Computational Tools I: Introduction to FEA and Implicit Simulations

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MECHANICAL & MATERIALS ENGINEERING
FOR PARTICLE ACCELERATORS AND DETECTORS

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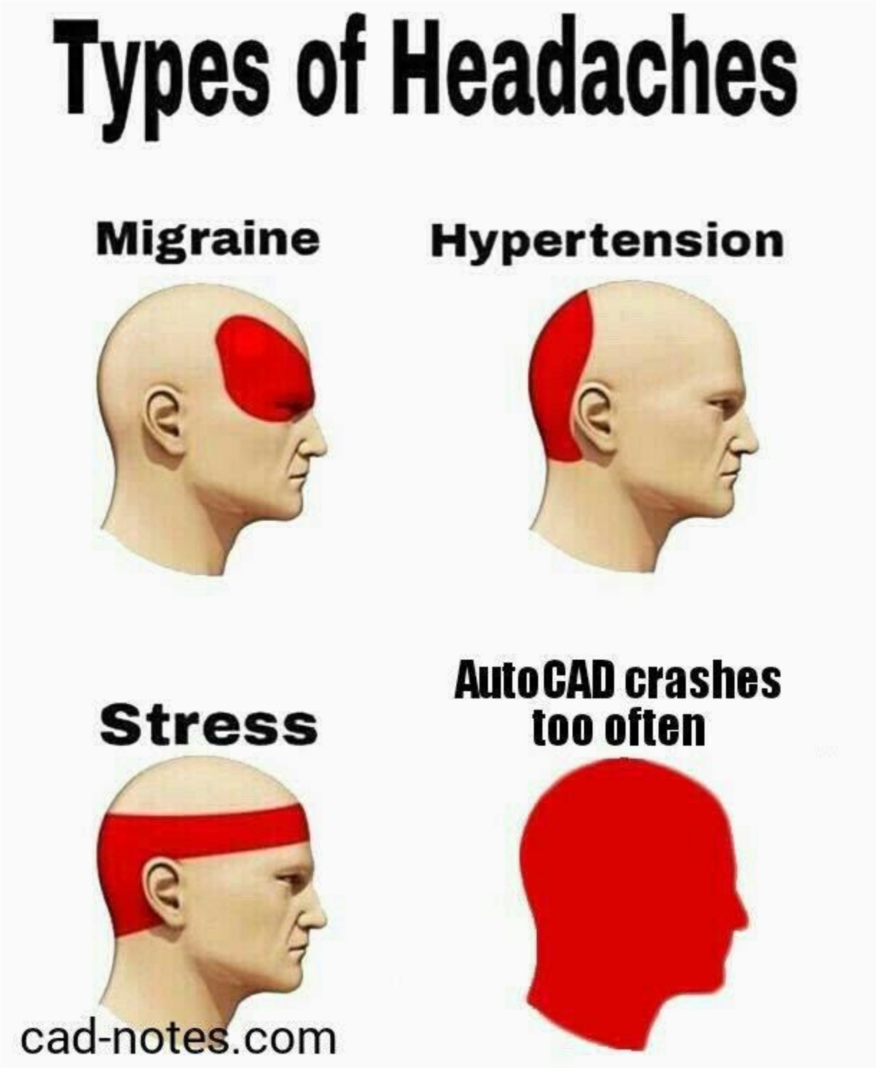
Outline

- What is CAE and why do we use it?
- FEM theory in a nutshell
- Finite Elements: Implicit vs. Explicit solvers
- Example of implicit simulations for CERN equipment
- Summary

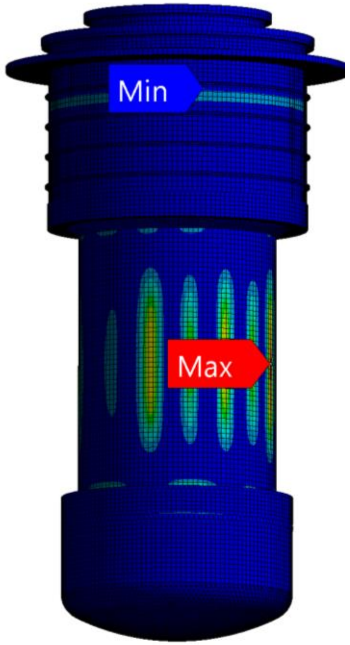
Computer-Aided Technologies (CAx)



CAx

omputer-Aided Engineering
(CAE – e.g. **FEA**, CFD)

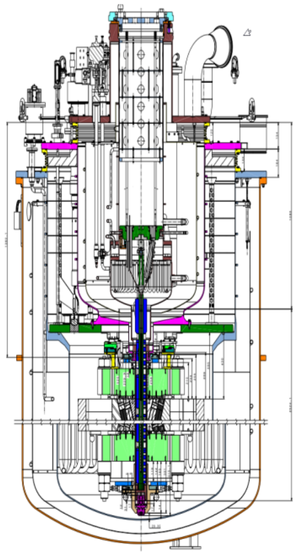
Computer-Aided Manufacturing
(CAM)



Computer-Aided Design and Engineering (CAD/CAE)

CAD/CAE

Design and Analysis* → Virtual Prototyping → Physical Prototyping → Final Product Testing



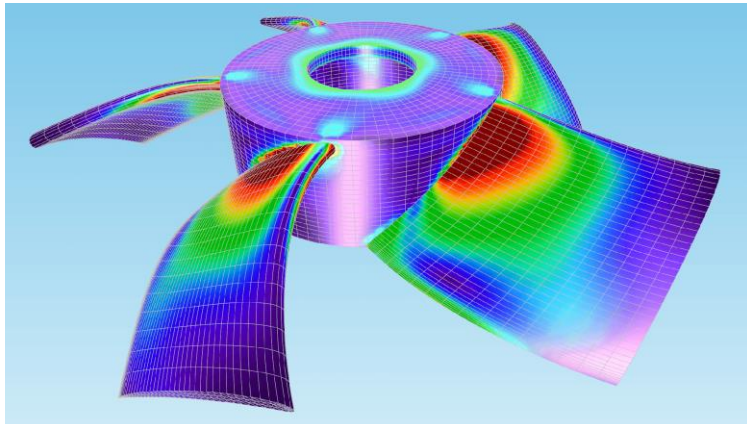
$ Relatively cheap

$$ Not so cheap

$$$ Really expensive

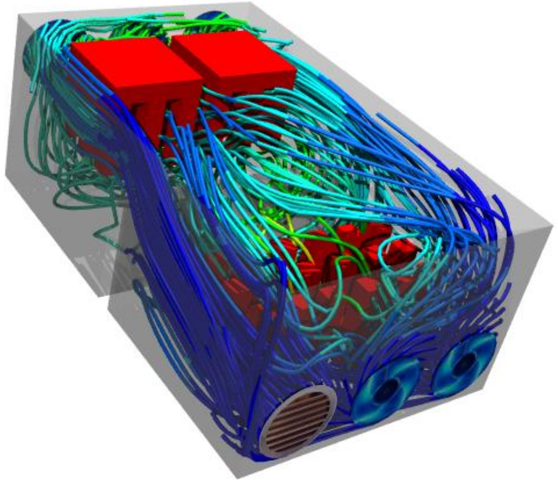
The more time spent here, the less money and time spent later

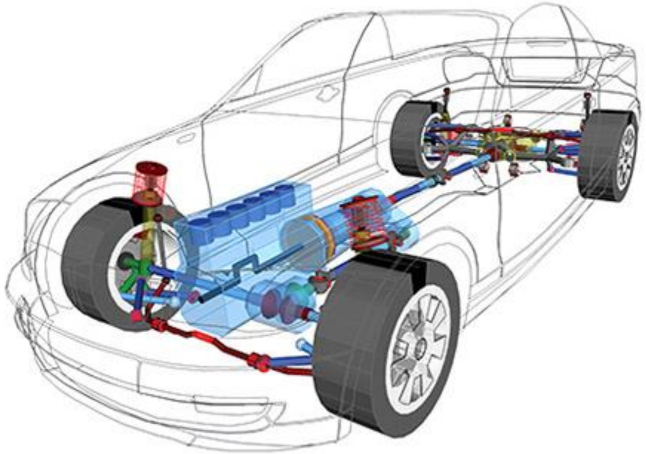
CAE Domains



Finite-Element Analysis
(FEA)

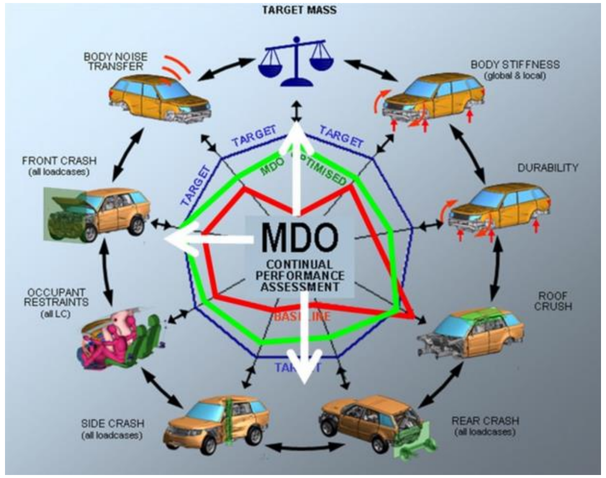
Computational Fluid Dynamics
(CFD)





