



## Central exclusive production of selected charmonia in the light-front $k_\perp$ -factorization approach

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# ExtreMe Matter Institute EMMI

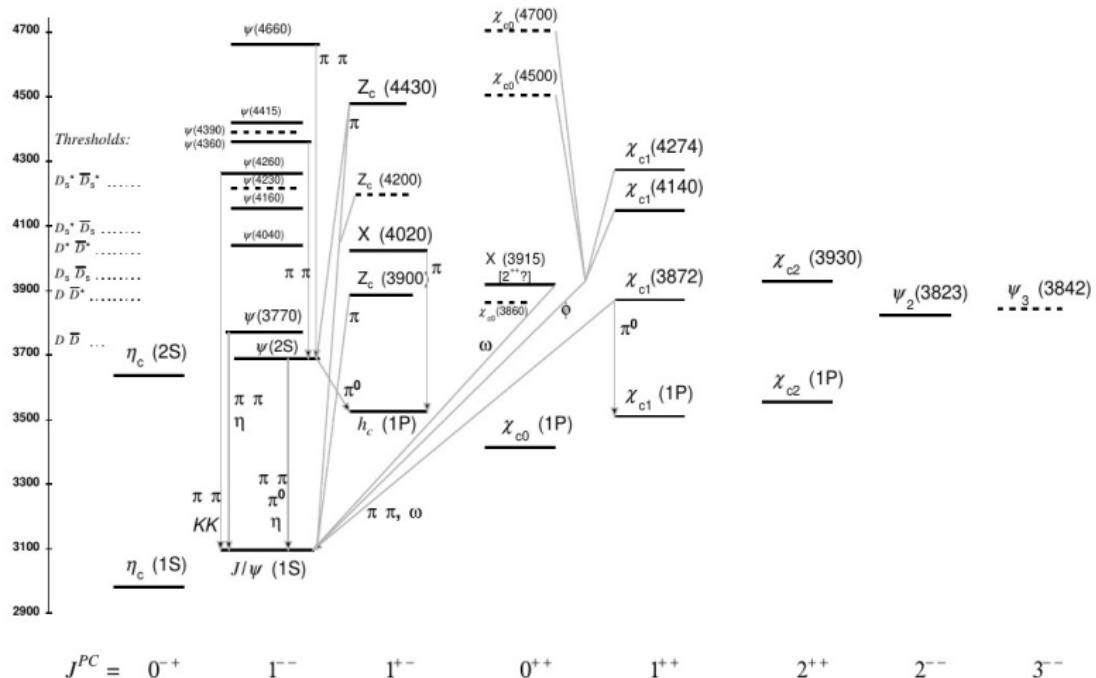
EMMI Workshop

## Forward Physics in ALICE 3

Physikalisches Institut, Heidelberg University  
October 18 - 20, 2023



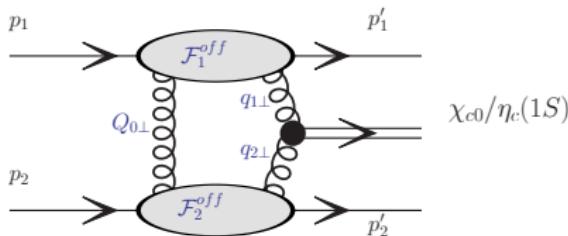
# Introduction $\Rightarrow \eta_c(1S), \chi_{c0} (1P)$



mass spectra of  $c\bar{c}$  states P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01

(2020)

# Introduction



Generic diagram for **central exclusive production** of  $\eta_c$  and  $\chi_{c0}$ ,  
[Phys. Rev. D 102, 114028 \(2020\)](#)

$\mathcal{V}^{c_1 c_2} \Rightarrow \mathcal{V}^{c_1 c_2}(g^* g^* \rightarrow \eta_c(1S))$   
Prompt hadroproduction of  $\eta_c(1S, 2S)$  in the  $k_T$ -factorization approach,  
[JHEP02, 037 \(2020\)](#),

$\mathcal{V}^{c_1 c_2} \Rightarrow \mathcal{V}^{c_1 c_2}(g^* g^* \rightarrow \chi_{c0}(1P))$   
Hadroproduction of scalar  $P$ -wave quarkonia in the light-front  $k_T$ -factorization approach

[JHEP06, 101 \(2020\)](#)

$$\mathcal{M} = \frac{s}{2} \pi^2 \frac{1}{2} \frac{\delta_{c_1 c_2}}{N_c^2 - 1} \int d^2 Q \mathcal{V}^{c_1 c_2} \frac{\mathcal{F}_g^{\text{off}}(x_1, x', \mathbf{Q}^2, \mathbf{q}_1^2, \mu^2, t_1) \mathcal{F}_g^{\text{off}}(x_2, x', \mathbf{Q}^2, \mathbf{q}_2^2, \mu^2, t_2)}{Q^2 \mathbf{q}_1^2 \mathbf{q}_2^2},$$

