Forward inclusive hadron/jet productions in pA collisions

QCD MASTER CLASS
SAINT-JACUT-DE-LA-MER, FRANCE

Lei Wang
Shandong University

28.Jun.2024

Collaborators: Lin Chen, Yu Shi, Zhan Gao, Shu-yi Wei, Bo-wen Xiao

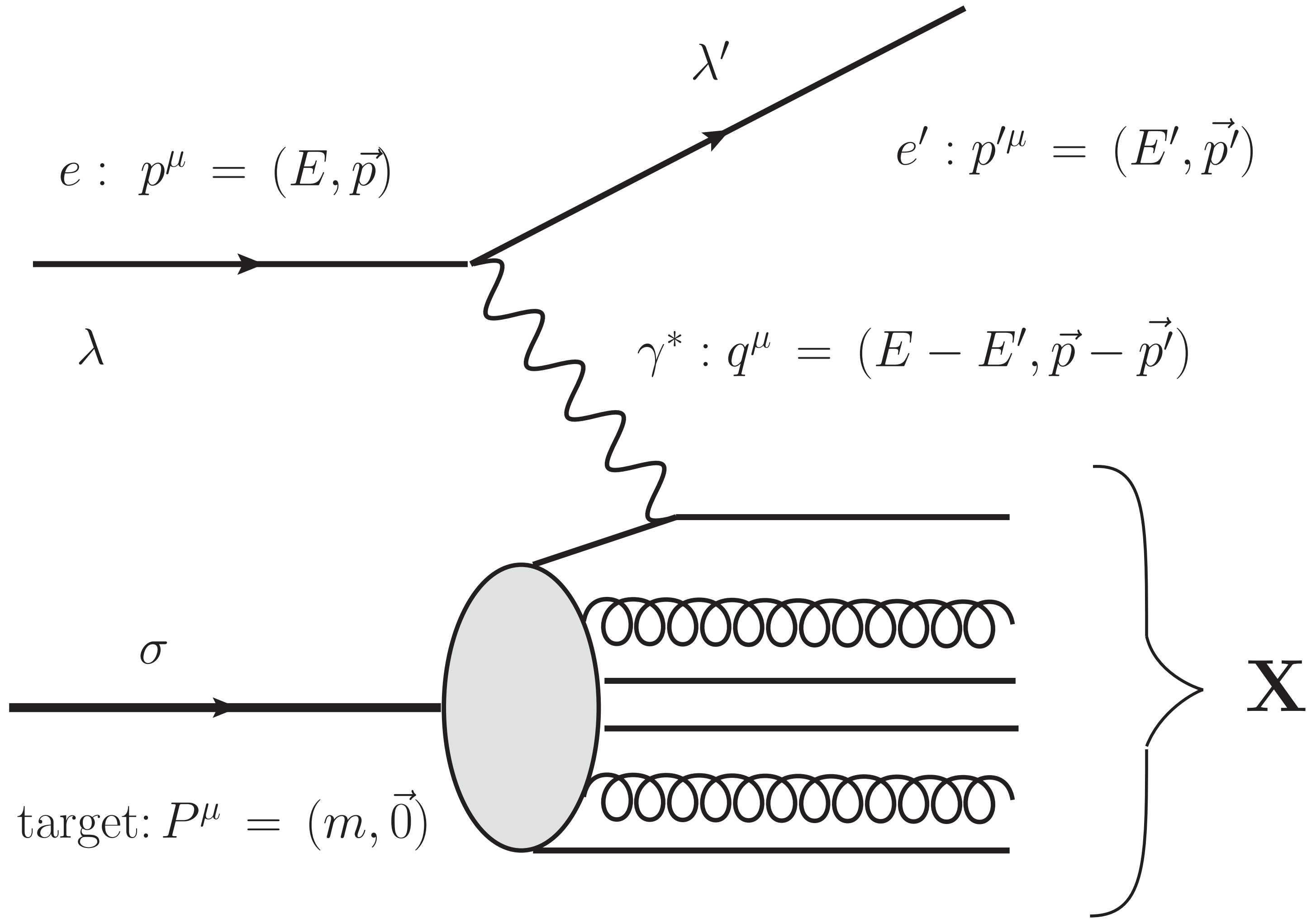
arXiv:2112.06975, 2211.08322

Outline

- **Introduction**
- **Precision study for forward hadron productions**
- **Forward single inclusive jet productions**
- **summary & outlook**

Color Glass Condensate

Deep inelastic scattering



$$Q^2 \equiv -q^2,$$

$$x_{Bj} \equiv \frac{Q^2}{2P\cdot q},$$

$$y \equiv \frac{P\cdot q}{P\cdot p}.$$

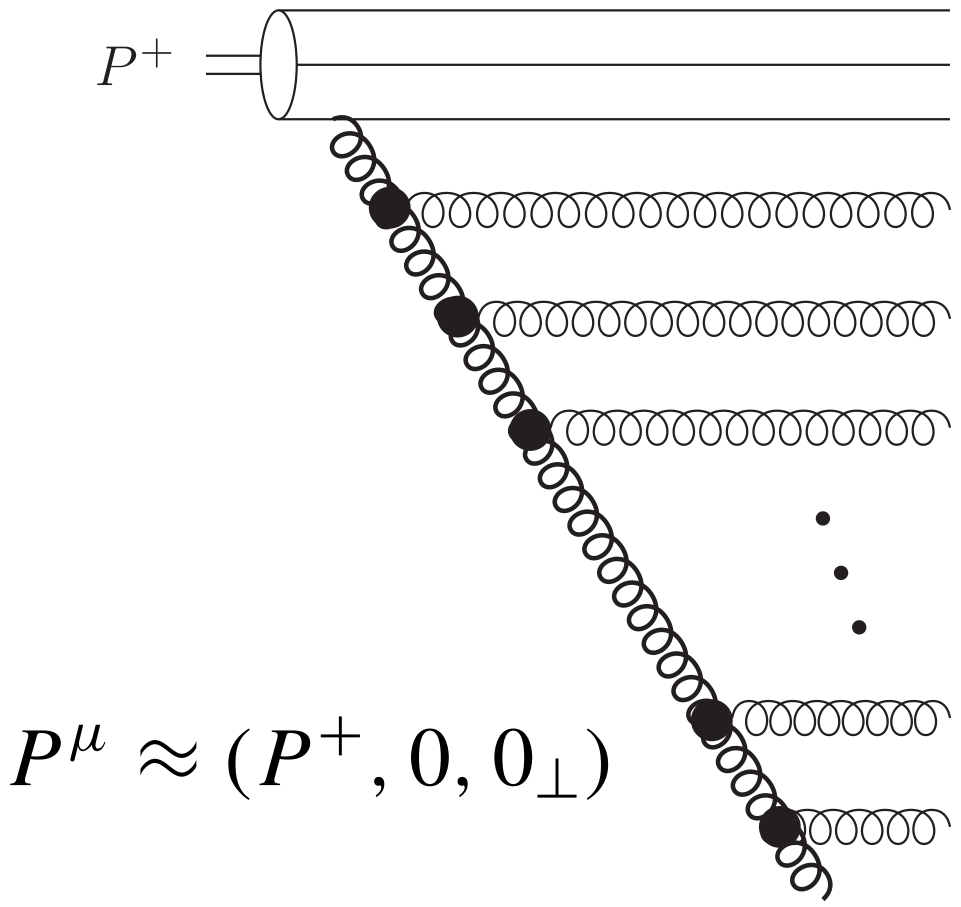
$$\nu \equiv \frac{P\cdot q}{m} = E - E',$$

$$x_{Bj} = \frac{Q^2}{\hat{s} + Q^2 - m^2} = \frac{Q^2}{2m\nu},$$

$$Q^2 = y x_{Bj}(s - m^2 - m_e^2) \approx y x_{Bj} s.$$

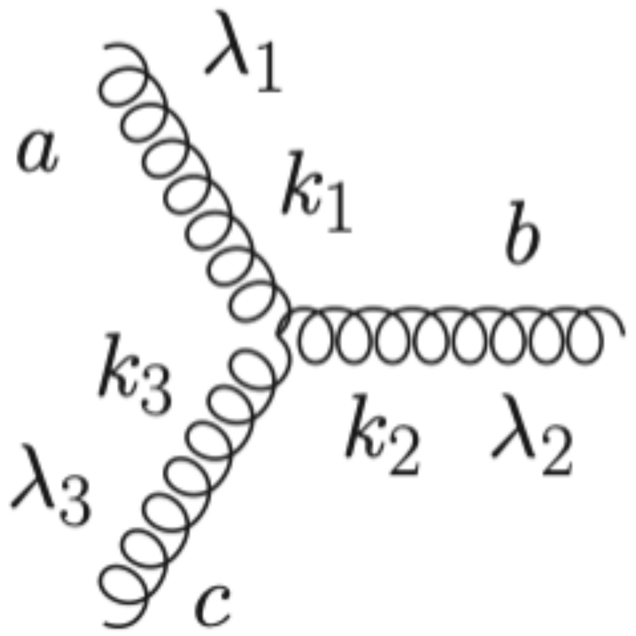
High energy=small x

Infinite momentum frame or Bjorken frame



linear evolution
(BFKL)

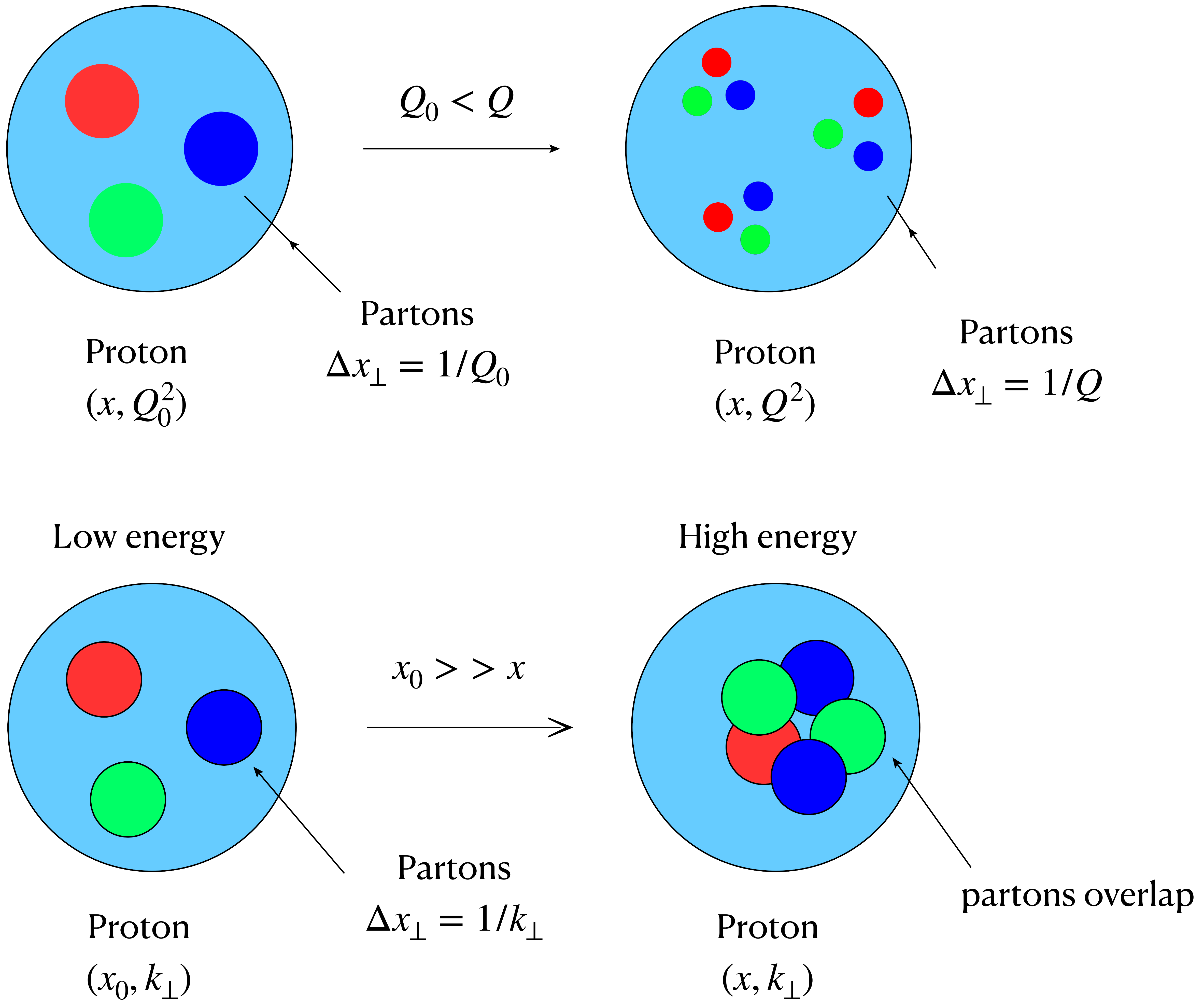
Three gluon vertex



Non-linear dynamics
(BK/JIMWLK)

