Last time

The Parton Model (cont'd)

We calculated structure functions of a single quark, obtaining

$$F_1^{quark}(x, Q^2) = \frac{Z_f^2}{2}\,\delta(1-x)$$

$$F_2^{quark}(x, Q^2) = Z_f^2\,\delta(1-x)$$

only x - dependent $\Rightarrow$ Bjorken scaling

In arriving at Eq. (2.17) we have neglected the mass of the electron m_e, to write

$$d^3 p' = p'^2 \, dp' \, d\Omega \approx E'^2 \, dE' \, d\Omega,$$

where Ω is the solid scattering angle. We have also used Eq. (2.3) to replace Q^2. Equation (2.17) demonstrates that the structure functions W_1 and W_2 can be measured experimentally by studying the angular dependence of the DIS cross section.

Note that the structure functions W_1 and W_2 have the dimension of inverse mass.[1] It is more convenient to define dimensionless structure functions F_1 and F_2, by

$$F_1(x_{Bj}, Q^2) \equiv m W_1(x_{Bj}, Q^2), \tag{2.18a}$$

$$F_2(x_{Bj}, Q^2) \equiv \nu W_2(x_{Bj}, Q^2) = \frac{Q^2}{2 m x_{Bj}} W_2(x_{Bj}, Q^2). \tag{2.18b}$$

All the QCD physics in DIS is contained in F_1 and F_2. We will now attempt to calculate these structure functions.

2.2 Parton model and Bjorken scaling

To find the structure functions F_1 and F_2 it is easier to change the frame in which we are working. Instead of the proton's rest frame we will now use a frame in which the proton is ultrarelativistic. Such a frame is usually referred to as the *infinite momentum frame* (IMF) or Bjorken frame. The proton is taken to be moving along the z-axis, and its momentum in this frame is

$$P^\mu \approx \left(P + \frac{m^2}{2P}, 0, 0, P \right) \tag{2.19}$$

in the (P^0, P^1, P^2, P^3) notation. We assume that the proton's momentum is much larger than its mass, $P \gg m$. The virtual photon in the IMF has $q^3 = 0$, so that

$$q^\mu = (q^0, q^1, q^2, 0). \tag{2.20}$$

The part of the DIS process relevant for the calculation of the structure functions, virtual photon–proton scattering, is depicted in Fig. 2.3. Note that, unlike Fig. 2.2, we now draw the proton at the top of the diagram. In fact, in our normal convention a proton at rest (or any other target) is drawn at the bottom of the diagram, while a proton (or any other projectile) moving at high energy is shown at the top of the diagram.

2.2.1 Warm-up: DIS on a single free quark

As a warm-up calculation in preparation for the full *parton model*, let us simply assume that the proton consists of noninteracting quarks and gluons, which we will refer to as *partons*. As we will see below in Sec. 2.3, this is not such a bad approximation as in the IMF the

[1] Our single-particle states are normalized such that $\langle p|p'\rangle = (2\pi)^3 \, 2E_p \, \delta^3(\vec{p} - \vec{p}\,')$, which allows one to see that the dimension of $W^{\mu\nu}$ in Eq. (2.14) is that of inverse mass.

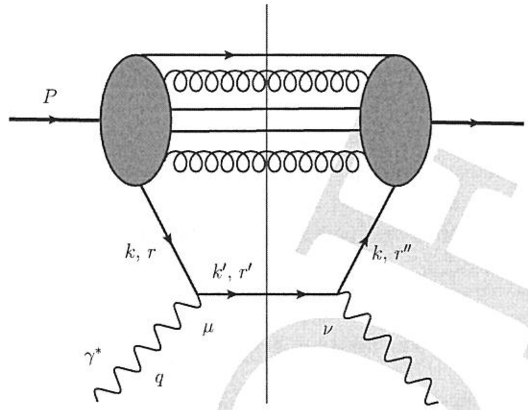

Fig. 2.3. Virtual photon–proton scattering in the IMF.

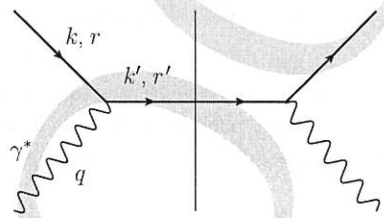

Fig. 2.4. Interaction of a virtual photon with one point-like particle (a parton), as the basic ingredient of the parton model. As usual, the vertical solid line denotes the final-state cut.

typical time scale of the quark and gluon interactions inside the proton is much longer than the time scale of DIS. Hence for the duration of the virtual photon–proton scattering we can assume that the quarks and gluons do not interact with each other. Thus the photon simply interacts with a quark in the proton. To better understand photon–quark scattering let us assume that we simply have one free quark instead of the proton. The diagram giving the cross section of the DIS process is shown in Fig. 2.4.

