University of Nottingham
UK | CHINA | MALAYSIA

MIDLANDS CENTRE
For Data-Driven Metrology

Measurement uncertainty

Prof Samanta Piano

Lecture outline

- **Metrology** and importance of measurements
- **Measurement uncertainty**: definition and standards
- Types of uncertainty
- **Errors** in measurements
- **Sources** of uncertainty
- Measurement uncertainty methods:
  - **GUM**
  - **Spread sheet**
  - **Monte Carlo**
- **CMM** measurement uncertainty
- Conclusions

What is metrology?

***Metrology* is the**
"Science of measurement and its application"
(International Vocabulary of Metrology-VIM 2012)

Measurement is part of our everyday lives

- **Food** and raw materials are bought by weight or size
- **Water, electricity** and **heat** are metered
- **Health care** relies upon medicine dosages
- Law and order have measured tolerances from **speed** of vehicles to finger print recognition
- Your safety relies upon setting smoke alarms at appropriate level or being able to see and read **safety sign**
- Our environment is constantly monitored from air quality to **weather** conditions

Why to measure?

Ideal World

- all workpieces produced correctly first time to the specified size

Real World

- no matter how well a manufacturing process is designed and operated variations will occur

Process variability

- size of a feature may vary in an unpredictable manner
- the size of the feature may be biased so that all workpieces deviate by a similar amount from a desired size

Quality control

- Traditional inspection method: after the manufacturing process
- During the manufacturing process: measure as quickly as possible

Measurement uncertainty

There is some uncertainty about every measurement result

… a “margin of doubt”

We need to ask:

“How big is the margin?”

“How bad is the doubt?”

Measurement uncertainty: definition

What is uncertainty?

- **VIM**: “A non-negative parameter characterising the dispersion of the quantity values being attributed to a measurand, based on the information used”
- **ISO GUM**: “parameter associated with the result of a measurement, that characterises the dispersion of the values that could reasonably be attributed to the measurand”
- **In short:** “it is an interval in which we are confident that the true value lies within. It represents our missing knowledge about the measurement”
- Simplified definition: “***quantified doubt about the result of a measurement***”

Note:

Uncertainty is attributed to a measurement result (not an instrument).
For instruments, we use the term “error”
GUM: Guide to the expression of uncertainty in measurement

Types of uncertainty

Type A : based on statistical calculation

- Using common standard deviation formula: $S = \sqrt{\sum_{i=1}^{N}(Xi-\bar{X})^2 / _{N-1}}$
- Repeatability

Type B: based on another methods

- Expert judgment, past experience of the measurements
- Calibration certificate,
- Tolerance in the design, drawing, historical data
- Commonly uses **Rectangular, U-shape and Triangular distribution**

Model for measurement uncertainty?

Ideal situation

True value = instrument reading – sum of all errors in the measurement

i.e. knowable, correctable errors, such as calibration error and unknown errors (uncertainties)

Reality

True value = instrument reading + known errors ± uncertainty

Errors in measurements

Error = difference between the measured value and the true value *(a calibrated value)*

Total error (ε)= Systematic error + Random error

Systematic error: Bias, can be compensated

Random error: Cannot be compensated

- a **systematic error**: deviation from the 'true' value in a measurement and remains constant in repeated measurements.

- a **random error**: varies in an unpredictable manner when the measurement is repeated (there is no constant pattern).

Systematic *vs* Random error

Systematic error	Random error
Can be compensated	Cannot be compensated (except with a real-time compensation method with close-loop control)
Cannot be detected easily	Can be detected easily
Cannot be reduced by repeated measurements	Can be minimised by several measurements
Can be calculated easily	Requires statistical analysis
Accuracy improvement after minimising systematic error	Repeatability improvement after minimising random error
Calibration and adjustment reduce the systematic error	Calibration and adjustment do not affect the random error
Characterisation is not needed	Need characterisation in terms of probability distribution, mean, variance
Inconsistency occurs along a direction (has a pattern)	Inconsistency occurs randomly (does not have a pattern)

Sources of uncertainty

Man
- Operator's skill
- Operator's knowledge
- Operator physical condition
- Etc ...

Machine
- Geometric and kinematic error
- Sensor noise
- Abbe error
- Cosine error
- Deformation
- Etc ...

Part
- Geometric error (e.g flatness, roundness, cylindricity)
- Surface defects
- Etc ...

Method
- Sampling strategy
- Measurement procedure
- Measurement setting
- Measurement algorithm
- Etc ...

Environment
- Temperature variation
- Vibration (e.g. ground, accoustic)
- Pressure variation
- Humidity variation
- Etc ...

Uncertainty sources

Eight steps to estimate uncertainty

1. Think ahead (procedure, caution, safety aspects)

2. Measure

3. Estimate uncertainty contributors

4. Consider correlation (In many cases it is assumed not exists)

5. Calculate results (including known corrections)

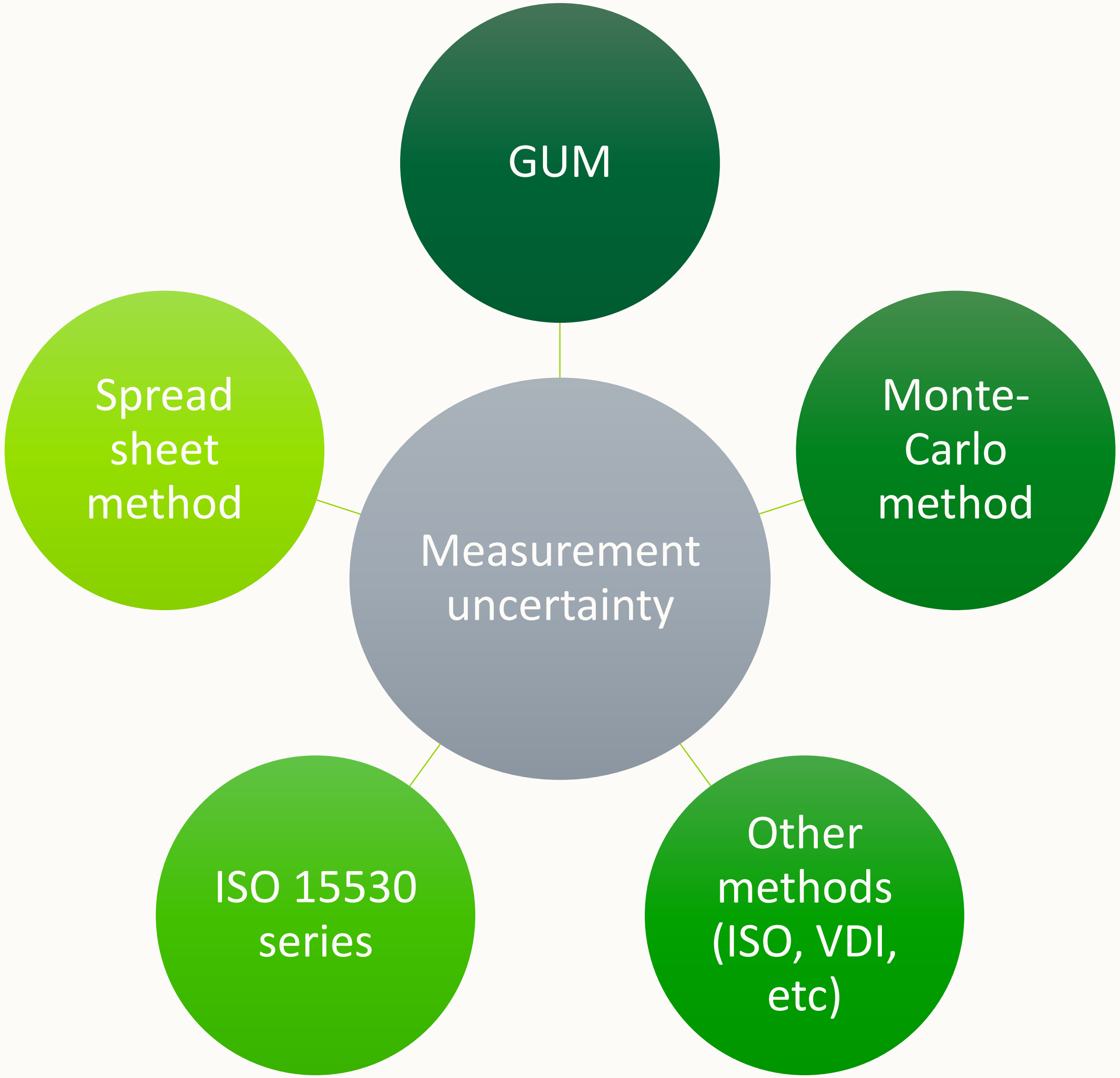
6. Find the combined uncertainty

7. Express the results (confidence interval, coverage factor: commonly 95%)

8. Record and document it!

How to estimate uncertainty

<u>Uncertainty</u> has to be estimated, but..
it is not an easy task!