Durham model of CEP, [Int. J. Mod. Phys. A 29, 1430031 \(2014\)](#), [arXiv:1405.0018 [hep-ph]]

$$\sigma = \frac{1}{2s} \frac{1}{2^8 \pi^4 s} \int |\mathcal{M}|^2 dt_1 dt_2 dy d\phi.$$

$t_1 = (p_1 - p'_1)^2$ ,  $t_2 = (p_2 - p'_2)^2$  and  $\phi \in (0, 2\pi)$

R. S. Pasechnik, A. Szczurek, and O. V. Teryaev,  
[Phys. Rev. D 78, 014007 \(2008\)](#),

## Off-shell vertices

$$\mathcal{V}_{\mu\nu}^{ab}(g^* g^* \rightarrow \chi_{c0}) = 4\pi\alpha_s \frac{\text{Tr}[t^a t^b]}{\sqrt{N_c}} 2\mathcal{T}_{\mu\nu} = \frac{4\pi\alpha_s}{\sqrt{N_c}} \delta^{ab} \mathcal{T}_{\mu\nu},$$

$$\mathcal{T} = n_\nu^+ n_\mu^- \mathcal{T}_{\mu\nu} = |\mathbf{q}_1| |\mathbf{q}_2| G_1(\mathbf{q}_1^2, \mathbf{q}_2^2) + (\mathbf{q}_1 \cdot \mathbf{q}_2) G_2(\mathbf{q}_1^2, \mathbf{q}_2^2),$$

$$G_1(\mathbf{q}_1^2, \mathbf{q}_2^2) = |\mathbf{q}_1| |\mathbf{q}_2| \frac{4m_c}{\mathbf{q}_2^2} \int \frac{dz d^2k}{z(1-z)16\pi^3} \psi_{\chi_{c0}}(z, \mathbf{k}) 2z(1-z)(2z-1) \left[ \frac{1}{I_A^2 + \varepsilon^2} - \frac{1}{I_B^2 + \varepsilon^2} \right]$$

$$\begin{aligned} G_2(\mathbf{q}_1^2, \mathbf{q}_2^2) &= 4m_c \int \frac{dz d^2k}{z(1-z)16\pi^3} \psi_{\chi_{c0}}(z, \mathbf{k}) \left[ \frac{1-z}{I_A^2 + \varepsilon^2} + \frac{z}{I_B^2 + \varepsilon^2} \right] \\ &+ \frac{4m_c}{\mathbf{q}_2^2} \int \frac{dz d^2k}{z(1-z)16\pi^3} \psi_{\chi_{c0}}(z, \mathbf{k}) 4z(1-z) \left[ \frac{\mathbf{q}_2 \cdot I_A}{I_A^2 + \varepsilon^2} - \frac{\mathbf{q}_2 \cdot I_B}{I_B^2 + \varepsilon^2} \right], \end{aligned}$$

$$\mathcal{V}_{\mu\nu}^{ab}(g^* g^* \rightarrow \eta_c) = (-i) 4\pi\alpha_s \epsilon_{\mu\nu\alpha\beta} q^\alpha q^\beta \frac{\delta^{ab}}{2\sqrt{N_c}} 2I(\mathbf{q}_1^2, \mathbf{q}_2^2),$$

where  $I(\mathbf{q}_1^2, \mathbf{q}_2^2) = F_{\gamma^* \gamma^* \rightarrow \eta_c}(\mathbf{q}_1^2, \mathbf{q}_2^2) / (e_c^2 \sqrt{N_c})$ .

$$\begin{aligned} I(\mathbf{q}_1^2, \mathbf{q}_2^2) &= 4m_c \int \frac{dz d^2k}{z(1-z)16\pi^3} \psi_{\eta_c}(z, \mathbf{k}) \left\{ \frac{1-z}{(\mathbf{k} - (1-z)\mathbf{q}_2)^2 + z(1-z)\mathbf{q}_1^2 + m_c^2} \right. \\ &\quad \left. + \frac{z}{(\mathbf{k} + z\mathbf{q}_2)^2 + z(1-z)\mathbf{q}_1^2 + m_c^2} \right\}, \end{aligned}$$

# Off-diagonal gluons

KMR off-diagonal gluon:

[Int. J. Mod. Phys. A 29, 1430031 \(2014\)](#),

[arXiv:1405.0018 [hep-ph]],

[Eur. Phys. J. C 35, 211–220 \(2004\)](#)

$$\mathcal{F}_{g,\text{KMR}}^{\text{off}}(x_i, x', Q_\perp^2, q_{i\perp}^2, \mu^2, t_i) = R_g \frac{d}{d \ln q_\perp^2} \left[ x g(x, q_\perp^2) \sqrt{T_g(q_\perp^2, \mu^2)} \right]_{q_\perp^2 = Q_\perp^2} F(t),$$

