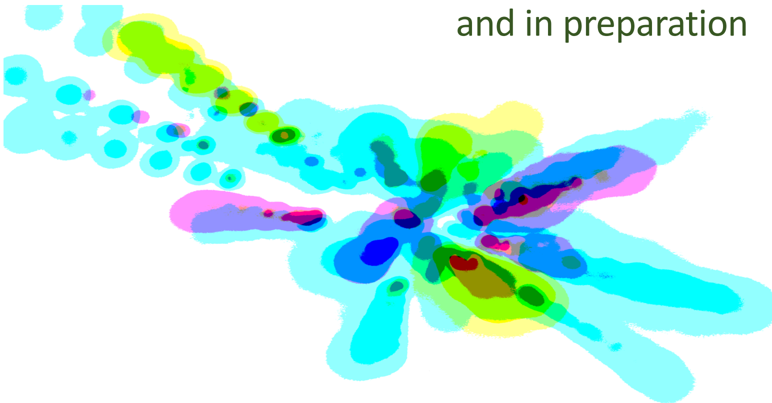
Topological screening and sphaleron transitions in polarized DIS at the Electron-Ion Collider

Talk based on work with A. Tarasov,
arXiv: 2008.08104 (PRD 2021), 2109.10370 (PRD 2022)
and in preparation



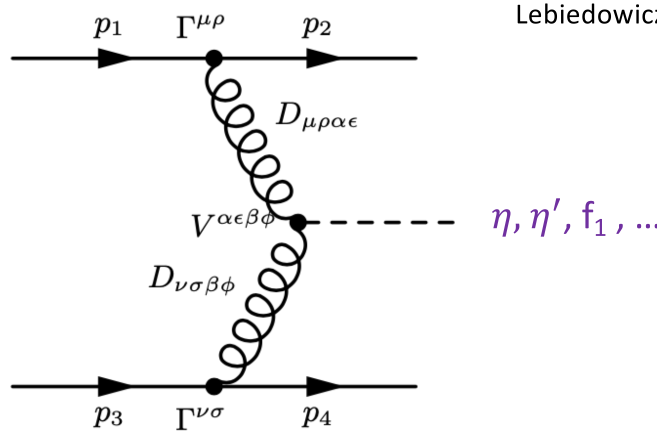
Raju Venugopalan

Brookhaven National Laboratory

Forward physics in ALICE 3 Workshop, October 23, 2023, Heidelberg

Prolegomena

Shuryak, Zahed, arXiv: hep-ph/0302231
Anderson,Domokos,Harvey,Mann,arXiv:1406.7010
Lebiedowicz,Leutgeb,Nachtmann,Rebhan,Szczurek,PRD (2020)



Considerable work in exploring the role of topology in proton-proton collisions:
Central Exclusive Production especially promising…

Much of the work is in intrinsically nonperturbative kinematics – very interesting application of holography methods in this context

My focus is this talk is on DIS – where a hard scale is present – and the role of topology there

There are interesting lessons – but I have no idea of how these can be quanitatively ported back to p+p and p+A - this workshop is motivating in that regard

Talk outline

The proton's spin puzzle and the chiral anomaly

Fun with worldlines: the anomaly pole dominates at large and small x

WZW term for a prodigal ninth Goldstone → an axionlike effective action

Spin and the $U_A(1)$ problem: The Goldberger- Treiman relation and topological mass generation of the η'

Spin damping at small x: sphaleron transitions induced by gluon saturation

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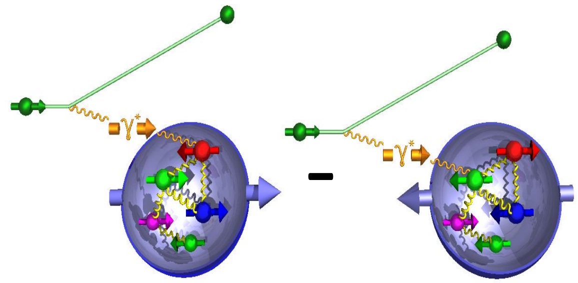
Takeaway: The proton's spin is deeply influenced by the topology of the QCD vacuum
- in particular, its features that are responsible for the large mass of the η' meson

Polarized DIS at the Electron-Ion Collider has the potential for
discovery of real-time topological (sphaleron-like) transitions

Resolving the proton's spin puzzle: the g_1 structure function

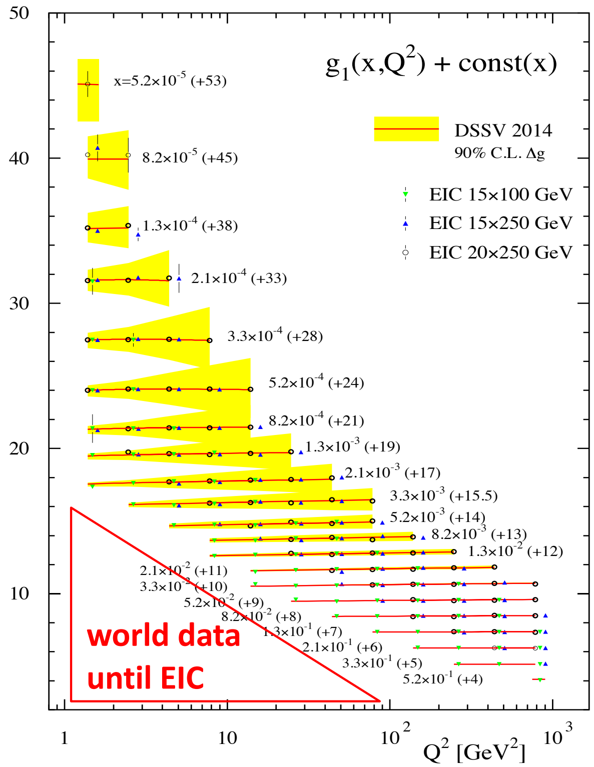
g_1 extracted from longitudinal spin asymmetry in polarized DIS

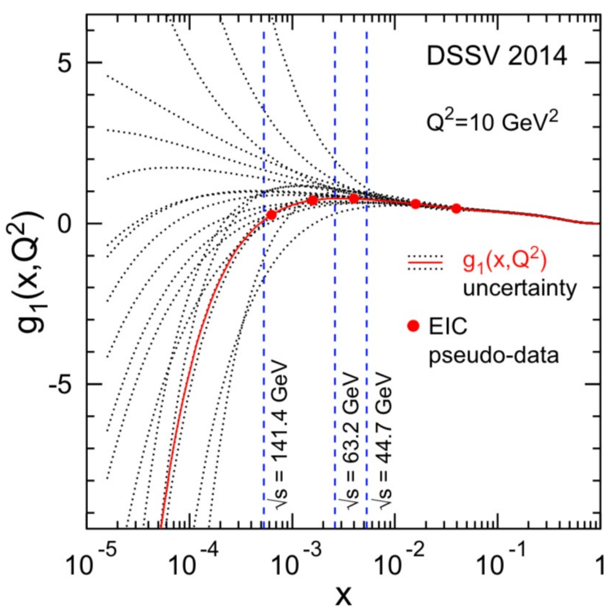


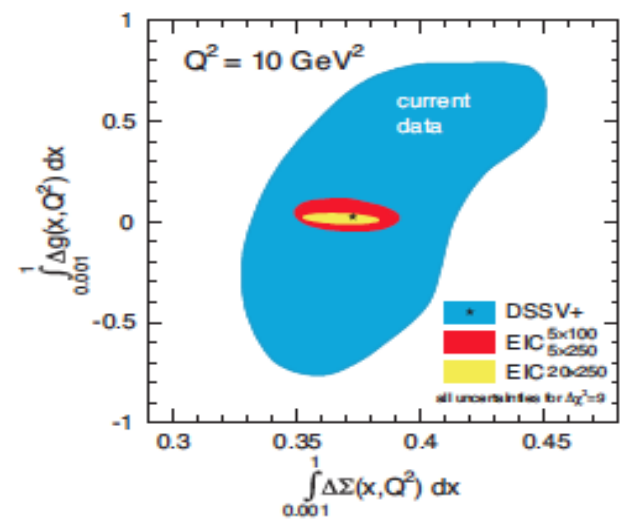
Proton's helicity: isosinglet first moment

