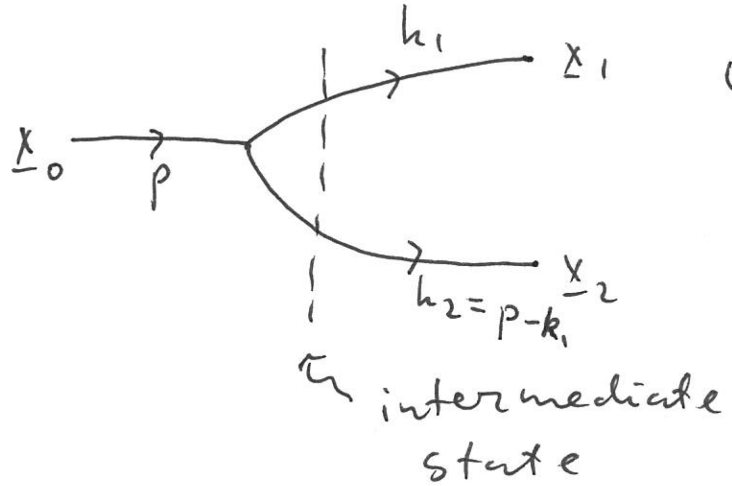
Last time | Calculated a sample LC wave function in φ^3 theory:



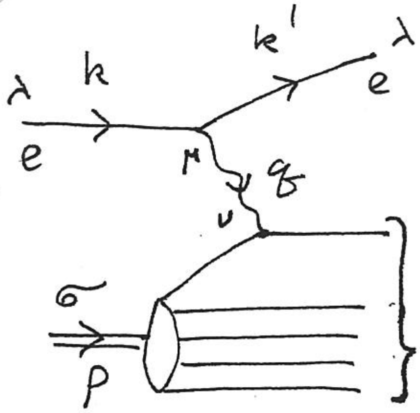
$$\psi(k_1, p-k_1) = -\frac{\lambda z_1(1-z_1)\,\theta(z_1)\,\theta(1-z_1)}{(\underline{k}_1 - z_1 \underline{p})^2 + m^2(1 - z_1 + z_1^2)}$$

where $z_1 = k_1^+/p^+$.

(Longitudinal momentum fraction carried by the particle k_1.)

Parton Model and Deep Inelastic Scattering

DIS



$e(k) + \text{proton}(p) \to$

$\to e(k') + X$

$$\begin{array}{cccc} t & x & y & z \end{array}$$

Rest frame of the proton: $p = (m_p, 0, 0, 0)$

$k = (\varepsilon, 0, 0, k) \simeq (\varepsilon, 0, 0, \varepsilon)$ $\left(\begin{array}{l}\text{energy} \sim \text{many GeV}\\ \text{neglect } m_e\end{array}\right)$

$k' = (\varepsilon', \varepsilon' \sin\theta, 0, \varepsilon' \cos\theta)$

Define:

$\to Q^2 \equiv -q^2 = -(k-k')^2 \simeq 2k\cdot k' = 2\varepsilon\varepsilon'(1-\cos\theta) =$

$= 4\varepsilon\varepsilon' \sin^2\frac{\theta}{2}$

$\nu \equiv \dfrac{p\cdot q}{m_p} = \varepsilon - \varepsilon'$ ← only in p's rest frame

energy lost by e^-. $\left(y = \dfrac{p\cdot q}{p\cdot k} = \dfrac{\varepsilon-\varepsilon'}{\varepsilon} = \dfrac{\nu}{\varepsilon}\right.$; y ~ another variable choice)

$\to x \equiv \dfrac{Q^2}{2p\cdot q} = \dfrac{Q^2}{2m_p\nu}$ Bjorken x variable

$\hat{s} \equiv (p+q)^2 \Rightarrow x = \dfrac{Q^2}{Q^2+\hat{s}}$

$x = \dfrac{Q^2}{2p\cdot q} = \dfrac{-q^2}{(p+q)^2 - p^2 - q^2}$

$= \dfrac{Q^2}{\hat{s} - m_p^2 + Q^2} \simeq \dfrac{Q^2}{\hat{s}+Q^2}$

if $Q^2 \gg m$

Q^2 and x are important / independent !

$\left(\nu = \dfrac{Q^2}{2xm}\right)$

Interaction amplitude:

$$T_{\sigma,\lambda,\lambda'}(n) = +ie\, \bar{u}_{\lambda'}(k')\, \gamma_\mu\, u_\lambda(k)\, \frac{-ig^{\mu\nu}}{q^2} \cdot$$

$$\cdot ie \langle n | j_\nu(0) | p, \sigma \rangle$$

where $j_\nu(x) = \sum_f e_f\, \bar{q}_f(x)\, \gamma_\nu\, q_f(x)$

with $e_f = +2/3, -1/3, \ldots$ (quark flavors) electric charges

j_ν is EM current

Let's calculate the total cross-section ($\underbrace{|\vec{v}_1 - \vec{v}_2|}_{= |\vec{c} - \vec{0}|} = 1$):

$$d\sigma = \underbrace{\frac{1}{4}}_{\text{spin averaging}} \sum_{\sigma,\lambda,\lambda'} \sum_n |T_{\sigma,\lambda,\lambda'}(n)|^2 (2\pi)^4 \delta^4(q + p - p_n) \cdot$$