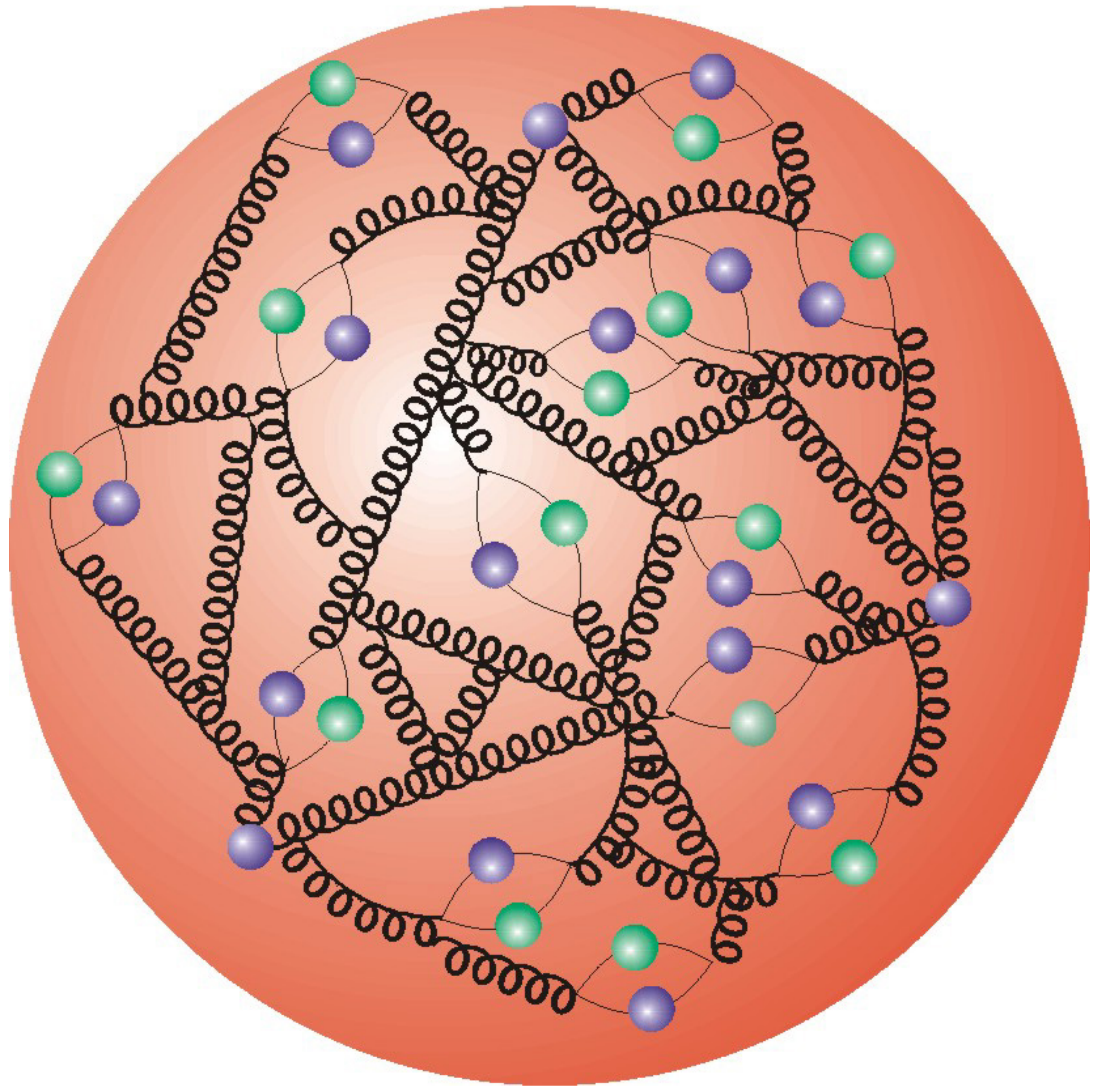
Color Glass Condensate

The proton wave function is determined by x and Q

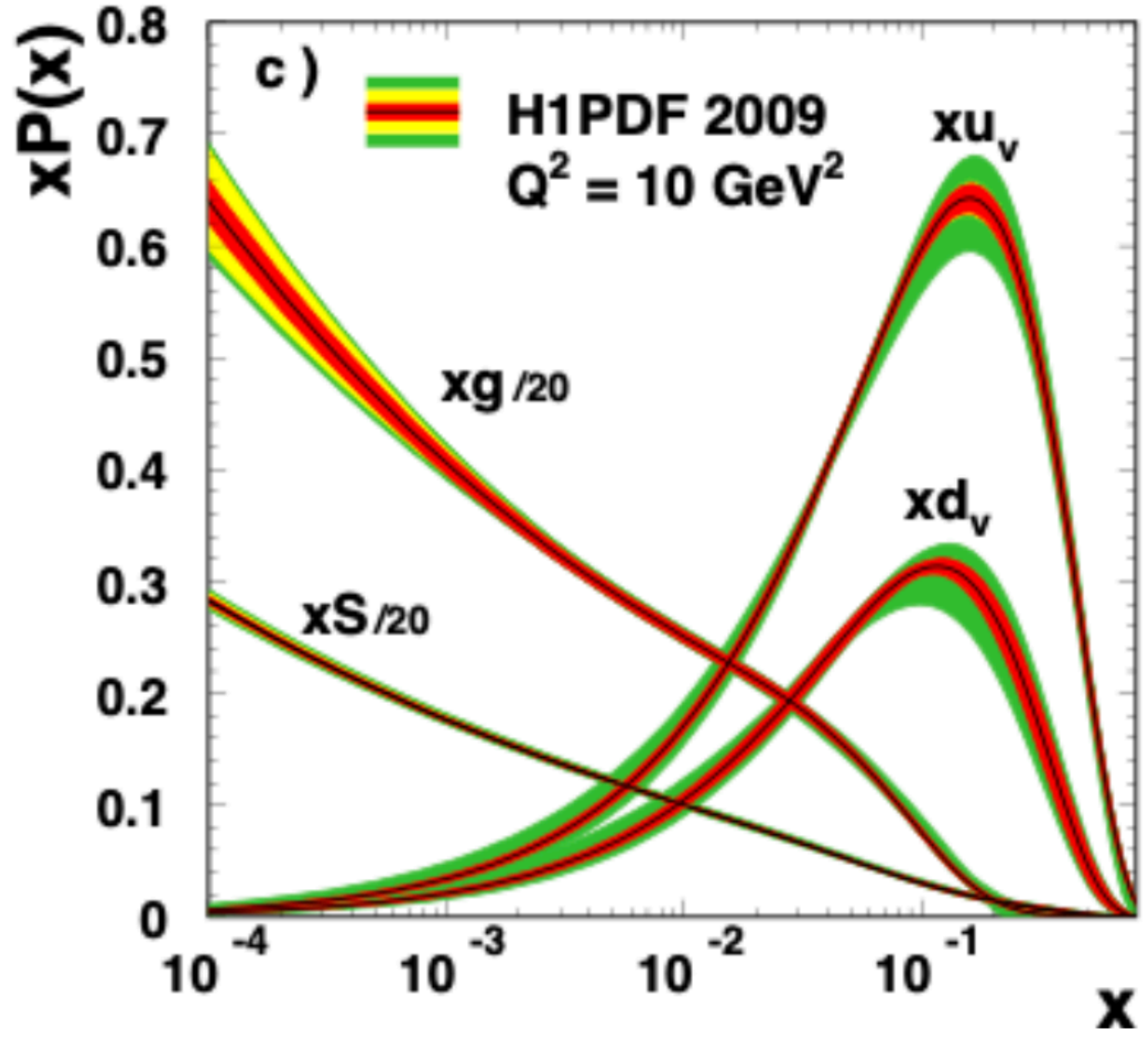


High number of gluons populate the transverse extend of the proton or nucleus, leading to a very dense saturated wave function. Color Glass Condensate (CGC) is the effective theory to study this kind of dense gluon system.

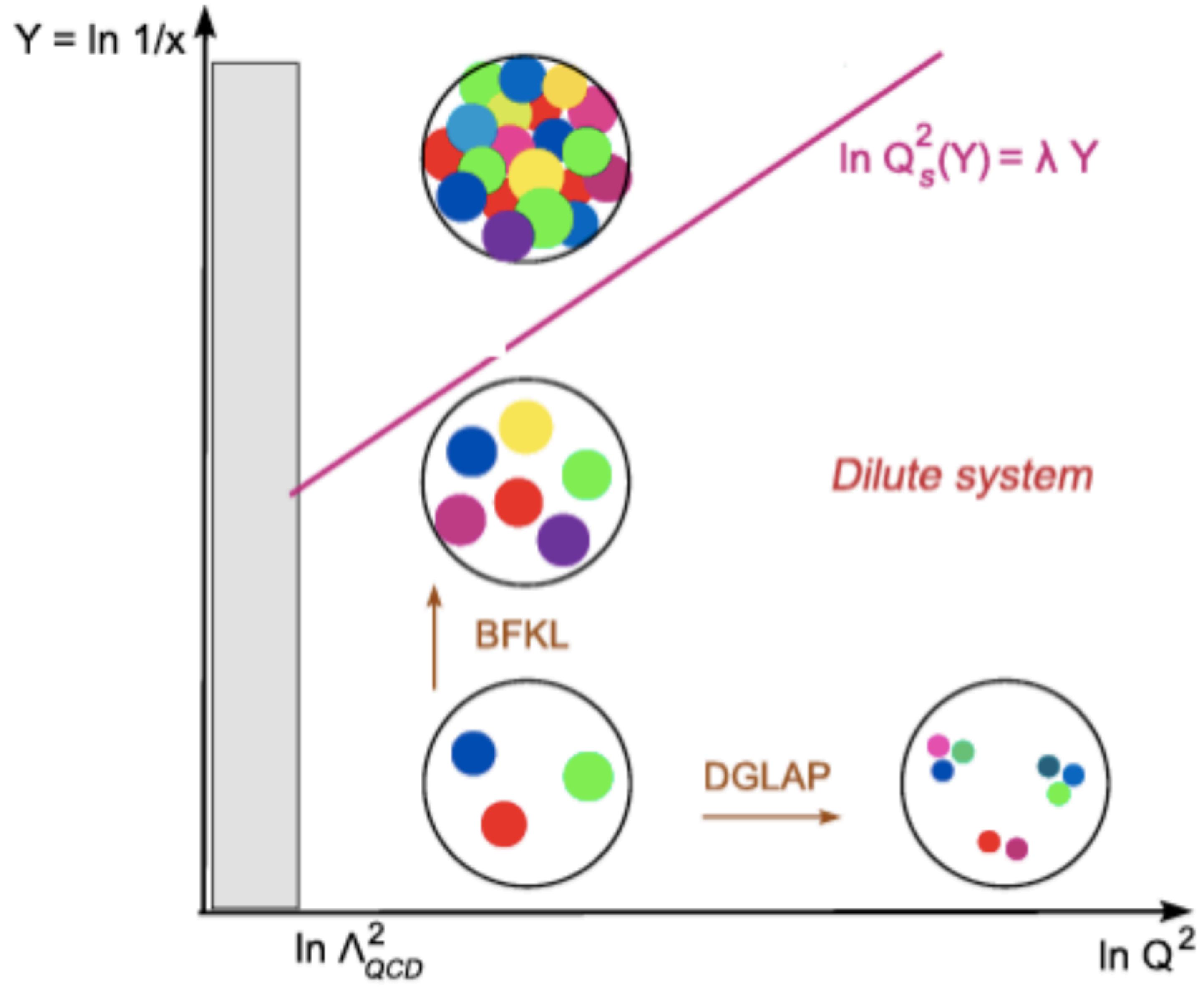
Color Glass Condensate



H1 and ZEUS



High energy QCD map



- Partons in the low-x region is dominated by gluons
- Collinear evolution: BFKL
- Low Q^2 and low x region $\Rightarrow$ Non-linear evolution: BK/JIMWLK
- Saturation momentum: $Q_s(Y)$

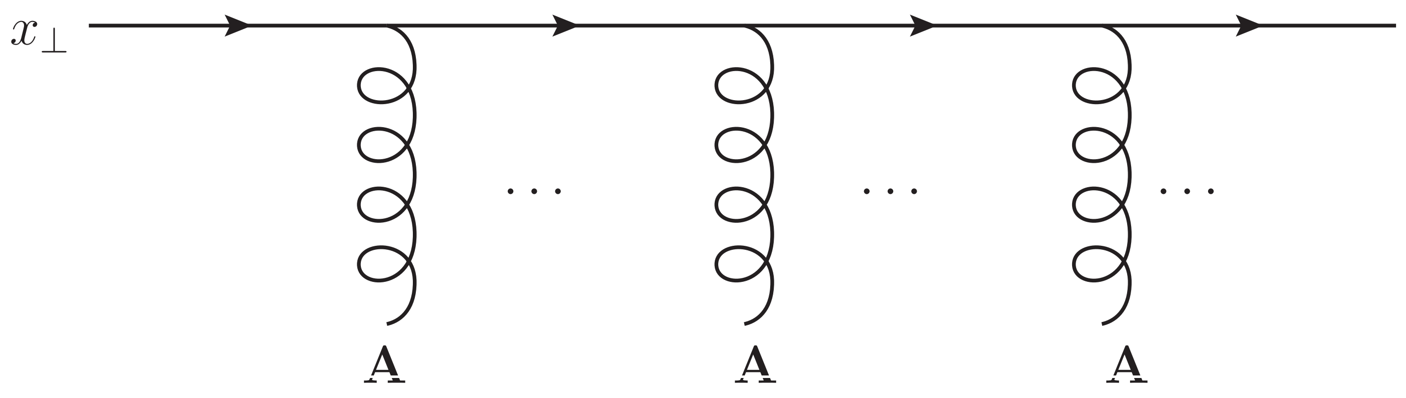
$Q_s^2 \sim A^{1/3} x^{-\lambda}$. For EIC, $A \simeq 200$, $\lambda \simeq 0.2$, $Q_s^2 \simeq 1 \sim 2 Gev^2$

Color Glass Condensate

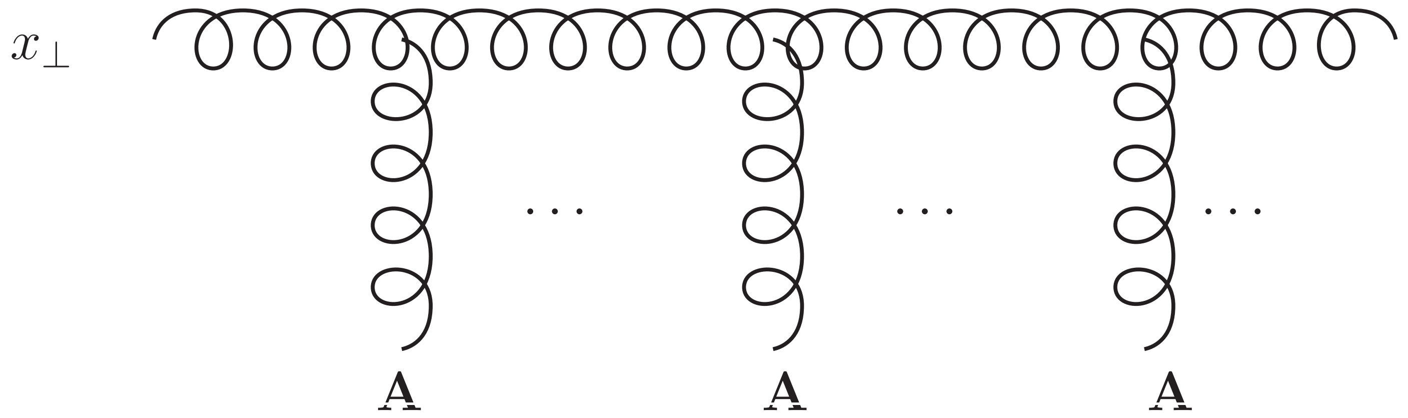
We use Wilson line to represent the multiple scattering between the fast moving parton and target background fields!

Fundamental representation-multiple scattering between fast moving quark and target dense gluons

Wilson Line $$U(x_\perp) = P\exp\left\{-ig_S\int_{-\infty}^{+\infty} \mathrm{d}x^+ A_a^-(x^+, x_\perp)t^a\right\}$$



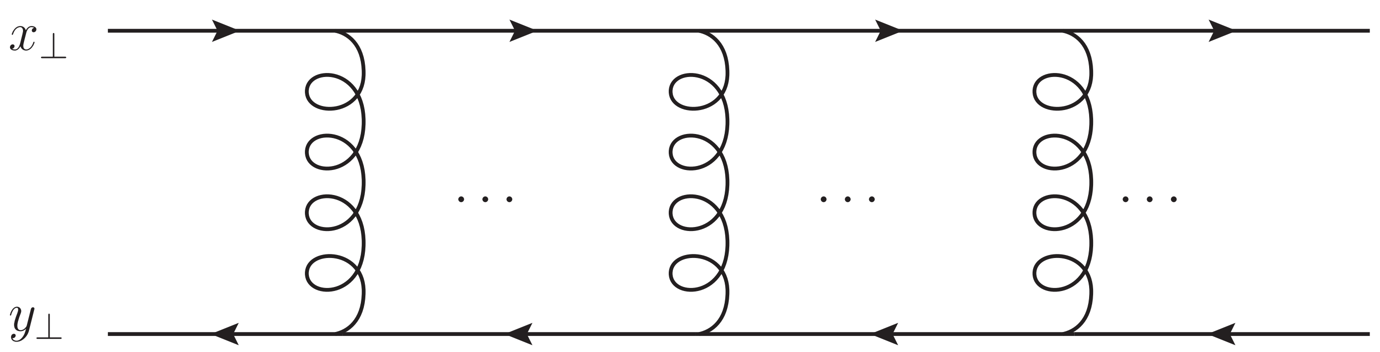
$$W^{ab}(x_\perp) = 2\mathrm{Tr}[t^a U(x_\perp) t^b U^\dagger(x_\perp)]$$



The color dipole in McLerran-Venugopalan model

$$S(x_\perp - y_\perp) = \frac{1}{N_c}\mathrm{Tr}\langle U(x_\perp)U^\dagger(y_\perp)\rangle = e^{-\frac{Q_s^2(x_\perp - y_\perp)^2}{4}}$$

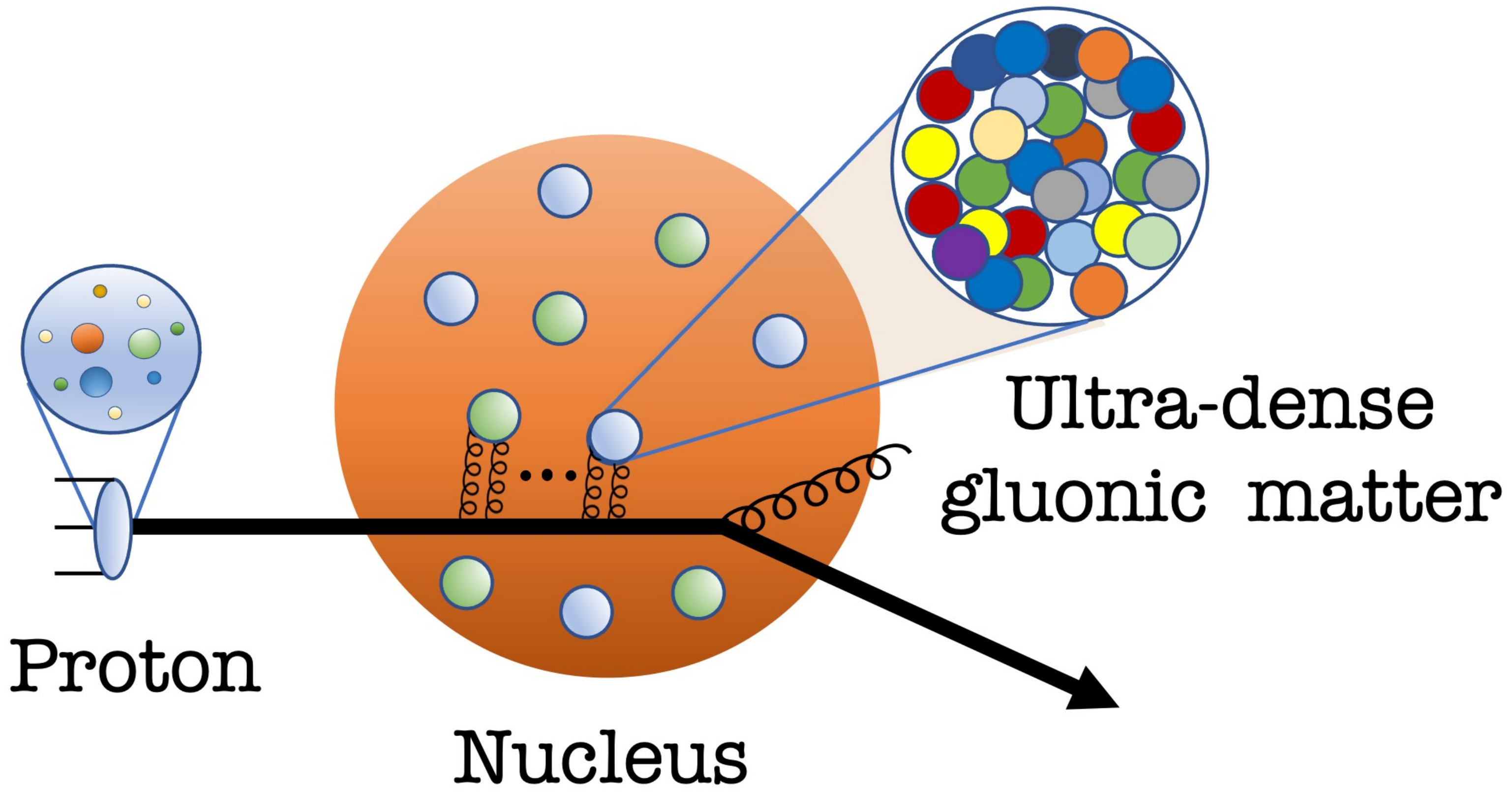
$$\mathcal{F}(k_\perp) \equiv \frac{dN}{d^2k_\perp} = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp - y_\perp)} \frac{1}{N_c}\left\langle \mathrm{Tr}U(x_\perp)U^\dagger(y_\perp)\right\rangle$$