$$\Delta\Sigma(Q^2) \propto \int_0^1 dx\, g_1(x, Q^2)$$

Spin "puzzle": why is measured ΔΣ much smaller than quark model expectations

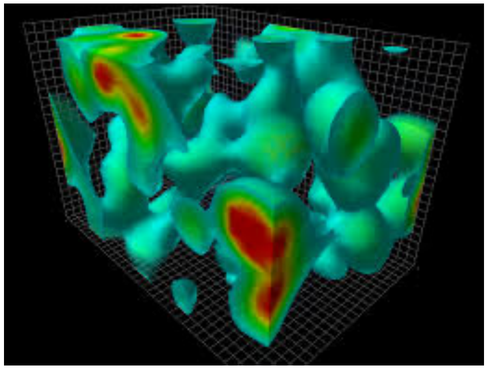






Iso-singlet axial vector current and the chiral anomaly

$$S^{\mu}\,\Delta\Sigma \;=\; \langle P,S\,|\,\bar{\psi}\,\gamma^{\mu}\gamma_5\,\psi\,|\,P,S\rangle \;\equiv\; \langle P,S\,|\,j_5^{\mu}\,|\,P,S\rangle$$



$U_A(1)$ violation from the chiral anomaly:

$$\partial_\mu J_5^\mu = 2n_f\partial_\mu K^\mu + \sum_{i=1}^{n_f} 2im_i\bar{q}_i\gamma_5 q_i$$

where the Chern-Simons current $K_\mu = \frac{g^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\left[A_a^\nu\left(\partial^\rho A_a^\sigma - \frac{1}{3}g f_{abc}A_b^\rho A_c^\sigma\right)\right]$

One suggested explanation of the "spin puzzle" of small $\Delta\Sigma$ is the identification of K_μ with ΔG

ca., 1988
Efremov, Teryaev
Altarelli, Ross
Carlitz, Collins, Mueller

However K_μ is local but not gauge-invariant (under large gauge transformations) while ΔG is non-local but gauge-invariant - - strongly hints key role of topology

Jaffe, 2007 Varenna lectures
Review:S.D. Bass, RMP, hep-ph/0411005

The resolution of the $U_A(1)$ problem regularizes the triangle

UA(1) problem: why is there no isosinglet Goldstone boson
or why is the η' so massive ?

t'Hooft (1976); Witten; Veneziano (1979)

R. L. Jaffe

A. Manohar

From a classic paper by Jaffe and Manohar on the proton's spin

> The authors of refs. [12, 13] suggest that the triangle diagram provides a *local* probe of the gluon distribution in the target. If this were true, $\Delta\Gamma$ would be protected from infrared problems and the calculation would be reliable in the usual sense. However, we believe there are strong arguments that the triangle is not local in the sense required. It is therefore not necessarily protected from infrared effects, in particular from the non-perturbative effects which give the η' a mass★.

The "spin problem" is deeply tied to the "$U_A(1)$ problem"

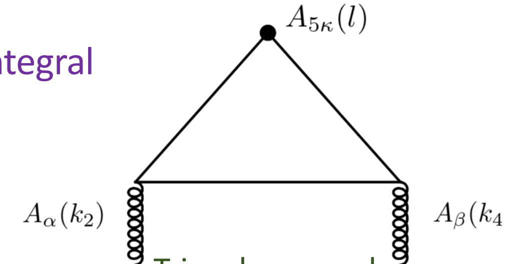
Almost completely ignored in recent perturbative QCD literature
(only ~25 citations since 2000 until our work)

Veneziano

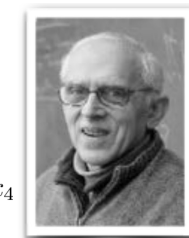
Adler-Bell-Jackiw chiral anomaly from worldlines

Key insight from Fujikawa:
Anomaly arises from the non-invariance of the path integral measure under chiral (γ_5) rotations

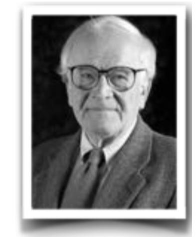
One loop QCD worldline action:
Axial vector source term resolves imaginary part-phase of the Dirac determinant



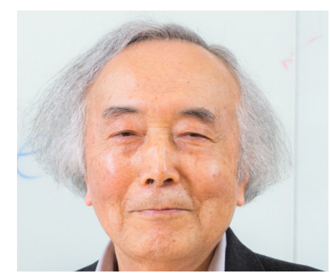
Triangle anomaly


Steven Adler


John S. Bell


Roman Jackiw


William A. Bardeen


Kazuo Fujikawa

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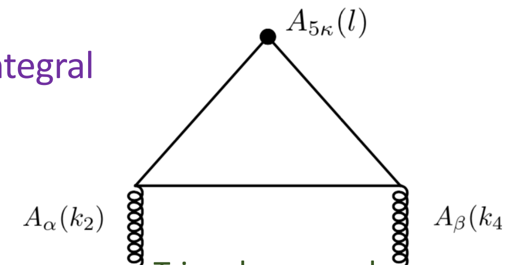
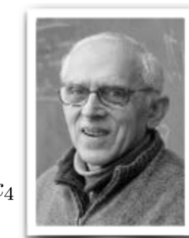


Triangle anomaly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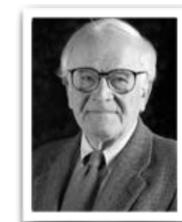

Roman Jackiw

Point particle Bose and Grassmann path integrals

McKeon,Schubert, PLB (1998)

$$\Gamma[A,A_5] = -\frac{1}{2}\mathrm{Tr}_c\int_0^\infty \frac{dT}{T}\int \mathcal{D}x\int_{AP}\mathcal{D}\psi$$

$$\times\exp\Big\{-\int_0^T d\tau\Big(\frac{1}{4}\dot{x}^2+\frac{1}{2}\psi_\mu\dot{\psi}^\mu+ig\dot{x}^\mu A_\mu-ig\psi^\mu\psi^\nu F_{\mu\nu}-2i\psi_5\dot{x}^\mu\psi_\mu\psi_\nu A_5^\nu+i\psi_5\partial_\mu A_5^\mu+(D-2)A_5^2\Big)\Big\}$$

