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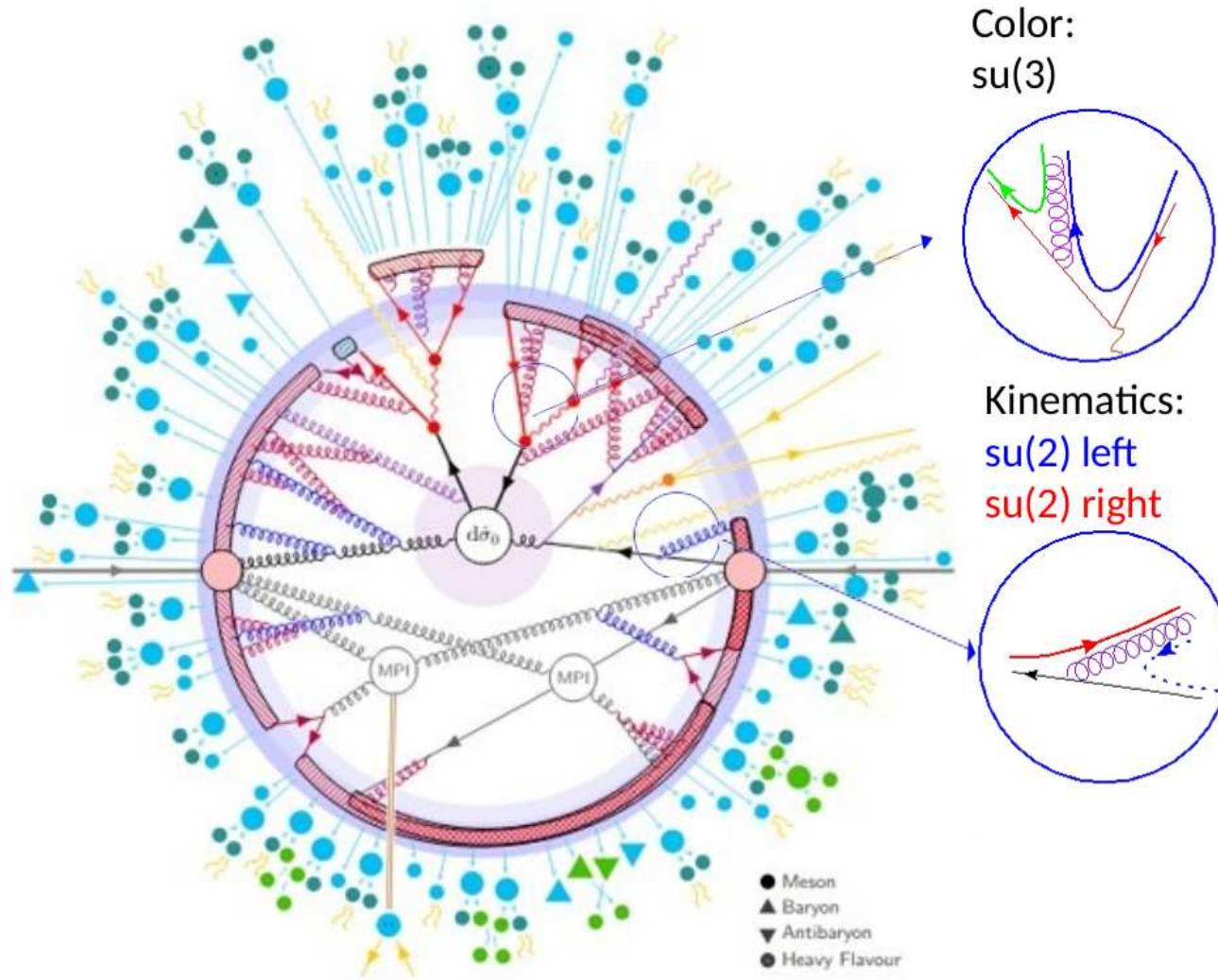
Chirality flow

Thanks to my collaborators:

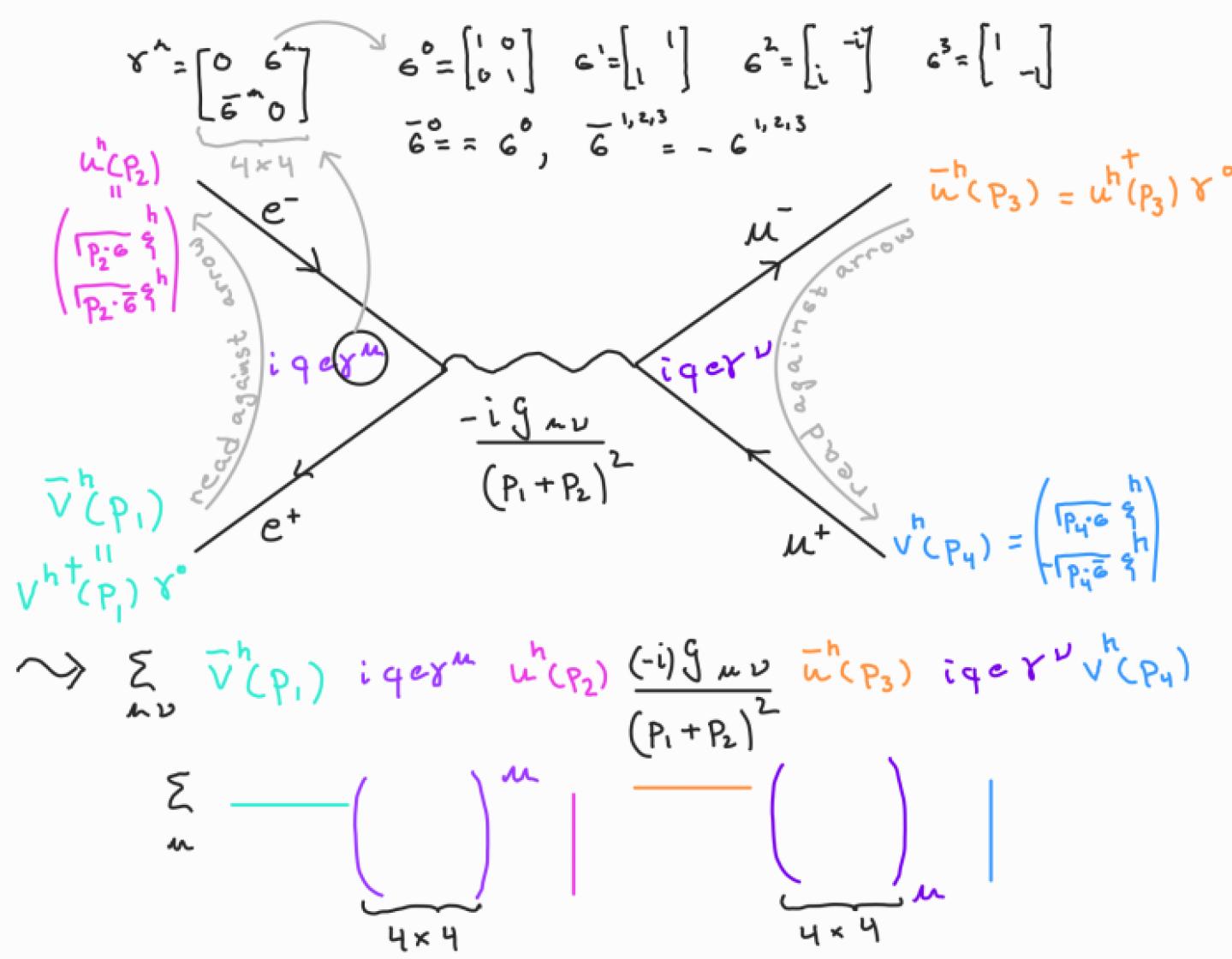
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Christian Reuschle, Simon Plätzer, Adam
Warnerbring, Zenny Wettersten

- Inspiration from QCD color
- Dissection of spacetime
- Simplification of Feynman rules, examples
- Conclusion and outlook

The bigger picture



A simple calculation (?)



