Politecnico di Torino

Introduction to Mechanics and Structures II

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Pressure vessels: theory

Pressure vessel: EN 13455

PRESSURE VESSEL – THEORY

Introduction

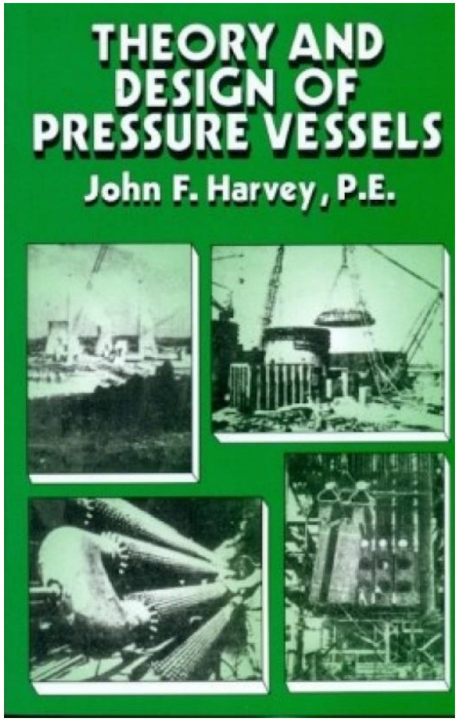
Axisymmetric shell

Membrane state of stress

Examples

Buckling

Discontinuity stresses

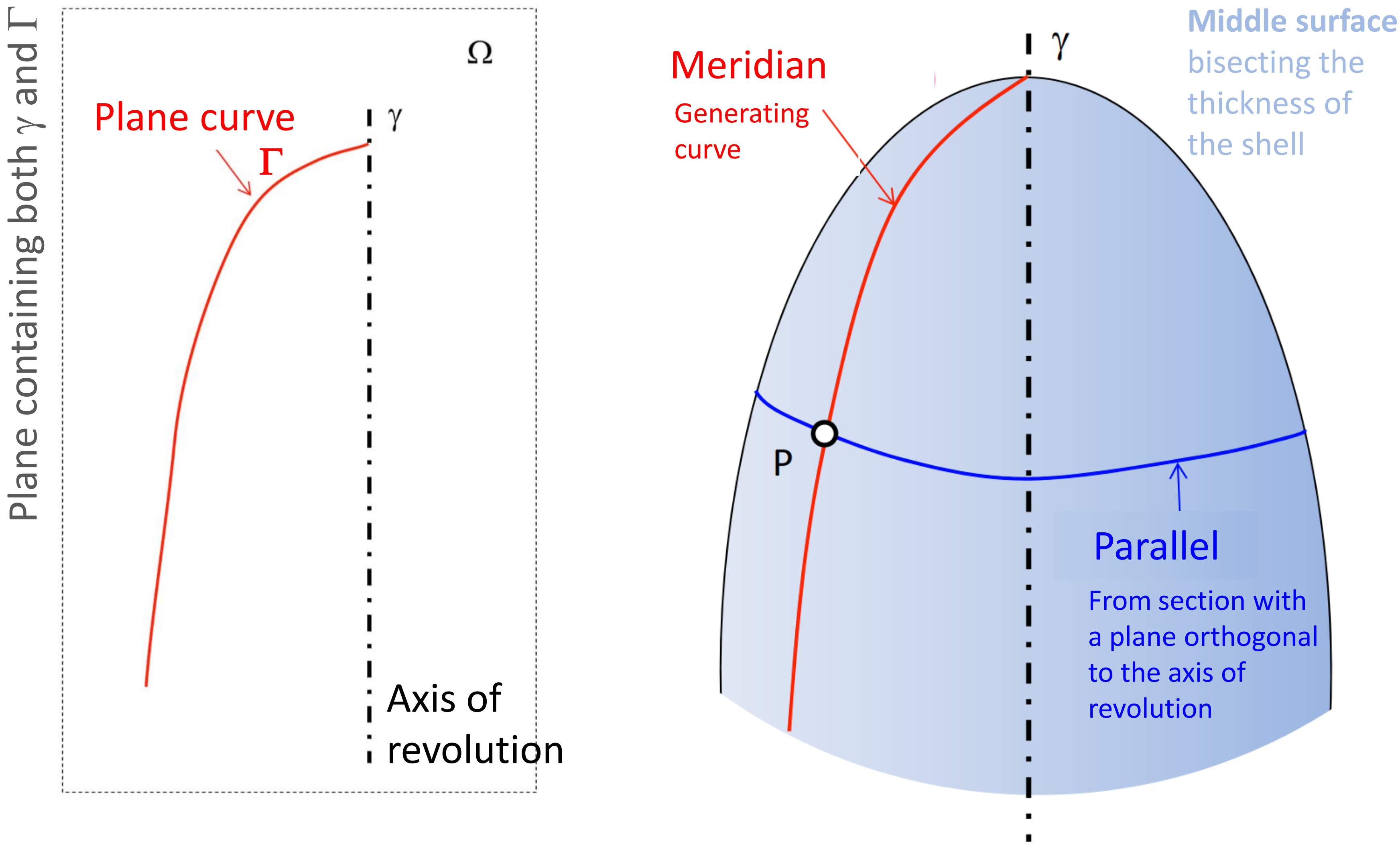


Introduction

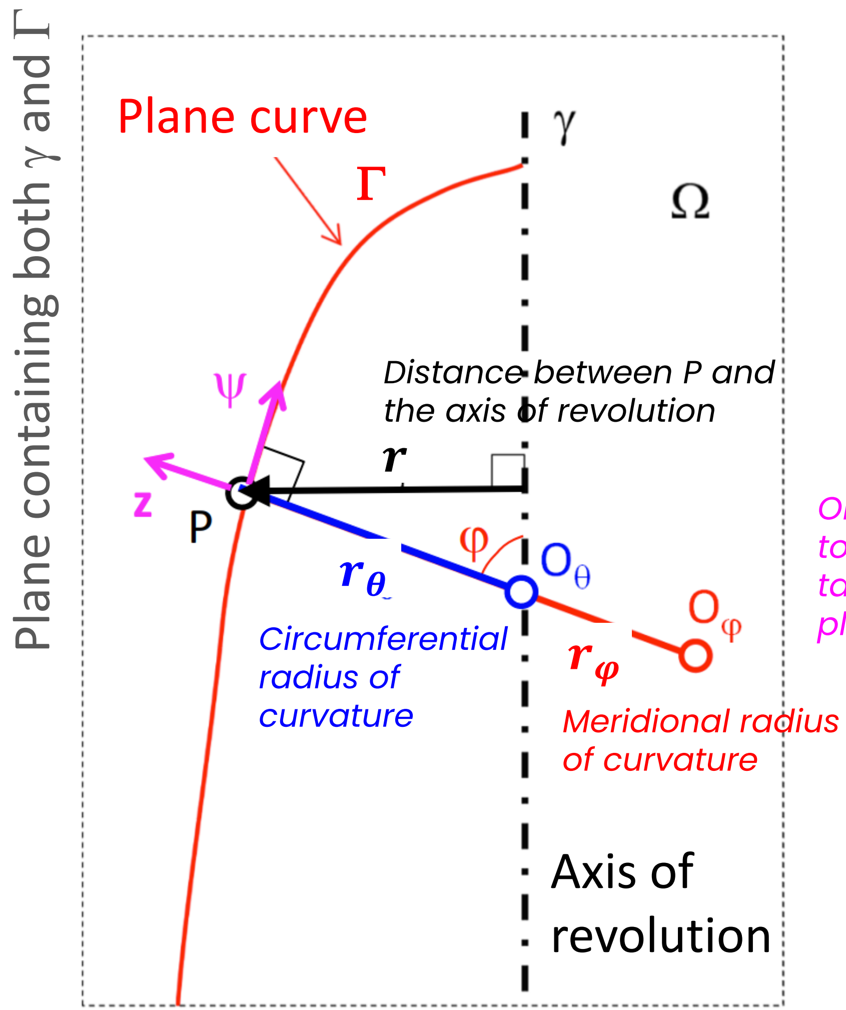
Pressure vessels are leakproof containers. They may be of any shape: they commonly have the form of spheres, cylinder, cone, ellipsoids or some composite of them. A common design is a cylinder with end caps called heads.
Vessels or shells are considered to be formed of curve plate in which the **thickness is small** in comparison with the other dimensions, offering little resistance to bending perpendicular to their surface: they are mainly subjected to a **membrane state of stress**.
The membrane stresses are *average* stresses over the thickness of the vessel and are considered to act tangent to its surface. Except for limited portions, bending is not necessary to equilibrate the pressure, differently from what happens in case of planar plates.

