Confronting Jet-Medium Interactions in a Weakly Coupled QGP

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Based on 2312.11731
with **Jacopo Ghiglieri, Philipp Schicho** and **Niels Schlusser**
See also PhD thesis 2311.04988

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Subatech

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QCD Masterclass – Saint Jacut de la Mer

Outline

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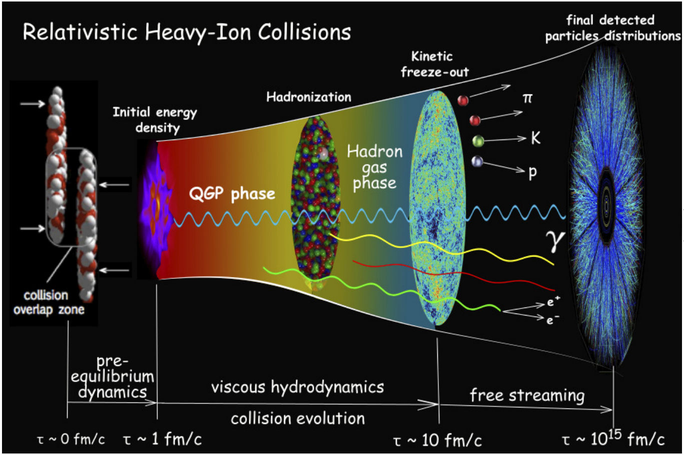
Gold, Lead ions smashed together at Large Hadron Collider (LHC) and Relativistic Heavy Ion Collider (RHIC)
$\Rightarrow$Heavy Ion Collisions (HIC)



[Heinz, 2013]

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  $\Rightarrow$ Probe created at beginning of collision $\Rightarrow$ **Jets!**

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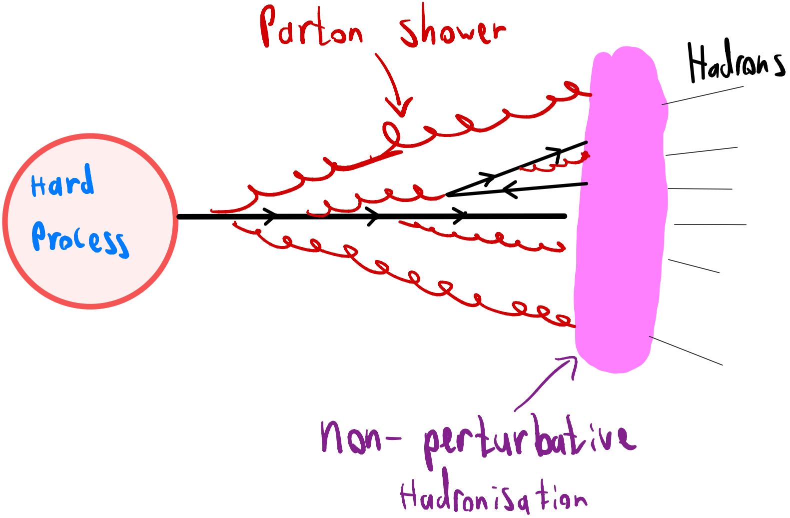
- Next best thing?
  $\Rightarrow$ Probe created at beginning of collision $\Rightarrow$ **Jets!**
- Can also use other hard probes such as **quarkonium, photons, dileptons**

Jets in "Vacuum"



Jets in "Vacuum"

Jets are relatively well-understood in the vacuum – used in searches for physics beyond the standard model

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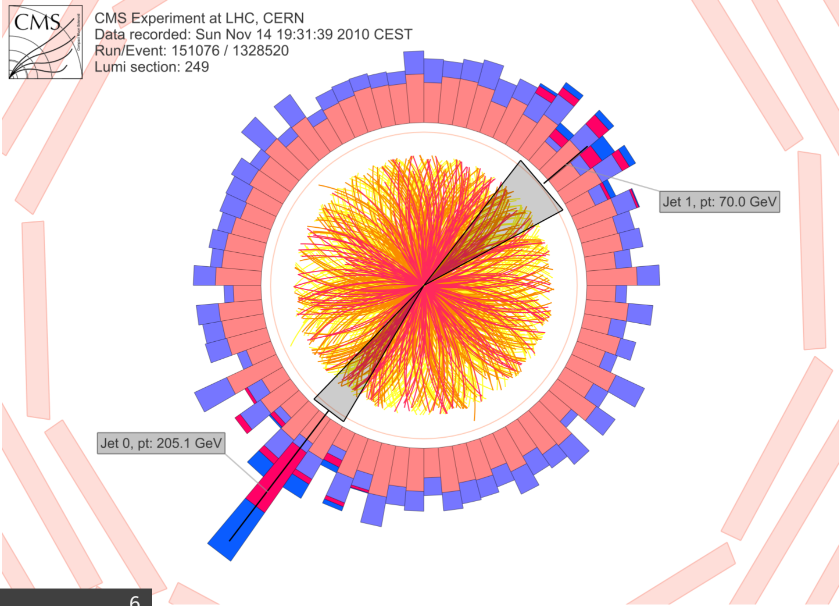
⇒ Provides nice benchmark to understand how jets are **quenched** in **heavy-ion collisions**!


CMS
CMS Experiment at LHC, CERN
Data recorded: Sun Nov 14 19:31:39 2010 CEST
Run/Event: 151076 / 1328520
Lumi section: 249
Jet 1, pt: 70.0 GeV
Jet 0, pt: 205.1 GeV

Moving Forward: Theory

Want to extract QGP properties
$\Rightarrow$ Need more precise theory calculations

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$\Rightarrow$ Need more precise theory calculations

- Include contributions from pre-equilibrium and Glasma phases
- Inclusion of sub-eikonal corrections
- Relaxation of static medium assumption
- Inclusion of **higher order corrections**

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Want to maintain connection with first principles

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- Finite-temperature lattice QCD should be preferred choice
  $\Rightarrow$ Well-suited for computation of thermodynamic (Euclidean) quantities

  $\Rightarrow$ In general, **not** well-suited to computation of real-time (Minkowskian) quantites,
  (see [Boguslavski et al., 2023] for recent progress)

What is Thermal Field Theory?

