The Three-loop Jet Function for Top Quarks

(Soon to appear in arXiv)

Alberto Martín Clavero
Vicent Mateu Barreda
Maximilian Stahlhofen
Robin Brüser

June 26, 2024

Overview

1. **Introduction to EFTs**

2. **Factorisation theorem**

3. **The 3-loop jet function**

4. **Conclusions**

Introduction to EFTs

What's an EFT?

- Low energy observables are insensitive to high energy physics.

What's an EFT?

- Low energy observables are insensitive to high energy physics.
- We expand in powers of a small parameter λ and compute quantities until some order n.

$$O_{QCD}(s,Q) = \sum_n C_n(Q,\mu) O_{EFT,n}(s,\mu) \left(\frac{s}{Q}\right)^n$$

What's an EFT?

- Low energy observables are insensitive to high energy physics.
- We expand in powers of a small parameter λ and compute quantities until some order n.

$$O_{QCD}(s,Q) = \sum_n C_n(Q,\mu) O_{EFT,n}(s,\mu) \left(\frac{s}{Q}\right)^n$$

- EFTs are a natural way of introducing separation of scales.

$$\langle O_{QCD}\rangle \sim \log\left(\frac{s}{Q}\right) \qquad C_n \sim \log\left(\frac{\mu}{Q}\right) \qquad \langle O_{EFT,n}\rangle \sim \log\left(\frac{s}{\mu}\right)$$

What's an EFT?

- Low energy observables are insensitive to high energy physics.
- We expand in powers of a small parameter λ and compute quantities until some order n.

$$O_{QCD}(s,Q) = \sum_n C_n(Q,\mu) O_{EFT,n}(s,\mu) \left(\frac{s}{Q}\right)^n$$

- EFTs are a natural way of introducing separation of scales.

$$\langle O_{QCD} \rangle \sim \log\left(\frac{s}{Q}\right) \qquad C_n \sim \log\left(\frac{\mu}{Q}\right) \qquad \langle O_{EFT,n} \rangle \sim \log\left(\frac{s}{\mu}\right)$$

- We can set the value of μ to make the logs in $\langle O_{EFT,n} \rangle$ of order 1 and then resum the logs in $C_n(Q,\mu)$ with an RGE.

HQET

[Manohar, Wise Camb.Monogr.Part.Phys.Nucl.Phys.Cosmol. 10 (2000) 1-191]

Consider a single (top) quark with offshellness

$$p_t = m_t v + k, \qquad v^2 = 1 \quad \& \quad k^2 \ll m_t^2$$

We can expand in $\lambda = (k^2/m_t^2) \ll 1$.

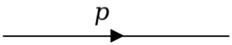
HQET

[Manohar, Wise Camb.Monogr.Part.Phys.Nucl.Phys.Cosmol. 10 (2000) 1-191]

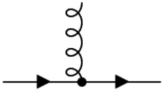
Consider a single (top) quark with offshellness

$$p_t = m_t v + k, \qquad v^2 = 1 \quad \& \quad k^2 \ll m_t^2$$

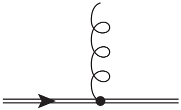
We can expand in $\lambda = (k^2/m_t^2) \ll 1$.



$$i\frac{\not{p}_t + m_t}{p_t^2 - m_t^2 + i0^+}\delta_{\alpha\beta} \to i\frac{1}{2\, v\cdot k + i0^+}\delta_{\alpha\beta}$$



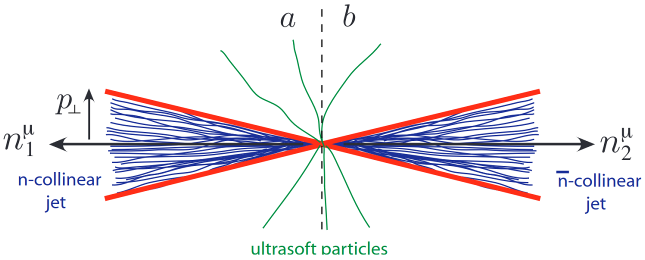
$$i\gamma^\mu T^a_{\alpha\beta} \to i v^\mu T^a_{\alpha\beta}$$



The basics of SCET I

[Bauer, Fleming, Luke Phys.Rev.D 63 (2000) 014006], [Bauer, Fleming, Pirjol, Stewart Phys.Rev.D 63 (2001) 114020], [Bauer, Pirjol, Stewart Phys.Rev.D 65 (2002) 054022]

- Consider the process $e^+e^- \to 2$ top jets, with CM energy $Q^2 \gg m_i^2$.
- We can regard each jet as a (anti)quark + other particles going roughly in the same direction.