In QCD we translate color to flows

- For the strong force (QCD) we have color as well: For each quark-gluon vertex a factor t_{ij}^a , a generator of $SU(3)$, i.e., a matrix giving an infinitesimal rotation of a complex three-component vector
- Fierz identity (here $T_R = 1$)

$$\underbrace{\begin{array}{c} i \rightarrow j \\ k \leftarrow l \\ \text{---} \\ t_{ij}^g t_{lk}^g \end{array}}_{t_{ij}^g t_{lk}^g} = \underbrace{\begin{array}{c} i \rightarrow j \\ k \leftarrow l \\ \text{---} \\ \delta_{ik} \delta_{lj} \end{array}}_{\delta_{ik} \delta_{lj}} - \frac{1}{N} \underbrace{\begin{array}{c} i \rightarrow j \\ k \leftarrow l \\ \text{---} \\ \delta_{ij} \delta_{lk} \end{array}}_{\delta_{ij} \delta_{lk}}$$

$SU(N=3)$ remove gluon indices



- The Dirac spinor structure transforms under the direct sum

representation $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ in the chiral/Weyl basis

$$\begin{pmatrix} u_L \\ u_R \end{pmatrix} \rightarrow \begin{pmatrix} e^{-i\bar{\theta} \cdot \frac{\vec{\sigma}}{2} + \bar{\eta} \cdot \frac{\vec{\sigma}}{2}} & 0 \\ 0 & e^{-i\bar{\theta} \cdot \frac{\vec{\sigma}}{2} - \bar{\eta} \cdot \frac{\vec{\sigma}}{2}} \end{pmatrix} \begin{pmatrix} u_L \\ u_R \end{pmatrix}$$

i.e. actually two copies of $\mathbf{SL}(2, \mathbb{C})$, infinitesimally two $\mathbf{su}(2)$

- Consider the matrix

$$p^{\dot{\alpha}\beta} = (p_\mu \sigma^\mu)^{\dot{\alpha}\beta} = \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix}^{\dot{\alpha}\beta}$$

If we transform this as indicated by the indices $\Lambda^{\dot{\alpha}}{}_{\dot{\beta}} \Lambda^{\alpha}{}_{\beta} p^{\dot{\beta}\dot{\beta}}$, we recover the transformation of a four-vector!

- Lorentz group \sim two copies of $\mathbf{su}(2)$, $so(3, 1) \cong su(2) \oplus su(2)$
- Can we do something similar for the Lorentz structure?



- Consider massless particles: chirality \sim helicity
- Spinors (in notation from the spinor-helicity formalism)

$$u^+(p) = v^-(p) = \begin{pmatrix} 0 \\ |p\rangle \end{pmatrix} \quad u^-(p) = v^+(p) = \begin{pmatrix} |p] \\ 0 \end{pmatrix}$$

$$\bar{u}^+(p) = \bar{v}^-(p) = \begin{pmatrix} [p|, & 0 \end{pmatrix} \quad \bar{u}^-(p) = \bar{v}^+(p) = \begin{pmatrix} 0, & \langle p| \end{pmatrix}$$

Note that incoming particles and antiparticles have bracket type corresponding to their helicity, but outgoing have the opposite



- Amplitudes have to be Lorentz invariant
- Lorentz inner products formed using **the only $SL(2, \mathbb{C})$ invariant object** $\epsilon^{\alpha\beta}$, $\epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12}$

$$\underbrace{\epsilon^{\alpha\beta}|i\rangle_\beta}_{\equiv \langle i|^\alpha} |j\rangle_\alpha = \langle i|^\alpha |j\rangle_\alpha = \langle ij\rangle, \quad \underbrace{\epsilon_{\dot{\alpha}\dot{\beta}}|i]^\dot{\beta}}_{\equiv [i|_{\dot{\alpha}}} |j]^{\dot{\alpha}} = [i]_{\dot{\alpha}} |j]^{\dot{\alpha}} = [ij],$$

where $|j]^{\dot{\alpha}} = |p_j]^{\dot{\alpha}}$ etc.

- \Rightarrow Amplitudes are built up of contractions of form $\langle ij\rangle, [ij] \sim \sqrt{s_{ij}}$
- If we manage to create a flow picture, the “flow” must contract dotted and undotted indices separately



- First step: Notation for spinor inner products (only possible Lorentz invariants)

$$\langle i |^\alpha | j \rangle_\alpha \equiv \langle ij \rangle = -\langle ji \rangle = i \xrightarrow{\hspace{1cm}} j$$

$$[i |_{\dot{\beta}} | j]{}^{\dot{\beta}} \equiv [ij] = -[ji] = i \xrightarrow{\dots} j$$

- Spinors and Kronecker deltas

$$\langle i |^\alpha = \text{circle with red arrow pointing left} \quad , \quad | j \rangle_\alpha = \text{circle with red arrow pointing right}$$

$$[i |_{\dot{\beta}} = \text{circle with blue dashed arrow pointing left} \quad , \quad | j]^{\dot{\beta}} = \text{circle with blue dashed arrow pointing right}$$

$$\delta_\alpha{}^\beta \equiv \mathbb{1}_\alpha{}^\beta = \text{circle with red arrow from } \alpha \text{ to } \beta \quad , \quad \delta^{\dot{\beta}}{}_{\dot{\alpha}} \equiv \mathbb{1}^{\dot{\beta}}{}_{\dot{\alpha}} = \text{circle with blue dashed arrow from } \dot{\beta} \text{ to } \dot{\alpha}$$



Towards chirality flow: Photon exchange

- Recall color, a single $SU(N)$: generators $t^a \rightarrow \delta$'s

$$\begin{array}{c}
 i \xrightarrow{\hspace{1cm}} j \\
 k \xleftarrow{\hspace{1cm}} l
 \end{array} =
 \underbrace{\begin{array}{c}
 i \xrightarrow{\hspace{1cm}} j \\
 k \xleftarrow{\hspace{1cm}} l
 \end{array}}_{t_{ij}^g t_{lk}^g} - \frac{1}{N} \underbrace{\begin{array}{c}
 i \xrightarrow{\hspace{1cm}} j \\
 k \xleftarrow{\hspace{1cm}} l
 \end{array}}_{\delta_{ij} \delta_{lk}}$$

- For the Lorentz structure $\gamma^\mu = \sqrt{2} \begin{pmatrix} 0 & \tau^\mu \\ \bar{\tau}^\mu & 0 \end{pmatrix}$ in vertices

$$\begin{array}{c}
 \alpha \quad \dot{\beta} \\
 \text{---} \quad \text{---} \\
 \text{---} \quad \text{---} \\
 \eta \quad \dot{\gamma}
 \end{array} =
 \underbrace{\begin{array}{c}
 \alpha \quad \dot{\beta} \\
 \text{---} \quad \text{---} \\
 \text{---} \quad \text{---} \\
 \eta \quad \dot{\gamma}
 \end{array}}_{\bar{\tau}_{\alpha \dot{\beta}}^\mu \tau_{\mu}^{\dot{\gamma} \eta}} - \frac{1}{N} \underbrace{\begin{array}{c}
 \alpha \quad \dot{\beta} \\
 \text{---} \quad \text{---} \\
 \text{---} \quad \text{---} \\
 \eta \quad \dot{\gamma}
 \end{array}}_{\delta_{\alpha}^{\eta} \delta_{\dot{\beta}}^{\dot{\gamma}}}$$



Non-matching arrows?

