Forward inclusive hadron/jet productions in pA collisions



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- Introduction
- Precision study for forward hadron productions
- Forward single inclusive jet productions
- summary & outlook

Outline





$$Q^{2} \equiv -q^{2},$$

$$x_{Bj} \equiv \frac{Q^{2}}{2P \cdot q},$$

$$y \equiv \frac{P \cdot q}{P \cdot p}.$$

$$x_{Bj} = \frac{Q^2}{\hat{s} + Q^2 - m^2} = \frac{Q^2}{2m\nu},$$

$$Q^2 = y x_{Bj} (s - m^2 - m_e^2) \approx y x_{Bj} s.$$

$$\nu \equiv \frac{P \cdot q}{m} = E - E',$$

High energy=small x



Infinite momentum frame or Bjorken frame



Three gluon vertex



Non-linear dynamics (BK/JIMWLK)





The proton wave function is determined by x and Q





High number of gluons populate the transverse extend of the proton or nucleus, leading to a very dense saturated wave function. Color Glass Condensate (CGC) is the effective theory to study this kind of dense gluon system.





Partons in the low-x region is dominated by gluons Collinear evolution: **BFKL** Low Q^2 and low x region \Rightarrow Non-linear evolution: **BK/JIMWLK** Saturation momentum: $Q_s(Y)$

 $Q_s^2 \sim A^{1/3} x^{-\lambda}$. For EIC, $A \simeq 200$, $\lambda \simeq 0.2$, $Q_s^2 \simeq 1 \sim 2 Gev^2$



Wilson Line
$$U(x_{\perp}) = P \exp \left\{ -ig_S \int_{-\infty}^{+\infty} dx^+ A_a^-(x^+, x_{\perp}) t \right\}$$

$$W^{ab}(x_{\perp}) = 2 \operatorname{Tr}[t^{a} U(x_{\perp}) t^{b} U^{\dagger}(x_{\perp})]$$

The color dipole in McLerran-Venugopalan model

$$S(x_{\perp} - y_{\perp}) = \frac{1}{N_c} \operatorname{Tr} \langle U(x_{\perp}) U^{\dagger}(y_{\perp}) \rangle = e^{-\frac{Q_s^2(x_{\perp} - y_{\perp})^2}{4}}$$
$$\mathcal{F}(k_{\perp}) \equiv \frac{dN}{d^2 k_{\perp}} = \int \frac{d^2 x_{\perp} d^2 y_{\perp}}{(2\pi)^2} e^{-ik_{\perp} \cdot (x_{\perp} - y_{\perp})} \frac{1}{N_c} \left\langle \operatorname{Tr} U(x_{\perp}) U^{\dagger}(y_{\perp}) \right\rangle$$



- We use Wilson line to represent the multiple scattering between the fast moving parton and target background fields!
- Fundamental representation-multiple scattering between fast moving quark and target dense gluons









Single inclusive hadron productions in pA collisions



LO calculation





Searching for Parton saturation in dilute-dense scatterings: ep & eA & pA.

$$\frac{d\sigma_{\text{LO}}^{pA \to hX}}{d^2 p_{\perp} dy_h} = \int_{\tau}^{1} \frac{dz}{z^2} \sum_{f} x_p q_f(x_p) F(k_{\perp}) D_{h/q}(z)$$

$$F(k_{\perp}) = \int \frac{d^2 x_{\perp} d^2 y_{\perp}}{(2\pi)^2} e^{-ik_{\perp} \cdot (x_{\perp} - y_{\perp})} S_Y^{(2)}(x_{\perp}, y_{\perp})$$

Dumitru and Jalilian-Marian, PRL 2002







Albacete and Marquet, PLB 2010

Higher order calculations are needed!

Incuding higher order corrections may give more accurate answers. • Going to higher orders can open new channels and can sometimes result in large corrections. • Going to higher orders cancels some but not all of factorization scale dependence.



They both used the K factor!





NLO diagrams in the $q \rightarrow q$ channel

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NLO Calculation and Factorization

Probing Saturation Physics in pА Collisions

Bo-Wen Xiao 肖博文

N Introduction

> Forward Hadron Productions in *pA* Collisions

Sudakov Factor

Summary

• Factorization is about separation of short distant physics (perturbatively calculable hard factor) from large distant physics (Non perturbative).

 $\sigma \sim xf(x) \otimes \mathcal{H} \otimes D_h(z) \otimes \mathcal{F}(k_{\perp})$

- NLO (1-loop) calculation always contains various kinds of divergences.
 - Some divergences can be absorbed into the corresponding evolution equations.
 - The rest of divergences should be cancelled.

Hard factor

 $\mathcal{H} = \mathcal{H}_{\rm LO}^{(0)} + \frac{\alpha_s}{2\pi} \mathcal{H}_{\rm NLO}^{(1)} + \cdots$

should always be finite and free of divergence of any kind.

NLO vs NLL Naive α_s expansion sometimes is not sufficient!

IO



Lost of contributions for NLO

u, Hayashigakia and Jalilian-Marianb, Altinoluk and Kovner, PRD, 2011 Chirilli, Xiao and Yuan, PRL, 2011 Chirilli, Xiao and Yuan, PRD, 2012

Kang, Vitev, Xing, PRL, 2014

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NLO NNLO
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Rapidity Divergence

Collinear to the target nucleus \Rightarrow BK evolution equation! 2. Collinear to the initial state quark \Rightarrow DGLAP evolution of PDFs! 3. Collinear to the final state quark \Rightarrow DGLAP evolution of FFs!