$Q_{i\perp}^2 = \min(Q_\perp^2, q_{i\perp}^2)$ ,  $i = 1, 2$  - Durham  
prescription

$Q_{i\perp}^2 = \sqrt{Q_\perp^2 q_{i\perp}^2}$   $i = 1, 2$  - BPSS prescription

CDHI off-diagonal gluon:

[Eur. Phys. J. C 61, 369–390 \(2009\)](#)

$$\mathcal{F}_{g,\text{CDHI}}^{\text{off}}(x_i, x', Q_\perp^2, q_{i\perp}^2, \mu^2, t_i) = R_g \left[ \frac{\partial}{\partial \log \bar{Q}^2} \sqrt{T_g(\bar{Q}^2, \mu^2)} x g(x, \bar{Q}^2) \right] \cdot \frac{2Q_\perp^2 q_{i\perp}^2}{Q_\perp^4 + q_{i\perp}^4} \cdot F(t),$$

$$F(t) = \exp\left(\frac{bt}{2}\right), \quad b = 4 \text{ GeV}^{-2},$$

PST off-diagonal:

[Phys. Rev. D 78, 014007 \(2008\)](#).

$$\mathcal{F}_{g,\text{PST}}^{\text{off}}(x_i, x', Q_\perp^2, q_{i\perp}^2, \mu^2, t_i) = \sqrt{Q_\perp^2 f_g^{\text{GBW}}(x', Q_\perp^2) q_{i\perp}^2 f_g^{\text{GBW}}(x, q_\perp^2)} \sqrt{T_g(q_\perp^2, \mu^2)} F(t),$$

$$f_g^{\text{GBW}}(x, q_\perp^2) = \frac{3 \sigma_0}{4 \pi^2 \alpha_s} R_0^2 q_\perp^2 \exp[-R_0^2 q_\perp^2],$$

[J.HighEnergyPhys.03\(2018\)102.](#)

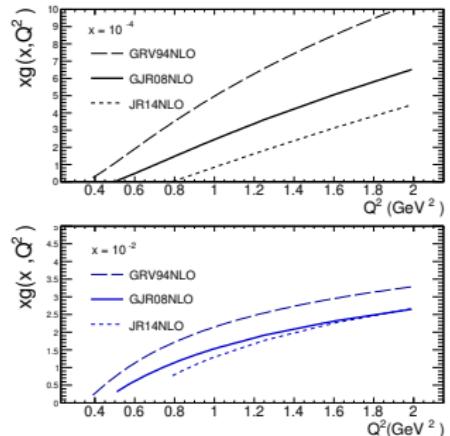
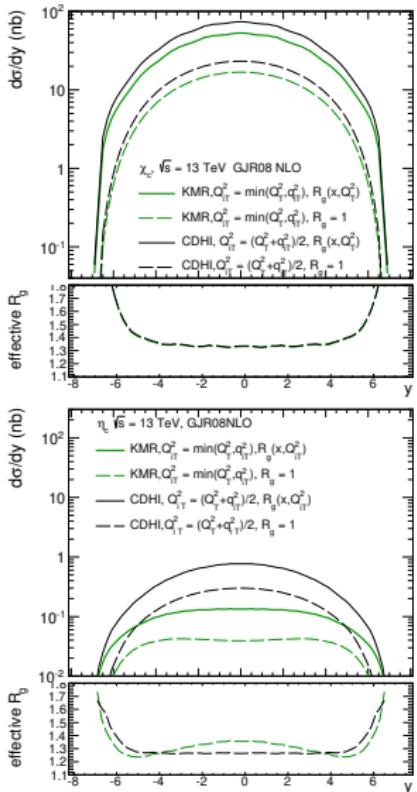
$$f_g^{RS}(x, |\mathbf{q}|) = \mathbf{q}^2 \frac{\sigma_0}{\alpha_s} \frac{N_c}{8\pi^2} \int_0^\infty r dr J_0(|\mathbf{q}|r) \left(1 - \frac{\sigma(x, r)}{\sigma_0}\right).$$