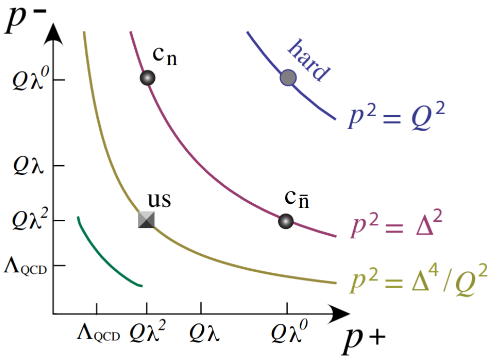
Separation of scales in SCET I

Two energy scales:

- Q = CM energy.
- Δ = energy scale in t rest frame.

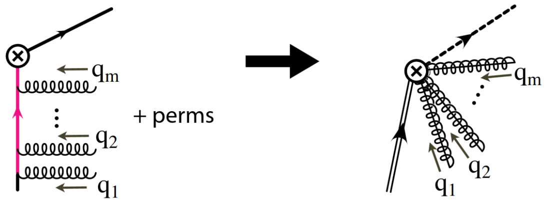
$\lambda = (\Delta/Q) \ll 1$ separates:

- Hard modes $p_h^2 = Q^2$.
- Collinear modes $p_n^2 = Q^2(\Delta/Q)^2$.
- Soft modes $p_s^2 = Q^2(\Delta/Q)^4$.



Separation of scales in SCET I

Wilson lines naturally appear in SCET:

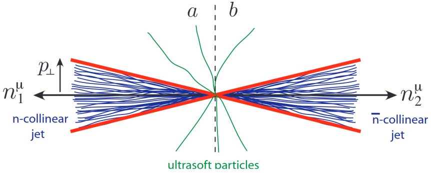


$$W_n = \sum_k \sum_{\text{perp}} \frac{(-g)^k}{k!} \left(\frac{\bar{n}\cdot A_n(q_1)\ldots\bar{n}\cdot A_n(q_k)}{[\bar{n}\cdot q_1]\ldots[\bar{n}\cdot\sum_{i=1}^k q_i]} \right)$$

Factorisation theorem

The QCD cross section

- We are interested in $e^+e^- \to 2$ jets + soft radiation.

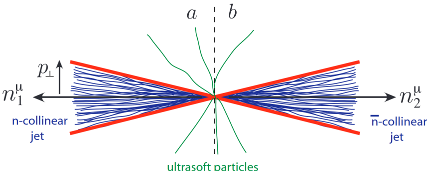


- The QCD cross-section is

$$\sigma = \sum_{X_{\text{jets}}} (2\pi)^4 \delta^4(q - p_X) \sum_{i=a,v} L^i_{\mu\nu} \langle 0|\mathcal{J}_i^{\nu\dagger}|X\rangle \langle X|\mathcal{J}_i^{\mu}|0\rangle$$

The QCD cross section

- We are interested in $e^+e^- \to 2$ jets $+$ soft radiation.



- The QCD cross-section is

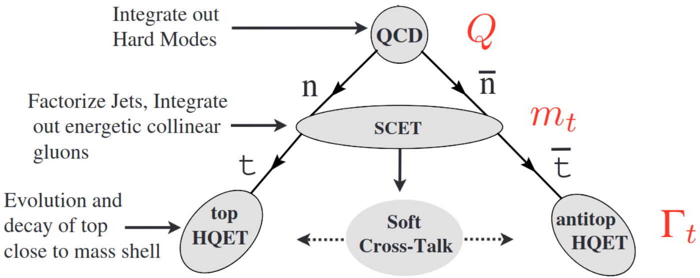
$$\sigma = \sum_{X_{\text{jets}}} (2\pi)^4 \delta^4(q - p_X) \sum_{i=a,v} L^i_{\mu\nu} \langle 0|\mathcal{J}_i^{\nu\dagger}|X\rangle \langle X|\mathcal{J}_i^{\mu}|0\rangle$$

- Relevant energy scales: $Q \gg m_t \gg \Gamma_t > \Lambda_{QCD}$

EFT approach

[Fleming, Hoang, Mantry, Stewart Phys.Rev.D 77 (2008) 074010]

$$Q \gg m_t \gg \Gamma_t > \Lambda_{QCD}$$



$$\begin{cases} M_t \sim m_t \\ \dfrac{M_t}{Q} \ll 1 \end{cases}$$

$$\begin{cases} \hat{s}_t \equiv \dfrac{M_t^2 - m_t^2}{m_t} \sim \Gamma_t \\ \dfrac{M_t}{Q} \ll 1 \quad \dfrac{\hat{s}_t}{m_t} \ll 1 \end{cases}$$

The factorised cross-section

After some hard work...

$$\frac{d^2\sigma}{dM_t^2 dM_{\bar{t}}^2} = \sigma_0 H_Q(Q,\mu_m) H_m(m_t, \frac{Q}{m_t}, \mu_m, \mu)$$
$$\times \int dl^+ dl^- B_+\left(\hat{s}_t - \frac{Ql^+}{m_t}, \Gamma_t, \delta m_t, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Ql^-}{m_t}, \Gamma_t, \delta m_t, \mu\right) S(l^+, l^-, \mu)$$

The factorised cross-section

After some hard work...

