level of the ground vibration.