Axisymmentric shell

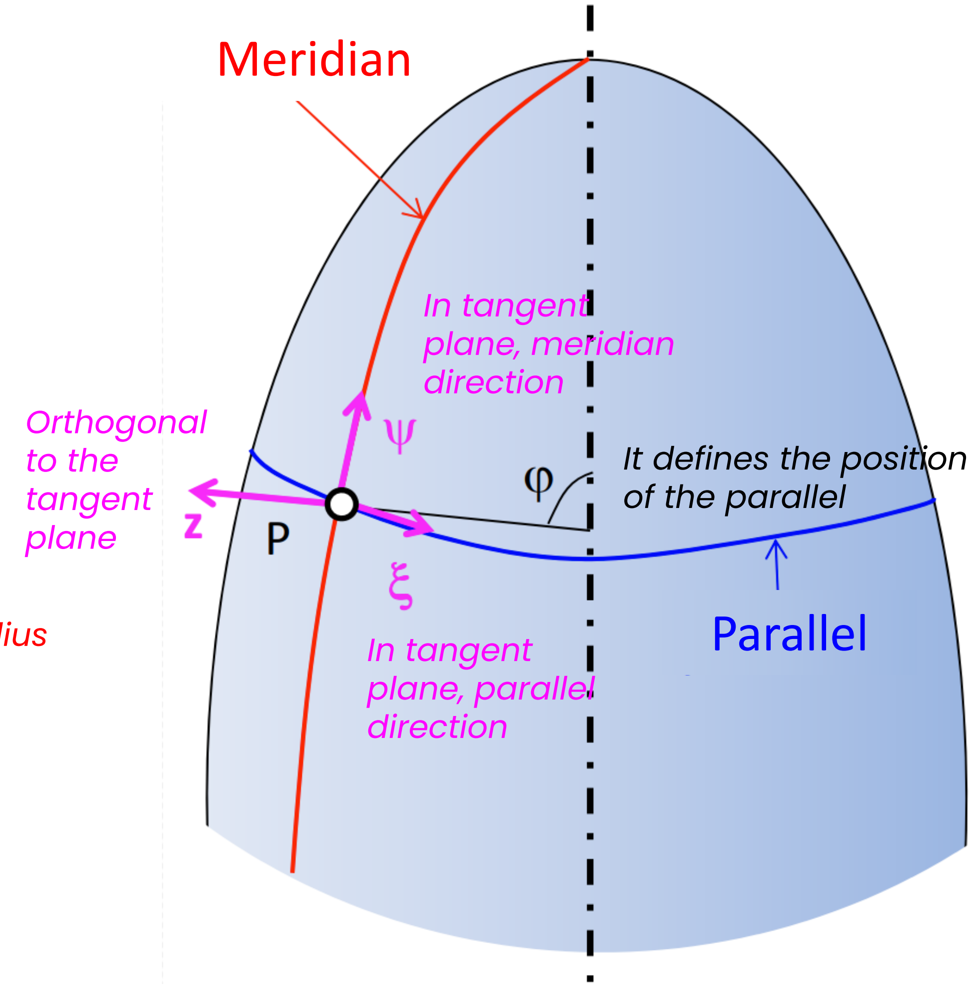
Shell of revolution: axisymmetric geometry defined by middle surface



Axisymmentric shell

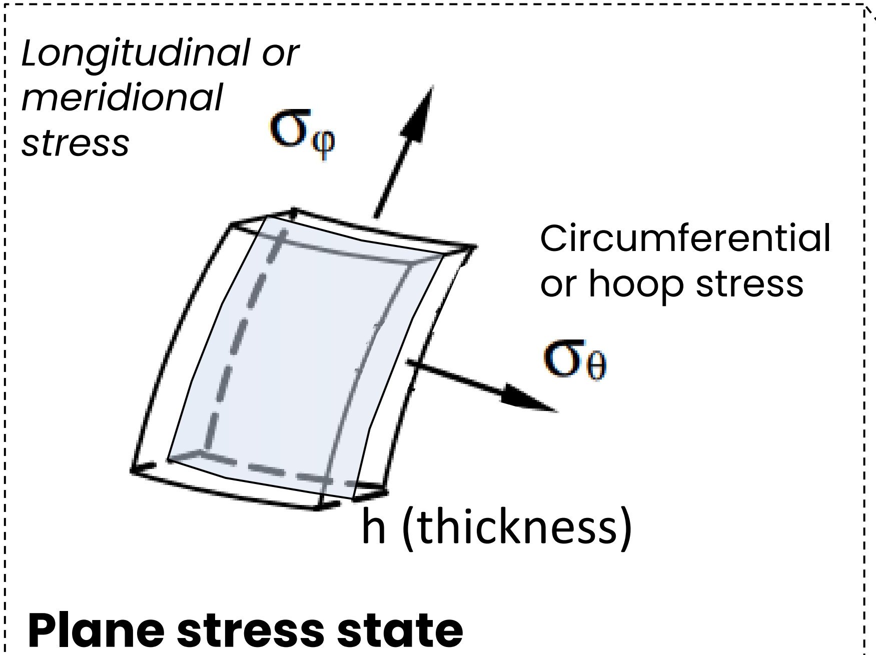


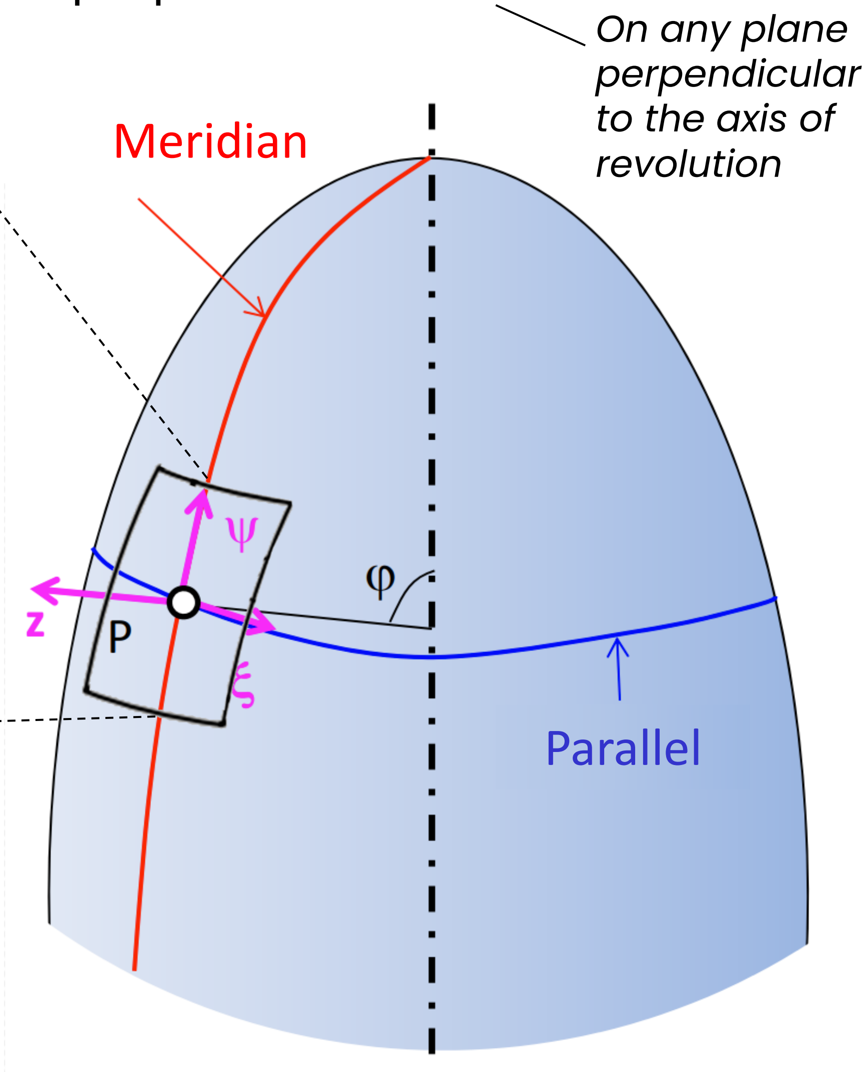
Curvinlinear coordinate system ψ, ξ, z centered in the point P