The hadronic tensor $W_{\mu\nu}$ for the interaction of the virtual photon with the point-like particle (a single quark) has a structure similar to $L_{\mu\nu}$ in Eq. (2.11), namely

$$
\begin{aligned}
W^{quark}_{\mu\nu} &= \frac{Z_f^2}{2} \sum_{r=\pm 1} \sum_{r'=\pm 1} \bar{u}_{r'}(k')\, \gamma_\mu\, u_r(k)\, \left[\bar{u}_{r'}(k')\, \gamma_\nu\, u_r(k)\right]^* \frac{1}{2m_q}\, \delta\!\left(k'^2 - m_q^2\right) \\
&= \frac{Z_f^2}{2} \operatorname{Tr}\left[(\not{k}' + m_q)\gamma_\mu(\not{k} + m_q)\gamma_\nu\right] \frac{1}{2m_q}\, \delta\!\left(k'^2 - m_q^2\right), \qquad (2.21)
\end{aligned}
$$

where $k' = k + q$ while r and r' are the quark helicities (see Fig. 2.4) and m_q is the quark mass. Equation (2.21) can be obtained from Eq. (2.13) by replacing X in it by a single

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particle (a quark), so that

$$\sum_{X=\text{one particle}} = \int \frac{d^3k'}{2k'^0\,(2\pi)^3} \sum_{r'=\pm 1}$$

along with $p_X \to k'$ and $P \to k$. It is then easy to show that

$$\frac{1}{4\pi m_q} \int \frac{d^3k'}{2k'^0\,(2\pi)^3}\,(2\pi)^4\,\delta^4(k+q-k') = \frac{1}{2m_q}\,\delta\left((k+q)^2 - m_q^2\right), \tag{2.22}$$

justifying the delta function factor in Eq. (2.21).

We can rewrite $\delta((k+q)^2 - m_q^2)$ as follows:

$$\delta\left((k+q)^2 - m_q^2\right) = \delta\left(2k\cdot q - Q^2\right) = \frac{1}{2k\cdot q}\,\delta\left(1 - \frac{Q^2}{2k\cdot q}\right), \tag{2.23}$$

where we have used the fact that the incoming quark is on mass shell.

Calculating the trace in Eq. (2.21), comparing the result with Eq. (2.16), and using Eqs. (2.18a) and (2.18b) with P replaced by k we obtain for DIS on a point-like particle (a quark)

$$F_1^{quark}\left(x_{Bj}, Q^2\right) = m_q W_1^{quark}\left(x_{Bj}, Q^2\right) = \frac{Z_f^2}{2}\,\delta\left(1 - x_{Bj}\right) \tag{2.24}$$

$$F_2^{quark}\left(x_{Bj}, Q^2\right) = \frac{Q^2}{2m_q x_{Bj}}\,W_2^{quark}\left(x_{Bj}, Q^2\right) = Z_f^2\,\delta\left(1 - x_{Bj}\right). \tag{2.25}$$

We have used the fact that, for DIS on a single quark, $x_{Bj} = Q^2/(2k\cdot q)$. We see that for DIS on a point-like particle the structure functions F_1 and F_2 turn out to depend only on one variable, x_{Bj}. This behavior is known as *Bjorken scaling* (Bjorken 1969).

2.2.2 Full calculation: DIS on a proton

The idea that the actual interaction in DIS occurs with the point-like constituents of a hadron (the partons) can be illustrated by studying the full DIS process. Let us consider DIS on the whole proton, as shown in Fig. 2.3. We want to calculate the diagram in Fig. 2.3 using the rules of light cone perturbation theory (LCPT) outlined in Sec. 1.3 (see also Sec. 1.4). We first rewrite all four-momenta in the light cone $(+, -, \perp)$ notation. In the IMF/Bjorken frame the proton has a very large momentum. The proton's momentum in Eq. (2.19) becomes, in light cone notation,

$$P^\mu \approx (P^+, 0, 0_\perp) \tag{2.26}$$

with very large $P^+ \approx 2P$. Quarks and gluons in such an ultrarelativistic proton also have very large light cone plus momenta. The quark in Fig. 2.3 has four-momentum $k^\mu = (k^+, (\vec{k}_\perp^2 + m_q^2)/k^+, \vec{k}_\perp)$; we assume that it has a large k^+ component. We define the Feynman-x variable as the fraction of the light cone momentum of the proton carried by

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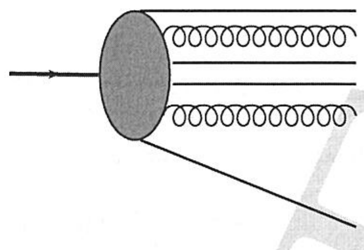

Fig. 2.5. Light cone wave function of the proton.

this quark[2]

$$x \equiv \frac{k^+}{P^+}, \tag{2.27}$$

writing $k^\mu = (x P^+, (\vec{k}_\perp^2 + m_q^2)/(x P^+), \vec{k}_\perp)$.

