(ii) Light-cone gauge

Define light-cone variables: $A^{\pm} = A^0 \pm A^3$

(choose a "preferred direction" $\sim x3$)

$A^+ = 0$ gauge is called the light-cone (LC) gauge

Write the gauge condition as

$$\eta \cdot A = 0 \quad \text{with} \quad \eta^- = 2,\ \eta^+ = 0,\ \eta^1 = \eta^2 = 0$$

$$A_\mu B^\mu = \frac{1}{2} A^+ B^- + \frac{1}{2} A^- B^+ - A^1 B^1 - A^2 B^2$$

(check)

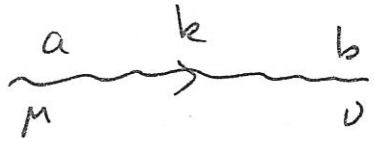
$$\eta \cdot A = \frac{1}{2} \underset{\overset{\shortparallel}{0}}{\eta^+} A^- + \underbrace{\frac{1}{2}\eta^-}_{=1} A^+ - \underset{\overset{\shortparallel}{0}}{\eta^1} A^1 - \underset{\overset{\shortparallel}{0}}{\eta^2} A^2 = A^+$$

$\Rightarrow$ there is <u>no ghost</u> in LC gauge!

<u>Feynman rules</u>: the same, but no ghost

$\Rightarrow$ no ghost propagator, no ghost-gluon vertex

$\Rightarrow$ gluon propagator is different:



$$\frac{-i}{k^2 + i\varepsilon}\, \delta^{ab} \left[g_{\mu\nu} - \frac{\eta_\mu k_\nu + \eta_\nu k_\mu}{\eta \cdot k} \right].$$

Some properties of light-cone coordinates $A^\pm$:

Boost along the z axis:

$$\begin{pmatrix} A'^0 \\ A'^1 \\ A'^2 \\ A'^3 \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} A^0 \\ A^1 \\ A^2 \\ A^3 \end{pmatrix}$$

$$\Rightarrow A'^+ = A'^0 + A'^3 = \gamma A^0 + \beta\gamma A^3 + \beta\gamma A^0 + \gamma A^3 =$$

$$= \gamma(1+\beta) A^+$$

$$\Rightarrow \begin{cases} A'^+ = \gamma(1+\beta) A^+ \\ A'^- = \gamma(1-\beta) A^- \\ A'^1 = A^1 \\ A'^2 = A^2 \end{cases} \quad \Rightarrow A^+ A^- \to \gamma^2 (1+\beta)(1-\beta) A^+ A^- = A^+ A^- \Rightarrow \text{invariant}$$

$\Rightarrow$ any $A^+ B^-$ is invariant under boosts in z-direction

also $\dfrac{A^+}{B^+}$ is invariant under —"—

Light Cone Perturbation Theory (LCPT)

$\Rightarrow$ use $x^+ = t + z$ instead of time

$\Rightarrow$ do a time-ordered (x^+-ordered) perturbation theory instead of the standard (covariant) one.

$\Rightarrow$ LCPT was first derived by Lepage & Brodsky in 1980.
(see the review by Brodsky, Pauli & Pinsky, 1998)

$\Rightarrow$ for time-ordered perturbation theory (but not LCPT), see Sterman, chapter 9.5

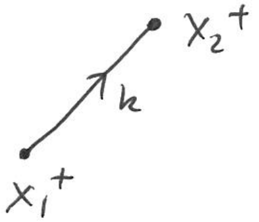
$\Rightarrow$ usually one works in $A^+ = 0$ gauge
(though $\partial_\mu A^\mu = 0$ gauge would work too)

$\Rightarrow$ A^- field component is expressed in terms of $\underline{A} = (A^1, A^2)$ ~ transverse field, using the equations of motion (EOM)

$\Rightarrow$ $q_\pm(x) = \frac{1}{2} \gamma^0 \gamma^\pm q(x)$ ~ quark fields, $\gamma^\pm = \gamma^0 \pm \gamma^3$.

$q_-(x)$ is also eliminated using EOM.

$\Rightarrow$ in the end one has the following LCPT rules in QCD (we are not going to derive them, see the above references if interested):



$$\sim \int_{-\infty}^{\infty} \frac{dk^-}{2\pi} e^{-i\frac{k^-}{2}(x_2^+ - x_1^+)} \frac{i}{k^2 - m^2 + i\varepsilon}$$