$$\frac{d^2\sigma}{dM_t^2 dM_{\bar{t}}^2} = \sigma_0 H_Q(Q,\mu_m) H_m\left(m_t, \frac{Q}{m_t}, \mu_m, \mu\right) \times \int dl^+ dl^- B_+\left(\hat{s}_t - \frac{Ql^+}{m_t}, \Gamma_t, \delta m_t, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Ql^-}{m_t}, \Gamma_t, \delta m_t, \mu\right) S(l^+, l^-, \mu)$$

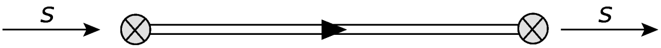
- $B_+ = B_- \equiv B$
- $\hat{s}_{\bar{t}} - \frac{Ql^-}{m_t} \equiv s$ is related to the offshellness.
- We can set $\Gamma_t = 0$, $\delta m_t = 0$ since they can be reintroduced by shifting s.

The bHQET jet-function

[Fleming, Hoang, Mantry, Stewart Phys.Rev.D 77 (2008) 094008]

$$B(s,\mu) \equiv \mathrm{Im}\left[\frac{-i}{4\pi N_c m_t}\int d^4x\, e^{i\,r\cdot x}\,\langle 0|T\{\bar{h}_{v_+}(0)W_n(0)W_n^\dagger(x)h_{v_+}(x)|0\rangle\right]$$

Where $v_+ \cdot r = s$

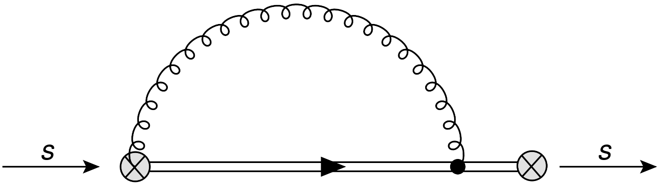


The bHQET jet-function

[Fleming, Hoang, Mantry, Stewart Phys.Rev.D 77 (2008) 094008]

$$B(s,\mu) \equiv \mathrm{Im}\left[\frac{-i}{4\pi N_c m_t}\int d^4x\, e^{i\,r\cdot x}\,\langle 0|T\{\bar{h}_{v_+}(0)W_n(0)W_n^\dagger(x)h_{v_+}(x)|0\rangle\right]$$

Where $v_+ \cdot r = s$

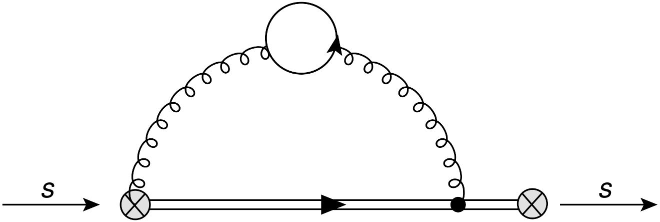


The bHQET jet-function

[Fleming, Hoang, Mantry, Stewart Phys.Rev.D 77 (2008) 094008]

$$B(s,\mu) \equiv \mathrm{Im}\left[\frac{-i}{4\pi N_c m_t}\int d^4x\, e^{i\,r\cdot x}\,\langle 0|T\{\bar{h}_{v_+}(0)W_n(0)W_n^{\dagger}(x)h_{v_+}(x)|0\rangle\right]$$

Where $v_+ \cdot r = s$

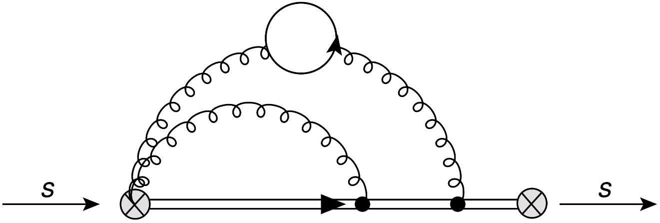


The bHQET jet-function

[Fleming, Hoang, Mantry, Stewart Phys.Rev.D 77 (2008) 094008]

$$B(s,\mu) \equiv \mathrm{Im}\left[\frac{-i}{4\pi N_c m_t}\int d^4x\, e^{i r\cdot x}\, \langle 0|T\{\bar{h}_{v_+}(0)W_n(0)W_n^\dagger(x)h_{v_+}(x)|0\rangle\right]$$

Where $v_+ \cdot r = s$



The bHQET jet-function

$$B(s,\mu) \equiv \mathrm{Im}\left[\frac{-i}{4\pi N_c m_t}\int d^4x\, e^{i r\cdot x}\langle 0|T\{\bar{h}_{v_+}(0)W_n(0)W_n^\dagger(x)h_{v_+}(x)|0\rangle\right]$$

Why computing the jet-function at higher perturbative orders in α_s is interesting?

- Precision measurements of the top mass in a renormalon-free scheme [Hoang Ann.Rev.Nucl.Part.Sci. 70 (2020) 225-255].
- Calibration of the MC top quark mass parameters [Butenschoen, Dehnadi, Hoang, Mateu, Preisser, Stewart Phys.Rev.Lett. 117 (2016) 23, 232001] , [Dehnadi, Hoang, Jin, Mateu JHEP 12 (2023) 065].
- It appears in other observables e.g. heavy jet mass, 2-jetiness and C-jetiness [Lepenik, Mateu JHEP 03 (2020) 024].