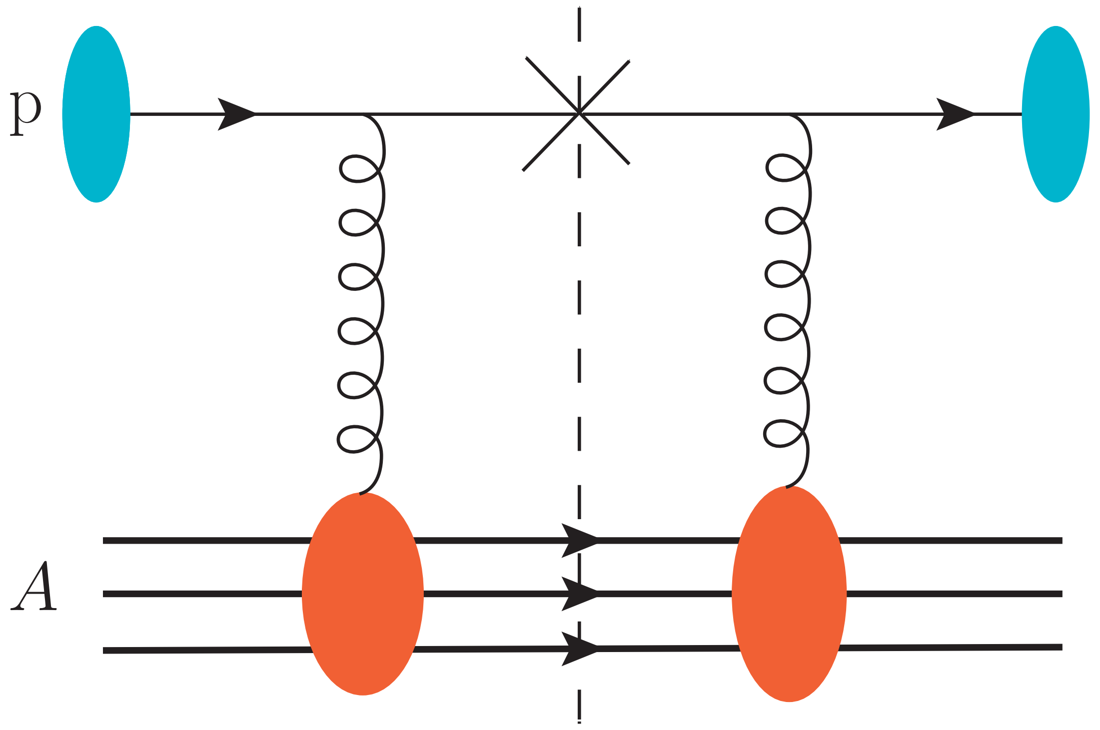
Proton-nucleus collisions

Searching for Parton saturation in dilute-dense scatterings: ep & eA & pA.

Single inclusive hadron productions in pA collisions



LO calculation



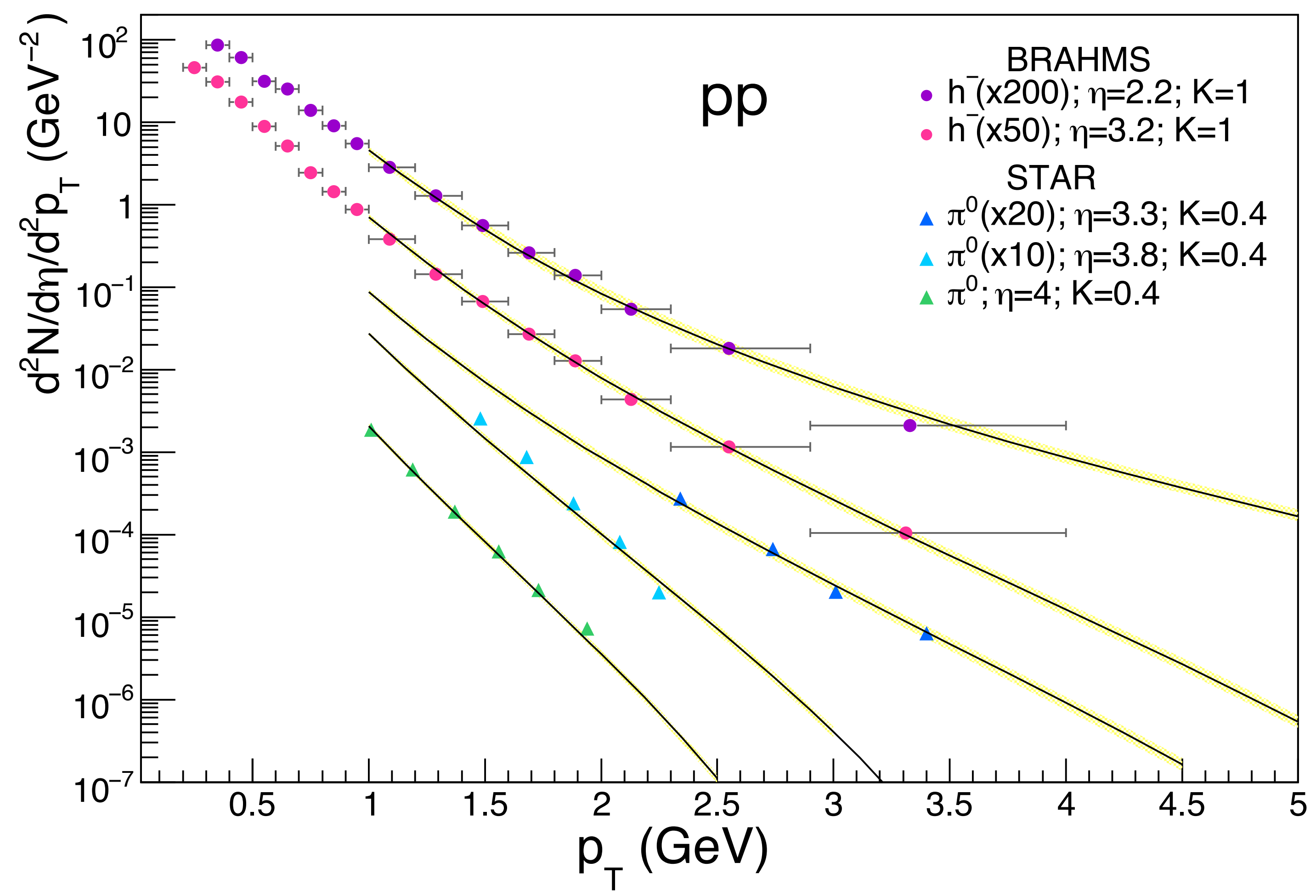
$$\frac{d\sigma_{\mathrm{LO}}^{pA\to hX}}{d^2p_\perp dy_h} = \int_\tau^1 \frac{dz}{z^2} \sum_f x_p q_f(x_p) F(k_\perp) D_{h/q}(z)$$

$$F(k_\perp) = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp - y_\perp)} S_Y^{(2)}(x_\perp, y_\perp)$$

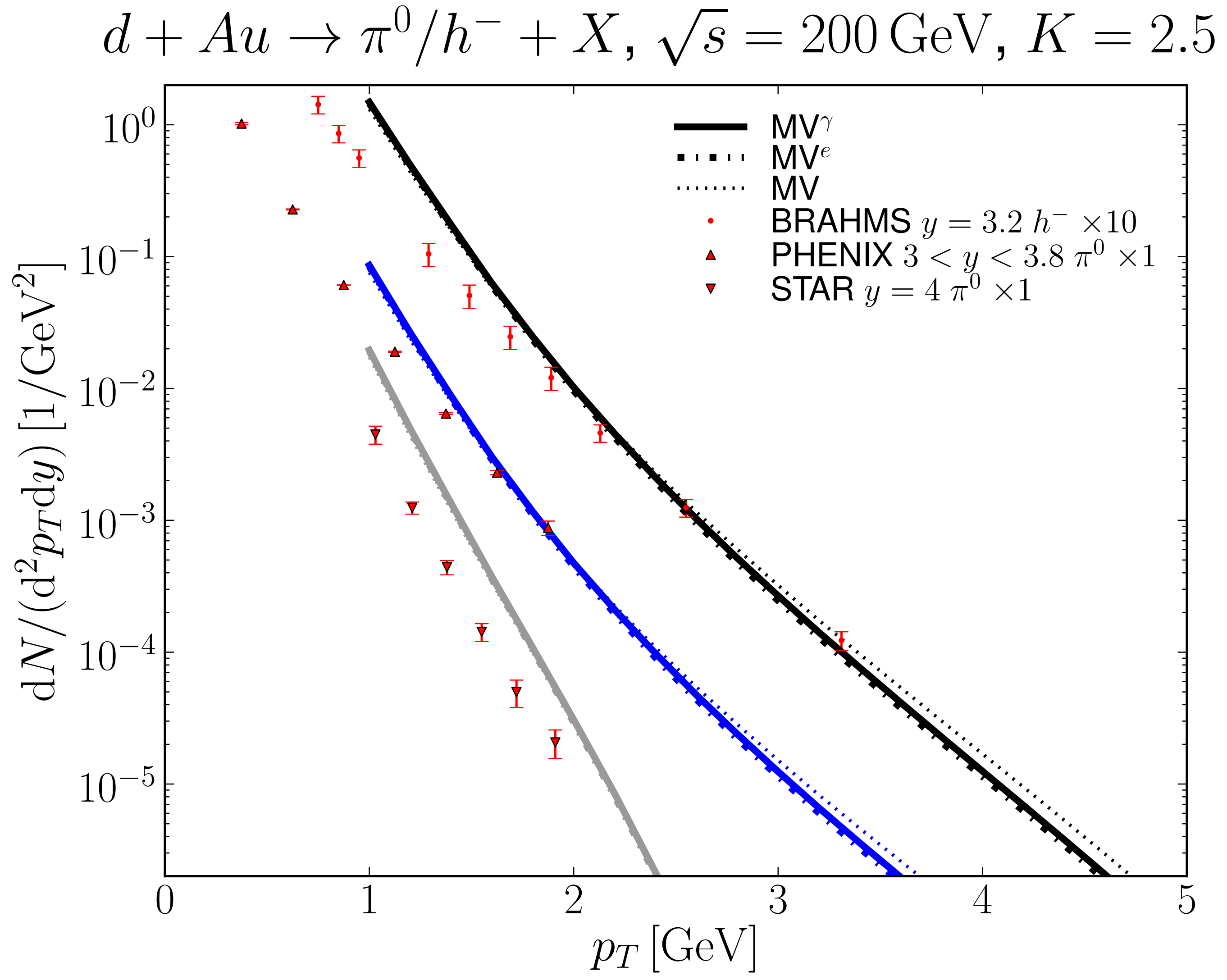
Dumitru and Jalilian-Marian, PRL 2002

Proton-nucleus collisions

LO calculation



Albacete and Marquet, PLB 2010



Lappi andM¨antysaari, PRD 2013

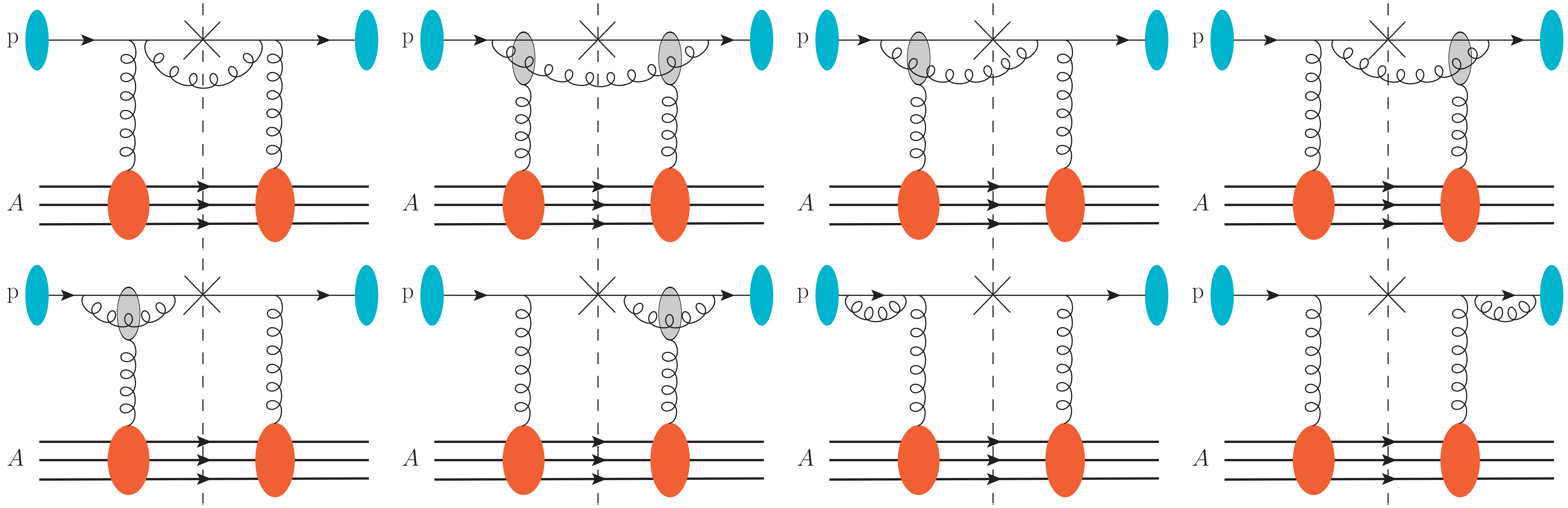
They both used the K factor!

Higher order calculations are needed!

- Incuding higher order corrections may give more accurate answers.
- Going to higher orders can open new channels and can sometimes result in large corrections.
- Going to higher orders cancels some but not all of factorization scale dependence.

Proton-nucleus collisions

NLO diagrams in the $q \rightarrow q$ channel



Lost of contributions for NLO

Dumitru, Hayashigakia and Jalilian-Marianb, NPA, 2006

Altinoluk and Kovner, PRD, 2011

Chirilli, Xiao and Yuan, PRL, 2011

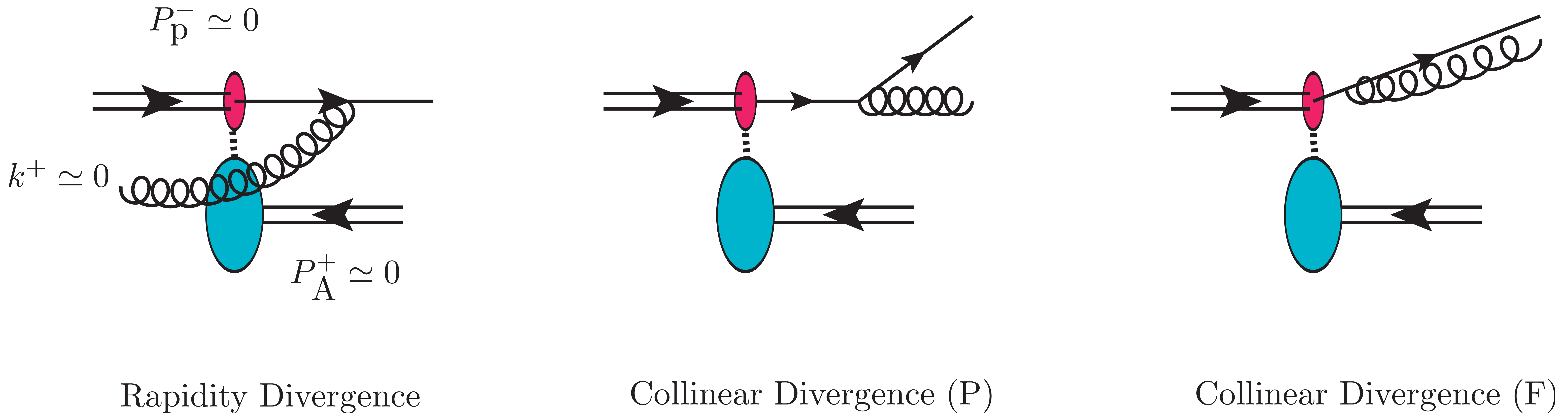
Chirilli, Xiao and Yuan, PRD, 2012

Kang, Vitev, Xing, PRL, 2014

1. Both real and virtual diagrams should be considered!
2. Grey blobs indicates the multiple interactions!
3. Integrate over gluon phase space $\Rightarrow$ divergences!

