National Aeronautics and Space Administration

Ames Discovery Innovations Solutions

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Effectively Leveraging NISQ Hardware for Quantum Simulations of Gauge Theories

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University of Tokyo and LBNL minisymposium

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Motivation

- Quantum Computers offer the possibility to tackle problems intractable by classical computers.
  - Time dynamics of quantum field theories
  - Study of electronic structure and reactions in chemistry
  - Solving traditionally hard optimization problems
- For time dynamics of physical systems classical methods encounter memory bottlenecks or sign problems.

Simulations of Compact Scalar QED

Why physics has a qudit bias:

[arXiv:2201.04546,](https://arxiv.org/abs/2201.04546)

Phys. Rev. D 103, 114505 (2021)

1+1d Scalar QED

- Two degrees of freedom
  - Photon field (U)
  - Compact Scalar $\phi = R\, e^{i\theta}$
- 1 dimensional system as initial study
  - 2d and 3d systems also possible
- Closely related to O(2) model

- Action:

$$S = S_{gauge} + S_{matter}$$

$$S_{gauge} = -\frac{1}{g^2 a_s a_\tau} \sum_x \sum_{\nu<\mu} ReTr\left(U_{x,\mu\nu}\right)$$

$$S_{matter} = -k_s \sum_x \phi^+ U_{x,s} \phi_{x+\hat{s}} + h.c. +$$

$$-k\_\tau \sum_x \phi_x^+ U_{x,\tau} \phi_{x+\hat{\tau}} + h.c$$

Time continuum limit and the Hamiltonian

$$\hat{H} = \frac{U}{2}\sum_i \left(\hat{L}_i^z\right)^2 + \frac{Y}{2}\sum_i \left(\hat{L}_i^z - \hat{L}_{i+1}^z\right)^2 + \frac{Y}{2}\left(\left(\hat{L}_1^z\right)^2 + \left(\hat{L}_N^z\right)^2 - X\sum_i \hat{U}_i^x\right.$$

Electric Field

$$\hat{L}^z = \sum_n n\,|n\rangle\langle n|$$

Magnetic Field

$$\hat{U}^x = \frac{1}{2}\left(\hat{U}^+ + \hat{U}^-\right) = \sum_n \frac{1}{2}\left(|n\rangle\langle n+1| + |n+1\rangle\langle n|\right)$$

Phys. Rev. D 98, 094511 (2018); Phys. Rev. D 92, 076003 (2015); PoS(LATTICE 2015)285

Time continuum limit and the Hamiltonian

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These operators are formally infinite dimensional

Electric Field

$$\hat{L}^z = \sum_n n\,|n\rangle\langle n|$$

Magnetic Field

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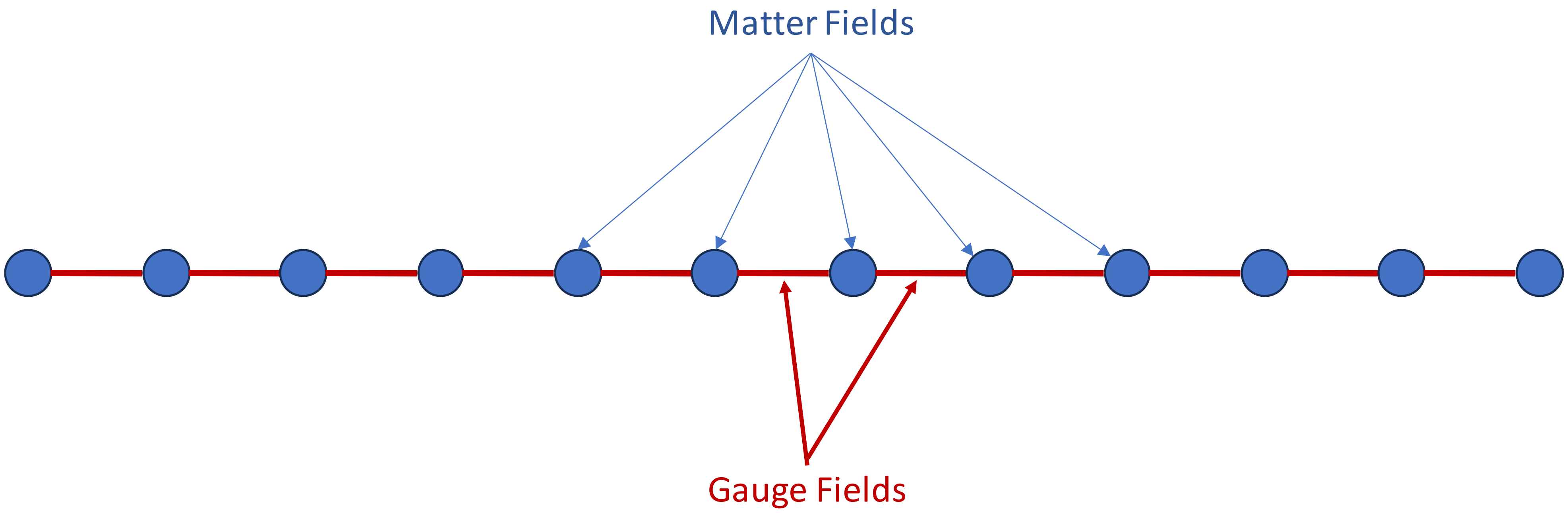
Model physically

$$\hat{H} = \frac{U}{2}\sum_i \left(\hat{L}_i^z\right)^2 + \frac{Y}{2}\sum_i \left(\hat{L}_i^z - \hat{L}_{i+1}^z\right)^2 + \frac{Y}{2}\left(\left(\hat{L}_1^z\right)^2 + \left(\hat{L}_N^z\right)^2\right) - X\sum_i \hat{U}_i^x$$



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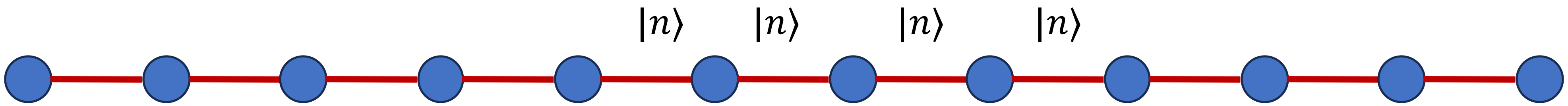
Measures Electric Field



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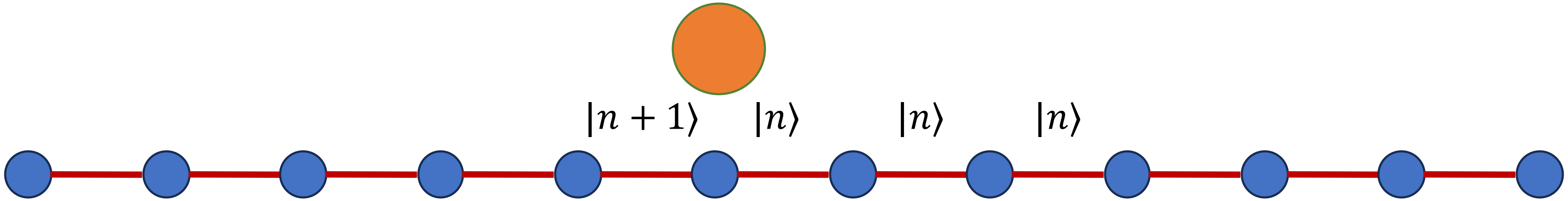
Mass term