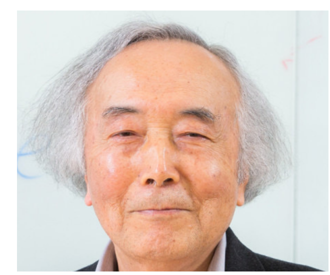
Wilson line — Spin precession — "re-exponentiated" axial vector couplings

$$\langle P',S|J_5^\kappa|P,S\rangle = \int d^4y\frac{\partial}{\partial A_{5\kappa}(y)}\Gamma[A,A_5]\Big|_{A_5=0}e^{ily}\equiv\Gamma_5^\kappa[l]$$

$$=\frac{1}{4\pi^2}\frac{l^\kappa}{l^2}\int\frac{d^4k_2}{(2\pi)^4}\int\frac{d^4k_4}{(2\pi)^4}\,\mathrm{Tr}_cF_{\alpha\beta}(k_2)\tilde{F}^{\alpha\beta}(k_4)\,(2\pi)^4\delta^4(l+k_2+k_4)$$

Famous infrared pole of anomaly. One loop exact: Adler-Bardeen theorem

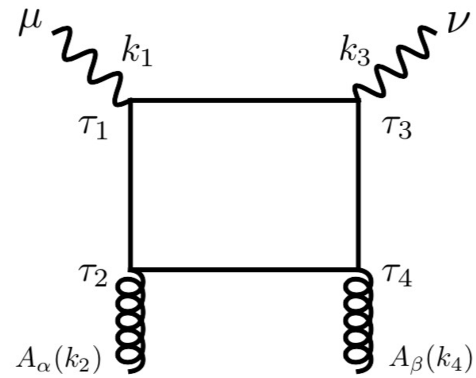

William A. Bardeen


Kazuo Fujikawa

Worldline formalism: box diagram for polarized DIS ($g_1(x,Q^2)$)

Hadron tensor in DIS: $W^{\mu\nu}(q,P,S) = \frac{1}{2\pi}\int d^4x\, e^{iqx}\langle P,S|j^\mu(x)j^\nu(0)|P,S\rangle$

Anti-symmetric part: $\tilde{W}_{\mu\nu}(q,P,S) = \frac{2M_N}{P\cdot q}\epsilon_{\mu\nu\alpha\beta}q^\alpha\left\{\boxed{S^\beta g_1(x_B,Q^2)} + \left[S^\beta - \frac{(S\cdot q)P^\beta}{P\cdot q}\right]g_2(x_B,Q^2)\right\}$



Isosinglet contribution to hadron tensor

$$g_1 \propto \Gamma_A^{\mu\nu}[k_1,k_3] = \int\frac{d^4k_2}{(2\pi)^4}\int\frac{d^4k_4}{(2\pi)^4}\,\Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4]\,\mathrm{Tr_c}(\tilde{A}_\alpha(k_2)\tilde{A}_\beta(k_4))$$

Polarization tensor (antisymmetric piece)

Box diagram

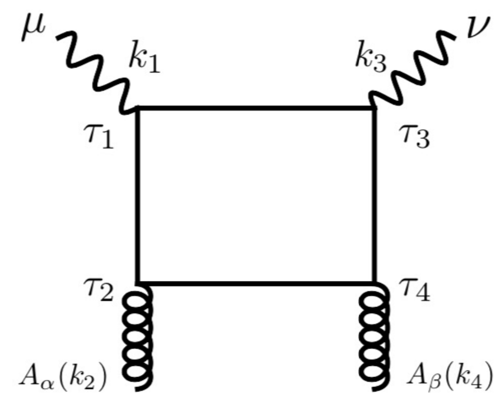
$$\Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4] = -\frac{g^2e^2e_f^2}{2}\int_0^\infty\frac{dT}{T}\int\mathcal{D}x\int\mathcal{D}\psi\,\exp\left\{-\int_0^T d\tau\left(\frac{1}{4}\dot{x}^2+\frac{1}{2}\psi\cdot\dot{\psi}\right)\right\}$$
$$\times\prod_{k=1}^4\int_0^T d\tau_k\,\boxed{\left[\sum_{n=1}^9 \mathcal{C}^{\mu\nu\alpha\beta}_{n;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4]-(\mu\leftrightarrow\nu)\right]}e^{i\sum_{i=1}^4 k_ix_i}\,.$$

Can compute exactly from *real* part of worldline effective action

DIS with worldlines:
Tarasov, RV, 1903.11624,
2008.08104, 2109.10370

The box diagram for polarized DIS ($g_1(x,Q^2)$)

$$\mathcal{C}^{\mu\nu\alpha\beta}_{1;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -4\dot{x}_3^{\nu}\psi_1^{\mu}\psi_1\cdot k_1\dot{x}_4^{\beta}\psi_2^{\alpha}\psi_2\cdot k_2;$$
$$\mathcal{C}^{\mu\nu\alpha\beta}_{2;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -4\dot{x}_3^{\nu}\psi_1^{\mu}\psi_1\cdot k_1\dot{x}_2^{\alpha}\psi_4^{\beta}\psi_4\cdot k_4;$$
$$\mathcal{C}^{\mu\nu\alpha\beta}_{3;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -4\dot{x}_1^{\mu}\psi_3^{\nu}\psi_3\cdot k_3\dot{x}_2^{\alpha}\psi_4^{\beta}\psi_4\cdot k_4;$$
$$\mathcal{C}^{\mu\nu\alpha\beta}_{4;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -4\dot{x}_1^{\mu}\psi_3^{\nu}\psi_3\cdot k_3\dot{x}_4^{\beta}\psi_2^{\alpha}\psi_2\cdot k_2$$
$$\mathcal{C}^{\mu\nu\alpha\beta}_{5;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -8i\dot{x}_3^{\nu}\psi_1^{\mu}\psi_1\cdot k_1\psi_2^{\alpha}\psi_2\cdot k_2\psi_4^{\beta}\psi_4\cdot k_4;$$
$$\mathcal{C}^{\mu\nu\alpha\beta}_{6;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -8i\dot{x}_1^{\mu}\psi_3^{\nu}\psi_3\cdot k_3\psi_2^{\alpha}\psi_2\cdot k_2\psi_4^{\beta}\psi_4\cdot k_4$$
$$\mathcal{C}^{\mu\nu\alpha\beta}_{7;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -8i\dot{x}_4^{\beta}\psi_2^{\alpha}\psi_2\cdot k_2\psi_1^{\mu}\psi_1\cdot k_1\psi_3^{\nu}\psi_3\cdot k_3;$$
$$\mathcal{C}^{\mu\nu\alpha\beta}_{8;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = -8i\dot{x}_2^{\alpha}\psi_4^{\beta}\psi_4\cdot k_4\psi_1^{\mu}\psi_1\cdot k_1\psi_3^{\nu}\psi_3\cdot k_3$$
$$\mathcal{C}^{\mu\nu\alpha\beta}_{9;(\tau_1,\tau_2,\tau_3,\tau_4)}[k_1,k_3,k_2,k_4] = 16\psi_1^{\mu}\psi_1\cdot k_1\psi_3^{\nu}\psi_3\cdot k_3\psi_2^{\alpha}\psi_2\cdot k_2\psi_4^{\beta}\psi_4\cdot k_4$$



The box diagram for polarized DIS ($g_1(x,Q^2)$)

