Status of phenomenological studies of GPDs

Paweł Sznajder
National Centre for Nuclear Research, Poland

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Outline

- Introduction
- Experimental campaign
- Recent progress
- New sources of GPD information
- Numerical tools
- Summary

Introduction

Nucleon is not a point-like particle, it is made out of partons:

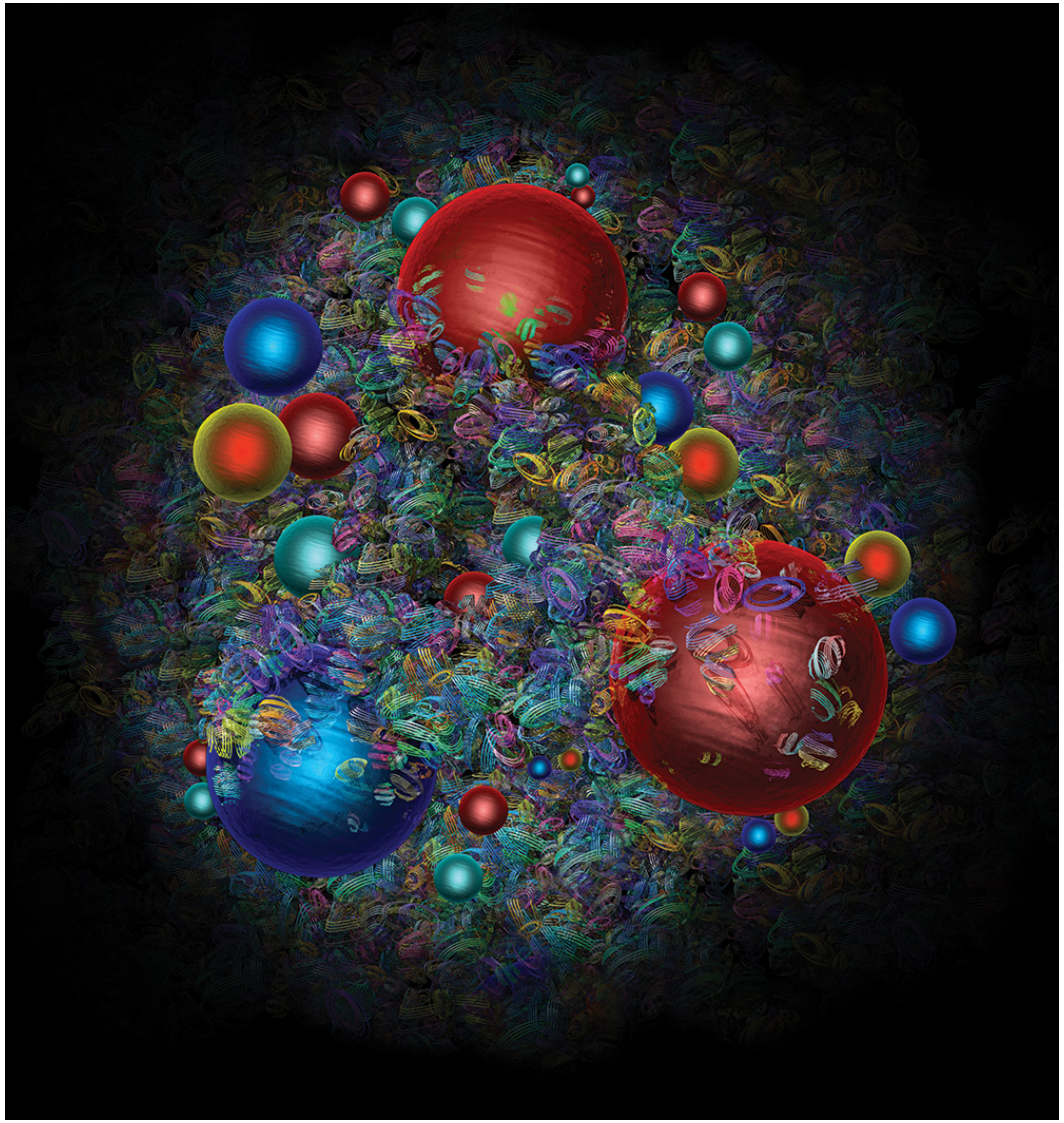
- quarks
- gluons

How can we recover basic properties of nucleon from those of its constituents?

- charge
- spin
- mass

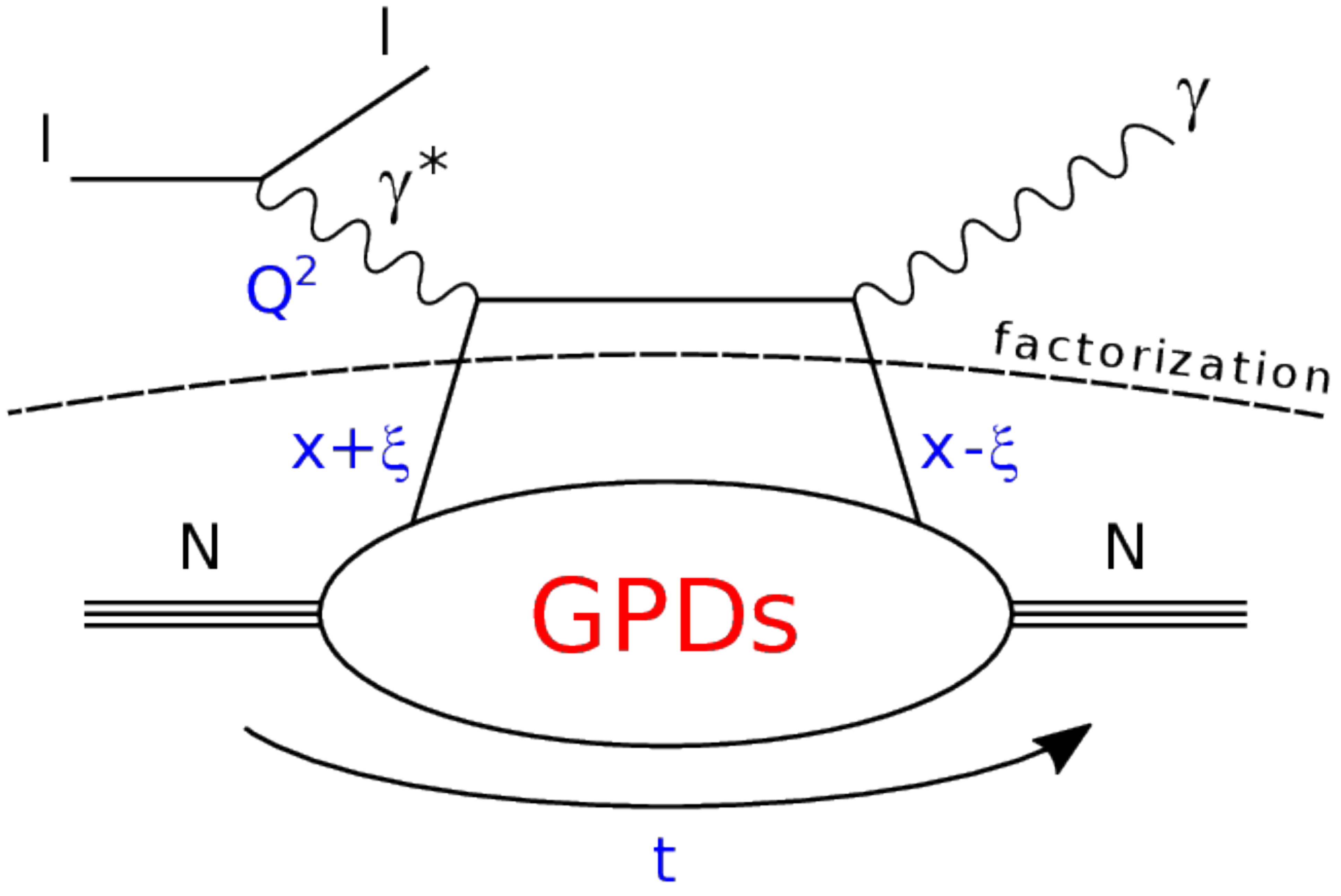
How partons are distributed inside nucleon?

- momentum (longitudinal and transverse)
- position
- polarisation
- "mechanical" properties



from CERN Courier / D. Dominguez

Deeply Virtual Compton Scattering (DVCS)



factorization for |t|/Q² « 1

Chiral-even GPDs:
(helicity of parton conserved)

$H^{q,g}(x,\xi,t)$	$E^{q,g}(x,\xi,t)$	*for sum over parton helicities*
$\widetilde{H}^{q,g}(x,\xi,t)$	$\widetilde{E}^{q,g}(x,\xi,t)$	*for difference over parton helicities*
nucleon helicity conserved	*nucleon helicity changed*	

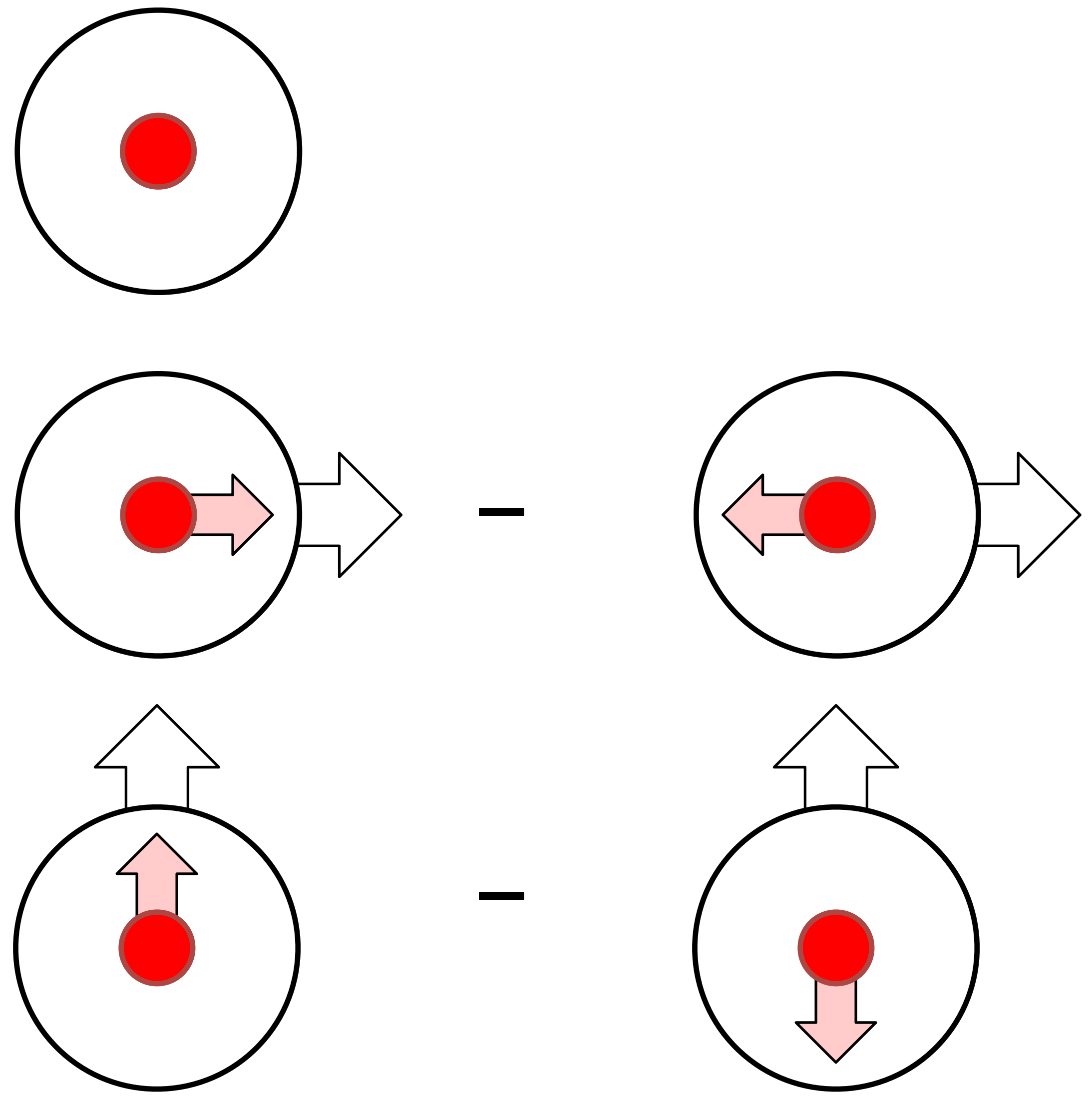
Introduction

- Reduction to PDFs:

$$H^q(x,0,0) \equiv q(x)$$

$$\widetilde{H}^q(x,0,0) \equiv \Delta q(x)$$

$$H_T^q(x,0,0) \equiv h_1(x)$$



no corresponding relations exist for other GPDs

- Reduction to Elastic Form Factors (EFFs):

$$\int_{-1}^{1} dx\, H^q(x,\xi,t) \equiv F_1^q(t) \qquad \int_{-1}^{1} dx\, E^q(x,\xi,t) \equiv F_2^q(t)$$

$$\int_{-1}^{1} dx\, \widetilde{H}^q(x,\xi,t) \equiv g_A^q(t) \qquad \int_{-1}^{1} dx\, \widetilde{E}^q(x,\xi,t) \equiv g_P^q(t)$$