Perturbative quantum field theory at finite temperature

Imaginary time formalism

Frequency integral becomes discrete sum over Matsubara modes $\omega_n^{\mathrm{B}} = 2\pi n T$ or $\omega_n^{\mathrm{F}} = 2\pi(n+1)T$

Real time formalism

Doubling of degrees of freedom

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Imaginary time formalism → Euclidean quantities e.g Thermodynamics

Real time formalism → Minkowskian quantities e.g Particle production rate

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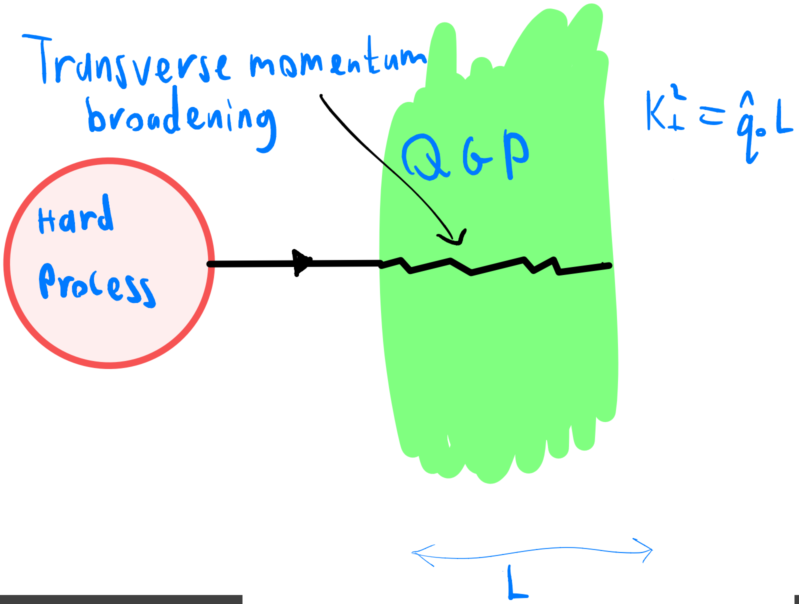
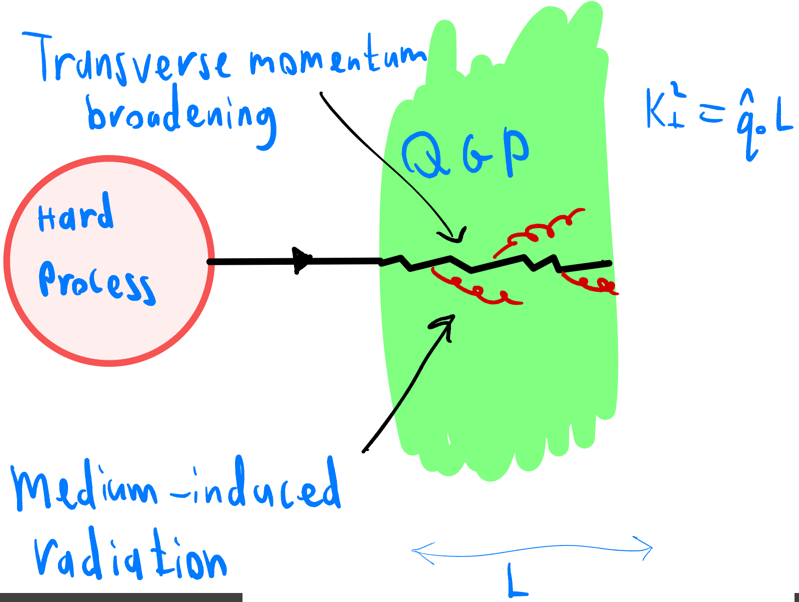
- Parton has energy, P, much greater than T, temperature of plasma


Transverse momentum
broadening
QGP
$k_\perp^2 \simeq \hat{q}_0 L$
Hard
Process
L

Jet Energy Loss



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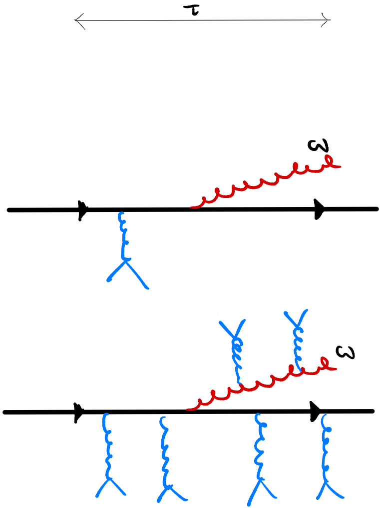
Not straightforward! But why??

How exactly is bremsstrahlung triggered?



Depends on quantum mechanical formation time, $\tau \sim \omega/k_\perp^2$ associated with the radiated gluon:

- Radiated gluon triggered by one collision with medium constituent
  - Bethe-Heitler or single scattering regime
- Many collisions with smaller momentum exchange effectively trigger gluon radiation
  - Multiple scattering regime

Two Approaches

- For single scattering, expand in number of scatterings between parton and medium constituents
  $\Rightarrow$ Opacity expansion [Gyulassy et al., 2001, Wiedemann, 2000]

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- For multiple scattering, need to account for LPM interference

  $\Rightarrow$ BDMPS-Z or AMY frameworks [Baier et al., 1995, Zakharov, 1997, Arnold et al., 2003]

  $\Rightarrow$ To get analytical solution, need to take **Harmonic Oscillator Approximation** (HOA)

Splitting Probability

Differential probability of parton with energy, P splitting into particles with energies zP and $(1-z)P$

$$\frac{d\mathcal{P}_{a\to b,c}}{dz} = \frac{g^2 p_{a\to b,c}(z)}{4\pi(z(1-z)P)^2} \mathrm{Re} \int_0^\infty dt_1 \int_{t_1}^\infty dt_2 \times \nabla_{\mathbf{B_1}} \cdot \nabla_{\mathbf{B_2}} \{G(\mathbf{B_2}, t_2; \mathbf{B_1}, t_1) - \mathrm{vac.}\}\Big|_{\mathbf{B}_1=\mathbf{B}_2=0} \quad (1)$$

$G(\mathbf{B_2}, t_2; \mathbf{B_1}, t_1)$ is Green's function of the Hamiltonian, $\mathcal{H}$ describing momentum diffusion in directions transverse to jet propagation.

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Both $m_\infty, \mathcal{C}(\mathbf{B})$ can be computed in **thermal field theory**!

Outline

In vacuum, going to higher orders in perturbation theory means adding loops

$\Rightarrow$ costs g^2

From Vacuum to Finite Temperature

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At finite temperature (or density), propagators come along with statistical functions (Bose-Einstein or Fermi-Dirac)

$$n_{B/F}(\omega) = \frac{1}{\exp \frac{\omega}{T} \mp 1}$$

$\Rightarrow$ Things become more complicated (and potentially problematic)

Thermal Scales in a Weakly Coupled QGP

- T, hard scale associated with energy of individual particles
  $\Rightarrow$ hard-hard interactions can be described perturbatively

- gT, soft scale associated with energy of collective excitations
  $\Rightarrow$ soft-soft interactions can also be described perturbatively

- g^2T, ultrasoft scale is associated with nonperturbative physics
  $\Rightarrow$ Loops can be added at no extra cost (Linde problem)
  $\Rightarrow$ **Cannot** use perturbation theory