Arrows opposed? Flip them (when contracted between external spinors) First use charge conjugation

$$\begin{array}{ccc}
 \text{Diagram: } i \xrightarrow{\mu} j & = & \text{Diagram: } i \xleftarrow{\mu} j \\
 \underbrace{i \xrightarrow{\alpha} \bar{\tau}_{\dot{\alpha}}^{\mu} |j\rangle_{\dot{\beta}}}_{\langle i|_{\alpha} \bar{\tau}_{\dot{\alpha}}^{\mu} |j\rangle_{\dot{\beta}}} & & \underbrace{|j\rangle_{\dot{\alpha}} \tau^{\mu, \dot{\alpha}}_{\beta} |i\rangle_{\beta}}_{\langle j|_{\dot{\alpha}} \tau^{\mu, \dot{\alpha}}_{\beta} |i\rangle_{\beta}}
 \end{array}$$

Then use Fierz identity $\bar{\tau}_{\alpha\dot{\beta}}^{\mu} \tau_{\mu}^{\dot{\gamma}\eta} = \delta_{\alpha}^{\eta} \delta_{\dot{\beta}}^{\dot{\gamma}}$ to remove vector index

$$\begin{array}{ccc}
 \text{Diagram: } 1 \xrightarrow{\mu} 2 & = & \text{Diagram: } 1 \xrightarrow{\mu} 2 \\
 \text{Diagram: } 4 \xleftarrow{\mu} 3 & & \text{Diagram: } 4 \xrightarrow{\mu} 3 \\
 \underbrace{(\langle 1|_{\alpha} \bar{\tau}_{\dot{\alpha}}^{\mu} |2\rangle_{\dot{\beta}})(\langle 3|_{\gamma} \bar{\tau}_{\mu, \gamma\dot{\eta}} |4\rangle_{\dot{\eta}})}_{(\langle 1|_{\alpha} \bar{\tau}_{\dot{\alpha}}^{\mu} |2\rangle_{\dot{\beta}})(\langle 3|_{\gamma} \bar{\tau}_{\mu, \gamma\dot{\eta}} |4\rangle_{\dot{\eta}})} & & \underbrace{(\langle 1|_{\alpha} \bar{\tau}_{\dot{\alpha}}^{\mu} |2\rangle_{\dot{\beta}})(\langle 4|_{\dot{\eta}} \tau_{\mu}^{\dot{\eta}\gamma} |3\rangle_{\gamma})}_{\text{flipped}} \\
 & & \text{Diagram: } 1 \xrightarrow{\mu} 2 \\
 & & \text{Diagram: } 4 \xrightarrow{\mu} 3 \\
 & & \underbrace{\langle 13 \rangle [42]}_{\langle 13 \rangle [42]}
 \end{array}$$



Fermion propagators

- We split $\not{p}_{4d} \equiv p_\mu \gamma^\mu = p_\mu \sqrt{2} \begin{pmatrix} 0 & \tau^\mu \\ \bar{\tau}^\mu & 0 \end{pmatrix}$ into two terms

$$\not{p} \equiv \sqrt{2} p^\mu \tau_{\mu}^{\dot{\alpha}\beta} = \text{---} \xrightarrow{-} \bullet \xrightarrow{p} \quad \bar{\not{p}} \equiv \sqrt{2} p_\mu \bar{\tau}_{\alpha\dot{\beta}}^\mu = \xrightarrow{p} \bullet \xrightarrow{-} \text{---}$$

- For massless momenta we have

$$\sqrt{2} p^\mu \tau_\mu \equiv \not{p} = |p] \langle p| , \quad \sqrt{2} p^\mu \bar{\tau}_\mu \equiv \bar{\not{p}} = |p\rangle [p|$$

- In a propagator, we have $p^\mu = \sum p_i^\mu$, $p_i^2 = 0$

$$\not{p} = \text{---} \xrightarrow{-} \bullet \xrightarrow{\sum_i p_i} = \sum_i |i] \dot{\alpha} \langle i|^\beta \quad \text{for} \quad p_i^2 = 0$$

$$\bar{\not{p}} = \xrightarrow{p} \bullet \xrightarrow{-} \text{---} = \sum_i |i\rangle_\alpha [i|_\beta \quad \text{for} \quad p_i^2 = 0$$



External gauge bosons

- In the spinor-helicity formalism

$$\epsilon_L^\mu(p, r) \rightarrow \frac{|r\rangle[p|}{\langle rp\rangle} \text{ or } \frac{|p|\langle r|}{\langle rp\rangle}, \quad \epsilon_R^\mu(p, r) \rightarrow \frac{|r]\langle p|}{[pr]} \text{ or } \frac{|p\rangle[r|}{[pr]}$$

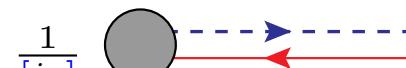
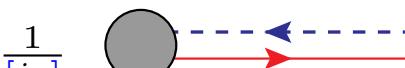
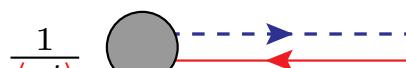
- \implies easy to translate to chirality flow

$$\begin{aligned} \epsilon_L^\mu(p, r) &\rightarrow \frac{1}{\langle rp\rangle} \text{ (grey circle)} \xrightarrow{\text{red arrow}} \begin{matrix} p \\ r \end{matrix} , \quad \text{or} \quad \epsilon_L^\mu(p, r) \rightarrow \frac{1}{\langle rp\rangle} \text{ (grey circle)} \xleftarrow{\text{red arrow}} \begin{matrix} p \\ r \end{matrix} \\ \epsilon_R^\mu(p, r) &\rightarrow \frac{1}{[pr]} \text{ (grey circle)} \xleftarrow{\text{red arrow}} \begin{matrix} r \\ p \end{matrix} , \quad \text{or} \quad \epsilon_R^\mu(p, r) \rightarrow \frac{1}{[pr]} \text{ (grey circle)} \xrightarrow{\text{blue arrow}} \begin{matrix} r \\ p \end{matrix} \end{aligned}$$

- In a Feynman diagram choose arrow directions which give aligned arrows through momentum dots, and opposing arrows for the two spinors of a gauge boson. (After careful consideration one can prove that this flow picture always works.)