Membrane state of stress

Axisymmetric in geometry, material properties and load





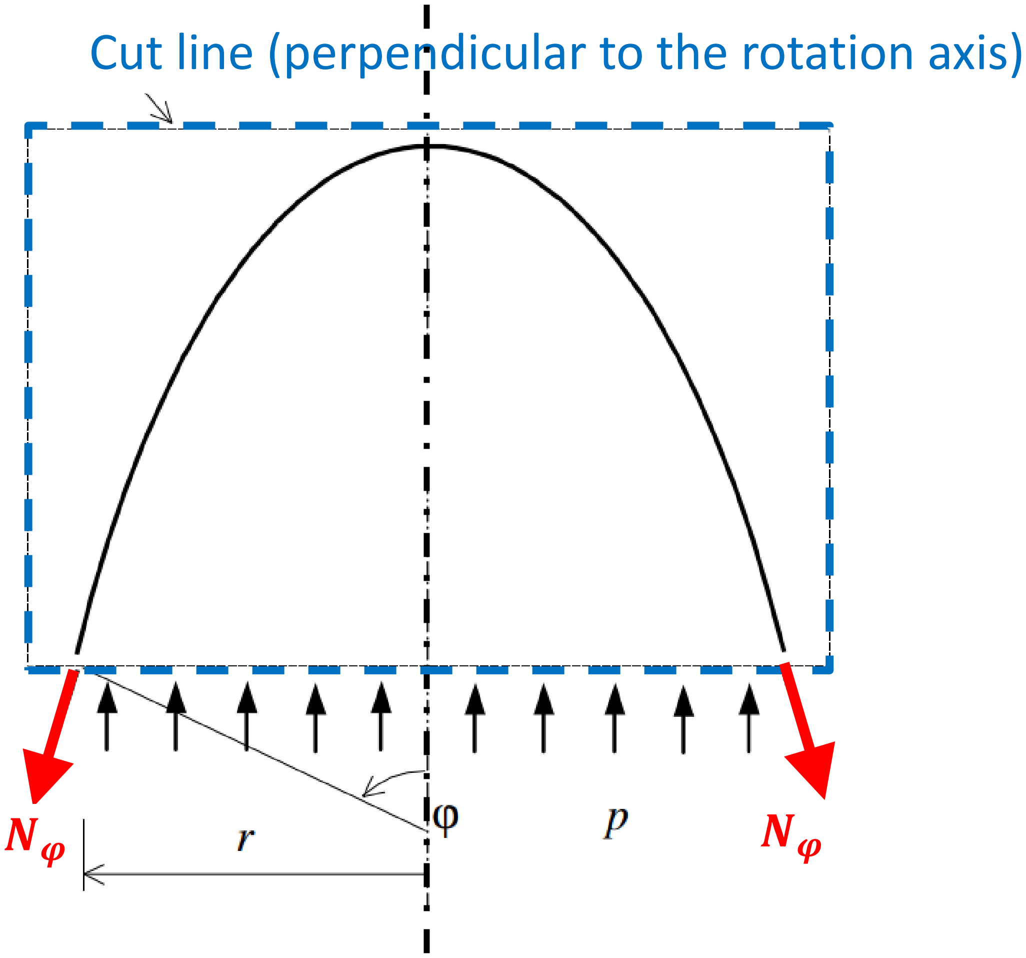
Forces for unit length

$$N_\varphi = \sigma_\varphi h$$

$$N_\theta = \sigma_\theta h$$

Membrane state of stress

Equilibrium equation along the axis of revolution (1)



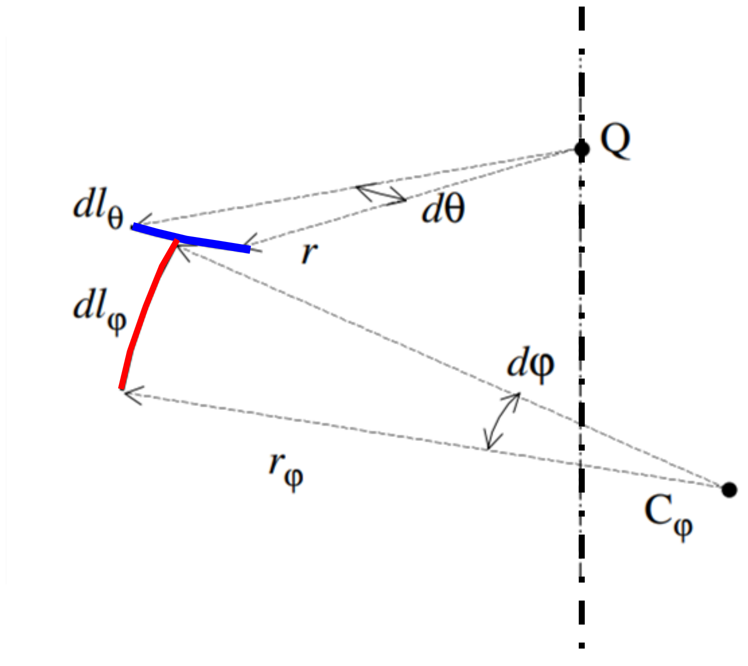
$$N_\varphi = \frac{p\pi r^2}{2\pi r \sin\varphi}$$

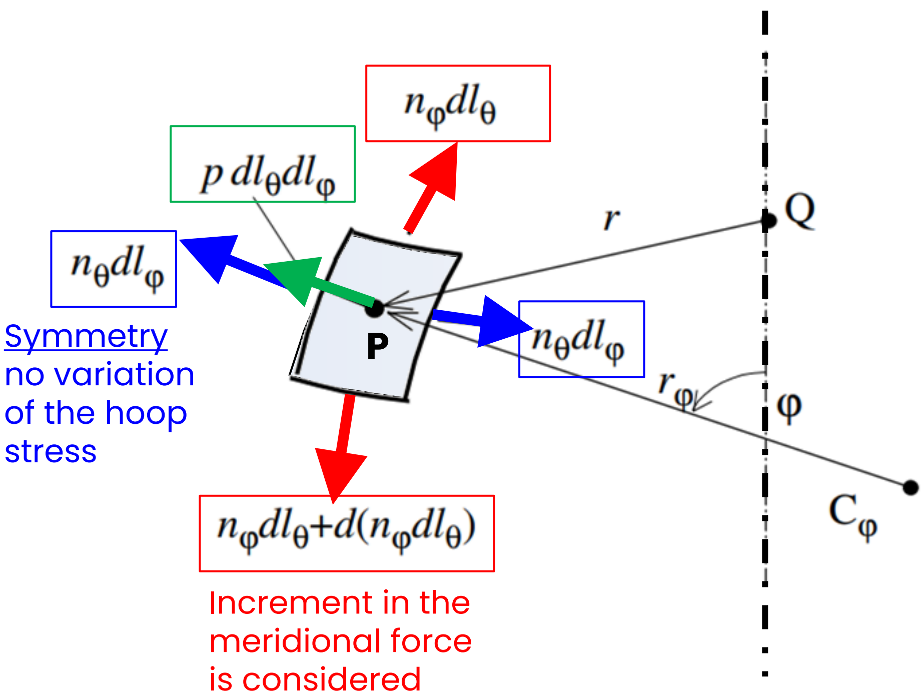
By neglecting the weight of the fluid and the weight of the vessel