Model physically

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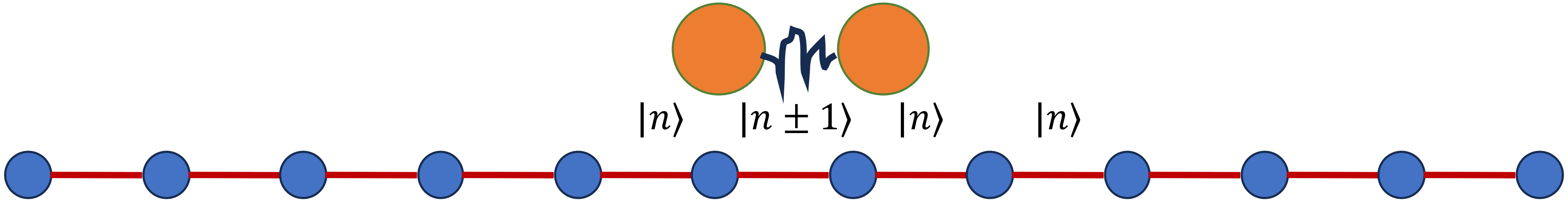
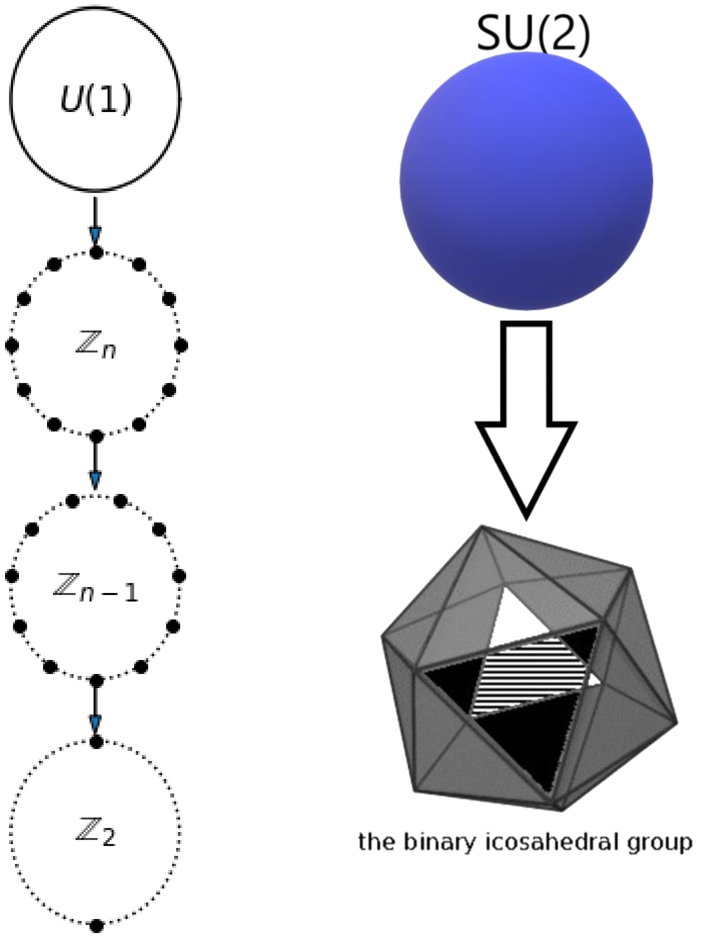
Hopping / Pair creation term



How does one regulate the field degrees of freedom?

- We regulate infinite degrees of freedom by approximating the theory with some truncation
  - Electric Field magnitude
  - Group Space Decimation
  - Quantum Links
  - Loop-String-Hadron
  - Other Novel methods



Constructing Gates

Qubits

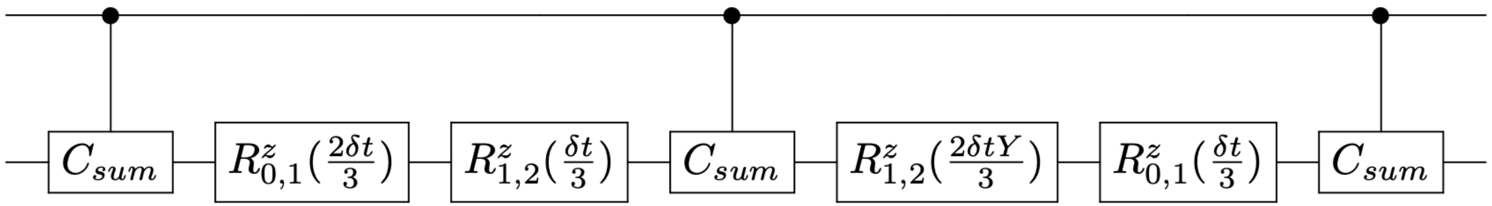
- Pros:
  - Easy to control
  - Lots of resources already built up
- Cons:
  - Possibly wasted Hilbert space
  - Moderately high interconnectivity required

Qudits

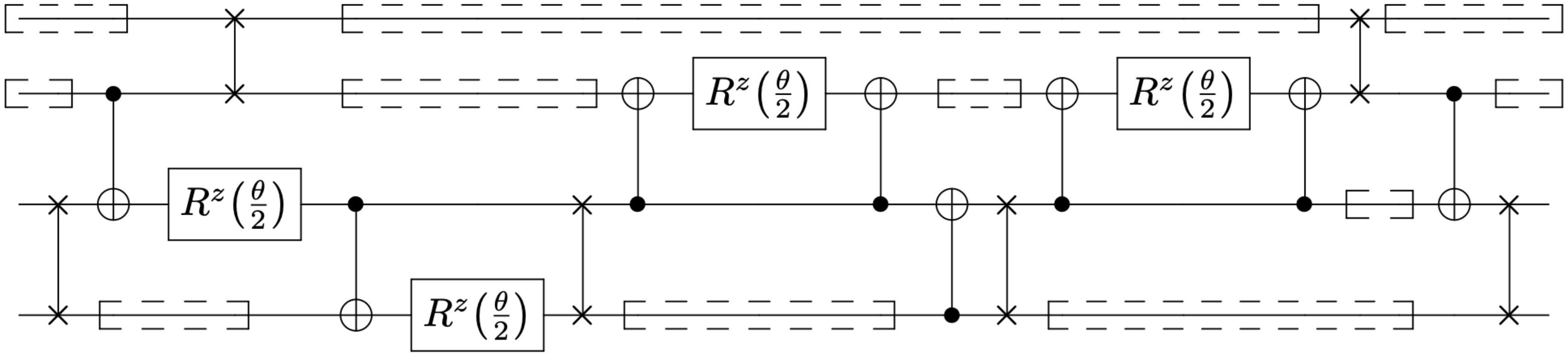
- Pros:
  - Hilbert space maps to hardware space
  - Gate operations more intuitive / physically motivated
- Cons:
  - Harder to control (Engineering)
  - More noisy

Constructing Gates: (Lz – Lz)