For the $q \rightarrow q$ channel, the factorization formula can be written as

$$\frac{d^{3}\sigma^{p+A\to h+X}}{dyd^{2}p_{\perp}} = \int \frac{dz}{z^{2}} \frac{dx}{x} \xi xq(x,\mu) D_{h/q}(z,\mu) \int \frac{d^{2}x_{\perp}d^{2}y_{\perp}}{(2\pi)^{2}} \left\{ S_{Y}^{(2)}(x_{\perp},y_{\perp}) \left[\mathcal{H}_{2qq}^{(0)} + \frac{\alpha_{s}}{2\pi} \mathcal{H}_{2qq}^{(1)} \right] + \int \frac{d^{2}b_{\perp}}{(2\pi)^{2}} S_{Y}^{(4)}(x_{\perp},b_{\perp},y_{\perp}) \frac{\alpha_{s}}{2\pi} \mathcal{H}_{4qq}^{(1)} \right\}$$

with $\mathcal{H}_{2aa}^{(0)} = e^{-ik} \perp \cdot r \perp \delta(1-\xi)$ and

$$\begin{aligned} \mathcal{H}_{2qq}^{(1)} &= C_F \mathcal{P}_{qq}(\xi) \ln \frac{c_0^2}{r_{\perp}^2 \mu^2} \left(e^{-ik_{\perp} \cdot r_{\perp}} + \frac{1}{\xi^2} e^{-i\frac{k_{\perp}}{\xi} \cdot r_{\perp}} \right) - 3C_F \delta(1-\xi) e^{-ik_{\perp} \cdot r_{\perp}} \ln \frac{c_0^2}{r_{\perp}^2 k_{\perp}^2} \\ &- (2C_F - N_c) e^{-ik_{\perp} \cdot r_{\perp}} \left[\frac{1+\xi^2}{(1-\xi)_{+}} \tilde{I}_{21} - \left(\frac{(1+\xi^2) \ln (1-\xi)^2}{1-\xi} \right)_{+} \right] \\ \mathcal{H}_{4qq}^{(1)} &= -4\pi N_c e^{-ik_{\perp} \cdot r_{\perp}} \left\{ e^{-i\frac{1-\xi}{\xi} k_{\perp} \cdot (x_{\perp} - b_{\perp})} \frac{1+\xi^2}{(1-\xi)_{+}} \frac{1}{\xi} \frac{x_{\perp} - b_{\perp}}{(x_{\perp} - b_{\perp})^2} \cdot \frac{y_{\perp} - b_{\perp}}{(y_{\perp} - b_{\perp})^2} \right. \\ &- \delta(1-\xi) \int_0^1 d\xi' \frac{1+\xi'^2}{(1-\xi')_{+}} \left[\frac{e^{-i(1-\xi')k_{\perp} \cdot (y_{\perp} - b_{\perp})}}{(b_{\perp} - y_{\perp})^2} - \delta^{(2)}(b_{\perp} - y_{\perp}) \int d^2r'_{\perp} \frac{e^{ik_{\perp} \cdot r'_{\perp}}}{r'_{\perp}^2} \right] \right\}, \end{aligned}$$
where
$$\tilde{I}_{21} = \int \frac{d^2b_{\perp}}{\pi} \left\{ e^{-i(1-\xi)k_{\perp} \cdot b_{\perp}} \left[\frac{b_{\perp} \cdot (\xi b_{\perp} - r_{\perp})}{b_{\perp}^2 (\xi b_{\perp} - r_{\perp})^2} - \frac{1}{b_{\perp}^2} \right] + e^{-ik_{\perp} \cdot b_{\perp}} \frac{1}{b_{\perp}^2} \right\}$$

$$\begin{aligned} & \mathcal{P}_{qq}(\xi) \ln \frac{c_0^2}{r_{\perp}^2 \mu^2} \left(e^{-ik_{\perp} \cdot r_{\perp}} + \frac{1}{\xi^2} e^{-i\frac{k_{\perp}}{\xi} \cdot r_{\perp}} \right) - 3C_F \delta(1-\xi) e^{-ik_{\perp} \cdot r_{\perp}} \ln \frac{c_0^2}{r_{\perp}^2 k_{\perp}^2} \\ & (2C_F - N_c) e^{-ik_{\perp} \cdot r_{\perp}} \left[\frac{1+\xi^2}{(1-\xi)_{+}} \tilde{p}_{21} - \left(\frac{(1+\xi^2) \ln (1-\xi)^2}{1-\xi} \right)_{+} \right] \\ & 4\pi N_c e^{-ik_{\perp} \cdot r_{\perp}} \left\{ e^{-i\frac{1-\xi}{\xi} k_{\perp} \cdot (x_{\perp} - b_{\perp})} \frac{1+\xi^2}{(1-\xi)_{+}} \frac{1}{\xi} \frac{x_{\perp} - b_{\perp}}{(x_{\perp} - b_{\perp})^2} \cdot \frac{y_{\perp} - b_{\perp}}{(y_{\perp} - b_{\perp})^2} \right. \\ & -\delta(1-\xi) \int_0^1 d\xi' \frac{1+\xi'^2}{(1-\xi')_{+}} \left[\frac{e^{-i(1-\xi')k_{\perp} \cdot (y_{\perp} - b_{\perp})}}{(b_{\perp} - y_{\perp})^2} - \delta^{(2)} (b_{\perp} - y_{\perp}) \int d^2r'_{\perp} \frac{e^{ik_{\perp} \cdot r'_{\perp}}}{r'_{\perp}^2} \right] \right\} . \end{aligned}$$