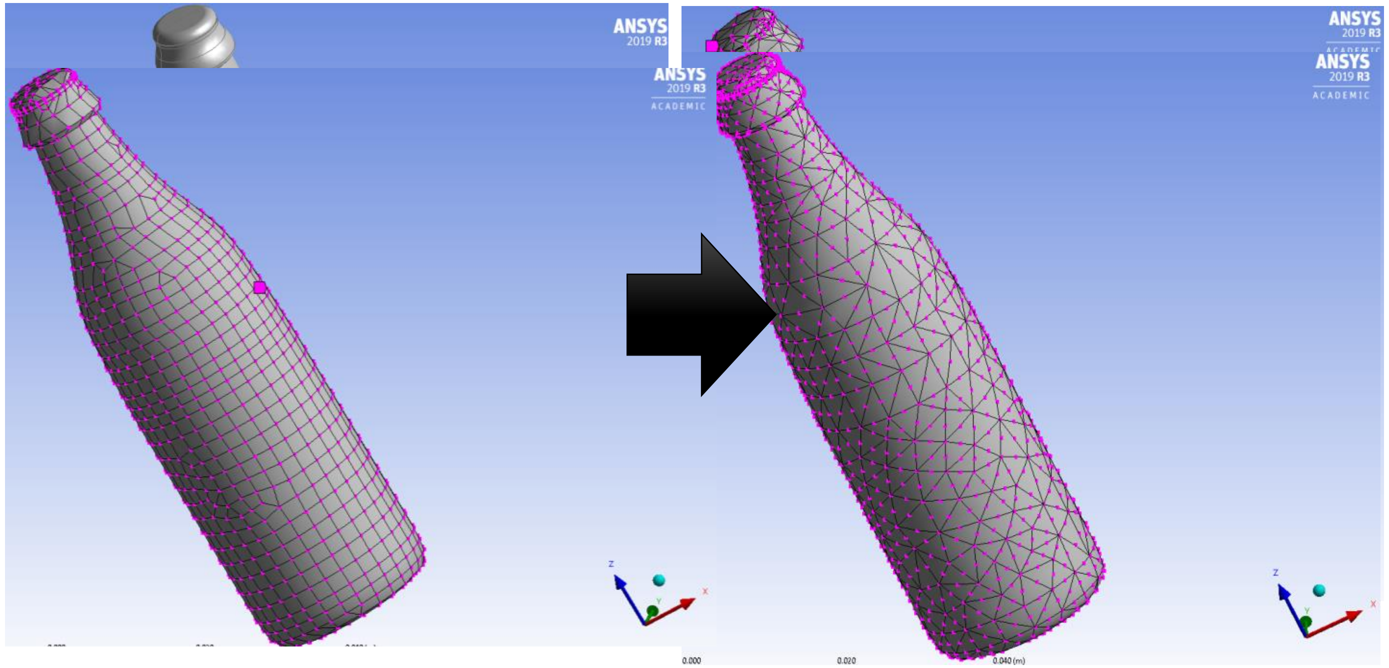
Multibody Dynamics
(MBD)

Multidisciplinary design optimization
(MDO)



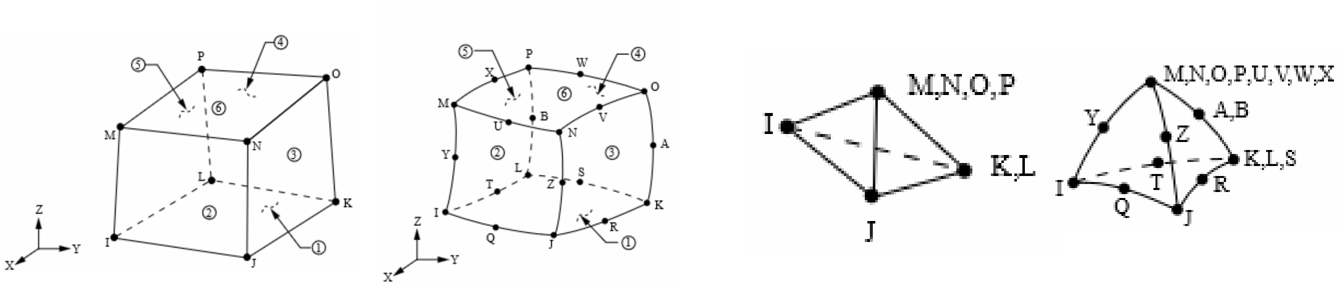
FEM Theory in a Nutshell

The displacements of all the points in a continuum under the action of external forces depends on the displacements of discrete points known as **nodes**.



This dependence is regulated by interpolating functions known as **shape functions**.

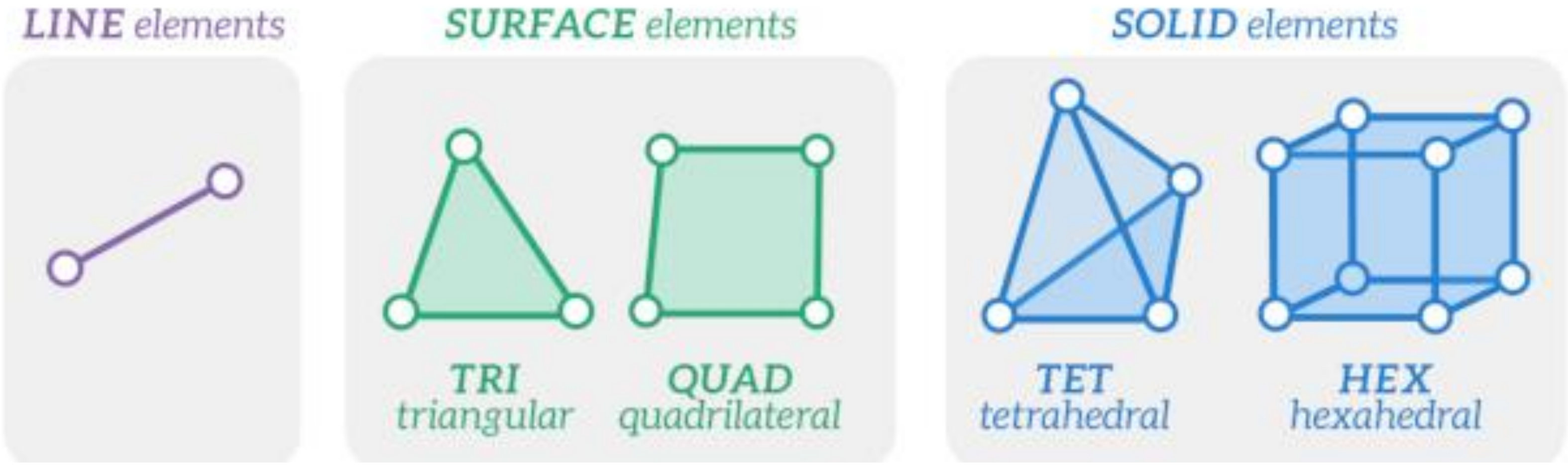
To study a body with FEM, we must thus **discretize the continuum in a finite number of elements**, each one featuring a number of nodes which depends on the type of element chosen.



Element Types

The shape functions depend on the element type:

- **Line elements** model 1D structures like beams, rods or pipes.
- **Surface elements** are used to model large and thin surfaces like shells, plates.
- **Solid elements** are used to model three-dimensional bodies.



FEM Theory in a Nutshell

FEM: solving for the nodal displacements $\{s\}$

$$\{s\} = [K]^{-1}\{F\}$$

$$[\partial] = \begin{bmatrix} \partial/\partial x & 0 & 0 \\ 0 & \partial/\partial y & 0 \\ 0 & 0 & \partial/\partial z \\ \partial/\partial y & \partial/\partial x & 0 \\ 0 & \partial/\partial z & \partial/\partial y \\ \partial/\partial z & 0 & \partial/\partial x \end{bmatrix}$$

After calculation of $\{s\}$:

$$\{u\} = [N]\{s\}$$

N Shape functions

$$\{\varepsilon\} = [\partial]\{u\} = [\partial][N]\{s\}$$

C Compatibility Equations

$$\{\sigma\} = [D]\{\varepsilon\} = [D][\partial][N]\{s\}$$

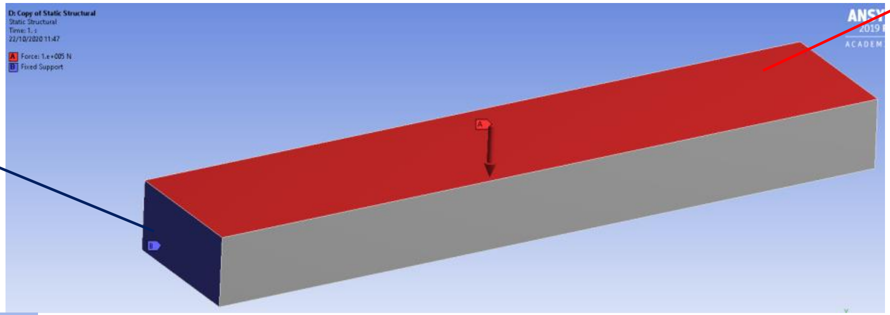
M Material Constitutive Law (e.g. Hooke's law)

N + C + M → **Solution can be obtained in all points of the structure (not only at the nodes!)**

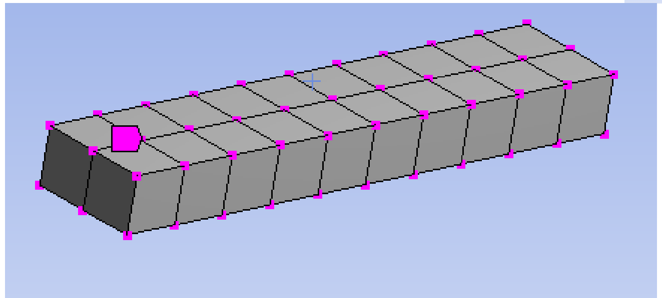
Linear vs. Quadratic Elements

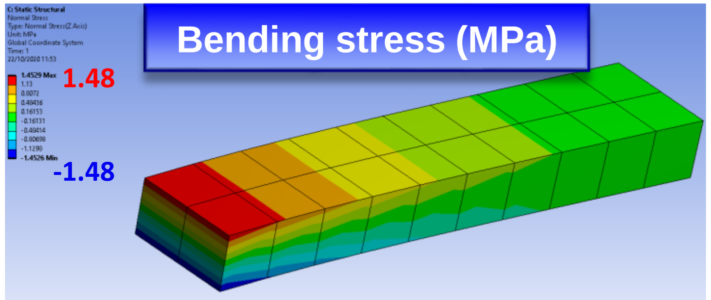
Distributed load

Fixed support

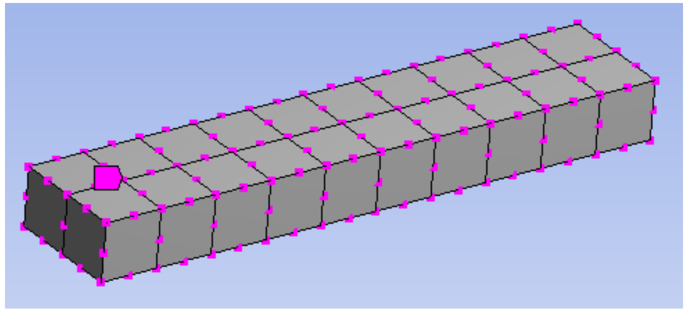


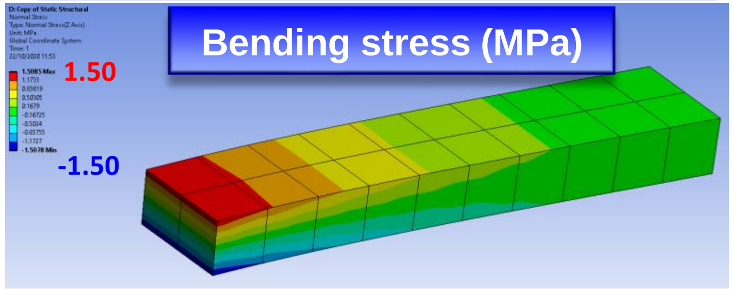
Linear elements





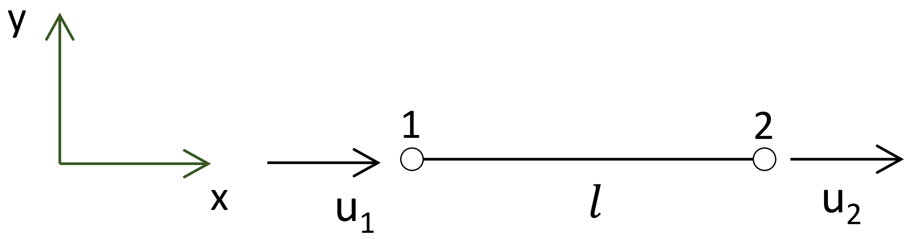
Quadratic elements





Linear elements: computationally more efficient, but when a nonlinear stress state is expected, use quadratic elements or more linear elements over the thickness