University of Nottingham
UK | CHINA | MALAYSIA

MIDLANDS CENTRE
For Data-Driven Metrology

ISO Guide to the Expression of Uncertainty in Measurement (GUM) method

GUM method

Based on BIPM JCGM:2008

The measurand Y is determined from N quantities: $X_1, X_2, \ldots, X_N$

$$Y = f(X_1, X_2, \ldots, X_N)$$

Measurement uncertainty should be propagated from the uncertainty of each of the measurement components:

$$u(Y) = \sqrt{\sum_{i}^{n}\left(\frac{\partial f}{\partial X_i}\right)^2 u^2(X_i) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{\partial f}{\partial X_i}\frac{\partial f}{\partial X_j}u(X_i, X_j)}$$

y can be linearised by Taylor-expansion only up-to first order

$$y = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

$c_i$is a Taylor-expansion coefficient

$$var(y) = var(c_1x_1) + var(c_1x_2) + \cdots + 2cov(c_1x_1, c_2x_2) + 2cov(c_1x_1, c_3x_3) + \cdots$$

where: $var(cy) = c^2\mu^2$, Hence:

$$\mu_y^2 = c_1^2\mu_1^2 + c_2^2\mu_2^2 + \ldots + 2c_1c_2\mu(x_1, x_2) + 2c_1c_3\mu(x_1, x_3) + \cdots$$

Probability density function

There are various ways to specify a random variable.

The probability density function describes the shape of the characteristic curve derived from numerous repeated measurements.

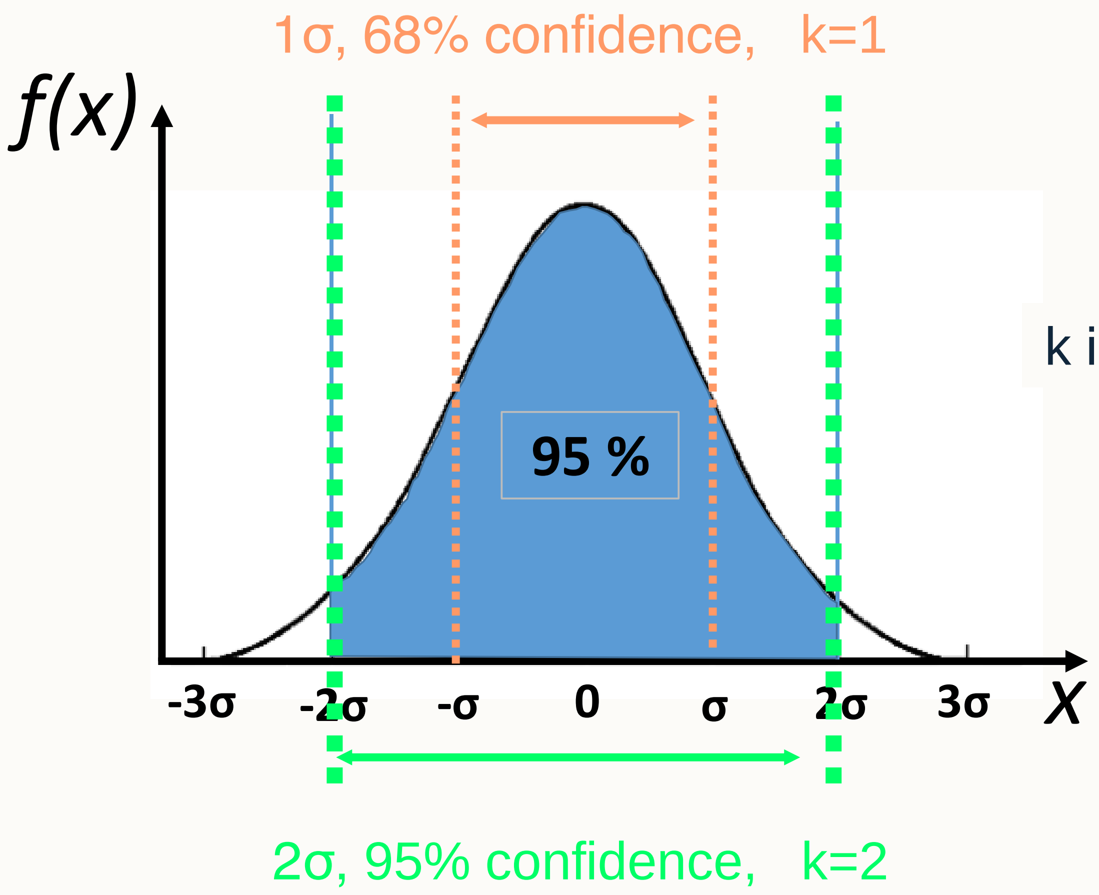
The probability density function represents how likely each value of the random variable is.

The normal distribution, also called the Gaussian distribution, is an extremely important probability distribution.

The distributions have the same general form, differing in their location and scale parameters: the mean (or average) and standard deviation (variability),

A plot of the distribution is often called the bell curve because the graph of its probability density resembles a bell

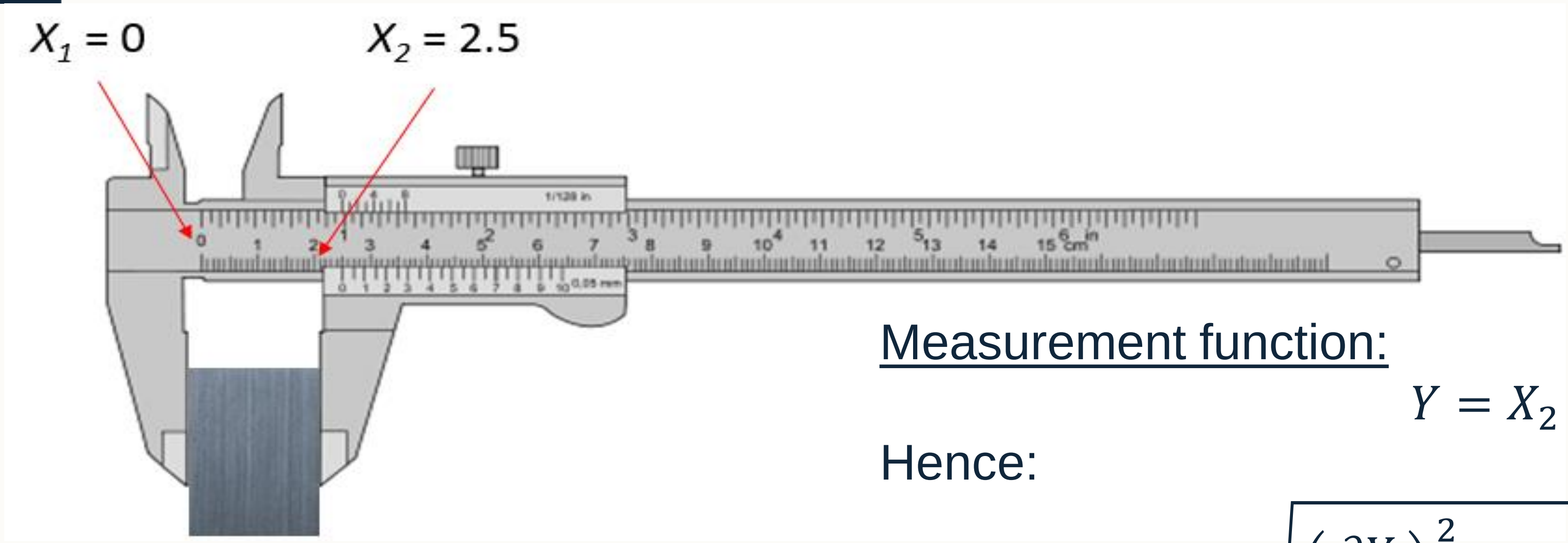
Coverage factor



k is “coverage factor”

Sample standard deviation s is an estimate of population standard deviation σ

Example: calliper measurement



Measurement function:

$$Y = X_2 - X_1$$

Hence:

$$\mu(Y) = \sqrt{\left(\frac{\partial Y}{\partial X_1}\right)^2 \mu(X_1)^2 + \left(\frac{\partial Y}{\partial X_2}\right)^2 \mu(X_2)^2}$$

$$\mu(Y) = \sqrt{(-1)^2\mu(X_1)^2 + (1)^2\mu(X_2)^2}$$

$$\mu(Y) = \sqrt{\mu(X_1)^2 + \mu(X_2)^2} = \sqrt{0.1^2 + 0.1^2}$$

$\mu(Y)$ = 0.21;

$U(Y) = 2\mu(Y)$=0.42 (95 % confidence interval)

Finally:

$$Y = (2.5 \pm 0.42)\ mm$$

Assume:

$\mu(X_1)$=$\mu(X_2)$=0.1

The instrument has equal uncertainty for each scale. The correlation is negligible.