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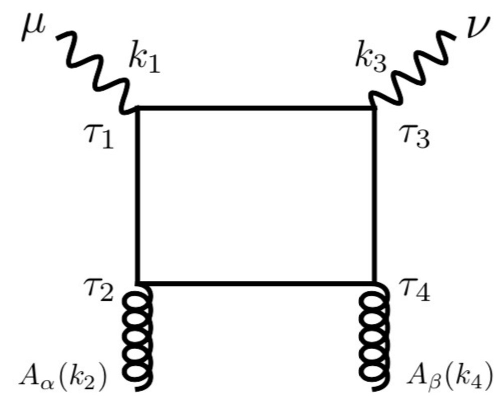
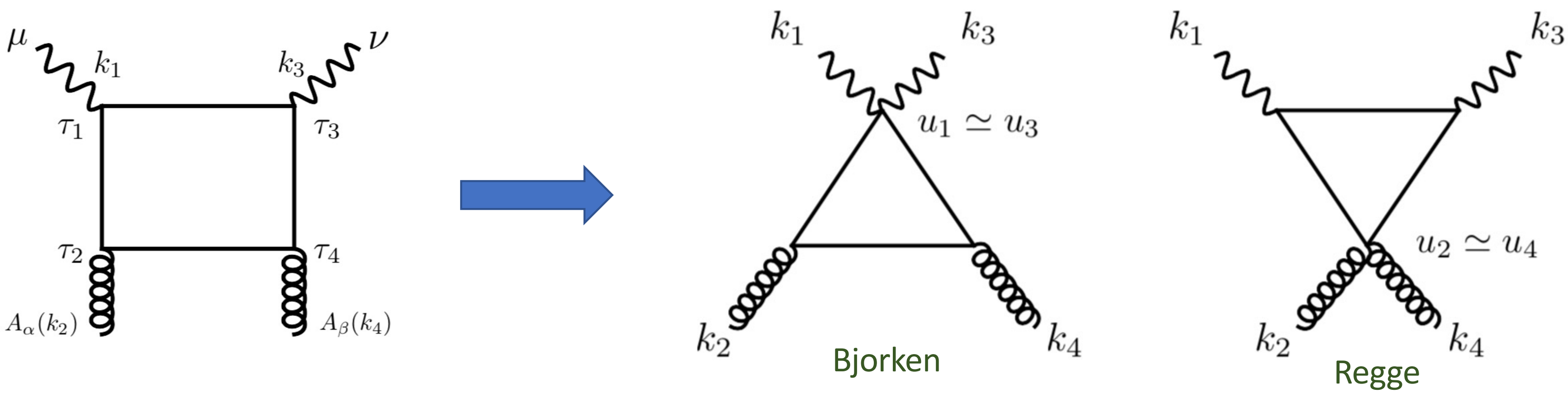


Tarasov, RV, 1903.11624, 2008.08104

Can compute corresponding boson and Grassmann integrals explicitly
using worldline integration techniques – equivalent to efficient Feynman diagram computation

Essential to perform computation in exact off-forward kinematics (with no kinematic approximations of internal variables)
to explore anomaly structure in both Bjorken ($Q^2 \to \infty, s \to \infty, x = \text{fixed}$) and Regge ($x \to 0, s \to \infty, Q^2 = \text{fixed}$) asymptotics

Finding triangles in boxes in Bjorken and Regge asymptotics



Remarkably, box diagram for $g_1(x_B,Q^2)$ has same structure in both limits - dominated by the triangle anomaly !

$$S^\mu g_1(x_B,Q^2)\Big|_{Q^2\to\infty} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N}\int_{x_B}^1 \frac{dx}{x}\left(1-\frac{x_B}{x}\right)\int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu \to 0}\frac{l^\mu}{l^2}\langle P', S|\mathrm{Tr_c} F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P,S\rangle$$ + non-pole contributions

$$S^\mu g_1(x_B,Q^2)|_{x_B\to 0} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N}\int_{x_B}^1 \frac{dx}{x}\int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu \to 0}\frac{l^\mu}{l^2}\langle P', S|\mathrm{Tr_c} F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P,S\rangle$$ + non-pole contributions

Hence g_1 is topological in both asymptotic limits of QCD…

Tarasov, RV, arXiv:2008.08104
See also Bhattacharya, Hatta, Vogelsang, arXiv:2210.13419
and arXiv:2305.09431

Global anomalies in the worldline formulation of QFT

Fermion action in background of scalar, pseudoscalar, vector and axial vector fields:
(with focus on $U_A(1)$ sector)

$$S_{\text{fermion}}[\bar{\Psi},\Phi,\Pi,A,B,\Psi] = \int d^4x\,\bar{\Psi}^I\left[i\partial\!\!\!/ - \Phi + i\gamma^5\Pi + A\!\!\!/ + \gamma^5 B\!\!\!/\right]^{IJ}\Psi^J$$

Effective action: $-\mathcal{W}[A,B,\Phi,\Pi] = \mathrm{Ln}\,\mathrm{Det}\,[\mathcal{D}]$ with $\mathcal{D} = p\!\!\!/ - i\Phi(x) - \gamma_5\,\Pi - A\!\!\!/ - \gamma_5 B\!\!\!/$

Split into real and imaginary parts: $\mathcal{W}_R = -\frac{1}{2}\mathrm{Ln}\left(\mathcal{D}^\dagger\mathcal{D}\right)$; $\mathcal{W}_I = \frac{1}{2}\mathrm{Arg}\,\mathrm{Det}\left(\mathcal{D}^2\right)$

Entire dynamics of the anomaly comes from W_I - the phase of the Dirac determinant

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Remarkable observation:

W_I can also be expressed as a worldline Lagrangian of 0+1- bosonic (coordinate) and Grassmann fields

D'Hoker, Gagne, hep-th/9508131

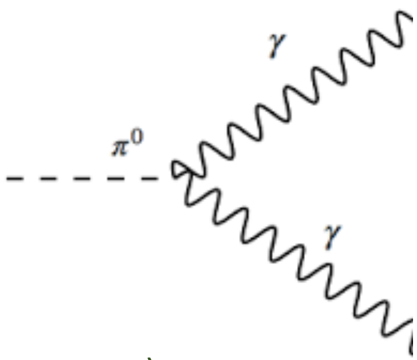
Jacobian for zero modes multiplied by G-parity factor

$$W_{\mathcal{I}} = -\frac{i}{32} \int_{-1}^{1} d\alpha \int_0^{\infty} dT\, \mathcal{N} \int_{\text{PBC}} \mathcal{D}x\, \mathcal{D}\psi\ \text{tr}\ \chi\, \bar{\omega}(0) \exp\left[-\int_0^T d\tau \mathcal{L}_{(\alpha)}(\tau) \right]$$

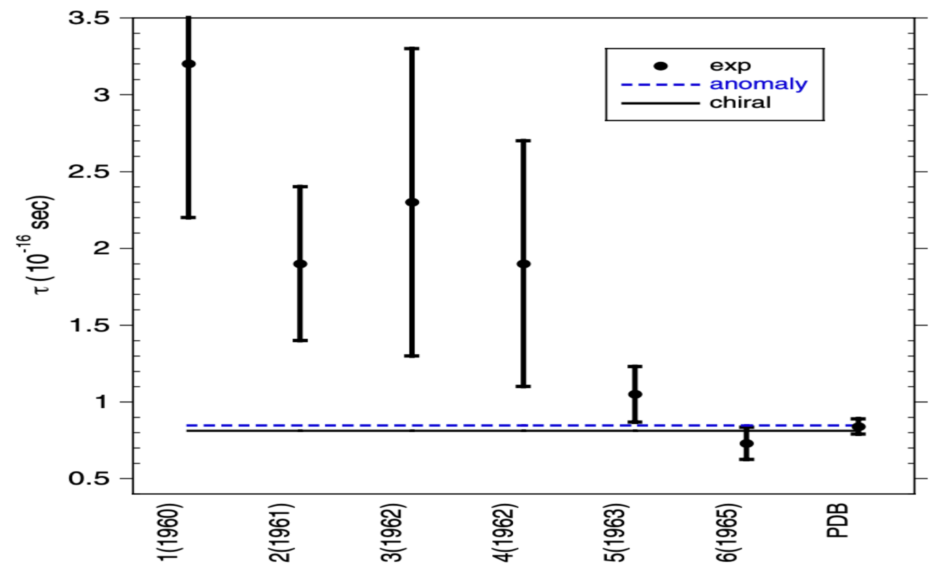
Worldline Lagrangian
with chiral symmetry breaking interpolating parameter α