Proton-nucleus collisions

Three kinds of divergences:



1. Collinear to the target nucleus ⇒ BK evolution equation!
2. Collinear to the initial state quark ⇒ DGLAP evolution of PDFs!
3. Collinear to the final state quark ⇒ DGLAP evolution of FFs!

Proton-nucleus collisions

For the $q \to q$ channel, the factorization formula can be written as

$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \int \frac{dz}{z^2}\frac{dx}{x}\xi xq(x,\mu)D_{h/q}(z,\mu)\int\frac{d^2x_\perp d^2y_\perp}{(2\pi)^2}\Big\{S_Y^{(2)}(x_\perp,y_\perp)\Big[\mathcal{H}_{2qq}^{(0)}+\frac{\alpha_s}{2\pi}\mathcal{H}_{2qq}^{(1)}\Big] + \int\frac{d^2b_\perp}{(2\pi)^2}S_Y^{(4)}(x_\perp,b_\perp,y_\perp)\frac{\alpha_s}{2\pi}\mathcal{H}_{4qq}^{(1)}\Big\}$$

with $\mathcal{H}_{2qq}^{(0)} = e^{-ik_\perp\cdot r_\perp}\delta(1-\xi)$ and

$$\mathcal{H}_{2qq}^{(1)} = C_F\mathcal{P}_{qq}(\xi)\ln\frac{c_0^2}{r_\perp^2\mu^2}\left(e^{-ik_\perp\cdot r_\perp}+\frac{1}{\xi^2}e^{-i\frac{k_\perp}{\xi}\cdot r_\perp}\right) - 3C_F\delta(1-\xi)e^{-ik_\perp\cdot r_\perp}\ln\frac{c_0^2}{r_\perp^2k_\perp^2} - (2C_F-N_c)e^{-ik_\perp\cdot r_\perp}\left[\frac{1+\xi^2}{(1-\xi)_+}\widetilde{I}_{21}-\left(\frac{(1+\xi^2)\ln(1-\xi)^2}{1-\xi}\right)_+\right]$$

$$\mathcal{H}_{4qq}^{(1)} = -4\pi N_c e^{-ik_\perp\cdot r_\perp}\Big\{e^{-i\frac{1-\xi}{\xi}k_\perp\cdot(x_\perp-b_\perp)}\frac{1+\xi^2}{(1-\xi)_+}\frac{1}{\xi}\frac{x_\perp-b_\perp}{(x_\perp-b_\perp)^2}\cdot\frac{y_\perp-b_\perp}{(y_\perp-b_\perp)^2} - \delta(1-\xi)\int_0^1 d\xi'\frac{1+\xi'^2}{(1-\xi')_+}\Big[\frac{e^{-i(1-\xi')k_\perp\cdot(y_\perp-b_\perp)}}{(b_\perp-y_\perp)^2}-\delta^{(2)}(b_\perp-y_\perp)\int d^2r'_\perp\frac{e^{ik_\perp\cdot r'_\perp}}{r'^2_\perp}\Big]\Big\};$$

where
$$\widetilde{I}_{21} = \int\frac{d^2b_\perp}{\pi}\Big\{e^{-i(1-\xi)k_\perp\cdot b_\perp}\Big[\frac{b_\perp\cdot(\xi b_\perp-r_\perp)}{b_\perp^2(\xi b_\perp-r_\perp)^2}-\frac{1}{b_\perp^2}\Big]+e^{-ik_\perp\cdot b_\perp}\frac{1}{b_\perp^2}\Big\}$$

$q \to qg$

$g \to gg$

$g \to q\bar{q}$

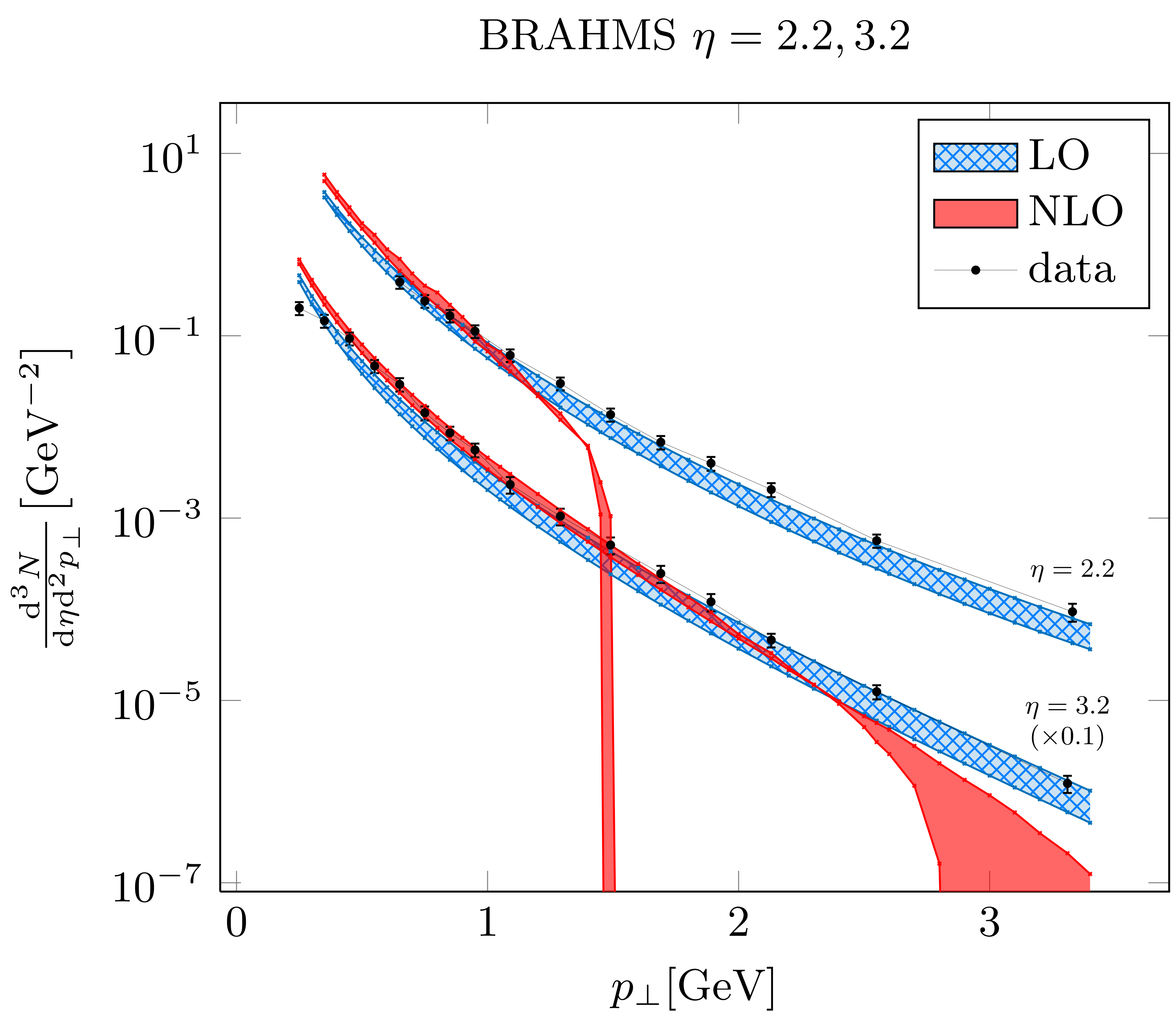
$q \to gq$

$\sigma \sim xf(x)\otimes\mathcal{H}\otimes D_h(z)\otimes\mathcal{F}(k_\perp)$

Need to go to momentum space for numerical evaluations!

Proton-nucleus collisions

Numerical calculations for NLO from SOLO (Saturation physics at One Loop Order)



Stasto, Xiao and Zaslavsky, PRL, 2013

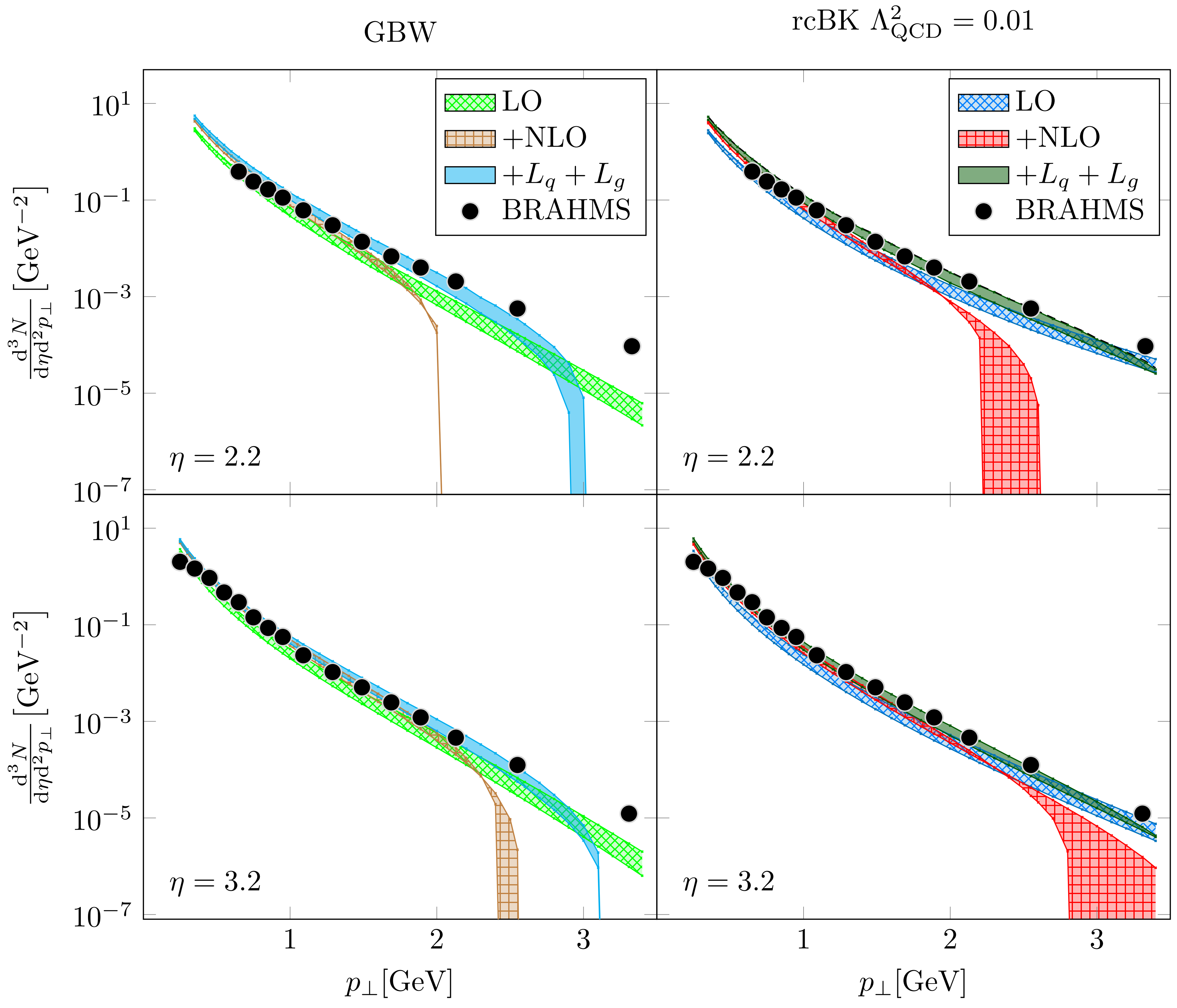
1.Perfect description at low p_T,
2. There is no K factor.

2.Cross-section turns negative at high p_T

Our motivation!

Proton-nucleus collisions

Lots of contributions for solving the negative puzzle



Watanabe, Xiao, Yuan and Zaslavsky, PRD, 2015

$$\ln \frac{1}{x_g} + \underbrace{\ln \frac{k_\perp^2}{q_\perp^2}}_{\text{missed earlier}} \Rightarrow$$

New terms: $L_q + L_g$.

Watanabe, Xiao, Yuan and Zaslavsky, PRD, 2015

Altinoluk, et al, PRD, 2015

Iancu, et al, JHEP, 2016

Ducloué, Lappi, Zhu, PRD, 2016, 2017

Ducloué, et al, PRD, 2018

Xiao, Yuan, PLB, 2019

Liu, Ma, Chao, PRD, 2019

Liu, Kang, Liu, PRD, 2020

Liu, Liu, Shi, Zheng, Zhou, 2022

factorization scheme ;

kinematic constraint ;

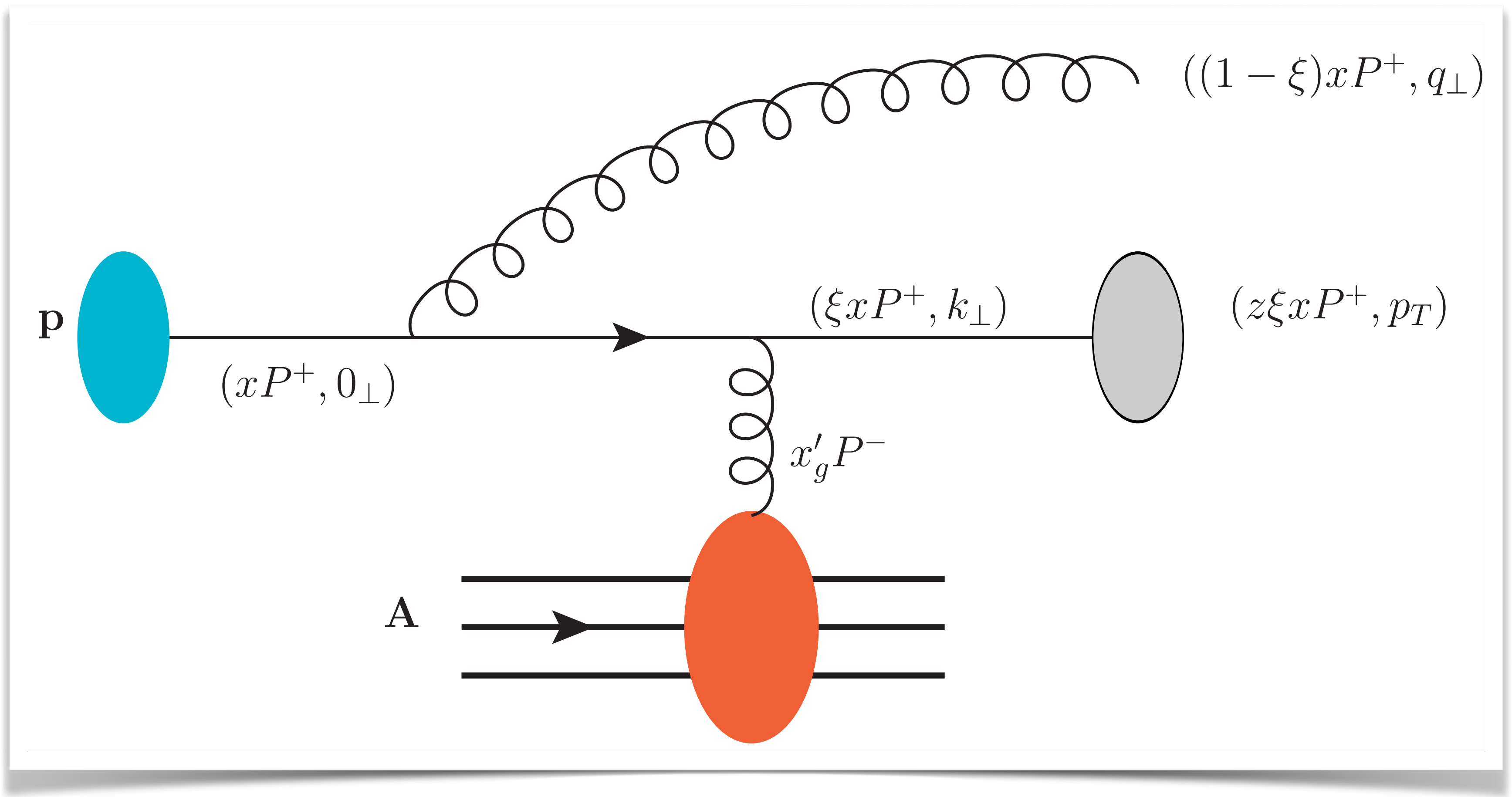
running coupling effect ;

Our approach! ← Threshold resummation

…

The origin of the negativity

Where is the negativity from?