GUM drawbacks

We require a functional relationship (the mathematical relationship between the output and the input)

Even if we have the functional relation, often it is not differentiable

Involves a cumbersome calculation, even in very simple cases

University of Nottingham
UK | CHINA | MALAYSIA

MIDLANDS CENTRE
For Data-Driven Metrology

Spreadsheet method

Spreadsheet method

- Using a table to list all significant (the most influential) uncertainty contributors to a measurand
- Important elements in the spread sheet model:
  1. Source of uncertainty
  2. Value (±) of an interval
  3. Conversion and description **(optional)**
  4. Probability distribution (related to decide divisor values)
  5. **Divisor** (depends on the probability distribution)
  6. $\boldsymbol{C_i}$ (sensitivity coefficient):

$$C_i = \frac{\partial Y_i}{\partial X_i}$$

  7. Standard uncertainty:

$$\mu_i = C_i \frac{Value_i\ (\pm)}{Divisor_i}$$

Distribution	Value	Divisor	standard uncertainty
Normal	± a	2	a/2
Rectangular	± a	√3	a/√3
Triangular	± a	√6	a/√6
U-shaped	± a	√2	a/√2

$\boldsymbol{C_i}$ (sensitivity coefficient) is usually set as 1 (especially for Type B uncertainty), unless we know the measurement function

Table of significant uncertainty contributors

Source of uncertainty	Value	Conversion and description	Probability distribution	Divisor	C_i	standard uncertainty
X_1	$u(X_1)$				C_1	u_1
X_2	$u(X_2)$				C_2	u_2
…	…				…	…
X_n	$u(X_n)$				C_n	u_n
Combined standard uncertainty μ_c			Assumed Normal			
Expanded uncertainty U = kμ_c (95% confidence interval)			Assumed Normal k = 2			$u(Y) = \sqrt{\sum_{i=1}^{N} u_i^2}$

Example: 50 mm gage block measurement by a micrometre

The measurement model is

$$L = Xread + X_{read} \alpha \Delta T$$

From the measurement model, we can see that scale resolution X_{read} , α and temperature *T* as well as the gauge block calibration are the uncertainty contributors.

Influential uncertainty sources:

- calibration uncertainty
- micrometre resolution
- standard uncertainty from measurements temperature variation
- coefficient of thermal expansion error

Spreadsheet model 1

Source of uncertainty	Value (±)	Conversion and description	Probability distribution	Divisor	$C_i = \frac{\partial Y_i}{\partial X}$	standard uncertainty $u_i = C_i u(X_i)$
Calibration uncertainty	0.03 µm	The value is from calibration certificate and is expanded uncertainty U with *k*=2. TYPE B.	Normal	2	1	0.015 µm
Resolution	0.005 mm	Resolution is assumed as a rectangular distribution with σ=half-resolution/√3. TYPE B.	Rectangular	√3	1	1.44 µm
Standard deviation from 10 measurements (repeatability)	10 µm	$\sigma = \sum_{i=1}^{n} \frac{X_i}{n}$ TYPE A.	Normal	1	1	10 µm
Temperature variation µ(temp)	0.5 ^{0}C	**DIRECT METHOD**	Normal	1	1	
Coefficient thermal expansion error µ(α) (assume 10% from the CTE)	10% of SS304 CTE	**DIRECT METHOD**	Normal	1	1	
Combined standard uncertainty μ_c			Assumed Normal			
Expanded uncertainty U = kμ_c (95 % confidence interval)			Assumed Normal k = 2			

Direct calculation method

Single variable

A measurement function $Y = f(X)$

The uncertainty: $u(Y) = |f(X + u(X)) - f(X)|$

Multiple variable

A measurement function $Y = f(X_1, X_2, \dots., Xn)$

The uncertainty:

$$u(Y_1) = |f(X_1 + u(X_1), X_2, \dots, Xn) - f(X1, X2, \dots, Xn)|$$

....

$$u(Y_n) = \left|f\left(X_1, X_2, \dots, Xn + u(X_n)\right) - f(X1, X2, \dots, Xn)\right|$$

Direct calculation method results

Temperature variation

$\mu(\varepsilon_{temp})$ = L×α×μ(temp) = 0.05 m × 17.3 μm/m^{0}C × 0.5 ^{0}C = **0.43 μm**

Coefficient of Thermal Expansion error

$$u(\varepsilon_\alpha) = \left|f\left(X_1, X_2, \ldots, Xn + u(X_n)\right) - f(X1, X2, \ldots, Xn)\right|$$

$$u(\varepsilon_\alpha) = |(Xread + Xread(\alpha + \varepsilon_\alpha)\Delta T) - (Xread + Xread.\alpha.\Delta T)|$$

$$u(\varepsilon_\alpha) = |(Xread.\Delta T)\varepsilon_\alpha|$$

$\mu(\varepsilon_\alpha)$=L×(t_{max}-20 ^{0}C)×μ(α)= 0.05 m × (20.5-20) ^{0}C× 10%×17.3 μm/m^{0}C = **0.043 μm**

Spreadsheet model 2

Source of uncertainty	Value (±)	Conversion and description	Probability distribution	Divisor	C_i	standard uncertainty
Calibration uncertainty	0.03 µm	The value is from calibration certificate and is expanded uncertainty U with *k*=2. TYPE B.	Normal	2	1	0.015 µm
Resolution	0.005 mm	Resolution is assumed as a rectangular distribution with σ=half-resolution/√3. TYPE B.	Rectangular	√3	1	1.44 µm
Standard deviation from 10 measurements (repeatability)	10 µm	$\sigma = \sum_{i=1}^{n} \frac{X_i}{n}$ TYPE A	Normal	1	1	10 µm
Temperature variation µ(temp)	0.5 ^{0}C	$\mu(\varepsilon_{temp}) = L \times \alpha \times \mu(temp)$ TYPE B	Normal	1	1	**0.43 µm**
Coefficient thermal expansion error µ(α) (assume 10% from the CTE)	10% of SS304 CTE	**0.043 µm** TYPE B	Normal	1	1	**0.043 µm**
Combined standard uncertainty μ_c			Assumed Normal			**10.11 µm**
Expanded uncertainty U = $k\mu_c$ (95 % confidence interval)			Assumed Normal k = 2			**20.22 µm**

Summary gage block measurement

- The **length of the gage block** = **(50 ± 0.02022) mm**
- The interval 20.22 µm is the ***expanded uncertainty*** with a coverage factor k = 2 (95% confidence interval, assumed Normal distribution)
- The length 50 mm is the **mean** from five times repeated measurements
- The uncertainty was estimated by **spread sheet method**
- The measurement was carried out in a room at (20 ± 0.5) °C by a trained operator

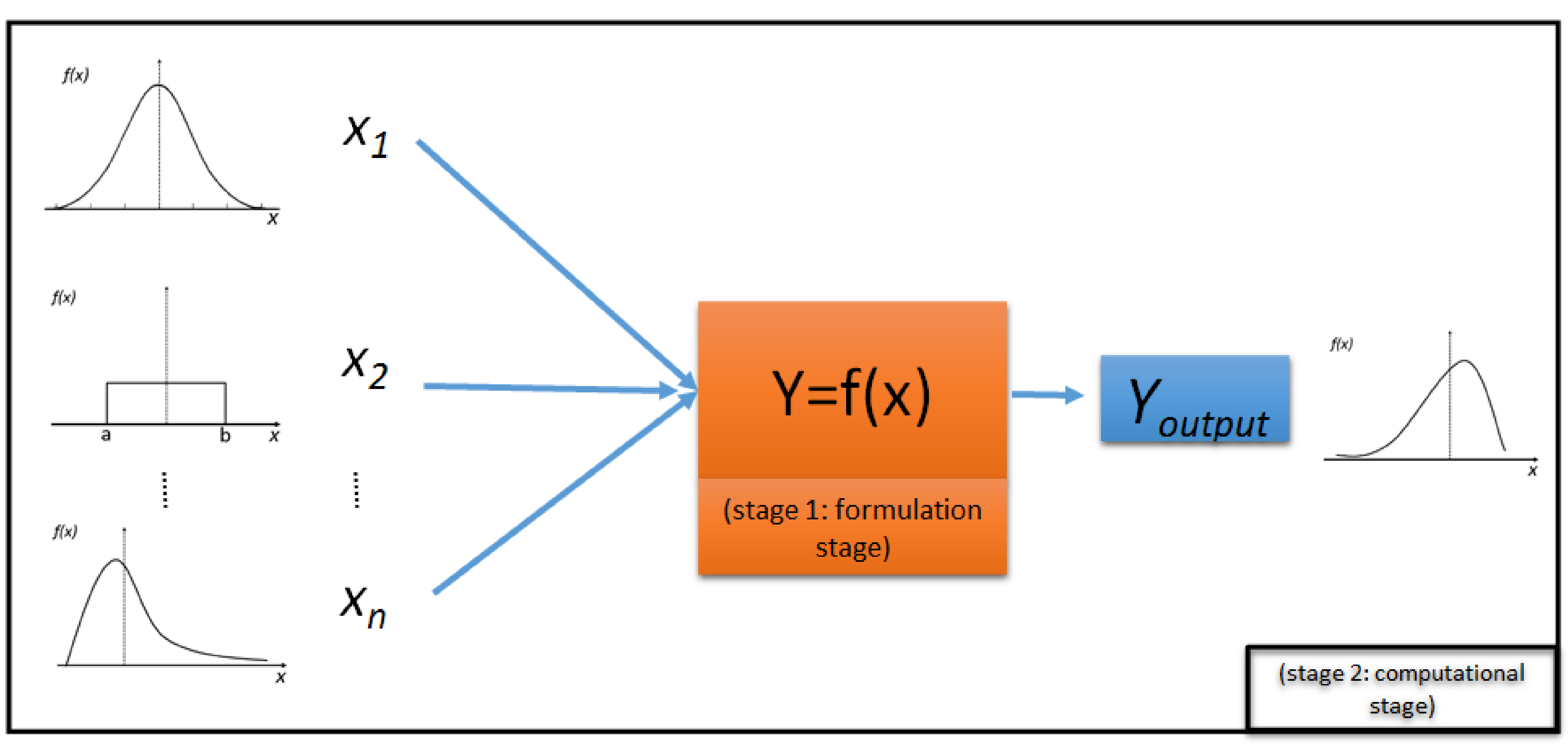
University of Nottingham
UK | CHINA | MALAYSIA