Polynomiality - non-trivial consequence of Lorentz invariance:

$$\int_{-1}^{1} \mathrm{d}x\ x^n H^q(x,\xi,t) = h_0^{q,n}(t) + \xi^2 h_2^{q,n}(t) + \ldots + \mathrm{mod}(n,2)\xi^{n+1} h_{n+1}^{q,n}(t)$$

$$\int_{-1}^{1} \mathrm{d}x\ x^n \widetilde{H}^q(x,\xi,t) = \tilde{h}_0^{q,n}(t) + \xi^2 \tilde{h}_2^{q,n}(t) + \ldots + \mathrm{mod}(n+1,2)\xi^{n} \tilde{h}_{n}^{q,n}(t)$$

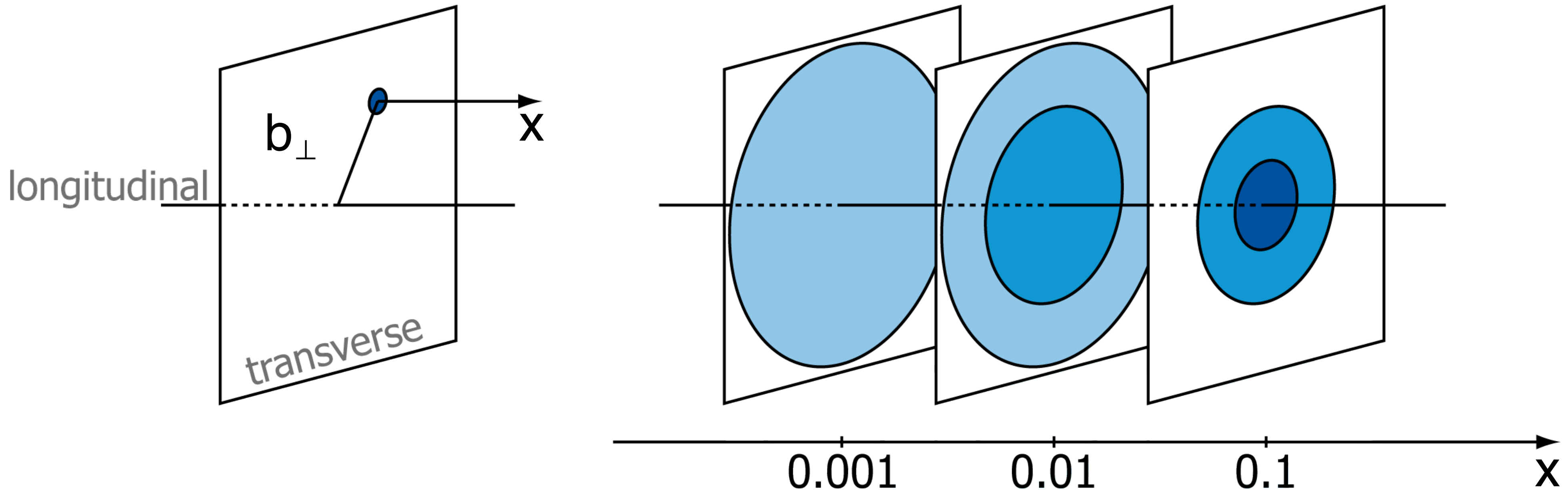
Positivity bounds - positivity of norm in Hilbert space, e.g.:

$$\left(1-\xi^2\right)\left(H^q - \frac{\xi^2}{1-\xi^2}E^q\right)^2 + \frac{t_0 - t}{4m^2}\left(E^q\right)^2 \leq q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)$$

strong constraint on GPD parameterizations!

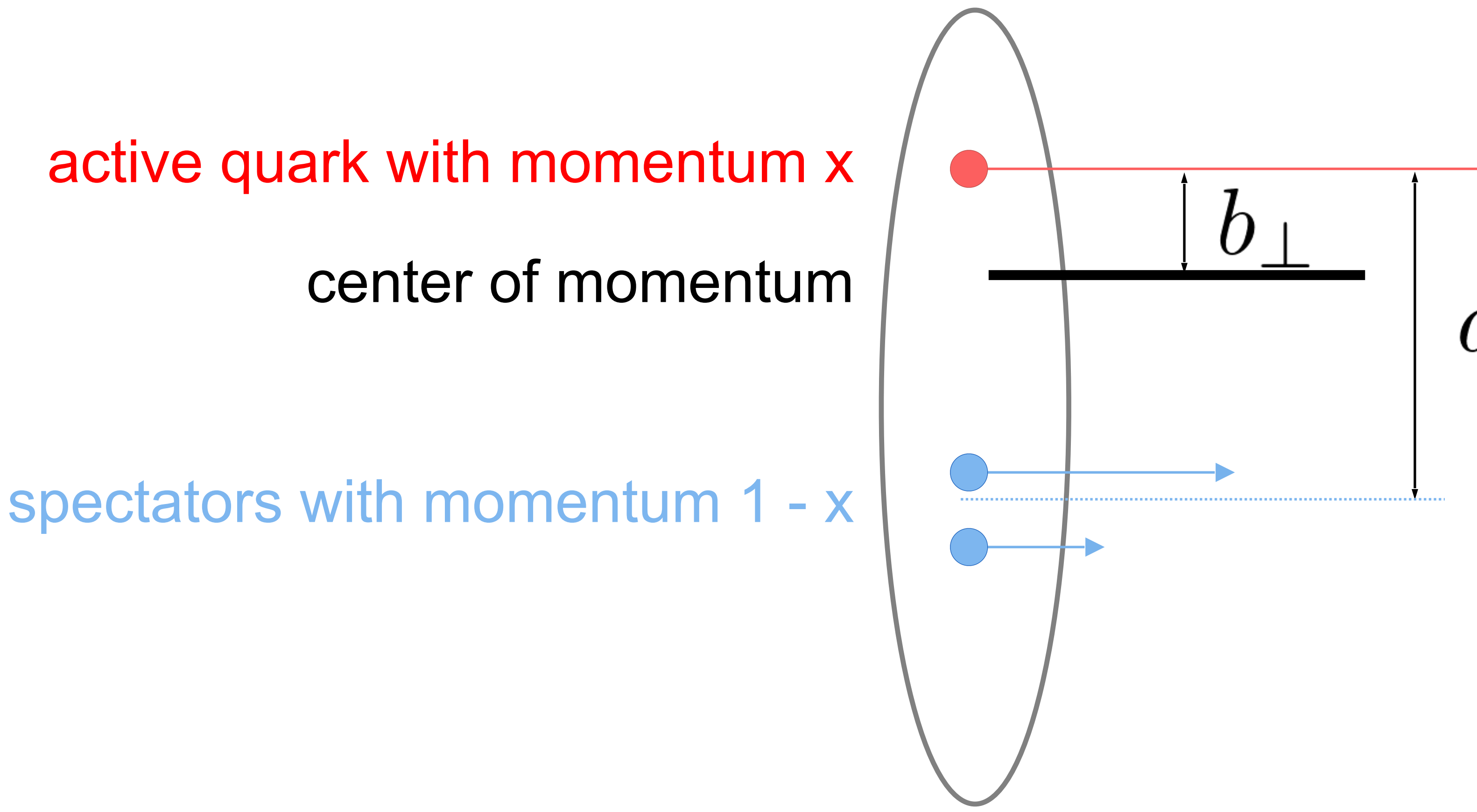
Nucleon tomography

$$q(x, \mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\mathbf{\Delta}}{4\pi^2} e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}} H^q(x, 0, t = -\mathbf{\Delta}^2)$$



Study of long. polarization with GPD $\widetilde{H}$
Study of distortion in transv. polarized nucleon with GPD E

Impact parameter $\mathbf{b}_\perp$ defined w.r.t. center of momentum, such as $\sum x\,\mathbf{b}_\perp = 0$



$$d_\perp = \frac{b_\perp}{1-x}$$

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

Energy density

Momentum density

Shear stress

Normal stress

Energy flux

Momentum flux

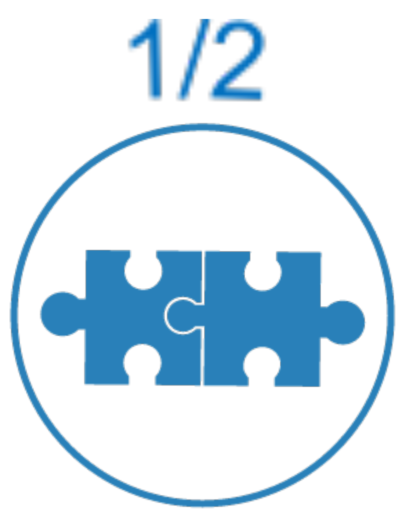
Energy momentum tensor in terms of form factors:

$$\langle p', s' | \widehat{T}^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \left[\frac{P^\mu P^\nu}{M} A(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} C(t) + M \eta^{\mu\nu} \bar{C}(t) + \right.$$

$$\left. \frac{P^\mu i \sigma^{\nu\lambda} \Delta_\lambda}{4M} \left[A(t) + B(t) + D(t) \right] + \frac{P^\nu i \sigma^{\mu\lambda} \Delta_\lambda}{4M} \left[A(t) + B(t) - D(t) \right] \right] u(p, s)$$

Total angular momentum:

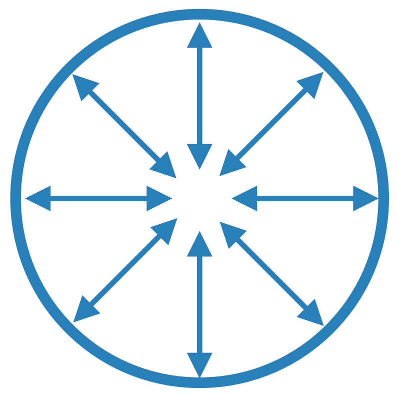
$$A^q(0) + B^q(0) = \int_{-1}^{1} x \left[H^q(x, \xi, 0) + E^q(x, \xi, 0)\right] = 2J^q$$