This operation has to be done once for each change in geometry or loading along the vessel

Membrane state of stress

Equilibrium equation in the normal direction (2)

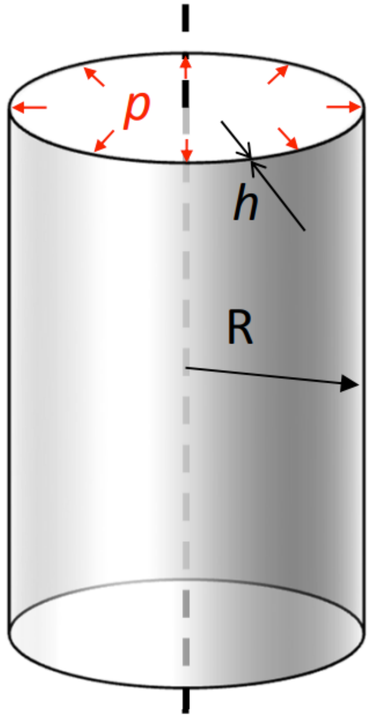




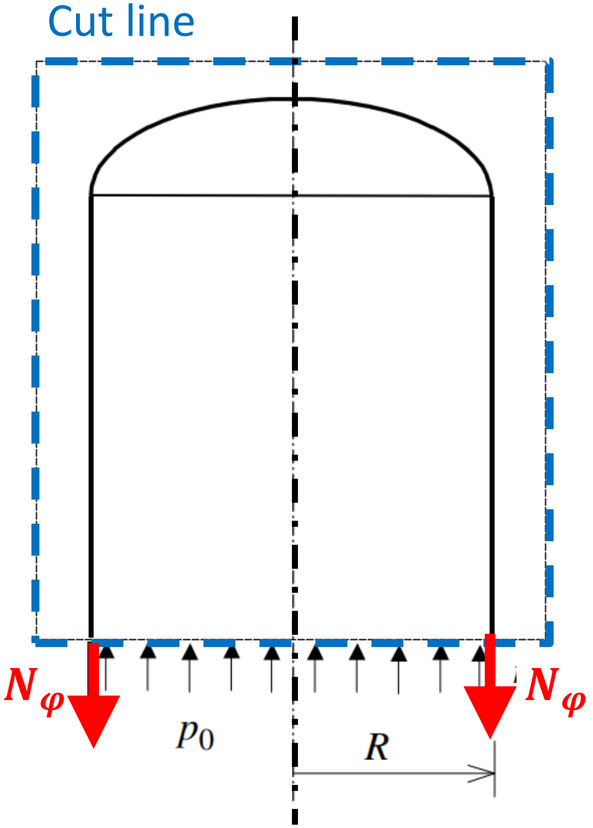
$$\frac{N_\varphi}{r_\varphi} + \frac{N_\theta}{r_\theta} = p$$

Example: cylinder

INTERNAL PRESSURE



Cut line



$$N_\varphi = \frac{p \cdot R}{2}$$

$$\sigma_\varphi = \frac{N_\varphi}{h} = \frac{p \cdot R}{2h}$$

$$N_\theta = p \cdot R$$

$$\sigma_\theta = \frac{N_\theta}{h} = \frac{p \cdot R}{h}$$

$$r_\theta = r = R$$

$$r_\varphi \to \infty$$

$$\varphi = \frac{\pi}{2}$$

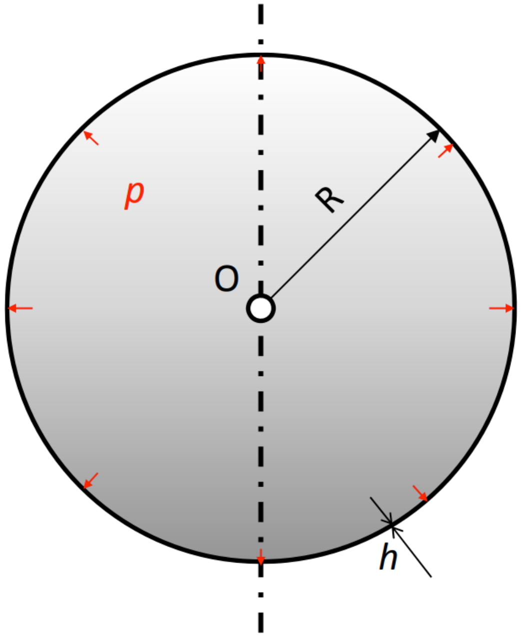
Tresca criterion

$$h_{\text{min}} = \frac{N_\theta}{\sigma_{\text{adm}}} = \frac{pR}{\sigma_{\text{adm}}}$$