$\hat{\mathcal{U}}_{zz}^{qutrit}(\delta t) =$

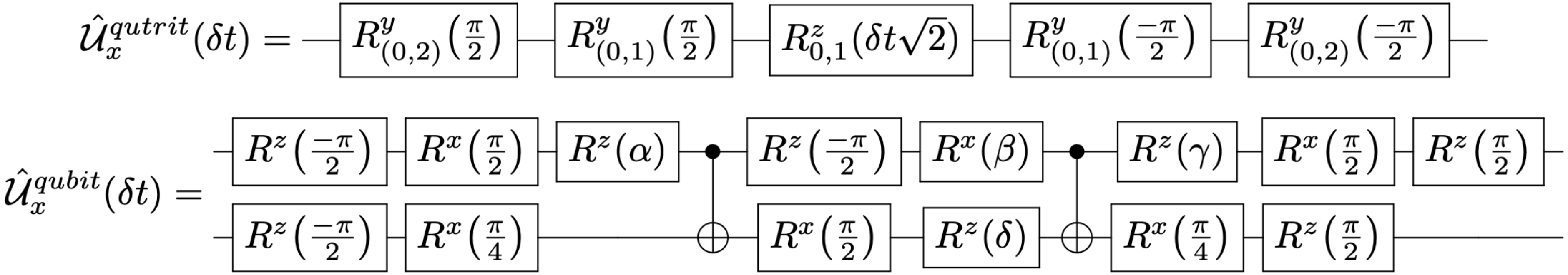


$\hat{\mathcal{U}}_{zz}^{qubit}(\delta t) =$



From Gustafson arXiv:2201.04546

Constructing Gates: Ux



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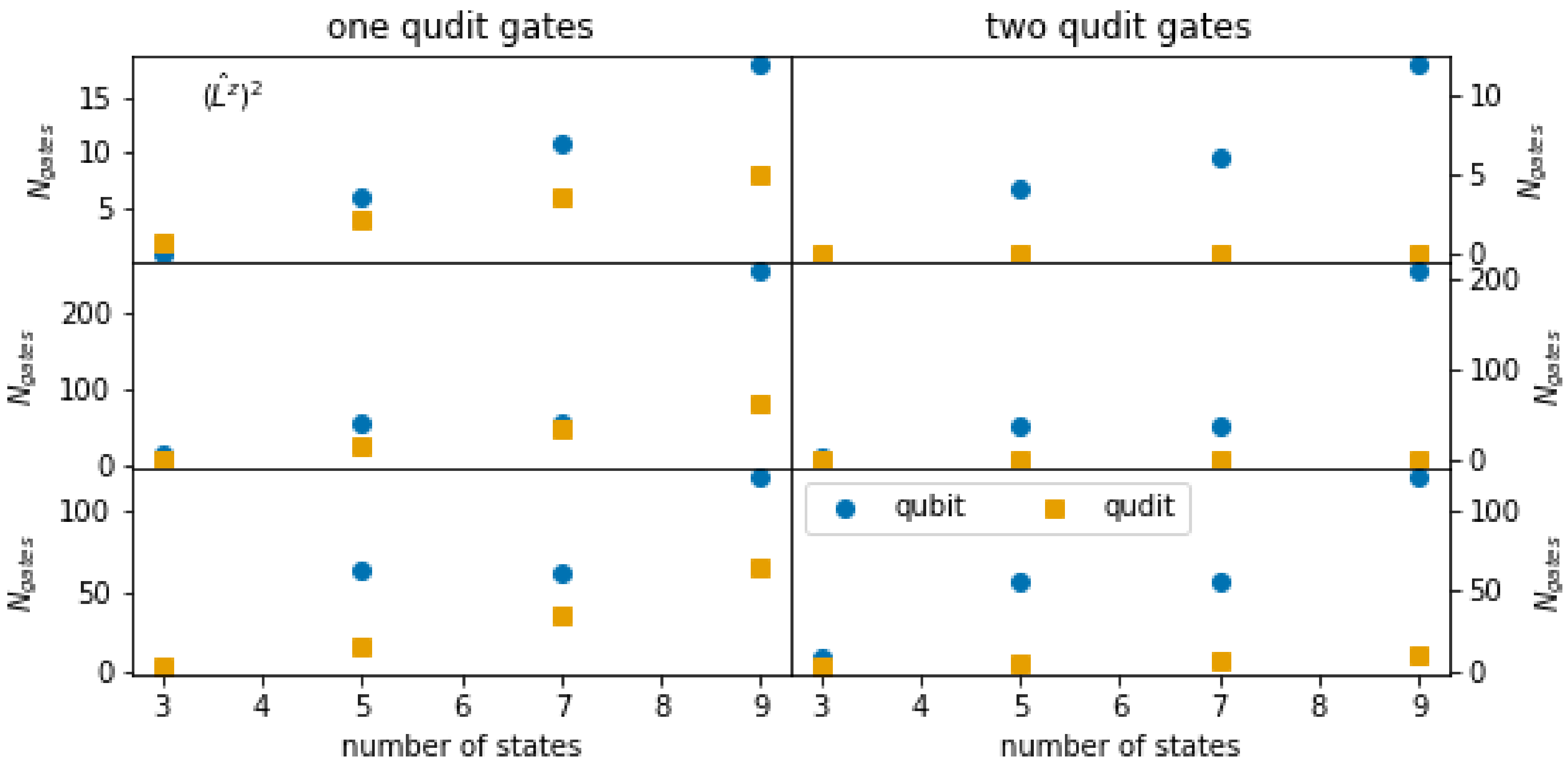
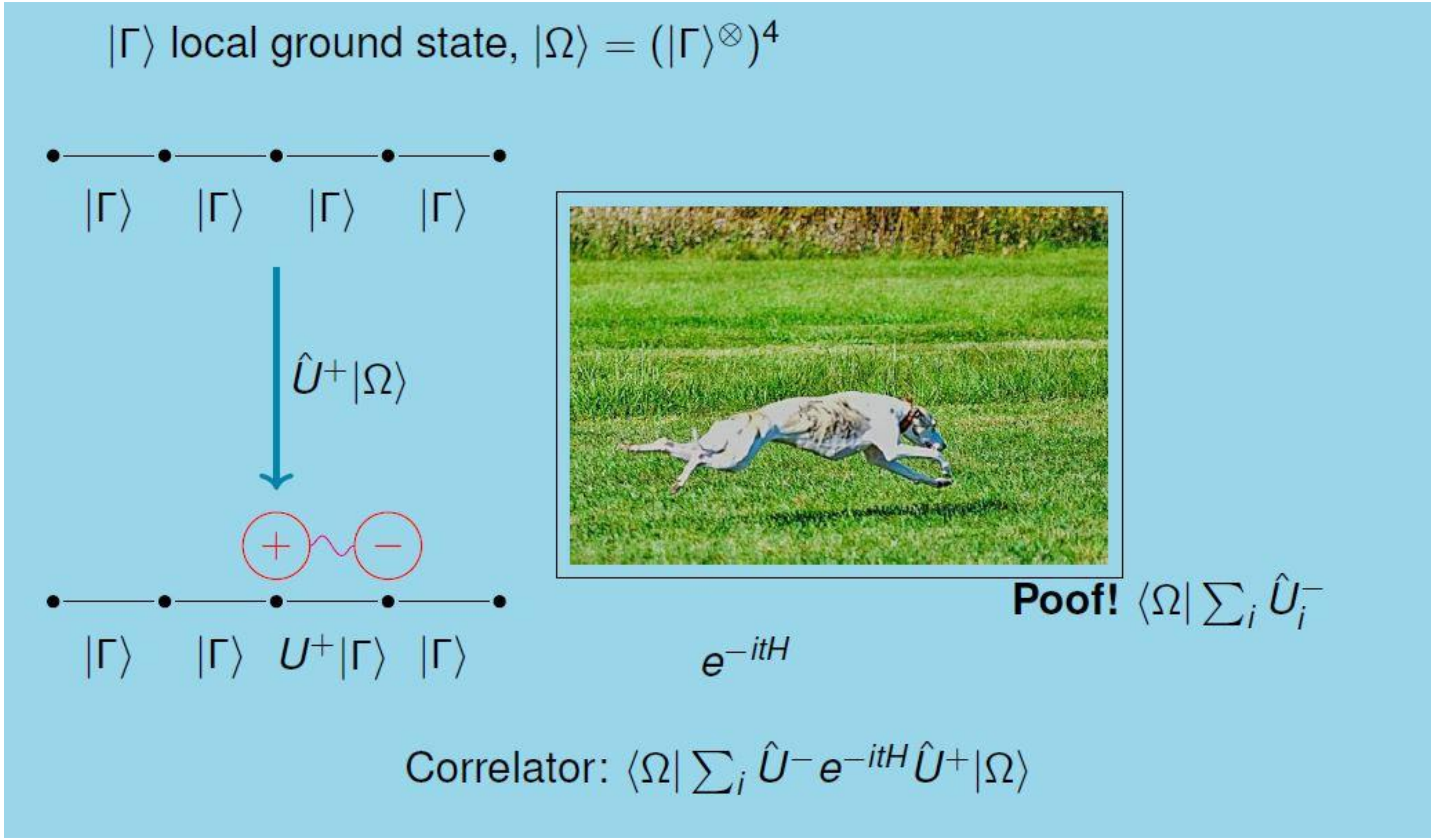
Extensions to larger truncations is doable

