Analytical Calculations of QCD Amplitudes - $Wt\bar{t}$ Production

TECHNISCHE UNIVERSITÄT MÜNCHEN

MAX-PLANCK-INSTITUT FÜR PHYSIK

SARA DITSCH

QCD MASTER CLASS 2024

Colliders and Scattering Amplitudes

Theory
QFT

- Exploration of new computational methods
- Insight into underlying theory
- Connection to interesting mathematics

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QFT

Experiment
LHC

SCATTERING AMPLITUDE

$$d\sigma \propto |A|^2$$

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- Full Run-3 data and HL phase at LHC
- Possibly find evidence for BSM physics
- Need theory to match the experimental precision

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→ High precision theory predictions needed

Precision Predictions

PERTURBATIVE EXPANSION IN COUPLING

$\alpha_s \ll 1$

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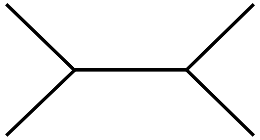
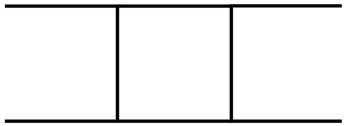
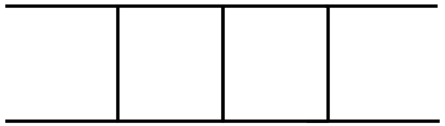
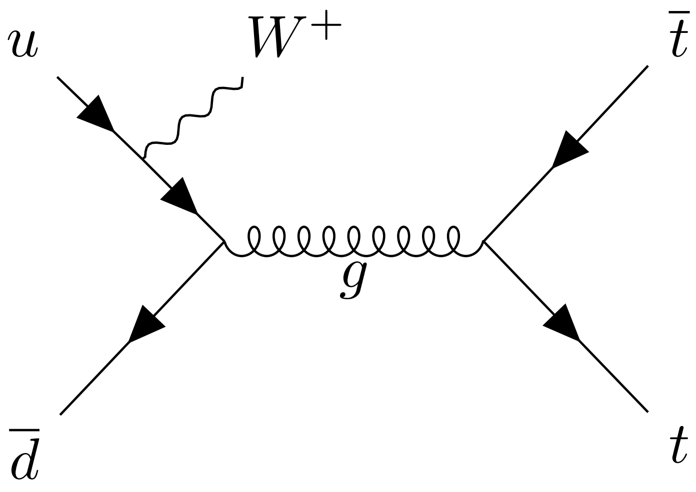
$A =$  $+$  $+$  $+ \cdots$

$$A = \alpha_s A^{(0)} + \alpha_s^2 A^{(1)} + \alpha_s^3 A^{(2)} + \mathcal{O}(\alpha_s^4)$$

Wttbar Production



$$\bar{d}(p_1) + u(p_2) \to t(-p_3) + \bar{t}(-p_4) + W^+(-p_5)$$

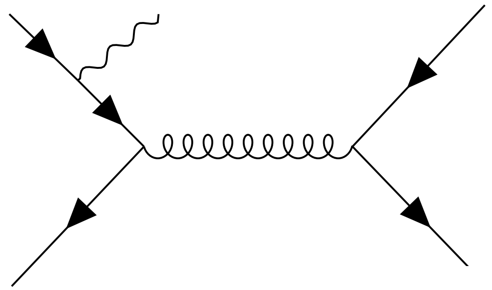
- Signature for BSM physics
- Background e.g. $Ht\bar{t}$ production
- Many scales, methodologically interesting

Representation of the Amplitude

FORMFACTORS IN THV [Peraro, Tancredi ’20]

$$A(\{p_k\}) = \sum_{i=1}^{N} F_i(\{x_k\}) T_i$$

$$\left(\overline{V}_1 ... U_2\right) \quad \left(\overline{V}_3 ... U_4\right) \quad \varepsilon_5^{*\mu}$$



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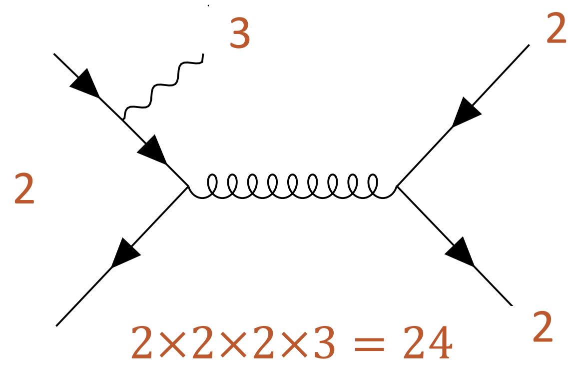
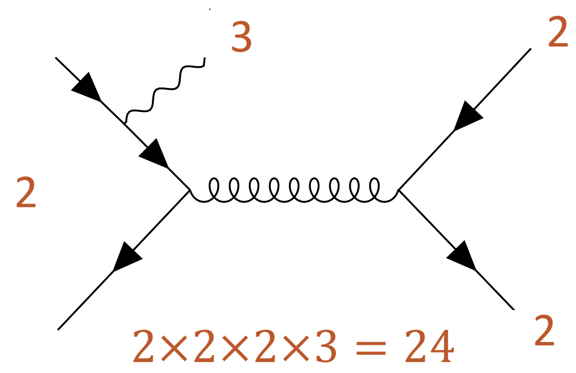