MIDLANDS CENTRE
For Data-Driven Metrology

Monte Carlo method

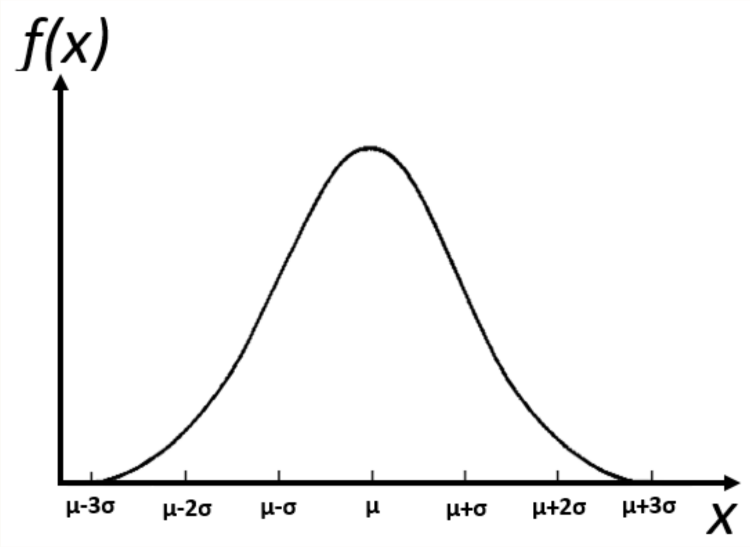
Monte-Carlo simulation stages

1. Formulation stage, including measurand definition
2. Computational stage (including propagation and summarising results)



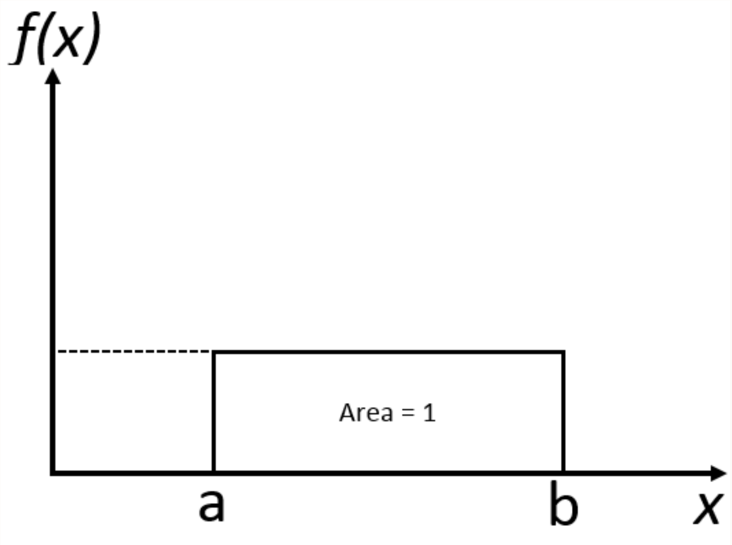
Probability distribution

Normal density function



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$$

Rectangular density function



$$f(x) = \begin{cases} 0, x \leq a \\ \frac{1}{b-a}, a < x < b \\ 0, x > a \end{cases}$$

Monte-Carlo process

Procedure:

1. Select a number of *n* runs (usually a big number)
2. Repeat *n* times:

   2.1 Randomly generated all inputs $X= \{X_1, X_2, \ldots, X_n\}$ randomly drawn from their probability density function (e.g. normal, rectangular, exponential)

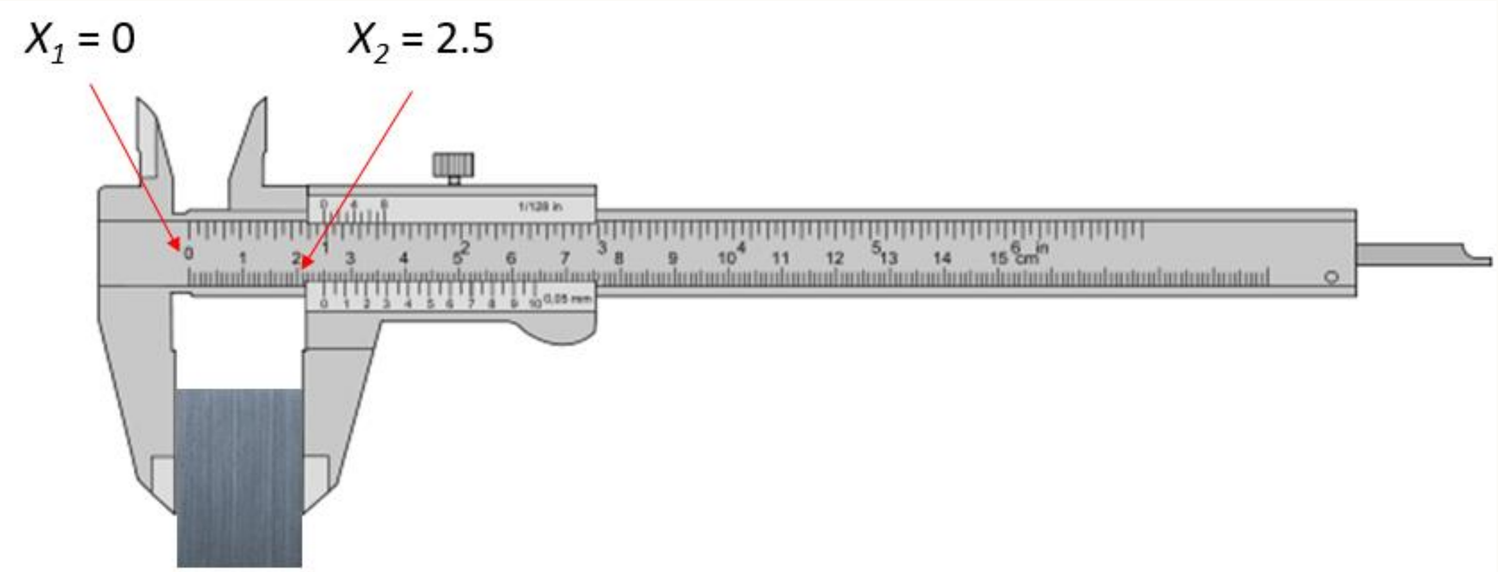
   2.2 Apply the measurement calculation according to the measurement model (formulation) $Y_i=f(X_i)$
3. From all outputs of Y_i calculated for *n* times :

   - If the distribution is known (commonly assumed as a normal distribution), calculate the standard uncertainty from all outputs of Y_i.
   - Else if, sort the all values of Y_i into incremental order $G=\{Y_i, i=1 \ldots n\}$ and take the values of Y_i at the 2.5 % and 97.5 % position as the interval limits.

Validation:

1. Comparing the result of MC to the results of GUM
   - In many situations, we use MC because GUM is impractical
2. Compare with other measurement from a more accurate instrument (see ISO 15530-4)