In LCPT every particle is on mass shell. However, we want to calculate the virtual photon–proton scattering cross section for the process shown in Fig. 2.3. By the definition of the problem the incoming photon is virtual, $q^2 = -Q^2$. Hence in LCPT we can treat this virtual photon as having an imaginary mass $i Q$. The virtual photon momentum (2.20) becomes, in light cone notation,

$$q^\mu = \left(q^+, \frac{\vec{q}_\perp^{\,2} - Q^2}{q^+}, \vec{q}_\perp \right) \tag{2.28}$$

with $(q^+)^2 = \vec{q}_\perp^{\,2} - Q^2$ in the IMF.

In the calculations below we will assume that Q^2 is very large. First, for QCD perturbation theory to be applicable Q^2 has to be much larger than the confinement scale Λ_{QCD}: $Q^2 \gg \Lambda_{QCD}^2$. Second, for the parton model (which we are about to present) to be valid, Q has to be much larger than the transverse momentum of any other particle in the problem. This applies to the quark line carrying momentum k in Fig. 2.3, for which we have $Q^2 \gg \vec{k}_\perp^2, m_q^2$. If, for a particular wave function configuration the upper boxed part of Fig. 2.3 contains n partons with transverse momenta $\vec{k}_{i\perp}$ for $i = 1, \ldots, n$, then we will assume that $Q^2 \gg \vec{k}_{i\perp}^2$ for any i. Note that $\vec{q}_\perp^{\,2} = Q^2 + (q^+)^2 > Q^2$ is also very large.

Now let us assume that these n partons carry light cone momentum components k_i^+ or, equivalently, have Feynman-x values given by x_i for $i = 1, \ldots, n$. We can then define the light cone wave function of the $(n+1)$-parton Fock state of the proton and denote it by $\Psi_n^f(\{x_i, k_{i\perp}\}; x, k_\perp; r)$. The proton has n "spectator" partons (both quarks and gluons) and one quark carrying momentum k in Fig. 2.3 that interacts with the photon. This quark has helicity r and flavor f. The light cone wave function $\Psi_n^f(\{x_i, k_{i\perp}\}; x, k_\perp; r)$ is illustrated in Fig. 2.5. In our discussion and notation we will suppress the polarization indices of the

[2] The Feynman-x variable was originally defined as $x = 2k^3/\sqrt{s}$ in the center-of-mass frame with k^μ the momentum of the produced outgoing particle (Feynman 1969). Our definition here is different, but is also widely used in the community: it maps back onto the original definition at large x.

proton and the polarization, color, and flavor indices of the spectator partons: averaging over the proton polarizations and summation over the polarization, color, and flavor of the partons will always be implicitly assumed to be made after we have multiplied the wave function $\Psi_n^f(\{x_i, k_{i\perp}\}; x, k_\perp; r)$ by its complex conjugate. Note also that $k_\perp = |\vec{k}_\perp|$ (the same notation applies to the other transverse momenta).

Let us now calculate the proton's $W_{\mu\nu}$ using Eq. (2.13). Note that after the interaction the n spectator partons, along with the quark that interacts with the photon, together form what is denoted X in Eq. (2.13). Therefore, for n partons we have (see also Eq. (1.67))

$$\sum_{X=n \text{ partons}} = \int \frac{dk'^+}{k'^+} \frac{d^2k'_\perp}{2(2\pi)^3} \frac{1}{S_n} \sum_{r'=\pm 1} \prod_{i=1}^{n} \frac{dk_i^+}{k_i^+} \frac{d^2k_{i\perp}}{2(2\pi)^3}$$
$$= \int \frac{dk'^+}{k'^+} \frac{d^2k'_\perp}{2(2\pi)^3} \frac{1}{S_n} \sum_{r'=\pm 1} \prod_{i=1}^{n} \frac{dx_i}{x_i} \frac{d^2k_{i\perp}}{2(2\pi)^3}, \tag{2.29}$$

where for physical particles all integrals over the k_i^+ and k'^+ run from 0 to P^+, which translates into integrals over the x_i running from 0 to 1. Here $k'^+ = k^+ + q^+, \vec{k}'_\perp = \vec{k}_\perp + \vec{q}_\perp$, and r' is the helicity of the k' quark line (see Fig. 2.3). The symmetry factor S_n is defined after Eq. (1.67).

