Chiral Density Wave in dense QCD under neutron star conditions

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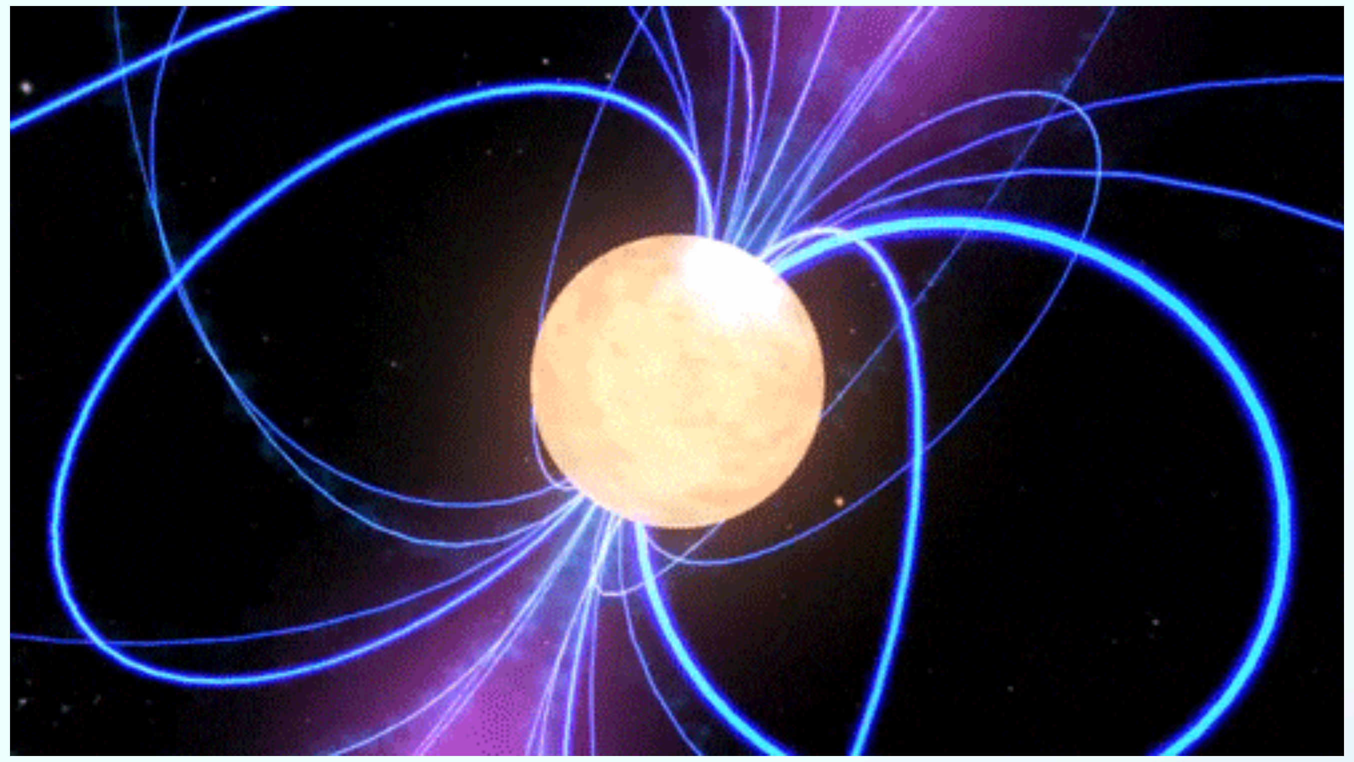
Background: Dense Matter in Neutron Stars



- Neutron stars → dense matter governed by QCD

- What do we expect to find in the interior of neutron stars?

1. Nucleons (protons, neutrons), leptons (electrons, muons), neutrinos

2. Deconfined quarks

- Star conditions: electric charge neutrality and in equilibrium with respect to β-decay.

- Small temperatures → Anisotropic phases?