Example: calliper measurement



From GUM:
$\mu(Y) = 0.21$

Monte-Carlo method simulation

```
n=10000;
```

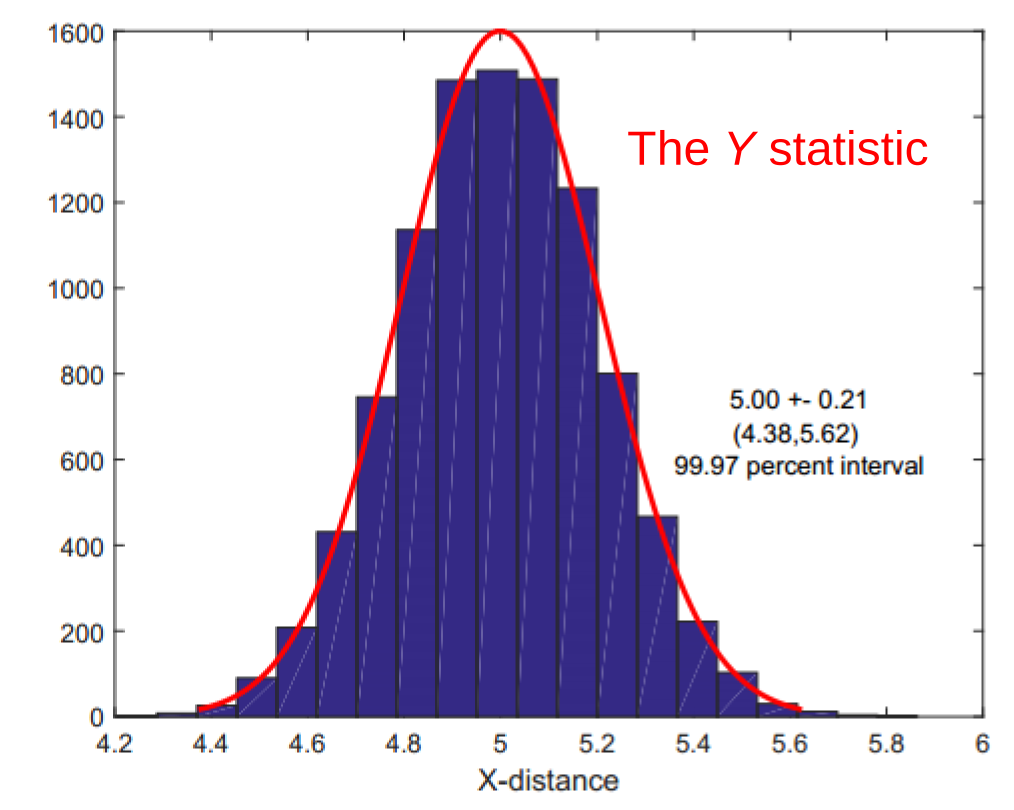
Number of repetitions

```
ux=0.15;
for i=1:n
    x2=5+normrnd(0,ux);
    x1=0+normrnd(0,ux);
    Y(i)=x2-x1;
end
```

Measurement function

From Simulation:

$\mu(Y) = 0.21$



Validation from GUM

$\mu(Y) = 0.21$

University of Nottingham
UK | CHINA | MALAYSIA

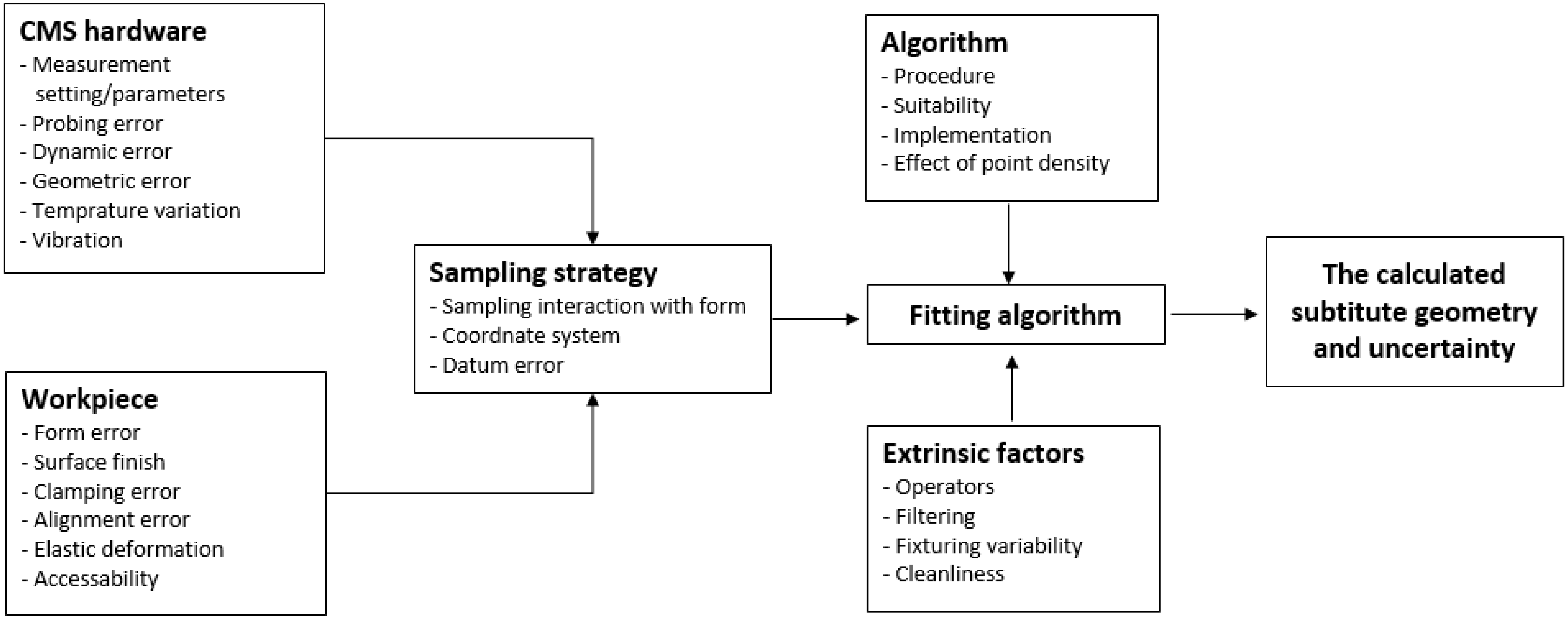
MIDLANDS CENTRE
For Data-Driven Metrology

ISO 15530

Coordinate measuring systems

Contact and non-contact **CMM**

Sources of uncertainty

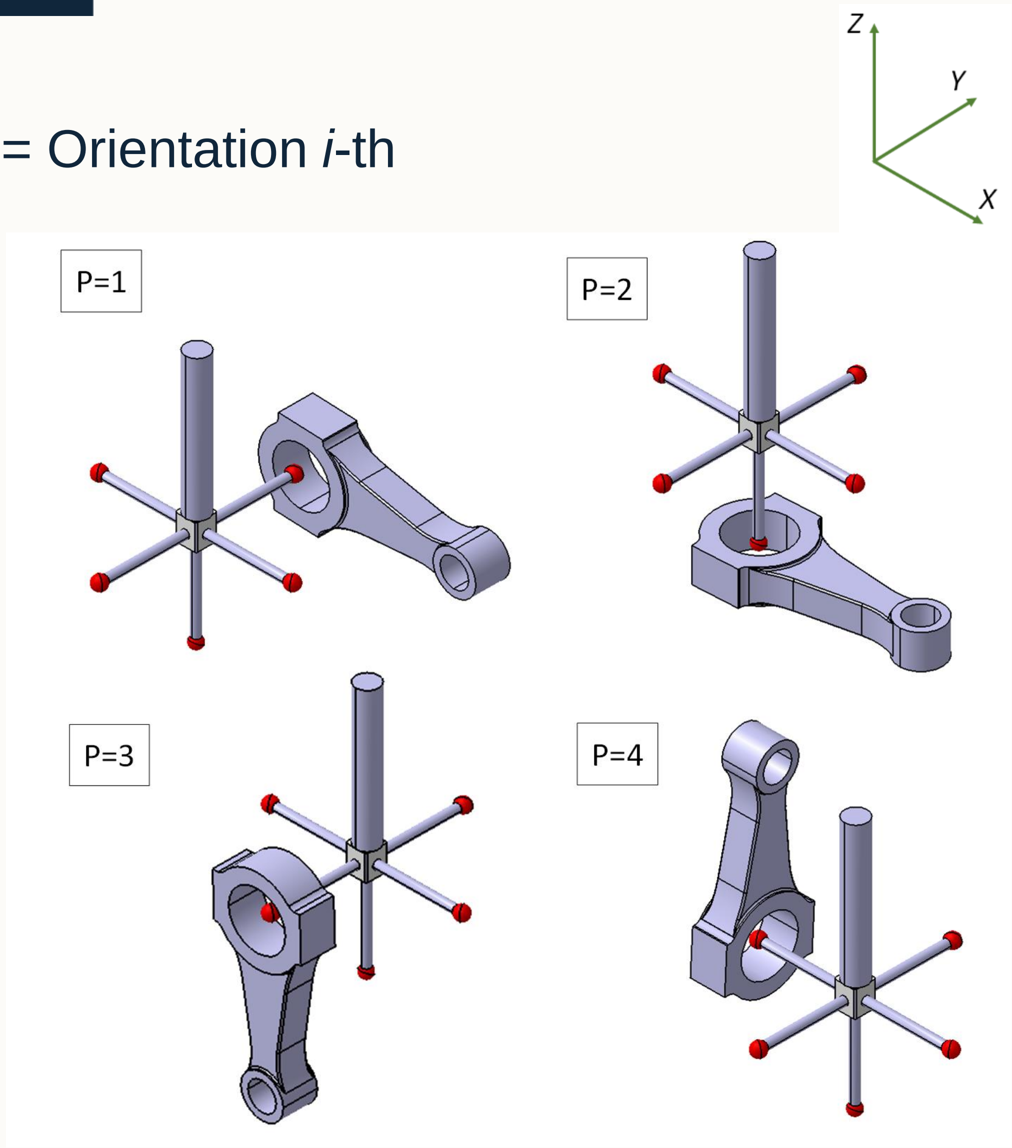


ISO DTS 15530-2 Multi-position method

- A method to calibrate and estimate uncertainty of a part (size and form measurement) by measuring an artefact with a **CMM** using multi-measurements
- The procedure needs (at least) total **20 measurements**: 4 different orientations and 5 repetitions for each orientation
- A performance verified CMM should be used
- The stylus of the CMM should be “qualified” first (not using previous recent qualification data)
- It is recommended to use different sampling strategies for each position