Example of Calculation of a Shape Function: Truss Element



$$u = a_1 + a_2 x = [1 \quad x] \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}$$

a_1, a_2 are coefficients that can be calculated imposing the b.c. $x_1 = 0, x_2 = l$

$$[N] = \left[\left(1 - \frac{x}{l}\right) \quad \frac{x}{l} \right]$$

- Best mathematical instrument to represent a shape function is a **polynomial**
- Displacements will be varying linearly over the length of the element, while **strains and stresses will be constant**
- Choose the right element for the right problem! In case of bending and shear, use a **beam element instead**

FEM Solvers

- **Explicit solvers**: derive the unknowns (displacements, velocities, accelerations) at a time instant *t+Δt* by
- **Explicit solvers**: suggested for fast transient and highly nonlinear problems
- **Implicit solvers**: suggested for slow transient and static problems
  equations at the time *t* and *t+Δt*.

Implicit codes

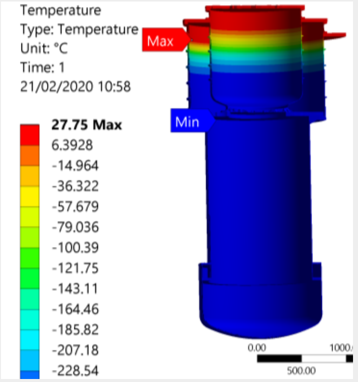
- Unconditionally stable
- Large time steps
- Matrix inversion
- Coupled equations
- Convergence problem

Explicit codes

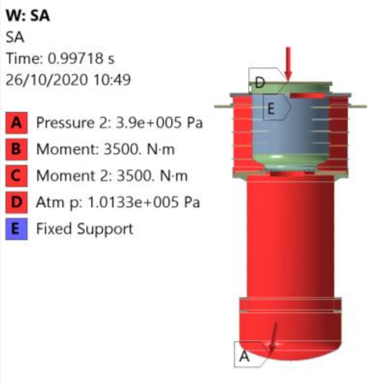
- Conditionally stable
- Small time steps
- «Lumped» matrix multiplication
- Uncoupled equations
- «Keep going»

Particle accelerator components: typical loads

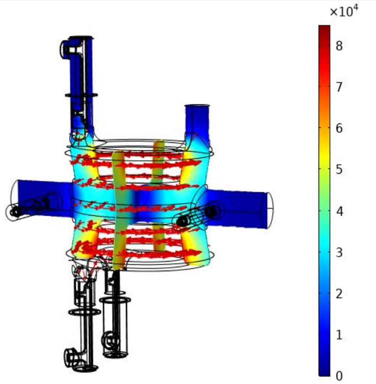
Temperature gradient



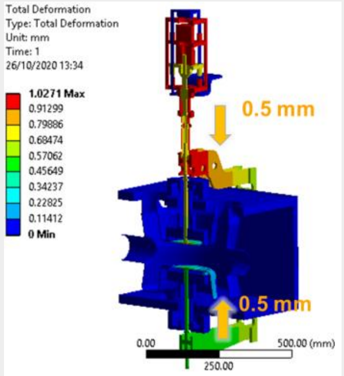
Mechanical forces



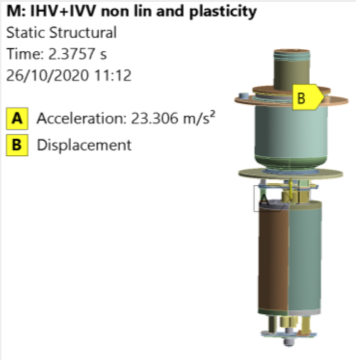
Electromagnetic forces



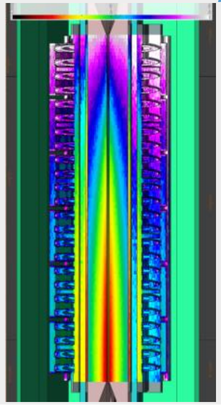
Displacements



Accelerations



Beam loads (RF & particle losses)



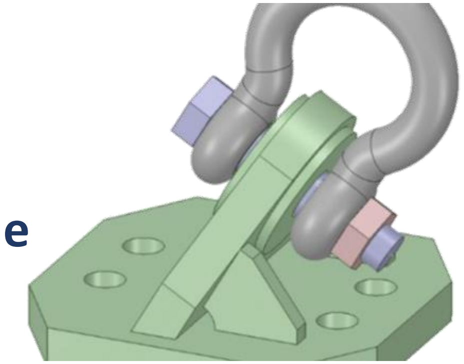
Can often be studied with **implicit FE codes**

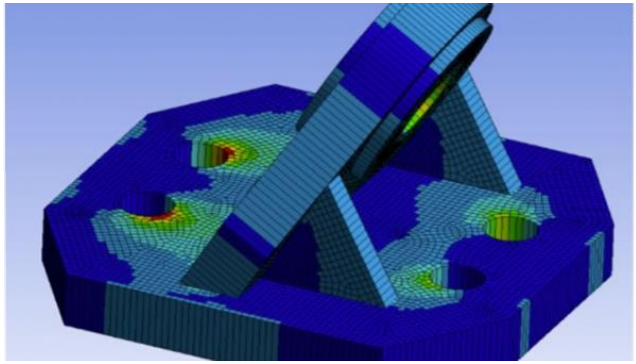
We resort to **explicit codes for special applications**

FEM tips: from reality to model

- **Simplification of the model**: removal of details not contributing to the solution of the problem under study
  - **Screws, welds** typically defeatured in the FEA, and calculated "by hand" extracting internal loads from FEA
  - **Chamfers, radii** can be verified via submodels
- **Loads and boundaries:**
  - As accurate as possible representation of the real working conditions
  - Compromise sometimes to be made to simplify the problem (*e.g.* nonlinear contacts, etc.)
  - **Most critical step of the process**
- **Safety factors!** (*i.e. factor of ignorance*)
- When approximating, always be on the **conservative side**
- **Start simple, complexify later**

…however…





However: what should we simulate?

- **1st thing to do when designing a component**: understand well (and write down!) the possible loading scenarios:
  - **How does it operate**? Are there more than one operating scenario?
  - How to **switch between different operating scenarios** (or from parking to operation and viceversa)? Slow transient / fast transient? Is it an issue?
  - **Which tests** should I foresee on the final component before operation to ensure that it fulfils its requirements? Are they more or less critical than the operating scenarios?
  - **How do I lift / handle / maintain it**?
  - **How many times (cycles)** all of these possible loading conditions are reproduced?
  - Are there **any other variables** possibly affecting the behaviour of the component? (chemical reactions, radiation, temperature, humidity, etc.)

However: what should we simulate?

- All of these questions need to be answered → **all answers need to be summarized in a "cahier des charges"**
  - Example: **FRESCA-2 Outer Helium Vessel (OHV)**
  - *(more details on it in a few slides)*

- **Main concept here**:
  - **Many** operational, exceptional and testing load cases can be defined
  - Often it is possible to **reduce these many load cases to very few ones** which are the most critical
  - You will then study / simulate **only those critical load cases!**

3 OUTER HELIUM VESSEL (OHV)

Nominal operation load cases

NLC1 - Transport
NLC2 - Installation in the pit
NLC3 - Assembly
NLC4 - Vacuum pumping
NLC5 - Pressurized
NLC6 - Cold
NLC7 - Powering
NLC8 - Quench
NLC9 - Vacuum loss
NLC10 - Purge with vacuum loss

Testing load case

TLC1 - Leak test during fabrication
TLC2 - Pressure test during fabrication
TLC3 - Pressure test in place

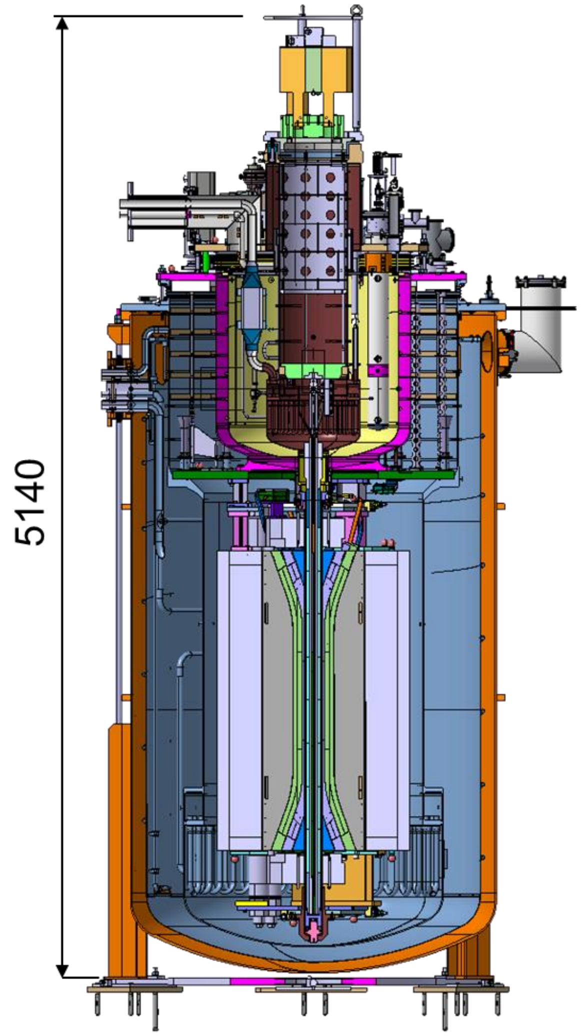
Table 2 - Applicable load cases for outer helium vessel