Model

- QCD hard to solve → revert to phenomenological models

- Nucleon-meson model:

S. Pitsinigkos and A. Schmitt. Phys. Rev. D, 109(1):014024, 2024.

Nucleons
Mesons
Interactions

$$\mathscr{L} = \bar{\psi}(i\gamma^\mu\partial_\mu + \gamma^0\hat{\mu})\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{4}\mathrm{Tr}[\partial_\mu\pi\partial^\mu\pi] - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} - \frac{1}{8}\mathrm{Tr}[\rho_{\mu\nu}\rho^{\mu\nu}] - U(\sigma,\pi)$$

$$+\frac{m_v^2}{2}\left(\omega_\mu\omega^\mu + \frac{1}{2}\mathrm{Tr}[\rho_\mu\rho^\mu]\right) + \frac{d}{4}\left\{\left(\omega_\mu\omega^\mu\right)^2 + \frac{1}{2}\mathrm{Tr}[\rho_\mu\rho^\mu]^2\right\}$$

$$-\bar{\psi}\left[g_\sigma(\sigma + i\gamma^5\pi) + \gamma_\mu\left(g_\omega\omega^\mu + g_\rho\rho^\mu\right)\right]\psi$$

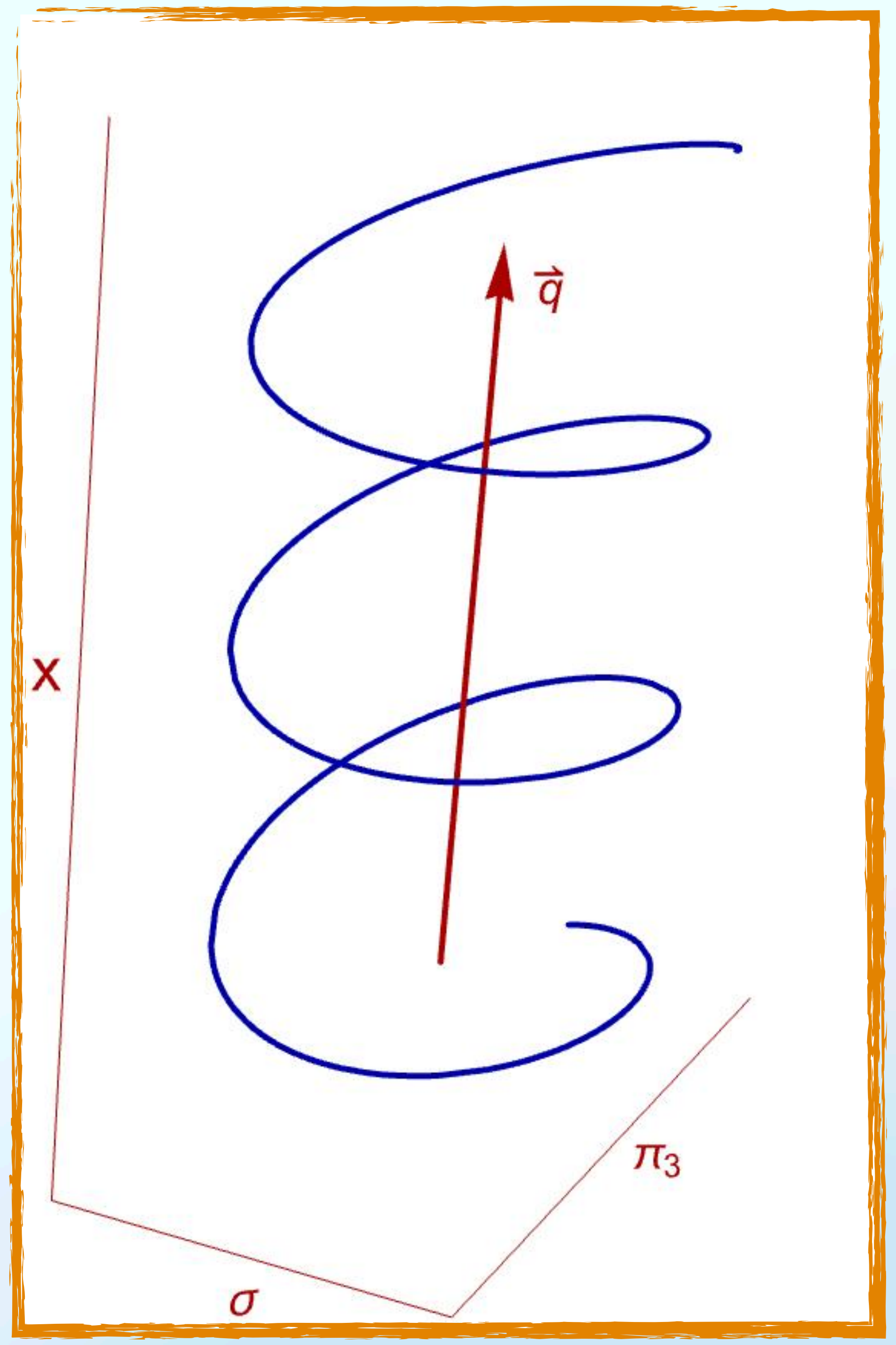
- Dynamical nucleon mass → accounts for both chirally broken and restored phases