The QED flow rules: massless particles

Species	Feynman	Flow
$\bar{u}^R(p_i)$		
$v^R(p_j)$		
$v^L(p_j)$		
$\bar{u}^L(p_i)$		
$\epsilon_R^\mu(p_i, r)$		 or 
$\epsilon_L^\mu(p_i, r)$		 or 

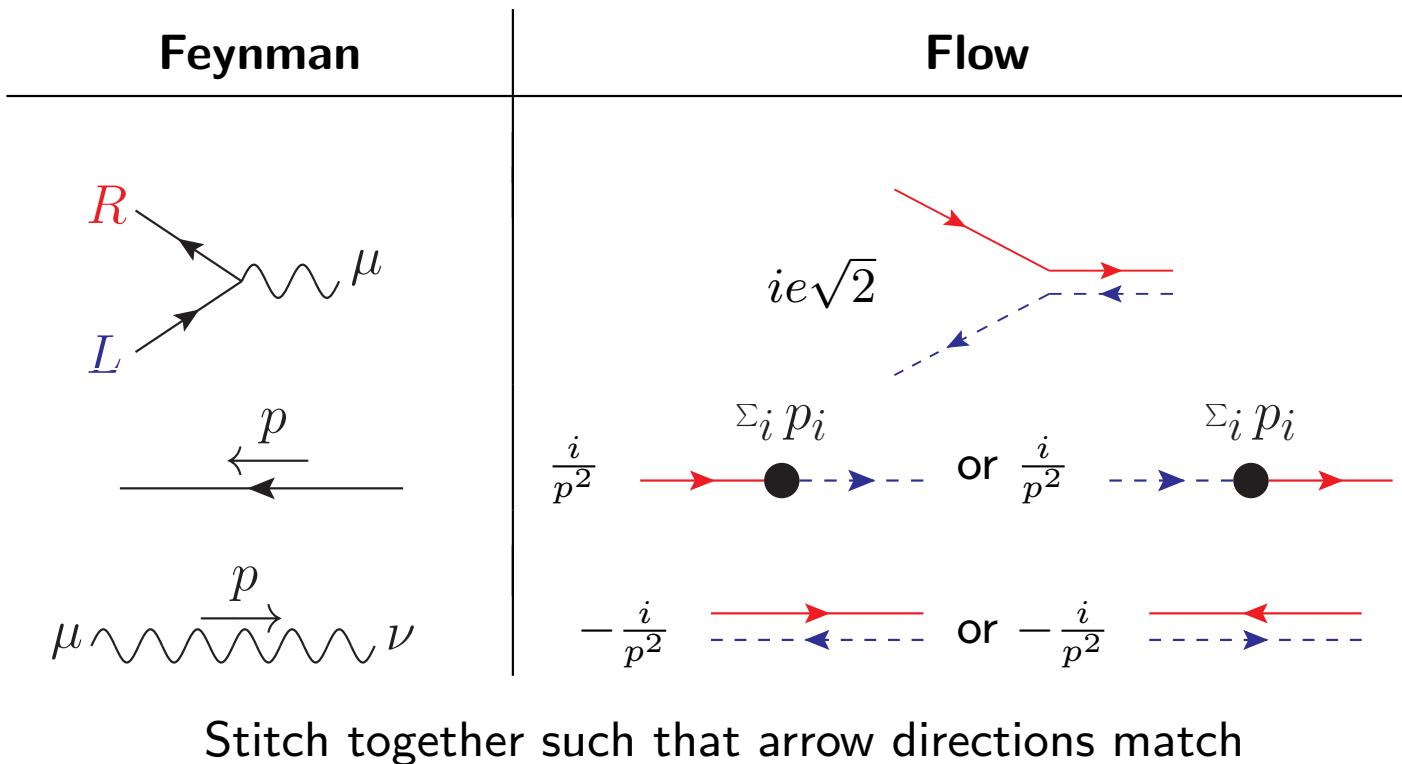
Use **left** and **right** chiral spinors $\underbrace{\text{su}(2)}$, $\underbrace{\text{su}(2)}$ and R/L for outgoing $-/+$

dotted undotted

helicity (A. Lifson, C. Reuschle and MS, 2003.05877 (EPJC))

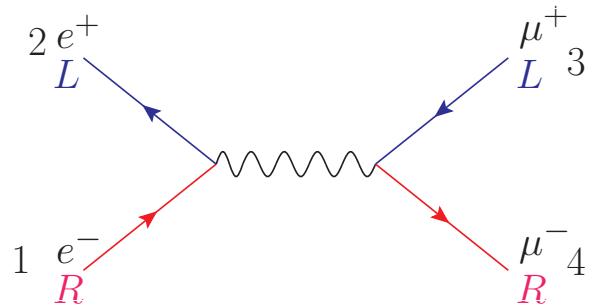


The QED flow rules: vertices and propagators



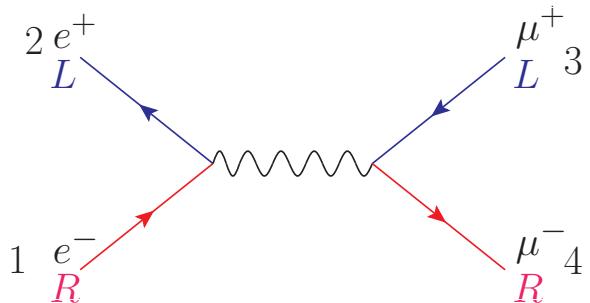
Simplest QED example

- Ordinary Feynman diagrams



$$= \frac{ie^2}{s_{e^+e^-}} \bar{v}^+(p_2) \gamma^\mu u^-(p_1) \bar{u}^+(p_4) \gamma_\mu v^-(p_3) = \dots$$

- Regular spinor-helicity, easy

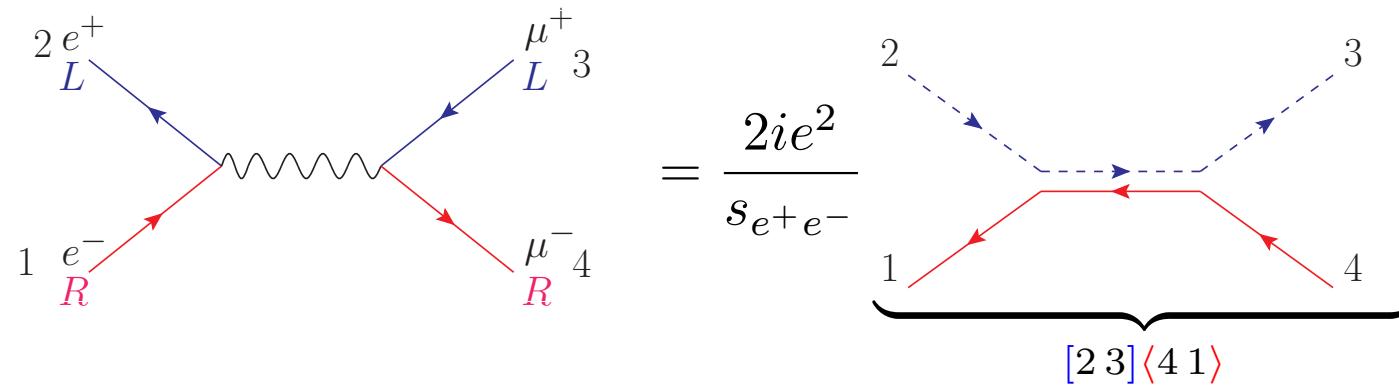


$$\begin{aligned} &= \frac{2ie^2}{s_{e^+e^-}} ([2]_{\dot{\alpha}} \tau_\mu^{\dot{\alpha}\beta} |1\rangle_\beta) (\langle 4|^\alpha \bar{\tau}_{\alpha\dot{\beta}}^\mu |3]^{\dot{\beta}}) \\ &= \frac{2ie^2}{s_{e^+e^-}} [2]_{\dot{\alpha}} [3]^{\dot{\alpha}} \langle 4|^\beta |1\rangle_\beta \equiv \frac{2ie^2}{s_{e^+e^-}} [2\ 3] \langle 4\ 1\rangle \end{aligned}$$