#Basis tensors = #Helicity amplitudes

→ One Basis for all Loop Orders

Basis Tensors for W$t\bar{t}$ Production

$$\left(\overline{V}_1 ... U_2\right) \quad \left(\overline{V}_3 ... U_4\right) \quad \varepsilon_5^{*\mu}$$



Basis Tensors for Wtt̄ Production

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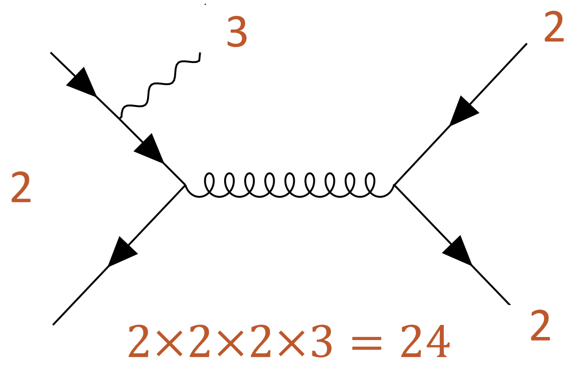

- $\sum_i p_i = 0$
- $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{1}$
- $(\not{p} - m)U(p) = 0 \Rightarrow \not{p}U(p) = mU(p) \sim U(p)$
- $\not{p}\not{p} = p^2$

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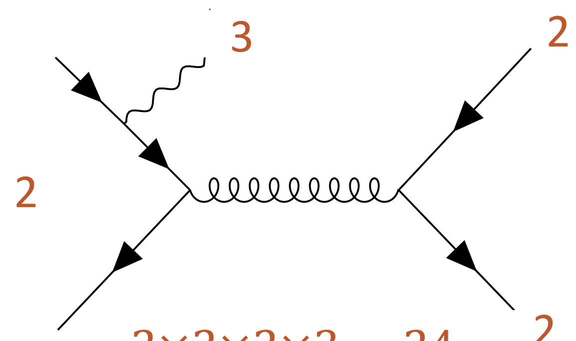

2×2×2×3 = 24

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$$T_i \equiv \left(\overline{V}_1\{\not{p}_3, \not{p}_4\}U_2\right) \left(\overline{V}_3\{\mathbb{1}, \not{p}_1, \not{p}_2, \not{p}_1\not{p}_2\}U_4\right) \left(\varepsilon_5^* \cdot \{p_1, p_2, p_3\}\right)$$

Basis Tensors for $Wt\bar{t}$ Production

WHAT IS A GOOD TENSOR BASIS?

- No spurious poles,
  gram determinant cancels
- Simple Forms Factors

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Basis Tensors for $Wt\bar{t}$ Production

WHAT IS A GOOD TENSOR BASIS?