- Potential U includes term $\epsilon\sigma$ → chiral symmetry explicitly broken for non-zero pion mass

Ansatz & Free Energy

- Chiral Density Wave Ansatz

$$\sigma = \langle\sigma\rangle\cos(2\vec{q}\cdot\vec{x}),$$
$$\pi_3 = \langle\sigma\rangle\sin(2\vec{q}\cdot\vec{x})$$



- Mean-field approximation
- Free energy $\Omega \equiv T/V \ln Z$
- $T \to 0$
- $\Omega = \Omega_{mes} + \Omega_{nuc} + \Omega_{lep}$

Leptons added here, electrons, muons

$$\Omega_{vac} + \Omega_{mat}$$

- Vacuum part divergent; renormalise and not throw it away

F. Dautry and E. Nyman. Nuclear Physics A, 319(3):323–348, 1979.

Stationarity Equations

- Stationary point of free energy

$$\frac{\partial \Omega}{\partial \sigma} = 0, \ \frac{\partial \Omega}{\partial \omega} = 0, \ \frac{\partial \Omega}{\partial \rho} = 0, \ \frac{\partial \Omega}{\partial q} = 0$$

- Neutron Star Conditions:

A. Schmitt. Physical Review D, 101(7), April 2020.

$$n \rightarrow p + l + \bar{\nu}_l$$

$$p + l \rightarrow n + \nu_l$$

1. Charge neutrality → $n_p = n_e + n_\mu$
2. β-equilibrium → $\mu_n = \mu_p + \mu_e$, $\mu_e = \mu_\mu$

Parameter Fit

- What we have to fit:

$$a_1, a_2, a_3, a_4, d, \epsilon$$

Parameters in potential U

$$g_\rho, g_\sigma, g_\omega$$

Yukawa coupling constants

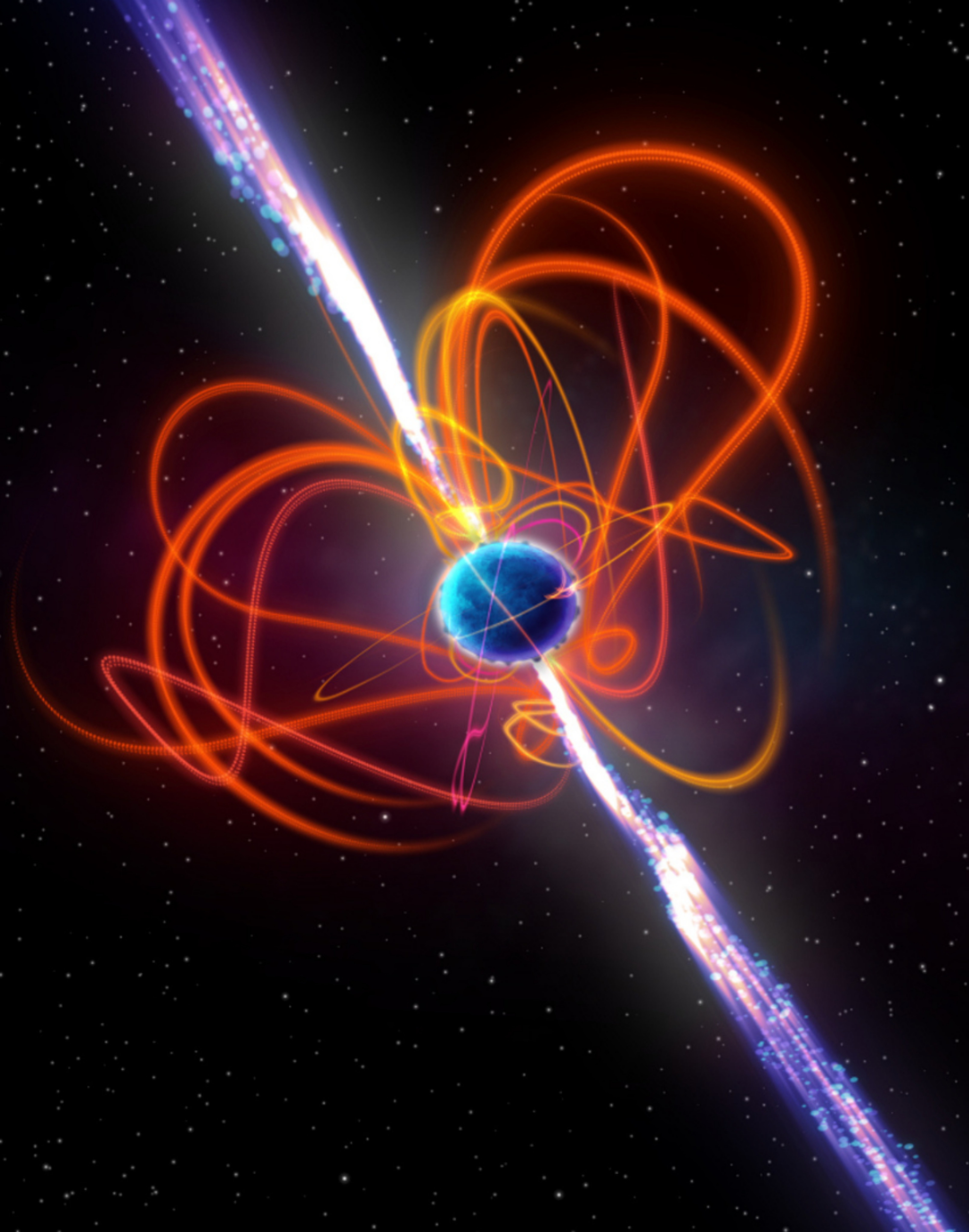
- Use properties at saturation density ($n_0 \equiv 0.153 \text{ fm}^{-3}$)