Simplest QED example

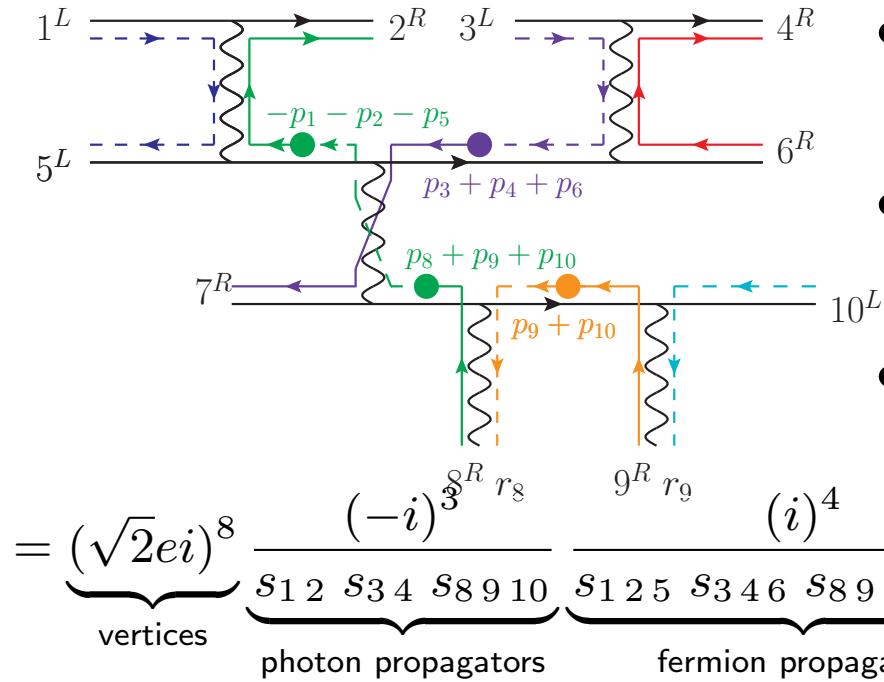
- Chirality flow, super easy and intuitive



- TODO: A slightly more complicated example on the board



A complicated QED example



- Pick any consistent arrow direction
- Compare to standard QFT: 12 γ^μ matrices
- Here all particles crossed to outgoing

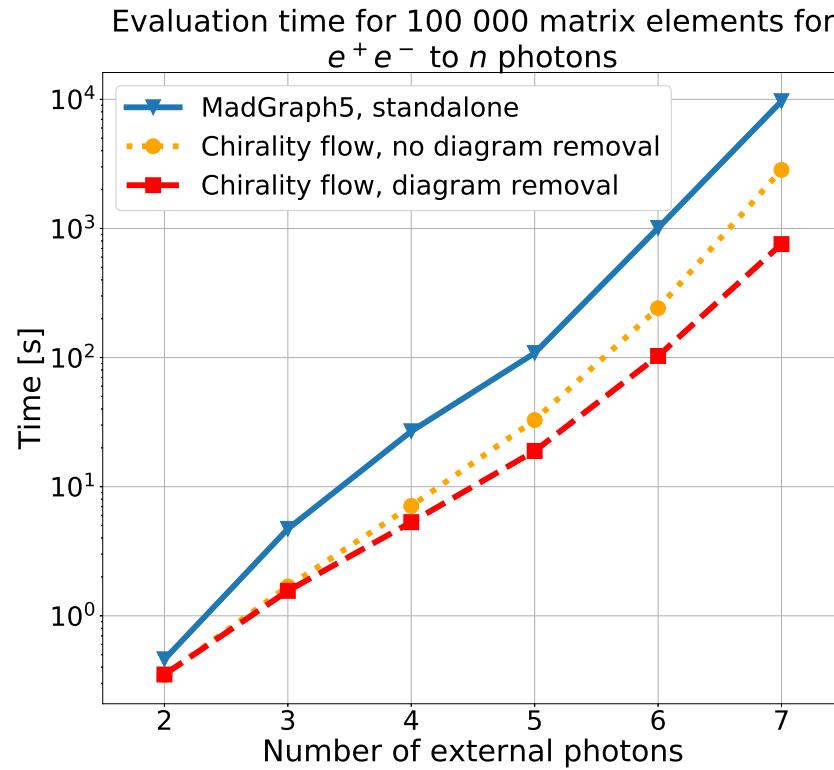
$$= \underbrace{(\sqrt{2}ei)^8}_{\text{vertices}} \frac{(-i)^{\sum_{i=1}^8 r_i}}{\underbrace{s_{12}s_{34}s_{8910}}_{\text{photon propagators}}} \frac{(i)^4}{\underbrace{s_{125}s_{346}s_{8910}s_{910}}_{\text{fermion propagators}}} \frac{1}{\underbrace{[8r_8][9r_9]}_{\text{polarization vectors}}} [15]\langle 64 \rangle [10] r_9$$

$$\times \left(\underbrace{\langle 99 \rangle [9r_8] + \langle 910 \rangle [10r_8]}_0 \right) \left(\underbrace{[33]\langle 37 \rangle + [34]\langle 47 \rangle + [36]\langle 67 \rangle}_0 \right)$$

$$\times \left(-\langle 89 \rangle [91]\langle 12 \rangle - \langle 89 \rangle [95]\langle 52 \rangle - \langle 810 \rangle [101]\langle 12 \rangle - \langle 810 \rangle [105]\langle 52 \rangle \right)$$



Implementation



Evaluation time in MadGraph5_aMC@NLO, $e^+ e^- \rightarrow n$ photons
(A. Lifson, M. Sjödahl and Z. Wettersten 2203.13618 (EPJC))



Massive momenta and spinors

- Decompose massive momentum p as sum of massless

$$p^\mu = p^b, \mu + \alpha q^\mu , \quad (p^b)^2 = q^2 = 0 , \quad \alpha = \frac{p^2}{2p \cdot q} = \frac{m^2}{2p \cdot q}$$

$$\not{p} \equiv \sqrt{2} p^\mu \tau_\mu = |p^b| \langle p^b | + \alpha |q| \langle q |$$
- Massive spinors and polarization vectors written in terms of massless Weyl spinors of momentum p^b, q ,

$$u^+(p) = \begin{pmatrix} -\frac{m}{[qp^b]} & \text{---} \xrightarrow{\hspace{1cm}} & q \\ & \text{---} \xrightarrow{\hspace{1cm}} & \\ & \text{---} \xrightarrow{\hspace{1cm}} & p^b \end{pmatrix}, \text{ etc.}$$

(All spinors in 2011.10075 eq. 3.11-3.16)
- q is arbitrary but physical, as it defines the spin direction s^μ

$$s^\mu = \frac{1}{m}(p^\mu - 2\alpha q^\mu) = \frac{1}{m}(p^\mu - \frac{m^2}{p \cdot q} q^\mu)$$



- Can also measure spin along the direction of motion

$$p^\mu = p_f^\mu + p_b^\mu$$

$$\alpha \rightarrow 1,$$

$$p^b \rightarrow p_f = \frac{p^0 + |\vec{p}|}{2}(1, \hat{p})$$

$$q \rightarrow p_b = \frac{p^0 - |\vec{p}|}{2}(1, -\hat{p})$$

Spin measured along $s^\mu = \frac{1}{m}(p_f^\mu - p_b^\mu) = \frac{1}{m}(|\vec{p}|, p^0 \hat{p})$, spatial components along the direction of motion

(Full Standard Model in J. Alnefjord, A.Lifson, C.Reuschle and M. Sjodahl 2011.10075 (EPJC))



What is spin?