HTL Effective Theory

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  Turns out that one can add loops for free
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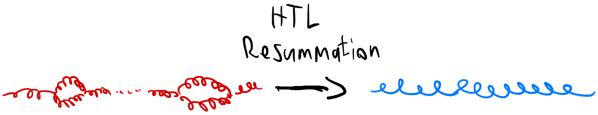
- Integrate out scale T to get Hard Thermal Loop (HTL) effective theory
  $\Rightarrow$ EFT for momenta $gT \sim m_D$ allows us to resum these loops
  $\Rightarrow$ typically used in jet energy loss calculations



Outline

Reminder: Consider jet as a hard parton with momentum $P \gg T$ propagating through the medium, with interactions controlled by Hamiltonian

- Hard parton undergoes forward scattering with the medium, induces shift in dispersion relation, **asymptotic mass** i.e $\omega^2 = \mathbf{k}^2 + m_\infty^2$

- **Transverse scattering rate,** $\mathcal{C}(k_\perp)$ describes damping in transverse momentum space due to interactions with medium,
  Related to transverse momentum broadening coefficient
  $\hat{q}(\mu) \equiv \int^\mu \frac{d^2k_\perp}{(2\pi)^2} k_\perp^2 \mathcal{C}(k_\perp)$

Classical Corrections

- Both m_∞ and $\mathcal{C}(k_\perp)$ can be expressed in terms of correlators on the lightcone
- Both receive **classical** contributions
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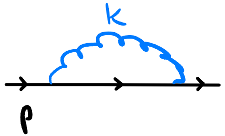
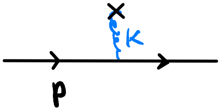
$$\implies \quad n_B(\omega) \equiv \frac{1}{\exp\left(\frac{\omega}{T}\right) - 1} \gg 1$$

$\implies$ corrections are enhanced!





$p \gg T$ $\quad k \sim gT \quad k = (\omega, \vec{k})$

Mapping onto EQCD

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- Breakthrough: classical corrections to thermal correlators on the lightcone can be computed in **Electrostatic QCD** (EQCD) [Caron-Huot, 2009]

What is EQCD?

- Integrate out hard scale, T from QCD

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- Arrive at **3 dimensional gluon effective field theory** for momenta, $gT \sim m_D$, inverse screening length for chromoelectric fields

- EQCD Lagrangian

$$\mathcal{L}_{\mathrm{EQCD}} = \frac{1}{2g_{3\mathrm{d}}^2}\,\mathrm{Tr}\,F_{ij}F_{ij} + \mathrm{Tr}\,D_i\Phi D_i\Phi + m_D^2\,\mathrm{Tr}\,\Phi^2 + \lambda\left(\mathrm{Tr}\,\Phi^2\right)^2$$

where Φ is adjoint scalar, $i, j = 1, 2, 3$

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- $\Rightarrow$ **“Dimensional Reduction”**

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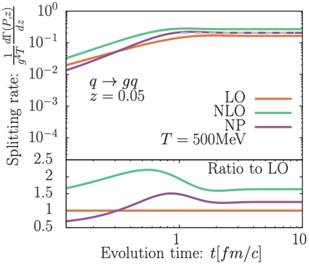
$\Rightarrow$ EQCD can be studied using **Lattice QCD**!

$\Rightarrow$ Captures (numerically) all classical contributions from soft and ultrasoft scales!!!

Non-perturbative Momentum Broadening

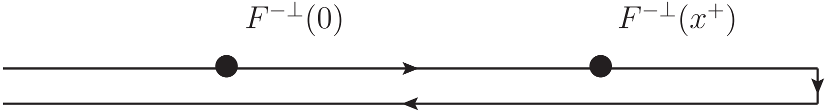
- Paved way for **non-perturbative** (NP) determination of classical corrections to $\mathcal{C}(k_\perp)$
- Series of papers culminated with determination of in-medium splitting rate for medium of finite size [Panero et al., 2014, Moore et al., 2021, Schlichting and Soudi, 2021]
- Difference between rate from LO kernel and NP kernel up to 50%!



Definition of m_∞

- Masses are given by $m^2_\infty = g^2 C_R(Z_g + Z_f)$, where classical corrections are contained in the part

$$Z_g \approx \int_0^\infty dx^+ x^+ \mathrm{Tr}\langle U_F(-\infty; x^+) F^{-\perp}(x^+) U_F(x^+; 0) F^{-\perp}(0) U_F(0; -\infty)\rangle$$



Same idea here, lattice calculation has been completed [Moore and Schlusser, 2020, Ghiglieri et al., 2022]

Outline

Road to Non-Perturbative Evaluation of Z_g

$$
\begin{aligned}
&\qquad\quad \text{scale } T \qquad\qquad\qquad \text{scale } gT \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{scale } g^2T \\
Z_g =& \left[\frac{T^2}{6} - \frac{T\mu_{\rm h}}{\pi^2} \right] \\
&+ \left[-\frac{Tm_{\rm D}}{2\pi} + \frac{T\mu_{\rm h}}{\pi^2} \right] \\
&+ \left[c_{\rm h}^{\ln} \ln\frac{T}{\mu_{\rm h}} + c_{\rm T} + c_{\rm h}^{\ln} \ln\frac{\mu_{\rm h}}{m_{\rm D}} + c_{\rm s}^{\ln} \ln\frac{m_{\rm D}}{\mu_{\rm s}} + c_{gT} + c_{\rm s}^{\ln} \ln\frac{\mu_{\rm s}}{g^2T} + c_{gT^2} \right] \\
&+ \mathcal{O}(g^3) \,.
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- Match lattice evaluation to perturbative EQCD evaluation ✓

[Ghiglieri et al., 2022]

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ROAD TO NON-PERTURBATIVE EVALUATION OF Z_g

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- Match perturbative EQCD evaluation to (4D) QCD evaluation

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  [Ghiglieri et al., 2022]
- Match perturbative EQCD evaluation to (4D) QCD evaluation
- Supply entire $\mathcal{O}(g^2)$ correction coming from thermal scale