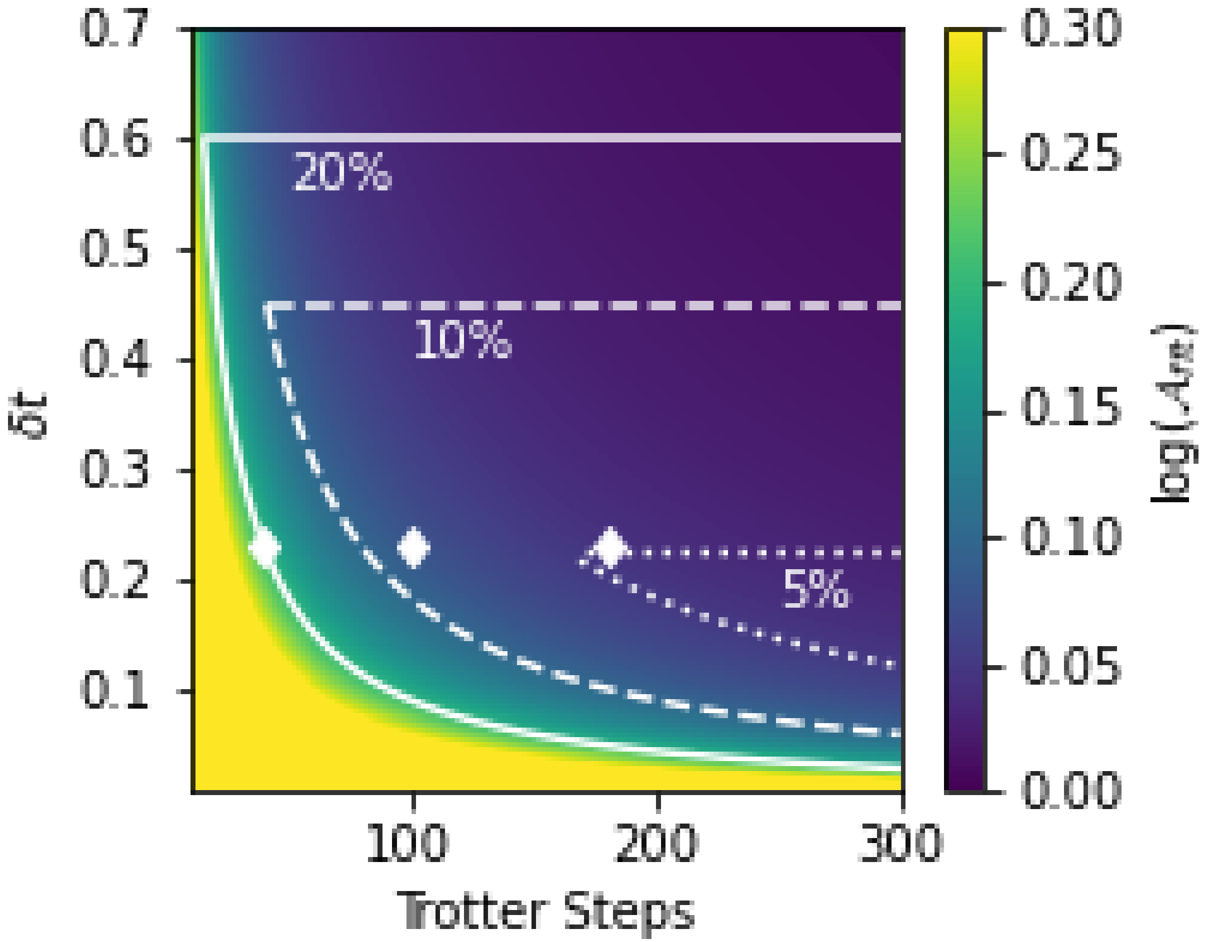


Fiducial simulation: two-point correlator

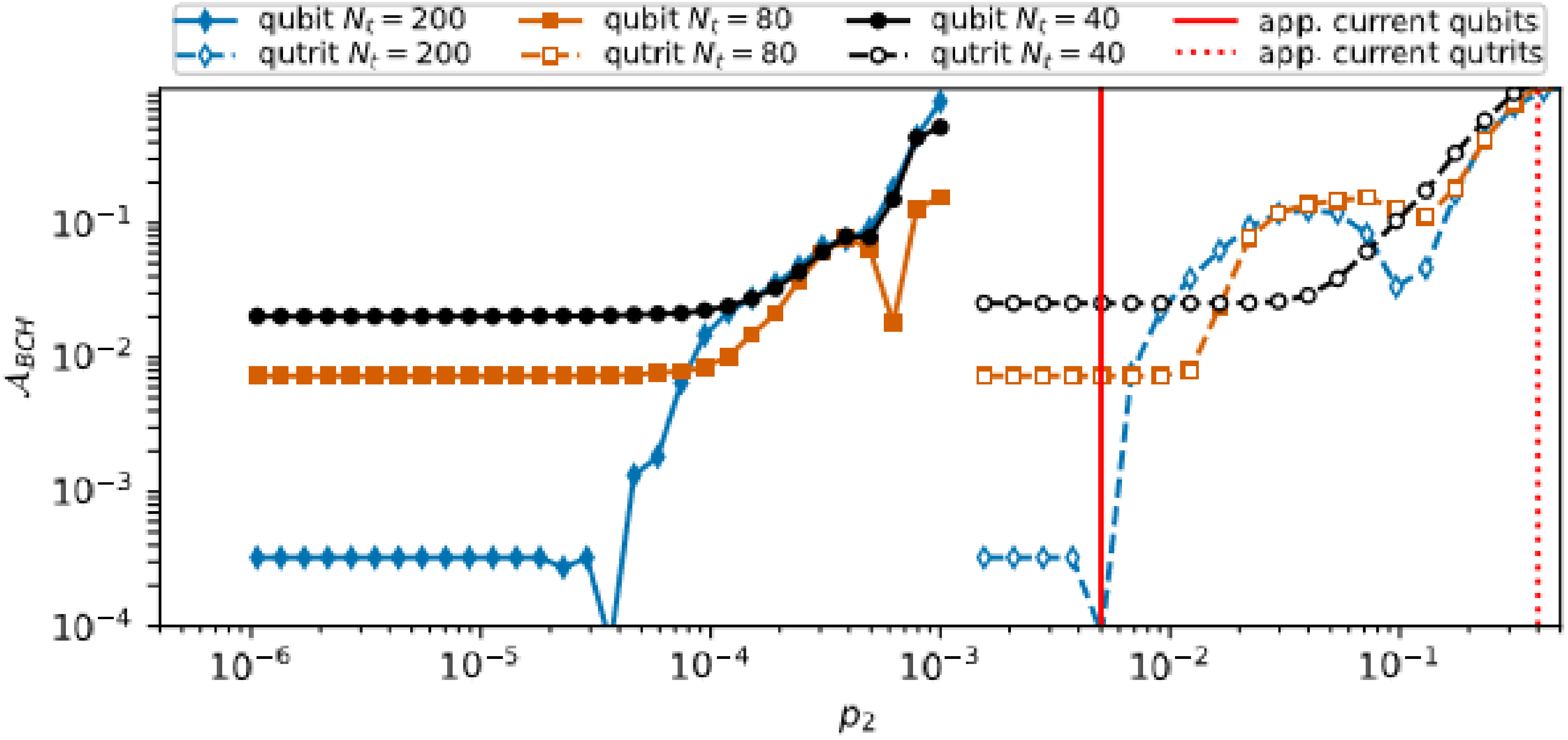


Parameter Selection

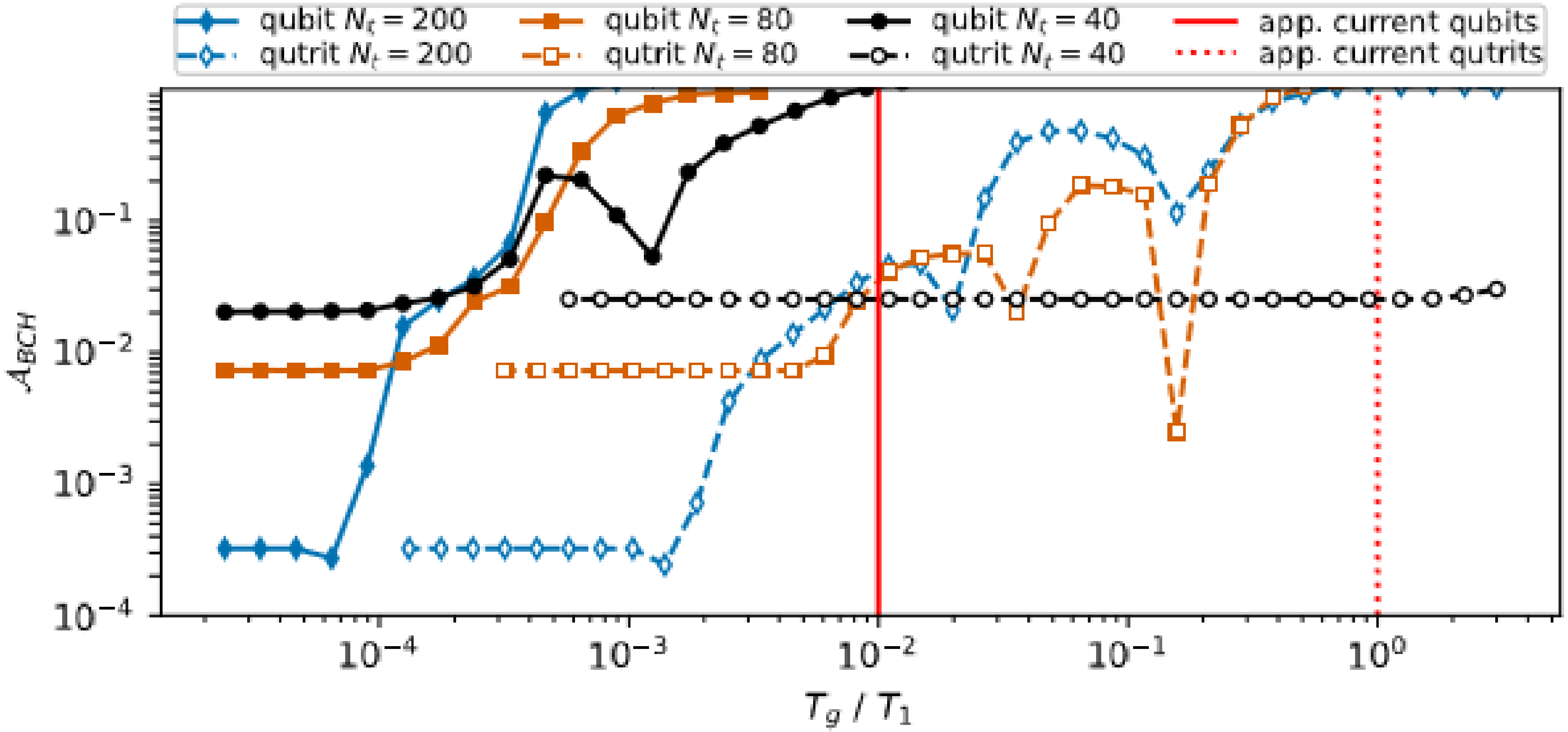
- $G^2a = 5$
- 4 Sites