Ji's sum rule

"Mechanical" forces acting on quarks, e.g. pressure in nucleon center

$$p(0) = \frac{1}{6\pi^2 M}\int_{-\infty}^{0} dt\sqrt{-t}\,tC(t)$$



Introduction

$\mathrm{GTMD}(x, \xi, \mathbf{k}_\perp, \Delta)$

$\xi = \Delta = 0$

$\int d^2\mathbf{k}_\perp$

$\mathrm{TMD}(x, \mathbf{k}_\perp)$

SIDIS and DY

$\mathrm{GPD}(x, \xi, t = \Delta^2)$

exclusive

$\int d^2\mathbf{k}_\perp$

$\xi = t = 0$

$\int dx$

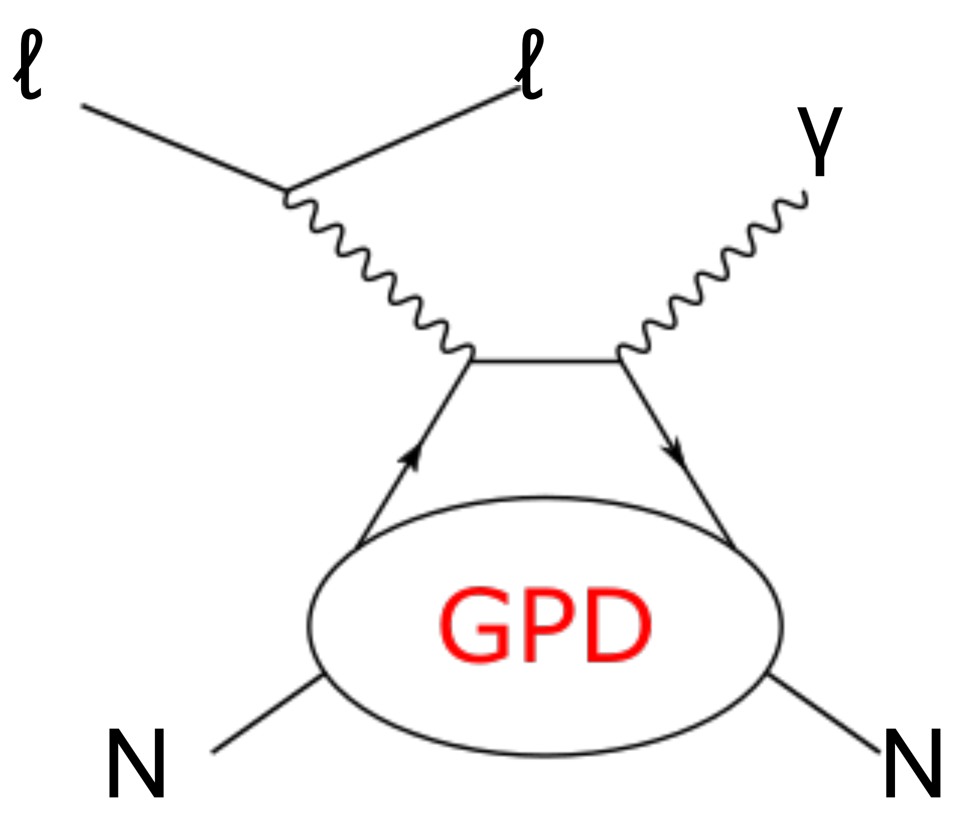
$\mathrm{PDF}(x)$

DIS, SIDS and pp

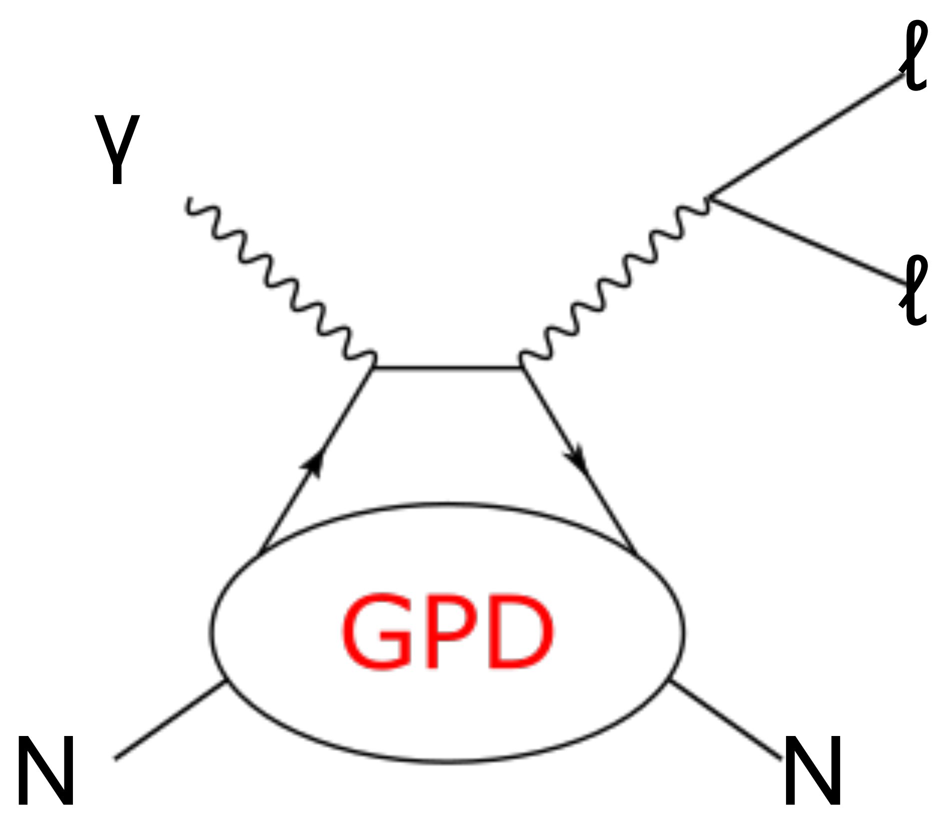
$\mathrm{EFF}(\mathrm{t})$

elastic

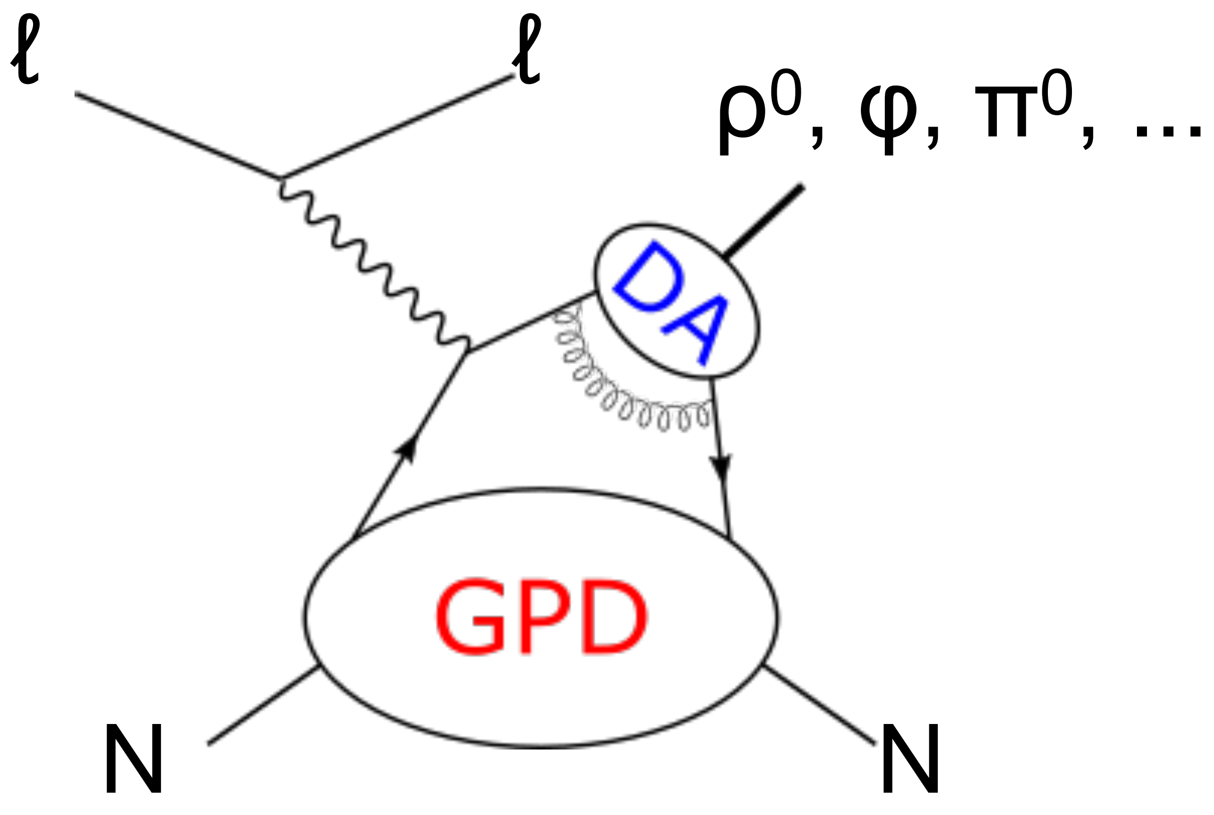
GPDs accessible in various production channels and observables
→ experimental filters



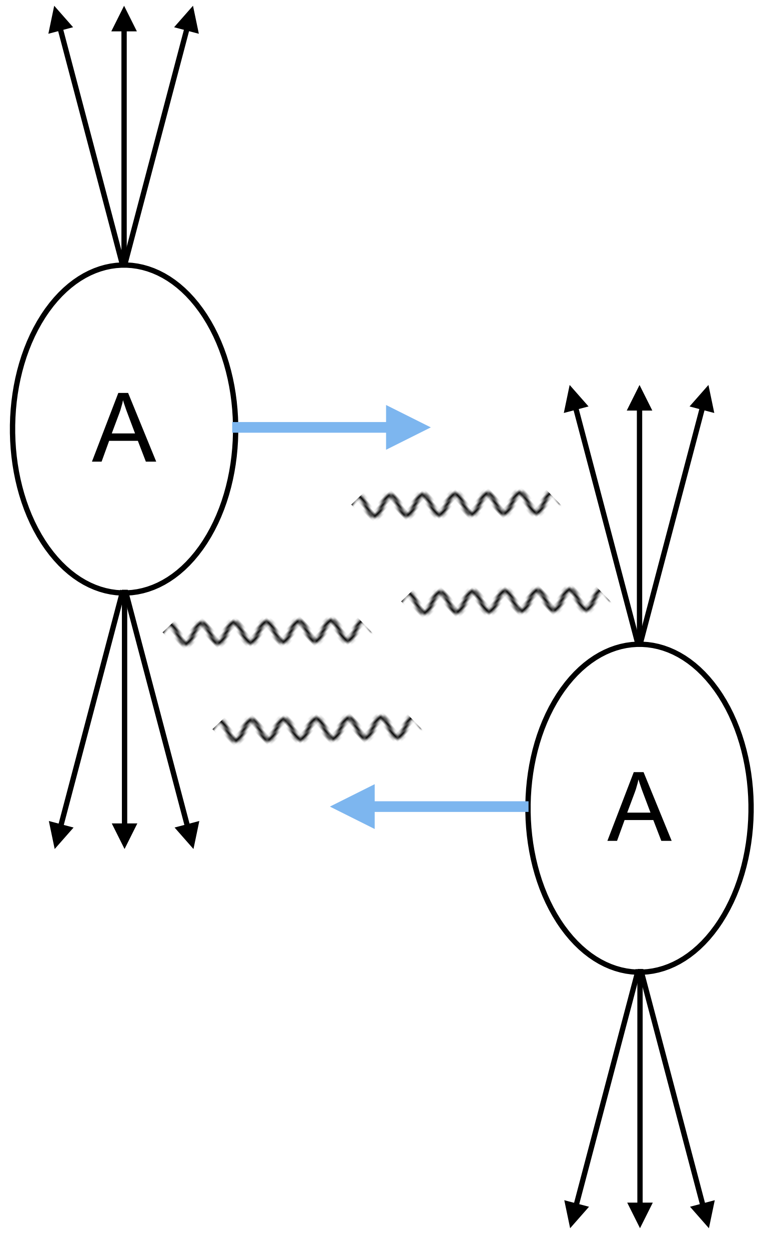
DVCS
Deeply Virtual Compton Scattering



TCS
Timelike Compton Scattering



HEMP
Hard Exclusive Meson Production



UPC
Ultra Peripheral Collisions

more production channels sensitive to GPDs exist!