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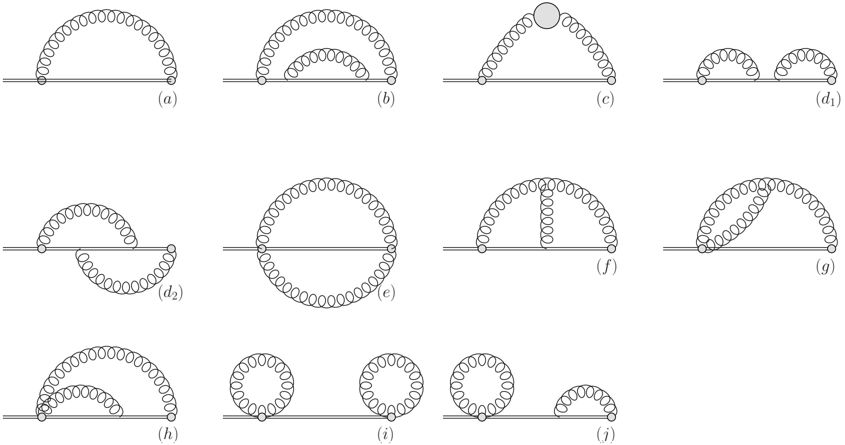
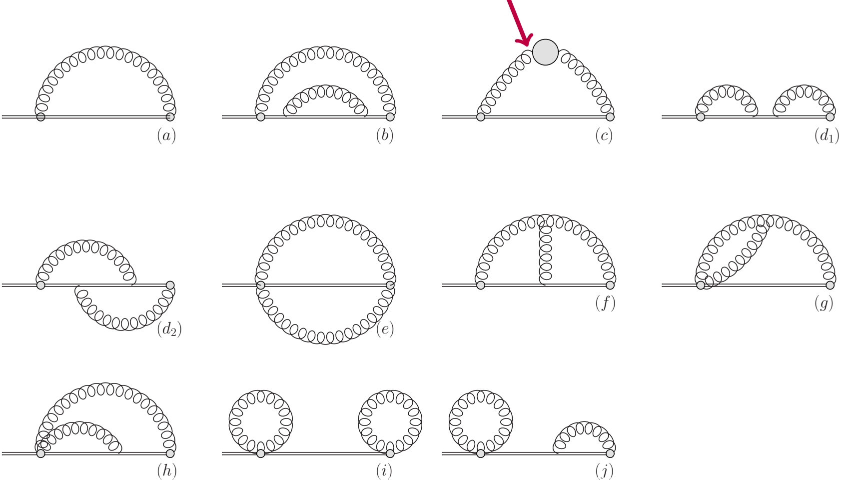
Z_g Diagrams in QCD



Z_g Diagrams in QCD

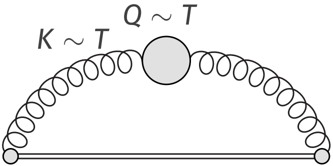
Only need to compute (c)

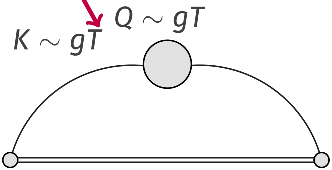


Matching to EQCD

EQCD equivalent

Two loop momenta: K, Q





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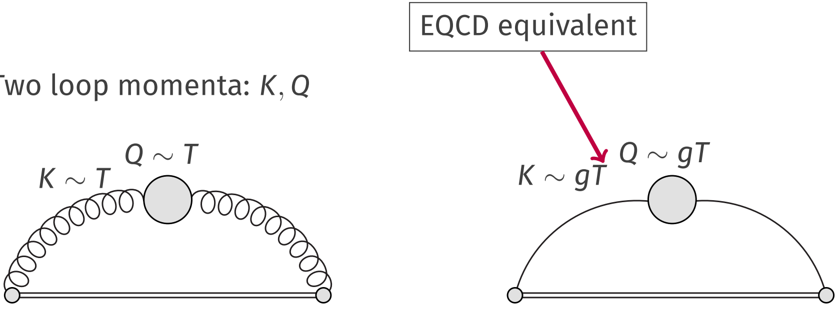

Isolate K zero-mode contribution

Take $K \gg gT$ limit

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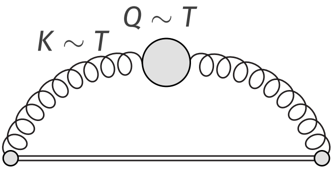
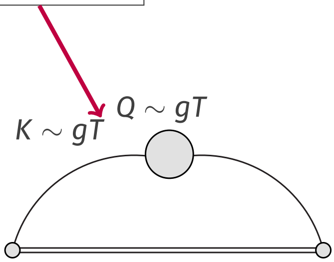


EQCD equivalent

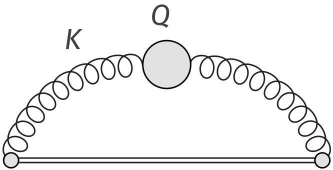


Isolate K zero-mode contribution

Take $K \gg gT$ limit

Find that logarithmic divergences cancel
$\Rightarrow$ UV behaviour of NP evaluation is cured!
[Ghiglieri et al., 2024]

Naively, should be finite...



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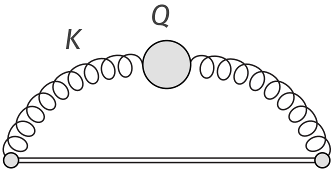

Find outstanding double-logarithmic divergence

in part of phase space: $k^+ \sim T,\ k^- \sim g^2 T,\ k_\perp \sim gT \Rightarrow K^2 \sim 0$

$$Z_{g\,n\neq 0\,\mathrm{div}} = i\int_K \frac{1+n_{\mathrm{B}}(k^0)}{(k^- - i\varepsilon)^2}\frac{k_\perp^2}{k^2}\left[\frac{\Pi_L^R(K)-\Pi_T^R(K)}{K^2+i\varepsilon k^0} - \text{adv.}\right]$$

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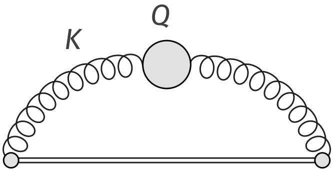

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Conclusion/Summary

- Want to improve accuracy of theoretical jet quenching calculations

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Conclusion/Summary

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  $\Rightarrow$ Finite temperature perturbation theory

- Jet evolution governed by asymptotic mass, m_∞ and transverse scattering rate, $\mathcal{C}(k_\perp)$

  $\Rightarrow$ Important to include higher-order corrections!
  $\Rightarrow$ In some cases we can even do better using lattice EQCD to get non-perturbative **classical corrections**

Thanks for listening!

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Numerical Results

T	$\frac{Z_g^{\text{non-pert class}}}{T^2}$	$\frac{Z_{\text{g,LO}}^{\text{3d}}}{T^2}$
250 MeV	$-0.513(138)(45)(7)$	-0.376
500 MeV	$-0.619(99)(39)(3)$	-0.324
1 GeV	$-0.462(71)(9)(7)$	-0.305
100 GeV	$-0.327(16)(5)(2)$	-0.223

Table:

DEFINING $\hat{q}(\mu)$

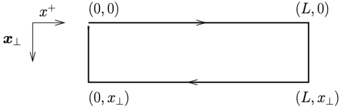
- Intuitively, transport coefficient describing transverse diffusion of jet: $k_\perp^2 = \hat{q}L$
- Can be related to the transverse scattering rate, $\mathcal{C}(k_\perp)$

$$\hat{q}(\mu) = \int^{\mu} \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 \mathcal{C}(k_\perp)$$

$$\lim_{L\to\infty} \langle W(x_\perp) \rangle = \exp(-\mathcal{C}(x_\perp)L)$$

- $W(x_\perp)$ is a Wilson loop defined in the $(x^+, x_\perp)$ plane

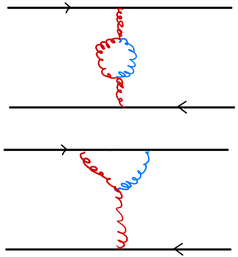
[Casalderrey-Solana and Teaney, 2007, D'Eramo et al., 2011, Benzke et al., 2013]