- dt = 0.235
- Truncate to 3 states



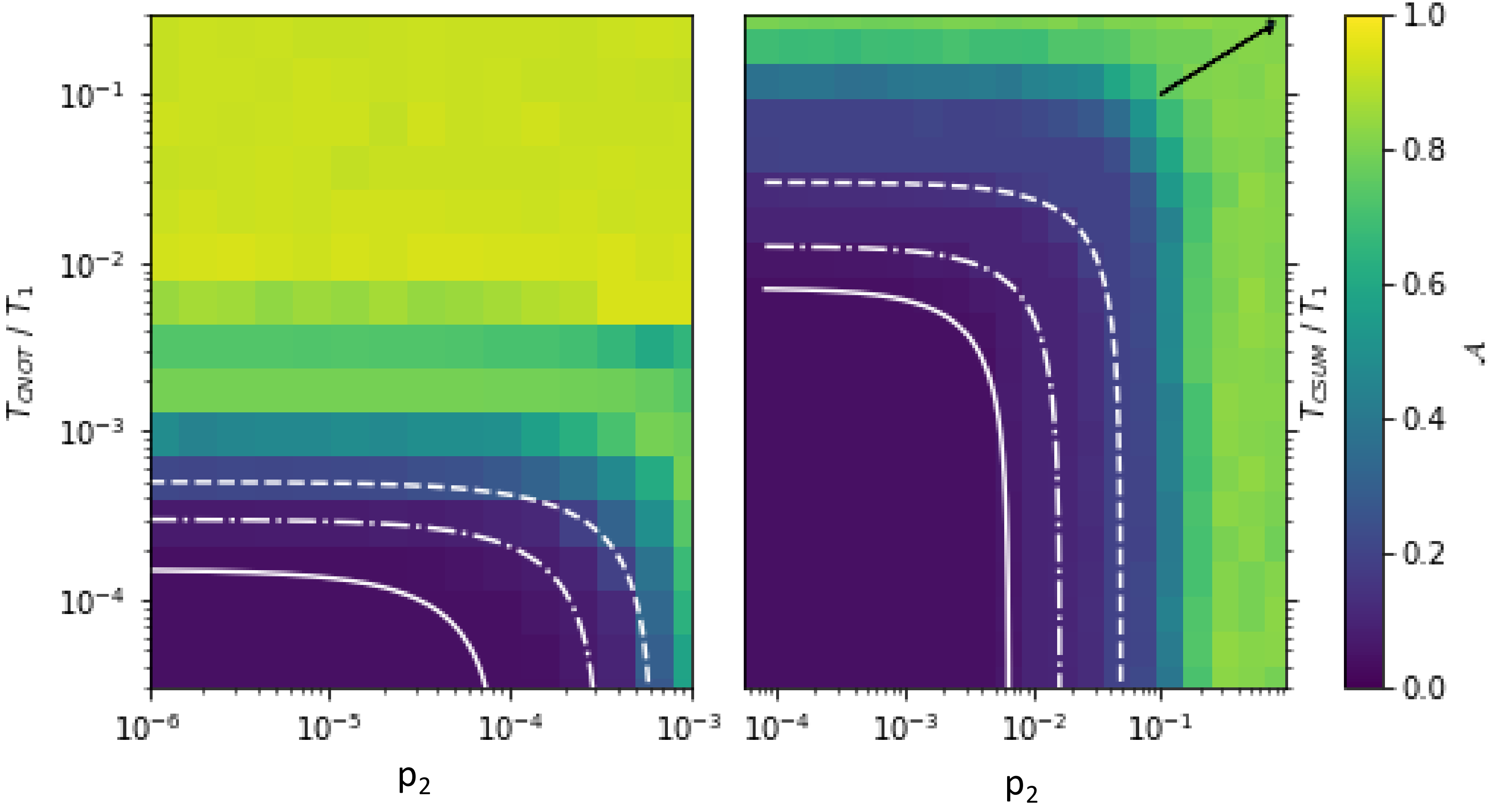
Accuracy versus Two qubit Pauli Error p2



Accuracy versus Amplitude damping time, T1



Combined Comparison



So where could we go from here?