Experimental campaign

GPDs studied in various laboratories
→ need to cover a broad kinematic range

experiments

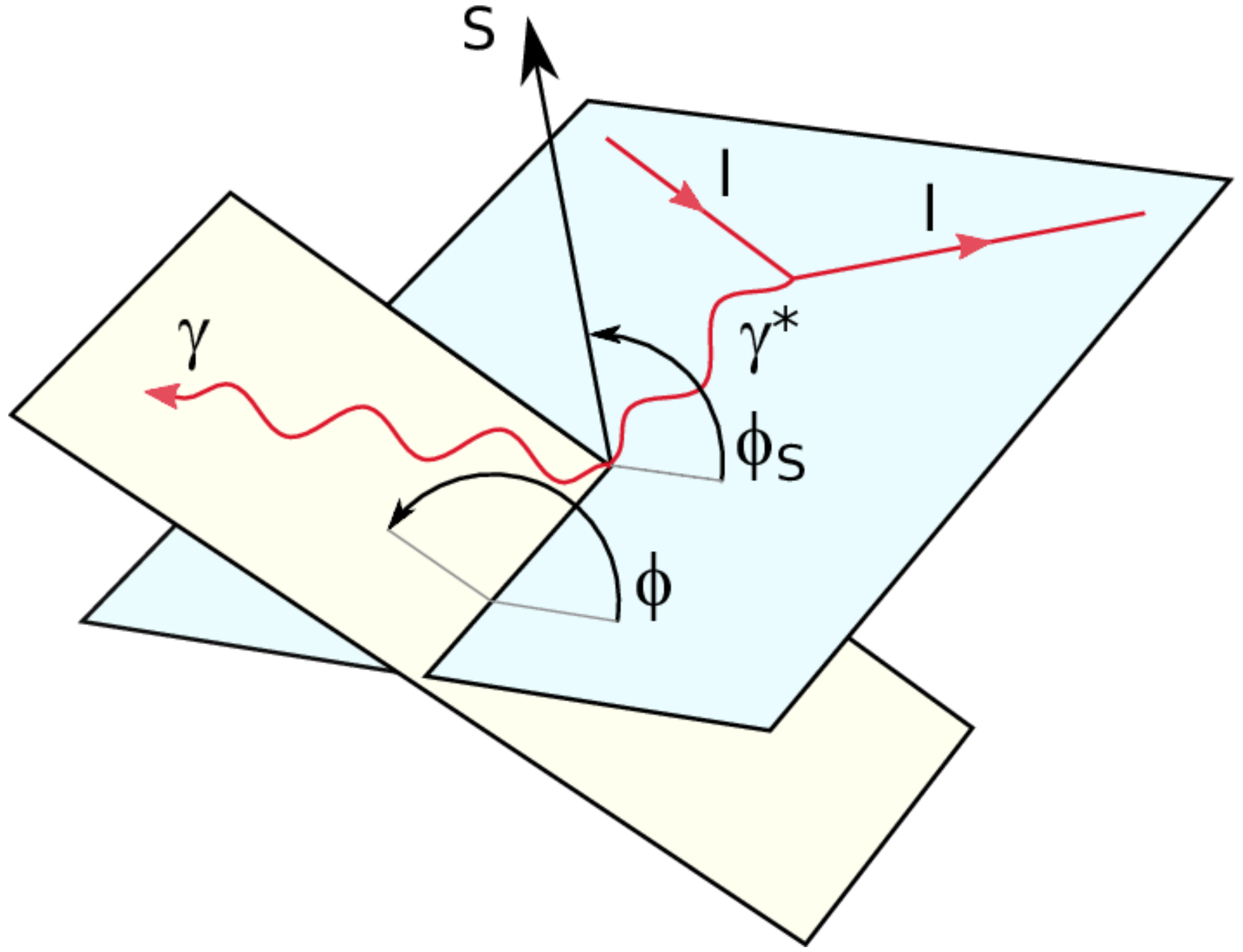
closed **active** **planned**

DVCS data

Kinematic cuts
used in our analyses:

$$Q^2 > 1.5\ \mathrm{GeV}^2$$
$$-t/Q^2 < 0.2$$

Angles:



No.	Collab.	Year	Observable		Kinematic dependence	No. of points used / all
1	HERMES	2001	A^{+}_{LU}		ϕ	10 / 10
2		2006	$A_{C}^{\cos i\phi}$	$i=1$	t	4 / 4
3		2008	$A_{C}^{\cos i\phi}$	$i=0,1$	x_{Bj}	18 / 24
			$A_{UT,\mathrm{DVCS}}^{\sin(\phi-\phi_S)\cos i\phi}$	$i=0$		
			$A_{UT,\mathrm{I}}^{\sin(\phi-\phi_S)\cos i\phi}$	$i=0,1$		
			$A_{UT,\mathrm{I}}^{\cos(\phi-\phi_S)\sin i\phi}$	$i=1$		
4		2009	$A_{LU,\mathrm{I}}^{\sin i\phi}$	$i=1,2$	x_{Bj}	35 / 42
			$A_{LU,\mathrm{DVCS}}^{\sin i\phi}$	$i=1$		
			$A_{C}^{\cos i\phi}$	$i=0,1,2,3$		
5		2010	$A_{UL}^{+,\sin i\phi}$	$i=1,2,3$	x_{Bj}	18 / 24
			$A_{LL}^{+,\cos i\phi}$	$i=0,1,2$		
6		2011	$A_{LT,\mathrm{DVCS}}^{\cos(\phi-\phi_S)\cos i\phi}$	$i=0,1$	x_{Bj}	24 / 32
			$A_{LT,\mathrm{DVCS}}^{\sin(\phi-\phi_S)\sin i\phi}$	$i=1$		
			$A_{LT,\mathrm{I}}^{\cos(\phi-\phi_S)\cos i\phi}$	$i=0,1,2$		
			$A_{LT,\mathrm{I}}^{\sin(\phi-\phi_S)\sin i\phi}$	$i=1,2$		
7		2012	$A_{LU,\mathrm{I}}^{\sin i\phi}$	$i=1,2$	x_{Bj}	35 / 42
			$A_{LU,\mathrm{DVCS}}^{\sin i\phi}$	$i=1$		
			$A_{C}^{\cos i\phi}$	$i=0,1,2,3$		
8	CLAS	2001	$A_{LU}^{-,\sin i\phi}$	$i=1,2$	—	0 / 2
9		2006	$A_{UL}^{-,\sin i\phi}$	$i=1,2$	—	2 / 2
10		2008	A^{-}_{LU}		ϕ	283 / 737
11		2009	A^{-}_{LU}		ϕ	22 / 33
12		2015	A^{-}_{LU}, A^{-}_{UL}, A^{-}_{LL}		ϕ	311 / 497
13		2015	$d^4\sigma^{-}_{UU}$		ϕ	1333 / 1933
14	Hall A	2015	$\Delta d^4\sigma^{-}_{LU}$		ϕ	228 / 228
15		2017	$\Delta d^4\sigma^{-}_{LU}$		ϕ	276 / 358
16	COMPASS	2018	$d^3\sigma^{\pm}_{UU}$		t	2 / 4
17	ZEUS	2009	$d^3\sigma^{+}_{UU}$		t	4 / 4
18	H1	2005	$d^3\sigma^{+}_{UU}$		t	7 / 8
19		2009	$d^3\sigma^{\pm}_{UU}$		t	12 / 12
					SUM:	2624 / 3996

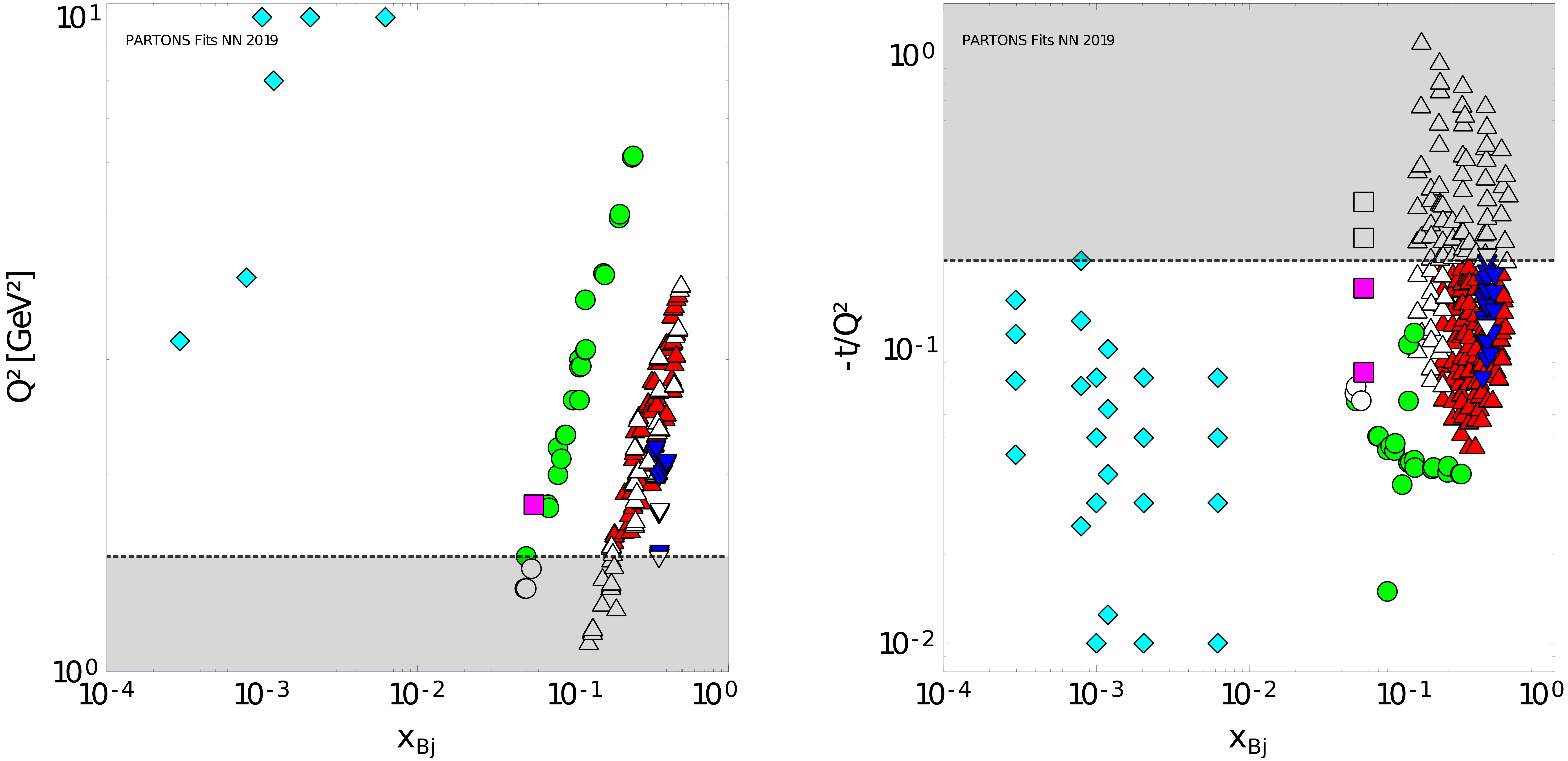
DVCS data

Kinematic cuts
used in our analyses:

$$Q^2 > 1.5 \text{ GeV}^2$$

$$-t/Q^2 < 0.2$$

- HALL A
- CLAS
- HERMES
- COMPASS
- H1 and ZEUS

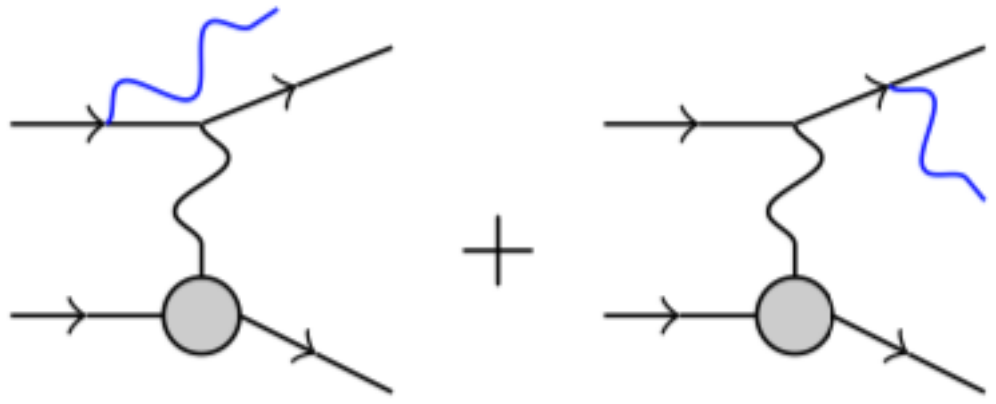


DVCS Compton Form Factors

Cross-section for single photon production ($l + N \to l + N + \gamma$):

$$\sigma \propto |\mathscr{A}|^2 = |\mathscr{A}_{BH} + \mathscr{A}_{DVCS}|^2 = |\mathscr{A}_{BH}|^2 + |\mathscr{A}_{DVCS}|^2 + \mathscr{I}$$

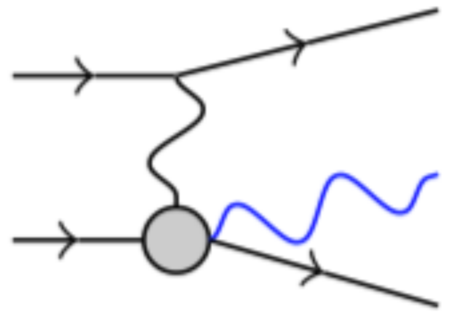
Bethe-Heitler process



calculable within QED

parametrised by elastic FFs

DVCS



calculable within QCD

parametrised by CFFs

see e.g. NPB 878 (2014) 214 for more details

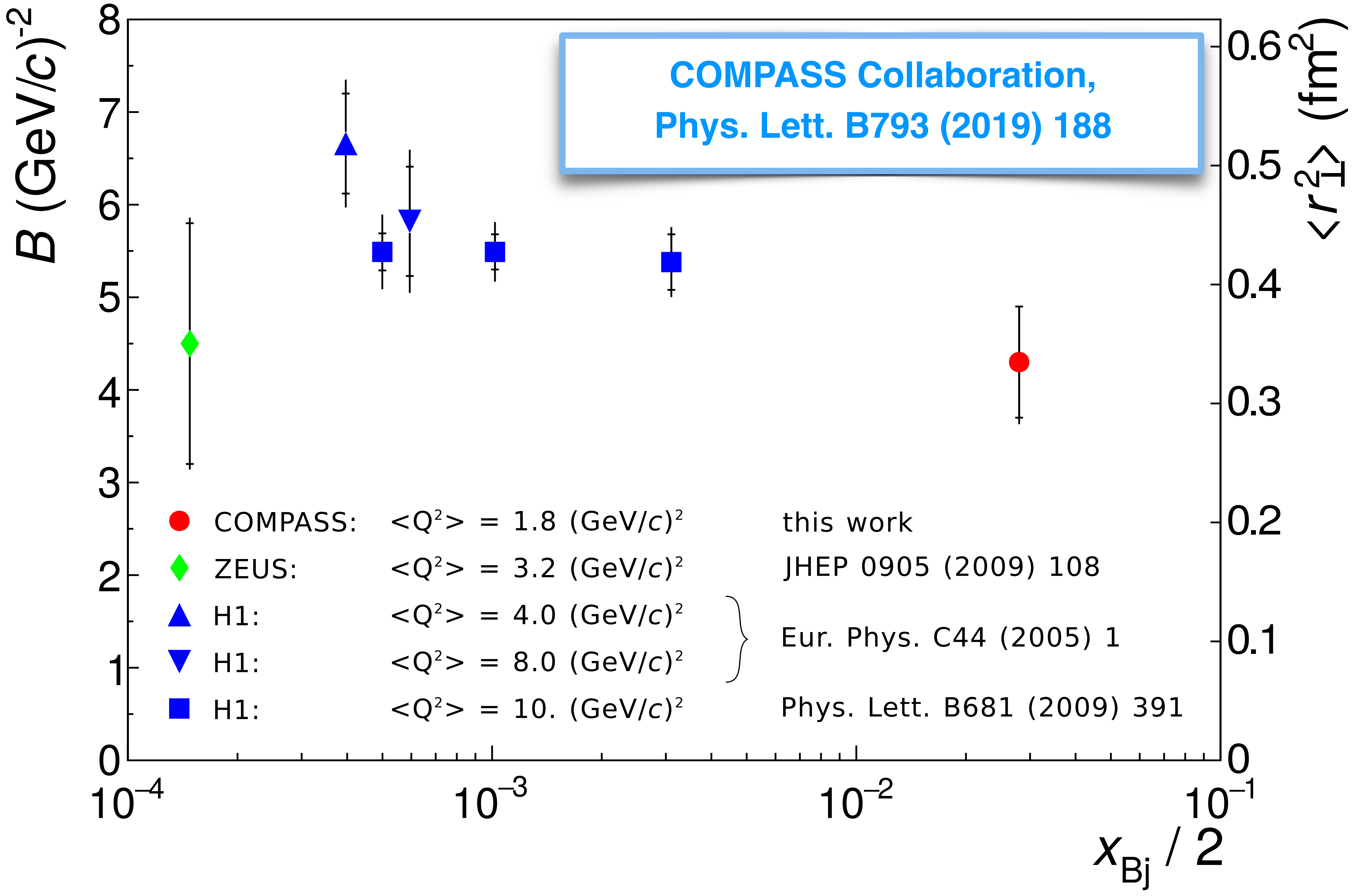
$$\mathrm{Im}\mathscr{H}(\xi, t) \stackrel{\text{LO}}{=} \pi \sum_q e_q^2 H^{q(+)}(\xi, \xi, t)$$

$$\mathrm{Re}\mathscr{H}(\xi, t) = \mathrm{PV}\int_0^1 \frac{d\xi'}{\pi} \mathrm{Im}\mathscr{H}(\xi', t)\left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'}\right) + C_H(t)$$

GPD phenomenology (amplitude level)

What we can learn from DVCS amplitudes?

- nucleon tomography at low-xB
- "mechanical" properties



Under following assumptions:

- single-exponential dependence
- dominance of CFF Im$\mathcal{H}$
- negligible “skewness effect”
  H(x, x, t) ~ H(x, 0, t)

From t-dependance of cross-section:

$$\frac{d\sigma^{\gamma^* p \to \gamma p}}{dt} \propto e^{-Bt}$$

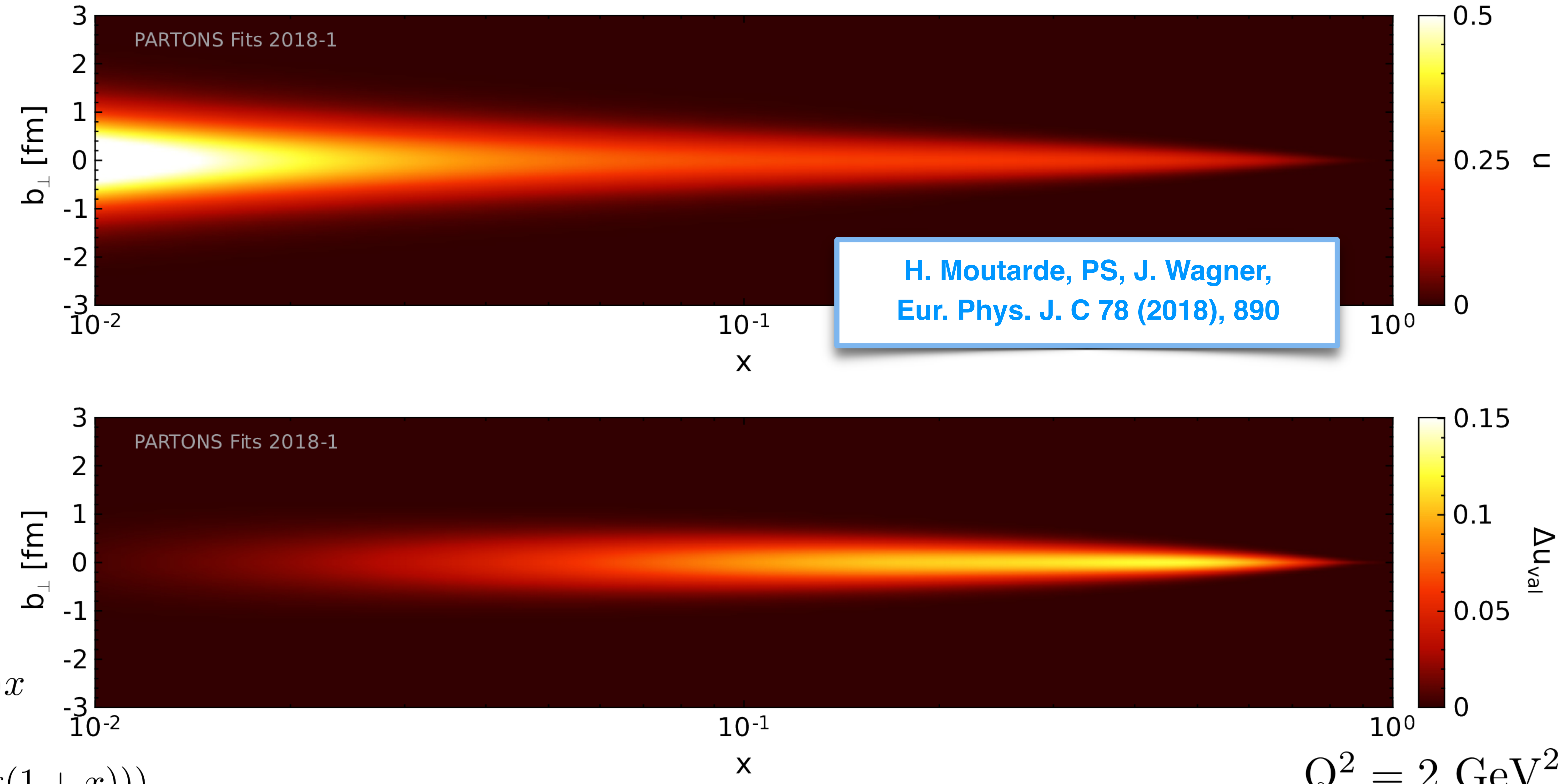
related to transverse extension of quarks:

$$\langle r_\perp^2(x_{\mathrm{Bj}}) \rangle \approx 2 \langle B(x_{\mathrm{Bj}}) \rangle$$

What we can learn from DVCS amplitudes?