$$\mathcal{H}_{2qq}^{(1)} = C_F \mathcal{P}_{qq}(\xi) \ln \frac{c_0^2}{r_{\perp}^2 \mu^2} \left(e^{-ik_{\perp} \cdot r_{\perp}} + \frac{1}{\xi^2} e^{-i\frac{k_{\perp}}{\xi} \cdot r_{\perp}} \right) - 3C_F \delta(1-\xi) e^{-ik_{\perp} \cdot r_{\perp}} \ln \frac{c_0^2}{r_{\perp}^2 k_{\perp}^2} \\ - (2C_F - N_c) e^{-ik_{\perp} \cdot r_{\perp}} \left[\frac{1+\xi^2}{(1-\xi)_{+}} \tilde{I}_{21} - \left(\frac{(1+\xi^2) \ln (1-\xi)^2}{1-\xi} \right)_{+} \right] \\ \mathcal{H}_{4qq}^{(1)} = -4\pi N_c e^{-ik_{\perp} \cdot r_{\perp}} \left\{ e^{-i\frac{1-\xi}{\xi} k_{\perp} \cdot (x_{\perp} - b_{\perp})} \frac{1+\xi^2}{(1-\xi)_{+}} \frac{1}{\xi} \frac{x_{\perp} - b_{\perp}}{(x_{\perp} - b_{\perp})^2} \cdot \frac{y_{\perp} - b_{\perp}}{(y_{\perp} - b_{\perp})^2} \right. \\ \left. -\delta(1-\xi) \int_0^1 d\xi' \frac{1+\xi'^2}{(1-\xi')_{+}} \left[\frac{e^{-i(1-\xi')k_{\perp} \cdot (y_{\perp} - b_{\perp})}}{(b_{\perp} - y_{\perp})^2} - \delta^{(2)}(b_{\perp} - y_{\perp}) \int d^2r'_{\perp} \frac{e^{ik_{\perp} \cdot r'_{\perp}}}{r'_{\perp}^2} \right] \right\},$$
where
$$\tilde{I}_{21} = \int \frac{d^2b_{\perp}}{\pi} \left\{ e^{-i(1-\xi)k_{\perp} \cdot b_{\perp}} \left[\frac{b_{\perp} \cdot (\xi b_{\perp} - r_{\perp})}{b_{\perp}^2 (\xi b_{\perp} - r_{\perp})^2} - \frac{1}{b_{\perp}^2} \right] + e^{-ik_{\perp} \cdot b_{\perp}} \frac{1}{b_{\perp}^2} \right\}$$

$$\mathcal{P}_{qq}(\xi) \ln \frac{c_{0}^{2}}{r_{\perp}^{2} \mu^{2}} \left(e^{-ik_{\perp} \cdot r_{\perp}} + \frac{1}{\xi^{2}} e^{-i\frac{k_{\perp}}{\xi} \cdot r_{\perp}} \right) - 3C_{F}\delta(1-\xi)e^{-ik_{\perp} \cdot r_{\perp}} \ln \frac{c_{0}^{2}}{r_{\perp}^{2}k_{\perp}^{2}}$$

$$(2C_{F} - N_{c}) e^{-ik_{\perp} \cdot r_{\perp}} \left[\frac{1+\xi^{2}}{(1-\xi)_{+}} \tilde{I}_{21} - \left(\frac{(1+\xi^{2})\ln(1-\xi)^{2}}{1-\xi} \right)_{+} \right]$$

$$\pi N_{c}e^{-ik_{\perp} \cdot r_{\perp}} \left\{ e^{-i\frac{1-\xi}{\xi}k_{\perp} \cdot (x_{\perp} - b_{\perp})} \frac{1+\xi^{2}}{(1-\xi)_{+}} \frac{1}{\xi} \frac{x_{\perp} - b_{\perp}}{(x_{\perp} - b_{\perp})^{2}} \cdot \frac{y_{\perp} - b_{\perp}}{(y_{\perp} - b_{\perp})^{2}} \right.$$

$$-\delta(1-\xi) \int_{0}^{1} d\xi' \frac{1+\xi'^{2}}{(1-\xi')_{+}} \left[\frac{e^{-i(1-\xi')k_{\perp} \cdot (y_{\perp} - b_{\perp})}}{(b_{\perp} - y_{\perp})^{2}} - \delta^{(2)}(b_{\perp} - y_{\perp}) \int d^{2}r_{\perp}' \frac{e^{ik_{\perp} \cdot r_{\perp}'}}{r_{\perp}'^{2}} \right] \right\},$$

$$p_{1} = \int \frac{d^{2}b_{\perp}}{\pi} \left\{ e^{-i(1-\xi)k_{\perp} \cdot b_{\perp}} \left[\frac{b_{\perp} \cdot (\xi b_{\perp} - r_{\perp})}{b_{\perp}^{2} (\xi b_{\perp} - r_{\perp})^{2}} - \frac{1}{b_{\perp}^{2}} \right] + e^{-ik_{\perp} \cdot b_{\perp}} \frac{1}{b_{\perp}^{2}} \right\}$$