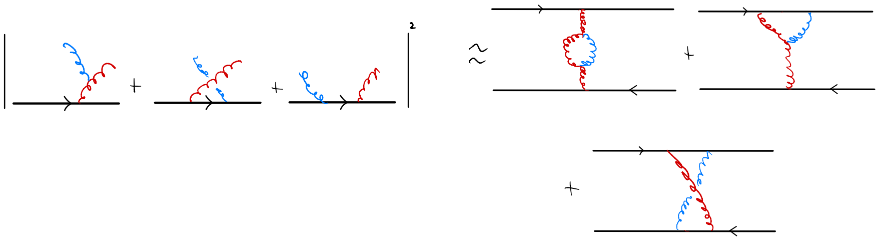
[Ghiglieri and Teaney, 2015]

Contributing diagrams

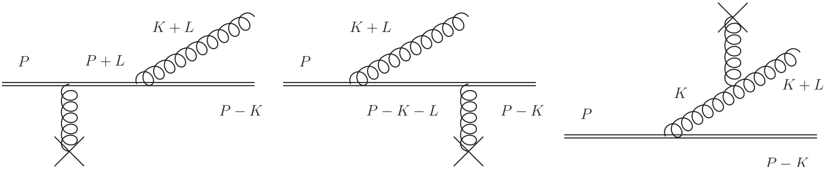
- Can think of sticking together amplitude and conjugate amplitude to get diagrams on the right
- **Black lines** represent hard parton in the amplitude and conjugate amplitude
- Red gluons are bremsstrahlung, represented by thermal propagators
- Blue gluons are those that are exchanged with the medium and are represented by HTL propagators



Where do these diagrams come from?



Double Logs from the literature



$N = 1$ term in opacity expansion emerges from dipole picture

$$\delta\mathcal{C}(k_\perp)_{\rm LMW} = 4\alpha_s C_R \int \frac{d\omega}{\omega} \int \frac{d^2 l_\perp}{(2\pi)^2} \mathcal{C}_0(l_\perp) \frac{l_\perp^2}{k_\perp^2 (\mathbf{k}_\perp + \mathbf{l}_\perp)^2} \qquad (2)$$

Double Logs from the literature

$$\delta\mathcal{C}(k_\perp,\rho)_{\text{LMW}} = 4\alpha_s C_R \int \frac{d\omega}{\omega} \int^\rho \frac{d^2 l_\perp}{(2\pi)^2} \mathcal{C}_0(l_\perp) \frac{l_\perp^2}{k_\perp^2 (\mathbf{k}_\perp + \mathbf{l}_\perp)^2}$$

$|\mathbf{k}_\perp + \mathbf{l}_\perp| \gg l_\perp \Rightarrow$ Single Scattering

$$\delta\mathcal{C}(k_\perp,\rho)_{\text{LMW}} = 4\alpha_s C_R \int \frac{d\omega}{\omega} \int^\rho \frac{d^2 l_\perp}{(2\pi)^2} \mathcal{C}_0(l_\perp) \frac{l_\perp^2}{k_\perp^4}$$

$\hat{q}_0(\rho) \to \hat{q}_0 \Rightarrow$ HOA

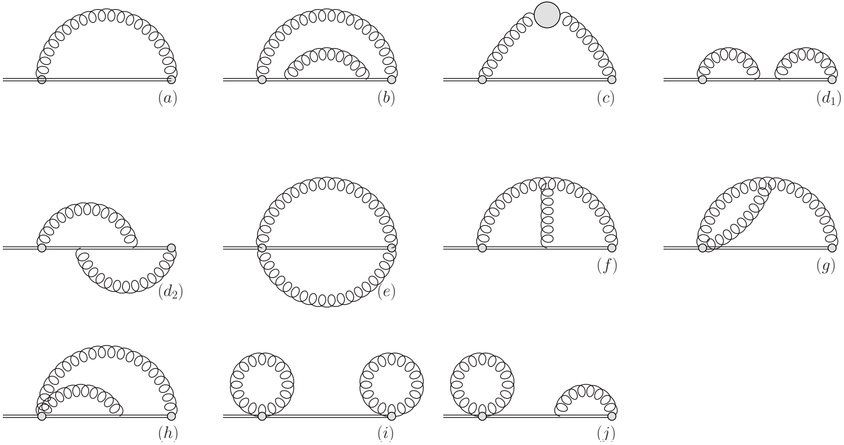
$$\delta\mathcal{C}(k_\perp)_{\text{LMW}} = 4\alpha_s C_R \hat{q}_0 \frac{1}{k_\perp^4} \int \frac{d\omega}{\omega}$$

Reminder: $\hat{q}(\mu) = \int^\mu \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 \mathcal{C}(k_\perp)$

From Static Scattering to a Weakly Coupled QGP

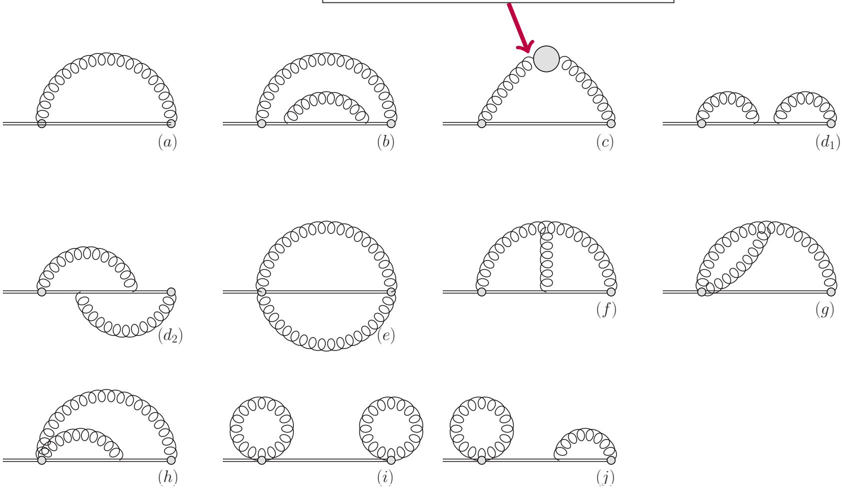
$2n_B(\omega)$ accounts for stimulated emission and absorption of thermal gluons

Z_g Diagrams in QCD



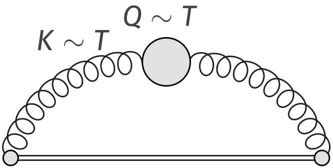
Z_g DIAGRAMS IN QCD

Only need to compute (c)