[Phys. Rev. D 88, 074016 \(2013\)](#)

$$T_g(q_\perp^2, \mu^2) = \exp \left[ - \int_{q_\perp^2}^{\mu^2} \frac{d\mathbf{k}_\perp^2}{\mathbf{k}_\perp^2} \frac{\alpha_s(k_\perp^2)}{2\pi} \right. \\ \times \left. \int_0^{1-\Delta} \left[ z P_{gg}(z) + \sum_q P_{qg}(z) \right] dz \right],$$

$$\mu^2 = M^2 + q_\perp^2.$$

# Effective skewedness correction



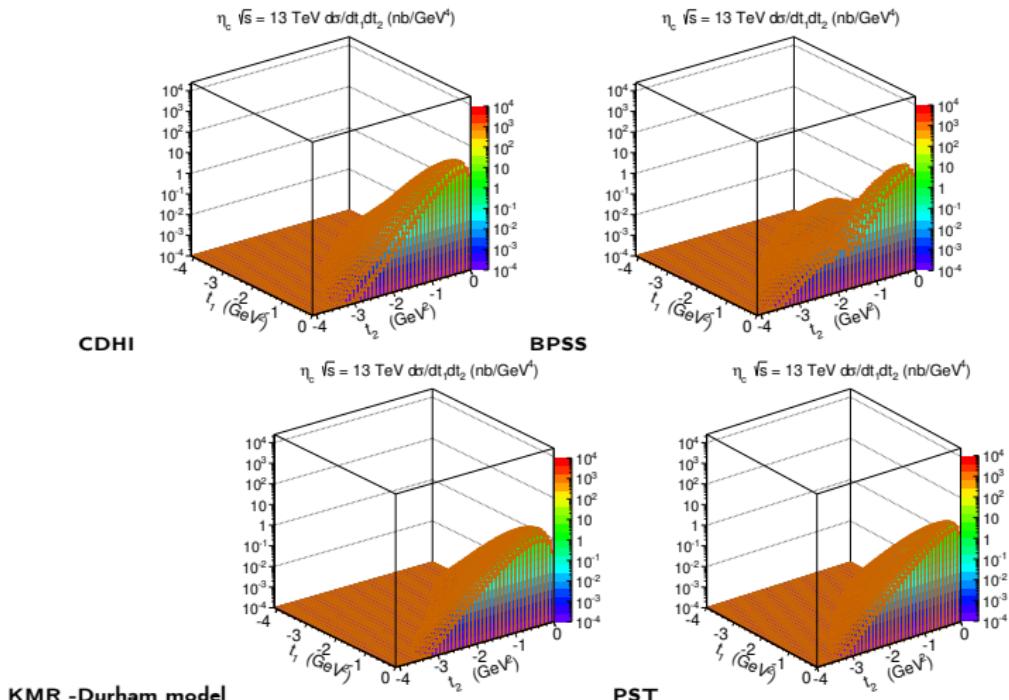
Shuvaev prescription

$$R_g = \frac{2^{2\lambda+3}}{\sqrt{\pi}} \frac{\Gamma(\lambda + 5/2)}{\Gamma(\lambda + 4)},$$

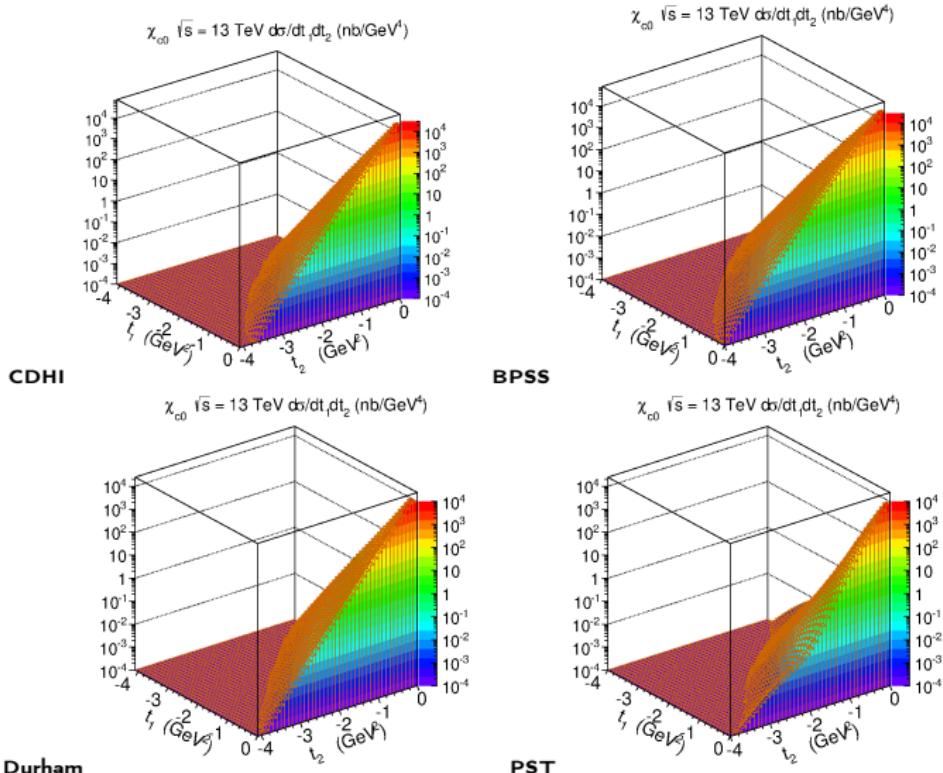
$$\lambda = \frac{d}{d \ln(1/x)} \left[ \ln \left( xg(x, q_\perp^2) \right) \right].$$