Following the definition of the hadronic tensor in Eq. (2.13) and with the help of the diagram in Fig. 2.3 we can write, using the LCPT rules presented in Secs. 1.3 and 1.4,

$$W_{\mu\nu} = \frac{1}{4\pi m} \sum_{n,f} \int \frac{dk'^+ \, d^2k'_\perp}{2k'^+ (2\pi)^3} \frac{1}{S_n} \sum_{r,r',r''} \prod_{i=1}^{n} \frac{dx_i}{x_i} \frac{d^2k_{i\perp}}{2(2\pi)^3}$$
$$\times \frac{P^+}{k^+} \Psi_n^f(\{x_i, k_{i\perp}\}; x, k_\perp; r) \left[\frac{P^+}{k^+} \Psi_n^f\left(\{x_i, k_{i\perp}\}; x, k_\perp; r''\right) \right]^* Z_f^2$$
$$\times \bar{u}_{r'}(k') \gamma_\mu u_r(k) \left[\bar{u}_{r'}(k') \gamma_\nu u_{r''}(k) \right]^* (2\pi)^4 \delta^4\left(P + q - k' - \sum_{j=1}^{n} k_j \right). \tag{2.30}$$

The labeling of the quark helicities r, r', and r'' is defined in Fig. 2.3. Note that, unlike in the simple case of DIS on a single quark considered above, the helicity of the quark line k in Fig. 2.3 is different on the left and on the right of the final-state cut. The factors P^+/k^+ multiplying the wave functions in Eq. (2.30) appear for two reasons. A factor $1/k^+$ has to be included, by the rules of LCPT from Sec. 1.3, is due to the internal quark line carrying momentum k and is not included in our definition of the light cone wave function outlined in Sec. 1.4. The same definition from Sec. 1.4 dictates that each light cone wave function contains a factor $1/P^+$ for each incoming line but, as the general LCPT rules in Sec. 1.3 prescribe no such factor for the full diagram for the scattering process, we need to remove this factor by multiplying the wave functions by P^+.

The delta function in Eq. (2.30) imposes the conservation of the transverse and "+" components of momenta. However, of particular importance is the conservation of the light cone energy that is also imposed by this delta function. Using Eq. (2.26) and rewriting the

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light cone energies of all partons in terms of the transverse and "+" components of their momenta we obtain

$$\frac{1}{k'^+}\,\delta\left(P^- + q^- - k'^- - \sum_{j=1}^{n} k_j^-\right)$$
$$= \frac{1}{k^+ + q^+}\,\delta\left(q^- - \frac{(\vec{k}_\perp + \vec{q}_\perp)^2}{k^+ + q^+} - \sum_{j=1}^{n} \frac{k_{j\perp}^2}{k_j^+}\right). \tag{2.31}$$

For simplicity we will now assume that all the partons are massless. This assumption also applies to the quark that interacts with the photon, for which we now put $m_q = 0$.

Since $Q^2,\ \vec{q}_\perp^{\,2} \gg \vec{k}_\perp^2, k_{i\perp}^2$ for any i we approximate $(\vec{k}_\perp + \vec{q}_\perp)^2$ as $\vec{q}_\perp^{\,2}$ and so neglect all $k_{j\perp}^2/k_j^+$ in the argument of the delta function in Eq. (2.31). This leaves us with

$$\frac{1}{k^+ + q^+}\,\delta\left(q^- - \frac{(\vec{k}_\perp + \vec{q}_\perp)^2}{k^+ + q^+} - \sum_{j=1}^{n} \frac{k_{j\perp}^2}{k_j^+}\right) \approx \delta\big((k^+ + q^+)q^- - \vec{q}_\perp^{\,2}\big)$$
$$= \delta\big(k^+ q^- - Q^2\big) = \delta\big(x\,P^+ q^- - Q^2\big) \approx \delta\big(x\,2P\cdot q - Q^2\big), \tag{2.32}$$

where the last approximation was made using Eq. (2.26). Using the definition of x_{Bj} the last delta function can be rewritten as

$$\delta\left(x\,2P\cdot q - Q^2\right) = \frac{1}{2P\cdot q}\,\delta(x - x_{Bj}) = \frac{x_{Bj}}{Q^2}\,\delta(x - x_{Bj}). \tag{2.33}$$

We see that Feynman x is identical to Bjorken x. The physical meaning of x_{Bj} becomes clear: *it is the fraction of the light cone momentum of the proton carried by the struck quark!*

Since the two quantities are equal, below we will use x and x_{Bj} interchangeably, using the notation with a subscript (x_{Bj}) only in cases when we need to avoid the potential confusion of x with other quantities.