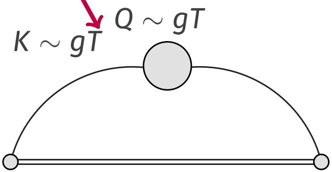


Matching to EQCD

Two loop momenta: K, Q



EQCD equivalent



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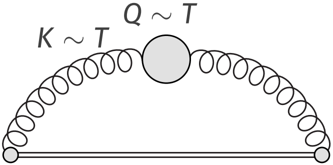
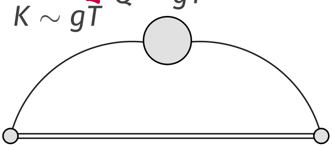


Isolate K zero-mode contribution



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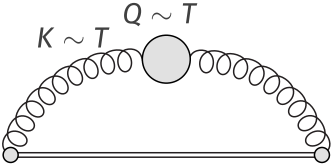

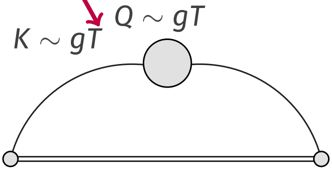

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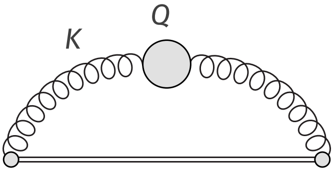
Rest of $\mathcal{O}(g^2)$ Contribution

Naively, should be finite...



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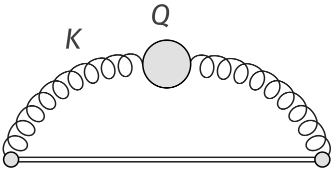

Find outstanding double-logarithmic divergence

in part of phase space: $k^+ \sim T,\ k^- \sim g^2T,\ k_\perp \sim gT \Rightarrow K^2 \sim 0$

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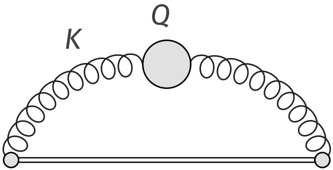

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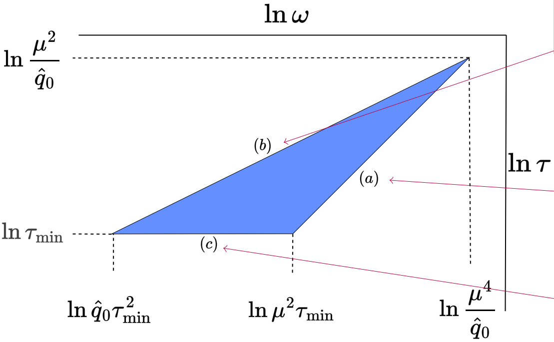


$$\tau < \sqrt{\frac{\omega}{\hat{q}_0}}$$

Keeps us away from multiple-scattering

$$\tau > \frac{\omega}{\mu^2}$$

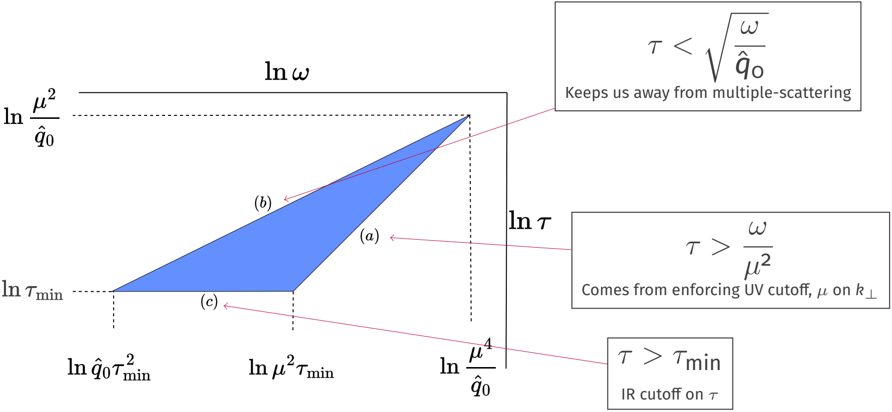
Comes from enforcing UV cutoff, μ on $k_\perp$

$$\tau > \tau_{\min}$$

IR cutoff on τ

$$\delta\hat{q}_{\mathrm{LMW}}(\mu) = \frac{\alpha_s C_R}{\pi}\hat{q}_0 \int_{\tau_{\min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{d\omega}{\omega} = \frac{\alpha_s C_R}{2\pi}\hat{q}_0 \ln^2 \frac{\mu^2}{\hat{q}_0\tau_{\min}}$$

Double Logs from the literature



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How to adapt BDIM/LMW result to weakly coupled QGP?

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$$\downarrow$$

$$\delta\hat{q}_{\rm LMW}(\mu) = \frac{\alpha_s C_R}{\pi}\hat{q}_0 \int_{\tau_{\min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{d\omega}{\omega}(1+2n_B(\omega))$$

$\Longrightarrow$ $n_B(\omega) \equiv \frac{1}{e^{\frac{\omega}{T}}-1}$ can be ignored iff $\omega_{\rm min} = \hat{q}_0\tau_{\rm min}^2 \gg T$

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$$\downarrow$$

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$\implies n_B(\omega) \equiv \frac{1}{e^{\frac{\omega}{T}} - 1}$ can be ignored iff $\omega_{\min} = \hat{q}_0\tau_{\min}^2 \gg T$

But is this consistent with single scattering?

From Static Scattering to a Weakly Coupled QGP

$$\delta\hat{q}_{\mathrm{LMW}}(\mu) = \frac{\alpha_s C_R}{\pi}\hat{q}_0 \int_{\tau_{\min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{d\omega}{\omega}$$

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$$\boxed{\hat{q}_0 \sim g^4 T^3}$$

Need to demand $g^4 T^3 \tau_{\min}^2 \gg T$

$\Rightarrow \tau_{\min}$ should be $\gg \dfrac{1}{g^2 T}$

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$\boxed{\hat{q}_0 \sim g^4 T^3}$

Need to demand $g^4 T^3 \tau_{\min}^2 \gg T$

$\Rightarrow$ $\tau_{\min}$ should be $\gg \dfrac{1}{g^2 T}$

But $\dfrac{1}{g^2 T}$ is the mean free time between multiple scatterings!

$\Rightarrow$ Would lead us away from single scattering regime!