The 3-loop jet function

The strategy to compute the jet function.

$$B(s,\mu) \equiv \mathrm{Im}\left[\frac{-i}{4\pi N_c m_t}\int d^4x\, e^{i r\cdot x}\,\langle 0|T\{\bar{h}_{v_+}(0)W_n(0)W_n^\dagger(x)h_{v_+}(x)|0\rangle\right]$$

1. Draw all possible Feynman diagrams.
2. Translate from diagrams to expressions contributing to the matrix element.
3. Work out the color, Lorentz and Dirac algebra.
4. (?)
5. Compute all integrals.
6. Substitute the results for the integrals and sum over all diagrams.

Why going to higher powers of α_s is not easy?

- **Tree level:**
  1 Feynman diagram $\rightarrow \cdots \rightarrow$ 0 scalar loop integrals.

Why going to higher powers of α_s is not easy?

- **Tree level:**
  1 Feynman diagram $\rightarrow \cdots \rightarrow$ 0 scalar loop integrals.
- **1 loop:**
  4 Feynman diagrams $\rightarrow \cdots \rightarrow$ 3 scalar loop integrals.

Why going to higher powers of α_s is not easy?

- **Tree level:**
  1 Feynman diagram $\rightarrow \cdots \rightarrow$ 0 scalar loop integrals.
- **1 loop:**
  4 Feynman diagrams $\rightarrow \cdots \rightarrow$ 3 scalar loop integrals.
- **2 loops:**
  54 Feynman diagrams $\rightarrow \cdots \rightarrow$ 74 scalar loop integrals.

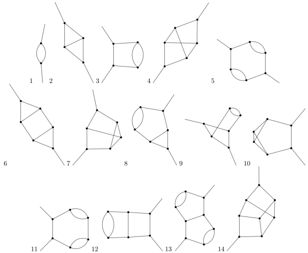
Why going to higher powers of α_s is not easy?

- **Tree level:**
  1 Feynman diagram $\rightarrow \cdots \rightarrow$ 0 scalar loop integrals.
- **1 loop:**
  4 Feynman diagrams $\rightarrow \cdots \rightarrow$ 3 scalar loop integrals.
- **2 loops:**
  54 Feynman diagrams $\rightarrow \cdots \rightarrow$ 74 scalar loop integrals.
- **3 loops:**
  1134 Feynman diagrams $\rightarrow \cdots \rightarrow$ 5406 scalar loop integrals.

Computing the jet function beyond tree level (1,2,3)

1. **Generate Feynman diagrams**

   Easily done thanks to `qgraf` [Nogueira J.Comput.Phys. 105 (1993) 279-289].



Computing the jet function beyond tree level (1,2,3)

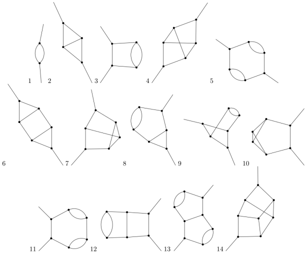
1. **Generate Feynman diagrams**

   Easily done thanks to `qgraf` [Nogueira J.Comput.Phys. 105 (1993) 279-289].

2. **Translate to expressions**

3. **Work out the color/Lorentz/Dirac algebra** and apply partial fractioning.

   We use the private code `Looping` [Brüser].



Generated diagrams (`qgraf`)
Feynman Rules (`FORM`, `color.h`) [Kuipers, Ueda, Vermaseren, Vollinga Comput.Phys.Commun. 184 (2013) 1453-1467] [Ritbergen, Schellekens, Vermaseren Int.J.Mod.Phys.A 14 (1999) 41-96] $\longrightarrow$ Diagrams in terms of scalar integrals

Computing the jet function beyond tree level (4)

4. **(?)**

Computing the jet function beyond tree level (4)

4. **Reducing the number of integrals**

   The key ingredient is using IBP (integration-by-parts) identities:

$$\int \prod_{r=1}^{n_\ell} \left(\frac{d^d k_r}{i\pi^{d/2}} \right) \frac{\partial}{\partial k_i^\mu} q^\mu \prod_j^{n_{\text{int}}} \frac{1}{D_j^{\nu_j}} = 0$$

   With q^μ any external vector or loop momentum.