Material limit divided by a safe coefficient

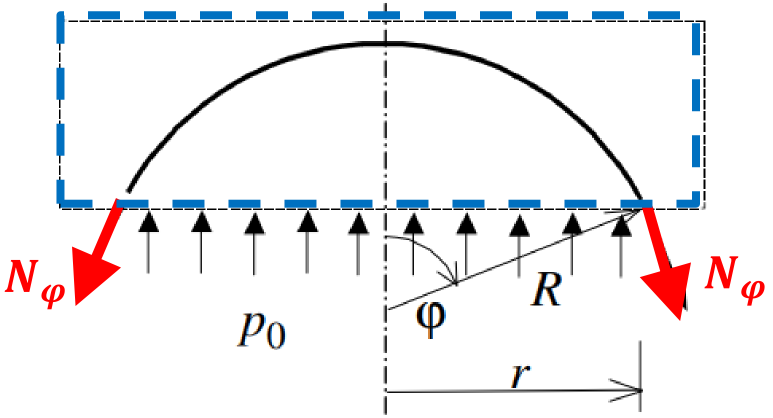
Example: sphere

INTERNAL PRESSURE



$$r_\theta = r_\varphi = R$$

Cut line



$$N_\theta = N_\varphi = \frac{p \cdot R}{2}$$

$$\sigma_\theta = \sigma_\varphi = \frac{p \cdot R}{2h}$$

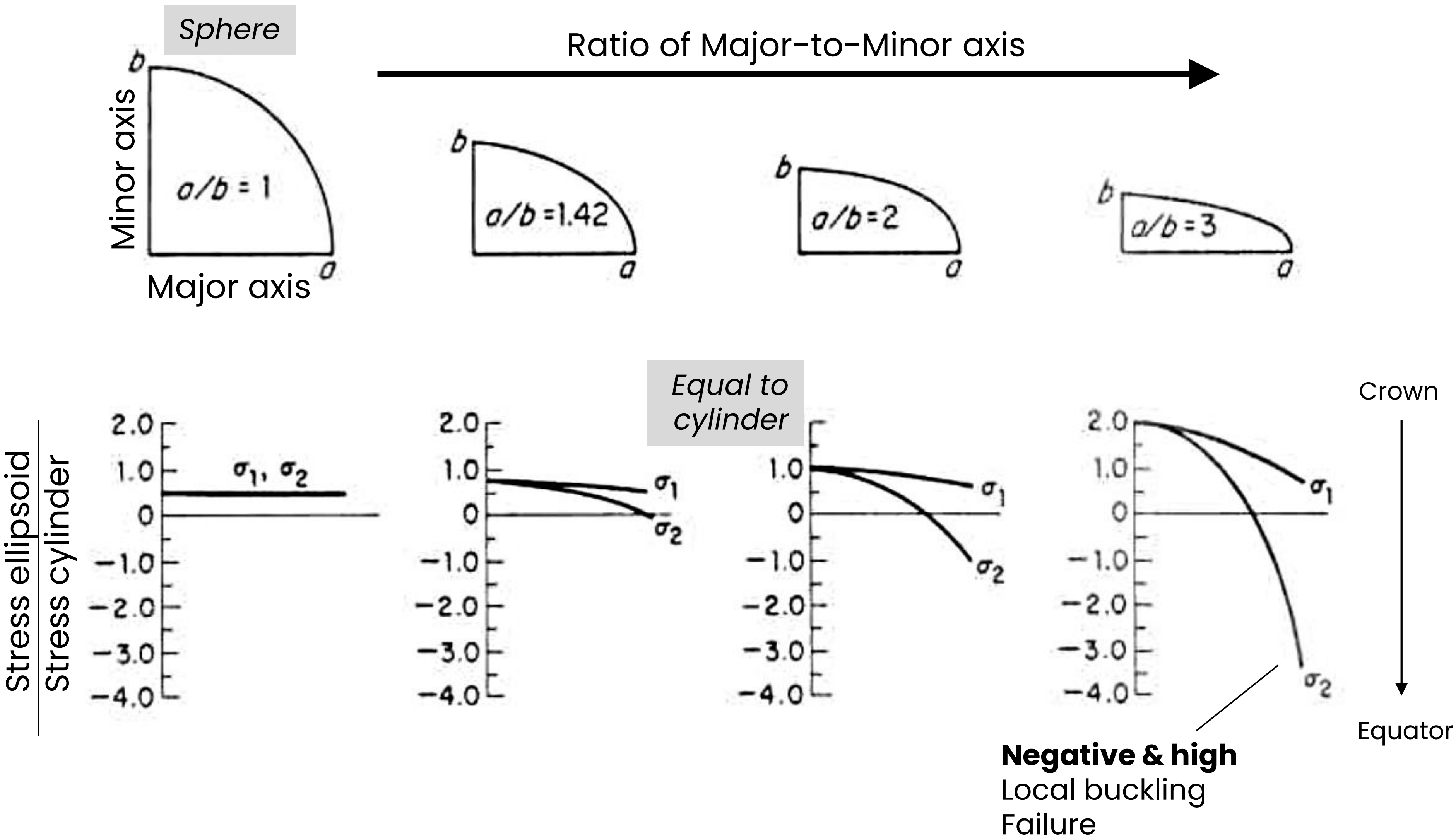
Tresca criterion

$$h_{\min} = \frac{N_\theta}{\sigma_{\text{adm}}} = \frac{pR}{2\sigma_{\text{adm}}}$$

Half respect to cylinder!

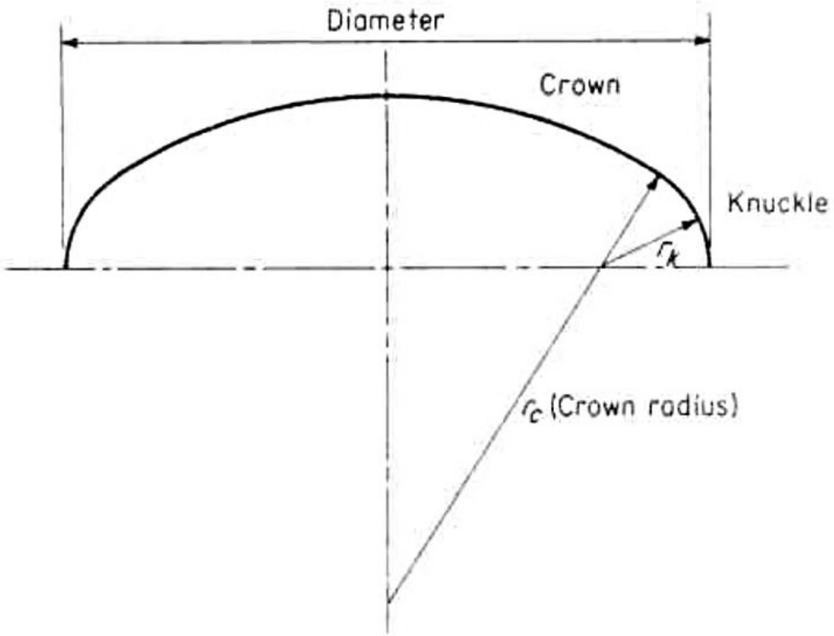
Example: ellipsoid

INTERNAL PRESSURE

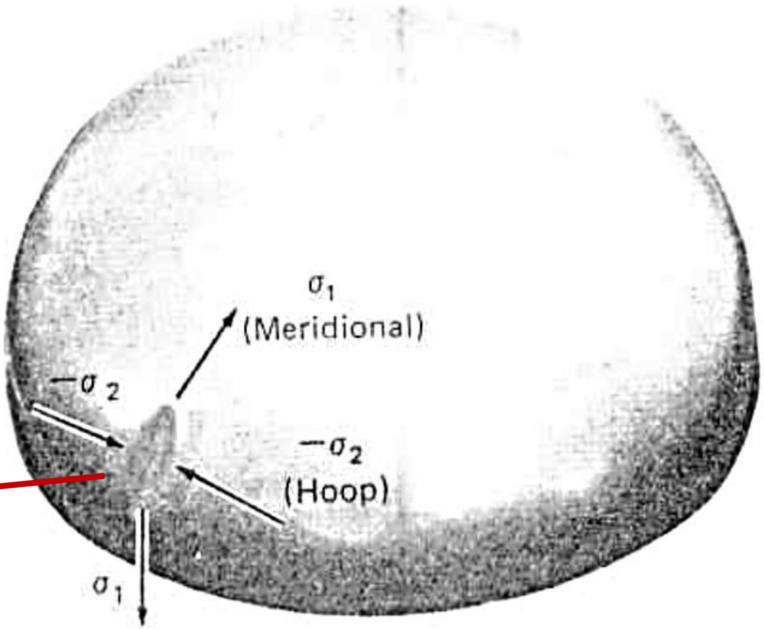


Example: torospherical

INTERNAL PRESSURE



Toroidal link between the sphere and the cylinder to simulate ellipsoidal head

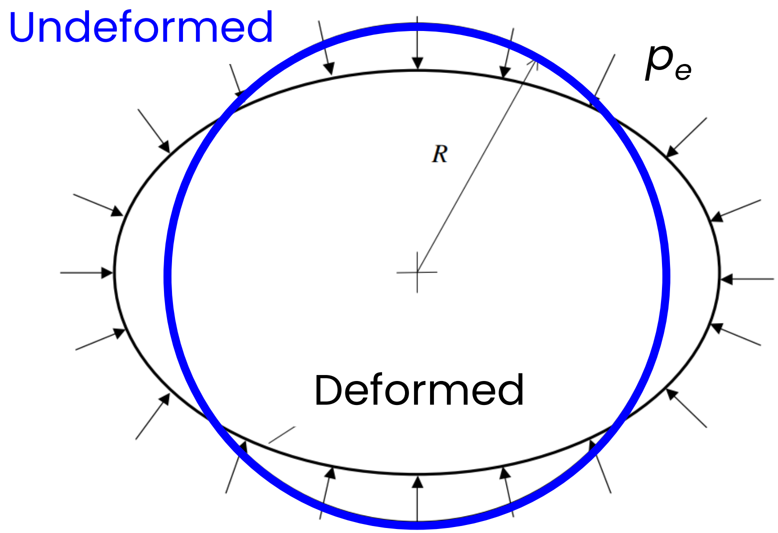


Buckling in the toroidal region

Buckling

EXTERNAL PRESSURE

The primary stresses become compressive and buckling instability can occur at stress well below the elastic limit giving little warning of the impending collapse