1. Zero pressure
2. Binding energy $E_B = -16.3$ MeV
3. Nucleon mass $M_0 = 0.7\ m_N - 0.8\ m_N$

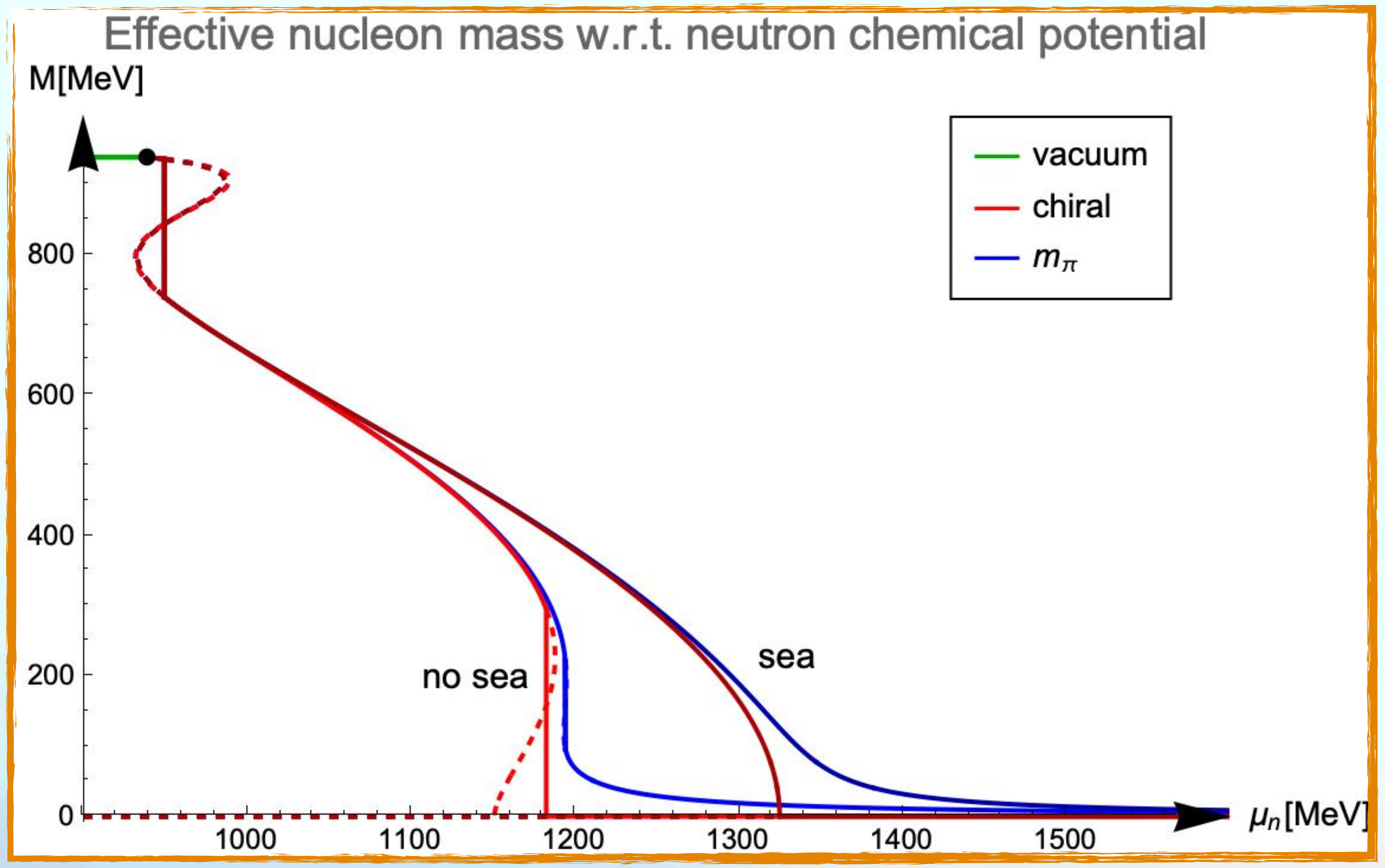
   Vacuum nucleon mass $m_N = 939$ MeV