# $\eta_c$ - distribution in $t_1 \times t_2$ , $\sqrt{s} = 13 \text{ TeV}$

- in the limit  $t_1, t_2 \rightarrow 0$  the CEP vertex vanishes  $\Rightarrow$  the cross-section is heavily suppressed relatively to  $\chi_{c0}$
- On the other hand the cross-section strongly depends on  $\Psi_S(0)$ , while  $\chi_{c0} \Psi'_P(0)$



# $\chi_{c0}$ - distribution in $t_1 \times t_2$ , $\sqrt{s} = 13 \text{ TeV}$



# Total cross-section $pp \rightarrow p\eta_c p$

$\text{KMR Skewed gluon, } 0.8\text{GeV}^2 \leq Q_{0T}^2, \text{ JR14NLO}$	$\sigma_{\text{tot}} [\text{nb}], R_g = 1.0$	$\sigma_{\text{tot}} [\text{nb}], R_g(x, Q_{iT}^2)$
CDHI, $Q_{iT}^2 = (Q_T^2 + q_{iT}^2)/2.$	1.1	2.4
KMR, $Q_{iT}^2 = \sqrt{Q_T^2 \cdot q_{iT}^2}$	0.39	1.2
KMR, $Q_{iT}^2 = \min(Q_T^2, q_{iT}^2)$	0.13	0.25
$\text{KMR Skewed gluon, } 0.5\text{GeV}^2 \leq Q_{0T}^2, \text{ GJR08NLO}$	$\sigma_{\text{tot}} [\text{nb}], R_g = 1.0$	$\sigma_{\text{tot}} [\text{nb}], R_g(x, Q_{iT}^2)$
CDHI, $Q_{iT}^2 = (Q_T^2 + q_{iT}^2)/2.$	2.2	5.6
KMR, $Q_{iT}^2 = \sqrt{Q_T^2 \cdot q_{iT}^2}$	0.52	2.1
KMR, $Q_{iT}^2 = \min(Q_T^2, q_{iT}^2), 0.5\text{GeV}^2 \leq Q_{0T}^2$	0.44	1.3
KMR, $Q_{iT}^2 = \min(Q_T^2, q_{iT}^2), 0.8\text{GeV}^2 \leq Q_{0T}^2$	0.22	0.45
$\text{KMR Skewed gluon, } 0.4\text{GeV}^2 \leq Q_{0T}^2, \text{ GRV94NLO}$	$\sigma_{\text{tot}} [\text{nb}], R_g = 1.0$	$\sigma_{\text{tot}} [\text{nb}], R_g(x, Q_{iT}^2)$
CDHI, $Q_{iT}^2 = (Q_T^2 + q_{iT}^2)/2.$	$1.2 \cdot 10^2$	$7.8 \cdot 10^3$
KMR, $Q_{iT}^2 = \sqrt{Q_T^2 \cdot q_{iT}^2}$	2.2	$1.3 \cdot 10^3$
KMR, $Q_{iT}^2 = \min(Q_T^2, q_{iT}^2), 0.4\text{GeV}^2 \leq Q_{0T}^2$	2.8	$1.0 \cdot 10^1$
KMR, $Q_{iT}^2 = \min(Q_T^2, q_{iT}^2), 0.8\text{GeV}^2 \leq Q_{0T}^2$	1.25	2.9
PST Skewed gluon	$\sigma_{\text{tot}} [\text{nb}]$	-
PST, GBW	1.9	-
PST, RS	4.1	-