$$\tilde{H}_{2qq}^{(1)} = C_F \mathcal{P}_{qq}(\xi) \ln \frac{c_0^2}{r_{\perp}^2 \mu^2} \left(e^{-ik_{\perp} \cdot r_{\perp}} + \frac{1}{\xi^2} e^{-i\frac{k_{\perp}}{\xi} \cdot r_{\perp}} \right) - 3C_F \delta(1-\xi) e^{-ik_{\perp} \cdot r_{\perp}} \ln \frac{c_0^2}{r_{\perp}^2 k_{\perp}^2} \\ - (2C_F - N_c) e^{-ik_{\perp} \cdot r_{\perp}} \left[\frac{1+\xi^2}{(1-\xi)_{+}} \tilde{I}_{21} - \left(\frac{(1+\xi^2) \ln (1-\xi)^2}{1-\xi} \right)_{+} \right] \\ \tilde{H}_{4qq}^{(1)} = -4\pi N_c e^{-ik_{\perp} \cdot r_{\perp}} \left\{ e^{-i\frac{1-\xi}{\xi} k_{\perp} \cdot (x_{\perp} - b_{\perp})} \frac{1+\xi^2}{(1-\xi)_{+}} \frac{1}{\xi} \frac{x_{\perp} - b_{\perp}}{(x_{\perp} - b_{\perp})^2} \cdot \frac{y_{\perp} - b_{\perp}}{(y_{\perp} - b_{\perp})^2} \\ -\delta(1-\xi) \int_0^1 d\xi' \frac{1+\xi'^2}{(1-\xi')_{+}} \left[\frac{e^{-i(1-\xi')k_{\perp} \cdot (y_{\perp} - b_{\perp})}}{(b_{\perp} - y_{\perp})^2} - \delta^{(2)} (b_{\perp} - y_{\perp}) \int d^2r'_{\perp} \frac{e^{ik_{\perp} \cdot r'_{\perp}}}{r'_{\perp}^2} \right] \right\} .$$
where
$$\tilde{I}_{21} = \int \frac{d^2b_{\perp}}{\pi} \left\{ e^{-i(1-\xi)k_{\perp} \cdot b_{\perp}} \left[\frac{b_{\perp} \cdot (\xi b_{\perp} - r_{\perp})}{b_{\perp}^2 (\xi b_{\perp} - r_{\perp})^2} - \frac{1}{b_{\perp}^2} \right] + e^{-ik_{\perp} \cdot b_{\perp}} \frac{1}{b_{\perp}^2} \right\}$$

Need to go to momentum space for numerical evaluations!



$q \rightarrow qg$ $g \rightarrow gg$ $g \rightarrow q\bar{q}$ $q \rightarrow gq$



Numerical calculations for NLO from SOLO (Saturation physics at One Loop Order)

BRAHMS $\eta = 2.2, 3.2$



Stasto, Xiao and Zaslavsky, PRL, 2013



STAR $\eta = 4$





and Ladue to the kinematical constraint. The error and is obtained to the constraint of the constraint



Watanabe, Xiao, Yuan and Zaslavsky, PRD, 2015

Altinoluk, et al, PRD, 2015

Iancu, et al, JHEP, 2016

Ducloué, Lappi, Zhu, PRD, 2016, 2017

Ducloué, et al, PRD, 2018

Xiao, Yuan, PLB, 2019 Liu, Ma, Chao, PRD, 2019 Liu, Kang, Liu, PRD, 2020 Liu, Liu, Shi, Zheng, Zhou, 2022

factorization scheme; kinematic constraint; running coupling effect; Threshold resummation

. . .



The origin of the negativity Where is the negativity from?



$$H_{ab} \sim P_{qq}(\xi) = \frac{1+\xi^2}{(1-\xi)_+} + \frac{3}{2}\delta(1-\xi)$$

$$P_{gg}(\xi) = 2\left[\frac{\xi}{(1-\xi)_{+}} + \frac{1-\xi}{\xi} + \xi(1-\xi)\right] + \left(\frac{11}{6} - \frac{2N_{f}T_{R}}{3N_{C}}\right)\delta(1-\xi)$$





$$\int_{\tau}^{1} d\xi \frac{1}{(1-\xi)_{+}} f(\xi) = \int_{\tau}^{1} d\xi \frac{f(\xi) - f(1)}{1-\xi} + f(1)\ln(1-\tau)$$



The origin of the negativity The plots of τ as a function of rapidity and transverse momentum



- The upper-most solid line z = 1.
- The lower-most solid line is corresponding to the boundary given by $x_{\rho} = 0.01$. • The region in the red and yellow indicates where the threshold resummation
- becomes important.





We then combine the second term in Eq. (16), which is p It is impossible to the numerical calc We the help combined to be condeded to the second term in the second term is the second term in the second term is the second term in the second term in the second term in Eq. (16), which is the second term in Eq. (16), which is possible to the second to $-\ln \frac{d\varphi}{d\mathcal{P}^k \mathcal{S}}$, and $\int b_{tain} \frac{d^2 r}{dt} \frac{d^2 r}{dt$ $3\frac{\alpha_s}{2\pi} S_{\perp} C_F \int_{\tau}^{1} \frac{\mathrm{d}z}{z^2 \mu^2} (x_p, \mu^2) B_h \\ P(\xi) \otimes \ln \frac{\pi^2}{2} \int_{\tau}^{\pi} \frac{z^2 \mu^2}{2} (x_p, \mu^2) B_h \\ \sigma_0$ The Eq. The Fourier transform of this term is the straightf Eq. (16)), the derivation is a bit more involved. We $\int \frac{\mathrm{d}^{2}r_{\perp}}{(2\pi)^{2}} e^{-ik_{\perp}\cdot r_{\perp}} S^{(2)}(r_{\perp}) \lim_{\substack{\to 10^{02} \\ \to 7^{2}_{\perp} \mu^{2} \\ \to 10^{-4}}} =$ 2. cross section in the momentation 10^{-8} $\eta = 6$ $P(\xi) \otimes \left[\ln \frac{\mu^2}{\Lambda^2} + I_1(\Lambda)\right] \qquad 10^{-10} \underbrace{\lim_{\eta \to 0^+} \eta = 0}_{10^{-10} \text{ pow-fit}}$ $P(\xi) \otimes \left[\ln \frac{\mu^2}{\Lambda^2} + I_1(\Lambda)\right] \qquad \sigma_0 \otimes \left[\ln \frac{\mu^2}{\Lambda^2} + I_1(\Lambda)\right]$

$$\frac{r}{r_{\perp}} \frac{\mu}{\rho} \frac{D_{h/q}(z,\mu')}{(z,\mu')} \frac{1}{(1-\xi)_{\perp}} \int \frac{1}{(2\pi)^{2}} \frac{1}{r_{\perp}^{2}\mu^{2}} \frac{1}{r_{\perp}^{2}\mu^{2}} \frac{1}{r_{\perp}^{2}\mu^{2}} \frac{S^{(2)}(r_{\perp})}{(16)} \frac{1}{r_{\perp}^{2}\mu^{2}} \int \frac{1}{(16)} \frac{1}{r_{\perp}^{2}\mu^{2}} \frac{1}{r_{\perp}^{2}\mu^{2$$