A big role for a phase in pole cancellation: the WZW isosinglet term

Famous WZW term from imaginary part of the worldline action: $\pi^0 \rightarrow 2\,\gamma$ d'Hoker,Gagne (1995,1996)



Bernstein and Holstein,RMP (2013)



Compute explicitly, from phase of determinant, the WZW term in the isosinglet channel

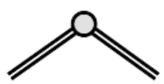
$$S^{\bar{\eta}}_{\rm WZW} = -i\frac{\sqrt{2\,n_f}}{F_{\bar{\eta}}}\int d^4x\,\bar{\eta}\,\Omega$$

Ω is the topological charge density and F_η is the $\bar{\eta}$ decay constant

Agrees exactly with expression in nonet Ch.PT. Kaiser, Leutwler (2000)

A big role for a phase in pole cancellation: the WZW isosinglet term

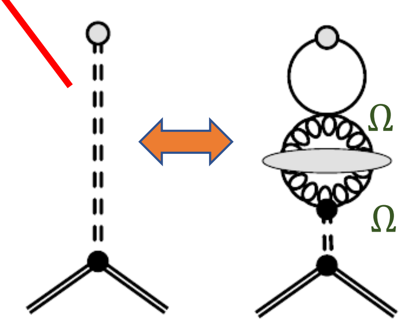
$$\langle P', S | J_5^\mu | P, S \rangle = \bar{u}(P', S)\left[\gamma^\mu \gamma_5 G_A(l^2) + l^\mu \gamma_5 G_P(l^2)\right] u(P, S)$$



Direct axial vector coupling of J_5^μ to polarized proton

Massless "prodigal Goldstone" $\bar{\eta}$ field

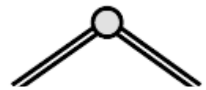
Triangle coupling to QCD topological susceptibility



Pseudoscalar coupling of polarized proton to J_5^μ

Goldberger-Treiman relation and anomaly cancellation

I) Consider first the direct axial vector coupling:



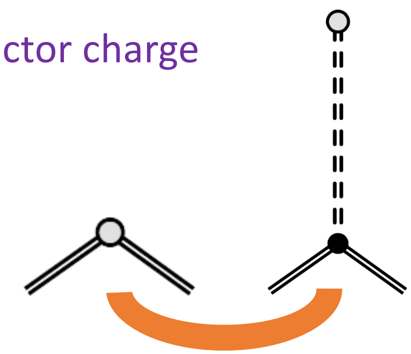
Since $G_P(0)$ cannot have a pole, trivially, $\langle P,S|J_5^\mu|P,S\rangle = \langle P,S|J_5^\mu|P,S\rangle|_{\mathrm{Fig.2a}} = 2M_N\,G_A(0)\,S^\mu$

$$\Sigma(Q^2) = 2\,G_A(0)$$

The helicity of the proton is twice its axial vector charge

II) The anomaly equation + the Dirac equation link the axial vector and pseudoscalar channels: Goldberger-Treiman relation

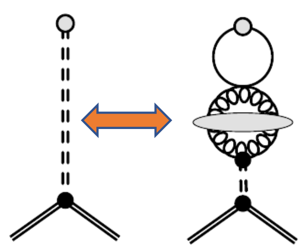
$$G_A(0) = \frac{\sqrt{2\tilde{n}_f}}{2M_N}\,F_{\tilde{\eta}}\,g_{\eta_0 NN}$$



Veneziano (1989)

$g_{\eta_0 \mathrm{NN}}$ is coupling of isosinglet field to proton

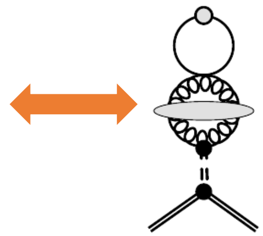
III) Absence of a pseudoscalar pole also implies…



$$\sqrt{2\tilde{n}_f}\,F_{\tilde{\eta}} = 2n_f\,\lim_{l\to 0} i\,\langle 0|T\,\Omega\eta_0|0\rangle$$

Topological mass generation from the WZW term

Can write $\langle 0|T\Omega\eta_0|0\rangle = -i\frac{1}{l^2}\frac{\sqrt{2\tilde{n}_f}}{F_{\bar{\eta}}}\chi(l^2)$



with QCD topological susceptibility

$$\chi(l^2) = i\int d^4x\, e^{ilx}\langle 0|T\,\Omega(x)\Omega(0)|0\rangle$$

1/N corrections to YM topological susceptibility induced by WZW coupling
... generates QCD top. susceptibility

$$\chi(l^2) = l^2\frac{1}{l^2 - m_{\eta'}^2}\chi_{\rm YM}(l^2) \quad \text{with} \quad m_{\eta'}^2 \equiv -\frac{2\,n_f}{F_{\bar{\eta}}^2}\chi_{\rm YM}(0)$$

Witten-Veneziano formula

Vanishes when $l^2 \to 0$

The QCD topological susceptibility is zero in the chiral limit

Topological mass generation from the WZW term

Since $\langle 0|T\Omega\eta_0|0\rangle = -i\frac{1}{l^2}\frac{\sqrt{2\tilde{n}_f}}{F_{\tilde{\eta}}}\chi(l^2)$ and $\chi(l^2) \to 0$ when $l^2 \to 0$, we obtain

$F_{\tilde{\eta}}^2 = 2n_f\chi'(0)$ where $\chi'(0)$ is the slope of the topological susceptibility in the forward limit

From the GT relation, it then follows that

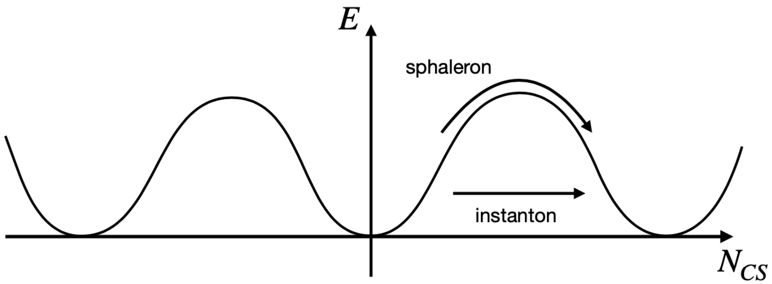
$$\Sigma(Q^2) = \sqrt{\frac{2}{3}}\frac{2n_f}{M_N} g_{\eta_0 NN}\sqrt{\chi'(0)}$$

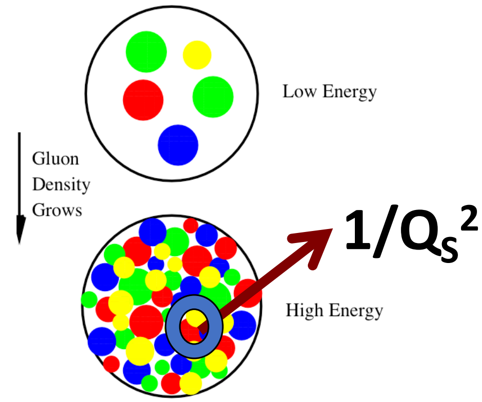
The proton's helicity is determined by the QCD topological susceptibility

Independent derivation of a result obtained by Shore & Veneziano (1992)

Comments: i) Limited number of studies of χ' on the lattice – see for instance Schierholz et al., arXiv:1012.1383
ii) Sum rule evaluation by Shore, Narison and Veneziano compatible with COMPASS and HERMES data for Σ

What about g_1 at small x_{Bj} ?