Using Eq. (2.33) in Eq. (2.30) and summing over the helicities r' yields

$$W_{\mu\nu} = \frac{1}{4m}\sum_{n,f}\int dk^+\,d^2k_\perp\,\frac{1}{S_n}\sum_{r,r''}\prod_{i=1}^{n}\frac{dx_i}{x_i}\frac{d^2k_{i\perp}}{2(2\pi)^3}$$
$$\times\,\Psi_n^f\left(\{x_i, k_{i\perp}\}; \frac{k^+}{P^+}, k_\perp; r\right)\left[\Psi_n^f\left(\{x_i, k_{i\perp}\}; \frac{k^+}{P^+}, k_\perp; r''\right)\right]^* Z_f^2$$
$$\times\,\bar{u}_{r''}(k)\,\gamma_\nu\,(\not{k} + \not{q})\,\gamma_\mu\,u_r(k)\,\delta\left(P^+ - k^+ - \sum_{l=1}^{n} k_l^+\right)\delta^2\left(\vec{k}_\perp + \sum_{j=1}^{n}\vec{k}_{j\perp}\right)$$
$$\times\left(\frac{P^+}{k^+}\right)^2\frac{x_{Bj}}{Q^2}\,\delta\left(x_{Bj} - \frac{k^+}{P^+}\right), \tag{2.34}$$

where we have switched from integration variables k'^+ and $\vec{k}'_\perp$ to k^+ and $\vec{k}_\perp$.

An important assumption of the parton model is that the integrals in Eq. (2.34) are convergent even if we impose no integration limit on the transverse momentum integrals.

Consider W_{ij}:

$$W_{ij} = \frac{1}{4m_p} \sum_{n,f} \int dk^+ d^2k_T \frac{1}{S_n} \sum_{r,r''} \prod_{i=1}^{n} \frac{dx_i}{x_i} \frac{d^2k_i}{2(2\pi)^3} Z_f^2 .$$

$$\cdot \psi_n^f(\ldots,r)\, \psi_n^{f\,*}(\ldots,r'')\, \bar{u}_{r''}(k)\, \gamma_j (\not{k}+\not{q})\, \gamma_i\, u_r(k)$$

$$\delta(p^+ - k^+ - \sum_\ell k_\ell^+)\, \delta^2(\underline{k} + \sum_j \underline{k}_j) \left(\frac{p^+}{k^+}\right)^2 \frac{x}{Q^2}\, \delta\left(x - \frac{k^+}{p^+}\right).$$

The leading contribution of the matrix element $\bar{u}_{r''}(k)\, \gamma_j (\not{k}+\not{q}) \gamma_i u_r(k)$ comes from

$(\not{k}+\not{q}) = \frac{1}{2}\gamma^+ (k+q)^-$ as γ^+ becomes the large momentum p^+ after integration.

Now, $(k+q)^- = \frac{(\underline{k}+\underline{q})^2}{k^+ + q^+} \simeq \frac{Q^2}{k^+}$.

We write

$$\bar{u}_{r''}(k)\, \gamma_j (\not{k}+\not{q})\, \gamma_i\, u_r(k) = \frac{Q^2}{2k^+}\, \bar{u}_{r''}(k)\, \gamma_j \gamma^+ \gamma_i\, u_r(k)$$

Further, $W_{\mu\nu}$ is symmetric, W_{ij} is too $\Rightarrow$ symmetrize the matrix element under $i \leftrightarrow j$:

$$\frac{Q^2}{2k^+} \frac{1}{2} \left[\bar{u}_{r''}(k)\, \gamma_j \gamma^+ \gamma_i\, u_r(k) + \bar{u}_{r''}(k)\, \gamma_i \gamma^+ \gamma_j\, u_r(k) \right] =$$

$$= \frac{Q^2}{4k^+} 4 \delta^{ij} \delta_{rr'} k^+ = Q^2 \delta^{ij} \delta_{rr'} = -Q^2 g^{ij} \delta_{rr'}.$$