From Static Scattering to a Weakly Coupled QGP

$\Longrightarrow$ In order to stay away from multiple scattering regime, must account for thermal effects

$$\delta\hat{q}_{\mathrm{LMW}}(\mu) = \frac{\alpha_s C_R}{\pi}\hat{q}_0 \int_{\tau_{\min}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{d\omega}{\omega}$$

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Introduce intermediate regulator

$\tau_{\rm int} \ll 1/g^2T$

$$\delta\hat{q}_{1+2}(\mu) = \frac{\alpha_s C_R}{\pi}\hat{q}_0 \int_{\tau_{\rm int}}^{\mu^2/\hat{q}_0} \frac{d\tau}{\tau} \int_{\hat{q}_0\tau^2}^{\mu^2\tau} \frac{d\omega}{\omega}(1+2n_B(\omega))$$

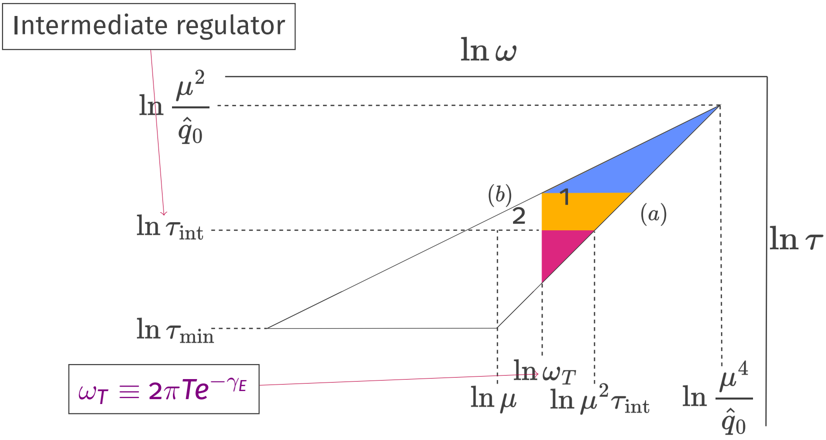
$$= \frac{\alpha_s C_R}{2\pi}\hat{q}_0\left\{ \ln^2 \frac{\mu^2}{\hat{q}_0\tau_{\rm int}} - \frac{1}{2}\ln^2 \frac{\omega_T}{\hat{q}_0\tau_{\rm int}^2}\right\}$$

$\omega_T \equiv 2\pi T e^{-\gamma_E}$

From Static Scattering to a Weakly Coupled QGP

$2n_B(\omega)$ accounts for stimulated emission and absorption of thermal gluons

Double Logs in a Weakly Coupled QGP



$$\delta\hat{q}_{1+2}(\mu) = \frac{\alpha_s C_R}{2\pi}\hat{q}_0\left\{\ln^2\frac{\mu^2}{\hat{q}_0\tau_{\rm int}} - \frac{1}{2}\ln^2\frac{\omega_T}{\hat{q}_0\tau_{\rm int}^2}\right\}$$

Strict Single Scattering

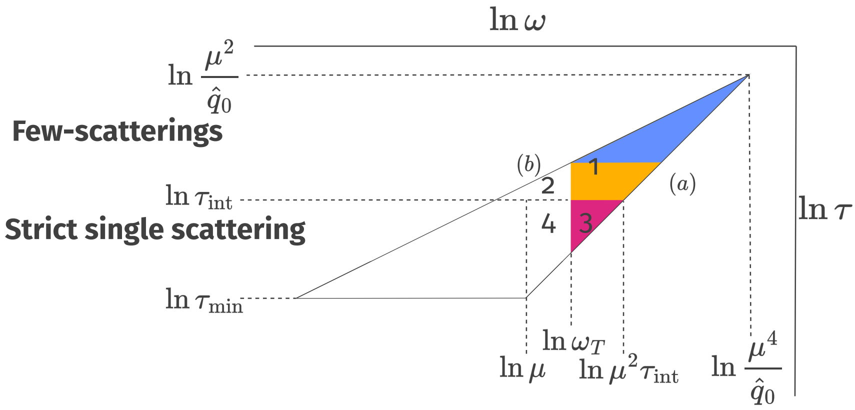
- Investigate which logs are produced by *soft, collinear* modes through a *semi-collinear* process associated with formation time $\tau_{\text{semi}} \sim 1/gT$ [Ghiglieri et al., 2013, Ghiglieri et al., 2016]

Only spacelike interactions with medium

Now timelike interactions are allowed too

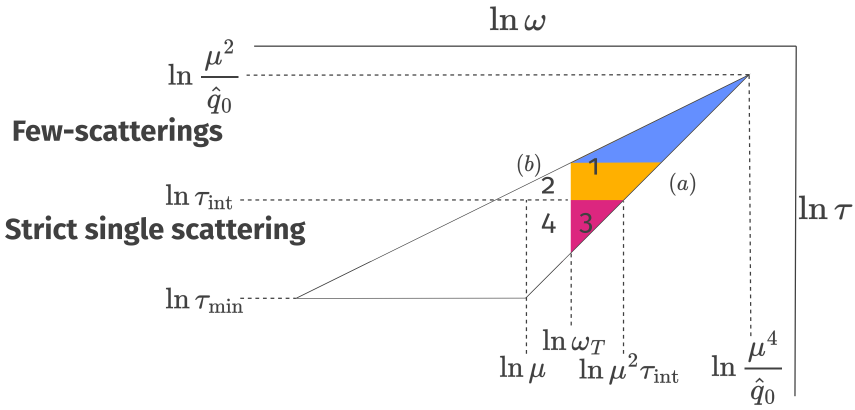
$\Rightarrow$ Going beyond instantaneous approximation!

Double Log with Single Scattering



$$\delta \hat{q}_{3+4}(\mu) = \frac{\alpha_s C_R}{2\pi} \hat{q}_0 \ln^2 \frac{\mu^2 \tau_{\rm int}}{\omega_T} + \text{subleading logs}$$

Double Log with Single Scattering



$$\delta \hat{q}_{GW}(\mu) = \delta \hat{q}_{1+2}(\mu) + \delta \hat{q}_{3+4}(\mu) = \frac{\alpha_s C_R}{4\pi} \hat{q}_0 \ln^2 \frac{\mu^4}{\hat{q}_0 \omega_T}$$

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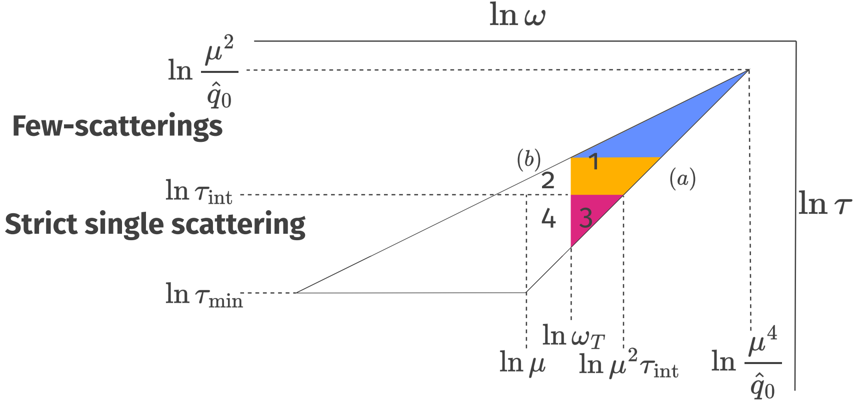


Why is it that region 2 and 4 do not contribute to the double Logs?