- If we are looking toward QCD
  - Use both qubits and qudits, fermions and gauge fields
  - Use a qudit to describe all color / spin indices for quarks.
- Pure gauge theories
  - Use qudits to help represent Hilbert space
- Could we use non-error corrected qubit-qudits to perform some smaller computation quickly that can be extrapolated to large scale fault-tolerant qubit hardware.

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Leveraging error mitigation strategies to improve quantum simulations

Gustafson et al. “Simulating Z_2 Lattice gauge theory on a quantum computer”
arXiv:2305.02361

Collaborators

Elizabeth Hardt:
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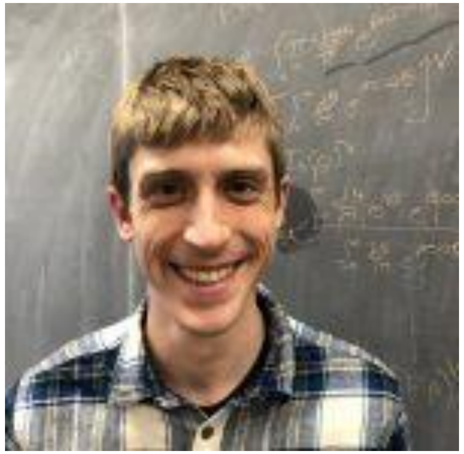
Sara Staracheski:
Sarah Lawrence College

Hank Lamm

Ruth van Der Water

Clement Charles
University of the West Indies

Norman Hogan:
North Caroline State University



Mike Wagman

Noise will ruin a quantum simulation

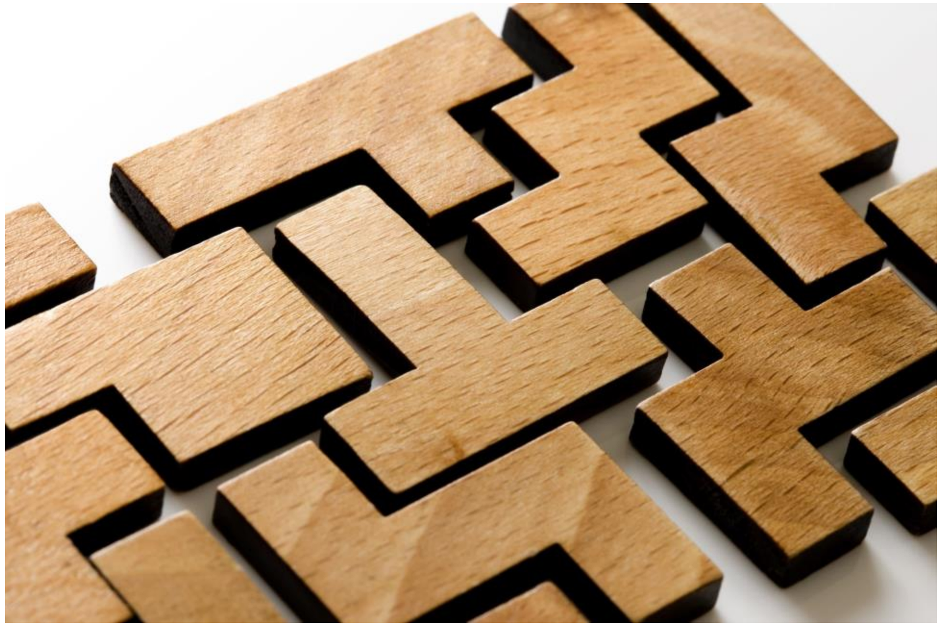
- Environmental couplings
- Gate imperfections
- Readout errors

Noise will ruin a quantum simulation

- ~~Environmental couplings~~
  - Dynamic Decoupling
- ~~Gate imperfections~~
  - Randomized Compiling
  - Zero Noise Extrapolation
  - Improved Tuning
- ~~Readout errors~~
  - Whole Suite of post processing tools

Other fields have started leveraging error mitigation

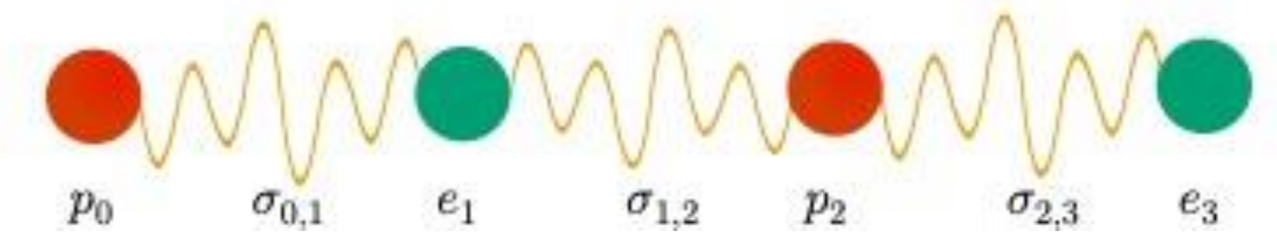
- Quantum Simulations for particle physics is lagging behind other fields.
- We need to have a set of bare minimum best practices for quantum simulations
  - Caveat: different hardware has different requirements
- We need to understand how each mitigation technique will interact with each other piece and the simulation as a whole.