↑ Table A.1 in KL

We arrive at

$$W_{ij} = \frac{-g_{ij}}{4 m_p X} \sum_{n,f} Z_f^2 \int dk^+ d^2k_\perp \frac{1}{S_n} \sum_r \prod_{i=1}^{n} \frac{dx_i}{x_i} \frac{d^2k_{i\perp}}{2(2\pi)^3}$$

$$\cdot |\psi_n^f|^2 \, \delta(p^+ - k^+ - \sum_\ell k_\ell^+) \, \delta^2(\underline{k} + \sum_j \underline{k}_j) \, \delta\left(x - \frac{k^+}{p^+}\right)$$

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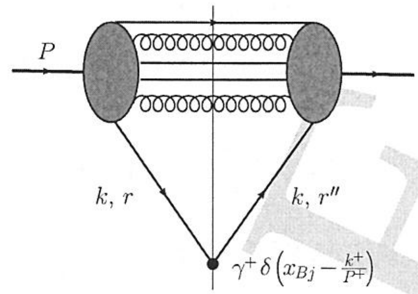

Fig. 2.6. Cut (Mueller) vertex in DIS, denoted by the solid circle.

$W^{-i} = W^{i-} \propto k_\perp^i$, which is much smaller than W^{ij} and integrates out to zero in Eq. (2.35) owing to the absence of a preferred transverse direction in the problem.)

From Eqs. (2.37) and (2.35) we see that, in the usual Feynman diagram language, the quark–photon part of the diagram in Fig. 2.3 can be replaced by a single effective vertex containing $\gamma^+\delta(x_{Bj} - k^+/P^+)$, as shown in Fig. 2.6. This effective vertex is known as a *cut vertex* or *Mueller vertex* (Mueller 1978, 1981).

From the general decomposition of $W^{\mu\nu}$ in Eq. (2.16) and using the fact that, by our frame choice, $\vec{P}_\perp = 0$ we can write

$$W^{ij} = -W_1(x_{Bj}, Q^2)\, g^{ij} + \frac{q^i q^j}{q^2}\left[W_1(x_{Bj}, Q^2) + \frac{W_2(x_{Bj}, Q^2)}{m^2}\frac{(P\cdot q)^2}{q^2}\right]. \quad (2.38)$$

Comparing Eq. (2.38) with Eq. (2.37), for which we showed that $W^{ij} \propto g^{ij}$, we see that the hadronic tensor is given by the first term in Eq. (2.38):

$$W^{ij} = -W_1(x_{Bj}, Q^2)\, g^{ij}. \quad (2.39)$$

Substituting Eq. (2.37) into Eq. (2.35), summing over r'', and comparing the result with Eq. (2.39) we can read off the structure function W_1:

$$W_1(x_{Bj}, Q^2) = \frac{1}{4 m x_{Bj}} \sum_{n,f} Z_f^2 \int dk^+ d^2k_\perp \frac{1}{S_n} \sum_r \prod_{i=1}^n \frac{dx_i}{x_i}\frac{d^2k_{i\perp}}{2(2\pi)^3}$$
$$\times \left|\Psi_n^f\left(\{x_i, k_{i\perp}\}; \frac{k^+}{P^+}, k_\perp; r\right)\right|^2 \delta\left(P^+ - k^+ - \sum_{l=1}^n k_l^+\right)$$
$$\times\, \delta^2\left(\vec{k}_\perp + \sum_{j=1}^n \vec{k}_{j\perp}\right) \delta\left(x_{Bj} - \frac{k^+}{P^+}\right). \quad (2.40)$$

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Let us now define the *quark distribution function* by

$$q^f(x_{Bj}) = \frac{1}{2x_{Bj}} \sum_n \int d\xi \, d^2k_\perp \, \frac{1}{S_n} \sum_r \prod_{i=1}^{n} \frac{dx_i}{x_i} \frac{d^2k_{i\perp}}{2(2\pi)^3}$$

$$\times \left| \Psi_n^f \left(\{x_i, k_{i\perp}\}; \frac{k^+}{P^+}, k_\perp; r \right) \right|^2 \delta\left(1 - \xi - \sum_{l=1}^{n} x_l\right)$$

$$\times \, \delta^2\left(\vec{k}_\perp + \sum_{j=1}^{n} \vec{k}_{j\perp}\right) \delta\left(x_{Bj} - \xi\right), \tag{2.41}$$

where $\xi = k^+/P^+$. With the help of Eq. (2.41) we can rewrite Eq. (2.40) as

$$W_1(x_{Bj}) = \frac{1}{2m} \sum_f Z_f^2 \, q^f(x_{Bj}). \tag{2.42}$$

Note that both the quark distribution function and the structure function W_1 are functions of Bjorken x only! Just as in the case of DIS on a single free quark, this is Bjorken scaling.