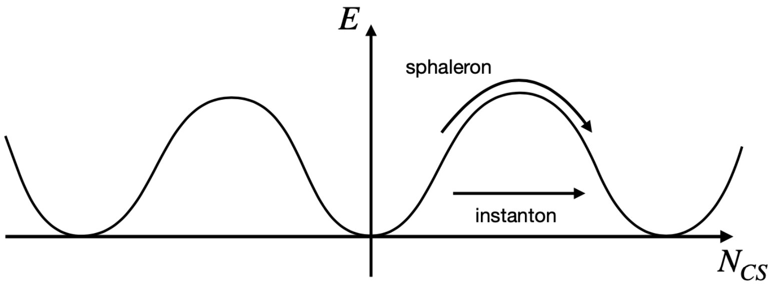


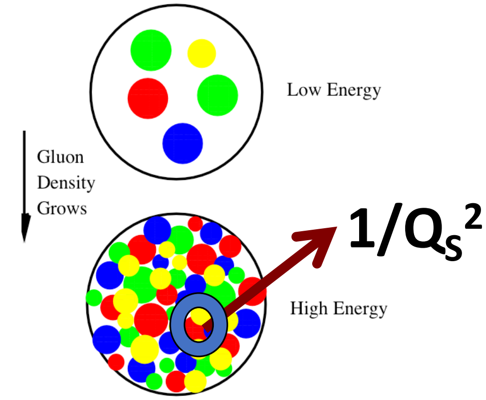


Gluon saturation: Maximal occupancy state in protons and nuclei at small x
Characterized by a semi-hard saturation scale: Many-body dynamics described by Color Glass Condensate EFT

Saturation induces over the barrier sphaleron-like transitions:
propagation of the $\bar{\eta}$ and its coupling to the CGC captured by an axion-like effective action

What about g_1 at small x_{Bj} ?





Distribution of large x color sources

$$g_1^{\rm Regge}(x_B,Q^2) = \left(\sum_f e_f^2\right)\frac{n_f\alpha_s}{\pi M_N} i\int d^4y\int_{x_B}^1\frac{dx}{x}\left(1-\frac{x_B}{x}\right)\int\frac{d\xi}{2\pi}e^{-i\xi x}\int\mathcal{D}\rho\, W_Y[\rho]\int D\bar\eta\,\tilde W_{P,S}[\bar\eta]\int[DA]$$

$$\times\,{\rm Tr}_c F_{\alpha\beta}(\xi n)\tilde F^{\alpha\beta}(0)\,\eta_0(y)\,\exp\left(iS_{\rm CGC}+i\int d^4x\left[\frac{1}{2}\left(\partial_\mu\bar\eta\right)\left(\partial^\mu\bar\eta\right)-\frac{\sqrt{2n_f}}{F_{\bar\eta}}\bar\eta\,\Omega\right]\right)$$

Generalization of $g_{\bar\eta NN}$

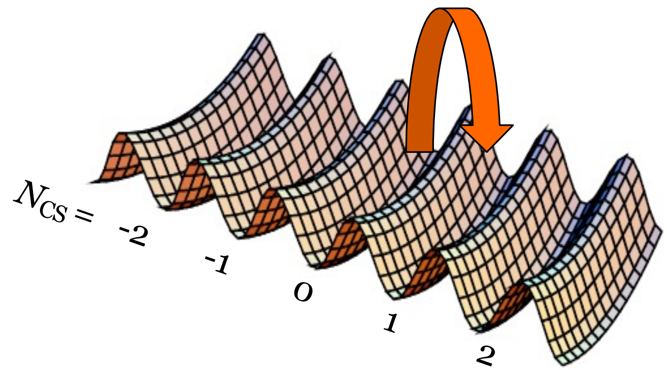
Axion-like action

$$S_{\rm CGC}[A,\rho] = -\frac{1}{4}\int d^4x F_a^{\mu\nu}F^a_{\mu\nu}+\frac{i}{N_c}\int d^2x_\perp\ {\rm tr}_c\left[\rho(x_\perp)\ln\left(U_{[\infty,-\infty]}(x_\perp)\right)\right]$$

Spin diffusion in topologically disordered media

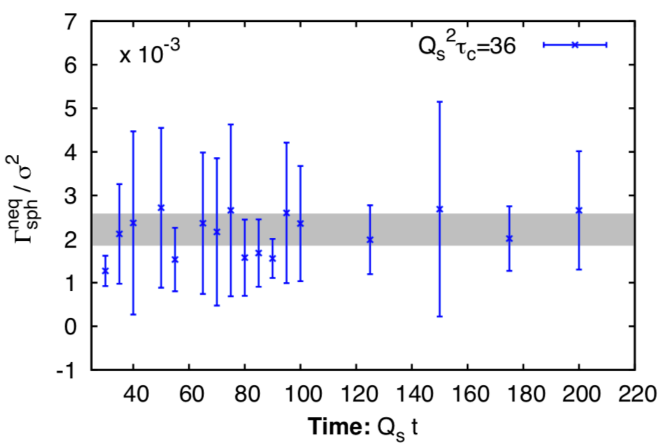
Two scales – the height of the barrier given by $m_{\eta'}^2 = 2n_f \frac{\chi_{\rm YM}}{F^2}$

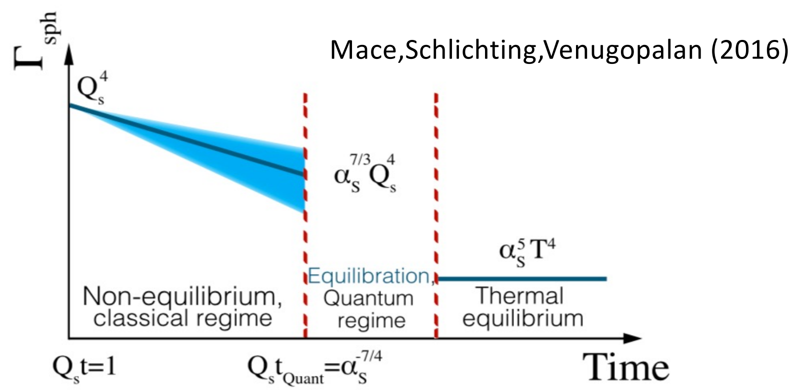
and the gluon saturation scale Q_S



When $Q_S^2 >> m_{\eta'}^2$ over the barrier sphaleron-like configurations dominate over instanton configurations