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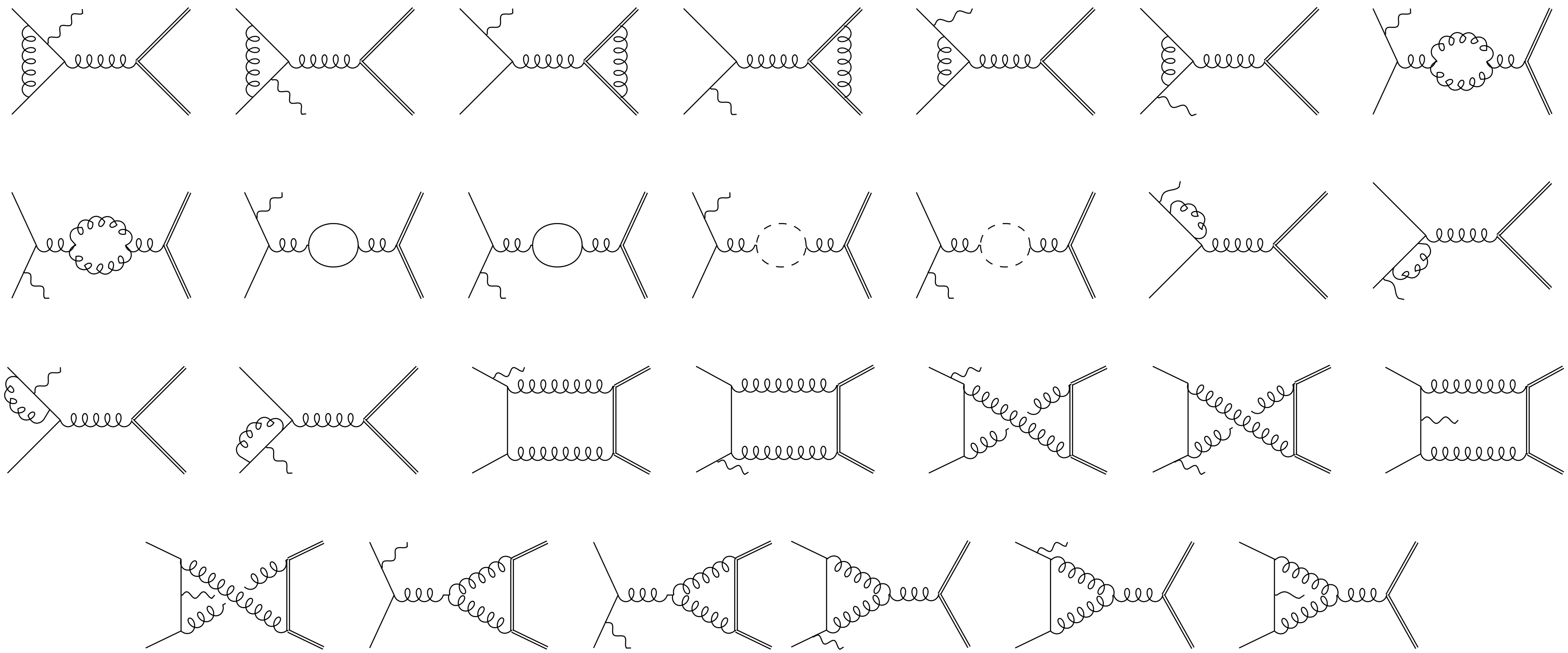
METHODS TO FIND NEW BASIS

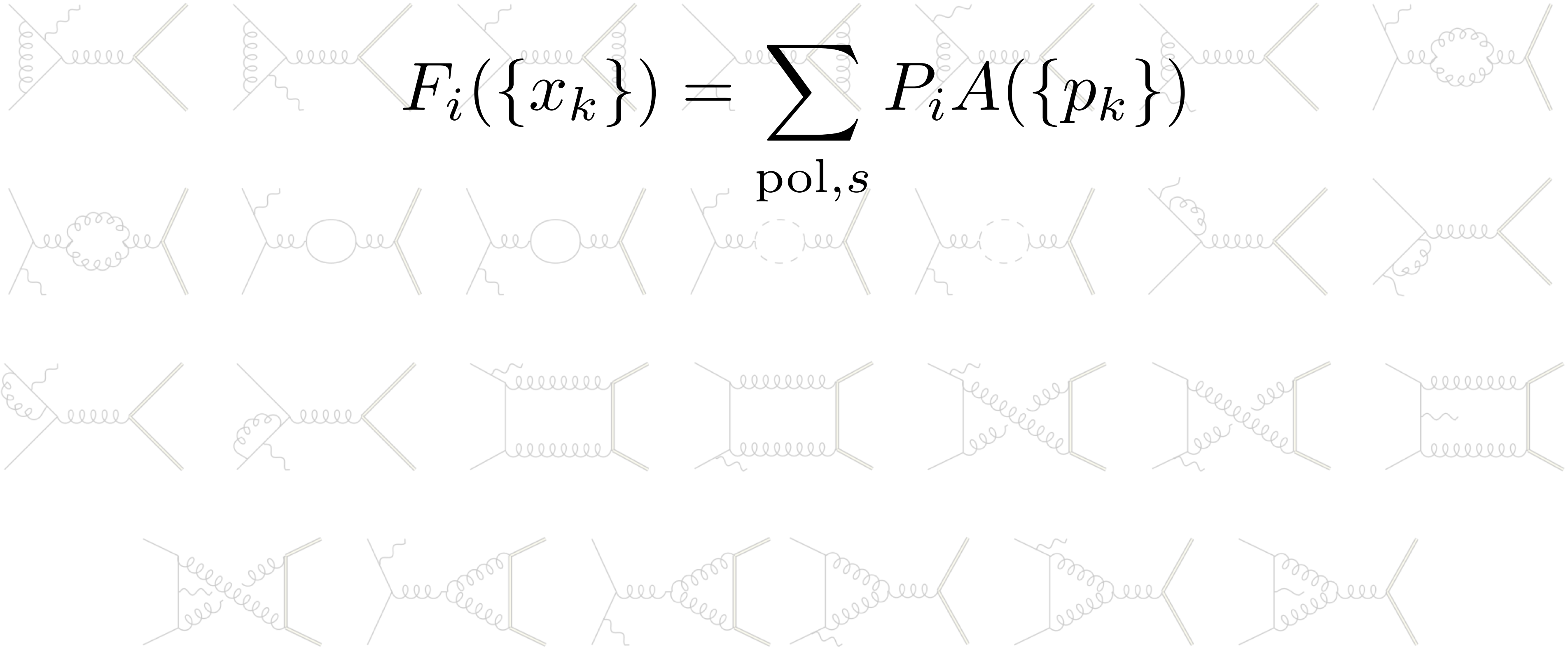
- Fixing helicities / massive chiralities
- Change polarization basis
- Impose cancelation of gram