Orientations

P_i = Orientation *i*-th



	Position p = 1	Position p = 2	Position p = 3	Position p = 4
Measurement i = 1	Y_{11}	Y_{12}	Y_{13}	Y_{14}
Measurement i = 2	Y_{21}	Y_{22}	Y_{23}	Y_{24}
Measurement i = 3	Y_{31}	Y_{32}	Y_{33}	Y_{34}
Measurement i = 4	Y_{41}	Y_{42}	Y_{43}	Y_{44}
Measurement i = 5	Y_{51}	Y_{52}	Y_{53}	Y_{54}
mean	Y_1	Y_2	Y_3	Y_4
Standard deviation	S_1	S_2	S_3	S_4

Uncertainty sources

- μ_{rep} = Uncertainty source from the CMM repeatability, part property (form, roughness, etc), sampling strategy, dirt on the surface, etc .
- μ_{geo} = Uncertainty source from CMM geometric error, stylus error, tip error, fixturing error and alignment error
- μ_{corr} = Uncertainty source from the error applied to the length measurement (only applied for distance/length and size measurement)
- μ_{temp} = Uncertainty source due to thermal variation and error in the coefficient of thermal expansion of the measured part
- k = The coverage factor for 95 % confidence interval (k=2 for normal distribution, also depends on the effective degrees of freedom)

Estimated uncertainty

- $U = k\sqrt{\frac{\mu_{rep}^2}{n_1} + \frac{\mu_{geo}^2}{n_2} + \mu_{corr}^2 + \mu_{temp}^2}$ for distance, length and size measurement
- $U = k\sqrt{\frac{\mu_{rep}^2}{n_1} + \frac{\mu_{geo}^2}{n_2}}$ for form measurement
- $U = k\sqrt{\frac{\mu_{rep}^2}{n_1} + \frac{\mu_{geo}^2}{n_2}}$ for angle measurement

n_1= number of measurement
n_2= number of position

ISO DTS 15530-2

Advantages:

- Applicable for low production parts
- Reducing the uncertainty of the CMM volumetric error by averaging from multi measurements in different orientations
- Very suitable for master part calibration

Disadvantages:

- The procedure is relatively longer compare to other ISO methods
- Relatively not efficient for a high or even medium volume part production

ISO DTS 15530-3 Substitution method

- The method use a calibrated artefact that has the similar shape (length, angle, form, etc) with respect to the measured part
- The calibrated artefact is measured at very similar condition with the real measurement of the part
- Has been adopted by industries having an inline CMM in their workshop and with medium-large production volume

Measuring uncertainty

$$U = k \times \sqrt{\mu_{cal}^2 + \mu_p^2 + \mu_b^2 + \mu_w^2}$$

- μ_{cal} = Uncertainty source from the calibrated part's certificate
- μ_p = Uncertainty source from CMM geometric error, stylus error, and sampling error.
- μ_b = Uncertainty source from the error for the applied size (e.g. length, diameter) corrections
- μ_w = Uncertainty source from the part-to-part variation, e.g. due to manufacturing error (manufacturing variation), temperature variation, environment effects
- *k* = The coverage factor for 95 % confidence interval (*k=2* for normal distribution, also depends on the effective degrees of freedom)
- <u>*Strategy*</u>*:* The calibrated artefact is measured at least 10 times with different sampling strategies and different time and different parts are measured from different time (to include temperature variation effect)

ISO DTS 15530-3

Advantages:

- The procedure only needs to be applied once for subsequent measurements of similar parts
- All the measurements are carried out under the same measurement conditions (environment, sampling strategy, etc)

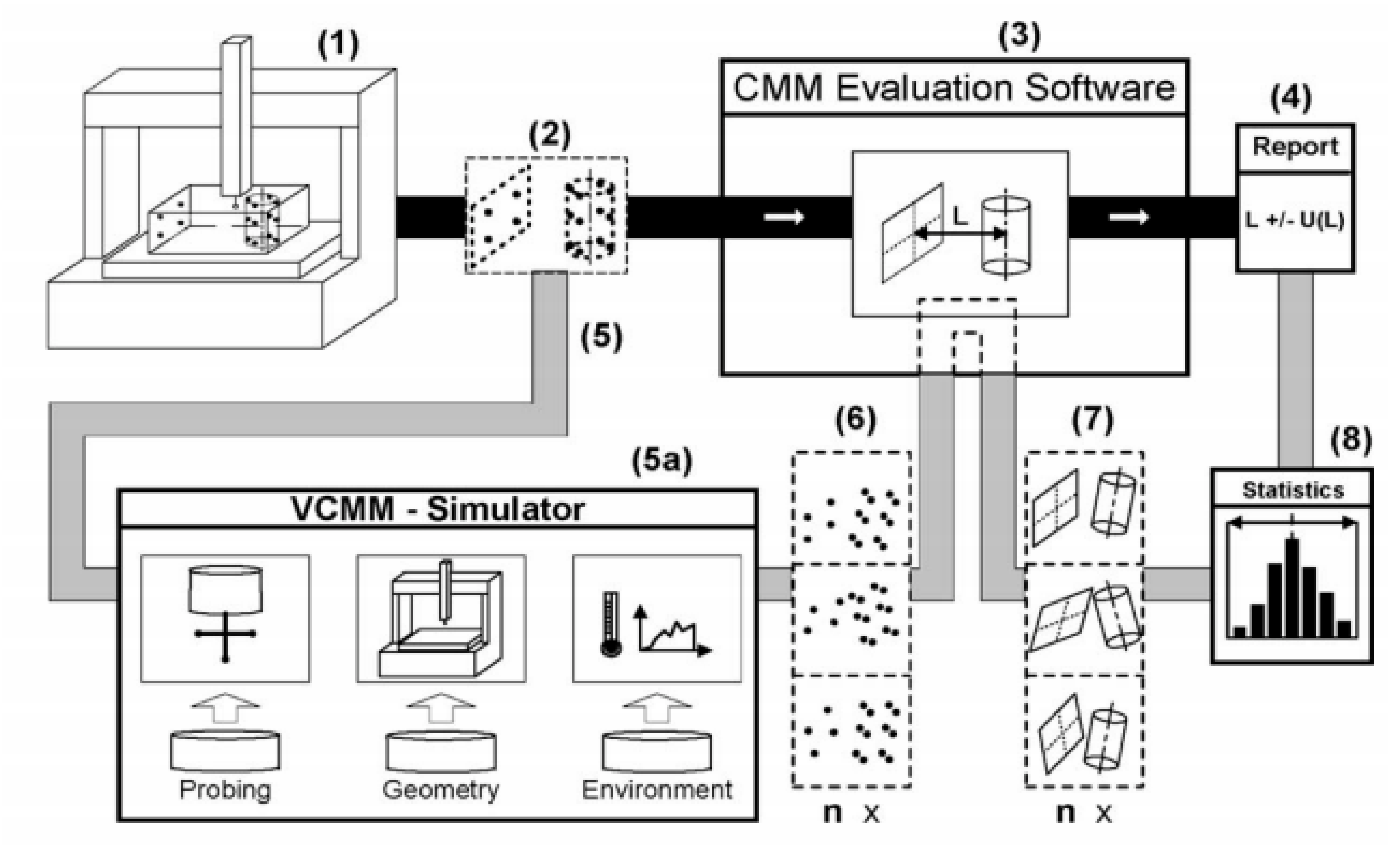
Disadvantages:

- The need of a calibrated artefact of the same size (length, diameter, freeform, etc) for each measurand (this will significantly increase the cost to estimate uncertainty)
- Very difficult for a free-form complex surface
- The cost is justified only for medium-to-large volume production