- nucleon tomography at low-xB
- "mechanical" properties

This is example of **model dependent** analysis, based on specific Ansatz of GPD



H. Moutarde, PS, J. Wagner, Eur. Phys. J. C 78 (2018), 890

$Q^2 = 2\ \mathrm{GeV}^2$

$$G^q(x,x,t) = G^q(x,0,t)\ g^q_G(x,x,t)$$

$$G^q(x,0,t) = \mathrm{pdf}^q_G(x)\ \ \exp(f^q_G(x)t)$$

$$f^q_G(x) = A^q_G \log(1/x) + B^q_G(1-x)^2 + C^q_G(1-x)x$$

$$g^q_G(x,x,t) = \frac{a^q_G}{(1-x^2)^2}\,(1 + t(1-x)(b^q_G + c^q_G \log(1+x)))$$

free parameters (for given GPD "G" and quark flavour "q") fixed by DVCS and elastic form factor data

What we can learn from DVCS amplitudes?

- nucleon tomography at low-xB
- "mechanical" properties

$$\mathcal{C}^q(t) \overset{\text{LO}}{=} 2\, e_q^2 \int_{-1}^{1} \mathrm{d}z\, \frac{D^q(z,t)}{1-z} \equiv 4D^q(t) \qquad z = \frac{x}{\xi}$$

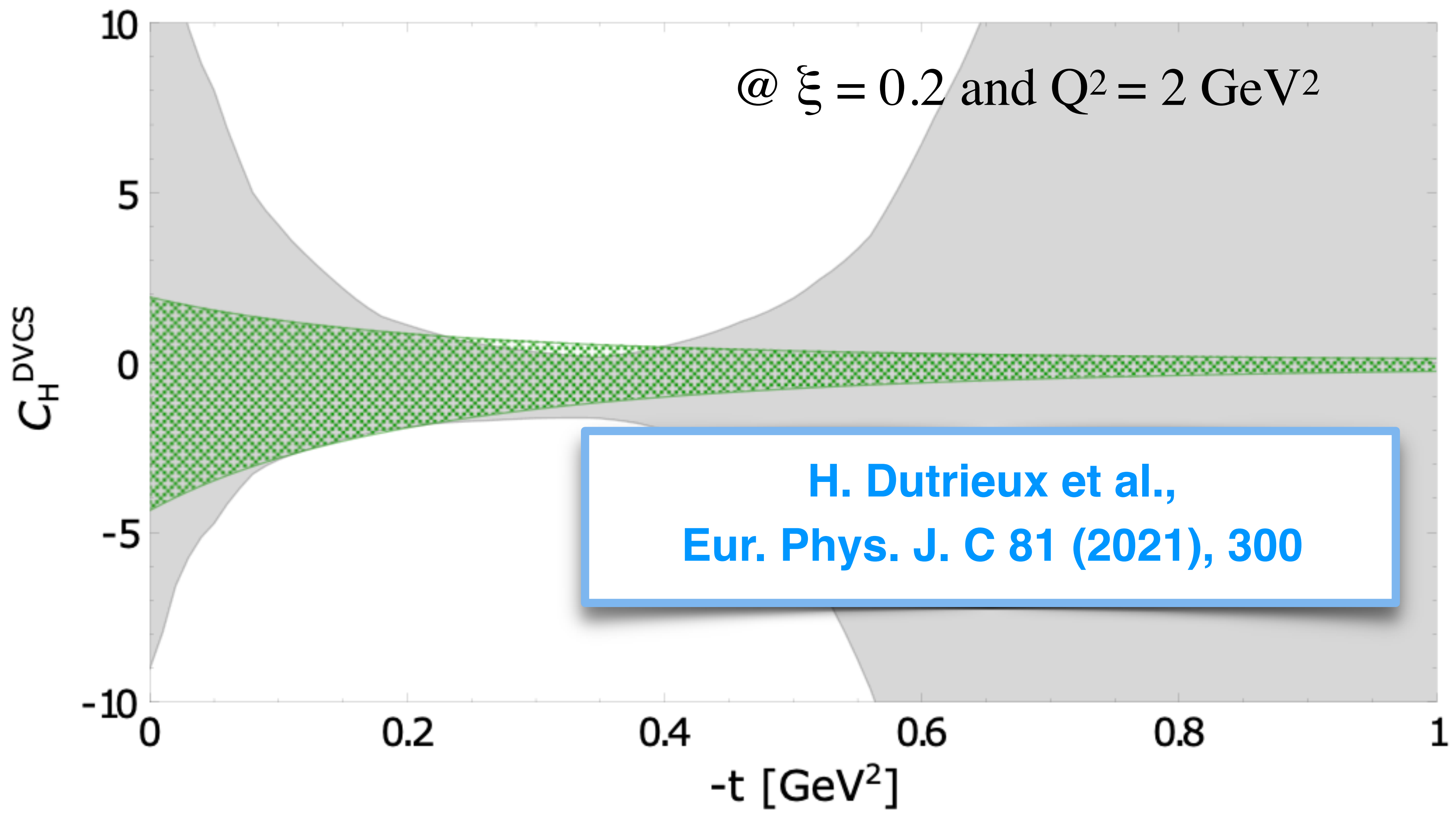
$$D^q(z,t) = (1-z^2) \sum_{i=0}^{\infty} d_i^q C_{2i+1}^{3/2}(z)$$

$$D^q(t) = \sum_{\substack{i=1\\ \text{odd}}}^{\infty} d_i^q(t) \qquad d_1^q(t) \equiv 5C_q(t)$$

ANN analysis

Model dependent extraction

$$d_1^{uds}(t,\mu_F^2) = d_1^{uds}(\mu_F^2)\left(1-\frac{t}{\Lambda^2}\right)^{-\alpha} \qquad \alpha = 3 \qquad \Lambda = 0.8\ \text{GeV}$$



H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252

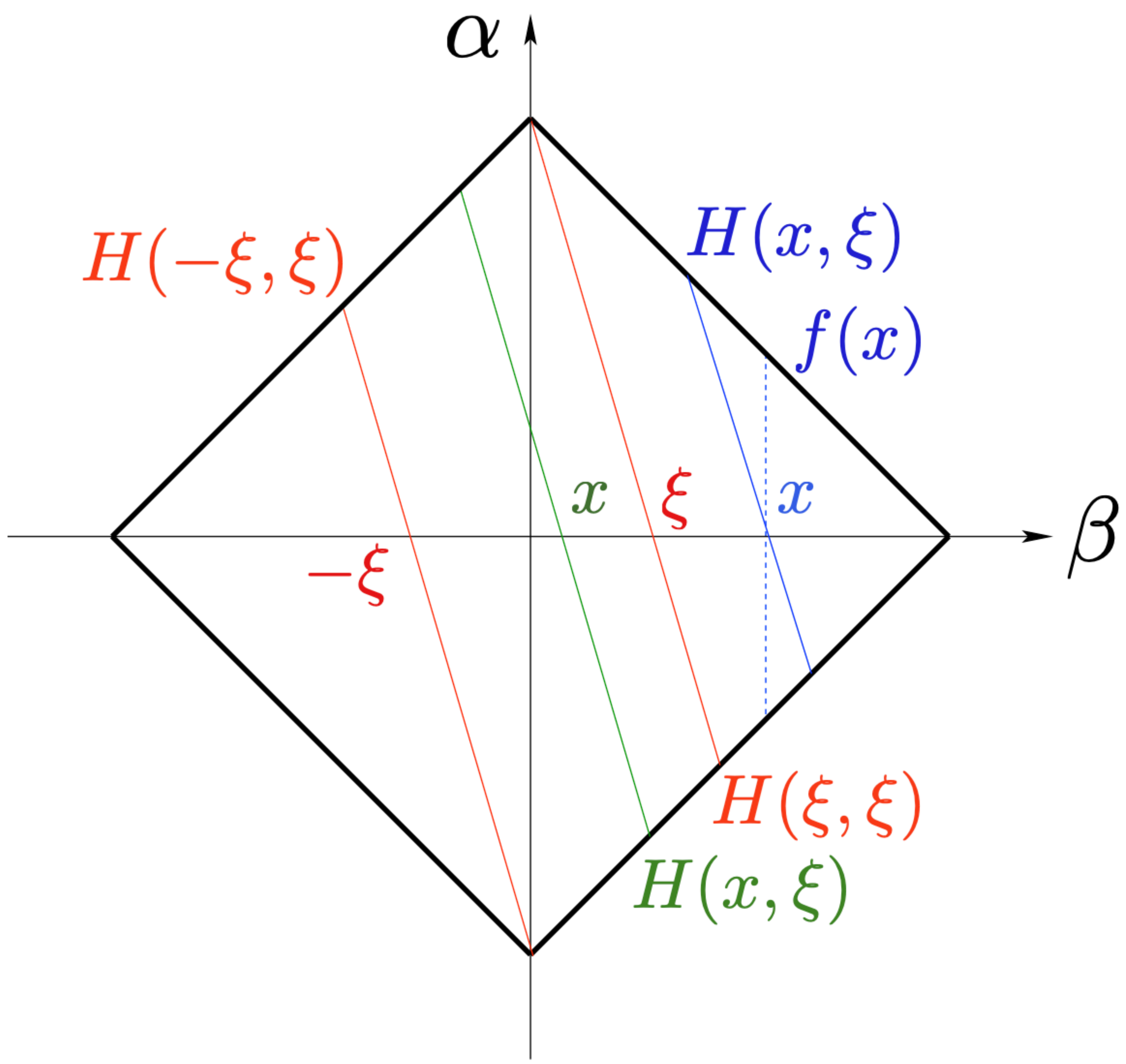
Double distribution:

$$H(x, \xi, t) = \int \mathrm{d}\Omega\, F(\beta, \alpha, t)$$

where:

$$\mathrm{d}\Omega = \mathrm{d}\beta\, \mathrm{d}\alpha\, \delta(x - \beta - \alpha\xi)$$

$$|\alpha| + |\beta| \leq 1$$



from PRD83, 076006, 2011

GPD phenomenology (GPD level)

H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252

Double distribution: $(1-x^2)F_C(\beta,\alpha) + (x^2-\xi^2)F_S(\beta,\alpha) + \xi F_D(\beta,\alpha)$

Classical term:

$$F_C(\beta,\alpha) = f(\beta)h_C(\beta,\alpha)\frac{1}{1-\beta^2}$$

$$f(\beta) = \mathrm{sgn}(\beta)q(|\beta|)$$

$$h_C(\beta,\alpha) = \frac{\mathrm{ANN}_C(|\beta|,\alpha)}{\int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \mathrm{ANN}_C(|\beta|,\alpha)}$$