	NLC1	NLC2	NLC3	NLC4	NLC5	NLC6	NLC7	NLC8	NLC9	NLC10	TLC1	TLC2	TLC3
Self-weight (1)	A	B	B	B	B	B	B	B	B	B	C	C	B
Temperature (2)	A	A	A	A	A	B	B	B	B	A	A	A	A
Internal pressure (3)	/	/	/	A	B	B	B	C	C	/	/	D	E
External pressure (4)	/	/	/	/	/	/	/	/	B	B	A	A	/
Magnet + IC weight	/	/	X	X	X	X	X	X	X	X	/	/	X
Torque	/	/	/	/	/	/	X	X	X	/	/	/	/

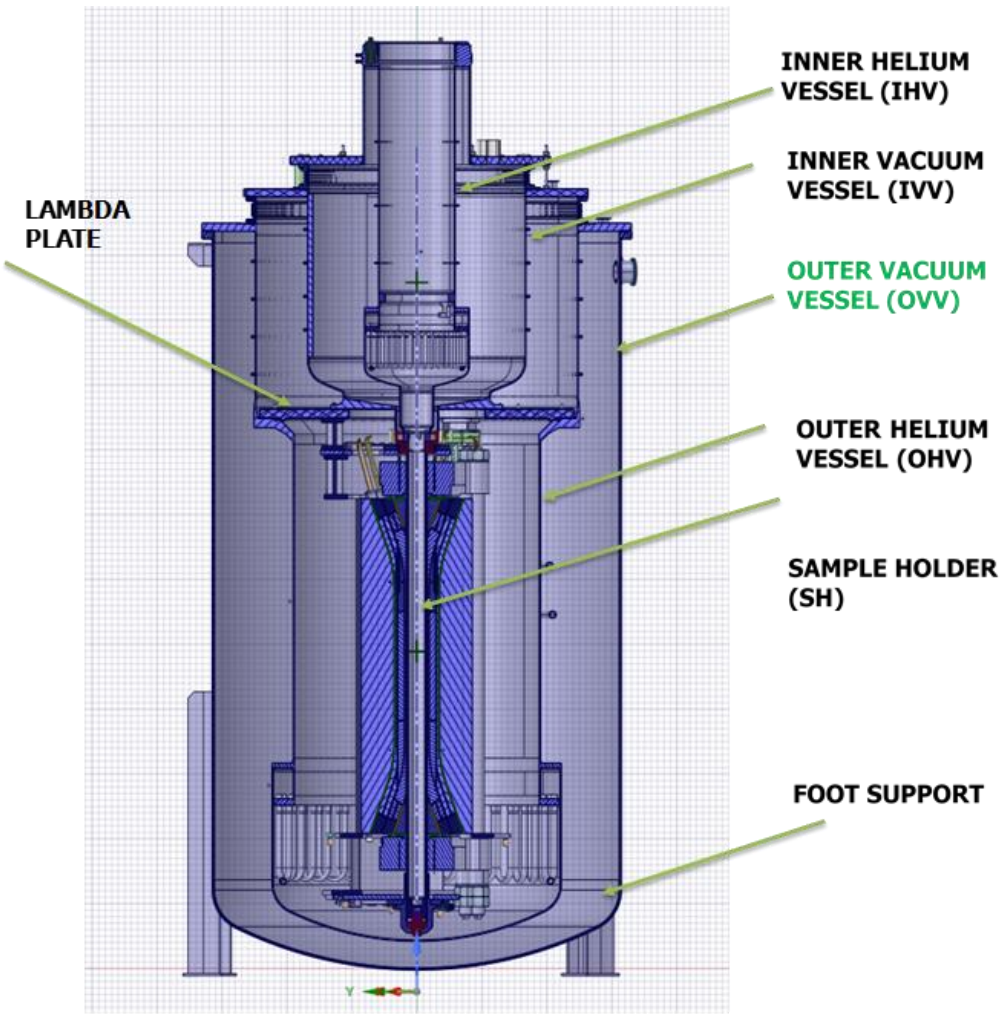
(1) A = self-weight supported by handling points; B = self-weight supported by top flange; C = self-weight on manufacturing supports
(2) A = 300 K; B = 4.5 – 300 K thermal gradient
(3) A = Atmospheric pressure; B = 1.3 bar (absolute); C = PS (3.9 bar absolute); D = Hydraulic Test pressure (1.43 x PS); E = Pneumatic test pressure (1.25 x PS)
(4) A = Atmospheric pressure; B = 1.5 bar (absolute)

Implicit simulations: an example

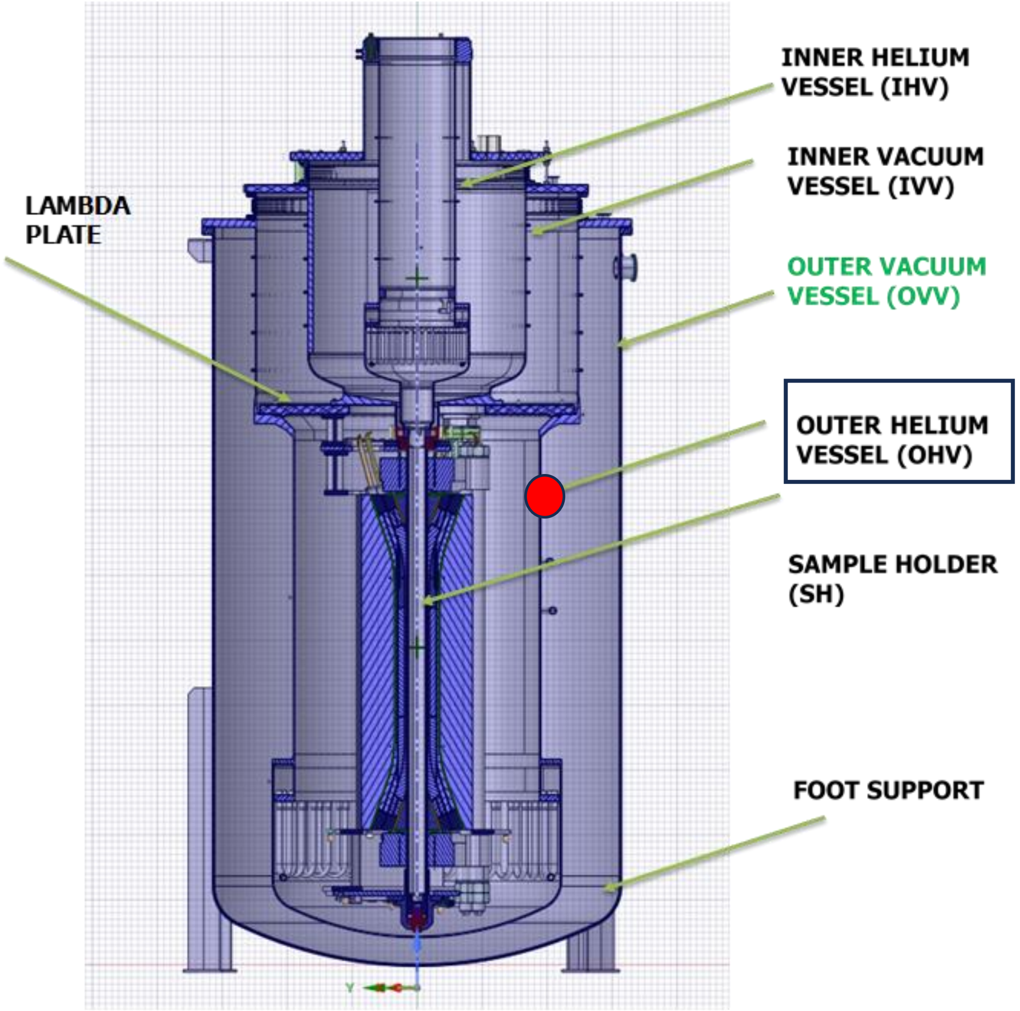
FRESCA2: a facility for testing SC samples



Simplified model for computations



FRESCA2: design of the OHV



1st step: definition of the **"cahier des charges"!**

3 OUTER HELIUM VESSEL (OHV)

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Self-weight (1)	A	B	B	B	B	B	B	B	B	B	C	C	B
Temperature (2)	A	A	A	A	A	B	B	B	B	A	A	A	A
Internal pressure (3)	/	/	/	A	B	B	B	C	C	/	/	D	E
External pressure (4)	/	/	/	/	/	/	/	/	B	B	A	A	/
Magnet + IC weight	/	/	X	X	X	X	X	X	X	X	/	/	X
Torque	/	/	/	/	/	/	X	X	X	/	/	/	/