$\tau = xz\xi = P_T e^y / \sqrt{s}$

$y \uparrow, p_T \uparrow$

$\tau \to 1$

$\xi \to 1$ → threshold regime

$H_{ab} \sim \quad P_{qq}(\xi) = \frac{1+\xi^2}{(1-\xi)_+} + \frac{3}{2}\delta(1-\xi)$

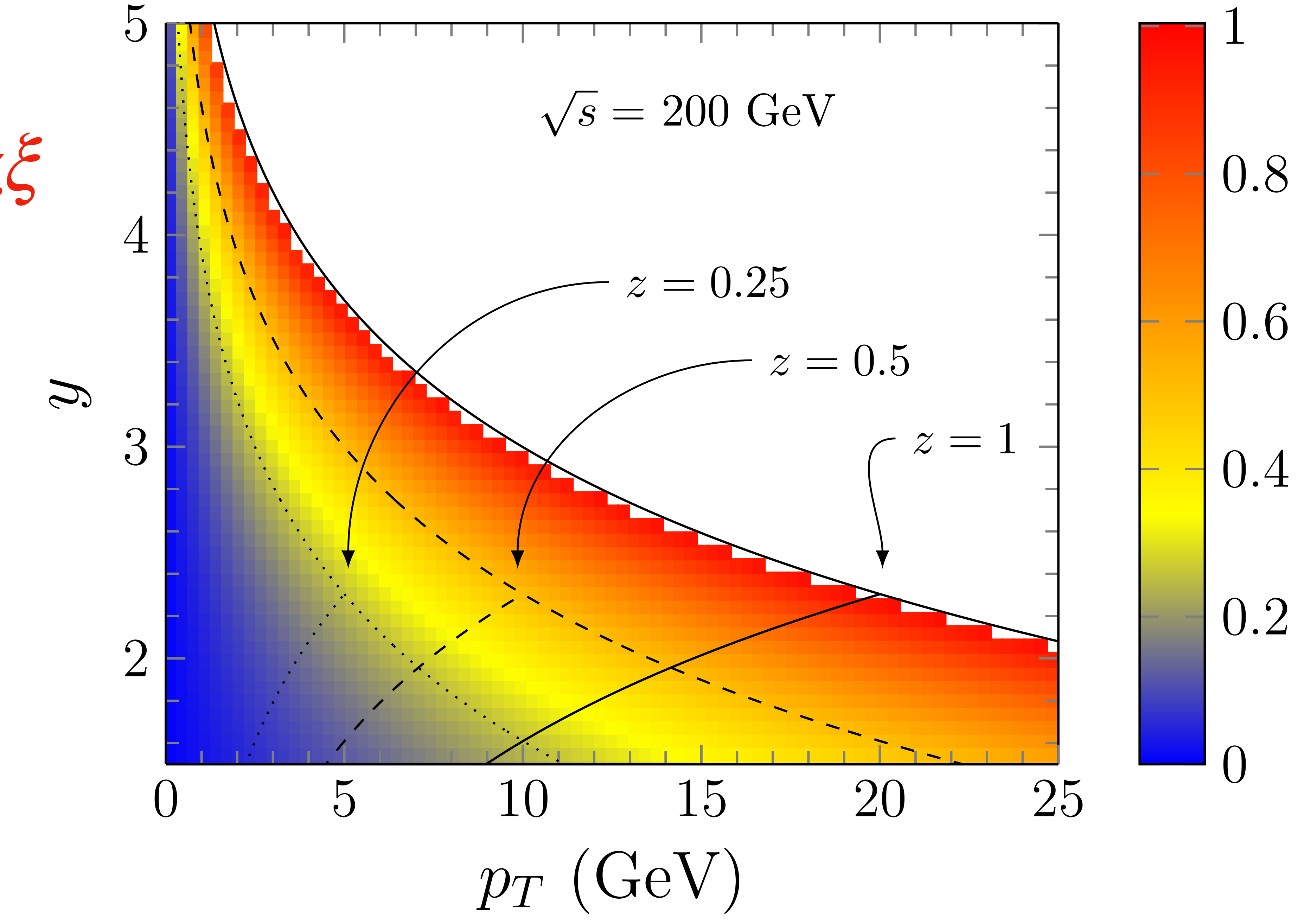
$$P_{gg}(\xi) = 2\left[\frac{\xi}{(1-\xi)_+} + \frac{1-\xi}{\xi} + \xi(1-\xi)\right] + \left(\frac{11}{6} - \frac{2N_f T_R}{3N_C}\right)\delta(1-\xi)$$

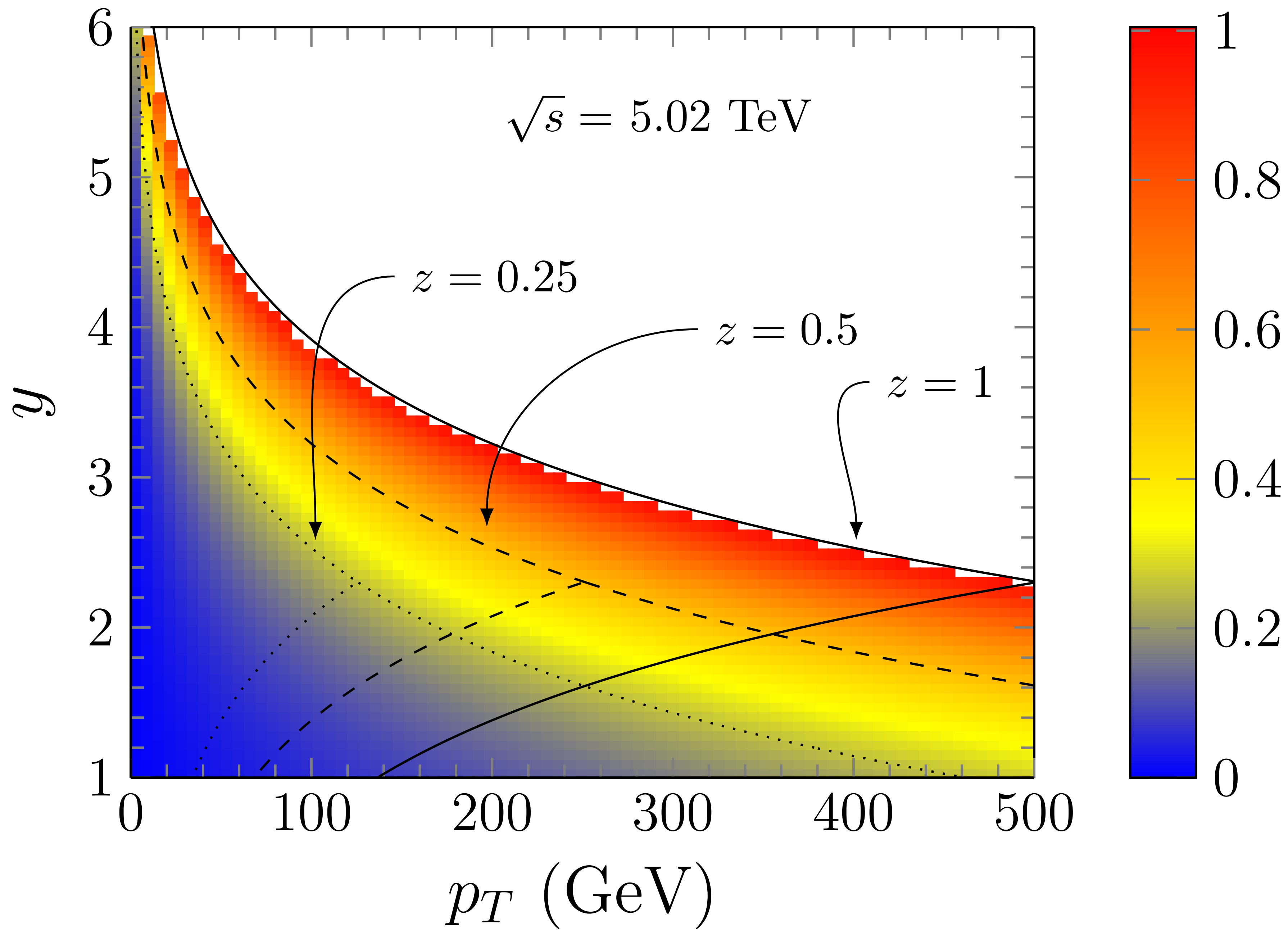
$$\int_\tau^1 d\xi \frac{1}{(1-\xi)_+} f(\xi) = \int_\tau^1 d\xi \frac{f(\xi) - f(1)}{1-\xi} + f(1)\ln(1-\tau)$$

The origin of the negativity

The plots of τ as a function of rapidity and transverse momemtum

$\tau = xz\xi$





- The upper-most solid line $z = 1$.
- The lower-most solid line is corresponding to the boundary given by $x_g = 0.01$.
- The region in the red and yellow indicates where the threshold resummation becomes important.

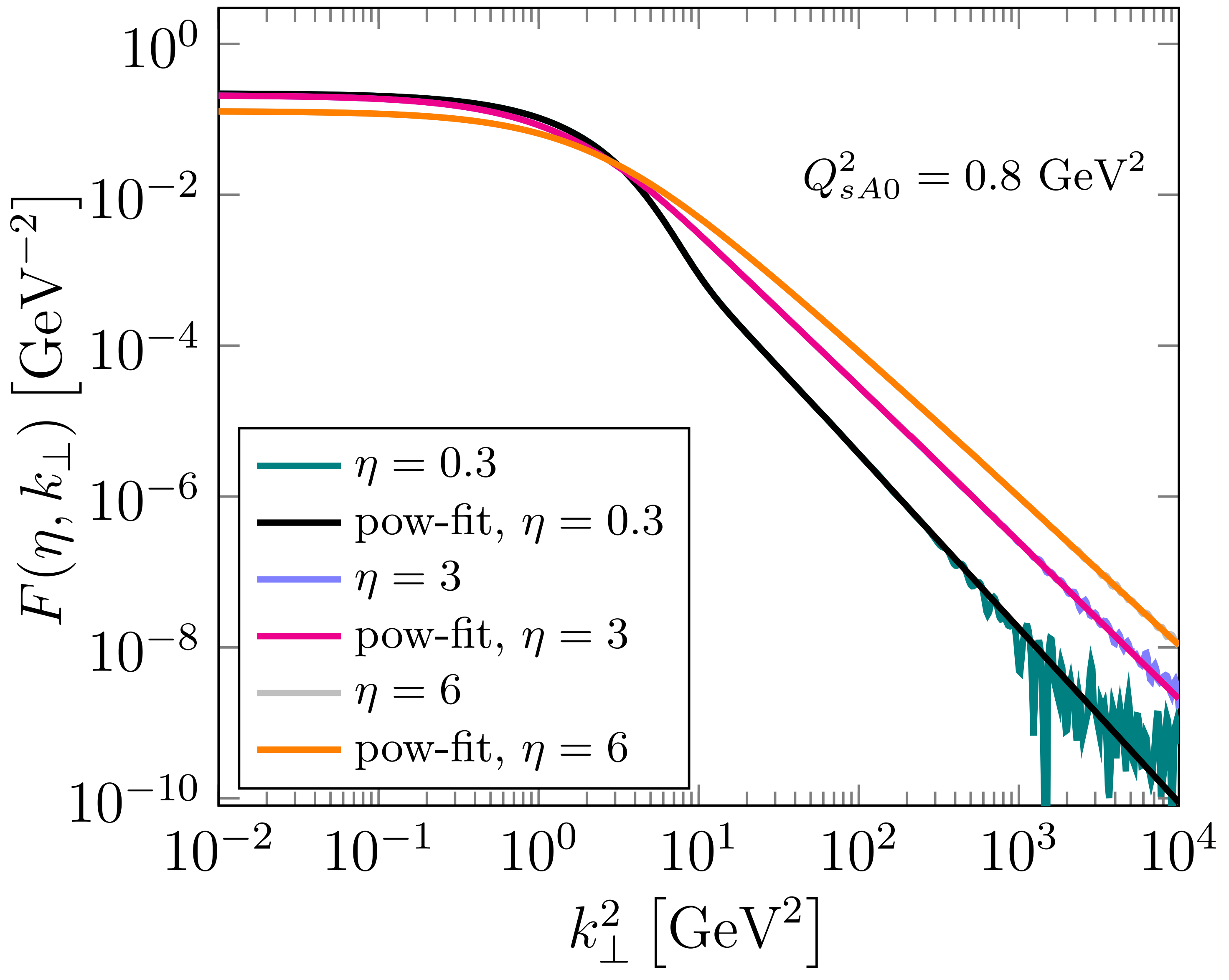
Fourier transform

It is impossible to do the numerical calculation in the coordinate space!

$$\frac{d\sigma}{d\mathcal{P}.\mathcal{S}.} \propto \int \frac{d^2 r_\perp}{(2\pi)^2} \exp[-i\vec{k}_T \cdot \vec{r}_\perp]$$

numerical FT becomes unstable at large k_T

$P(\xi) \otimes \ln \frac{\mu^2}{\mu_r^2}$ $\sigma_0 \otimes \ln \frac{k_T^2}{\mu_r^2}$ $\sigma_0 \otimes \ln^2 \frac{k_T^2}{\mu_r^2}$ $\mu_r \equiv c_0/r_\perp$



Fourier transform

Fourier transform cross-section into the momentum space

$$\int \frac{\mathrm{d}^2 r_\perp}{(2\pi)^2} e^{-ik_\perp \cdot r_\perp} S^{(2)}(r_\perp) \ln \frac{c_0^2}{r_\perp^2 \mu^2} = \frac{1}{\pi} \int \frac{\mathrm{d}^2 l_\perp}{l_\perp^2} \left[F(k_\perp - l_\perp) - J_0 \left(\frac{c_0}{\mu} |l_\perp| \right) F(k_\perp) \right]$$

$$= \frac{1}{\pi} \int \frac{\mathrm{d}^2 l_\perp}{l_\perp^2} \left[F(k_\perp - l_\perp) - \frac{\Lambda^2}{\Lambda^2 + l_\perp^2} F(k_\perp) \right] + F(k_\perp) \ln \frac{\Lambda^2}{\mu^2}$$

coordinate space: $P(\xi) \otimes \ln \frac{\mu^2}{\mu_r^2}$ | $\sigma_0 \otimes \ln \frac{k_T^2}{\mu_r^2}$ | $\sigma_0 \otimes \ln^2 \frac{k_T^2}{\mu_r^2}$

momentum space: $P(\xi) \otimes [\ln \frac{\mu^2}{\Lambda^2} + I_1(\Lambda)]$ | $\sigma_0 \otimes [\ln \frac{k_T^2}{\Lambda^2} + I_1(\Lambda)]$ | $\sigma_0 \otimes [\ln^2 \frac{k_T^2}{\Lambda^2} + I_2(\Lambda)]$

collinear logarithm (first box); soft logarithm (second and third boxes)

Threshold resummation

Two kinds of methods to resum the collinear logarithms

1. Reverse-evolution approach **DGLAP**

$$\begin{bmatrix} q(x_p,\mu) \\ g(x_p,\mu) \end{bmatrix} + \frac{\alpha_s}{2\pi}\ln\frac{\Lambda^2}{\mu^2}\int_{x_p}^{1}\frac{d\xi}{\xi}\begin{bmatrix} C_F\mathcal{P}_{qq}(\xi) & T_R\mathcal{P}_{qg}(\xi) \\ C_F\mathcal{P}_{gq}(\xi) & N_C\mathcal{P}_{gg}(\xi) \end{bmatrix}\begin{bmatrix} q(x_p/\xi,\mu) \\ g(x_p/\xi,\mu) \end{bmatrix} \Rightarrow \begin{bmatrix} q(x_p,\Lambda) \\ g(x_p,\Lambda) \end{bmatrix}$$