Shadow term:

$$F_S(\beta,\alpha) = f(\beta)h_S(\beta,\alpha)$$

$$f(\beta) = \mathrm{sgn}(\beta)q(|\beta|)$$

$$h_S(\beta,\alpha)/N_S = \frac{\mathrm{ANN}_S(|\beta|,\alpha)}{\int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \mathrm{ANN}_S(|\beta|,\alpha)} - \frac{\mathrm{ANN}_{S'}(|\beta|,\alpha)}{\int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \mathrm{ANN}_{S'}(|\beta|,\alpha)} .$$

$$\mathrm{ANN}_{S'}(|\beta|,\alpha) \equiv \mathrm{ANN}_C(|\beta|,\alpha)$$

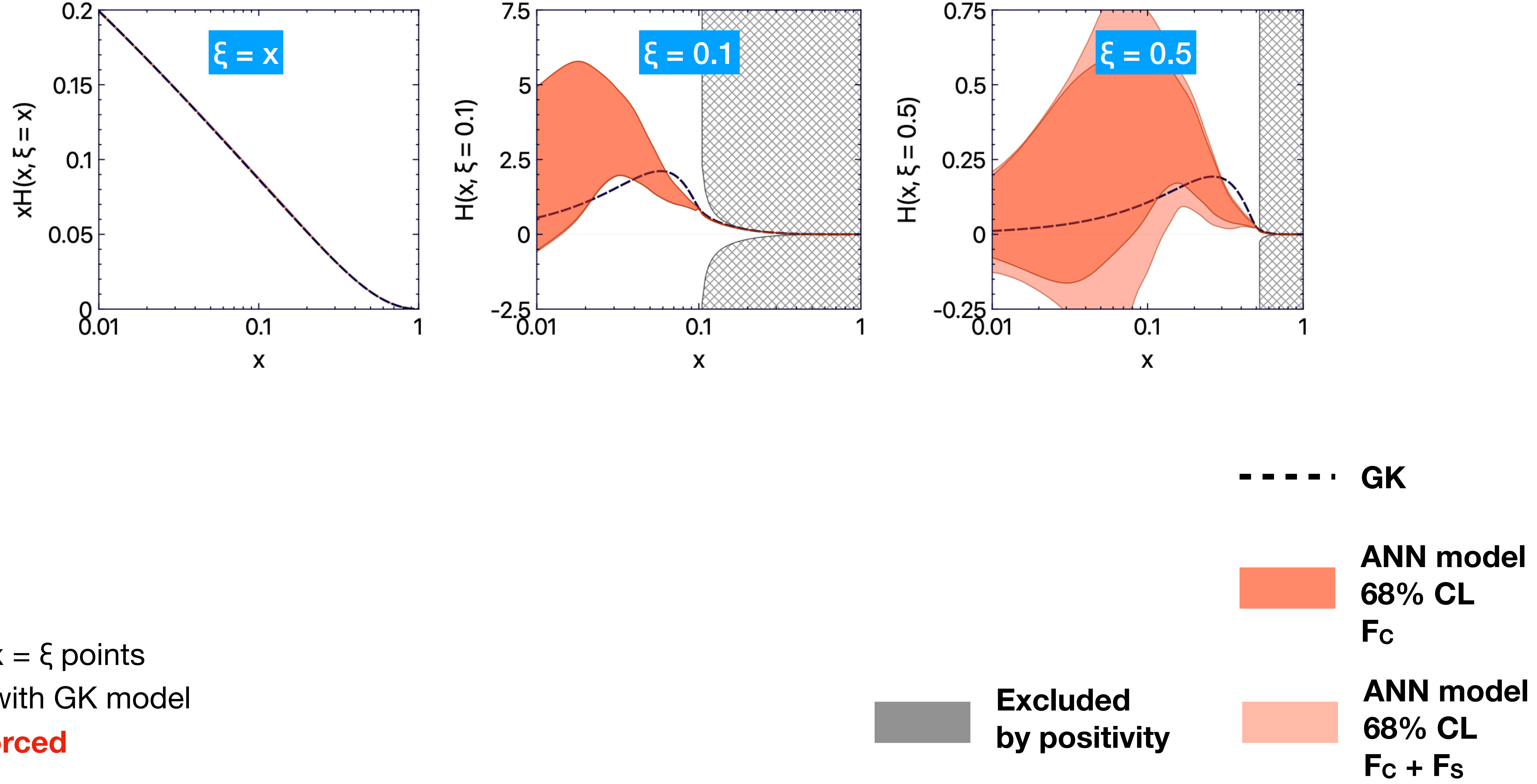
D-term:

$$F_D(\beta,\alpha) = \delta(\beta)D(\alpha)$$

$$D(\alpha) = (1-\alpha^2)\sum_{\substack{i=1\\ \mathrm{odd}}} d_i C_i^{3/2}(\alpha)$$

GPD phenomenology (GPD level)

H. Dutrieux et al.,
Eur. Phys. J. C 82 (2022) 3, 252



Conditions:

- Input: 200 x = ξ points generated with GK model
- Positivity **forced**

Double DVCS

K. Deja, V. Martínez-Fernández, B. Pire, PS, J. Wagner
Phys. Rev. D 107 (2023) 9, 094035

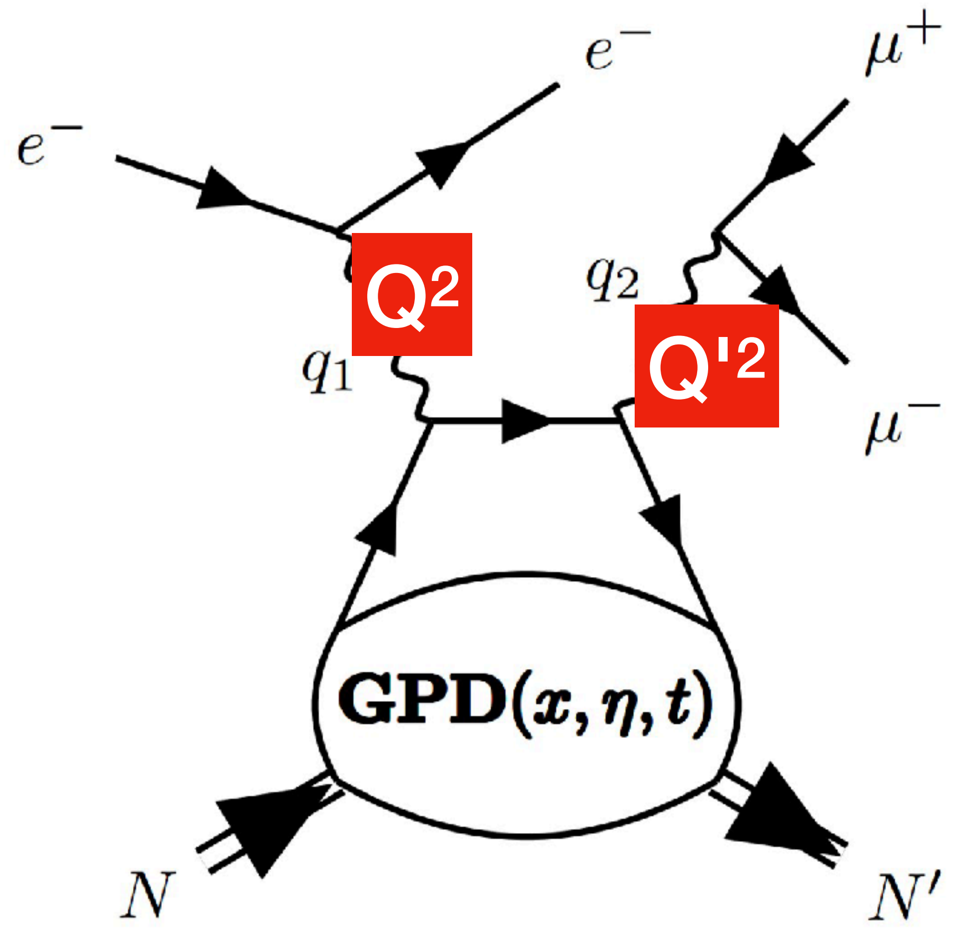
- The process allows to directly probe GPDs outside x=ξ line, but is much more challenging experimentally

$$\xi = \frac{Q^2 + Q'^2}{2Q^2/x_B - Q^2 - Q'^2}$$

$$\rho = \xi \frac{Q^2 - Q'^2}{Q^2 + Q'^2}$$

$$(\mathcal{H}, \mathcal{E})(\rho, \xi, t) = \sum_{f=\{u,d,s\}} \int_{-1}^{1} dx\; C_f^{(-)}(x, \rho)(H_f, E_f)(x, \xi, t)$$

$$C_f^{(\pm)}(x, \rho) \overset{LO}{=} \left(\frac{e_f}{e}\right)^2 \left(\frac{1}{\rho - x - i0} \pm \frac{1}{\rho + x - i0}\right)$$



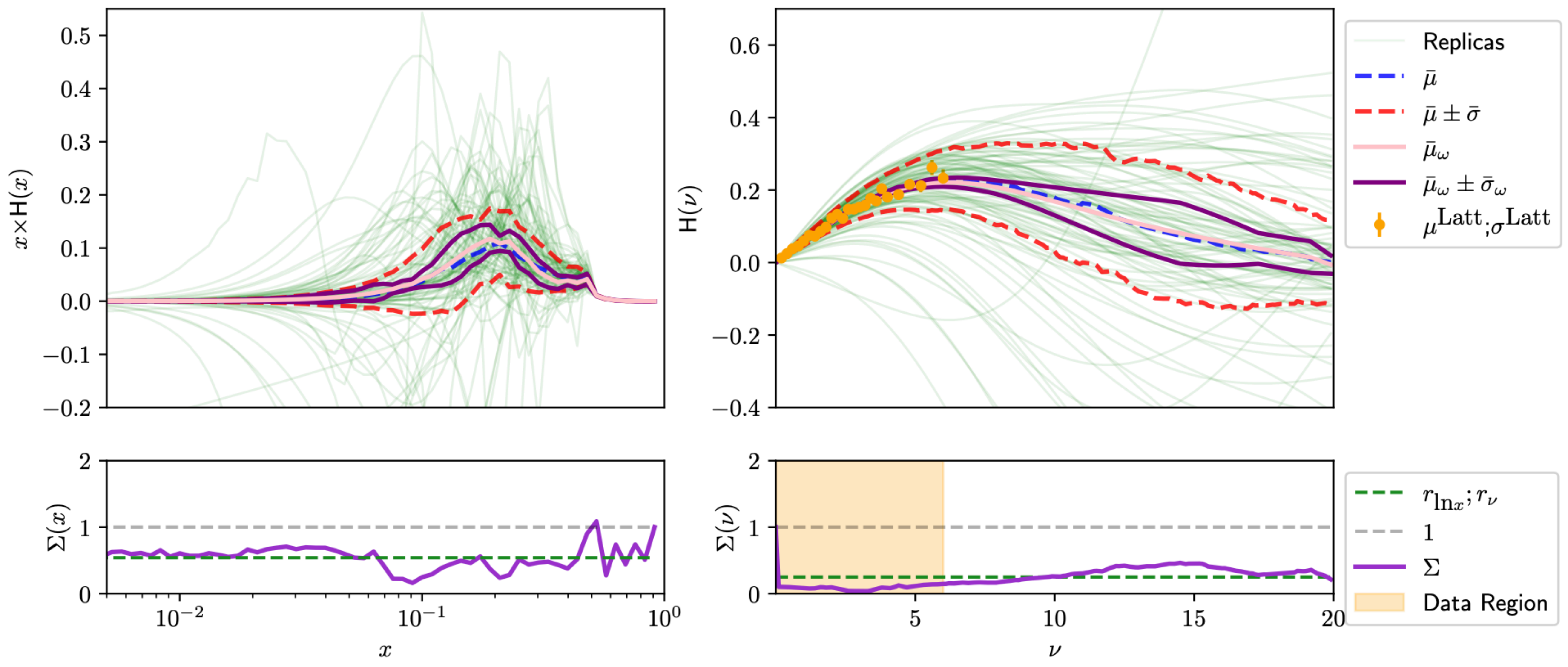
- We revisit DDVCS phenomenology in view of new experiments, including reevaluation of DDVCS and BH cross-sections with Kleiss-Stirling spinor techniques

- Obtained results are available in PARTONS and EpIC MC generator

M. J. Riberdy, H. Dutrieux, C. Mezrag, PS,
hep-ph/2306.01647

- Exploratory study to include lattice-QCD results!