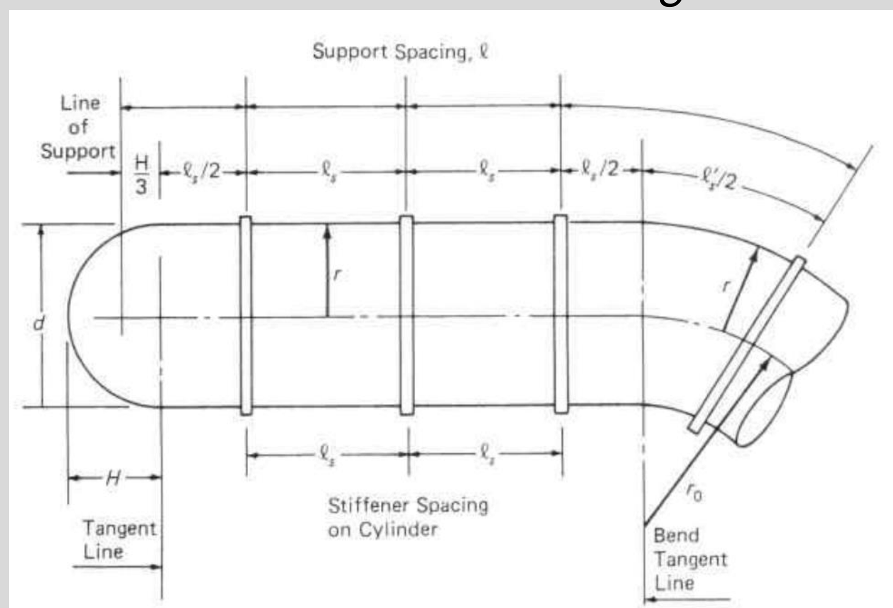


Elastic buckling in the form of two lobes

$$p_{cr} = \frac{E}{4\left(1-\nu^2\right)}\left(\frac{s}{R}\right)^3$$

Elastic deflection are not proportional to the loads: the approach is to obtain an expression for the *critical collapse pressure*

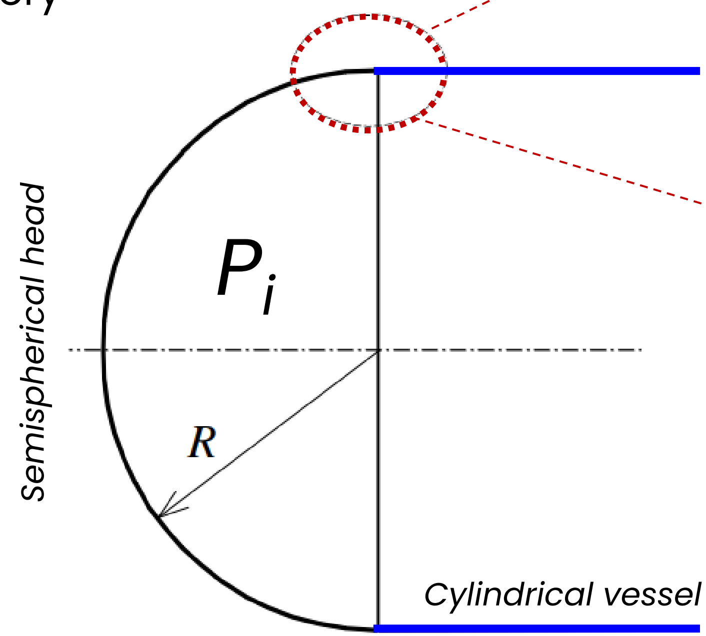
Ring stiffeners

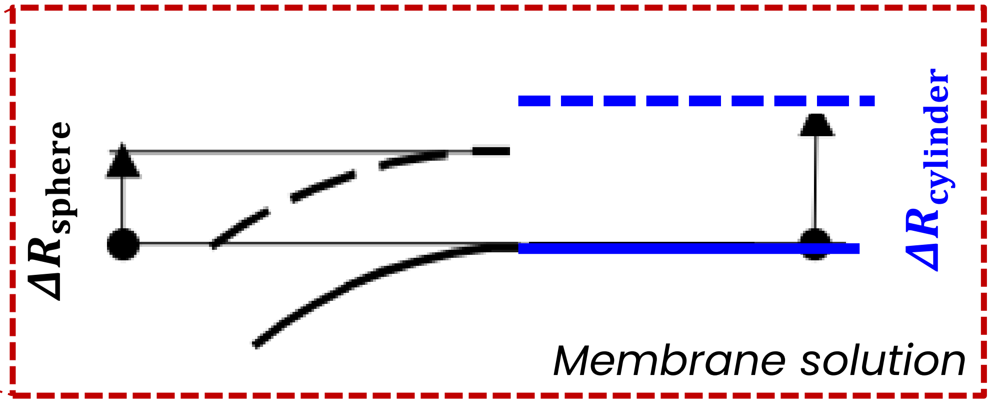


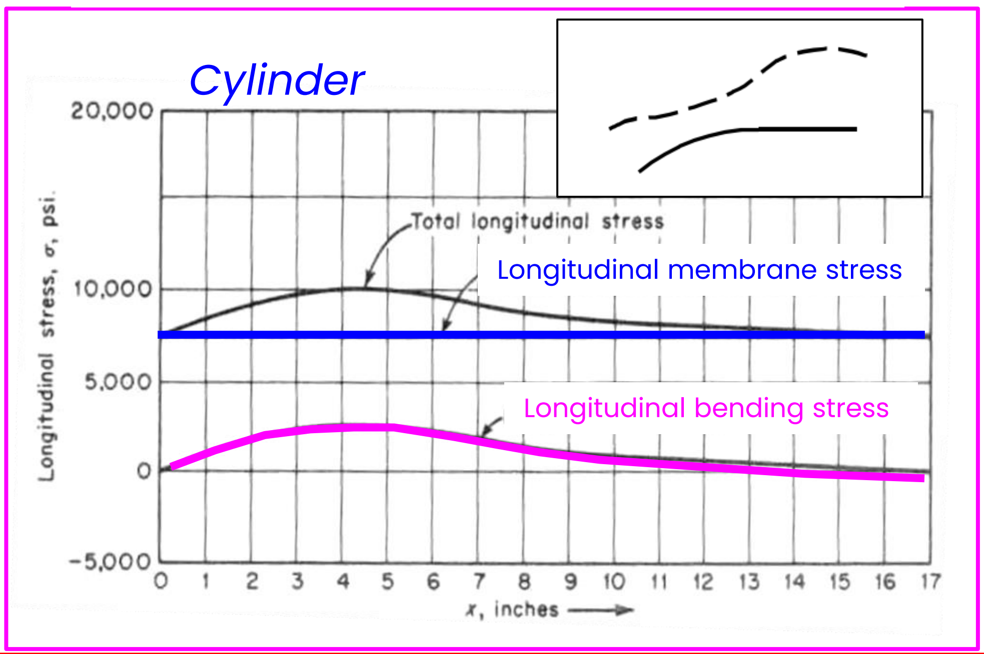
Discontinuity stresses

Secondary stresses are developed by the constraint of adjacent parts: they arises to compensate different dilatations at the juncture of a cylindrical vessel and its closure heads predicted by membrane theory

$$\Delta R = \varepsilon_\theta R = \frac{1}{E}\left(\sigma_\theta - \upsilon\sigma_\varphi\right)R$$