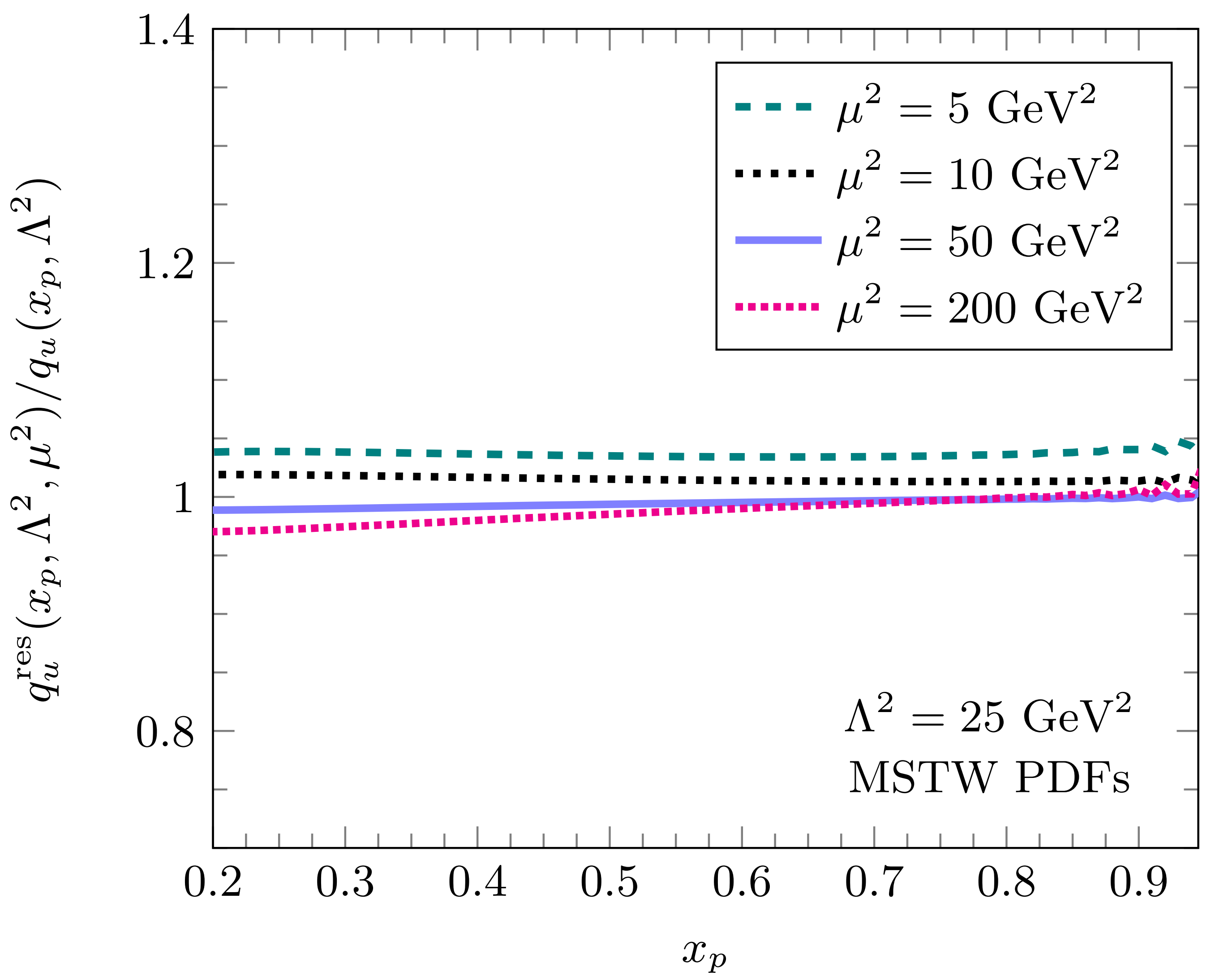
2. Renormalization group equation approach **RGE**

$$P_{qq}(N) = \int_0^1 d\xi\,\xi^{N-1}P_{qq}(\xi) = -2\gamma_E - 2\ln N + \frac{3}{2} + \mathcal{O}\left(\frac{1}{N}\right)$$

$$q^{\rm res}(N) = q(N)\exp\left[-\frac{\alpha_s}{\pi}C_F\ln\frac{\Lambda^2}{\mu^2}(\gamma_E - \frac{3}{4} + \ln N)\right]$$

$$\Delta^q(\Lambda^2,\mu^2,\omega) = \frac{e^{-\gamma^q_{\Lambda,\mu}(\gamma_E-\frac{3}{4})}}{\Gamma(\gamma^q_{\Lambda,\mu})}\omega^{\gamma^q_{\Lambda,\mu}-1}, \qquad \omega \equiv \ln\frac{1}{\xi}, \qquad \gamma^q_{\Lambda,\mu} = C_F\int_{\mu^2}^{\Lambda^2}\frac{d\mu'^2}{\mu'^2}\frac{\alpha_s(\mu'^2)}{\pi}$$

$$\frac{d\Delta^q(\Lambda^2,\mu^2,\omega)}{d\ln\mu^2} = -\frac{\alpha_s C_F}{\pi}\left[\ln\omega + \frac{3}{4}\right]\Delta^q(\Lambda^2,\mu^2,\omega) + \frac{\alpha_s C_F}{\pi}\int_0^\omega d\omega'\frac{\Delta^q(\Lambda^2,\mu^2,\omega) - \Delta^q(\Lambda^2,\mu^2,\omega')}{\omega-\omega'}$$



Two approaches are numerically equivalent!

Threshold resummation

Resummation of the soft/Sudakov logarithms

fixed coupling:

$$S_{\text{Sud}}^{qq} = -\frac{\alpha_s}{2\pi} C_F \ln^2 \frac{k_\perp^2}{\Lambda^2} + 3\frac{\alpha_s}{2\pi} C_F \ln \frac{k_\perp^2}{\Lambda^2}$$

running coupling:

$$S_{\text{Sud}}^{qq} = C_F \int_{\Lambda^2}^{k_\perp^2} \frac{\mathrm{d}\mu^2}{\mu^2} \frac{\alpha_s(\mu^2)}{\pi} \ln \frac{k_\perp^2}{\mu^2} - 3C_F \int_{\Lambda^2}^{k_\perp^2} \frac{\mathrm{d}\mu^2}{\mu^2} \frac{\alpha_s(\mu^2)}{2\pi}$$

NLO resummed

$$\frac{\mathrm{d}\sigma_{\text{resummed}}}{\mathrm{d}y\mathrm{d}^2p_\perp} = S_\perp \int_\tau^1 \frac{\mathrm{d}z}{z^2} x_p q(x_p, \Lambda^2) D_{h/q}(z, \Lambda^2) F(k_\perp) e^{-S_{\text{Sud}}^{qq}}$$

Final results

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\mathrm{d}^2p_\perp} = \frac{\mathrm{d}\sigma_{\text{resummed}}}{\mathrm{d}y\mathrm{d}^2p_\perp} + \frac{\mathrm{d}\sigma_{\text{NLO matching}}}{\mathrm{d}y\mathrm{d}^2p_\perp} + \frac{\mathrm{d}\sigma_{\text{Sud matching}}}{\mathrm{d}y\mathrm{d}^2p_\perp}$$

NLO small corrections from the hard factor

$$S_{\text{Sud}}^{qq} - C_F \frac{\alpha_s}{2\pi}\left(\ln^2 \frac{k_\perp^2}{\Lambda^2} - 3\ln \frac{k_\perp^2}{\Lambda^2}\right)$$

Choosing Λ^2

Saddle point approximation

$$\frac{d\sigma^{qq}_{\text{resummed}}}{dy d^2 p_T} = S_\perp \int_\tau^1 \frac{dz}{z^2} \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-ik_\perp \cdot r_\perp} S^{(2)}(r_\perp) e^{-S^{qq}_{\text{Sud}}} \int_{x_p}^1 \frac{dx}{x} q(x,\mu) \frac{e^{(3/4-\gamma_E)\gamma^q_{\mu_r,\mu}}}{\Gamma(\gamma^q_{\mu_r,\mu})} \left[\ln \frac{x}{x_p}\right]_*^{\gamma^q_{\mu_r,\mu}-1}$$

$$\times \int_z^1 \frac{dz'}{z'} D_{h/q}(z') \frac{e^{(3/4-\gamma_E)\gamma^q_{\mu_r,\mu}}}{\Gamma(\gamma^q_{\mu_r,\mu})} \left[\ln \frac{z'}{z}\right]_*^{\gamma^q_{\mu_r,\mu}-1},$$

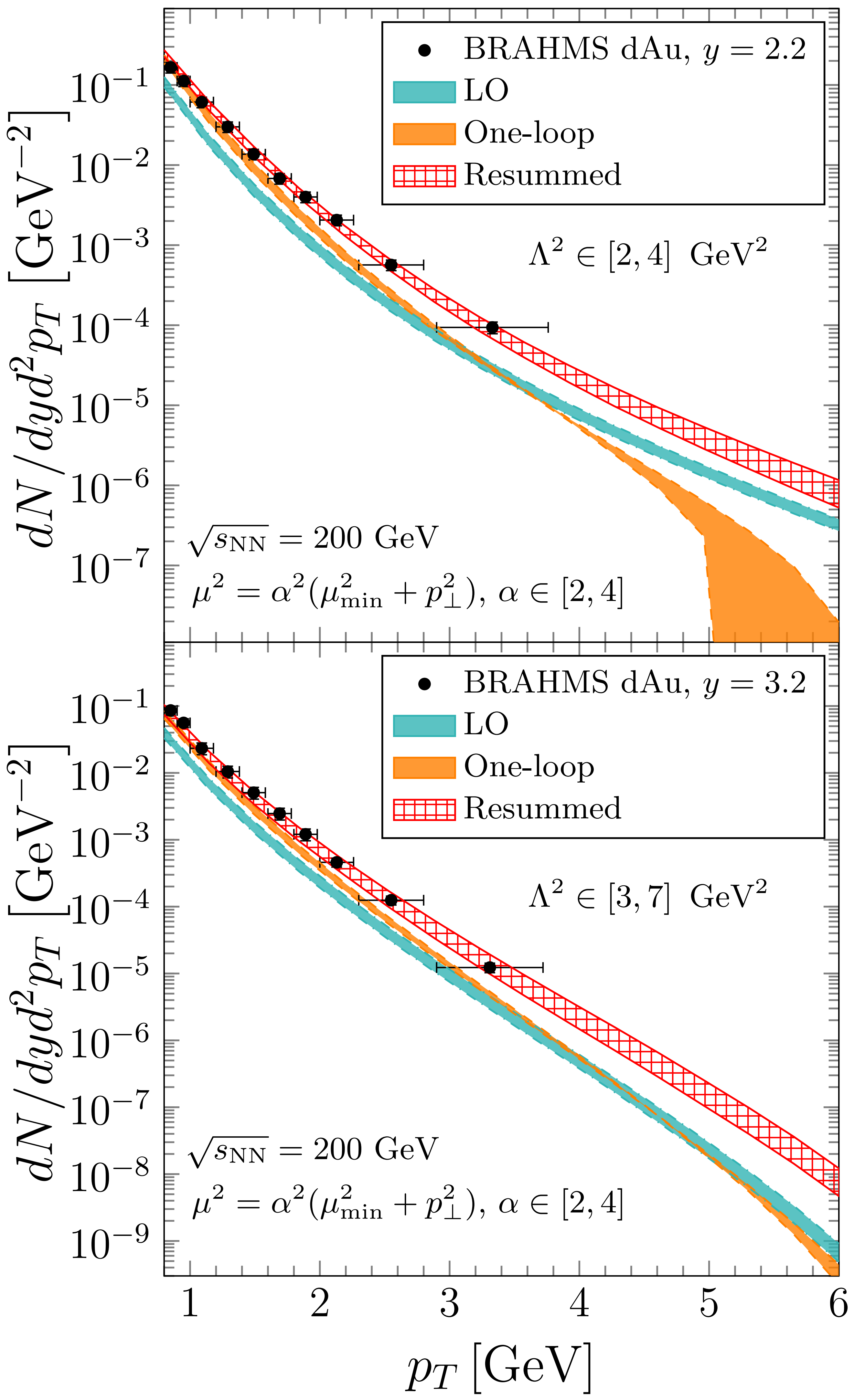
$$\Lambda \sim \mu_r = \frac{c_0}{r_\perp}$$

$$\Lambda^2 \approx \max\left\{\Lambda^2_{\text{QCD}} \left[\frac{k_\perp^2(1-\xi)}{\Lambda^2_{\text{QCD}}}\right]^{\frac{C_F}{C_F+N_c\beta_0}}, Q_s^2\right\}$$

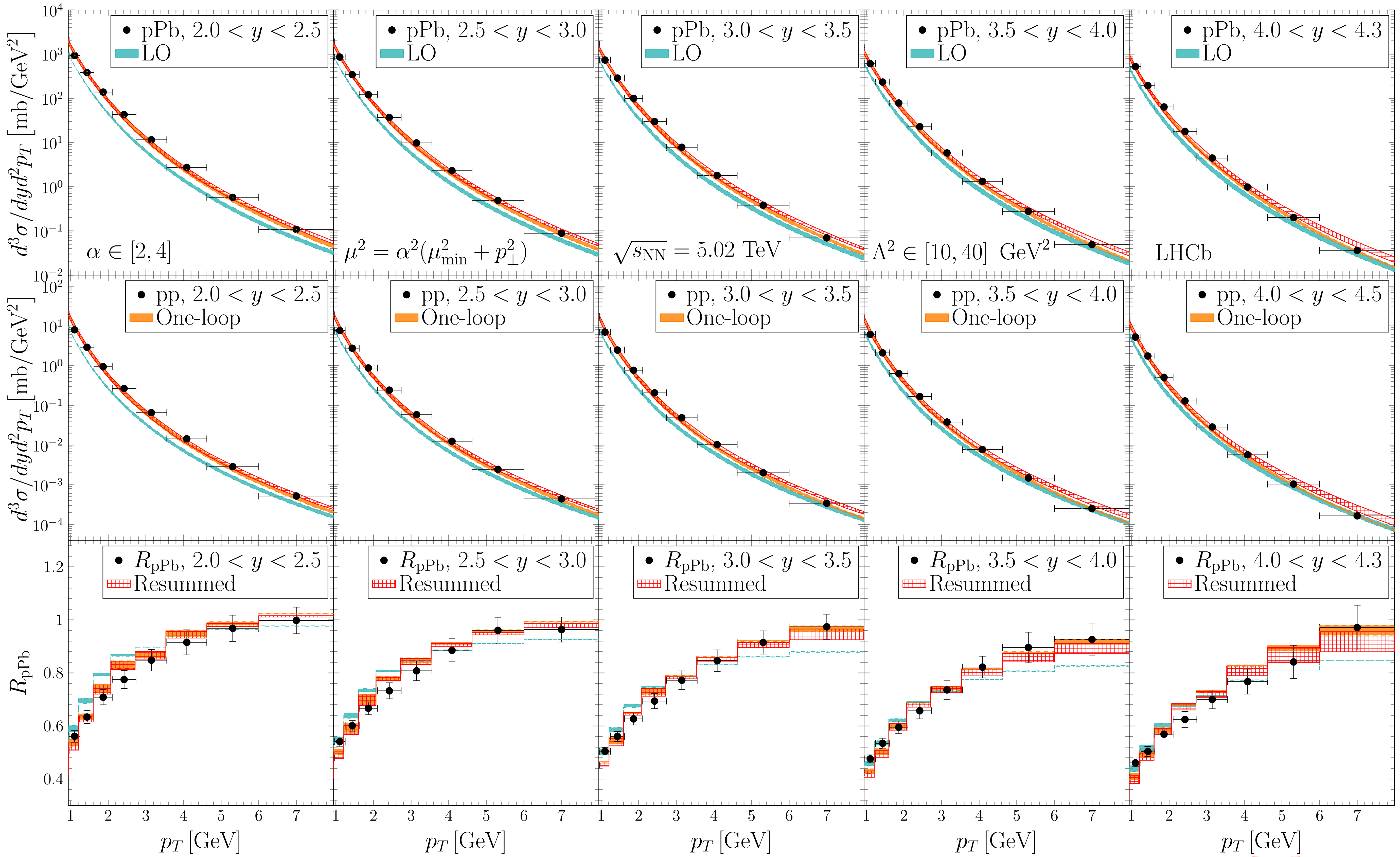
when the saturation effect is strong

Numerical results

BRAHMS



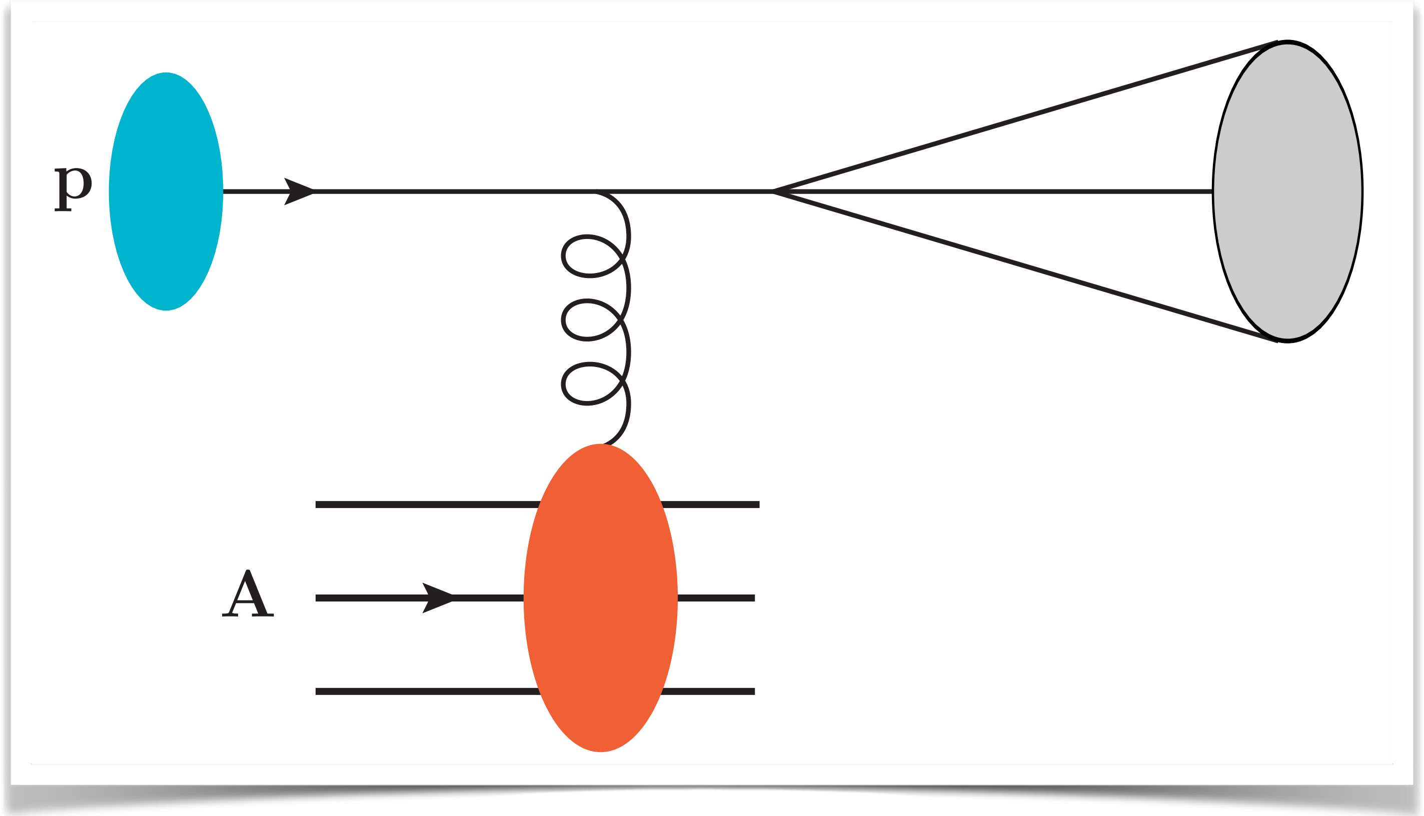
LHCb



- ☑ Threshold resummation solves the negative problem.
- ☑ Our calculation can also describe LHCb measurement.
- ☑ The suppression of R_{pPb} reflects the gluon saturation.