Computing the jet function beyond tree level (4)

4. **Reducing the number of integrals**
   The key ingredient is using IBP (integration-by-parts) identities:

$$\int \prod_{r=1}^{n_\ell} \left(\frac{d^d k_r}{i\pi^{d/2}} \right) \frac{\partial}{\partial k_i^\mu} q^\mu \prod_j^{n_{\rm int}} \frac{1}{D_j^{\nu_j}} = 0$$

With q^μ any external vector or loop momentum.

$$G[a_1, a_2, a_3] \equiv \int \frac{d^d k_1}{i\pi^{d/2}} \frac{1}{[-(2v \cdot k_1 - 1)]^{a_1} (-n \cdot k_1)^{a_2} (-k_1^2)^{a_3}}$$

$$\left(\frac{\partial}{\partial k_1^\mu} k_1^\mu \right) \quad G[1,1,1] \quad \rightarrow \quad (4-d) G[1,1,1] + G[2,1,1] = 0$$

For the IBP reduction we used `FIRE6` [Smirnov, Chuharev Comput.Phys.Commun. 247 (2020) 106877] and `LiteRed` [Lee J.Phys.Conf.Ser. 523 (2014) 012059].

Computing the jet function beyond tree level (4)

We can express all the integrals as a linear combination of **master integrals (MI)** (which we choose to be as simple as possible).

- **Tree level:**
  1 Feynman diagram $\rightarrow \cdots \rightarrow$ 0 scalar loop integrals.

Computing the jet function beyond tree level (4)

We can express all the integrals as a linear combination of **master integrals (MI)** (which we choose to be as simple as possible).

- **Tree level:**
  1 Feynman diagram $\rightarrow \cdots \rightarrow$ 0 scalar loop integrals.
- **1 loop:**
  4 Feynman diagrams $\rightarrow \cdots \rightarrow$ 3 scalar integrals $\rightarrow \cdots \rightarrow$ 1 MI.

Computing the jet function beyond tree level (4)

We can express all the integrals as a linear combination of **master integrals (MI)** (which we choose to be as simple as possible).

- **Tree level:**
  1 Feynman diagram $\rightarrow \cdots \rightarrow$ 0 scalar loop integrals.
- **1 loop:**
  4 Feynman diagrams $\rightarrow \cdots \rightarrow$ 3 scalar integrals $\rightarrow \cdots \rightarrow$ 1 MI.
- **2 loops:**
  54 Feynman diagrams $\rightarrow \cdots \rightarrow$ 74 scalar integrals $\rightarrow \cdots \rightarrow$ 3 MIs.

Computing the jet function beyond tree level (4)

We can express all the integrals as a linear combination of **master integrals (MI)** (which we choose to be as simple as possible).

- **Tree level:**
  1 Feynman diagram $\to \cdots \to$ 0 scalar loop integrals.
- **1 loop:**
  4 Feynman diagrams $\to \cdots \to$ 3 scalar integrals $\to \cdots \to$ 1 MI.
- **2 loops:**
  54 Feynman diagrams $\to \cdots \to$ 74 scalar integrals $\to \cdots \to$ 3 MIs.
- **3 loops:**
  1134 Feynman diagrams $\to \cdots \to$ 5406 scalar integrals $\to \cdots \to$ 21 MIs.

Computing the jet function beyond tree level (5)

5. **Calculating the MI**
   It is convenient to use the Feynman parametrization

$$\int \prod_{r=1}^{n_\ell} \left(\frac{d^d k_r}{i\pi^{d/2}} \right) \prod_{j=1}^{n_{\rm int}} \frac{1}{D_j^{\nu_j}} \rightarrow \frac{\Gamma\left(\nu - \frac{n_\ell d}{2}\right)}{\prod_{j=1}^{n_{\rm int}} \Gamma(\nu)} \int_{x_j \geq 0} d^{n_{\rm int}} x \, \delta(1 - \textstyle\sum_j^* x_j) \left(\prod^{n_{\rm int}} x_j^{\nu_j - 1} \right) \frac{[\mathcal{U}(x)]^{\nu - \frac{(n_\ell+1)d}{2}}}{[\mathcal{F}(x)]^{\nu - \frac{n_\ell d}{2}}}$$

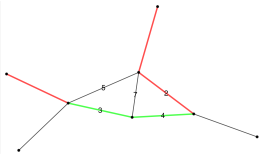
Computing the jet function beyond tree level (5)

5. **Calculating the MI**
   It is convenient to use the Feynman parametrization

$$\int \prod_{r=1}^{n_\ell} \left(\frac{d^d k_r}{i\pi^{d/2}} \right) \prod_{j=1}^{n_{\rm int}} \frac{1}{D_j^{\nu_j}} \to \frac{\Gamma\left(\nu - \frac{n_\ell d}{2}\right)}{\prod_{j=1}^{n_{\rm int}} \Gamma(\nu)} \int_{x_j \geq 0} d^{n_{\rm int}} x\, \delta(1 - \textstyle\sum_j^* x_j) \left(\prod^{n_{\rm int}} x_j^{\nu_j - 1} \right) \frac{[\mathcal{U}(x)]^{\nu - \frac{(n_\ell+1)d}{2}}}{[\mathcal{F}(x)]^{\nu - \frac{n_\ell d}{2}}}$$

In general, it is not trivial to compute the integral.

$$\Gamma^2(1-\varepsilon)\Gamma(-1+4\varepsilon) \int_0^\infty \int_0^\infty dx_3\, dx_4\, (1+x_3)^{-3\varepsilon}(1+x_4)^{-1+\varepsilon}(1+x_3+x_4)^{-1+\varepsilon}$$



We can numerically check the (ε expanded) result with FIESTA5 [Smirnov, Shapurov, Vysotsky Comput.Phys.Commun. 277 (2022) 108386] or PySecDec [Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke Comput.Phys.Commun. 222 (2018) 313-326].