- Spin operator Σ^μ and Pauli-Lubanski operator W^μ

$$\frac{1}{2}\Sigma^\mu = -\frac{1}{4m}\epsilon^{\mu\nu\lambda\omega}P_\nu\sigma_{\lambda\omega} = \frac{1}{m}W^\mu$$

- Here $W^2 = -m^2J(J+1)$, is the 2nd quadratic Casimir operators of the Poincaré algebra, along $P^2 = m^2$, ($P_\nu = i\partial/\partial x^\nu$), and $\sigma^{\mu\nu}$ (giving the Lorentz generators for the $(1/2, 1/2)$ -representation) is defined as $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$.
- The spin projected onto s^μ is given by the operator

$$\mathcal{O}_s = -\frac{\Sigma^\mu s_\mu}{2} = \frac{1}{4m}\epsilon^{\mu\nu\lambda\omega}s_\mu P_\nu\sigma_{\lambda\omega} ,$$

such that the spin direction, in this sense, is only defined up to a four-momentum proportional to p .



- Adding any linear combination of p to $s = (1/m)(p - 2\alpha q)$ leaves \mathcal{O}_s invariant
- Therefore, we could equally well have used $s' = -2\alpha q/m = -mq/(p \cdot q)$ for the spin direction, when expressed as above. When rewritten as $\mathcal{O}_s = \frac{1}{2}\gamma^5 s^\mu \gamma_\mu$ the spin vector must be taken to be s .
- Thus q plays the role of defining the *other* four-vector (aside from p) which determines the operator \mathcal{O}_s , and thereby what we mean with positive and negative spin.
- In the non-relativistic limit $p = (m, 0, 0, 0)$ and $s = (0, \hat{s})$. Clearly, adding any linear combination of p to s or q will leave \mathcal{O}_s invariant.



Fermion vertices

- Fermion-vector vertex

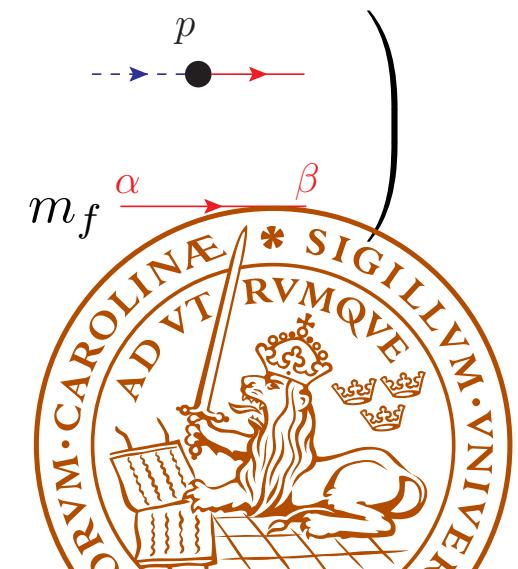
$$\begin{array}{ccc}
 \text{Diagram:} & = ie(P_L C_L + P_R C_R) \gamma^\mu = ie\sqrt{2} \left(\begin{array}{cc} 0 & \\ C_L & \end{array} \right. & \text{Diagram:} \\
 & & \left. \begin{array}{c} C_R \\ 0 \end{array} \right)
 \end{array}$$

The left diagram shows a fermion line (solid black) and a vector boson line (wavy) meeting at a vertex. The right diagram shows a matrix representation of the vertex function.

Left and right chiral couplings may differ, in particular $C_R = 0$
 for $W^\pm \Rightarrow$ **electroweak sector nice** in chirality flow

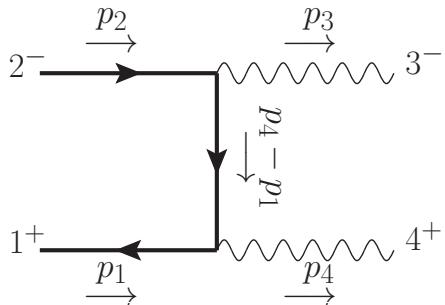
- Fermion propagator

$$\frac{i}{p^2 - m_f^2} \begin{pmatrix} m_f \delta_{\dot{\alpha}}^{\dot{\beta}} & \sqrt{2} p^{\dot{\alpha}\beta} \\ \sqrt{2} \bar{p}_{\alpha\dot{\beta}} & m_f \delta_{\alpha}^{\beta} \end{pmatrix} = \frac{i}{p^2 - m_f^2} \begin{pmatrix} m_f \overset{\dot{\alpha}}{\cdots} \overset{\dot{\beta}}{\cdots} \\ \overset{p}{\cdots} \end{pmatrix} \quad \text{Diagram:}$$



A massive example

Consider the diagram of $e_{1+}^+ e_{2-}^- \rightarrow \gamma_3^{-=R} \gamma_4^{+=L}$ and include the mass m_e



- Obtain 3 new terms
- Simplify with choices of q_1, q_2, r_3, r_4
 $q_2 = p_3, r_3 = p_2^\flat, \dots$

$$= \frac{-2ie^2}{((p_4 - p_1)^2 - m_e^2)[3r_3]\langle 4r_4 \rangle} \left\{ \begin{array}{c} \text{Diagram 1: } p_2^\flat \xrightarrow{r_3} 3 \\ \text{Diagram 2: } p_1^\flat \xrightarrow{r_4} 4 \\ \text{Diagram 3: } p_4 - p_1^\flat - q_1 \end{array} \right. - \frac{m_e}{[q_1 p_1^\flat]} \frac{m_e}{\langle p_2^\flat q_2 \rangle} \quad \text{Diagrams 4 and 5: } \begin{array}{c} q_2 \xleftarrow{r_3} 3 \\ q_1 \xleftarrow{r_4} 4 \\ p_4 - p_1^\flat - q_1 \end{array}$$

$$+ m_e \left(\frac{m_e}{\langle p_2^\flat q_2 \rangle} \text{Diagram 6: } \begin{array}{c} q_2 \xleftarrow{r_3} 3 \\ p_1^\flat \xrightarrow{r_4} 4 \end{array} - \frac{m_e}{[q_1 p_1^\flat]} \text{Diagram 7: } \begin{array}{c} p_2^\flat \xleftarrow{r_3} 3 \\ q_1 \xleftarrow{r_4} 4 \end{array} \right)$$



The non-abelian QCD vertices

$$\begin{aligned}
 & \text{Diagram: Three external gluon lines meeting at a vertex. The top line has momentum } p_1 \text{ and color index } a_1, \mu_1. \text{ The middle line has momentum } p_2 \text{ and color index } a_2, \mu_2. \text{ The bottom line has momentum } p_3 \text{ and color index } a_3, \mu_3. \\
 & = i \frac{g_s}{\sqrt{2}} i f^{a_1 a_2 a_3} \left(g^{\mu_1 \mu_2} (p_1 - p_2)^{\mu_3} + g^{\mu_2 \mu_3} (p_2 - p_3)^{\mu_1} + g^{\mu_3 \mu_1} (p_3 - p_1)^{\mu_2} \right) \\
 & \rightarrow i \frac{g_s}{\sqrt{2}} i f^{a_1 a_2 a_3} \frac{1}{\sqrt{2}} \left(\text{Diagram 1: } p_1 - p_2 \text{ (dashed blue), } p_2 - p_3 \text{ (dashed red), } p_3 - p_1 \text{ (solid red). Vertex at } p_2.} \right. \\
 & \quad \left. + \text{Diagram 2: } p_1 - p_2 \text{ (dashed blue), } p_2 - p_3 \text{ (dashed red), } p_3 - p_1 \text{ (solid red). Vertex at } p_3.} \right. \\
 & \quad \left. + \text{Diagram 3: } p_1 - p_2 \text{ (dashed blue), } p_2 - p_3 \text{ (dashed red), } p_3 - p_1 \text{ (solid red). Vertex at } p_1.} \right)
 \end{aligned}$$