LO jet production

LO contribution



$$\frac{d\sigma_{\mathrm{LO}}^{p+A\to \mathrm{jet}+X}}{d\eta d^2P_J} = \int_\tau^1 \frac{dz}{z^2} \sum_f x q_f(x) F(q_\perp) J_q(z)$$

VS

$$\frac{d\sigma_{\mathrm{LO}}^{pA\to hX}}{d^2p_\perp dy_h} = \int_\tau^1 \frac{dz}{z^2} \sum_f x_p q_f(x_p) F(k_\perp) D_{h/q}(z)$$

Collinear jet function $J_q^{(0)}(z) = \delta(1-z)$

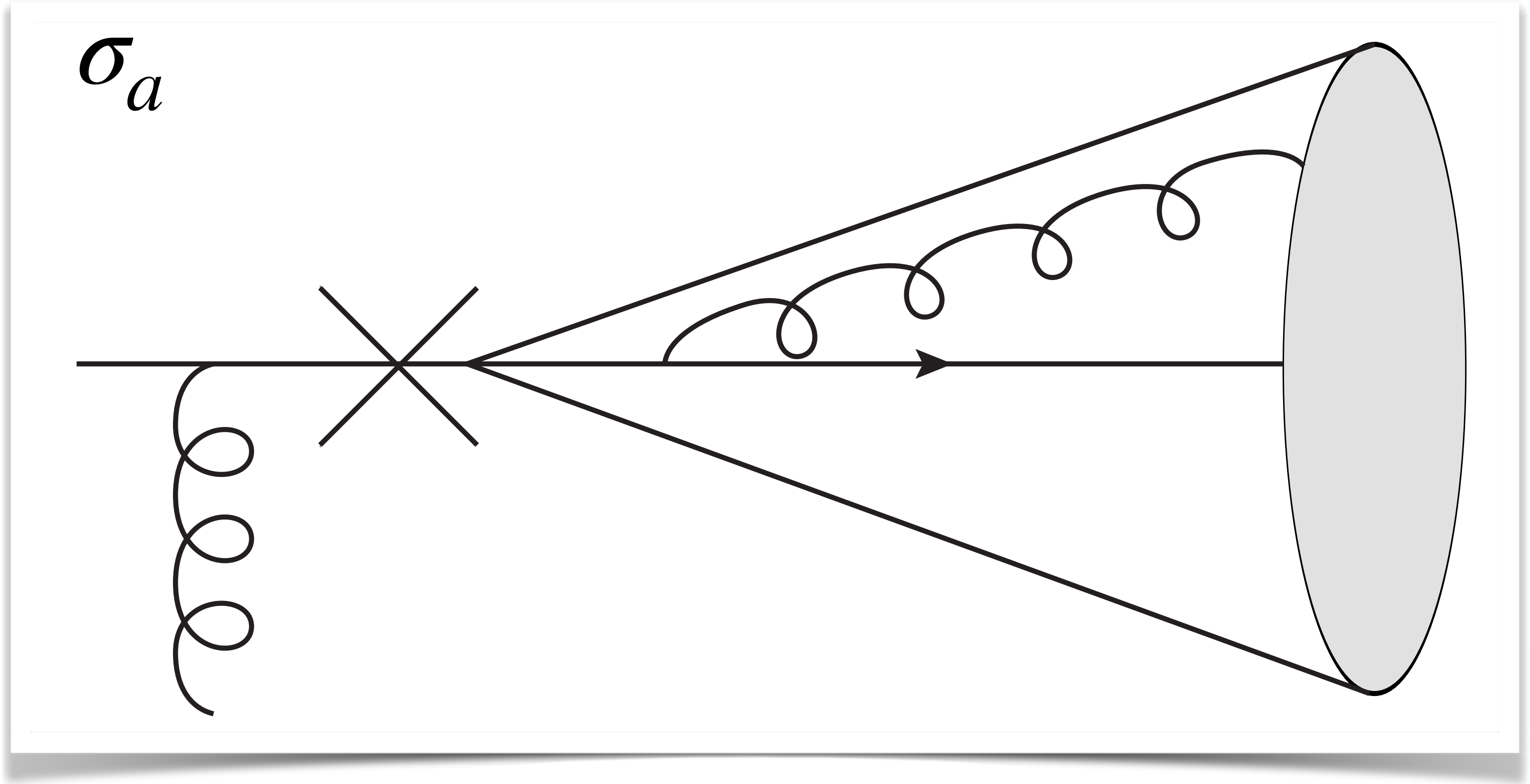
Initial condition

Hadron fragmentation function $D_{h/q}(z)$

NLO jet productions

NLO contribution of the final state radiation

In-cone contribution: when the final state quark and the radiated gluon stay close to each other



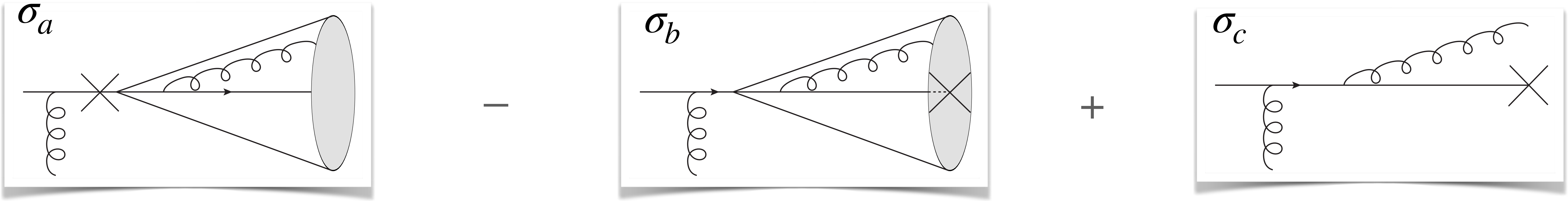
We need to integrate the relative momentum !

$$R_{qg} = \sqrt{\Delta\eta^2 + \Delta\phi^2}$$

Kinematic constraint: $R_{qg} \leq R$ → $\frac{p_\perp^2}{\xi(1-\xi)} \leq l_\perp k_\perp R^2 \simeq \frac{\xi(1-\xi)}{z^2} P_J^2 R^2 = \xi(1-\xi) q_\perp^2 R^2$

NLO jet productions

NLO contribution from the final state radiation



$$\sigma_{\text{final}}^{\text{jet}} = \sigma_a + (\sigma_c - \sigma_b) + \sigma_{\text{virt}}$$

$$\frac{d^3\sigma}{d\eta\, d^2P_J} = -\frac{\alpha_s C_F}{2\pi}\int_x^1 d\xi \frac{1}{\xi^2}\sigma_{\text{LO}}\left(\frac{x}{\xi},\frac{q_\perp}{\xi}\right)\left[(1+\xi^2)\left(\frac{\ln\frac{(1-\xi)^2}{\xi^2}}{1-\xi}\right)_+ - \frac{1+\xi^2}{(1-\xi)_+}\ln\frac{c_0^2}{q_\perp^2 r_\perp^2} - \mathcal{P}_{qq}(\xi)\ln\frac{1}{R^2}\right] + \frac{\alpha_s C_F}{2\pi}\sigma_{\text{LO}}(x,q_\perp)\left(6-\frac{4}{3}\pi^2\right),$$

Unique terms from jet algorithms

Comparing with hadron productions

$$\ln\frac{1}{R^2} \Leftrightarrow -\frac{1}{\epsilon} + \ln\frac{q_\perp^2}{\mu^2}$$

Our one-loop calculations are consistent with hadron calculations.

Chirilli, Xiao and Yuan, PRL, 2012

Our one-loop calculations are consistent with Liu's paper.

Liu, Xie, Kang, and Liu, JHEP, 2022

Threshold resummation

Threshold resummation is still needed!

process	collinear log(initial)	single log	double log	collinear log(final)
$q \to q$	$\mathcal{P}_{qq}(\xi) \ln \frac{\Lambda^2}{\mu^2}$	$\ln \frac{q_\perp^2}{\Lambda^2}$	$\ln^2 \frac{q_\perp^2}{\Lambda^2}$	$\frac{1}{\xi^2}\mathcal{P}_{qq}(\xi) \ln \frac{\Lambda^2}{\mu_J^2}$
$g \to g$	$\mathcal{P}_{gg}(\xi) \ln \frac{\Lambda^2}{\mu^2}$	$\ln \frac{q_\perp^2}{\Lambda^2}$	$\ln^2 \frac{q_\perp^2}{\Lambda^2}$	$\frac{1}{\xi^2}\mathcal{P}_{gg}(\xi) \ln \frac{\Lambda^2}{\mu_J^2}$
$q \to g$	$\mathcal{P}_{gq}(\xi) \ln \frac{\Lambda^2}{\mu^2}$	/	/	$\frac{1}{\xi^2}\mathcal{P}_{gq}(\xi) \ln \frac{\Lambda^2}{\mu_J^2}$
$g \to q$	$\mathcal{P}_{qg}(\xi) \ln \frac{\Lambda^2}{\mu^2}$	/	/	$\frac{1}{\xi^2}\mathcal{P}_{qg}(\xi) \ln \frac{\Lambda^2}{\mu_J^2}$

Reusmmation of the collinear logarithms from the final state radiation

$$\frac{\partial \mathcal{J}_q(z,\Lambda)}{\partial \ln \Lambda^2} = \frac{\alpha_s}{2\pi}\int_z^1 \frac{\mathrm{d}\xi}{\xi}\left[C_F \mathcal{P}_{qq}(\xi)\mathcal{J}_q\left(\frac{z}{\xi},\Lambda\right) + C_F \mathcal{P}_{gq}(\xi)\mathcal{J}_g\left(\frac{z}{\xi},\Lambda\right)\right]$$

Initial condition: $J_q^{(0)} = \delta(1-z)$ at scale $\mu_J = P_J R$

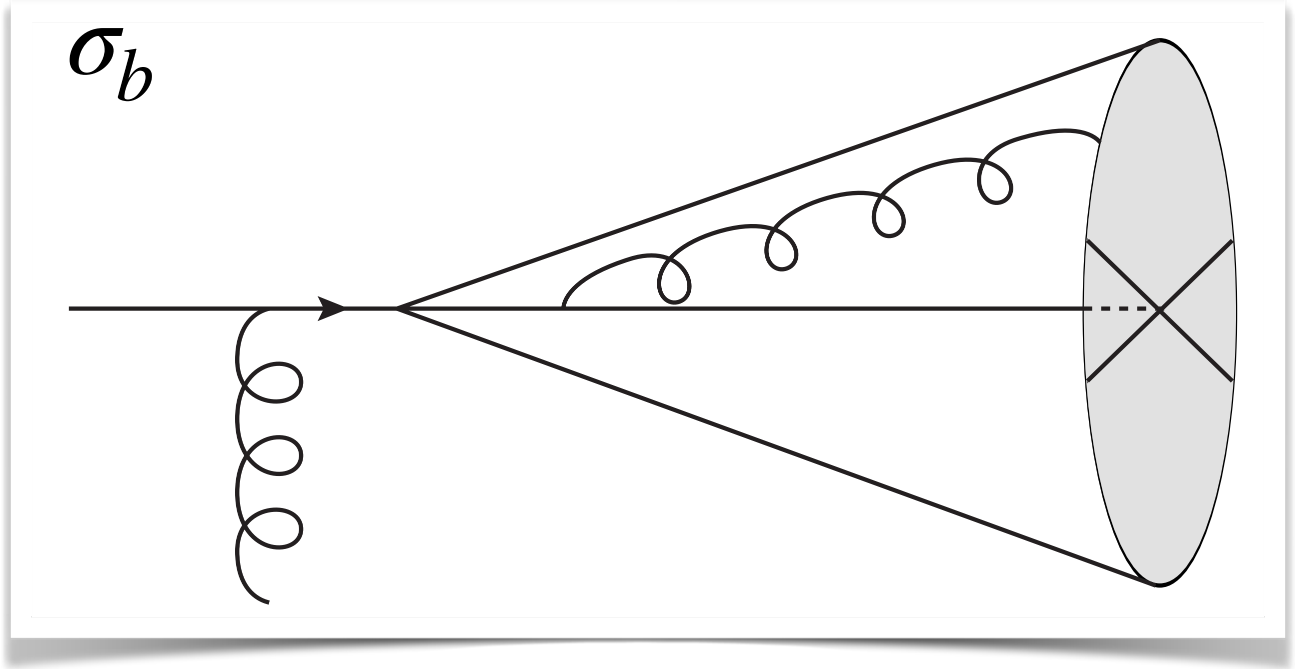
$$\begin{bmatrix} \mathcal{J}_q(z,\mu_J) \\ \mathcal{J}_g(z,\mu_J) \end{bmatrix} + \frac{\alpha_s}{2\pi}\ln\frac{\Lambda^2}{\mu_J^2}\int_z^1 \frac{\mathrm{d}\xi}{\xi}\begin{bmatrix} C_F\mathcal{P}_{qq}(\xi) & C_F\mathcal{P}_{gq}(\xi) \\ T_R\mathcal{P}_{qg}(\xi) & N_C\mathcal{P}_{gg}(\xi) \end{bmatrix}\begin{bmatrix} \mathcal{J}_q(z/\xi,\mu_J) \\ \mathcal{J}_g(z/\xi,\mu_J) \end{bmatrix} \Rightarrow \begin{bmatrix} \mathcal{J}_q(z,\Lambda) \\ \mathcal{J}_g(z,\Lambda) \end{bmatrix}$$

Threshold resummation

Resummation of the soft logarithms

- Sudakov logs from the jet contribution cancel.
- The term which proportional to $\left(\frac{\ln(1-\xi)}{1-\xi}\right)_+$,

  and this stems from final state gluon radiations.
- When $\xi \to 1$ $(\tau \to 1)$, these terms give us $\ln^2 N$.