Z_2 Gauge theory in (1+1)d

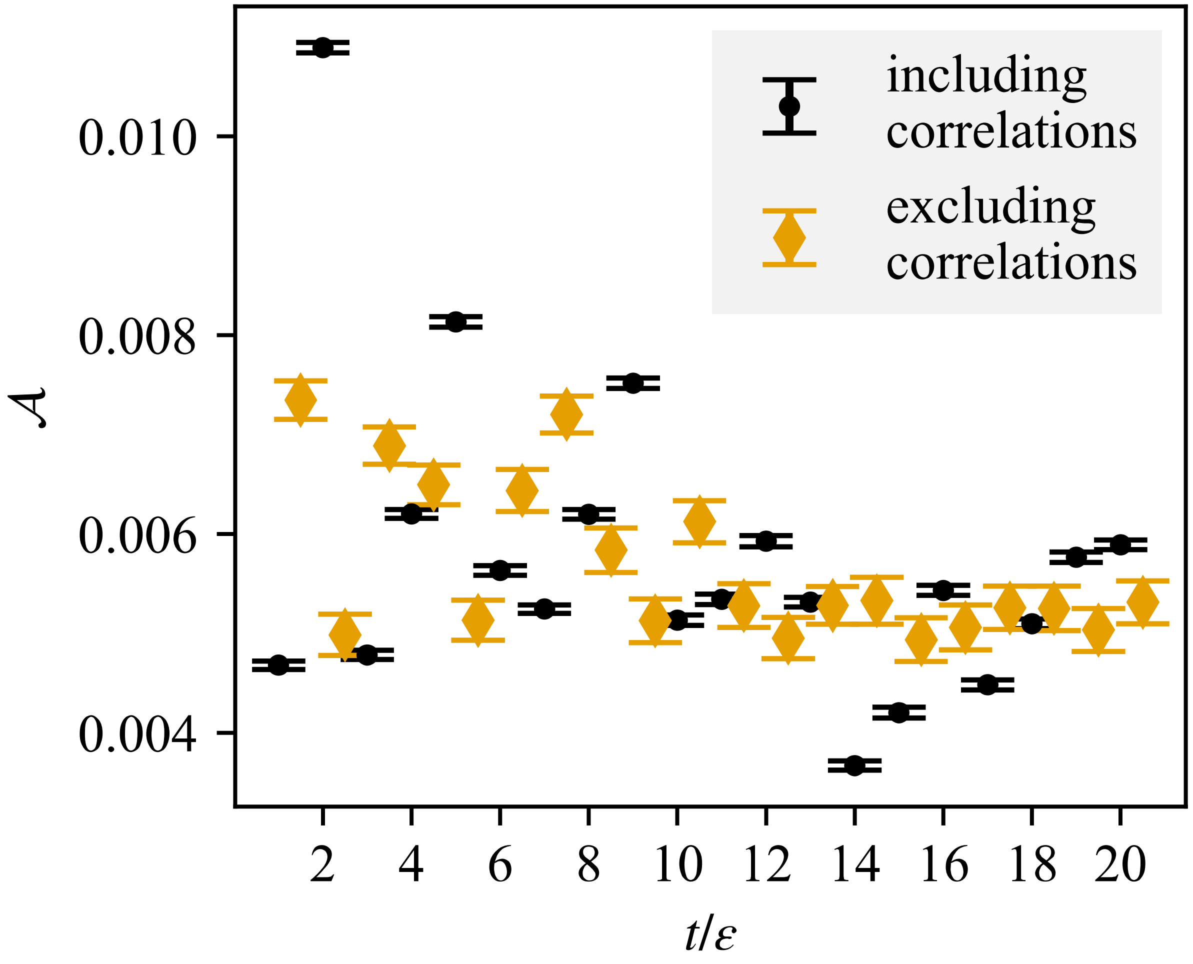
- Nice toy model that maps cleanly to qubit based hardware
- Has qualitative behaviors similar to Schwinger model
- Has explicit gauge fields which will be important in 2 and 3d simulations
- We want to measure: $C(t) = \langle\Omega|S^{+}(t)S(0)|\Omega\rangle$

$$\mathrm{H} = \frac{g^2}{2}\sum_{i=1}^{N_s-1}\sigma_i^x + \frac{a_s}{2}\sum_{i=1}^{N_s-1}\bar{\psi}_i\sigma_i^z\psi_{i+1} + h.c. + a_s m_0\sum_{i=1}^{N_s}(-1)^i\,\bar{\psi}_i\psi_i$$

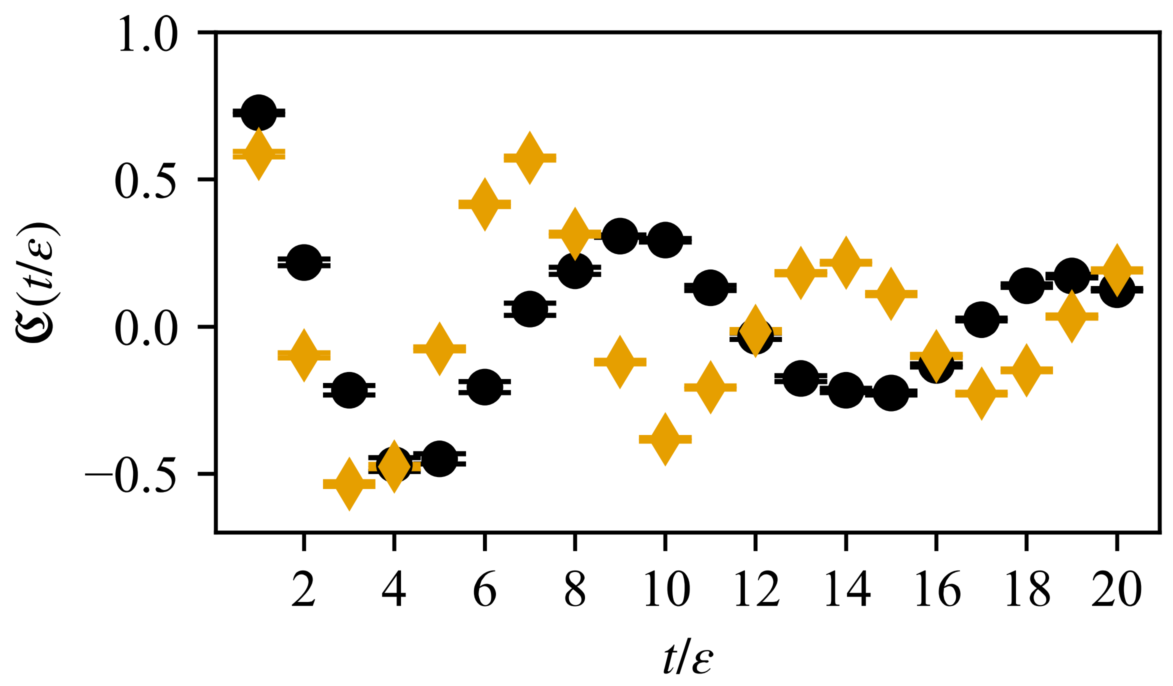


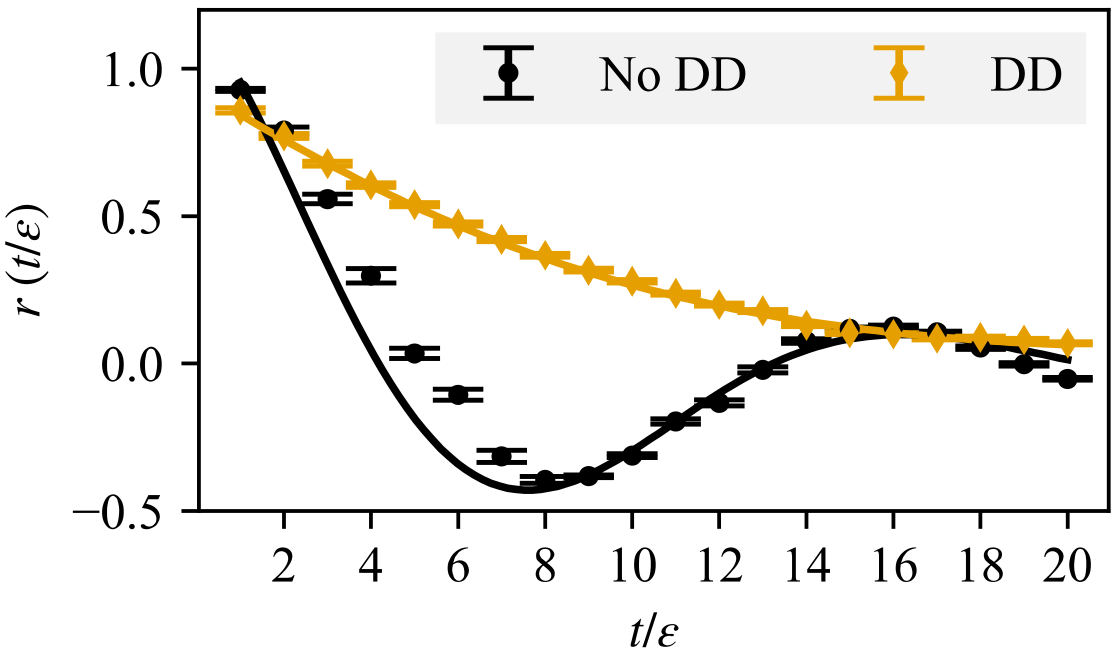
Readout Error Mitigation Induces Correlations

- $\tilde{B}\ M^{-1} \rightarrow B$
  - Raw bit string, $\tilde{B}$, is passed through an *estimated* filter M
- Figure to the right shows the errors on the absolute shift by neglecting and including correlations.
- If we want precision calculations these correlations can be important.