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$$\frac{dP.S.}{2} \int \frac{(2\pi)^{2}}{2\pi} S_{1}C_{k}} \int \frac{\frac{\pi}{2}}{2\pi} \frac{\pi}{2} g(q_{p},\mu^{2})}{\sigma_{0} \otimes \ln \frac{k_{2}}{2}} \int \frac{(2\pi)^{2}}{\sigma_{0} \otimes \ln^{2} \frac{k_{1}}{2}} \int \frac{(2\pi)^{2}}{\mu_{r}} \int \frac{(2\pi)^{2}}{\mu_{r}} \int \frac{(2\pi)^{2}}{\mu_{r}} \int \frac{(2\pi)^{2}}{\mu_{r}} \int \frac{(2\pi)^{2}}{\mu_{r}^{2}} \int \frac{(2\pi)^{2}}{\mu_{r}} \int \frac{(2\pi)^{2}}{\mu_{r}^{2}} \int \frac{(2\pi)^{2}}{\mu_{r}$$











gluon channel anomalous dimension reads $f(N) = \int dx x^{n-1} f(x)$, Threshold resum 24 12 24 0 m), ere \mathcal{C} stands for the proper contour which puts all the poles to its left. Collowing the same strategy developed in the last subsection, we resum the collinear logarithms associated with Fs and FFs seperately. For the first deform of dother was the taget font time Mellin transform as follows s to the jermination of the second the second of the second second of the second of t $\operatorname{ere} q(N) \equiv \int_{0}^{1} \mathrm{d}xx \begin{bmatrix} q(x_{p}, \mu) \\ g(x) \\ \varphi(x) \\ \varphi($ 2. Reported ind go in the contraction of the contr $\begin{array}{l} \Delta \left(\Lambda, \mu, \omega \right) = & \Gamma\left(\gamma_{A}^{g} \right) \\ \Lambda \left(D_{h/q}(N) \right) \equiv & \int_{0}^{1} \mathrm{d}z z^{N-1} D_{h} \left(z \right) \cdot \mathrm{d}Y \\ P_{q}(N) = & \left[\left(z \right) \cdot \mathrm{d}Y \\ \mathrm{d}\xi \xi^{N-1} P_{d}(\xi) = -2 \\ \overline{\gamma_{E}} = 2 \\ \overline{\gamma_{E} = 2 \\ \overline{\gamma_{E}} = 2 \\ \overline{\gamma_{E}} = 2 \\ \overline{\gamma_{$ tion can then be written as $q(N) \exp \left(\ln \frac{\mu^2}{2} C_F \ln \frac{\mu}{2} \right) (\mathcal{T}_E - \frac{\mu}{4} + \ln N)$ ere $\psi(N) = \ln N + \mathcal{O}(\frac{1}{N})$ is the polygamma function. We have taken the large-N limit in the last step. Due to the scale dependence in the anomalous dimensions, the flow directions of the reportation group equation³ n the threshold limit th exponential 1 83 - 90 - 17The resummed quark Philes and FFs in the Mellin Space can be pressed in the larger N²



(98)(112)

(9')







Threshold resummation Resummation of the soft/Sudakov logarithms

fixed coupling:

running coupling:

NLO resummed

 $\frac{\mathrm{d}\sigma_{\mathrm{resummed}}}{\mathrm{d}y\mathrm{d}^2p_{\perp}} = S_{\perp} \int_{\tau}^{1} \frac{\mathrm{d}z}{z^2} x_p dz$

Final results

 $\frac{\mathrm{d}\sigma}{\mathrm{d}y\mathrm{d}^2p_{\perp}} = \frac{\mathrm{d}\sigma_{\mathrm{resur}}}{\mathrm{d}y\mathrm{d}^2}$

NLO small corrections from the hard fac



$$S_{\text{Sud}}^{qq} = -\frac{\alpha_s}{2\pi} C_F \ln^2 \frac{k_\perp^2}{\Lambda^2} + 3\frac{\alpha_s}{2\pi} C_F \ln \frac{k_\perp^2}{\Lambda^2}$$
$$S_{\text{Sud}}^{qq} = C_F \int_{\Lambda^2}^{k_\perp^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s(\mu^2)}{\pi} \ln \frac{k_\perp^2}{\mu^2} - 3C_F \int_{\Lambda^2}^{k_\perp^2} \frac{d\mu^2}{\mu^2} \frac{\alpha_s(\mu^2)}{2\pi}$$

$$T_p q(x_p, \Lambda^2) D_{h/q}(z, \Lambda^2) F(k_\perp) e^{-S_{\text{Sud}}^{qq}}$$

$$\frac{d\sigma_{\rm NLO\ matching}}{dyd^2p_{\perp}} + \frac{d\sigma_{\rm Sud\ matching}}{dyd^2p_{\perp}} + \frac{d\sigma_{\rm Sud\ matching}}{dyd^2p_{\perp}}$$
etor
$$S_{\rm Sud}^{qq} - C_F \frac{\alpha_s}{2\pi} \left(\ln^2 \frac{k_{\perp}^2}{\Lambda^2} - 3 \ln \frac{k_{\perp}^2}{\Lambda^2} \right)$$