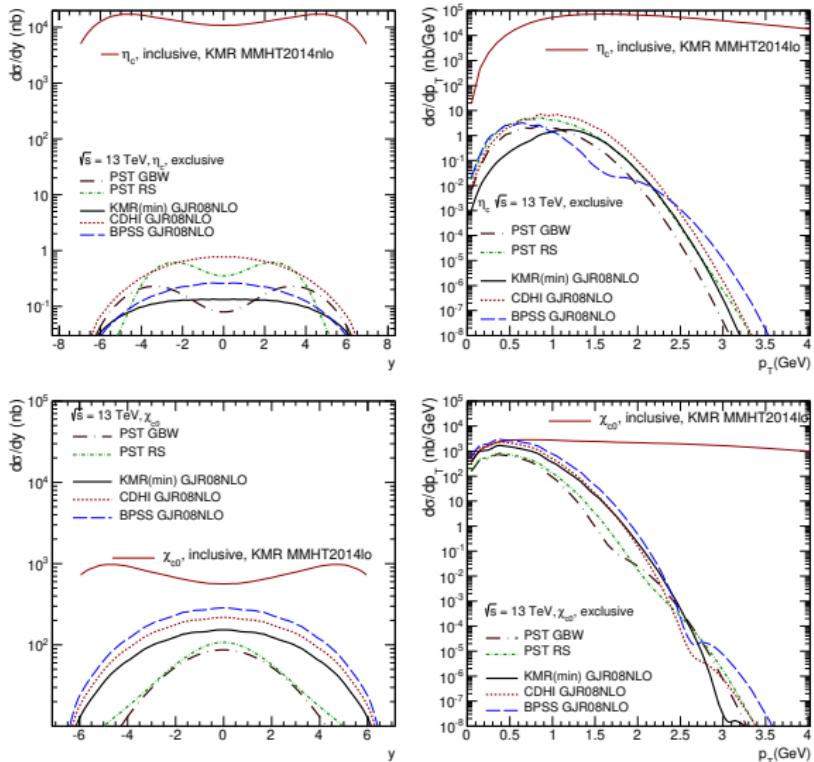
# Total cross-section $pp \rightarrow p\chi_{c0}p$

KMR Skewed gluon $0.8 \text{ GeV}^2 \leq Q_{0T}^2$ , JR14NLO CDHI, $Q_{iT}^2 = (Q_T^2 + q_{iT}^2)/2$ .	$\sigma_{\text{tot}} [\text{nb}], R_g = 1.0$ $0.42 \cdot 10^3$	$\sigma_{\text{tot}} [\text{nb}], R_g(x, Q_{iT}^2)$ $1.1 \cdot 10^3$
KMR, $Q_{iT}^2 = \sqrt{Q_T^2 \cdot q_{iT}^2}$	$0.36 \cdot 10^3$	$0.94 \cdot 10^3$
KMR, $Q_{iT}^2 = \min(Q_T^2, q_{iT}^2)$	$0.20 \cdot 10^3$	$0.52 \cdot 10^3$
KMR Skewed gluon $0.5 \text{ GeV}^2 \leq Q_{0T}^2$ , GJR08NLO CDHI, $Q_{iT}^2 = (Q_T^2 + q_{iT}^2)/2$ .	$\sigma_{\text{tot}} [\text{nb}], R_g = 1.0$ $0.46 \cdot 10^3$	$\sigma_{\text{tot}} [\text{nb}], R_g(x, Q_{iT}^2)$ $1.57 \cdot 10^3$
KMR, $Q_{iT}^2 = \sqrt{Q_T^2 \cdot q_{iT}^2}$	$0.64 \cdot 10^3$	$2.1 \cdot 10^3$
KMR, $Q_{iT}^2 = \min(Q_T^2, q_{iT}^2)$	$0.34 \cdot 10^3$	$1.1 \cdot 10^3$
KMR Skewed gluon $0.4 \text{ GeV}^2 \leq Q_{0T}^2$ , GRV94NLO CDHI, $Q_{iT}^2 = (Q_T^2 + q_{iT}^2)/2$ .	$\sigma_{\text{tot}} [\text{nb}], R_g = 1.0$ $1.88 \cdot 10^3$	$\sigma_{\text{tot}} [\text{nb}], R_g(x, Q_{iT}^2)$ $9.02 \cdot 10^3$
KMR, $Q_{iT}^2 = \sqrt{Q_T^2 \cdot q_{iT}^2}$	$3.03 \cdot 10^3$	$13.4 \cdot 10^3$
KMR, $Q_{iT}^2 = \min(Q_T^2, q_{iT}^2)$ , $0.4 \text{ GeV}^2 \leq Q_{0T}^2$	$1.4 \cdot 10^3$	$6.1 \cdot 10^3$
KMR, $Q_{iT}^2 = \min(Q_T^2, q_{iT}^2)$ , $0.8 \text{ GeV}^2 \leq Q_{0T}^2$	$0.75 \cdot 10^3$	$3.9 \cdot 10^3$
PST Skewed gluon	$\sigma_{\text{tot}} [\text{nb}]$	-
PST prescription, GBW	$0.44 \cdot 10^3$	-
PST prescription, RS	$0.52 \cdot 10^3$	-