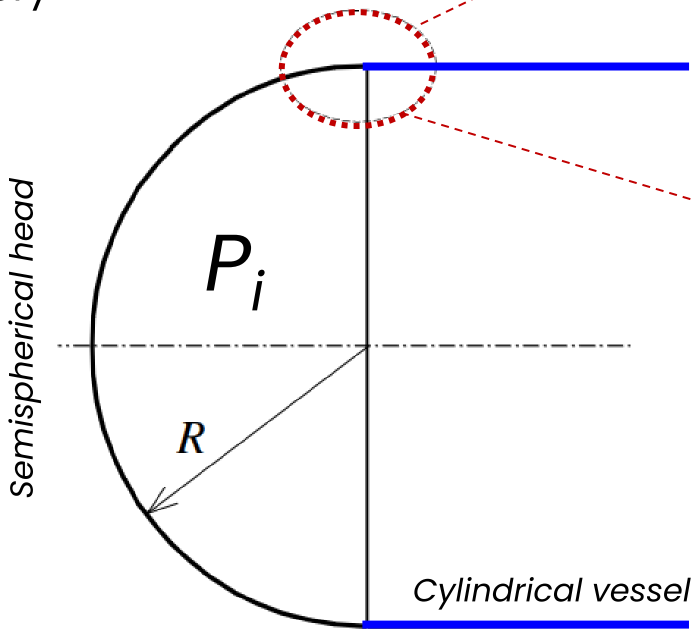


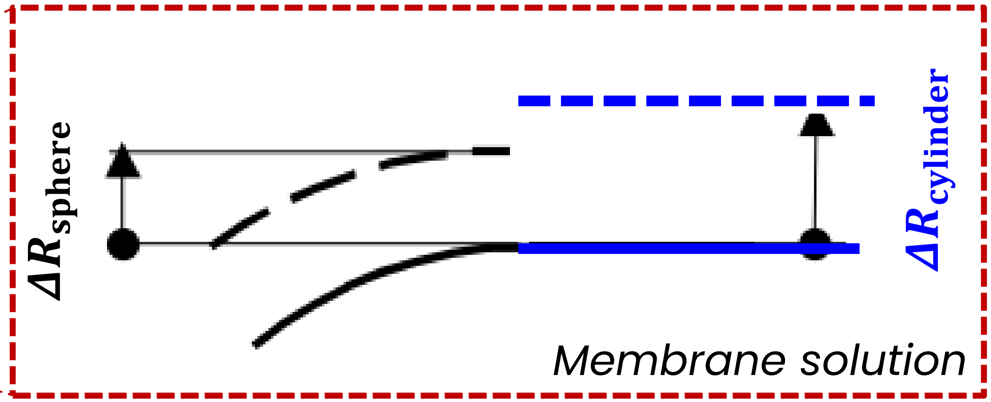
Local bending takes place to preserve continuity of the vessel wall

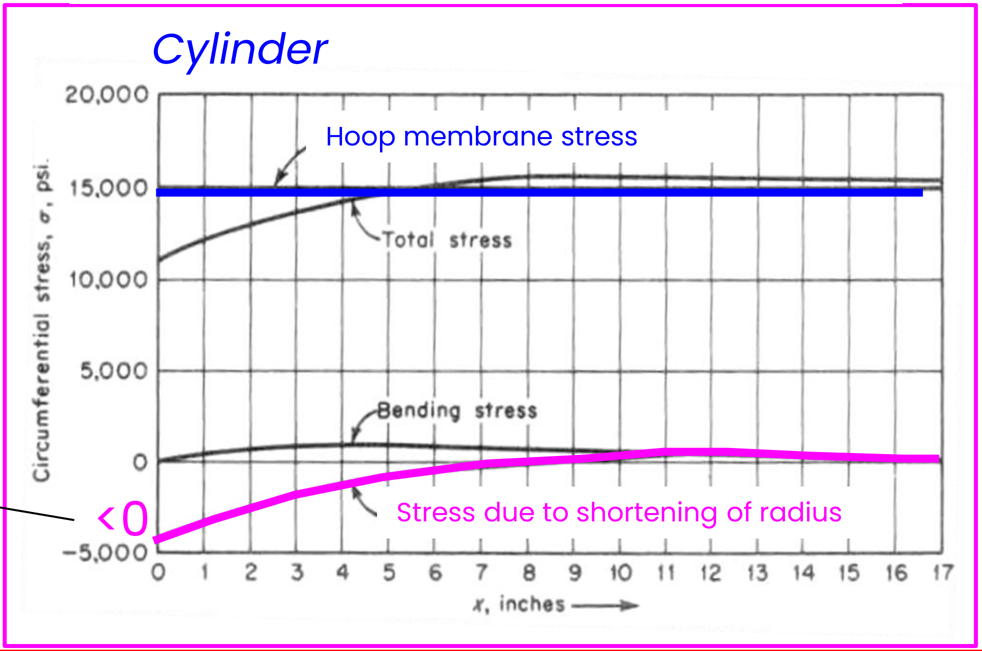
Discontinuity stresses

Secondary stresses are developed by the constraint of adjacent parts: they arises to compensate different dilatations at the juncture of a cylindrical vessel and its closure heads predicted by membrane theory

$$\Delta R = \varepsilon_\theta R = \frac{1}{E}\left(\sigma_\theta - \upsilon\sigma_\varphi\right)R$$







On the contrary, in the emispherical head, tension is induced by the extension of the radius

Other

- Corrosion
- Stress concentration
- Weldings
- Bolted joints and gaskets
- …

- Fatigue stress
- Temperature
- Irradiation damage
- …

PRESSURE VESSEL – EN 13445

Part 3: Design


BS EN 13445-1:2021

BSI Standards Publication

Unfired pressure vessels

Part 1: General

bsi.

EN 13445 – Part 3

5.2.2 Additional thickness to allow for corrosion

In all cases where reduction of the wall thickness is possible as a result of surface corrosion or erosion, of one or other of the surfaces, caused by the products contained in the vessel or by the atmosphere, a corresponding additional thickness sufficient for the design life of the vessel components shall be provided. The value shall be stated on the design drawing of the vessel. The amounts adopted shall be adequate to cover the total amount of corrosion expected on either or both surfaces of the vessel.

A corrosion allowance is not required when corrosion can be excluded, either because the materials, including the welds, used for the pressure vessel walls are corrosion resistant relative to the contents and the loading or are reliably protected (see 5.2.4).

No corrosion allowance is required for heat exchanger tubes and other parts in similar heat exchanger duty, unless a specific corrosive environment requires one.

This corrosion allowance does not ensure safety against the risk of deep corrosion or stress corrosion cracking, in these cases a change of material, cladding, etc. is the appropriate means.

Where deep pitting may occur, suitably resistant materials shall be selected, or protection applied to the surfaces.

5.6 Joint coefficient

For the calculation of the required thickness of certain welded components (e.g. cylinders, cones and spheres), the design formulae contain z, which is the joint coefficient of the governing welded joint(s) of the component.

Examples of governing welded joints are:

- longitudinal or helical welds in a cylindrical shell;
- longitudinal welds in a conical shell;
- any main weld in a spherical shell/head;
- main welds in a dished head fabricated from two or more plates.