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 $0 0 (1-\xi)x \neq q^{\mu}$ channel in the coordinate space choose this semi-hard scale Λ^{μ} to be $\lim_{(z \in xP^+, \mu)} \frac{d^2 r_{\mu}}{dr} = \lim_{x \to \infty} \frac{d^2 r_{\mu}}{dr} = \lim_{x \to \infty} \frac{d\xi}{dr} = \lim_{x \to \infty} \frac{k_{\perp}^2}{dr} + \lim_{x \to \infty} \frac{1}{r_{\perp}^2} \frac{d\xi}{dr} = \lim_{x \to \infty} \frac{k_{\perp}^2}{r_{\perp}^2} + \lim_{x \to \infty} \frac{1}{r_{\perp}^2} \frac{d\xi}{dr} = \lim_{x \to \infty} \frac{k_{\perp}^2}{r_{\perp}^2} + \lim_{x \to \infty} \frac{1}{r_{\perp}^2} \frac{d\xi}{dr} = \lim_{x \to \infty} \frac{k_{\perp}^2}{r_{\perp}^2} + \lim_{x \to \infty} \frac{1}{r_{\perp}^2} \frac{d\xi}{dr} = \lim_{x \to \infty} \frac{k_{\perp}^2}{r_{\perp}^2} + \lim_{x \to \infty} \frac{1}{r_{\perp}^2} \frac{d\xi}{dr} = \lim_{x \to \infty} \frac{k_{\perp}^2}{r_{\perp}^2} + \lim_{x \to \infty} \frac{1}{r_{\perp}^2} \frac{d\xi}{dr} = \lim_{x \to \infty} \frac{k_{\perp}^2}{r_{\perp}^2} + \lim_{x \to \infty} \frac{1}{r_{\perp}^2} \frac{d\xi}{dr} = \lim_{x \to \infty} \frac{k_{\perp}^2}{r_{\perp}^2} + \lim_{x \to \infty} \frac{1}{r_{\perp}^2} \frac{d\xi}{dr} = \lim_{x \to \infty} \frac{k_{\perp}^2}{r_{\perp}^2} + \lim_{x \to \infty} \frac{1}{r_{\perp}^2} \frac{d\xi}{dr} = \lim_{x \to \infty} \frac{1}{r_$ Ch p with finite longitudinal momentum y_{qv}^{1} , e_{ret}^{1} , h_{qv}^{2} , $h_{qv}^$ these two logarithms arise from two physical regions of interview of the corresponds to particle of the corresponds to the corresponds to particle of the entum q^- , one gets Sudakov logarithm $\ln \frac{k_{\perp}^2}{q_{\perp}^2}$ corresponding to the real gluon, there is no such requirement. If we introduce the semi-hard scale Λ^2 to represent the typical transverse moment d_{m} as sociated with the sed in the region 1 with lext, det/n = 0.000 m in the second in the second in the second of the $rtpal gluon there is vely ing the displayed gluon disprint the scare twith a quentuk his the larges effected is estimated to be the Besset <math>(1 - \tau)k_{\perp}^{2}$ since ard scale A' to represent the typical to answerse anneat the field of the statest to a filled in the the statest and the statest of the state and scale Λ^2 to represent the window of product the product of the state of the indicates that this expression distributions in the part of the solution of t In addition to the above intuitive derivation we can analytically stud g virtual contribution is found to this ically speaking When the final state r_{jet} transverse momentum k_{jed} mainly comes from the dipole gluon distribution, we find that the which can be identified as the Sudarsvie erapide of the most stribution we find that the second se Typical semie hard's cale should hopef theatide is Orev Near then then a the streshold is the first of the sector of the first of the sector of the first of the sector of ne Sudakov double heavipinal contribution from the representation ways and the state of the spheric factor of the second franker of the spheric for the second franker of the se ration of the kinematerbeytion, to the whole is to gradien and the transformed to the prophatemission and the set two or effects alloud we praking attents region where The next subsectionalleris important the interval of the second interval i $\neg \frac{\frac{c_F}{C_F + N_c \beta_0}}{\frac{C_F}{C_F + N_c \beta_0}} p_T$ oft gluon emission and the saturation effects when we try to locate the region where the s. In addition, for convenience, we use a fixed estimated value of $\Lambda^2 \Lambda_{\rm QCD}^2$ in the <u>function</u> Λ^2 $\bar{\Lambda^2_{
m QCD}}$ atic region. \sum_{s}







Numerical results

BRAHMS







LHCb



LO jet production LO contribution



$$\frac{d\sigma_{\text{LO}}^{p+A\to \text{jet}+X}}{d\eta d^2 P_J} = \int_{\tau}^{1} \frac{dz}{z^2} \sum_{f} x q_f(x) F(q_\perp) J_q(z)$$

Collinear jet function

 $J_q^{(0)}(z) = \delta(1-z)$

Initial condition



VS

$$\frac{d\sigma_{\text{LO}}^{pA \to hX}}{d^2 p_{\perp} dy_h} = \int_{\tau}^{1} \frac{dz}{z^2} \sum_{f} x_p q_f(x_p) F(k_{\perp}) D_{h/q}(z)$$

Hadron fragmentation function

 $D_{h/q}(z)$

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NLO jet productions

NLO contribution of the final state radiation



We need to integrate the relative momentum

Kinematic constraint: $R_{qg} \leq R$



In-cone contribution: when the final state quark and the radiated gluon stay close to each other

n!
$$R_{qg} = \sqrt{\Delta \eta^2 + \Delta \phi^2}$$

$$\frac{p_{\perp}^2}{\xi(1-\xi)} \le l_{\perp}k_{\perp}R^2 \simeq \frac{\xi(1-\xi)}{z^2}P_J^2R^2 = \xi(1-\xi)q_{\perp}^2R^2$$

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NLO contribution from the final state radiation





 $\sigma_{\text{final}}^{\text{jet}} = \sigma_a + (\sigma_c - \sigma_b) + \sigma_{\text{virt}}$

Comparing with hadron productions

$$\ln \frac{1}{R^2} \Leftrightarrow -\frac{1}{\epsilon} + \ln \frac{q_{\perp}^2}{\mu^2}$$

Our one-loop calculations are consistent with hadron calculations.