Only initial state radiation contribution!

$$S_{\text{Sud}}^{qq} = -\frac{\alpha_s}{2\pi}\frac{C_F}{2}\ln^2\frac{k_\perp^2}{\Lambda^2} + \frac{\alpha_s}{2\pi}\frac{3}{2}C_F\ln\frac{k_\perp^2}{\Lambda^2}$$

Note: for hadron productions we have $S_{\text{Sud}}^{qq} = -\frac{\alpha_s}{2\pi}C_F\ln^2\frac{k_\perp^2}{\Lambda^2} + 3\frac{\alpha_s}{2\pi}C_F\ln\frac{k_\perp^2}{\Lambda^2}$

Final results

Final resummed results

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\eta \mathrm{d}^2 P_J} = \frac{\mathrm{d}\sigma_{\text{resummed}}}{\mathrm{d}\eta \mathrm{d}^2 P_J} + \frac{\mathrm{d}\sigma_{\text{NLO matching}}}{\mathrm{d}\eta \mathrm{d}^2 P_J} + \frac{\mathrm{d}\sigma_{\text{Sud matching}}}{\mathrm{d}\eta \mathrm{d}^2 P_J}$$

$$\begin{aligned}\frac{\mathrm{d}\sigma_{\text{resummed}}}{\mathrm{d}\eta \mathrm{d}^2 P_J} =& S_\perp \int_\tau^1 \frac{\mathrm{d}z}{z^2} xq(x,\Lambda^2)\mathcal{J}_q(z,\Lambda^2)F(q_\perp)e^{-S_{\text{Sud}}^{qq}} \\ &+ S_\perp \int_\tau^1 \frac{\mathrm{d}z}{z^2} xg(x,\Lambda^2)\mathcal{J}_g(z,\Lambda^2)\int \mathrm{d}^2 q_{1\perp} F(q_{1\perp})F(q_\perp - q_{1\perp})e^{-S_{\text{Sud}}^{gg}}\end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}\sigma_{\text{Sud matching}}}{\mathrm{d}\eta \mathrm{d}^2 P_J} =& S_\perp \int_\tau^1 \frac{\mathrm{d}z}{z^2} xq(x,\mu^2)\mathcal{J}_q(z,\mu_J^2)F(q_\perp)\left\{S_{\text{Sud}}^{qq} - \left[C_F\frac{\alpha_s}{2\pi}\left(\frac{1}{2}\ln^2\frac{q_\perp^2}{\Lambda^2} - \frac{3}{2}\ln\frac{q_\perp^2}{\Lambda^2}\right)\right]\right\} \\ &+ S_\perp \int_\tau^1 \frac{\mathrm{d}z}{z^2} xg(x,\mu^2)\mathcal{J}_g(z,\mu_J^2)\int \mathrm{d}^2 q_{1\perp} F(q_{1\perp})F(q_\perp - q_{1\perp}) \\ &\times \left\{S_{\text{Sud}}^{gg} - \left[N_c\frac{\alpha_s}{2\pi}\left(\frac{1}{2}\ln^2\frac{q_\perp^2}{\Lambda^2} - 2\beta_0\ln\frac{q_\perp^2}{\Lambda^2}\right)\right]\right\}.\end{aligned}$$

We consider
- $q \to qg$
- $g \to gg$
- $g \to q\bar{q}$
- $q \to gq$

$$\frac{\mathrm{d}\sigma_{\text{NLO matching}}}{\mathrm{d}\eta \mathrm{d}^2 P_J}$$

Too long to show!

Summary

1. **We briefly introduce the color glass condensate effective theory.**
2. **By incorporating the threshold resummation in CGC formalism, we can describe the experimental data from RHIC and LHC.**
3. **We have systematically calculated the complete NLO cross-section for single inclusive jet production in pA collisions at forward rapidity region within the small-x framework.**

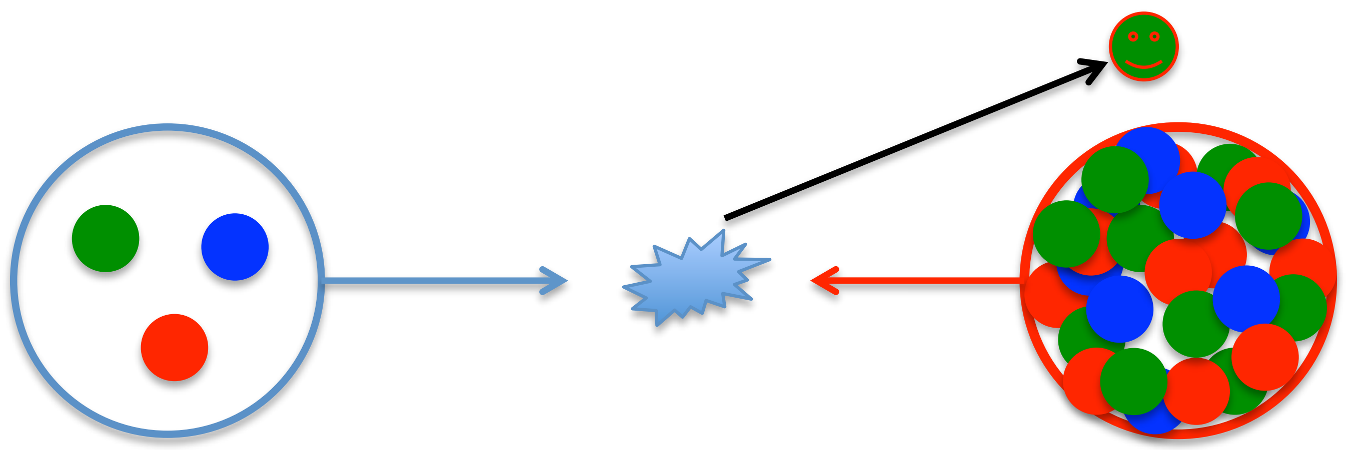
Outlook

1. **The numerical calculation of jet production is in progress.**
2. **We can apply the threshold resummation formalism on many other processes. Such as, the Muller-Navelet jet productions, diffractive processes …**

Thank you!

Backup

Dilute-Dense factorizations



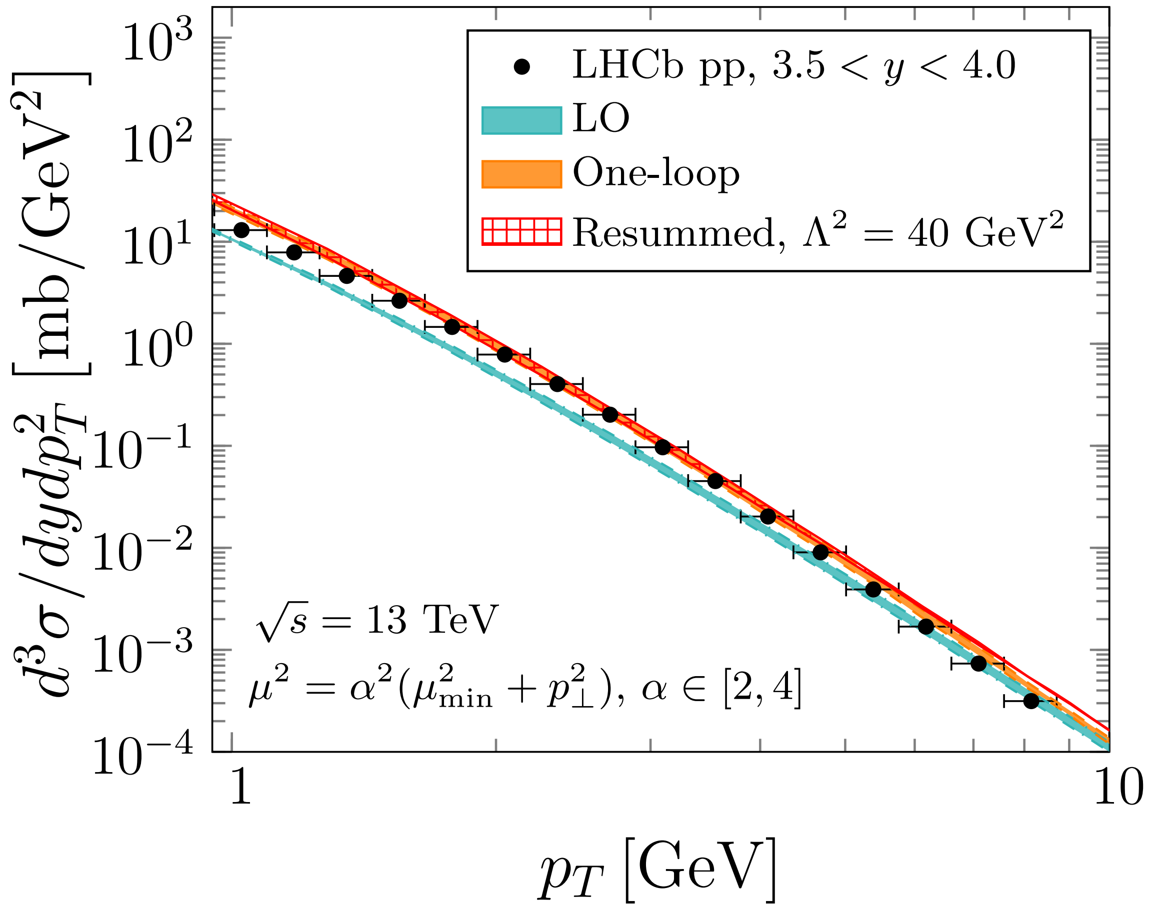
projectile: $x_1 \sim \frac{p_\perp}{\sqrt{s}} e^{+y} \sim 1$ valence

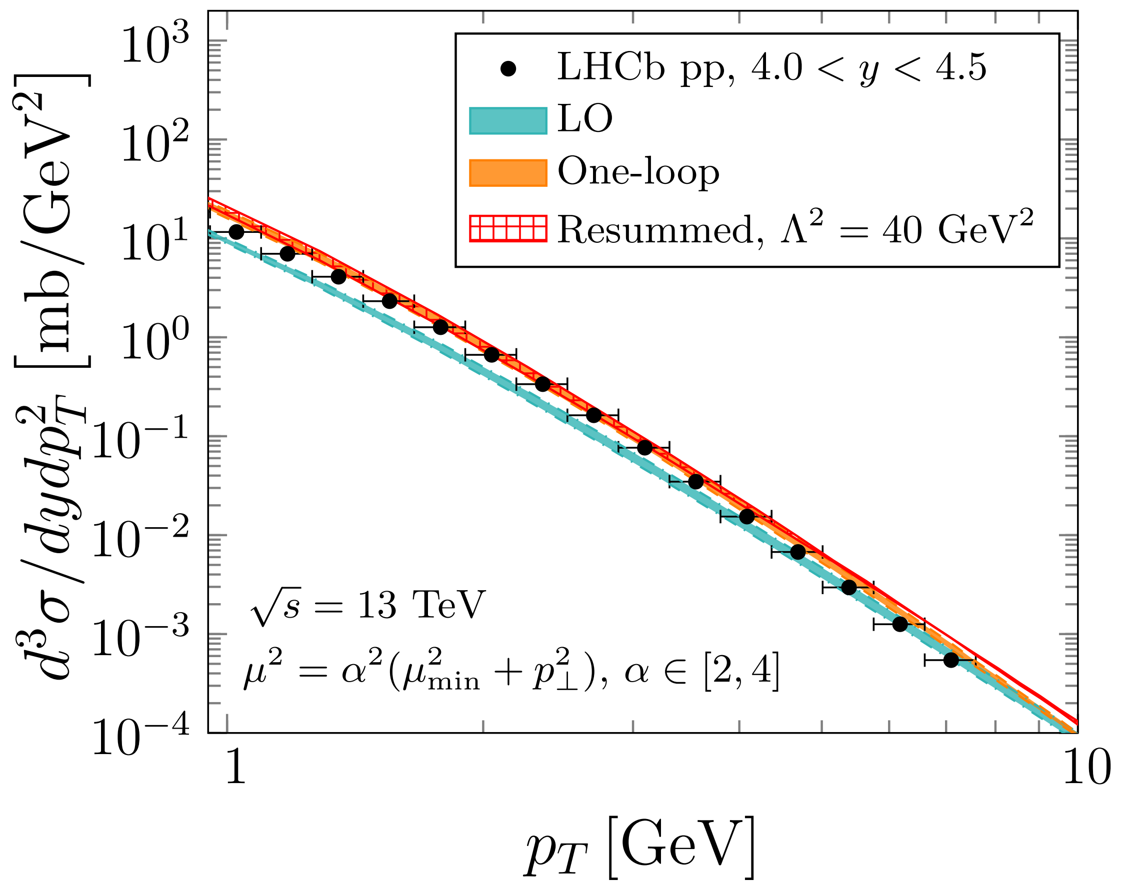
target: $x_2 \sim \frac{p_\perp}{\sqrt{s}} e^{-y} \ll 1$ gluon

- Due to quantum evolution, PDF and FF changes with scale. This introduces large theoretical uncertainties in $xf(x)$ and $D(z)$. Choice of the scale at LO requires information at NLO.
- LO cross section is always a monotonic function of μ, thus it is just order of magnitude estimate.
- NLO calculation significantly reduces the scale dependence. More reliable.

Backup

More results for forward rapidity region





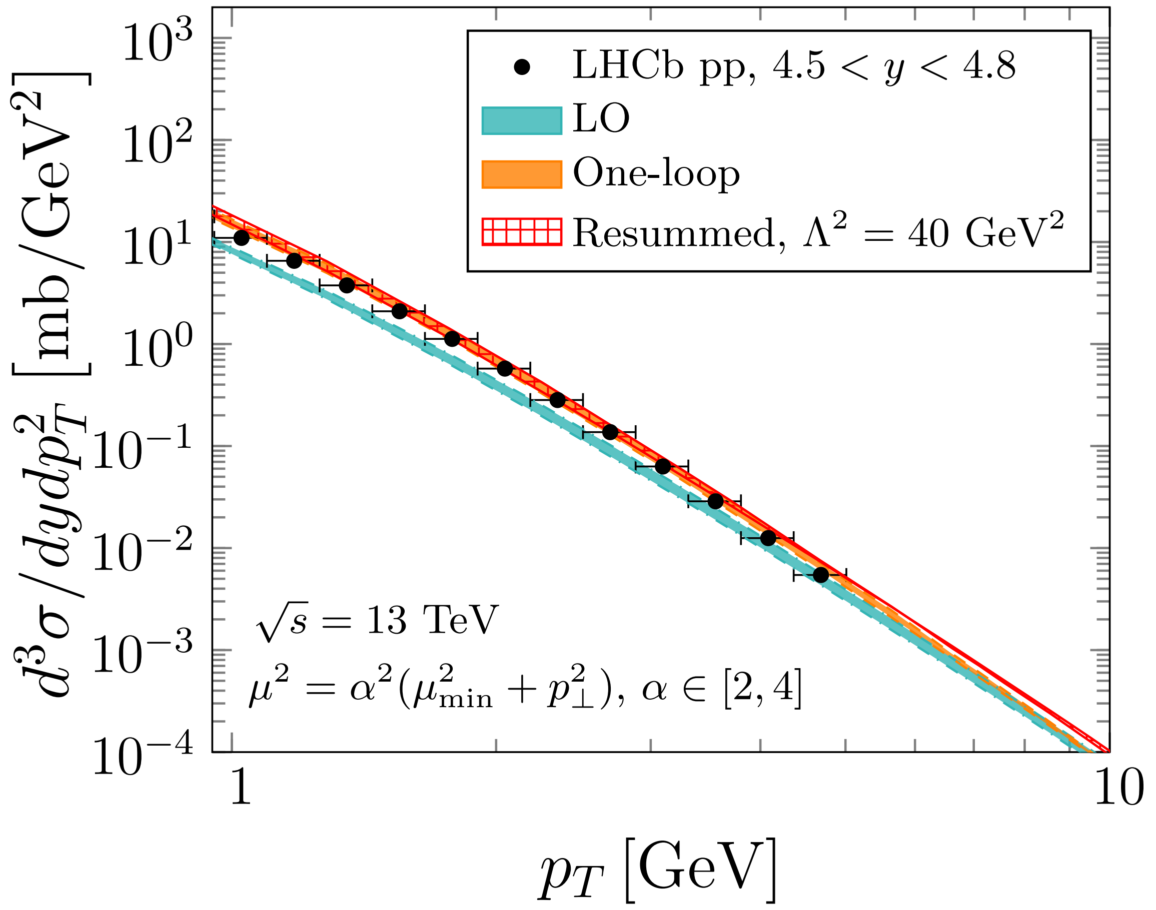


TABLE I. Values of Λ^2 in calculating the cross-sections at $\sqrt{s_{\rm NN}} = 200$ GeV at RHIC.

Rapidity	$y = 2.2$		$y = 3.2$		$y = 4$	
Collisional systerms	dAu	pp	dAu	pp	dAu	pp
Λ^2 (GeV^2)	$2 \sim 4$	~ 1	$3 \sim 7$	~ 2	$5 \sim 9$	~ 3

TABLE II. List of input values of parameters used in our numerical evaluations.

Experiment	BRAHMS	STAR	ATLAS/ALICE	LHCb
Hadron h	h^-	π^0	$h^\pm$	prompt $h^\pm$
σ^h/σ^π	1.3	1	2.2	2.2×0.85

Backup

Jet algorithm: k_t-type jet algorithm (base on the narrow jet approximation).

Indeed, we need to specify the jet algorithm. Nowadays, the popular jet algorithms are kt-type jet algorithms, such as, the kt, anti-kt, and Cambridge/Aachen algorithms. In our calculation, we computed the jet cross-section based on the narrow jet approximation [Phys.Rev.D 70 (2004) 034010] together with the kt-type jet algorithm [1209.1785]

SOLO=Saturation physics at One Loop Order

Backup

$$\frac{1}{2\pi p_T}\frac{\mathrm{d}^2 N^{\mathrm{pA}\to h\mathrm{X}}}{\mathrm{d}y\mathrm{d}p_T} = \frac{1}{\sigma_{\mathrm{inel}}}\frac{1}{2\pi p_T}\frac{\mathrm{d}^2\sigma^{\mathrm{pA}\to h\mathrm{X}}}{\mathrm{d}y\mathrm{d}p_T}$$

$$R_{pPb} = \frac{1}{A}\frac{d^2\sigma_{pPb}/dp_T dy}{d^2\sigma_{pp}/dp_T dy}.$$