(1) A = self-weight supported by handling points; B = self-weight supported by top flange; C = self-weight on manufacturing supports
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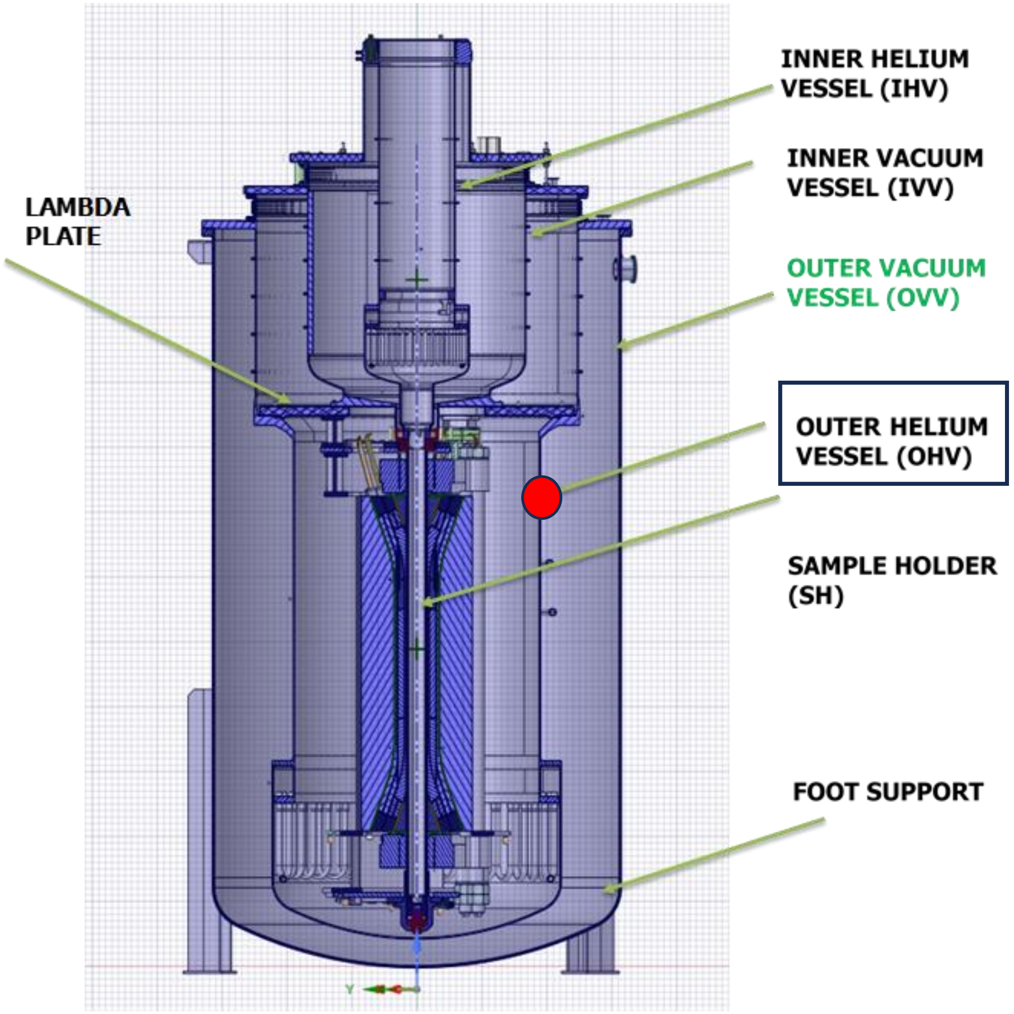
FRESCA2: design of the OHV

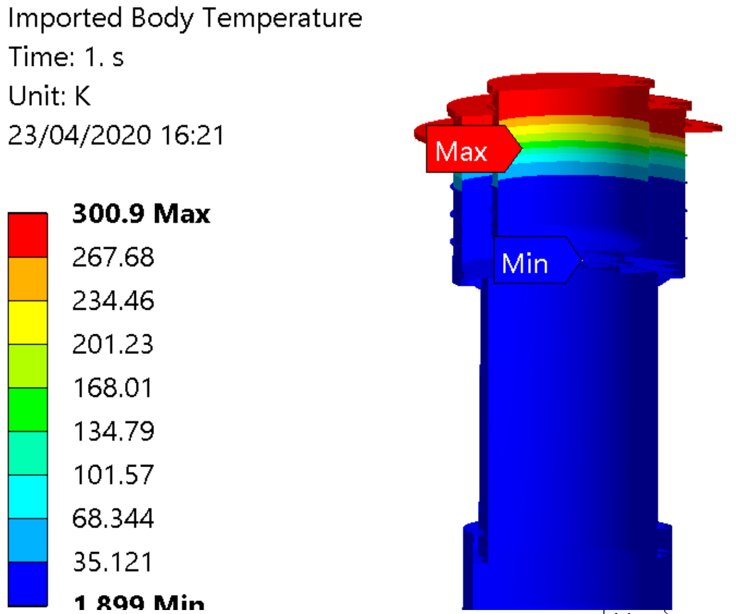


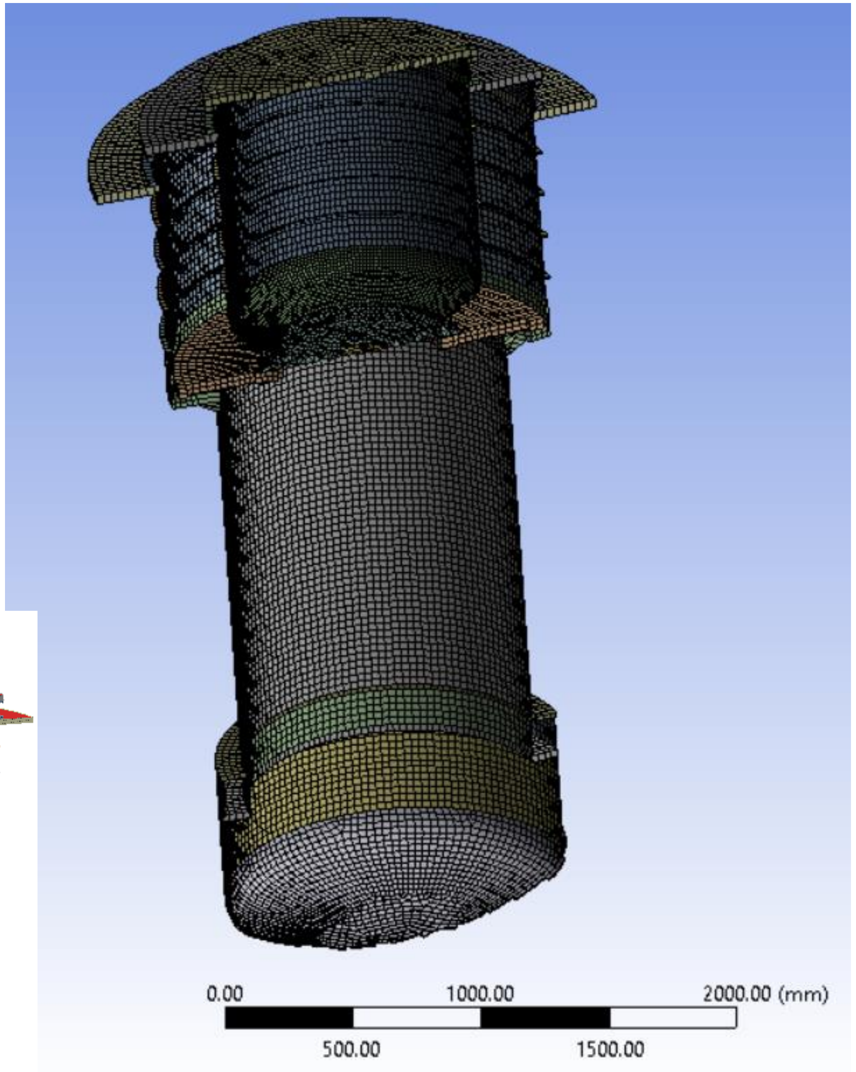
13 load cases reduced to **two design cases**:

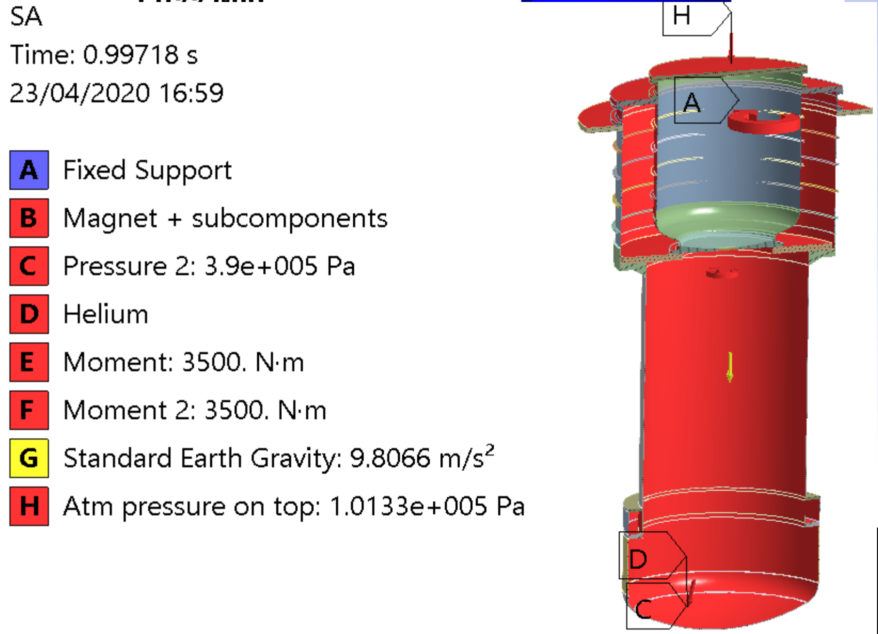
1. **Quench during operation**:
   - **Internal pressure** in the OHV 3.9 bara
   - **Thermal gradient** 4.5-300 K
   - **EM torque** 3500 Nm
   - Most likely failure scenario is by **plastic deformation**
2. **Vacuum loss during OHV purging**:
   - **External pressure** on the OHV 1.5 bara
   - Most likely failure scenario is by **buckling**

(we conservatively assume material properties at 300 K for all scenarios)

FRESCA2: quench during operation







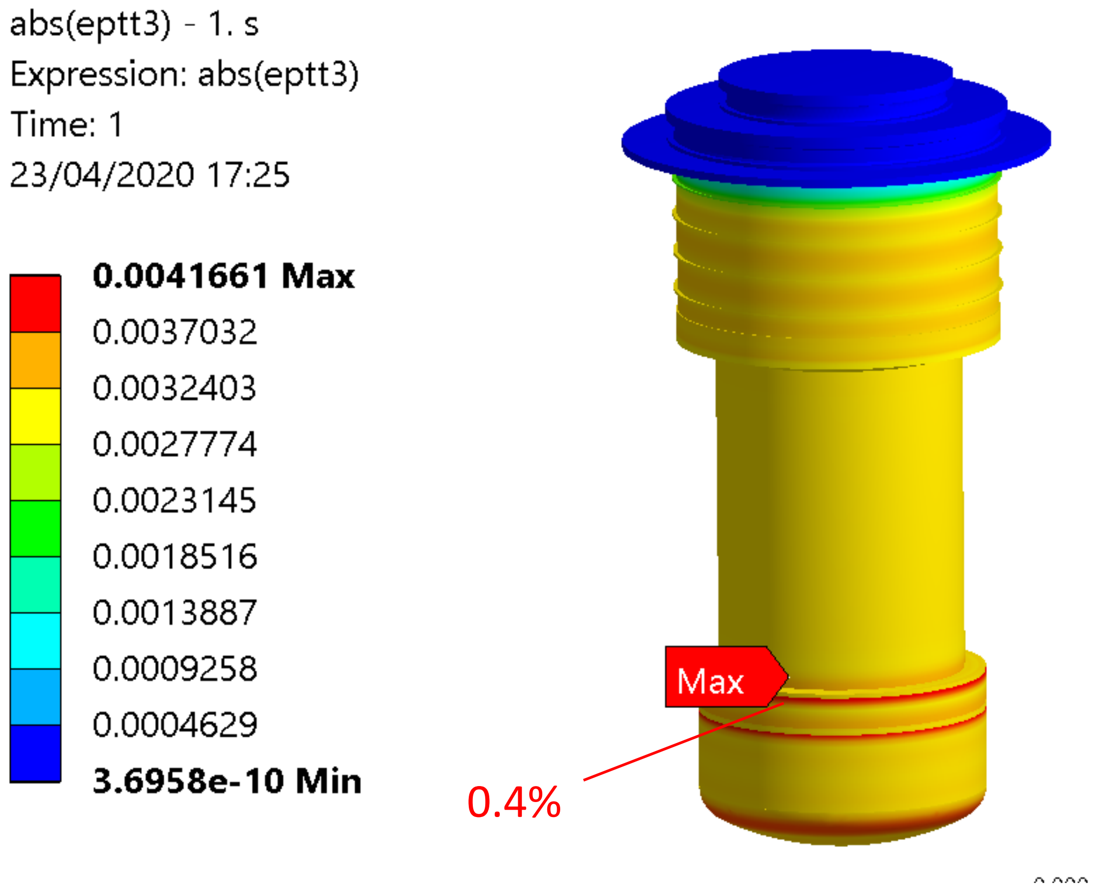
Suggestions:

- Use **shell elements** instead of solids wherever possible
- T field can be calculated in a separated **thermal analysis**, then imported into structural
- In the preliminary design phase, **start simple**, design for elasticity → linear elastic calculation

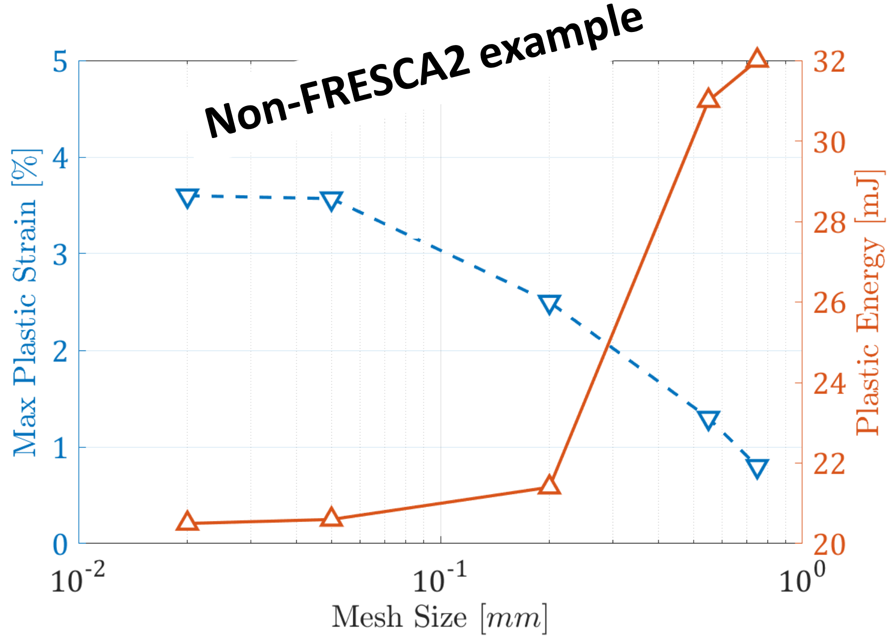
At a **later design stage**:

- **Nonlinearity of materials** (temperature, strain, …)
- Structure verified against EN-13445 Direct Route: **total strain must be less than 5%**

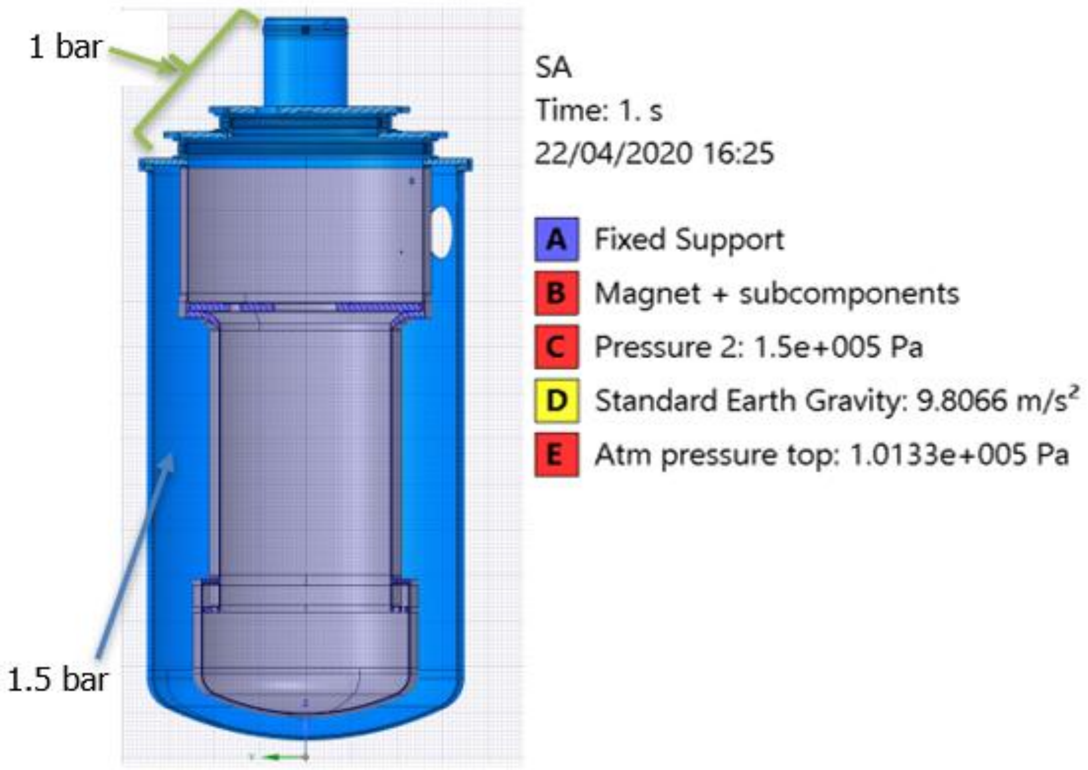
FRESCA2: quench during operation

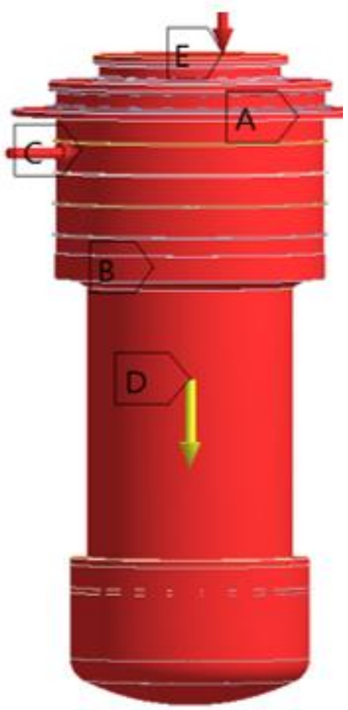


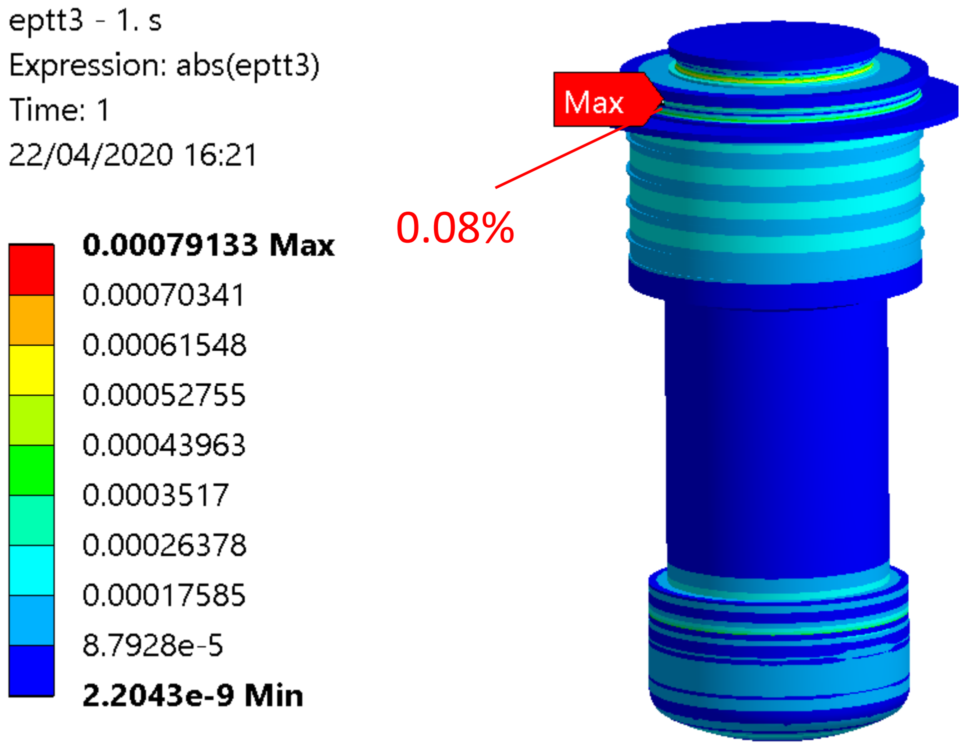
- Direct route requires $max(|\varepsilon_1|, |\varepsilon_2|, |\varepsilon_3|) < 5\%$
- How to make sure of accuracy of the results?
  - Convergence study
  - Submodeling