$$\frac{d^3k'}{\underbrace{2m\, 2\mathcal{E}}_{\text{incoming particles}}\, 2\mathcal{E}'(2\pi)^3} = \frac{e^4}{Q^4} \cdot$$

$$\overbrace{\cdot \frac{1}{2} \sum_{\lambda,\lambda'} [\bar{u}_{\lambda'}(k')\, \gamma_\mu\, u_\lambda(k)]^* [\bar{u}_{\lambda'}(k')\, \gamma_\nu\, u_\lambda(k)]}^{l_{\mu\nu}} \cdot$$

$$\underbrace{\cdot \frac{1}{2} \sum_{\sigma,n} \langle n | j^\mu(0) | p, \sigma \rangle^* \langle n | j^\nu(0) | p, \sigma \rangle \cdot (2\pi)^4 \delta(q + p - p_n)}_{= 4\pi m_p W^{\mu\nu}} \frac{d^3k'}{2m\, 2\mathcal{E}\, 2\mathcal{E}'} \frac{1}{(2\pi)^3}$$

Therefore

$$\boxed{\frac{d\sigma}{d^3k'} = \frac{\alpha_{EM}^2}{Q^4 \quad \varepsilon \cdot \varepsilon'} \quad l_{\mu\nu} \quad W^{\mu\nu}}$$ (rest frame of proton)

$$l_{\mu\nu} = \frac{1}{2} \sum_{\lambda,\lambda'} \left[\bar{u}_{\lambda'\alpha}(k') (\gamma_\mu)_{\alpha\beta} \, u_{\lambda\beta}(k) \right]^* \bar{u}_{\lambda'\alpha'}(k') \cdot$$

see next page

$$\cdot (\gamma_\nu)_{\alpha'\beta'} \, u_{\lambda\beta'}(k) = \frac{1}{2} \sum_{\lambda,\lambda'} \bar{u}_\lambda(k) \, \gamma_\mu \, u_{\lambda'}(k') \cdot$$

$$\cdot \bar{u}_{\lambda'}(k') \, \gamma_\nu \, u_\lambda(k) = \frac{1}{2} \mathrm{Tr}\left[\gamma_\mu \, \gamma\cdot k' \, \gamma_\nu \, \gamma\cdot k \right]$$

as $\sum_{\lambda'} u_{\lambda'}(k') \bar{u}_{\lambda'}(k') = \gamma\cdot k' + m_e \simeq \gamma \cdot k'$

Using $\mathrm{Tr}\left[\gamma_\alpha \gamma_\beta \gamma_\mu \gamma_\nu\right] = 4\left[g_{\alpha\beta} g_{\mu\nu} + g_{\alpha\nu} g_{\beta\mu} - g_{\alpha\mu} g_{\beta\nu} \right]$

We get

$$\boxed{l_{\mu\nu} = 2\left(k_\mu k_\nu' + k_\nu k_\mu' - g_{\mu\nu} \, k\cdot k' \right)}$$

$$W_{\mu\nu} = \frac{1}{4\pi m} \frac{1}{2} \sum_{\sigma, n} \langle n | j^\mu(0) | p, \sigma\rangle^* \langle n | j^\nu(0) | p, \sigma \rangle$$

$$\cdot (2\pi)^4 \, \delta^4(q + p - p_n) = \frac{1}{4\pi m} \frac{1}{2} \sum_{\sigma, n} \int d^4x \, e^{iq\cdot x} \cdot$$

$$\langle p, \sigma | j^\mu(x) | n \rangle \langle n | j^\nu(0) | p, \sigma \rangle$$

$$j^\mu(x) = e^{i\hat{p}\cdot x} \, j^\mu(0) \, e^{-i\hat{p}\cdot x}$$ (Heisenberg picture)

$$\left[\bar{u}_{\lambda'}(k')\gamma^\mu u_\lambda(k)\right]^* \underset{\substack{\uparrow \\ \text{as this is a scalar}}}{=} \left[u^\dagger_{\lambda'}\gamma^0\gamma^\mu u_\lambda\right]^\dagger =$$

$$= u^\dagger_\lambda \underbrace{(\gamma^0)^2}_{\mathbb{1}} \gamma^{\dagger\mu}\gamma^{\dagger 0} u_{\lambda'} = \bar{u}_\lambda(k)\gamma^0\gamma^{\dagger\mu}\gamma^{\dagger 0}u_{\lambda'}$$

now, $\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \Rightarrow \gamma^{\dagger 0} = \gamma^0 \Rightarrow \gamma^0\gamma^{\dagger\mu}\gamma^{\dagger 0} = \gamma^0\gamma^{\dagger\mu}\gamma^0$

Let's find $\gamma^0\gamma^{\dagger\mu}\gamma^0$:

$\mu = 0 \Rightarrow \gamma^0\gamma^{\dagger 0}\gamma^0 = \gamma^0$

$\mu = i \Rightarrow \gamma^0\gamma^{\dagger i}\gamma^0 = -\gamma^0\gamma^i\gamma^0 = \gamma^i(\gamma^0)^2$
$= \gamma^i$

$\Rightarrow \boxed{\gamma^0\gamma^{\dagger\mu}\gamma^0 = \gamma^\mu}$

$\Rightarrow$ get $\bar{u}_\lambda(k)\gamma^\mu u_{\lambda'}(k')$ as desired.