Computing the jet function beyond tree level (5)

Mellin-Barnes transform:

$$(A+B)^{-c} = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} d\sigma\, \frac{\Gamma(\sigma+c)\Gamma(-\sigma)}{\Gamma(c)} A^{\sigma} B^{-\sigma-c}$$

In our previous example, it returns

$$\frac{\Gamma(-1+4\varepsilon)\Gamma(1-\varepsilon)}{\Gamma(3\varepsilon)}\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dz\, \frac{\Gamma(-1+3\varepsilon-z)\Gamma(-z)\Gamma(1+z)\Gamma(1-\varepsilon+z)}{(1-2\varepsilon+z)}$$

Computing the jet function beyond tree level (5)

Mellin-Barnes transform:

$$(A+B)^{-c} = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} d\sigma \, \frac{\Gamma(\sigma+c)\Gamma(-\sigma)}{\Gamma(c)} A^{\sigma} B^{-\sigma-c}$$

In our previous example, it returns

$$\frac{\Gamma(-1+4\varepsilon)\Gamma(1-\varepsilon)}{\Gamma(3\varepsilon)} \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dz \, \frac{\Gamma(-1+3\varepsilon-z)\Gamma(-z)\Gamma(1+z)\Gamma(1-\varepsilon+z)}{(1-2\varepsilon+z)}$$

- It can be solved using the residues theorem.
- There are many useful `Mathematica` packages. We used `MB` [Czakon Comput.Phys.Commun. 175 (2006) 559-571] and `MBresolve` [Smirnov, Smirnov Eur.Phys.J.C 62 (2009) 445-449].

Computing the jet function beyond tree level (5)

Quasi-finite integrals are finite for $\varepsilon \to 0$ (or such that the divergence is a global factor). They are especially easy to integrate because we can expand the integrand in ε.

Computing the jet function beyond tree level (5)

Quasi-finite integrals are finite for $\varepsilon \to 0$ (or such that the divergence is a global factor).
They are especially easy to integrate because we can expand the integrand in ε.
The idea is to relate the quasifinite integrals with the original MI [Manteuffel, Panzer, Schabinger JHEP 02 (2015) 120]:

Computing the jet function beyond tree level (5)

Quasi-finite integrals are finite for $\varepsilon \to 0$ (or such that the divergence is a global factor). They are especially easy to integrate because we can expand the integrand in ε.
The idea is to relate the quasifinite integrals with the original MI [Manteuffel, Panzer, Schabinger JHEP 02 (2015) 120]:

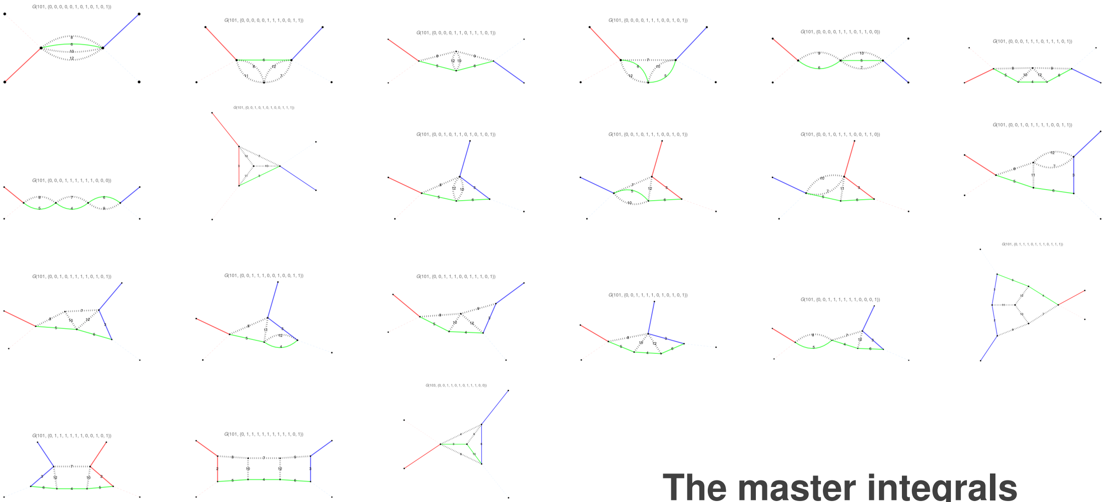
1. Look for quasi-finite integrals (`Reduze`):
   - Decrease the degree of IR divergence by increasing $d \to d + 2$.
   - Decrease the degree of UV divergence by raising the power of the denominators.
2. Apply IBP reductions and dimensional shifting relations to the quasi-finite integral. [Lee Nucl.Phys.B Proc.Suppl. 205-206 (2010) 135-140]
3. We express the original MI in terms of the quasi-finite integral.