To find the remaining structure function, W_2, we note that, as we have just shown in Eq. (2.37), $W^{ij} \propto g^{ij}$. Therefore the term in the large parentheses in Eq. (2.38) must be zero. Equating it to zero, and recalling the definitions of x_{Bj} and ν from Eqs. (2.2) and (2.5), we write

$$\nu \, W_2(x_{Bj}) = 2m x_{Bj} \, W_1(x_{Bj}). \tag{2.43}$$

Using the definitions in Eqs. (2.18a) and (2.18b) we can rewrite Eq. (2.43) as

$$F_2(x_{Bj}) = 2x_{Bj} \, F_1(x_{Bj}). \tag{2.44}$$

Equation (2.44) is known as the *Callan–Gross* relation (Callan and Gross 1969). This relation is characteristic of spin-1/2 partons, such as quarks, and would be different if the proton had constituents with a different spin interacting with the virtual photon.

Combining Eqs. (2.18a), (2.42), and the Callan-Gross relation we write

$$F_1(x_{Bj}) = \frac{1}{2} \sum_f Z_f^2 \, q^f(x_{Bj}), \tag{2.45}$$

$$F_2(x_{Bj}) = \sum_f Z_f^2 \, x_{Bj} \, q^f(x_{Bj}). \tag{2.46}$$

$\nu = \frac{Q^2}{2m_p x}$

$F_1 = m_p W_1$

$F_2 = \nu W_2$

We can see that both structure functions are independent of Q^2 and are functions of x_{Bj} only. Therefore, if we assume that some nonperturbative QCD effects lead to a natural UV cutoff on the transverse momenta of the partons then the DIS cross section can be described by two functions, F_1 and F_2, that are dependent on only one variable, x_{Bj}. This is *Bjorken scaling* (Bjorken 1969) but formulated in a more general case. We have now shown that Bjorken scaling results from a full parton model calculation.

~Jerome Friedman, Henry Kendall and Richard Taylor received the 1990 Nobel Prize in Physics for the DIS experiments which led to establishing quark model (and, hence, for Bjorken scaling). What about Bjorken

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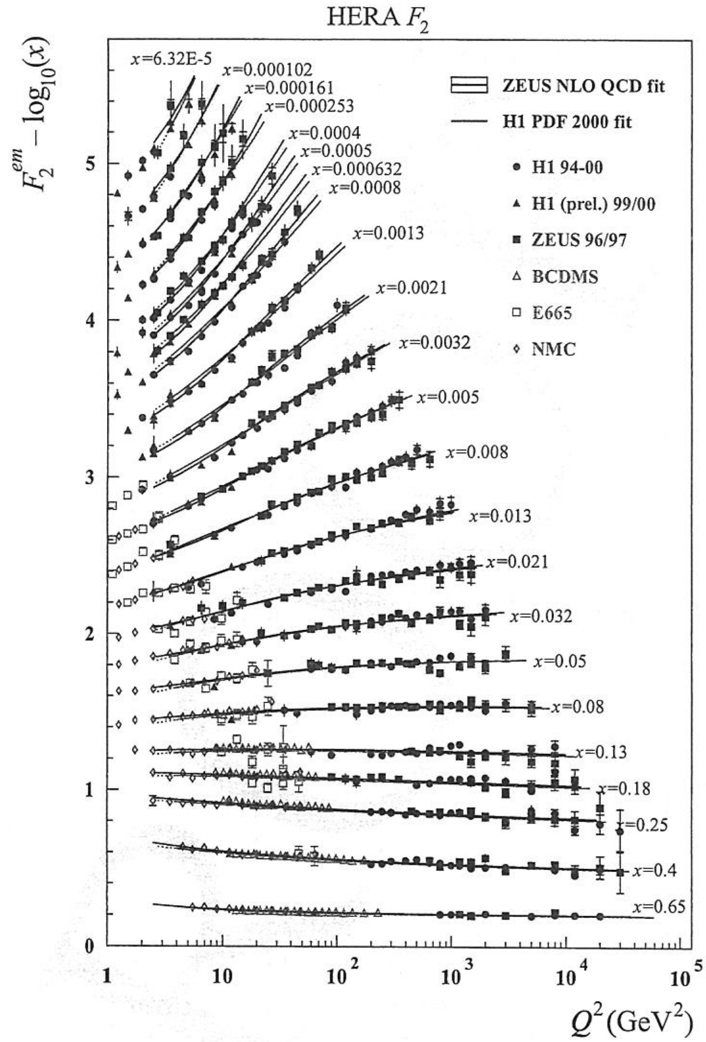