FRESCA2: vacuum loss during purging

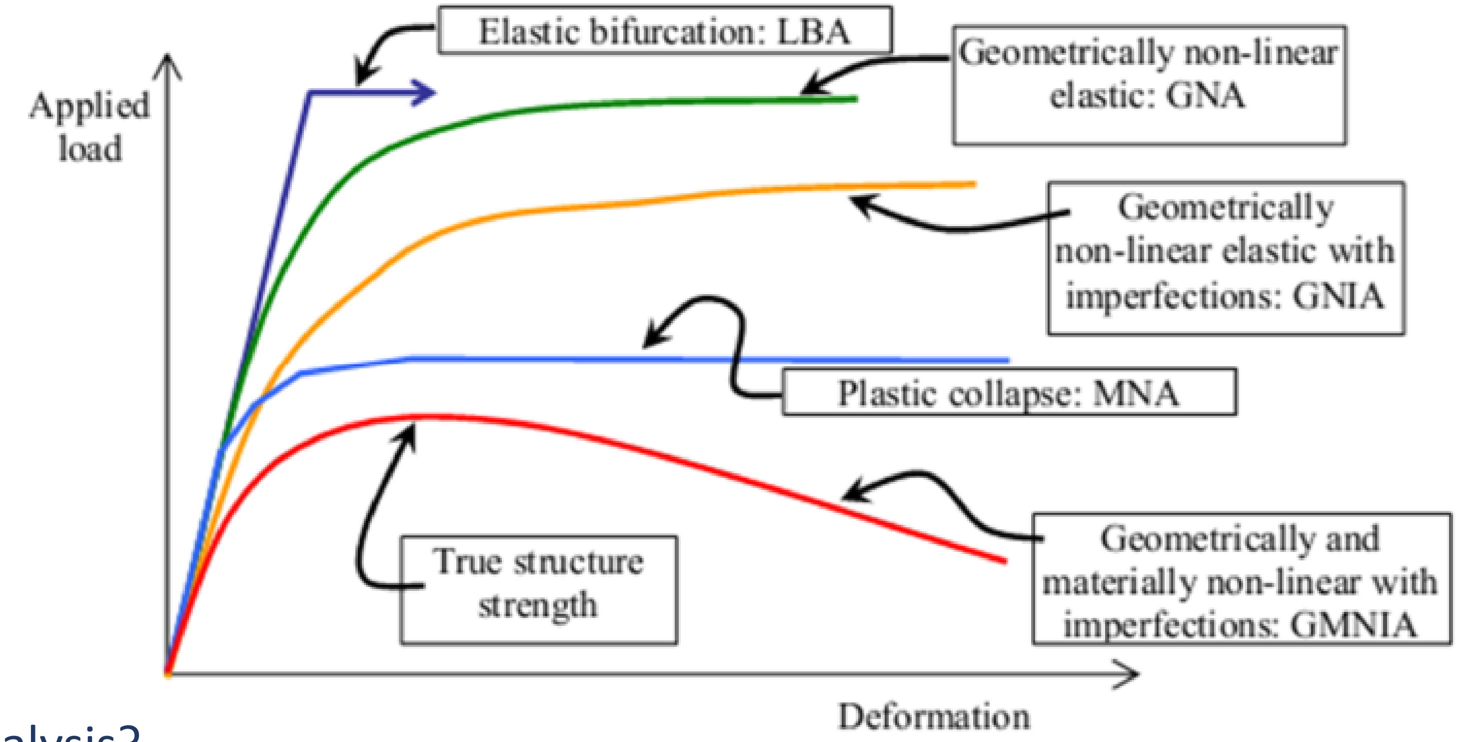






- Direct route check: $\mathbf{0.08\% < 5\%}$
- But: (especially) with external pressure, important to **verify buckling**

FRESCA2: vacuum loss during purging - Buckling



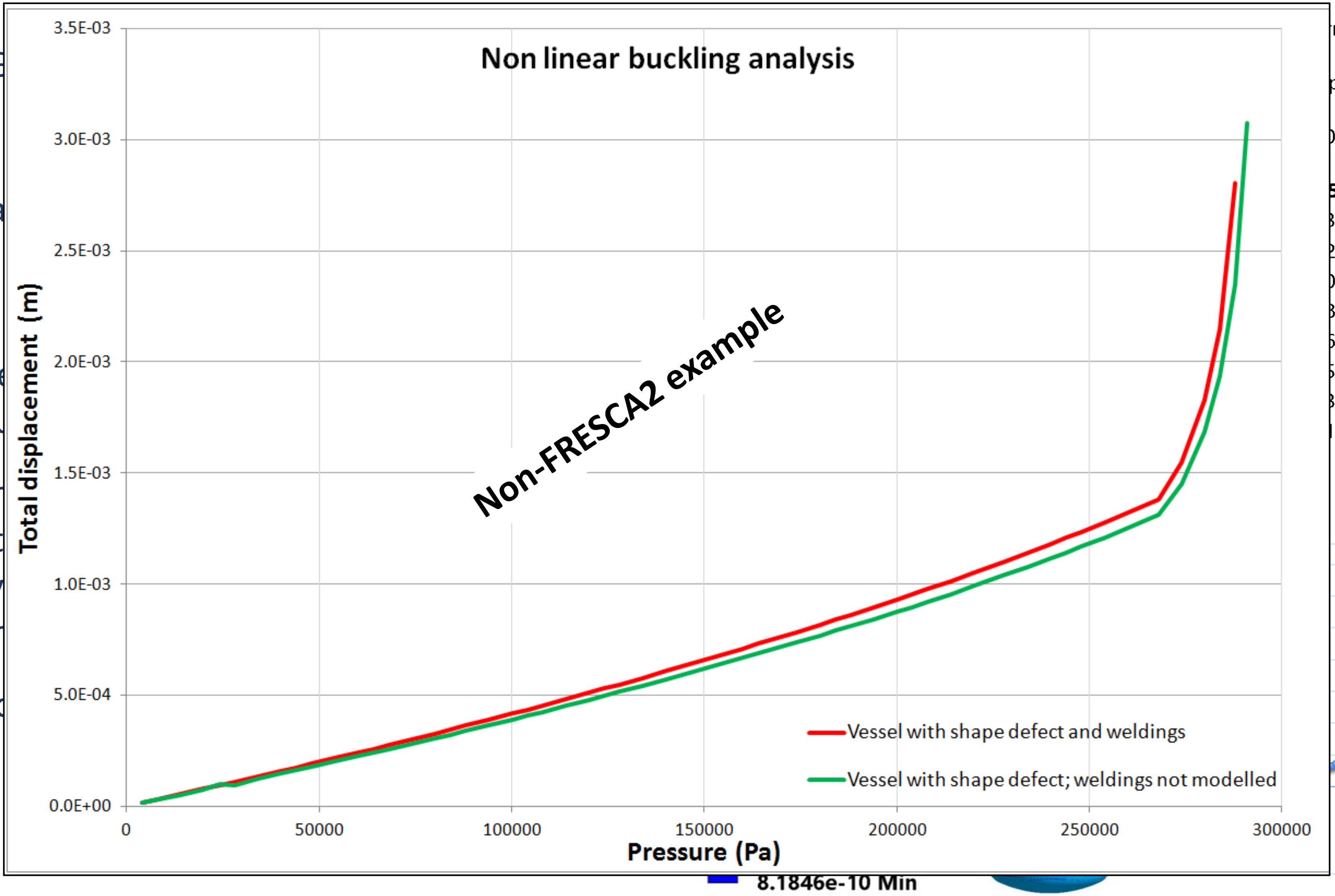
- Which kind of buckling analysis?
- **When going with FEM, better to directly take the most accurate one (GMNIA)**
- Also required by direct route. It accounts for large deformation theory, material nonlinearities, and initial geometry imperfections (e.g. shape errors, etc.)

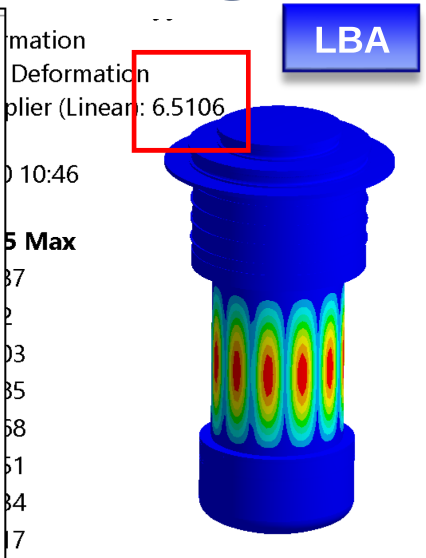
FRESCA2: vacuum loss during purging - Buckling

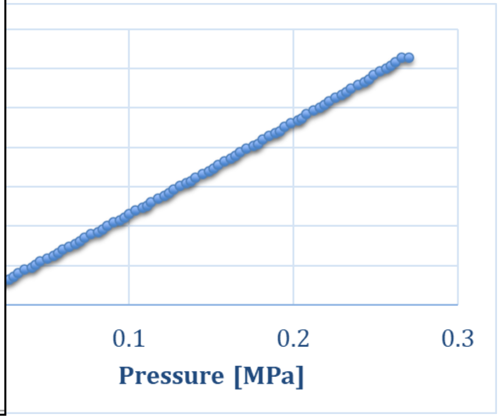
How to perform a G

Steps:

1. **LSA**: Run a linea imperfections),
2. **LBA**: Perform a buckling) and de and the load mu
3. **GMNIA**: Run a r analysis, import initial geometry progressively ur
   - Buckling oc
   - The safety reached





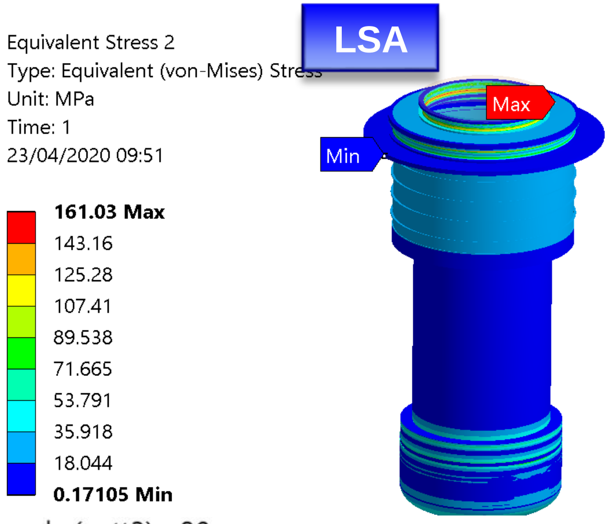


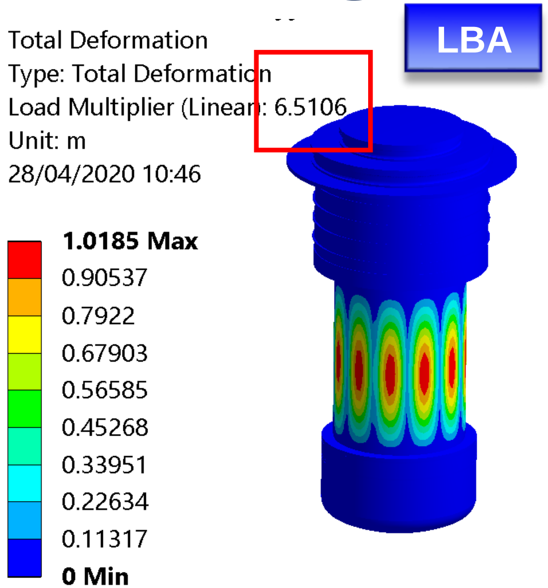
FRESCA2: vacuum loss during purging - Buckling