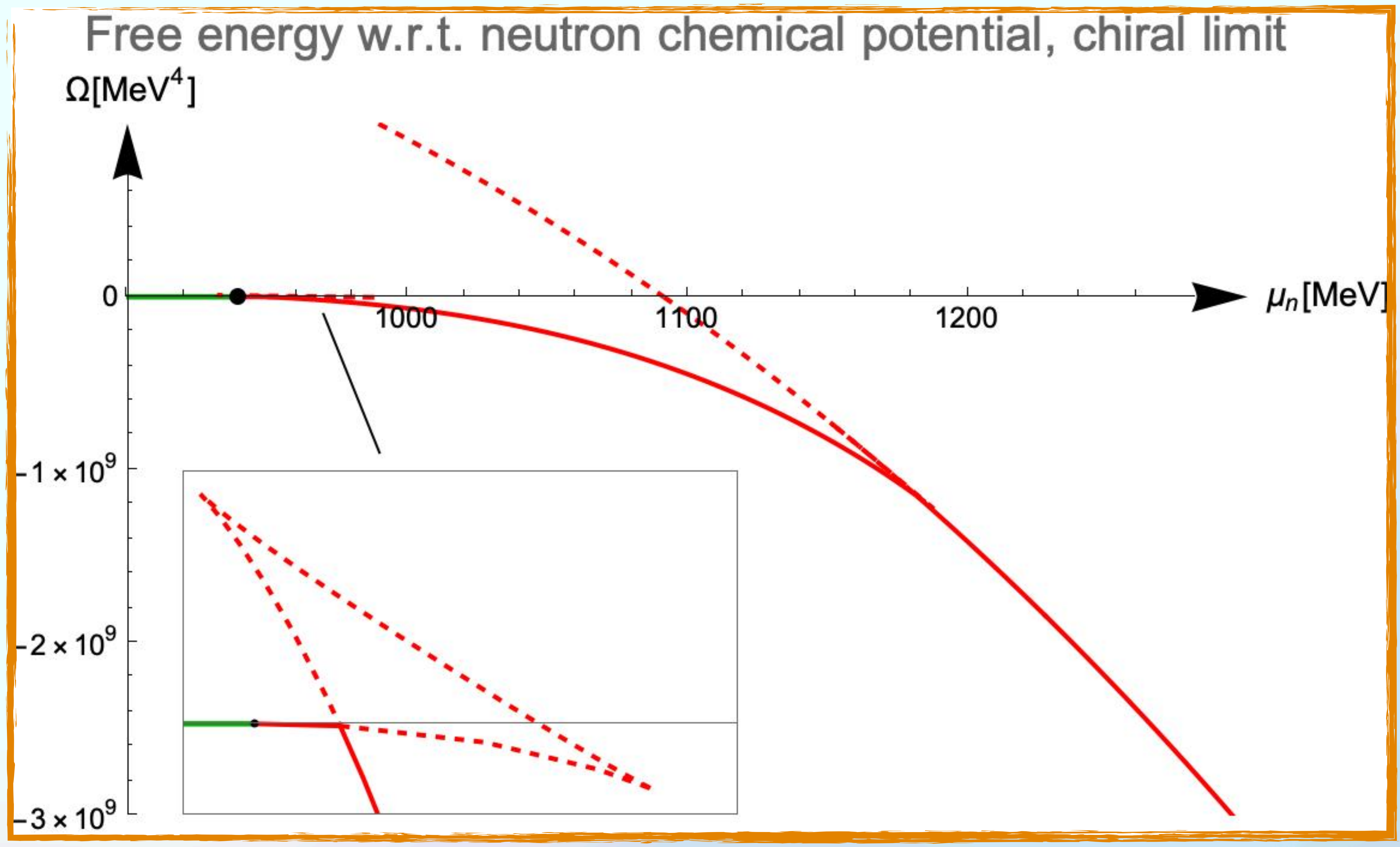
4. Incompressibility $K \equiv k_F^2 \dfrac{\partial^2 (E/n_B)}{\partial k_F^2} \approx 200 - 300$ MeV