Sphaleron transition rate off-equilibrium



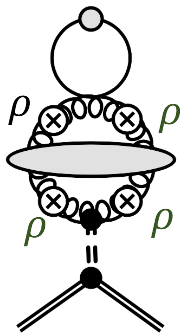


Numerical results of 3+1-D classical Yang-Mills of overoccupied configurations

- Sphaleron transition rate scales as the (time-dependent) string tension of spatial Wilson loops
- The rate is large $\propto\ Q_S^4$

g_1 at small x_{Bj} from sphaleron transitions

For $Q_S^2 \;< m_{\eta'}^2$
over the barrier transitions



"drag force" on "axion" propagation in the background of dense color sources impacts topological susceptibility

Drag force is proportional to sphaleron transition rate

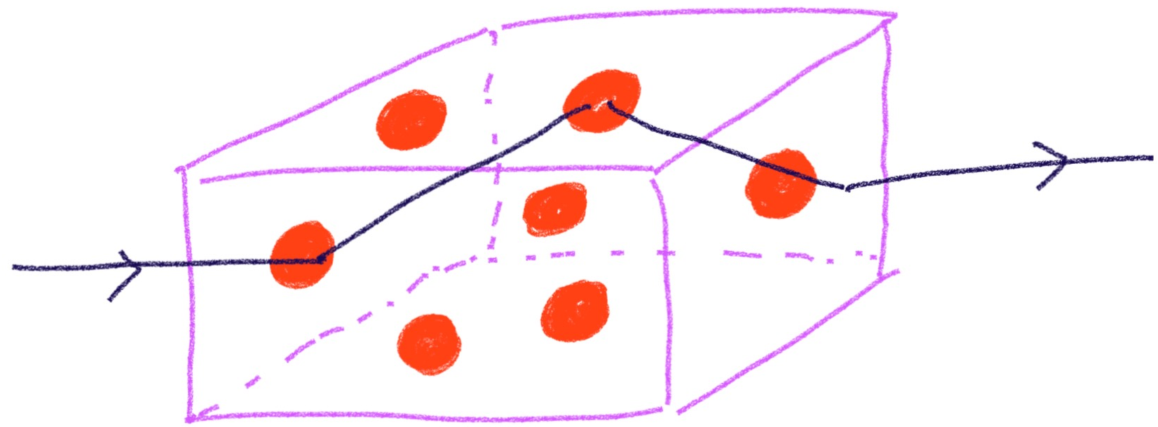
From small x_B axion effective action, $$\frac{\partial^2 \eta'}{\partial t^2} = -\gamma \frac{\partial \eta'}{\partial t} - m_{\eta'}^2 \eta'$$ $$\gamma = \frac{2n_f\,\Gamma_{sphaleron}}{F^2{}_{\eta'}\,Qs}$$

McLerran,Mottola,Shaposhnikov (1990)

$$g_1^{\rm Regge}(x_B, Q^2) \propto F(x_B) \times \frac{Q_S^2 m_{\eta'}^2}{F_{\tilde\eta}^3 M_N} \exp\left(-4\,n_f C\,\frac{Q_S^2}{F_{\tilde\eta}^2}\right)$$

Very rapid quenching of spin diffusion at small x_{Bj} !

Spin diffusion: sphaleron transitions in topologically disordered media

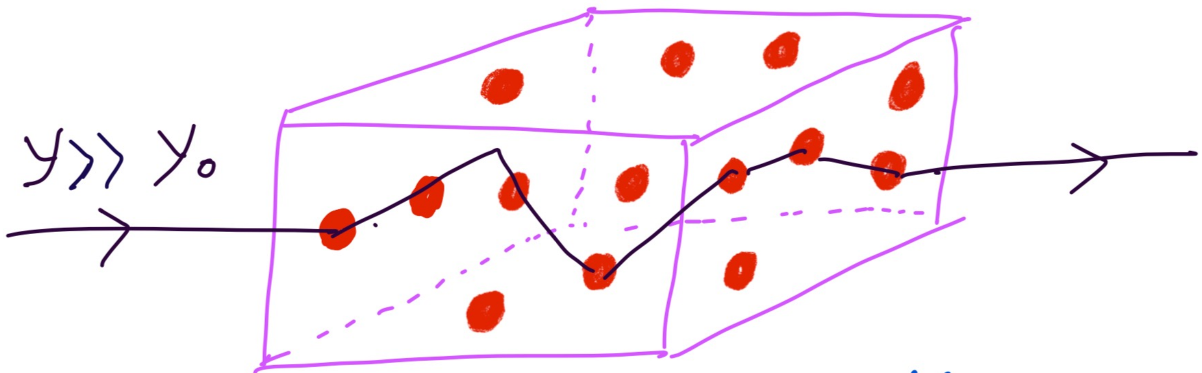


● Dense gluon blob of size $1/\sqrt{\sigma}$ given by $\Gamma^{Y}_{sphaleron} = \#\, \sigma^2$ & carrying topological charge

Atiyah-Singer index theorem

Helicity flip for massless quarks given by $n_L - n_R = n_f \nu$, where ν is the topological charge and $\Gamma^{Y}_{sphaleron} \propto \langle \nu^2 \rangle$

Spin diffusion: sphaleron transitions in topologically disordered media



As x decreases ($y \gg y_0$), the blobs become smaller ($Q_s(y) > Q_s(y_0)$) and denser with more topological charge

Expect very rapid quenching of g_1 at small x_B:
interplay between QCD evolution of the topological charge and the saturation scale

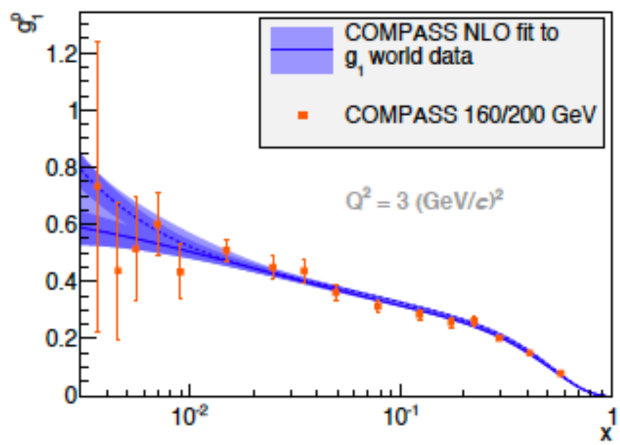
Takeaway: The proton's spin is deeply influenced by the topology of the QCD vacuum
- in particular, its features that are responsible for the large mass of the η' meson

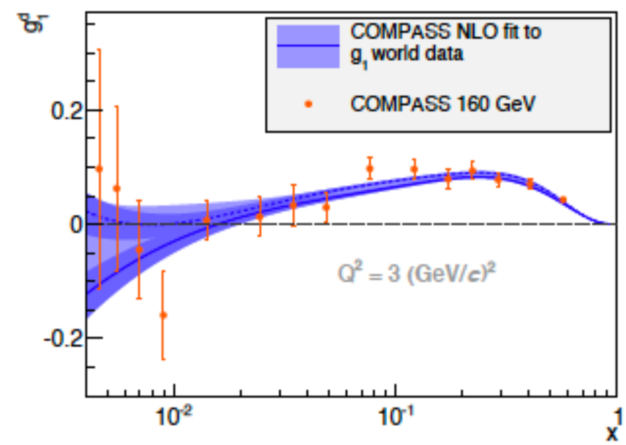
Polarized DIS at the Electron-Ion Collider has the potential for
discovery of real-time topological (sphaleron-like) transitions