Reduction of GPD model uncertainties due to inclusion of pseudo-latticeQCD results

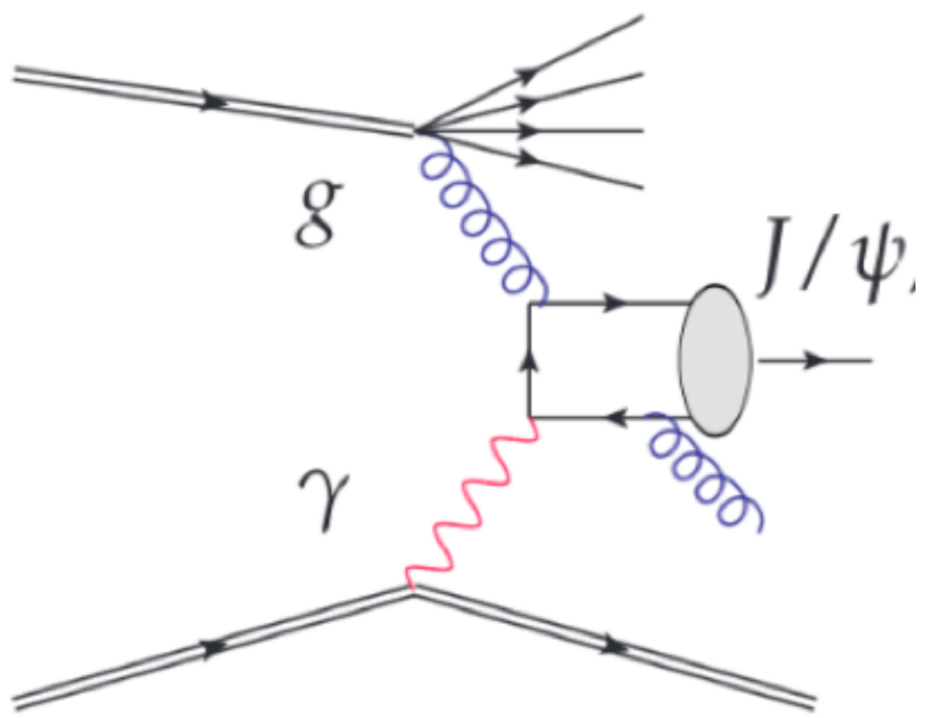


Ultra-peripheral collisions (UPCs)

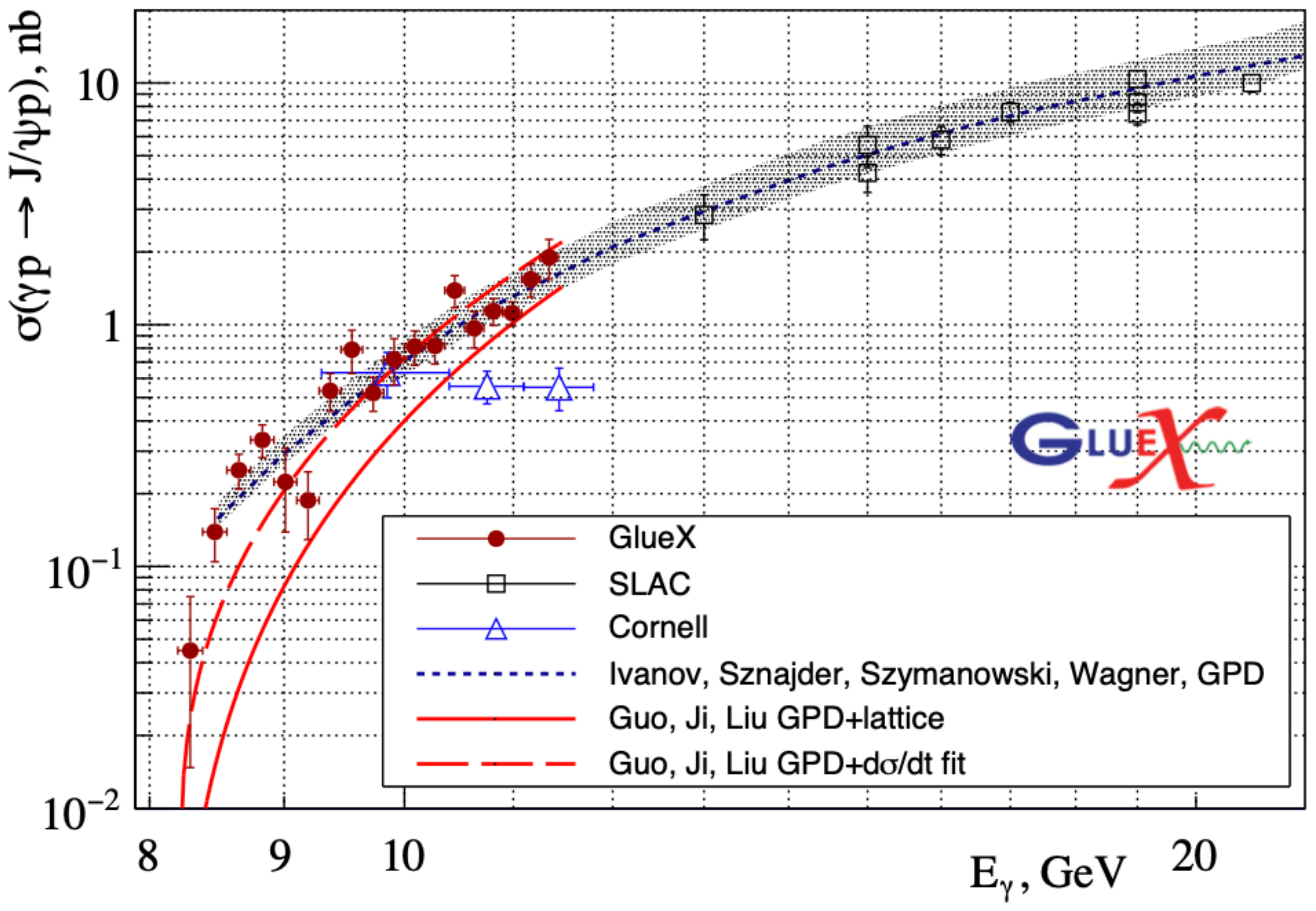
base on Spencer R. Klein's talk at SPIN'23
and Kate Lynch's talk (hal-03822190)

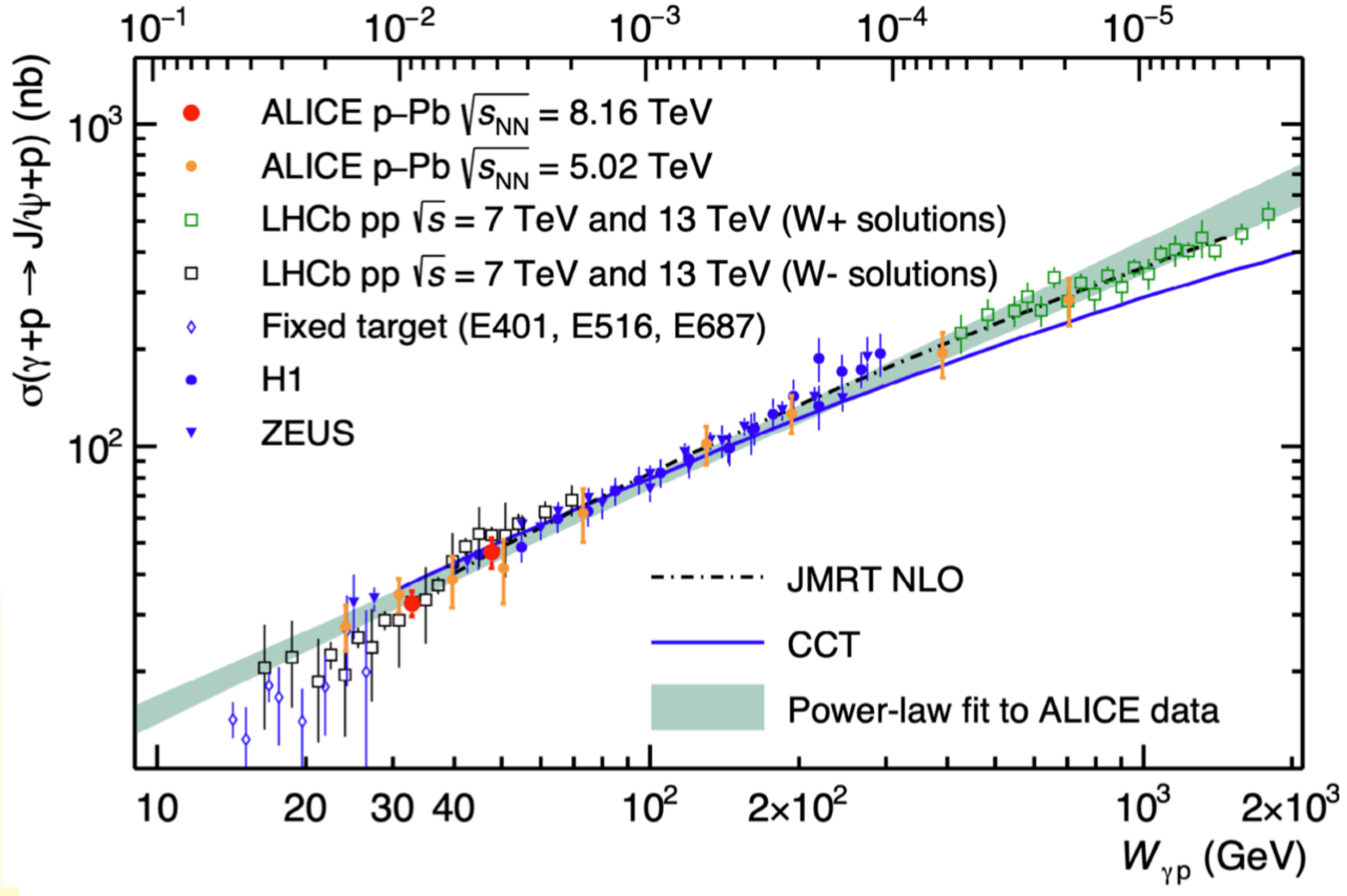
- The energy frontier for electromagnetic probes -> 10 times higher CM energy than available in HERA
- Allows to probe xBj down to a few 10^{-6} at moderate Q^2
- Electromagnetic probes have a_{EM} ~ 1/137, so are less affected by multiple interactions than hadronic interactions
- Also important source of PDF information

H. Dutrieux, M. Winn, V. Bertone,
Phys. Rev. D 107 (2023) 11, 114019



Exclusive photoproduction of J/Psi





PARTONS project

B. Berthou et al.,
Eur. Phys. J. C 78 (2018) 6, 478

- PARTONS - open-source framework to study GPDs
  → http://partons.cea.fr

- Come with number of available physics developments implemented

- Written in C++, also available via virtual machines (VirtualBox) and containers (Docker)

- Addition of new developments as easy as possible

- Developed to support effort of GPD community,
  can be used by both theorists and experimentalists

- v3 version of PARTONS is now available!

EpIC MC generator

E. C. Aschenauer et al.,
Eur. Phys. J. C 82 (2022) 9, 819

- Novel MC generator called EpIC released
  → *https://pawelsznajder.github.io/epic*

- EpIC is based on PARTONS

- EpIC is characterised by:
  - flexible architecture that utilises a modular programming paradigm
  - a variety of modelling options, including radiative corrections
  - multichannel capability (now: DVCS, TCS, DVπ^0P, diphoton; coming soon: DDVCS, J/ψ)

- This is the new tool to be use in the precision era commenced by the new generation of experiments

Summary

- Substantial progress in:
  - understanding of fundamental problems, like deconvolution of CFFs, and analysis methods
    → important for extraction of GPDs
  - modelling of GPD, fulfilling all theory-driven constraints (including positivity)
    → subject not touched enough in the current literature
    → developed in mind for easy inclusion of latticeQCD data
  - addressing the long-standing problem of model dependency of GPDs
    → nontrivial and timely analysis
  - description of exclusive processes
    → new sources of GPD information
  - delivering open-source tools for the community
    → to suport both experimentalists and theoreticians

This progress is important for the precision era of GPD extraction allowed by the new generation of experiments