Our one-loop calculations are consistent with Liu's paper.







Threshold resummation Threshold resummation is still needed!

process	collinear log(initial)	single log	double log	collinear log(final)
$q \rightarrow q$	$\mathcal{P}_{qq}(\xi) \ln rac{\Lambda^2}{\mu^2}$	$\ln \frac{q_{\perp}^2}{\Lambda^2}$	$\ln^2 \frac{q_\perp^2}{\Lambda^2}$	$\frac{1}{\xi^2} \mathcal{P}_{qq}(\xi) \ln \frac{\Lambda^2}{\mu_J^2}$
$g \rightarrow g$	$\mathcal{P}_{gg}(\xi) \ln rac{\Lambda^2}{\mu^2}$	$\ln \frac{q_{\perp}^2}{\Lambda^2}$	$\ln^2 \frac{q_\perp^2}{\Lambda^2}$	$\frac{1}{\xi^2} \mathcal{P}_{gg}(\xi) \ln \frac{\Lambda^2}{\mu_J^2}$
$q \rightarrow g$	$\mathcal{P}_{gq}(\xi) \ln rac{\Lambda^2}{\mu^2}$	/	/	$\frac{\frac{1}{\xi^2} \mathcal{P}_{gq}(\xi) \ln \frac{\Lambda^2}{\mu_J^2}}{\frac{1}{\mu_J^2}}$
$g \rightarrow q$	$\mathcal{P}_{qg}(\xi) \ln rac{\Lambda^2}{\mu^2}$	/	/	$\frac{1}{\xi^2} \mathcal{P}_{qg}(\xi) \ln \frac{\Lambda^2}{\mu_J^2}$

Reusemation of the collinear logarithms from the final state radiation

$$\frac{\partial \mathcal{J}_q(z,\Lambda)}{\partial \ln \Lambda^2} = \frac{\alpha_s}{2\pi} \int_z^1 \frac{\mathrm{d}\xi}{\xi} \left[C_F \mathcal{P}_{qq}(\xi) \mathcal{J}_q\left(\frac{z}{\xi},\Lambda\right) + C_F \mathcal{P}_{gq}(\xi) \mathcal{J}_g\left(\frac{z}{\xi},\Lambda\right) \right]$$

Initial condition: $J_q^{(0)} = \delta(1-z)$ at scale $\mu_J = P_J R$

$$\begin{bmatrix} \mathcal{J}_q(z,\mu_J) \\ \mathcal{J}_g(z,\mu_J) \end{bmatrix} + \frac{\alpha_s}{2\pi} \ln \frac{\Lambda^2}{\mu_J^2} \int_z^1 \frac{\mathrm{d}\xi}{\xi} \begin{bmatrix} C_F \mathcal{P}_{qq}(\xi) & C_F \mathcal{P}_{gq}(\xi) \\ T_R \mathcal{P}_{qg}(\xi) & N_C \mathcal{P}_{gg}(\xi) \end{bmatrix} \begin{bmatrix} \mathcal{J}_q(z/\xi,\mu_J) \\ \mathcal{J}_g(z/\xi,\mu_J) \end{bmatrix} \Rightarrow \begin{bmatrix} \mathcal{J}_q(z,\Lambda) \\ \mathcal{J}_g(z,\Lambda) \end{bmatrix}$$



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Threshold resummation Resummation of the soft logarithms

- Sudakov logs from the jet contribution cancel.
- The term which proportional to $\left(\frac{\ln(1-\xi)}{1-\xi}\right)$,

and this stems from final state gluon radiations. • When $\xi \to 1$ ($\tau \to 1$), these terms give us $\ln^2 N$.

> Only initial state radiation contribution!

$$S_{\text{Sud}}^{qq} = -\frac{\alpha_s}{2\pi} \frac{C_F}{2} \ln^2 \frac{k_\perp^2}{\Lambda^2} + \frac{\alpha_s}{2\pi} \frac{3}{2} C_F \ln \frac{k_\perp^2}{\Lambda^2}$$

Note: for hadron productions we have











Final results

Final resumed results $\frac{\mathrm{d}\sigma}{\mathrm{d}\eta\mathrm{d}^2P_J} = \frac{\mathrm{d}\sigma_{\mathrm{res}}}{\mathrm{d}\eta}$

$$\frac{\mathrm{d}\sigma_{\mathrm{resummed}}}{\mathrm{d}\eta\mathrm{d}^{2}P_{J}} = S_{\perp} \int_{\tau}^{1} \frac{\mathrm{d}z}{z^{2}} xq(x,\Lambda^{2}) \mathcal{J}_{q}(z,\Lambda^{2}) F(q_{\perp}) e^{-S_{\mathrm{Sud}}^{qq}} + S_{\perp} \int_{\tau}^{1} \frac{\mathrm{d}z}{z^{2}} xg(x,\Lambda^{2}) \mathcal{J}_{g}(z,\Lambda^{2}) \int \mathrm{d}^{2}q_{1\perp} F(q_{1\perp}) F(q_{\perp}-q_{1\perp}) e^{-S_{\mathrm{Sud}}^{gg}}$$