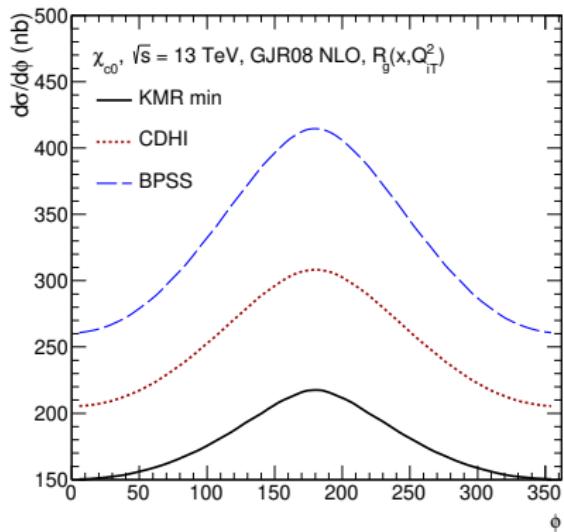
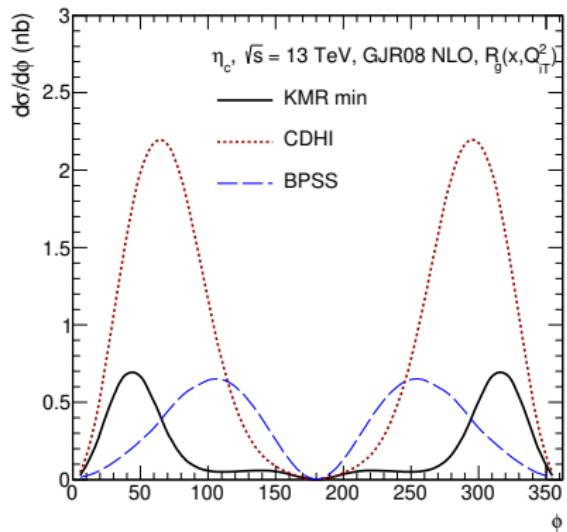
$$Br(\chi_{c0} \rightarrow J/\Psi \gamma) = (1.40 \pm 0.05)\%$$

$$Br(\chi_{c1} \rightarrow J/\Psi \gamma) = (34.3 \pm 1.0)\%$$

# Exclusive vs. inclusive distributions



# Azimuthal angle distribution



$$|\mathcal{M}(g^* g^* \rightarrow \eta_c)|^2 \sim \sin^2(\phi)$$

# Absorptive correction to $pp \rightarrow pVp$ processes

$$\mathcal{M}(Y, y, \mathbf{p}_1, \mathbf{p}_2) = \mathcal{M}^{(0)}(Y, y, \mathbf{p}_1, \mathbf{p}_2) - \delta\mathcal{M}(Y, y, \mathbf{p}_1, \mathbf{p}_2),$$

$$\begin{aligned}\mathcal{M}^{(0)}(Y, y, \mathbf{p}_1, \mathbf{p}_2) &= i s \Phi_1(\mathbf{p}_1) R_{\mathbf{P}}(Y-y, \mathbf{p}_1^2) \\ &\times V(\mathbf{p}_1, \mathbf{p}_2) R_{\mathbf{P}}(y, \mathbf{p}_2^2) \Phi_2(\mathbf{p}_2)\end{aligned}$$

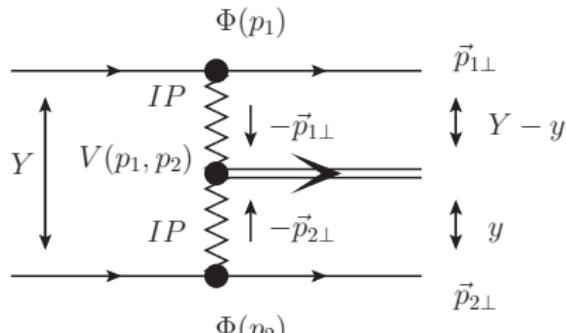
$$\begin{aligned}\delta\mathcal{M}(Y, 0, \mathbf{p}_1, \mathbf{p}_2) &= \\ &\int \frac{d^2 k}{2(2\pi)^2} T(s, k) \exp\left(-\frac{1}{2} B_D(\mathbf{p}_1 + \mathbf{k})^2\right) \\ &\exp\left(-\frac{1}{2} B_D(\mathbf{p}_2 - \mathbf{k})^2\right) \times V(\mathbf{p}_1 + \mathbf{k}, \mathbf{p}_2 - \mathbf{k})\end{aligned}$$

$$\begin{aligned}T(s, k) &= \sigma_{\text{tot}}^{pp}(s) \\ &\times \exp\left(-\frac{1}{2} B_{\text{el}}(s) k^2\right)\end{aligned}$$

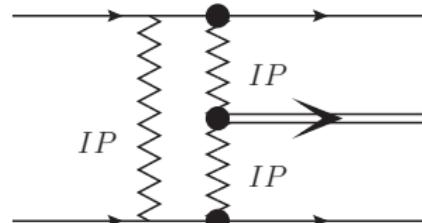
$$\sqrt{s} = 13 \text{ TeV} \Rightarrow \sigma_{\text{tot}}^{pp} = (110.6 \pm 3.4) \text{ mb}, B_{\text{el}} = (20.36 \pm 0.19) \text{ GeV}^{-2}$$

G. Antchev *et al.* [TOTEM Collaboration],

[Eur. Phys. J. C 79, no.2, 103 \(2019\)](#)

