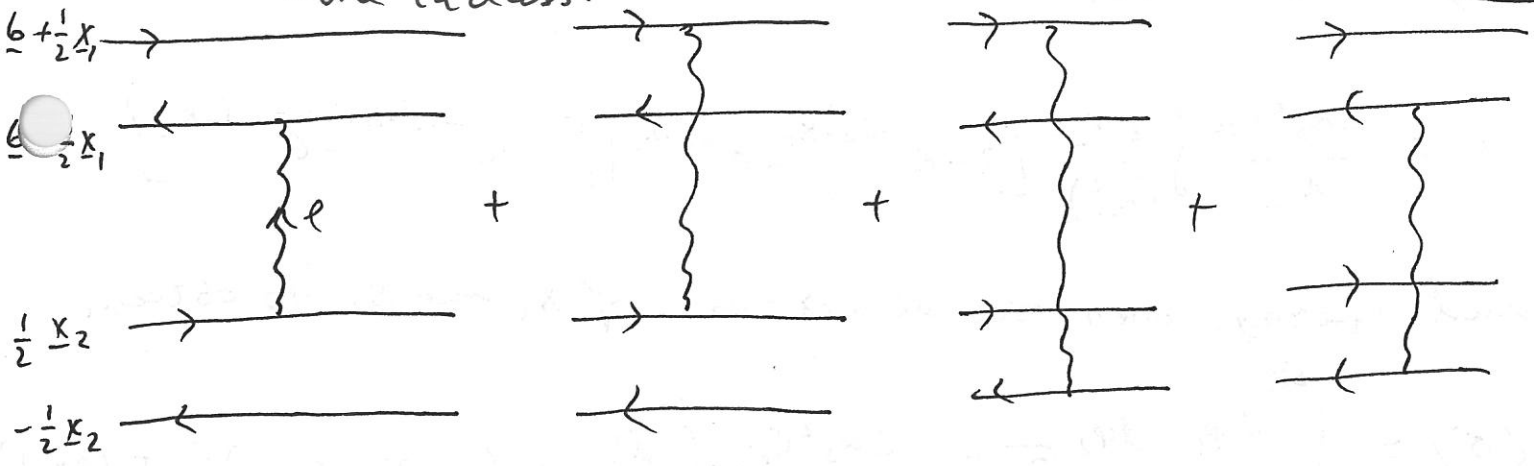


(a) Like done in class:

(2)

(121')



$$A = (-g^2) \int \frac{d^2 \ell_\perp}{(2\pi)^2} \frac{1}{\ell_\perp^2} \left[-e^{i\ell \cdot (b - \frac{1}{2} k_1 - \frac{1}{2} k_2)} + e^{i\ell \cdot (b + \frac{1}{2} k_1 - \frac{1}{2} k_2)} - e^{i\ell \cdot (b + \frac{1}{2} k_1 - (-\frac{1}{2} k_2))} + e^{i\ell \cdot (b - \frac{1}{2} k_1 - (-\frac{1}{2} k_2))} \right] =$$

$$= g^2 \int \frac{d^2 \ell_\perp}{(2\pi)^2} \frac{1}{\ell_\perp^2} e^{i\ell \cdot b} \left[e^{i\ell \cdot \frac{1}{2} k_1} - e^{-i\ell \cdot \frac{1}{2} k_1} \right] \left[e^{i\ell \cdot \frac{1}{2} k_2} - e^{-i\ell \cdot \frac{1}{2} k_2} \right]$$

$$\Rightarrow \int |A|^2 d^2 b = g^4 \int d^2 b \int \frac{d^2 \ell}{(2\pi)^2} \frac{1}{\ell_\perp^2} e^{i\ell \cdot b} \left[e^{i\ell \cdot \frac{1}{2} k_1} - e^{-i\ell \cdot \frac{1}{2} k_1} \right]$$