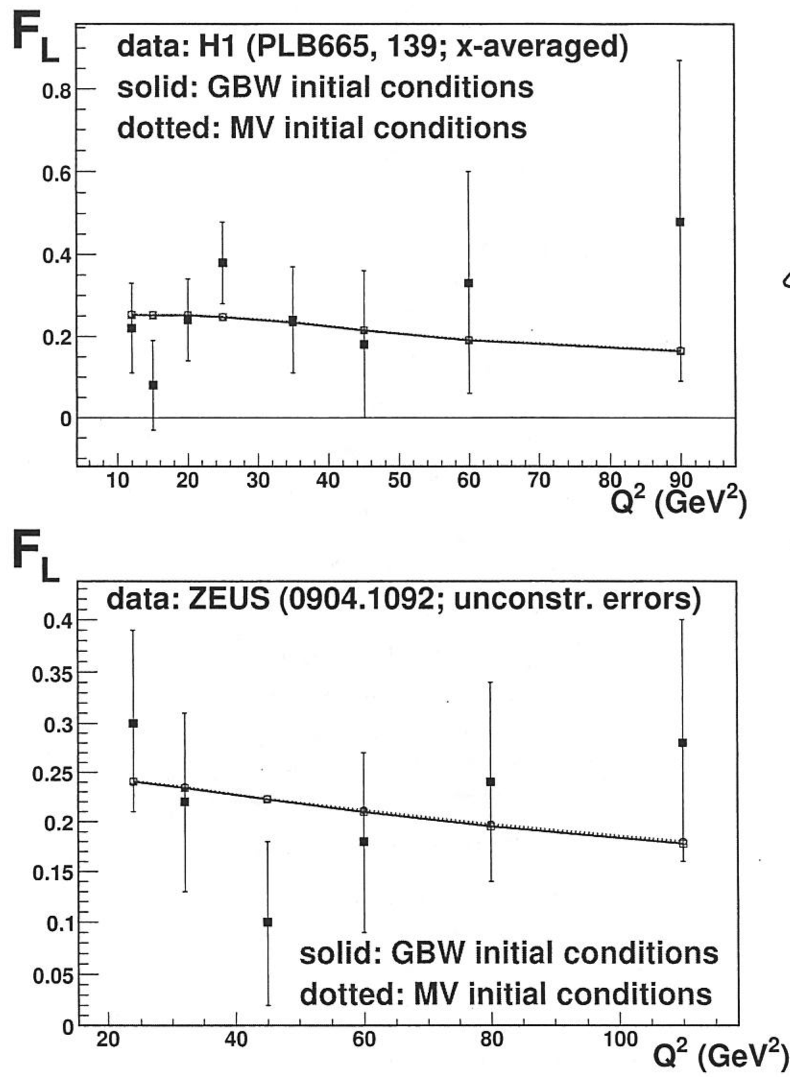
Fig. 2.7. Compilation of the world F_2 data for DIS on a proton. The proton F_2 structure function is plotted as a function of Q^2 for a range of values of x, as indicated next to the data. It can be seen that, except for very small x, F_2 is independent of Q^2, a manifestation of Bjorken scaling. (We thank Kunihiro Nagano for providing us with this figure.) A color version is available online at www.cambridge.org/9780521112574.

In Fig. 2.7 we show a summary of the world knowledge of the proton F_2 structure function. This structure function is plotted as a function of Q^2 for many different fixed values of Bjorken-x. One can clearly see that, when x is not too small, F_2 is independent of Q^2. This is the experimental manifestation of Bjorken scaling. We see that the theory we

(72)

$F_L \equiv F_2 - 2x F_1$, F_L is zero in the naive Parton model

arXiv: 0902.1112 [hep-ph]



F_L is non-zero, but small (e.g. compared to F_2) ⇓ Callan-Gross relationship works!

Figure 3: Comparison between experimental data from the H1 [17] (upper plot) and ZEUS [18] (lower plot) Collaborations and the predictions of our model for $F_L(x, Q^2)$. Red solid lines and open squares correspond to GBW i.c., and blue dotted lines and open circles to MV i.c. The theoretical results have been computed at the same $\langle x \rangle$ as the experimental data, and then joined by straight lines. The error bars correspond to statistical and systematic errors added in quadrature for those data coming from [17], while they correspond to the error quoted for the unconstrained fit for those data coming from [18].

to allow a discrimination of the different UV behaviors of the two employed i.c.

Second, the fits using GBW i.c. and obtained by letting γ vary as a free pa-