$$\frac{\mathrm{d}\sigma_{\mathrm{Sud\ matching}}}{\mathrm{d}\eta\mathrm{d}^{2}P_{J}} = S_{\perp} \int_{\tau}^{1} \frac{\mathrm{d}z}{z^{2}} xq(x,\mu^{2}) \mathcal{J}_{q}(z,\mu_{J}^{2}) F(q_{\perp}) \left\{ S_{\mathrm{Sud}}^{qq} - \left[C_{F} \frac{\alpha_{s}}{2\pi} \left(\frac{1}{2} \ln^{2} \frac{q_{\perp}^{2}}{\Lambda^{2}} - \frac{3}{2} \ln \frac{q_{\perp}^{2}}{\Lambda^{2}} \right) \right] \right\} \\ + S_{\perp} \int_{\tau}^{1} \frac{\mathrm{d}z}{z^{2}} xg(x,\mu^{2}) \mathcal{J}_{g}(z,\mu_{J}^{2}) \int \mathrm{d}^{2}q_{1\perp} F(q_{1\perp}) F(q_{\perp}-q_{1\perp}) \\ \times \left\{ S_{\mathrm{Sud}}^{gg} - \left[N_{c} \frac{\alpha_{s}}{2\pi} \left(\frac{1}{2} \ln^{2} \frac{q_{\perp}^{2}}{\Lambda^{2}} - 2\beta_{0} \ln \frac{q_{\perp}^{2}}{\Lambda^{2}} \right) \right] \right\}.$$

 $\frac{\mathrm{d}\sigma_{\mathrm{NLO matching}}}{\mathrm{d}\eta\mathrm{d}^2 P_J} \qquad \qquad \mathbf{Too lon}$



$$\frac{\mathrm{d}\sigma_{\mathrm{NLO matching}}}{\mathrm{d}\eta\mathrm{d}^{2}P_{J}} + \frac{\mathrm{d}\sigma_{\mathrm{NLO matching}}}{\mathrm{d}\eta\mathrm{d}^{2}P_{J}} + \frac{\mathrm{d}\sigma_{\mathrm{Sud matching}}}{\mathrm{d}\eta\mathrm{d}^{2}P_{J}}$$

We consider
$$\begin{cases} \frac{q}{g} \\ \frac{g}{g} \end{cases}$$

$$\begin{array}{c} q \rightarrow qg \\ g \rightarrow gg \\ g \rightarrow gg \\ q \rightarrow q\bar{q} \\ q \rightarrow gq \end{array}$$

Too long to show!



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Summary

- We briefly introduce the color glass condensate effective theory. 1.
- By incorporating the threshold resummation in CGC formalism, we can describe the 2. experimental data from RHIC and LHC.
- We have systematically calculated the complete NLO cross-section for single inclusive 3. jet production in pA collisions at forward rapidity region within the small-x framework. Outlook

- 1. The numerical calculation of jet production is in progress.
- 2. We can apply the threshold resummation formalism on many other processes. Such as, the **Muller-Navelet jet productions, diffractive processes** ...





Thank you!







• The following plot shows the type of behavior which is typical of LO and NLO calculations



radiative effect predicted by certainties in xf(x) and D(z). Choice of the scale at LO

$$x \sum_{i} e_i^2 q_i(x)$$

5 ULLAU ACHILLE ULLAS YUU actually is an implici ure from the very star (7) in the reference it to notice what the hich can be found as E lifference is that in [14]written as f/k_T^2 .) Prior to t the production of gluons in [14]. The exact statement ju form of Eq. (40) in [14] read





Backup

More results for forward rapidity region



TABLE I. Values of Λ^2 in calculating the cross-sections at $\sqrt{s_{\rm NN}} = 200$ GeV at RHIC.

Rapidity	y = 2.2		y = 3.2		y = 4	
Collisional systems	dAu	pp	dAu	pp	dAu	pp
$\Lambda^2 ({ m GeV}^2)$	$2 \sim 4$	~ 1	$3\sim7$	~ 2	$5 \sim 9$	~ 3

TABLE II. List of input values of parameters used in our numerical evaluations.

Experiment	BRAHMS	STAR	ATLAS/ALICE	LHCb
Hadron h	h^-	π^0	h^{\pm}	prompt h^{\pm}
σ^h/σ^π	1.3	1	2.2	2.2×0.85

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Backup

Jet algorithm: k_t -type jet algorithm (base on the narrow jet approximation).

Indeed, we need to specify the jet algorithm. Nowadays, the popular jet algorithms are kttype jet algorithms, such as, the kt, anti-kt, and Cambridge/Aachen algorithms. In our calculation, we computed the jet cross-section based on the narrow jet approximation [Phys.Rev.D 70 (2004) 034010] together with the kt-type jet algorithm [1209.1785]

SOLO=Saturation physics at One Loop Order

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Backup

$$\frac{1}{2\pi p_T} \frac{\mathrm{d}^2 N^{\mathrm{pA}\to hX}}{\mathrm{d}y \mathrm{d}p_T} = \frac{1}{\alpha}$$

$$R_{pPb} = \frac{1}{A} \frac{d^2 \sigma_{pPb}}{d^2 \sigma_{pp}}$$

 $\frac{1}{\sigma_{\text{inel}}} \frac{1}{2\pi p_T} \frac{\mathrm{d}^2 \sigma^{\mathrm{pA} \to hX}}{\mathrm{d}y \mathrm{d}p_T}$

 $\frac{b/dp_T dy}{/dp_T dy}.$

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