Thank you for your attention !

g_1 at small x_{Bj} from sphaleron transitions

COMPASS: arXiv:1503.08935
arXiv: 1612.00620





The key feature of the topological screening picture is its target independence
However, as we have argued, the result is sensitive to the density of color sources, which is larger for the deuteron – one anticipates the same behavior for g_1^p as g_1^d at even smaller x_B

Other observables: semi-inclusive DIS, g_1^γ DeFlorian, Shore, Veneziano, hep-ph/9711353

Of particular interest is the g_2 structure function – in the naïve parton model, it is zero in the chiral limit. Turning on quark masses introduce non-trivial mixing between the UA(1) and SU(3) flavor sectors – which can be computed Bhattacharya, Hatta, Tarasov, RV, in progress

Low energy dynamics of η' in QCD

For N_f=3, dynamical variables of effective theory are massless modes in limit $N_C \to \infty$ and $m \to 0$

Symmetry group is G= $U_R(3)\times U_L(3)$

Spontaneous symmetry breaking: $U_R(3)\times U_L(3) \to U_V(3)$

The nine parameters of its coset space correspond to the nine pseudoscalar Goldstone bosons – including the prodigal $\eta' \to \eta_0$

Relative to the "standard" SU(3) framework, where $\det U(x) = e^{i\eta_0(x)}$ and η_0 transforms as

$\eta_0' = \eta_0 - i\,\mathrm{Ln}\det V_R + i\,\mathrm{Ln}\det V_L$

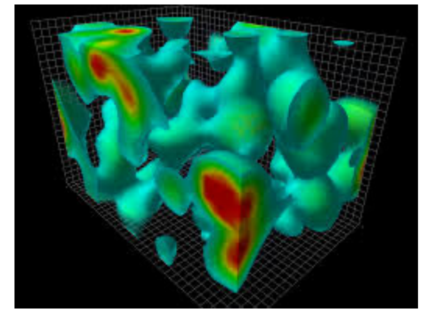
For non-zero quark masses, expansion in # of derivatives, powers of m and 1/N_c

Wess-Zumino-Witten terms for the SU(3) and U(1) sectors correspond to the "un-natural parity"part of the effective Lagrangian

Leutwyler, hep-ph/9601234
Herrera-Siklody et al, hep-ph/9610549
Kaiser, Leutwyler, hep-ph/0007101

Iso-singlet axial vector current and the chiral anomaly

$$S^\mu \, \Delta\Sigma \; = \; \langle P, S \,|\, \bar{\psi} \, \gamma^\mu \gamma_5 \, \psi \,|\, P, S \rangle \; \equiv \; \langle P, S \,|\, j_5^\mu \,|\, P, S \rangle$$



$U_A(1)$ violation from the anomaly:

$$\partial_\mu J_5^\mu = 2n_f \partial_\mu K^\mu + \sum_{i=1}^{n_f} 2im_i \bar{q}_i \gamma_5 q_i$$

where the Chern-Simons current $K_\mu = \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \left[A_a^\nu \left(\partial^\rho A_a^\sigma - \frac{1}{3} g f_{abc} A_b^\rho A_c^\sigma \right) \right]$

But, identification of CS charge with ΔG is intrinsically ambiguous
… *the latter is gauge invariant, the former is not*

Jaffe, Manohar (1990)

$$K_\mu \to K_\mu \; + \; i \frac{g}{8\pi^2} \epsilon_{\mu\nu\alpha\beta} \partial^\nu \left(U^\dagger \partial^\alpha U \, A^\beta \right)$$
$$+ \; \frac{1}{24\pi^2} \epsilon_{\mu\nu\alpha\beta} \left[(U^\dagger \partial^\nu U)(U^\dagger \partial^\alpha U)(U^\dagger \partial^\beta U) \right]$$

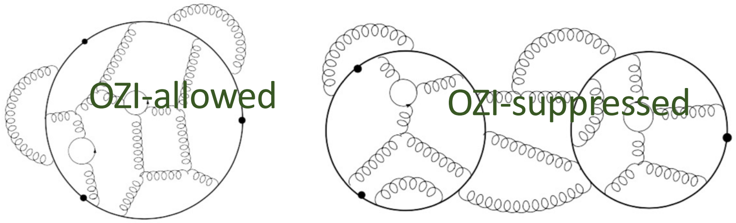
"Large gauge transformation"
- deep consequence of topology

R. Jaffe: identification of K^μ with ΔG
a source of much confusion
in the literature (Varenna lectures, 2007)

Anomaly cancellation and topological screeening

$$\Sigma(Q^2) = \sqrt{\frac{2}{3}}\,\frac{2n_f}{M_N}\,g_{\eta_0 NN}\sqrt{\chi'(0)}\,.$$

Magnitude of of OZI violation $\frac{a^0(Q^2)}{a^8} \simeq \frac{\sqrt{6}}{f_\pi}\sqrt{\chi'(0)}$



Computations on the lattice…

Bali et al., arXiv:2106.05398

$G_A|_{model}$ = 0.33±0.05

Sum rule analysis in good agreement with HERMES and COMPASS data

Narison,Shore,Veneziano (1998)

HERMES (Q^2= 5 GeV^2) $0.330 \pm 0.011(th) \pm 0.025(exp) \pm 0.028(evol)$

COMPASS (Q^2=3 GeV^2) $0.35 \pm 0.03(stat) \pm 0.05(syst)$

Axion-like effective action

As suggested by Shore and Veneziano, and following from our discussion as well,

$$S_{\bar\eta} = \int d^4x \left[\frac{1}{2}(\partial_\mu \bar\eta)(\partial^\mu \bar\eta) + \left(\theta - \frac{\sqrt{2n_f}}{F_{\bar\eta}}\bar\eta\right)\Omega + \frac{\chi_{\rm YM}}{2}\theta^2\right]$$

Since θ *is not dynamical, can get rid of it from the equations of motion,*

$$S_{\bar\eta} = \int d^4x \left[\frac{1}{2}(\partial_\mu \bar\eta)(\partial^\mu \bar\eta) - \frac{\sqrt{2n_f}}{F_{\bar\eta}}\bar\eta\,\Omega - \frac{\Omega^2}{2\,\chi_{\rm YM}}\right]$$

Axion-like effective action for $\bar\eta$

Defining $\eta' = \frac{F_{\eta'}}{F_{\bar\eta}}\bar\eta$ *and* $G = \Omega + \frac{\sqrt{2n_f}}{F_{\eta'}}\chi_{\rm YM}\,\eta'$

$$S_{\eta'} = \int d^4x \left[-\frac{1}{2}\eta'\,(\partial^2 + m_{\eta'}^2)\,\eta' - \frac{G^2}{2\chi_{\rm YM}}\right]$$

Re-express in terms of the η' and a non-propagating glueball that decouples from the physical spectrum

Shore,Veneziano (1990); Hatsuda (1990)
Dvali,Jackiw,Pi (1995)

In the instanton framework, χ_{YM} is saturated by such classical configurations

t'Hooft (1976); Schafer-Shuryak (1996)

Several spin discussions by multiple groups in this framework:

Forte, Shuryak (1990); Qian, Zahed (2016); …