Computing the jet function beyond tree level (5)

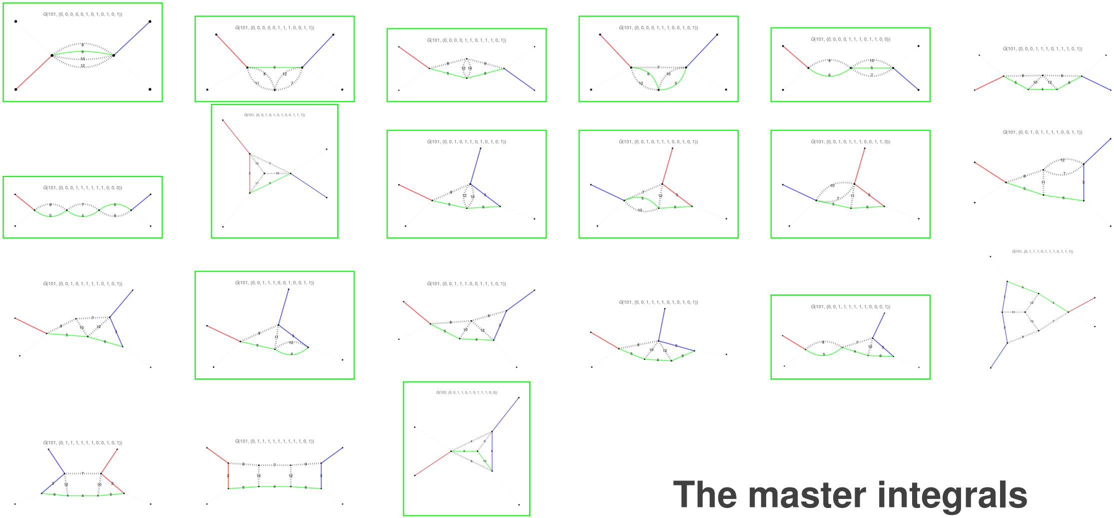
Quasi-finite integrals are finite for $\varepsilon \to 0$ (or such that the divergence is a global factor). They are especially easy to integrate because we can expand the integrand in ε.
The idea is to relate the quasifinite integrals with the original MI [Manteuffel, Panzer, Schabinger JHEP 02 (2015) 120]:

1. Look for quasi-finite integrals (`Reduze`):
   - Decrease the degree of IR divergence by increasing $d \to d+2$.
   - Decrease the degree of UV divergence by raising the power of the denominators.
2. Apply IBP reductions and dimensional shifting relations to the quasi-finite integral. [Lee Nucl.Phys.B Proc.Suppl. 205-206 (2010) 135-140]
3. We express the original MI in terms of the quasi-finite integral.

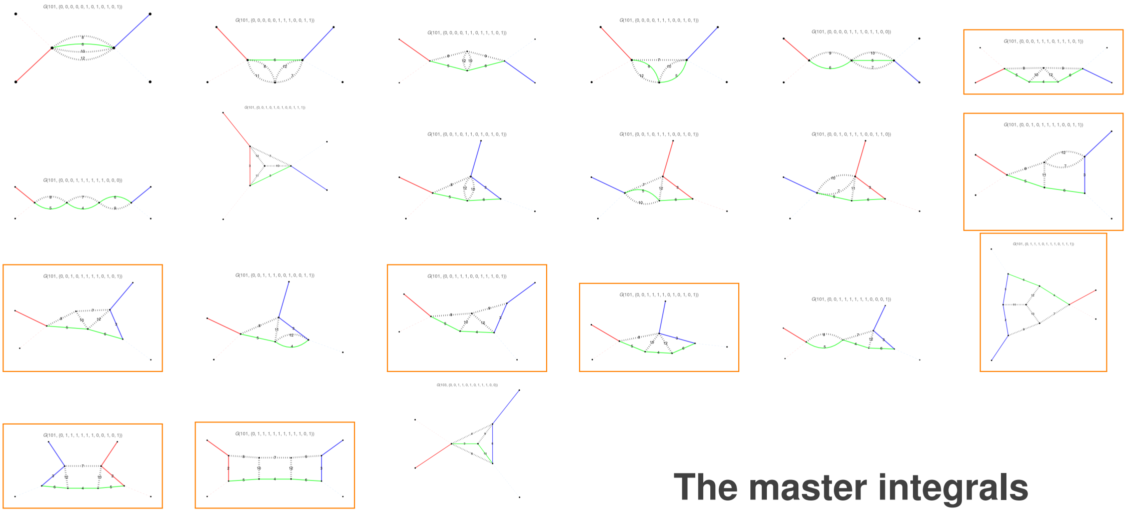
We can use `HyperInt` [Panzer Comput.Phys.Commun. 188 (2015) 148-166] to compute the finite integral.