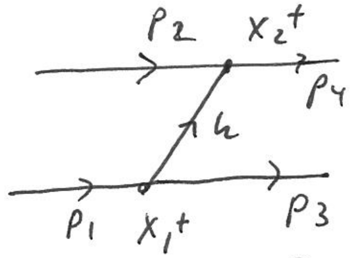
Suppose I want to integrate out k^- in a given propagator, which is a part of a larger Feynman diagram:

$$\int_{-\infty}^{\infty} \frac{dk^-}{2\pi} e^{-i\frac{k^-}{2}(x_2^+ - x_1^+)} \frac{i}{k^+k^- - k_\perp^2 - m^2 + i\varepsilon} = \theta(x_2^+ - x_1^+)\,\frac{1}{k^+}\cdot\theta(k^+)$$

$$\cdot\, e^{-i\frac{k_\perp^2 + m^2}{2k^+}(x_2^+ - x_1^+)} - \theta(x_1^+ - x_2^+)\,\frac{\theta(-k^+)}{k^+}\, e^{-i\frac{k_\perp^2 + m^2}{2k^+}(x_2^+ - x_1^+)}$$

$$= e^{-i\frac{k_\perp^2 + m^2}{2k^+}(x_2^+ - x_1^+)}\ \frac{1}{k^+}\Big[\theta(x_2^+ - x_1^+)\theta(k^+) - \theta(x_1^+ - x_2^+)\cdot\theta(-k^+)\Big].$$

Then one can integrate over x_1^+ and x_2^+: imagine a diagram like this in φ^3 theory



$\Rightarrow$

$$\int_{-\infty}^{\infty} dx_1^+ \, dx_2^+ \; e^{-i\frac{1}{2}(p_1^- - p_3^-)x_1^+ \; - i\frac{1}{2}(p_2^- - p_4^-)x_2^+} \cdot e^{-i\,\frac{k_\perp^2+m^2}{2k^+}(x_2^+-x_1^+)}$$

$$\cdot \frac{1}{k^+}\Big[\underset{1}{\theta(x_2^+-x_1^+)\,\theta(k^+)} - \underset{2}{\theta(x_1^+-x_2^+)\,\theta(-k^+)}\Big]$$

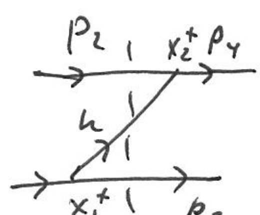
$$1 = \frac{\theta(k^+)}{k^+}\int_{-\infty}^{\infty} dx_2^+ \, e^{-i\frac{1}{2}\left(p_2^- - p_4^- + \frac{k_\perp^2+m^2}{k^+}\right)x_2^+} \cdot \int_{-\infty}^{x_2^+} dx_1^+ \,\cdot$$

$$\cdot\, e^{-i\frac{1}{2}\left(p_1^- - p_3^- - \frac{k_\perp^2+m^2}{k^+}\right)x_1^+} = \frac{\theta(k^+)}{k^+}\int_{-\infty}^{\infty} dx_2^+ \, e^{-i\frac{1}{2}\left(p_2^- - p_4^- + \cancel{\frac{k_\perp^2+m^2}{k^+}}\right)x_2^+}$$

$$\cdot \frac{i}{\frac{1}{2}\left(p_1^- - p_3^- - \frac{k_\perp^2+m^2}{k^+} + i\varepsilon\right)} \, e^{-i\frac{1}{2}\left(p_1^- - p_3^- - \cancel{\frac{k_\perp^2+m^2}{k^+}}\right)x_2^+} = \frac{\theta(k^+)}{k^+}\,\cdot$$

$+i\varepsilon$ ~ if we insert proper regulator at $x_1^+ \to -\infty$

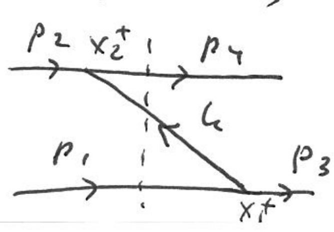
$$\cdot \underbrace{\frac{2i}{p_1^- - p_3^- - \frac{k_\perp^2+m^2}{k^+} + i\varepsilon}}_{\sum p^-_{inc} - \sum p^-_{interm} + i\varepsilon} \cdot 2\pi \cdot 2\, \underbrace{\delta(p_1^- + p_2^- - p_3^- - p_4^-)}_{\text{overall momentum conservation}}$$



$x_1^+ < x_2^+$

$$2 = -\frac{\theta(-k^+)}{k^+}\int_{-\infty}^{\infty} dx_2^+ \, e^{-i\frac{1}{2}\left(p_2^- - p_4^- + \frac{k_\perp^2+m^2}{k^+}\right)x_2^+} \int_{x_2^+}^{\infty} dx_1^+ \, e^{-i\frac{1}{2}\left(p_1^- - p_3^- - \frac{k_\perp^2+m^2}{k^+}\right)x_1^+}$$

$$= \underbrace{-\frac{\theta(-k^+)}{k^+}}_{\frac{\theta(-k^+)}{-k^+}} \; \underbrace{\frac{-2i}{p_1^- - p_3^- - \frac{k_\perp^2+m^2}{k^+} - i\varepsilon}}_{\frac{2i}{p_2^- - p_4^- - \frac{k_\perp^2+m^2}{-k^+} + i\varepsilon}} \, 4\pi\, \delta(p_1^- + p_2^- - p_3^- - p_4^-)$$



$x_1^+ > x_2^+$