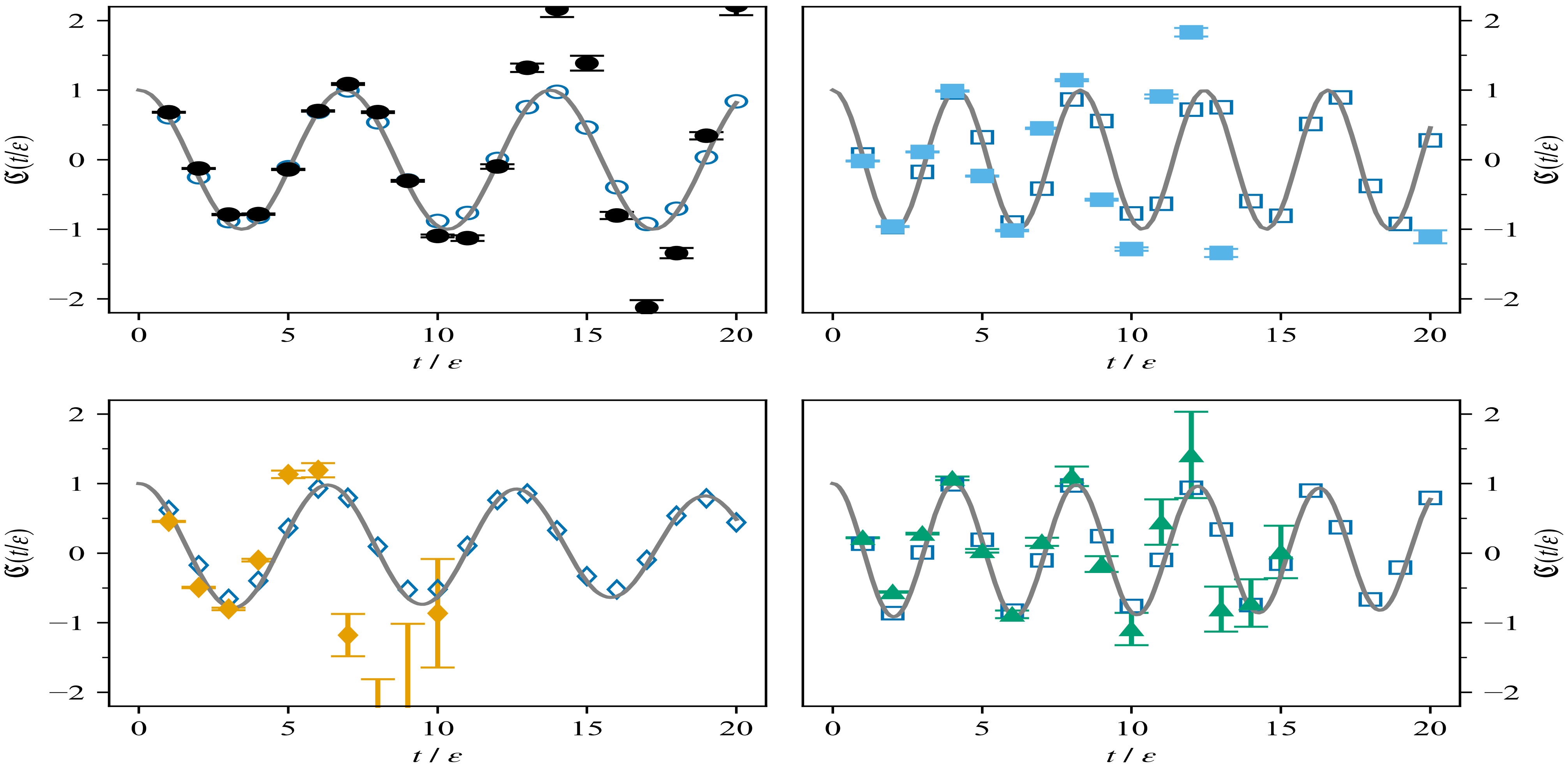


Dynamic decoupling removes oscillations



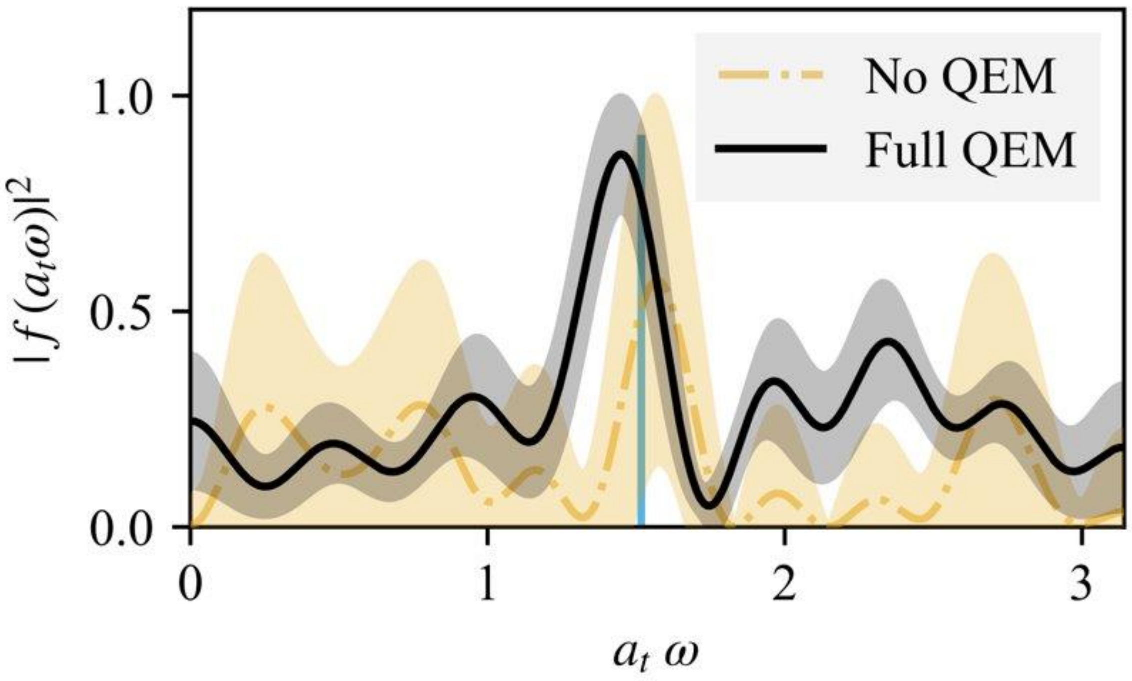


Putting all the pieces together



Fourier Spectrum Example

- Error Mitigation allows a resolvable signal on the Fourier Spectrum
- Ringing is an artifact of interpolation
- Error mitigation is crucial.



Outlook and future?

- Need to quantify systematic errors for precision calculations
  - Address bias from gate and machine errors
  - Understand possible correlations induced by machine errors
  - Effects of inexact state preparation (VQE, Adiabatic)
- Can we use gauge symmetries to perform some level of error correction or mitigation?
- Are error mitigation techniques going to be scalable or how do we make them scalable for quantum simulations?

Physics has a qudit bias

Error mitigation is technical and important

Acknowledgements: Simulations of Compact Scalar QED

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