ISO DTS 15530-4 Monte Carlo method

The need of a method that is

1. Flexible
2. Cheap
3. Easy to apply
4. Less operator involvement
5. Fast

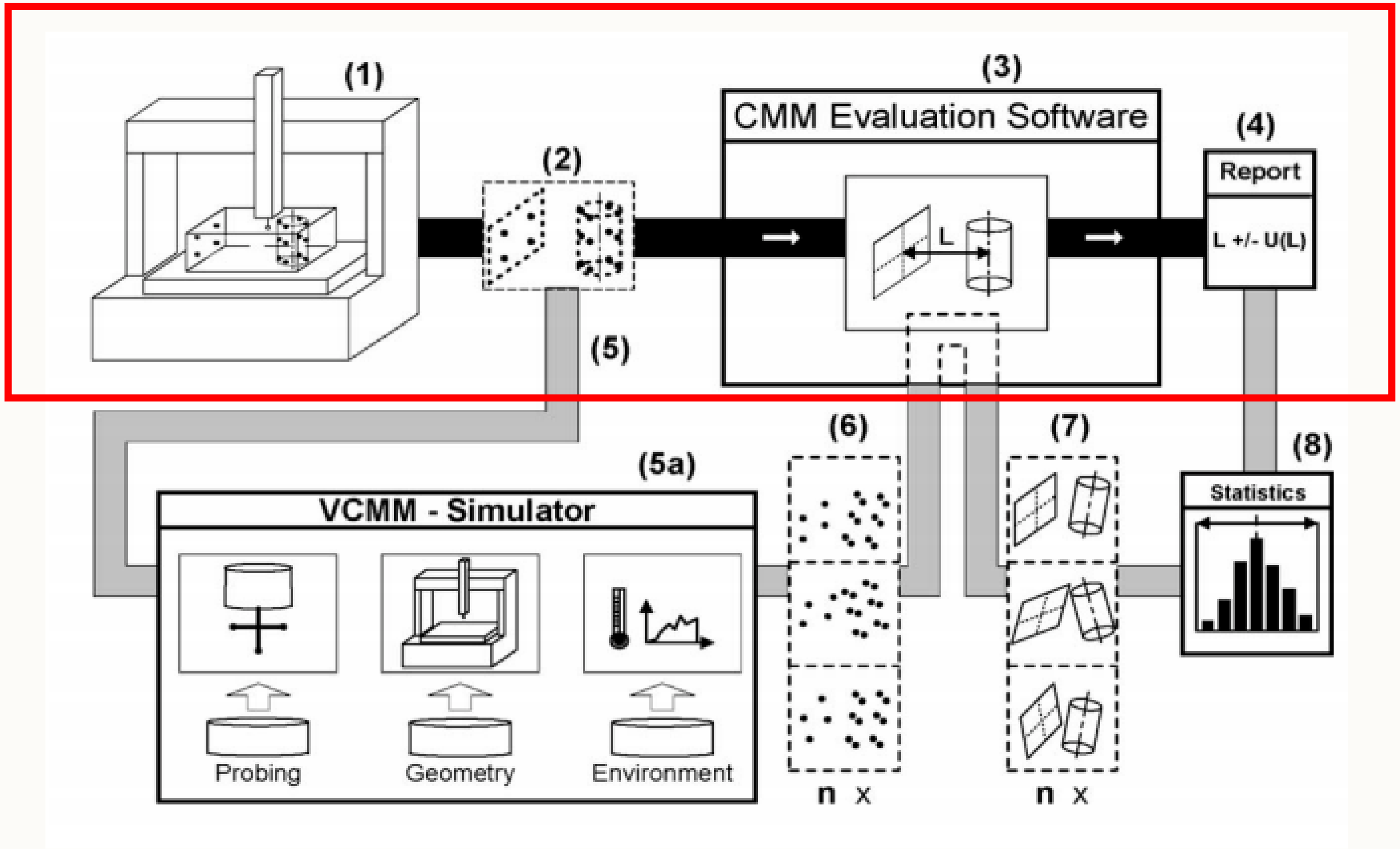


Wilhelm RG, Hocken R, Schwenke H 2011 Task specific uncertainty in coordinate measurement Annal. CIRP 50 553-563

ISO DTS 15530-4 Monte Carlo method

First stage

- A normal measurement by a CMS (contact or non-contact)
- E.g. measurement of flatness, perpendicularity, length, diameter
- The measurement result ***Y*** is taken

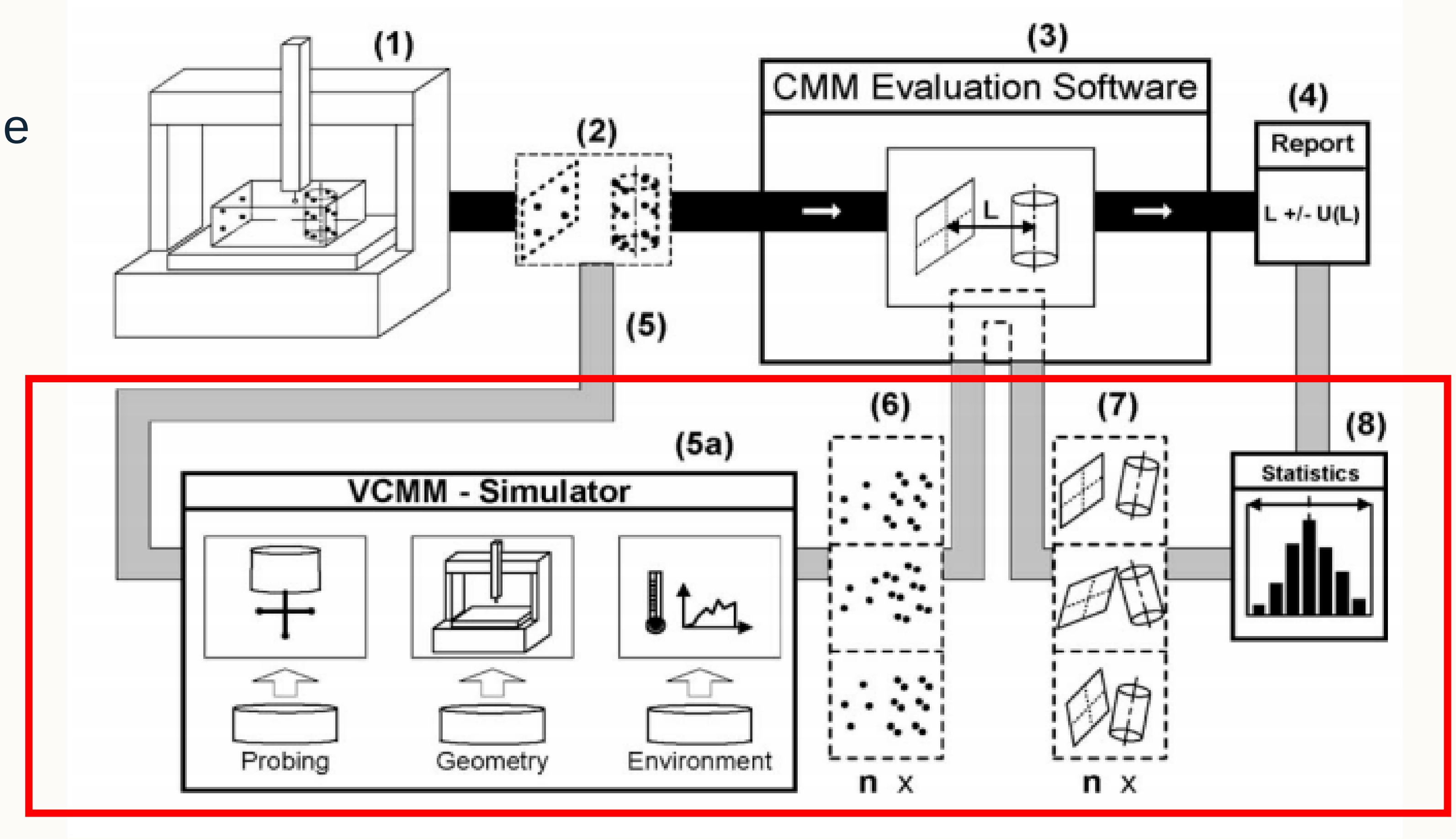


Wilhelm RG, Hocken R, Schwenke H 2011 Task specific uncertainty in coordinate measurement Annal. CIRP 50 553-563

ISO DTS 15530-4 Monte Carlo method

Second stage

- From the points obtained by the first measurement, the points are perturbed, considering the uncertainty contributors, and are processed with the same algorithm
- The points perturbation is repeated many times
- The uncertainty ***U*** by Monte-Carlo simulation is estimated



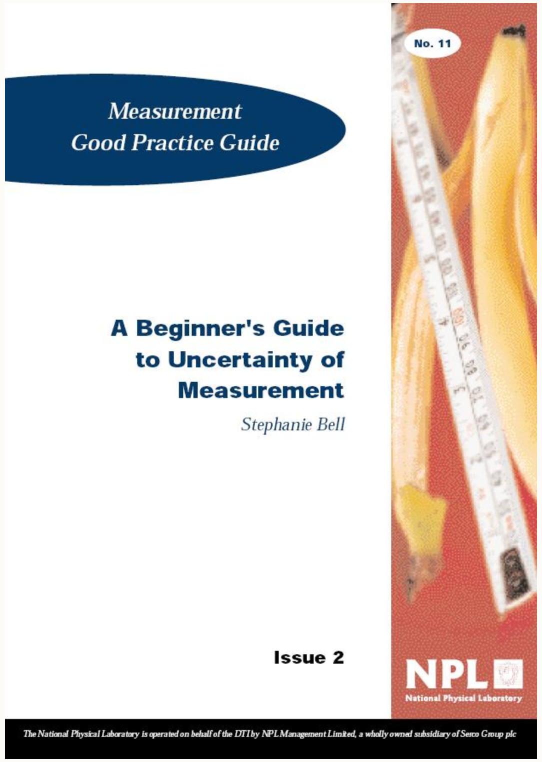
$$u_c = \sqrt{\mu_{sim}^2 + \sum_{i=1}^{n} \mu_i^2}$$