Vacuum and Quantum Correction Cancellation

First, note that

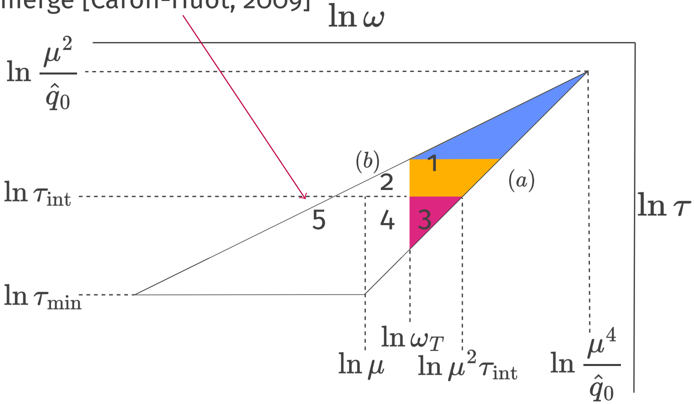
$$\lim_{\frac{\omega}{T}->0}\Big(1+2n_B(\omega)\Big)=1+\frac{2T}{\omega}-1 \tag{3}$$

The absence of the IR scale in any logarithms can then be seen by looking at the following integral, with $\nu_{IR} \ll T \ll \nu_{UV}$

$$\begin{aligned}\int_{\nu_{IR}}^{\nu_{UV}}\frac{d\omega}{\omega}\Big(\underbrace{1}_{\text{vacuum}}+\underbrace{2n_B(\omega)}_{\text{thermal}}\Big) &= \underbrace{\ln\frac{\nu_{uv}}{\nu_{IR}}}_{\text{vacuum}}+\underbrace{\frac{2T}{\nu_{IR}}-\ln\frac{2\pi T}{\nu_{IR}e^{\gamma_E}}+\ldots}_{\text{thermal}} \\ &= \frac{2T}{\nu_{IR}}+\ln\frac{\nu_{UV}e^{\gamma_E}}{2\pi T}+\ldots\end{aligned} \tag{4}$$

Relation to Classical Corrections

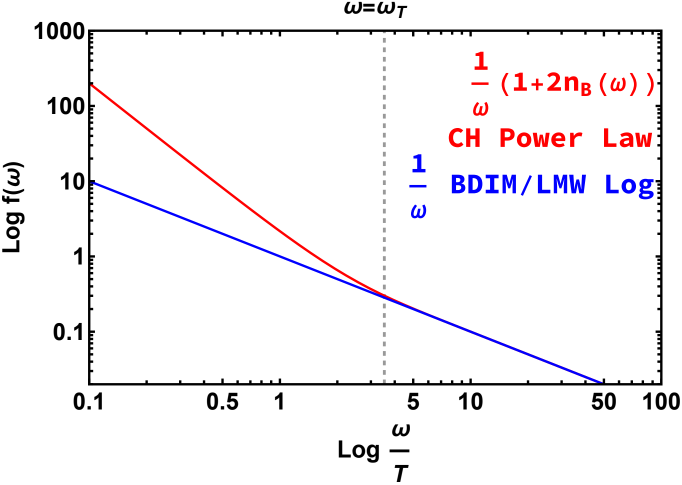
Region of phase space from which classical $\mathcal{O}(g)$ corrections emerge [Caron-Huot, 2009]



How can we understand the transition to power law enhancement in regions 2 and 4?

Relation to Classical Corrections

Can understand transition by looking at ω integrand, $f(\omega)$



$$\omega_T \equiv 2\pi T e^{-\gamma_E}$$

$$n_B(\omega) \equiv \frac{1}{e^{\frac{\omega}{T}} - 1}$$

Relation to Classical Corrections

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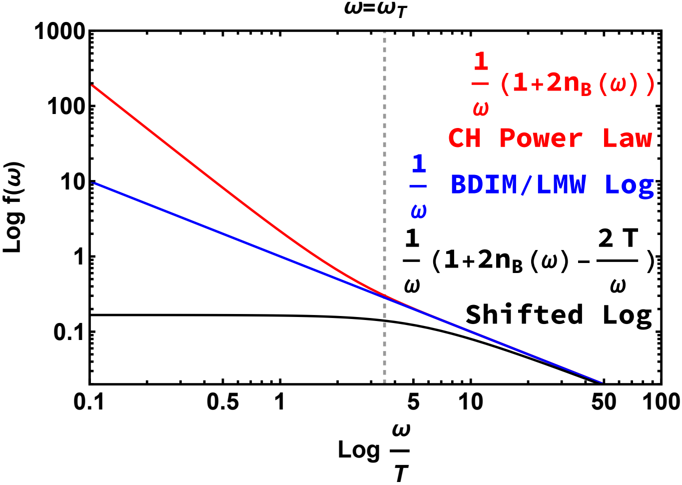

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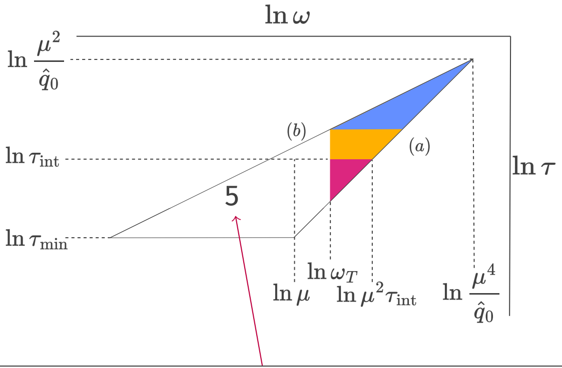
Relation to Classical Corrections

Our results include power law corrections depending on our IR cutoff



They cancel against cutoff-dependent corrections computed from [Caron-Huot, 2009]$\Rightarrow$ Non-trivial check!

Going Beyond Harmonic Oscillator Approximation – Outlook

HOA not well-suited to single-scattering

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So how can we go beyond it?

$$\delta\hat{q}_{\text{GW}}(\mu) = \frac{\alpha_s C_R}{4\pi}\hat{q}_0 \ln^2 \frac{\mu^4}{\hat{q}_0\omega_T} \longrightarrow \frac{\alpha_s C_R}{4\pi}\hat{q}(\rho)\ln^2 \frac{\#}{\omega_T}$$

where $\hat{q}(\rho) \propto \ln \frac{\rho^2}{m_D^2}$

ρ separates us from neighbouring region
with simultaneously single-scattering and multiple scatterings

Appearance of $\hat{q}_0$ in double log signifies lack of understanding of transition between single scattering and multiple scattering regimes

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Need to solve transverse momentum-dependent LPM equation without **HOA** [Ghiglieri and Weitz, 2022] in order to shed light on how these issues could be addressed

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Improved Opacity Expansion [Barata et al., 2021]
could be used to solve resummation equation in order to better understand transition from single scattering to multiple scattering