$$T_i \equiv \left(\overline{V}_1\{\not{p}_3, \not{p}_4\}U_2\right) \left(\overline{V}_3\{\mathbb{1}, \not{p}_1, \not{p}_2, \not{p}_1\not{p}_2\}U_4\right) \left(\varepsilon_5^* \cdot \{p_1, p_2, p_3\}\right)$$

One-Loop Amplitude



One-Loop Form Factors



$$F_i(\{x_k\}) = \sum_{\text{pol},s} P_i A(\{p_k\})$$

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$$\sum_{\text{pol},s} |A|^2 = \vec{F}^\dagger M \vec{F}$$

One-Loop Form Factors

$$F = \sum_k r_k(\{s_{ij}\}) I_k$$

$$I_{(a_1,..,a_n)} = \int \frac{d^d k}{i\pi^{\frac{d}{2}}} \frac{1}{D_1^{a_1} D_2^{a_2} ... D_n^{a_n}}$$

$$D_i = q_i^2 - m_i^2$$

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$$I_{(1,0,1,0,0)} = \int \frac{d^d k}{i\pi^{\frac{d}{2}}} \frac{1}{D_1 D_3}$$

$$I_{(0,0,2,0,-1)} = \int \frac{d^d k}{i\pi^{\frac{d}{2}}} \frac{D_5}{D_3^2}$$

Reduction of Scalar Integrals

$$I = \int \frac{d^d k}{i\pi^{\frac{d}{2}}} \frac{1}{D_1^{a_1} D_2^{a_2} \dots D_n^{a_n}}$$

Integral families form a vector space with finite basis of integrals:

Master Integrals J [Smirnov, Petukhov '11]

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IBP REDUCTION (+ SYMMETRY RELATIONS)

$$0 = \int \frac{d^D k}{i\pi^{\frac{D}{2}}} \frac{\partial}{\partial k^\mu} \frac{v^\mu}{(q_1^2 - m_1^2)^{a_1} \ldots (q_n^2 - m_n^2)^{a_n}}$$

[Chetyrkin, Tkachov '81]

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[Chetyrkin, Tkachov '81]

$$I = \sum_i c_i(\{s_{ij}, d\}) J_i$$

$$F = \sum_k r_k(\{s_{ij}\}) I_k = \sum_{k,m} r_k(\{s_{ij}\}) c_m^{(k)}(\{s_{ij}\}, d) J_m$$

Computation of Master Integrals

METHODE OF DIFFERENTIAL EQUATIONS

$$\frac{\partial}{\partial x_i} J_j = \sum_k b_k I_k = \sum_n a_n J_n$$

$$\frac{\partial}{\partial x_i} \vec{J} = A_{x_i}(\{x_i\}, D)\vec{J}$$

[Kotikov ‘91] [Remiddi ‘97]

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CANONICAL DIFFERENTIAL EQUATIONS

$$\frac{\partial}{\partial x_i} \vec{J} = \varepsilon A_{x_i}(\{x_i\})\vec{J}$$

$$d\vec{J} = \varepsilon \sum_k A_k \mathrm{dlog}(\alpha_k)\vec{J}$$

[Henn ‘13]

Computation of Master Integrals

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[Henn '13]

+ BOUNDARY CONDITIONS

Outlook: Towards Two-Loops

FIND CANONICAL MASTER INTEGRALS

CHALLENGES

COMPUTE MASTER INTEGRALS

SIMPLIFY RATIONAL COEFFICIENTS