Here f^{abc} is the color factor in the vertex, the SU(3) structure constants (A.Lifson, C.Reuschle and M. Sjodahl 2011.10075 (EPJC))



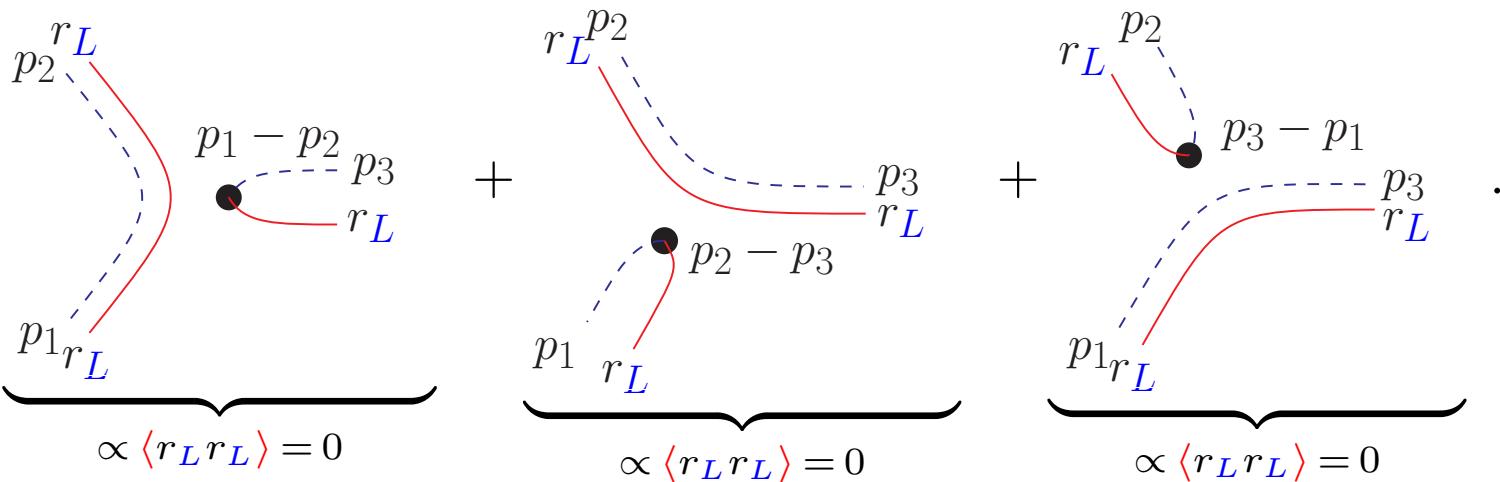
$$\begin{aligned}
 & \text{Diagram: Four external lines labeled } \mu_1, a_1, \mu_2, a_2 \text{ at top and } \mu_4, a_4, \mu_3, a_3 \text{ at bottom, forming a cross-like shape.} \\
 & = i \left(\frac{g_s}{\sqrt{2}} \right)^2 \sum_{Z(2,3,4)} i f^{a_1 a_2 b} i f^{b a_3 a_4} \left(g^{\mu_1 \mu_3} g^{\mu_2 \mu_4} - g^{\mu_1 \mu_4} g^{\mu_2 \mu_3} \right) \\
 & \rightarrow i \left(\frac{g_s}{\sqrt{2}} \right)^2 \sum_{Z(2,3,4)} i f^{a_1 a_2 b} i f^{b a_3 a_4} \left(\begin{array}{c} 1 \\ \diagdown \\ 4 \\ \diagup \\ 2 \\ \diagdown \\ 3 \\ \diagup \\ 4 \\ \end{array} - \begin{array}{c} 1 \\ | \\ 2 \\ | \\ 3 \\ | \\ 4 \\ \end{array} \right)
 \end{aligned}$$

where $Z(2, 3, 4)$ denotes the set of cyclic permutations of the integers 2, 3, and 4

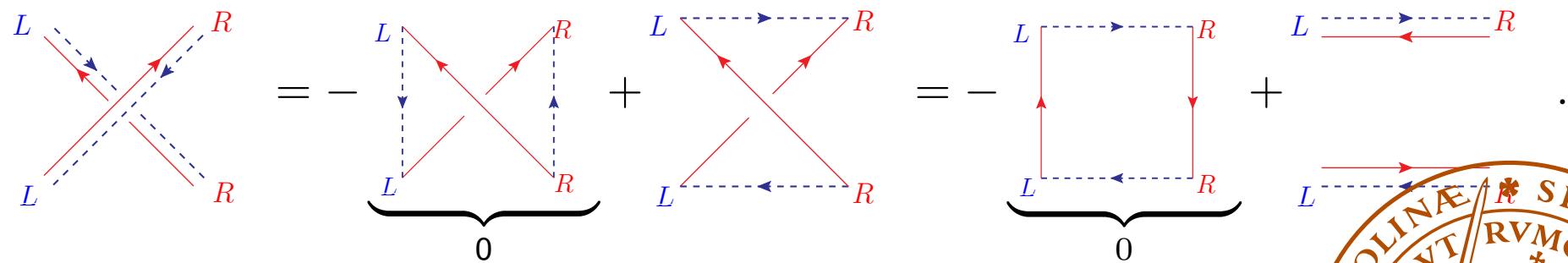
TODO: A QCD example on the board...



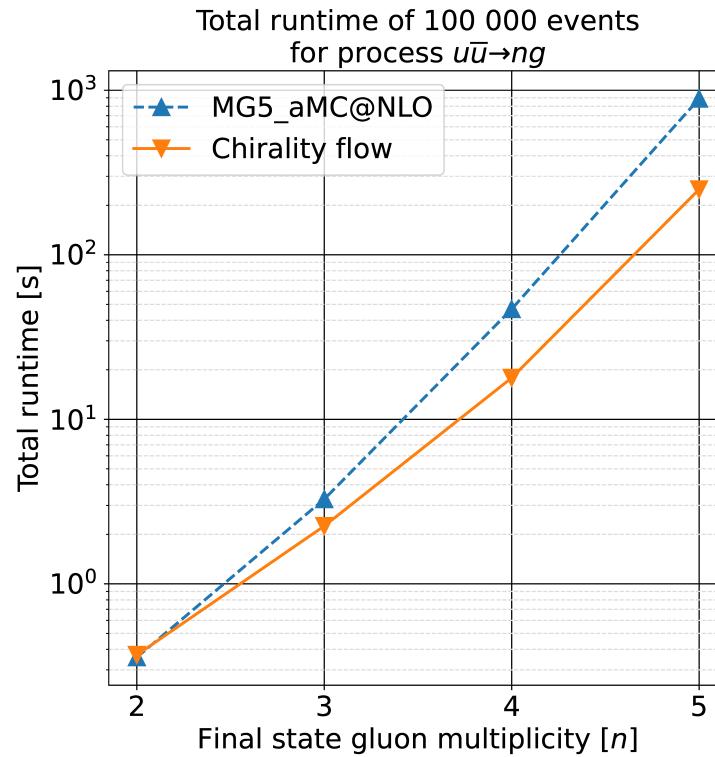
The gluon vertices can be simplified due to a nice gauge choice, for example, we can not have three left gluons