Results



Effect of the Dirac Sea



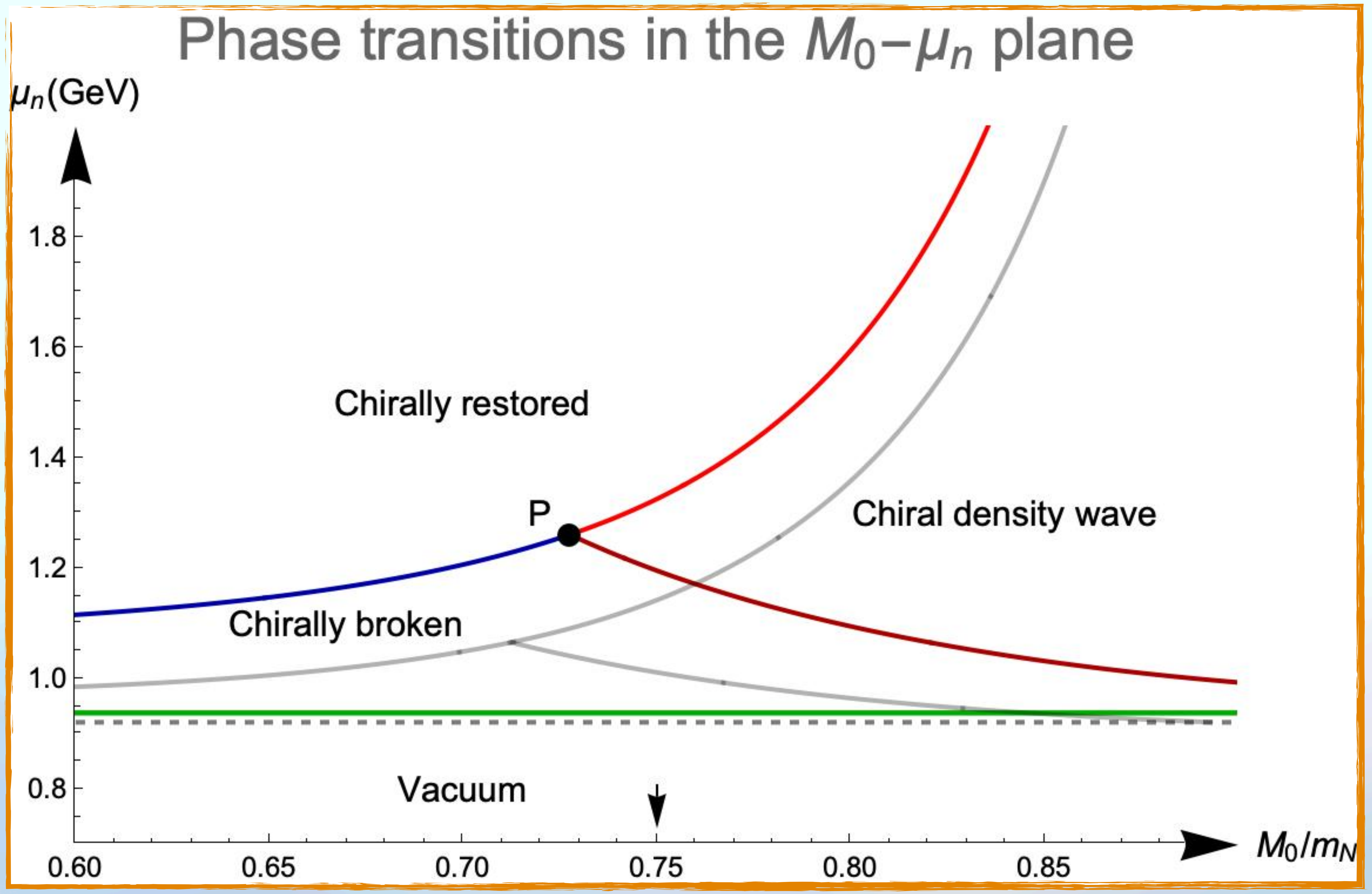


- 1st. order ph. transition → 2nd. order ph. transition
- 1st. order ph. transition → Crossover