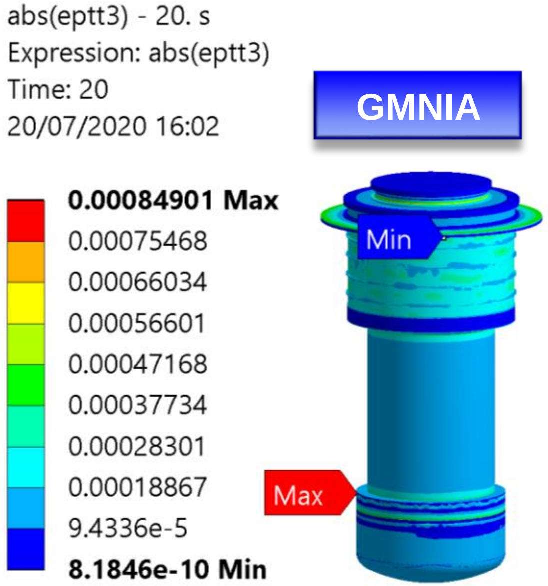
Again: in a preliminary design phase, **start simple:**

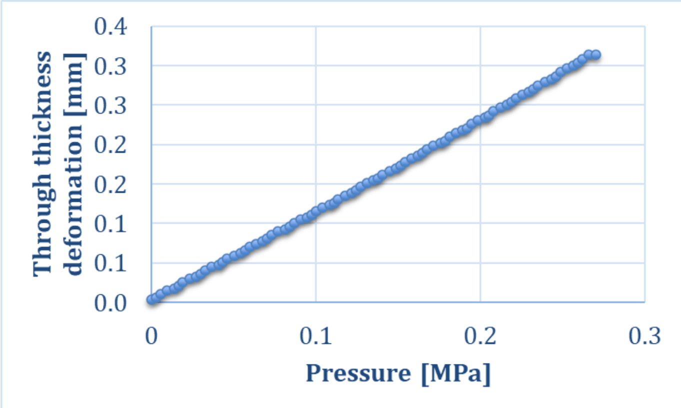
1. **LSA**: Run a linear elastic static analysis (no imperfections), with nominal loads
2. **LBA**: Perform a bifurcation analysis (eigenvalue buckling) and determine the linear buckling modes, and the load multipliers wrt (1)

Aiming at **large safety factors** (e.g. 3 against plasticity, 10-15 against eigenvalue buckling collapse)









Attention!

- **Large safety factors also have drawbacks** (increased weights, more difficult welds, lower material properties, costs, etc.)
- At a later design phase, best compromise between these parameters must be found, and the **more refined nonlinear analysis is necessary**

Computational Tools I - Summary

- Computer-Aided Engineering (CAE): **powerful tool** in the design phase of components, to decrease cost, time, risk for the project
- CAE require a number of iterations with CAD, with the goal of **optimizing the component**
- Also: combine CAD/CAE with **testing & prototyping** (calculation cannot replace everything!)
- Finite-Element Method (FEM) in the last years: **most adopted tool for CAE**
- When engineering particle accelerator components, we may often resort to **implicit codes**
- **Explicit codes become necessary** when dealing with short transient simulations (*e.g.* beam impact on dumps, windows, etc.) and with strongly nonlinear problems (e.g. fabrication technologies: cutting, welding, brazing, forming, etc.) → **examples in the next module!**
- **Graphical interfaces of FEM tools are becoming simpler: easier work, riskier if we do not well master the method!**

Symbols

- $[M]$: mass matrix $[kg]$
- $[C]$: damping matrix $[N/(^m/_s)]$
- $[K]$: stiffness matrix $[N/m]$
- $\{\ddot{u}\}$: acceleration vector $[m/s^2]$
- $\{\dot{u}\}$: velocity vector $[m/s]$
- $\{u\}$: displacement vector $[m]$
- $\{F\}$: external force vector $[N]$
- $\{s\}$: nodal displacements vector $[m]$
- $[N]$: shape functions matrix $[-]$
- $\{\varepsilon\}$: strain vector $[-]$
- $[\partial]$: strain-displacement matrix $[m^{-1}]$
- $\{\sigma\}$: stress vector $[Pa]$
- $[D]$: material constitutive matrix $[Pa]$
- $\{a\}$: polynomial coefficients vector $[-]$
- $[P]$: position matrix $[m]$
- $\{a\}$: nodal position matrix $[m]$
- ε_1: maximum principal strain $[-]$
- ε_2: middle principal strain $[-]$
- ε_3: minimum principal strain $[-]$

Bibliography

- O. C. Zienkiewicz, “The Finite Element Method: Its Basis and Fundamentals”, ISBN 978-1-85617-633-0.
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- D. Braess, “Finite Elements: Theory, Fast Solvers, and Applications in Solid Mechanics”, ISBN 978-0-52170-518-9.


Thanks for your attention!
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FOR PARTICLE ACCELERATORS AND DETECTORS

FEM Theory in a Nutshell

Physical phenomenon → Model → Governing equations & BCs → Approximation → Finite element equations

- Solid Mechanics
  e.g. Axially loaded elastic bar
- Fluid Mechanics
  e.g. Poiseuille flow in pipe
- Thermal Conduction
  e.g. 1-D heat flow
- Diffusion
  e.g. 1-D diffusion
- Electrical Conduction
  e.g. 1-D electric current flow

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(AE\frac{\mathrm{d}u}{\mathrm{d}x}\right)+b=0$$

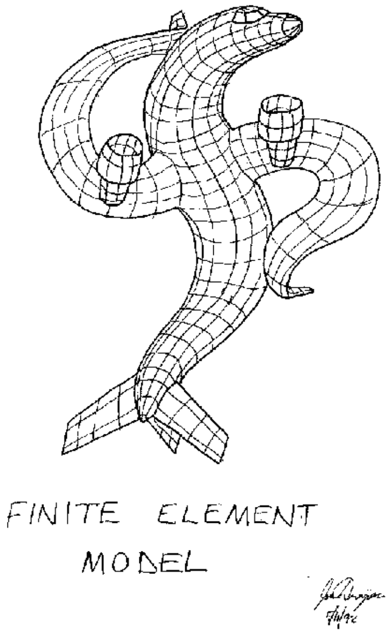
$$\frac{\mathrm{d}}{\mathrm{d}x}\left(A\frac{D^2}{32\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\right)+Q=0$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(Ak\frac{\mathrm{d}T}{\mathrm{d}x}\right)+Q=0$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(AD\frac{\mathrm{d}C}{\mathrm{d}x}\right)+Q=0$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(A\sigma\frac{\mathrm{d}V}{\mathrm{d}x}\right)+Q=0$$

+ BCs (Boundary Conditions)



$[K]\{u\} = \{F\}$ Static problems

$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F\}$ Dynamic problems

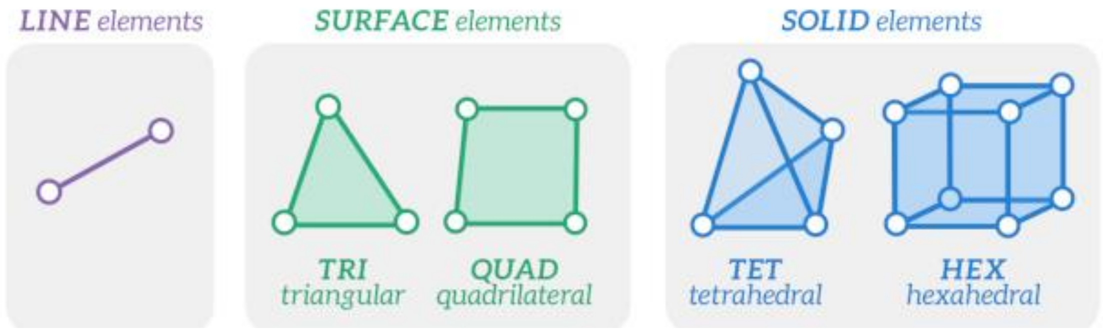
Linear problems: $[K], [C], [M] = [K_0], [C_0], [M_0]$

Nonlinear problems: $[K], [C], [M] = f(u, \dot{u}, \ddot{u})$

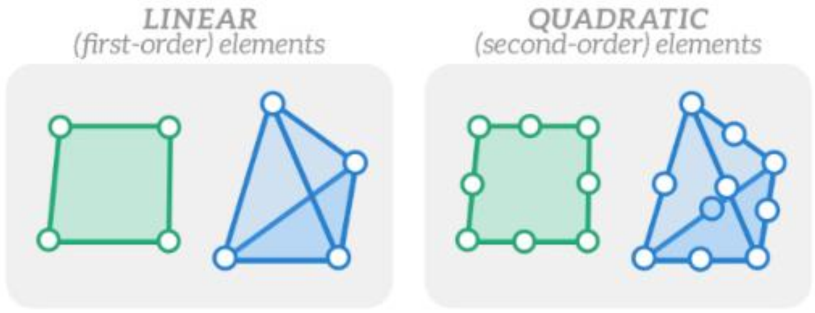
Element Types

The shape functions depend on the element type:

- **Line elements** model 1D structures like beams, rods or pipes.
- **Surface elements** are used to model large and thin surfaces like shells, plates.
- **Solid elements** are used to model three-dimensional bodies.



- 2D and 3D elements can be **linear** (first-order elements) or **quadratic** (second-order elements).
- Quadratic elements have additional mid-side nodes along each side of the element.
- Quadratic elements require **more computational power** but generally produce **more accurate results**.



Properties of the Shape Functions

1. It **must** be a continuous function, and must possess a derivative at least until to the *n-1* order required by the problem under study (*e.g.* n = 1 for a truss element, n = 2 for a beam or plane element, etc.)
2. It **must** reproduce rigid motion of the element with a null deformation energy (*i.e.* in an eigenvalue problem, the rigid motion d.o.f. gave a null eigenvalue → in a 3D space, for an unconstrained body there will be 6 null eigenvalues)
3. It **must** guarantee a constant deformation along the element (minimal condition when element size tends to zero)
4. It **must** guarantee continuity among elements (*i.e.* identical displacement field on a segment belonging to two adjacent elements)

Items 1–3: **Complete**; item 4: **Compatible**; items 1–4: **Conform**

5. It **should** be geometrically isotropic (*i.e.* displacement field is invariant wrt the reference system, not presenting preferential directions)

↓

Polynomials