The four-gluon vertex simplifies further due to the Schouten identity



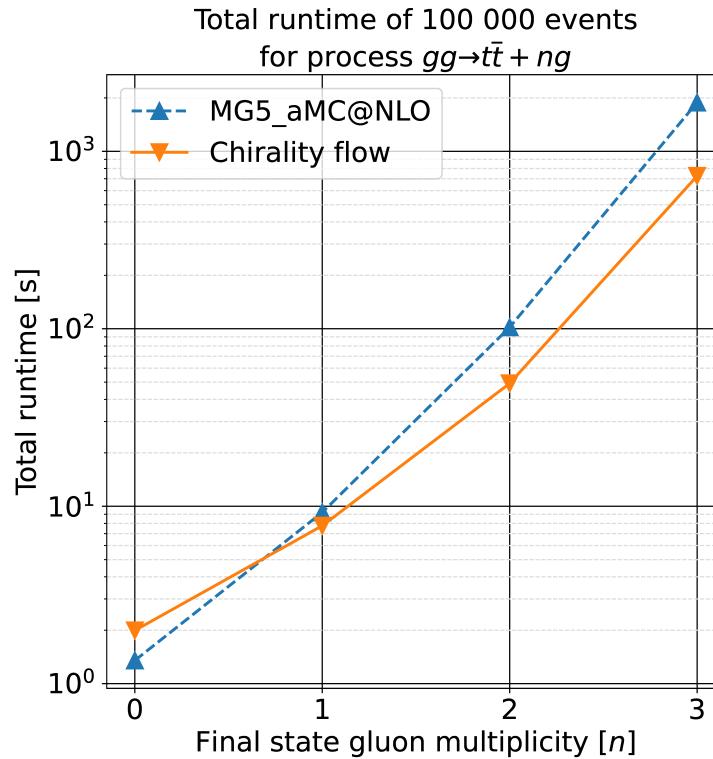
Implementation



Runtime for the Lorentz structure of $u\bar{u} \rightarrow ng$ for the standalone version of MadGraph5_aMC@NLO and for the chirality-flow branch.
(E. Boman, A. Lifson, M. Sjodahl, A. Warnerbring, Z. Wettersten,
2312.07447 (JHEP))



Implementation



Runtime for the Lorentz structure of $gg \rightarrow t\bar{t} + ng$, for the standalone version of MadGraph5_aMC@NLO and for the chirality-flow branch.
(E. Boman, A. Lifson, M. Sjodahl, A. Warnerbring, Z. Wettersten,
2312.07447 (JHEP))



Electroweak physics

- This sector is chiral → largest amount of simplification expected
- In particular the W^\pm vertex is simple
- TODO: Consider W exchange



Electroweak physics

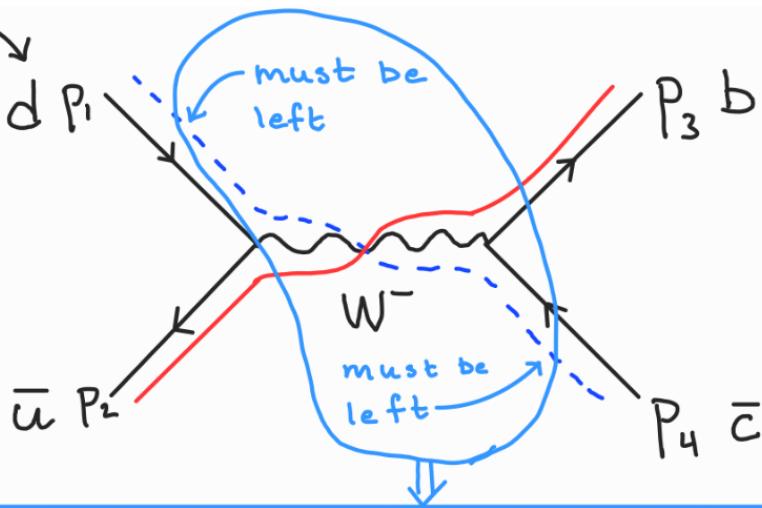
- This sector is chiral → largest amount of simplification expected
- In particular the W^\pm vertex is simple

Assume positive spin along $s_1^\mu = \frac{1}{m_1} (p_1^\mu - \frac{m_1^2}{p_1 \cdot q} q^\mu)$

$$u^+(p_1) = \begin{pmatrix} -\frac{m_1}{[q p_1]} & \text{---} \rightarrow q \\ \text{---} \rightarrow p_1^b & \times \end{pmatrix}$$

then particle 4 can not have q for its left spinor

$$\Rightarrow v^+(p_4) = \begin{pmatrix} \text{---} \rightarrow p_4^b \\ -\frac{m_4}{\langle p_4^\mu q \rangle} \end{pmatrix} \Rightarrow$$



only one possible chirality flow

particle 4 has positive spin along $s_4^\mu = \frac{1}{m_4} (p_4^\mu - \frac{m_4^2}{p_4 \cdot q} q^\mu)$



Conclusion and outlook

- Splitting Lorentz structure into $\text{su}(2)$, $\text{su}(2)$, we are able to recast all standard model Feynman rules to chirality-flow rules
- This gives a transparent and intuitive way of writing down values of Feynman diagrams, simplifying pen-and paper calculations to the extent that they are often trivial
- Vertices can be simplified by good gauge choices, and the four-gluon vertex further by the Schouten identity
- Significant speed-up for event generation



Backup: Massive polarization vectors

External gauge bosons, for helicity states $p^\flat \rightarrow p_f$, $\alpha \rightarrow 1$, $q \rightarrow p_b$

$$\epsilon_L^\mu(p) \rightarrow \frac{|p^\flat]\langle q|}{\langle qp^\flat\rangle} \quad \text{or} \quad \frac{|q\rangle[p^\flat|}{\langle qp^\flat\rangle} \quad \epsilon_R^\mu(p) \rightarrow \frac{|q]\langle p^\flat|}{[p^\flat q]} \quad \text{or} \quad \frac{|p^\flat\rangle[q|}{[p^\flat q]}$$

$$\epsilon_0^\mu(p) = s^\mu = \frac{1}{m}(p^{\flat,\mu} - \alpha q^\mu)$$

for incoming (outgoing) bosons, use L/R for $-/+$ ($-/+$) spin along s^μ -axis. Translate to chirality flow

$$\begin{aligned} \epsilon_L^\mu(p) &\rightarrow \frac{1}{\langle qp^\flat\rangle} \text{ (grey circle)} \xrightarrow{\text{dashed blue arrow}} \begin{matrix} p^\flat \\ q \end{matrix}, \quad \text{or} \quad \epsilon_L^\mu(p) \rightarrow \frac{1}{\langle qp^\flat\rangle} \text{ (grey circle)} \xrightarrow{\text{dashed blue arrow}} \begin{matrix} p^\flat \\ q \end{matrix} \\ \epsilon_R^\mu(p) &\rightarrow \frac{1}{[p^\flat q]} \text{ (grey circle)} \xleftarrow{\text{dashed blue arrow}} \begin{matrix} q \\ p^\flat \end{matrix}, \quad \text{or} \quad \epsilon_R^\mu(p) \rightarrow \frac{1}{[p^\flat q]} \text{ (grey circle)} \xleftarrow{\text{dashed blue arrow}} \begin{matrix} q \\ p^\flat \end{matrix} \\ \epsilon_0^\mu(p) &\rightarrow \frac{1}{m\sqrt{2}} \text{ (grey circle)} \xrightarrow{\text{dashed blue arrow}} \begin{matrix} p^\flat - \alpha q \\ \bullet \end{matrix}, \quad \text{or} \quad \epsilon_0^\mu(p) \rightarrow \frac{1}{m\sqrt{2}} \text{ (grey circle)} \xleftarrow{\text{dashed blue arrow}} \begin{matrix} p^\flat - \alpha q \\ \bullet \end{matrix} \end{aligned}$$