- Minimum free energy → phase transitions

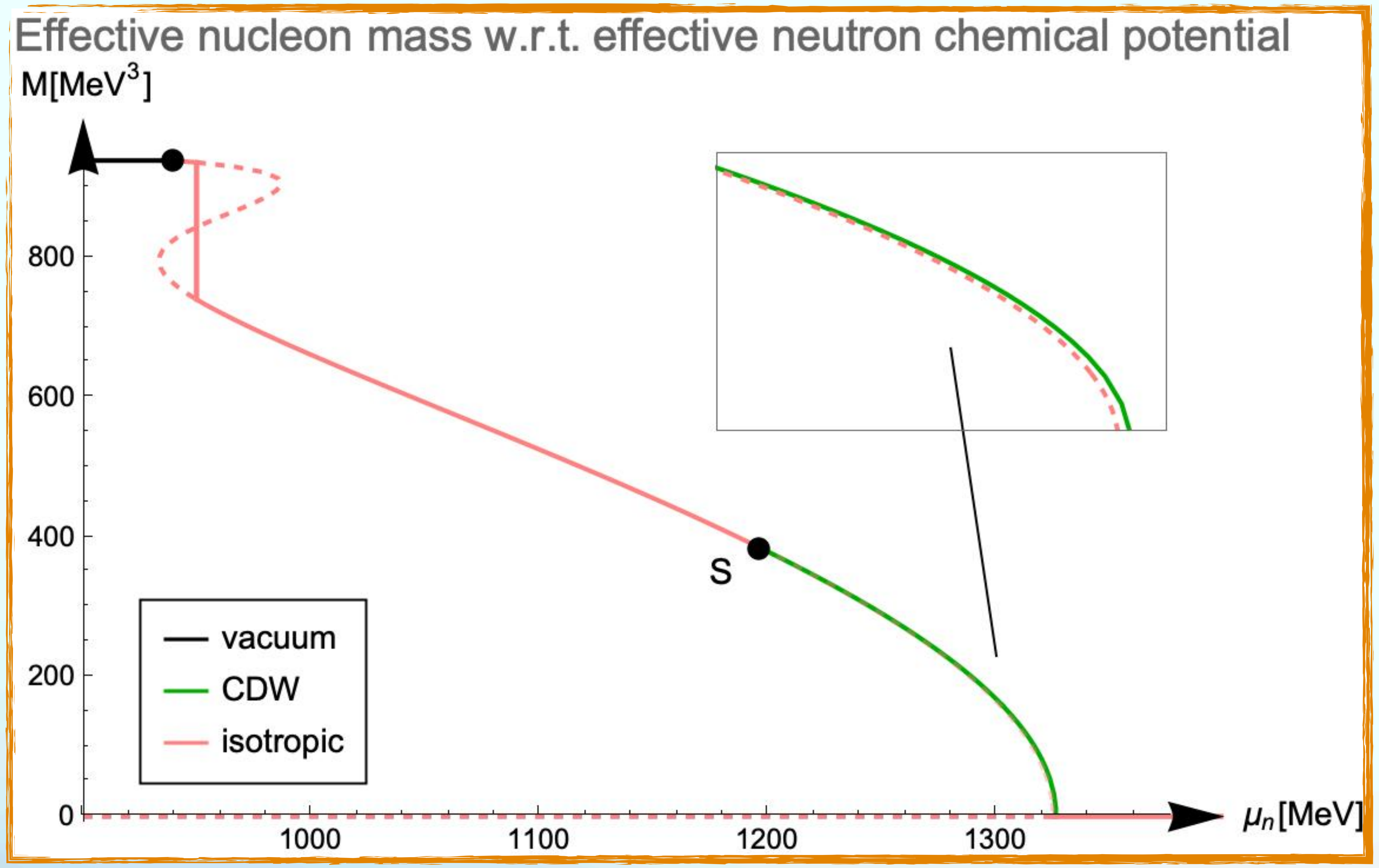
Phase Diagram including CDW

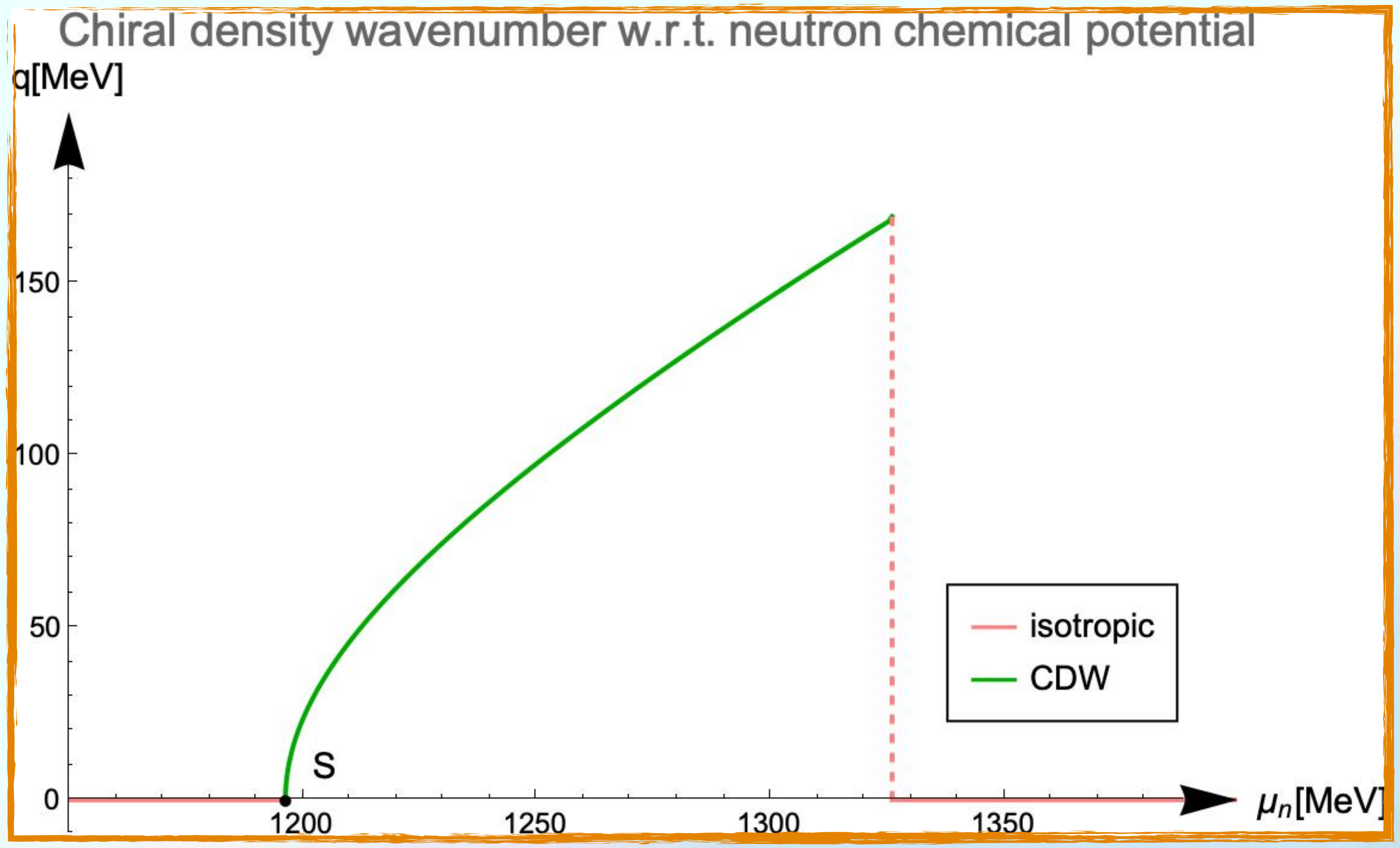
$m_\pi = 0$



- Grey → Isospin symmetric
- No chiral density wave solution left of P
- 2nd order ph. transitions
- Neutron star conditions → preference of CDW at higher neutron chemical potentials
- For $M_0 = 0.75 m_N$ → ...

Effective mass including CDW phase





- 2nd order transition for start of CDW solution
- Back to isotropic (chirally symmetric) after CDW's end.
- Large amplitude, large wavelength → small amplitude, small wavelength

Next Steps: Neutron Stars

- Chiral density wave with physical pion mass
- Isopsin-symmetric matter → density profile
- Obtain equation of state (soft?) → Use Tolman-Oppenheimer-Volkov equation
- Mass-Radius curves
- Large enough maximum mass ($2M_\odot$)?

Next Steps

- Explore parameter space
- Other models?
- Holographic Approach?

N. Kovensky, A. Poole, and A. Schmitt. SciPost Phys. Proc., 6:019, 2022.

T. Sakai and S. Sugimoto. Progress of Theoretical Physics, 113(4):843–882, April 2005.

Thank you and nice to meet you all!