Table 5.6-1 — Joint coefficient and corresponding testing group

z	1	0,85	0,7
Testing Group	1, 2	3	4

EN 13445 – Part 3

5.3.2.1 Normal operating load cases

5.3.2.2 Exceptional load cases

5.3.2.3 Testing load cases

Table 6-1 — Maximum allowed values of the nominal design stress for pressure parts other than bolts

Steel designation	Normal operating load cases[a b]	Testing and exceptional load cases[b c]
Steels other than austenitic, as per 6.2 $A < 30\ \%$ [d]	$f_d = \min\left(\frac{R_{p0,2/T}}{1,5}; \frac{R_{m/20}}{2,4}\right)$	$f_{test} = \left(\frac{R_{p0,2/T_{test}}}{1,05}\right)$
Steels other than austenitic, as per 6.3: Alternative route $A < 30\ \%$ [d]	$f_d = \min\left(\frac{R_{p0,2/T}}{1,5}; \frac{R_{m/20}}{1,875}\right)$	$f_{test} = \left(\frac{R_{p0,2/T_{test}}}{1,05}\right)$
Austenitic steels as per 6.4 $30\ \% \le A < 35\ \%$ [d]	$f_d = \left(\frac{R_{p1,0/T}}{1,5}\right)$	$f_{test} = \left(\frac{R_{p1,0/T_{test}}}{1,05}\right)$
Austenitic steels as per 6.5 $A \ge 35\ \%$ [d]	$f_d = \max\left[\left(\frac{R_{p1,0/T}}{1,5}\right); \min\left(\frac{R_{p1,0/T}}{1,2}; \frac{R_{m/T}}{3}\right)\right]$	$f_{test} = \max\left[\left(\frac{R_{p1,0/T_{test}}}{1,05}\right); \left(\frac{R_{m/T_{test}}}{2}\right)\right]$
Cast steels as per 6.6	$f_d = \min\left(\frac{R_{p0,2/T}}{1,9}; \frac{R_{m/20}}{3}\right)$	$f_{test} = \left(\frac{R_{p0,2/T_{test}}}{1,33}\right)$

EN 13445 – Part 3

INTERNAL PRESSURE

7.4.2 Cylindrical shells

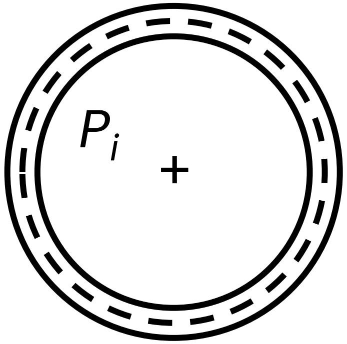
The required thickness shall be calculated from

$$e = \frac{P \cdot D_i}{2f \cdot z - P}$$

7.4.3 Spherical shells

The required thickness shall be calculated from

$$e = \frac{P \cdot D_i}{4f \cdot z - P}$$



7.5.3 Torispherical ends

$$e_s = \frac{P \cdot R}{2f \cdot z - 0{,}5P}$$

$$e_y = \frac{\beta \cdot P\left(0{,}75R + 0{,}2D_i\right)}{f}$$

To avoid yielding

$$e_b = \left(0{,}75R + 0{,}2D_i\right)\left[\frac{P}{111f_b}\left(\frac{D_i}{r}\right)^{0{,}825}\right]^{\left(\frac{1}{1{,}5}\right)}$$

To avoid plastic buckling

Membrane theory

$$h_{\min} = \frac{N_\theta}{\sigma_{\text{adm}}} = \frac{pR}{\sigma_{\text{adm}}}$$

$$h_{\min} = \frac{N_\theta}{\sigma_{\text{adm}}} = \frac{pR}{2\sigma_{\text{adm}}}$$

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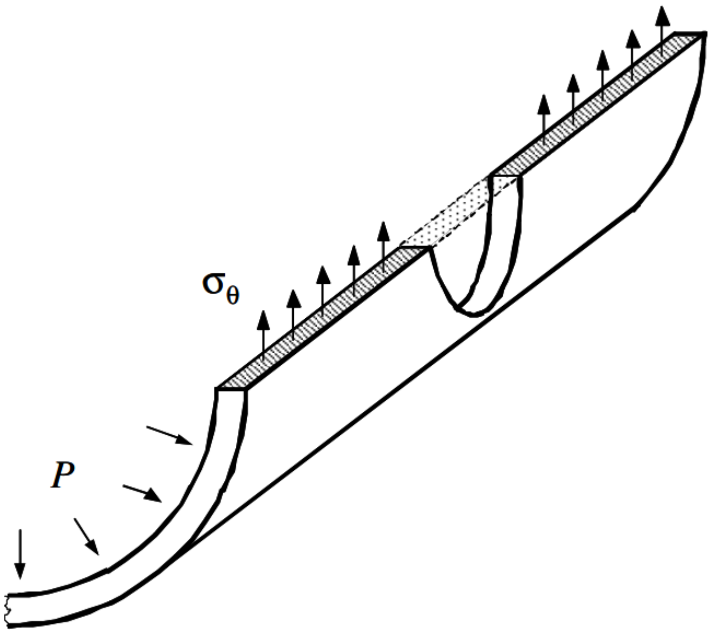

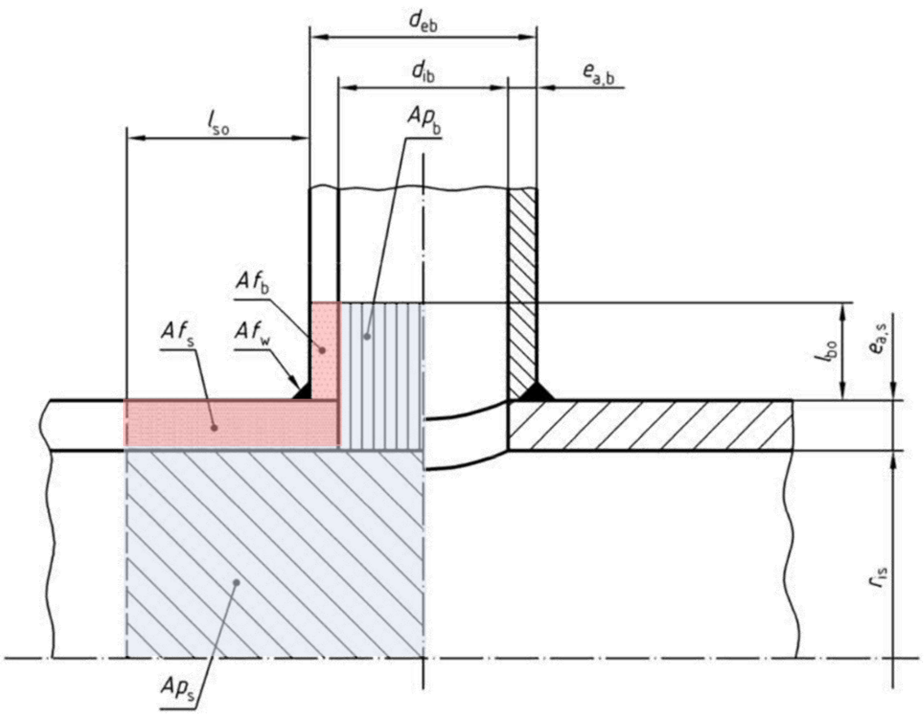

Figure 9.4-7 — Cylindrical shell with isolated opening and set-on nozzle

9.5.2.1.1 The general equation for the reinforcement of an isolated opening is given by

$$(Af_s + Af_w)(f_s - 0{,}5P) + Af_p(f_{op} - 0{,}5P) + Af_b(f_{ob} - 0{,}5P) \geq P(Ap_s + Ap_b + 0{,}5\,Ap_\varphi)$$

Reactive force *Load from pressure*

11.5.2 Bolt loads and areas

Bolt loads and areas shall be calculated for both the assembly and operating conditions as follows.

a) *Assembly condition*. The minimum bolt load is given by:

$$W_A = \pi b \cdot G \cdot y \qquad (11.5\text{-}7)$$

NOTE The minimum bolt loading to achieve a satisfactory joint is a function of the gasket and the effective gasket area to be seated.

b) *Operating condition*. The minimum bolt load is given by:

$$W_{op} = H + H_G \qquad (11.5\text{-}8)$$

$$H = \frac{\pi}{4} \cdot (G^2 \cdot P)$$

$$H_G = 2\pi \cdot G \cdot b \cdot m \cdot P$$

The required bolt area $A_{B,min}$ is given by:

$$A_{B,min} = \max\left(\frac{W_A}{f_{B,A}}; \frac{W_{op}}{f_B}\right) \qquad (11.5\text{-}9)$$

Bolting shall be chosen so that $A_B \geq A_{B,min}$

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Thank you for your attention

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