μ_i = Other uncertainty sources that are not considered in the simulation

Conclusions

- **Metrology** and importance of measurements
- **Measurement uncertainty**: definition and sources of uncertainty
- Measurement uncertainty methods:
  - **GUM**
  - **Spread sheet**
  - **Monte Carlo**
  - **ISO DTS 15530** series

References


No. 11

Measurement Good Practice Guide

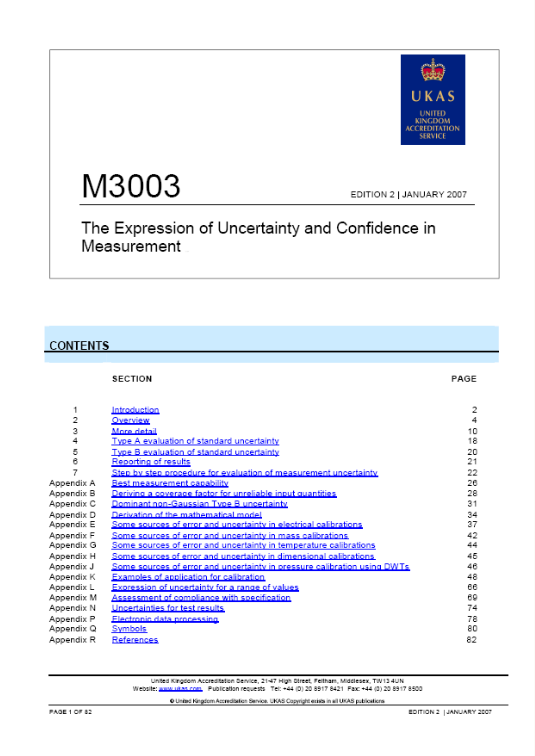
A Beginner's Guide to Uncertainty of Measurement

Stephanie Bell

Issue 2

NPL National Physical Laboratory

The National Physical Laboratory is operated on behalf of the DTI by NPL Management Limited, a wholly owned subsidiary of Serco Group plc


UKAS UNITED KINGDOM ACCREDITATION SERVICE

M3003 EDITION 2 | JANUARY 2007

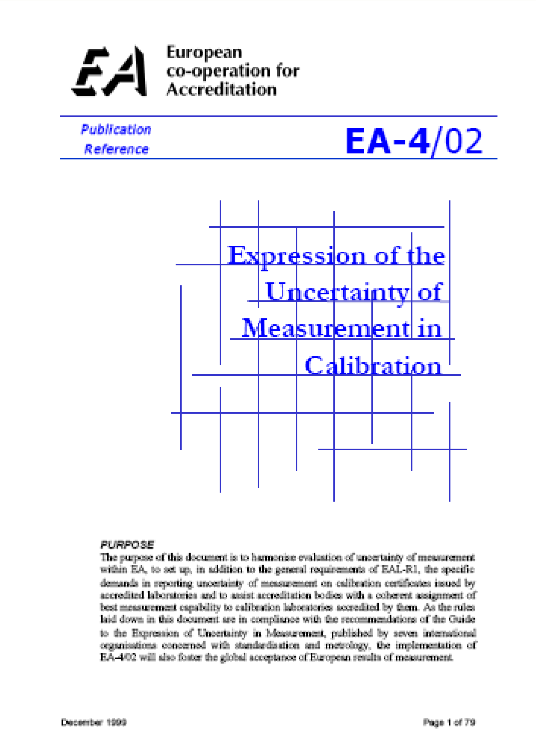
The Expression of Uncertainty and Confidence in Measurement

CONTENTS

United Kingdom Accreditation Service, 21-47 High Street, Feltham, Middlesex, TW13 4UN
Website: www.ukas.com Publication requests Tel: +44 (0) 20 8917 8421 Fax: +44 (0) 20 8917 8500

© United Kingdom Accreditation Service. UKAS Copyright exists in all UKAS publications

PAGE 1 OF 82 EDITION 2 | JANUARY 2007


EA European co-operation for Accreditation

Publication Reference EA-4/02

Expression of the Uncertainty of Measurement in Calibration

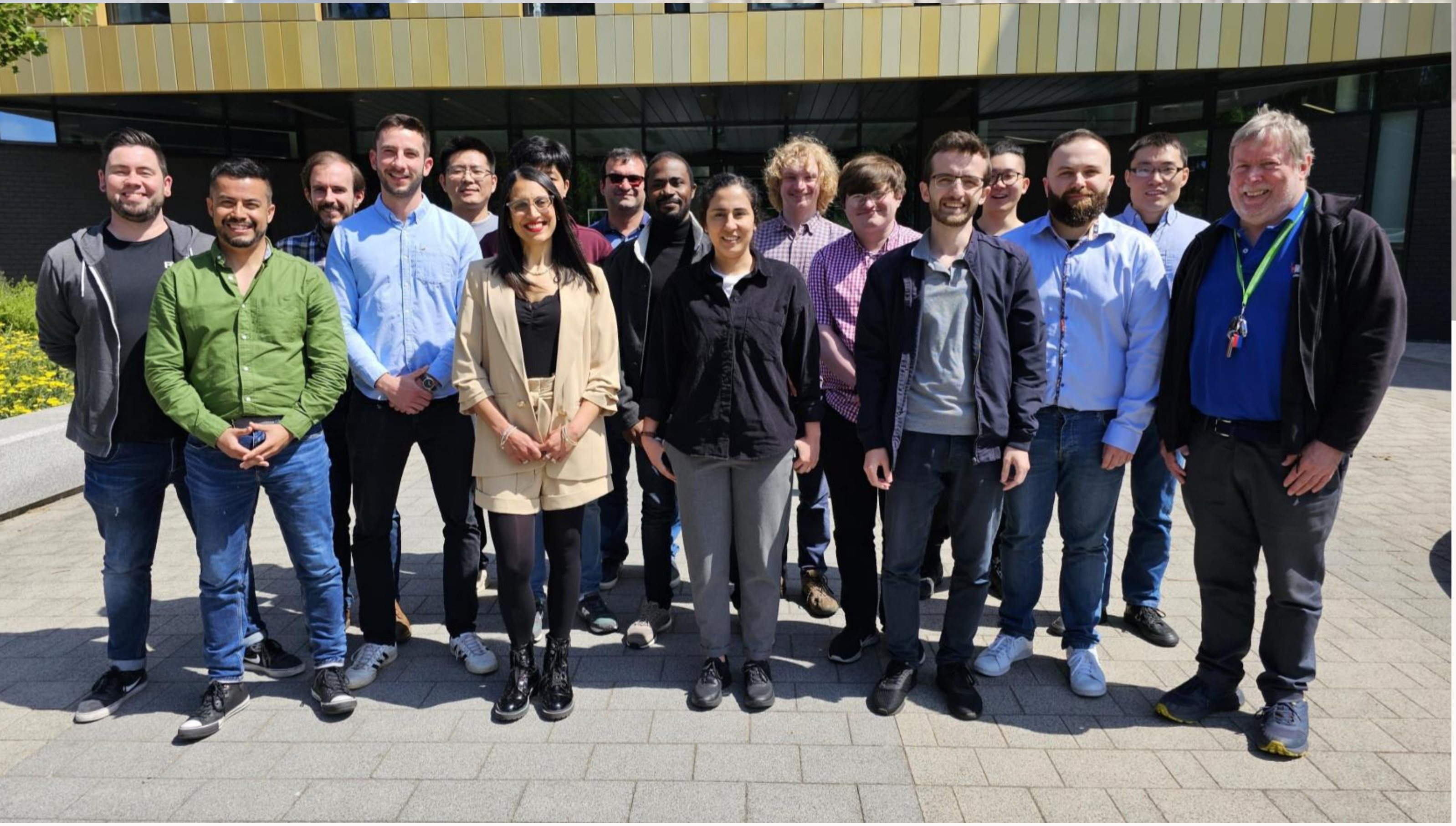
PURPOSE

The purpose of this document is to harmonise evaluation of uncertainty of measurement within EA, to set up, in addition to the general requirements of EAL-R1, the specific demands in reporting uncertainty of measurement on calibration certificates issued by accredited laboratories and to assist accreditation bodies with a coherent assignment of best measurement capability to calibration laboratories accredited by them. As the rules laid down in this document are in compliance with the recommendations of the Guide to the Expression of Uncertainty in Measurement, published by seven international organisations concerned with standardisation and metrology, the implementation of EA-4/02 will also foster the global acceptance of European results of measurement.

December 1999 Page 1 of 79

Manufacturing Metrology Team

Research focused on optimising and developing methods for **manufacturing metrology** to enable the development of more **sustainable** products.



- **AI-driven metrology**
- **In-process metrology**
- **Measurement uncertainty & traceability**
- **Sustainable metrology**
- **Quantum technology**

https://www.nottingham.ac.uk/research/groups/advanced-manufacturing-technology-research-group/research/manufacturing-metrology-team/index.aspx

University of Nottingham
UK | CHINA | MALAYSIA

Thanks for listening

Any question?