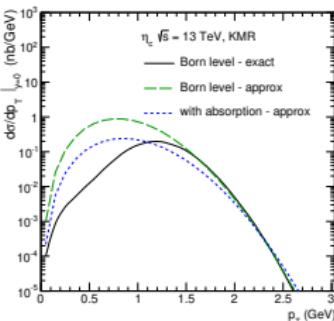
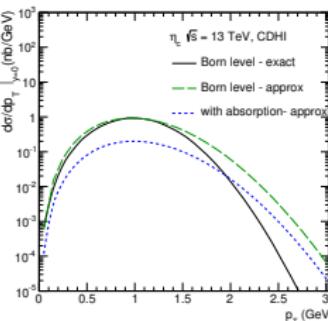
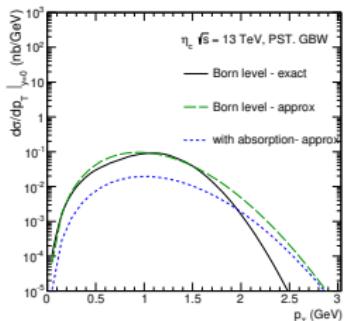
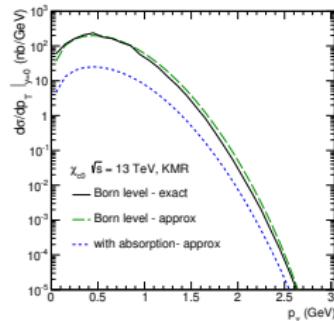
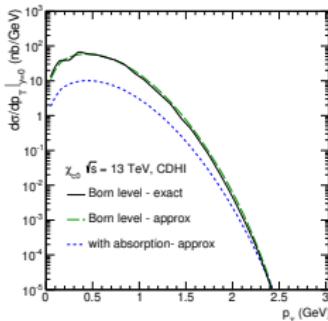
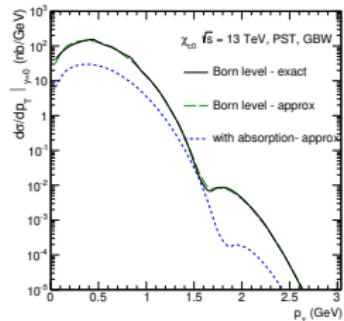


Born level diagram



absorptive correction

# Absorptive correction - results



# Gap survival probability at mid rapidity

$\chi_{c0}$	$\frac{d\sigma}{dy}_{tot} _{y=0}$ [nb]	$\frac{d\sigma}{dy}_{tot}^{abs} _{y=0}$ [nb]	$S^2_{y=0}$
PST GBW	17	3.7	0.22
PST RS	21	4.5	0.21
CDHI GJR08NLO	42	7.5	0.18
KMR GJR08NLO	29	3.7	0.13
BPSS GJR08NLO	61	8.0	0.13

$\eta_c$	$\frac{d\sigma}{dy}_{tot} _{y=0}$ [nb]	$\frac{d\sigma}{dy}_{tot}^{abs} _{y=0}$ [nb]	$S^2_{y=0}$
PST GBW	$1.8 \times 10^{-2}$	$3.9 \times 10^{-3}$	0.22
PST RS	$9.0 \times 10^{-3}$	$1.9 \times 10^{-3}$	0.21
CDHI GJR08NLO	$1.8 \times 10^{-1}$	$4.0 \times 10^{-2}$	0.22
KMR GJR08NLO	$1.3 \times 10^{-1}$	$3.0 \times 10^{-2}$	0.23
BPSS GJR08NLO	$5.8 \times 10^{-2}$	$2.2 \times 10^{-2}$	0.38

$$S^2 \equiv \frac{\frac{d\sigma}{dy}|_{y=0}}{\frac{d\sigma_{Born}}{dy}|_{y=0}}$$

- Transition amplitude for  $g^*g^* \rightarrow \eta_c$  and  $g^*g^* \rightarrow \chi_{c0}$  was calculated using light-cone wave functions of  $c\bar{c}$  states in the framework of potential models.
- We also proposed a way to calculate the soft effects (in the region of small gluon transverse momenta) using the GBW or RS UGDs, which were obtained from the respective color dipole cross sections and a simple (PST) prescription for its off-diagonal extrapolation.
- Central exclusive processes in proton-proton collisions are sensitive to the low scale, especially  $\eta_c$ , which is the main uncertainty in the results.
- In our calculation of absorptive corrections, we restricted ourselves to the so-called elastic rescattering correction.
- Depending on the gluon distribution used, we obtain for the  $\chi_c$  the gap survival values of  $S^2 = (0.13 - 0.21)$ , while for the  $\eta_c$  production, they are somewhat higher,  $S^2 = (0.21 - 0.38)$ .