The master integrals



The master integrals



The master integrals

Computing the jet function beyond tree level (6)

6. **Final steps:**
   - Sum all diagrams.
   - Take the imaginary part and renormalize the jet function

$$B(s,\mu) = \int ds' Z_B^{-1}(s-s',\mu) B^{\text{bare}}(s')$$

Computing the jet function beyond tree level (6)

6. **Final steps:**
   - Sum all diagrams.
   - Take the imaginary part and renormalize the jet function

$$B(s,\mu) = \int ds' Z_B^{-1}(s-s',\mu) B^{\text{bare}}(s')$$

Working in this way, we managed to . . .

- (Trivially) reproduce the 1 loop result.
- Reproduce the 2 loop result [Jain, Scimemi, Stewart Phys.Rev.D 77 (2008) 094008] (at arbitrary order in ε *new!*).
- Obtain the 3 loop result at order ε^1 *new!*.

The 3-loop jet function

The numeric value of our **analytic** result for the renormalized jet function for $n_f = 5$ is

$$
\begin{aligned}
m\tilde{B}(y,\mu) \simeq\, & 1.0000 + a_s\left(5.3333\, L^2 + 5.3333 L + 7.5266\right) \\
& + a_s^2\left(14.222\, L^4 + 55.704\, L^3 + 132.10\, L^2 + 122.65\, L + 183.80\right) \\
& + a_s^3\left(25.284\, L^6 + 221.23\, L^5 + 951.84\, L^4 + 2231.2\, L^3 + 3822.9\, L^2 \right. \\
& \left. + 6531.2\, L + 5827.8\right) + \mathcal{O}(a_s^4)
\end{aligned}
$$

Where

$$
a_s \equiv \frac{\alpha_s}{4\pi} \qquad L \equiv \ln\left(i e^{\gamma_E} y \mu\right)
$$

Testing our result

- The gauge parameter ξ drops out when summing over all diagrams, even before the evaluation of the MI.

Testing our result

- The gauge parameter ξ drops out when summing over all diagrams, even before the evaluation of the MI.
- All MI computed numerically up to order ε^3, and agree with our **analytic** results.

Testing our result

- The gauge parameter ξ drops out when summing over all diagrams, even before the evaluation of the MI.
- All MI computed numerically up to order ε^3, and agree with our **analytic** results.
- The jet function satisfies the non-abelian exponentiation theorem [Gatheral Phys.Lett.B 133 (1983) 90-94], [Frenkel, Taylor Nucl.Phys.B 246 (1984) 231-245] up to order α_s^3.

$$\tilde{B}(y,\mu) = \exp\left(\tilde{B}_{\text{exp}}(y,\mu)\right)$$

Testing our result

- The gauge parameter ξ drops out when summing over all diagrams, even before the evaluation of the MI.
- All MI computed numerically up to order ε^3, and agree with our **analytic** results.
- The jet function satisfies the non-abelian exponentiation theorem [Gatheral Phys.Lett.B 133 (1983) 90-94], [Frenkel, Taylor Nucl.Phys.B 246 (1984) 231-245] up to order α_s^3.

$$\tilde{B}(y,\mu) = \exp\left(\tilde{B}_{\text{exp}}(y,\mu)\right)$$

- Our direct computation of the anomalous dimension at α_s^3 (cusp and non-cusp) agrees with indirect/known computations

$$\frac{d\log B}{d\log\mu} = -\frac{d\log Z_B}{d\log\mu} = \gamma_B$$

Testing our result

- The gauge parameter ξ drops out when summing over all diagrams, even before the evaluation of the MI.
- All MI computed numerically up to order ε^3, and agree with our **analytic** results.
- The jet function satisfies the non-abelian exponentiation theorem [Gatheral Phys.Lett.B 133 (1983) 90-94], [Frenkel, Taylor Nucl.Phys.B 246 (1984) 231-245] up to order α_s^3.

$$\tilde{B}(y,\mu) = \exp\left(\tilde{B}_{\text{exp}}(y,\mu)\right)$$

- Our direct computation of the anomalous dimension at α_s^3 (cusp and non-cusp) agrees with indirect/known computations

$$\frac{d\log B}{d\log\mu} = -\frac{d\log Z_B}{d\log\mu} = \gamma_B$$

- The piece with $\alpha_s^3\beta_0^2$ agrees with [Mateu, Gracia JHEP 07 (2021) 229].

Conclusions

- We reviewed the basics of EFTs, and introduced SCET and bHQET.
- We presented the jet function as a common ingredient in the factorization theorem for several observables in heavy quark production.
  In particular, we discussed how it takes a role in determining the top quark mass in a renormalon-free scheme.
- We outlined some of the techniques required to solve the problems that appear when studying processes at more than one loop.
- **We computed the analytic result for the 3-loop jet function**: results are soon to appear in arXiv.

VNiVERSiDAD D SALAMANCA

Thank you for your attention!

Alberto Martín Clavero
Vicent Mateu Barreda
Maximilian Stahlhofen
Robin Brüser

June 26, 2024