$$\cdot \left[e^{i\ell \cdot \frac{1}{2} k_2} - e^{-i\ell \cdot \frac{1}{2} k_2} \right] \int \frac{d^2 \ell'}{(2\pi)^2} \frac{1}{\ell_\perp'^2} e^{-i\ell' \cdot b} \left[e^{-i\ell' \cdot \frac{1}{2} k_1} - e^{i\ell' \cdot \frac{1}{2} k_1} \right]$$

$$\cdot \left[e^{-i\ell' \cdot \frac{1}{2} k_2} - e^{i\ell' \cdot \frac{1}{2} k_2} \right] = g^4 \int \frac{d^2 \ell_\perp}{(2\pi)^2} \frac{1}{(\ell_\perp^2)^2} \left| e^{i\ell \cdot \frac{1}{2} k_1} - e^{-i\ell \cdot \frac{1}{2} k_1} \right|^2$$

$$\cdot \left| e^{i\ell \cdot \frac{1}{2} k_2} - e^{-i\ell \cdot \frac{1}{2} k_2} \right|^2 = g^4 \int \frac{d^2 \ell_\perp}{(2\pi)^2} \frac{1}{(\ell_\perp^2)^2} \underbrace{\left[2 - e^{i\ell \cdot k_1} - e^{-i\ell \cdot k_1} \right]}_{\text{impact factor}}$$

$$\cdot \underbrace{\left[2 - e^{i\ell \cdot k_2} - e^{-i\ell \cdot k_2} \right]}_{\text{impact factor}} \Rightarrow \text{leads to Eq. (3.24)}$$

(6) Start from

$$\sigma = \frac{2\alpha_s^2 C_F}{N_c} \int \frac{d^2 l_\perp}{(l_\perp^2)^2} [2 - e^{-i l \cdot x_1} - e^{i l \cdot x_1}] [2 - e^{-i l \cdot x_2} - e^{i l \cdot x_2}]$$

and average over the directions of x_1 and x_2 to obtain

$$\begin{aligned} \langle \sigma \rangle &= \int_0^{2\pi} \frac{d\varphi_1}{2\pi} \int_0^{2\pi} \frac{d\varphi_2}{2\pi} \sigma = \frac{2\alpha_s^2 C_F}{N_c} \int_0^\infty \frac{dl}{l^3} \cdot 4(1 - J_0(lx_1))(1 - J_0(lx_2)) \\ &= \frac{16\pi\alpha_s^2 C_F}{N_c} \lim_{\epsilon \rightarrow 0} \left\{ \int_0^\infty \frac{dl}{l^{3-\epsilon}} (1 - J_0(lx_1)) - \int_0^\infty \frac{dl}{l^{3-\epsilon}} J_0(lx_2) + \int_0^\infty \frac{dl}{l^{3-\epsilon}} J_0(lx_1) J_0(lx_2) \right\} \\ &= \frac{16\pi\alpha_s^2 C_F}{N_c} \lim_{\epsilon \rightarrow 0} \left\{ - \frac{2^{\epsilon-3} x_1^{2-\epsilon} \Gamma(-1 + \frac{\epsilon}{2})}{\Gamma(2 - \frac{\epsilon}{2})} - \frac{2^{\epsilon-3} x_2^{2-\epsilon} \Gamma(-1 + \frac{\epsilon}{2})}{\Gamma(2 - \frac{\epsilon}{2})} \right. \\ &\quad \left. + \frac{2^{\epsilon-3} x_>^{2-\epsilon} \Gamma(-1 + \frac{\epsilon}{2})}{\Gamma(2 - \frac{\epsilon}{2})} {}_2F_1\left(-1 + \frac{\epsilon}{2}, -1 + \frac{\epsilon}{2}; 1; \frac{x_c^2}{x_>^2}\right) \right\} = \\ &= \frac{4\pi\alpha_s^2 C_F}{N_c} x_c^2 \left[\ln\left(\frac{x_>}{x_c}\right) + 1 \right], \text{ which is exactly} \\ &\text{Eq. (3.25) in KL, as desired!} \end{aligned}$$