Quantum Entanglement and Bell Inequality Violation in Semi-Leptonic Top Decays

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2310.17696

Motivation

Questions:

1. How strong is the entanglement between $t\bar{t}$ in LHC?
2. Does $t\bar{t}$ violate Bell's inequality?

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Interesting property of quantum state

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↓

Refutes local hidden variable theory

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1. How strong is the entanglement between $t\bar{t}$ in LHC?
2. Does $t\bar{t}$ violates **Bell's inequality**?

$\Downarrow$

Refutes local hidden variable theory

$\Longrightarrow$ If try to understand quantum with classical theory

$\Rightarrow$ must be some nonlocal theory

Quantum state

Density matrix

$$\rho = \sum_i n_i \left|\phi_i\right\rangle \left\langle\phi_i\right| \qquad\qquad \langle\mathcal{O}\rangle = \mathrm{Tr}(\mathcal{O}\rho)$$

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Production density matrix for $t\bar{t}$

$$\rho_{ab,\bar{a}\bar{b}} \propto R_{ab,\bar{a}\bar{b}} = \overline{\sum_{\text{initial}}} \mathcal{M}(XY \to t_a \bar{t}_{\bar{a}})\, \mathcal{M}^*(XY \to t_b \bar{t}_{\bar{b}})$$

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Decomposition of two-qubit density matrix

$$\rho = \frac{1}{4}\Big(\mathbb{I}_4 + P_i^A\,(\sigma_i \otimes \mathbb{I}_2) + P_i^B\,(\mathbb{I}_2 \otimes \sigma_i) + C_{ij}\,(\sigma_i \otimes \sigma_j)\Big)$$

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Polarization vector

Spin correlation matrix

Entanglement

Concurrence

$$\mathcal{C}(\rho) = \begin{cases} \frac{1}{2}\max(|C_1 + C_2| - 1 - C_3, 0), & C_3 \leq 0 \\ \frac{1}{2}\max(|C_1 - C_2| - 1 + C_3, 0), & C_3 \geq 0 \end{cases}$$

C_i: eigenvalues of $\{C_{ij}\}$

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C_i: eigenvalues of $\{C_{ij}\}$

$$0 \le \mathcal{C}(\rho) \le 1$$

No entanglement

Maximumly entangled

Bell's inequality

$$\langle A_1 B_1 \rangle - \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle + \langle A_2 B_2 \rangle \leq 2$$

E.g., choosing

$$A_1 = \sigma_1, \quad A_2 = \sigma_3, \quad B_1 = \pm\frac{1}{\sqrt{2}}(\sigma_1 + \sigma_3), \quad B_2 = \pm\frac{1}{\sqrt{2}}(-\sigma_1 + \sigma_3)$$

$$\Rightarrow |C_{11} \pm C_{33}| \leq \sqrt{2}$$

How to measure $\rho_{ab,\bar{a}\bar{b}}$?

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$\rho_{ab,\bar{a}\bar{b}}$ can be extracted from the angular distribution of decay product

$$\sigma(XY \to t\bar{t} \to (A_1A_2A_3)(B_1B_2B_3)) = \int \mathrm{d}\Omega^A \mathrm{d}\Omega^B \left(\frac{\mathrm{d}\Gamma_{ab}}{\mathrm{d}\Omega^A}\right) R_{ab,\bar{a}\bar{b}} \left(\frac{\mathrm{d}\bar{\Gamma}_{\bar{a}\bar{b}}}{\mathrm{d}\Omega^B}\right)$$

$$\frac{\mathrm{d}\Gamma_{ab}}{\mathrm{d}\Omega} \propto \delta_{ab} + \kappa\, \sigma^i_{ab}\, \Omega^i$$

Spin analyzing power

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Spin analyzing power

$$\Rightarrow \frac{1}{\sigma}\frac{d\sigma}{d\Omega^A d\Omega^B} = \frac{1}{(4\pi)^2}\left(1 + \kappa^A P_i^A \Omega_i^A + \kappa^B P_i^B \Omega_i^B + \kappa^A \kappa^B \Omega_i^A C_{ij} \Omega_j^B\right)$$

Direction of A, B

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Spin analyzing power

$$\Rightarrow \frac{1}{\sigma} \frac{\mathrm{d}\sigma}{\mathrm{d}(\cos\theta^A_i \cos\theta^B_j)} = -\frac{1 + \kappa^A \kappa^B C_{ij} \cos\theta^A_i \cos\theta^B_j}{2} \log\left|\cos\theta^A_i \cos\theta^B_j\right|$$

Polar angle of A with respect to the i-th axis

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$$\frac{\mathrm{d}\Gamma_{ab}}{\mathrm{d}\Omega} \propto \delta_{ab} + \kappa\, \sigma^i_{ab}\, \Omega^i$$

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$$\Rightarrow \quad C_{ij} = \frac{4}{\kappa^A \kappa^B} \frac{N(\cos\theta_i^A \cos\theta_j^B > 0) - N(\cos\theta_i^A \cos\theta_j^B < 0)}{N(\cos\theta_i^A \cos\theta_j^B > 0) + N(\cos\theta_i^A \cos\theta_j^B < 0)}$$

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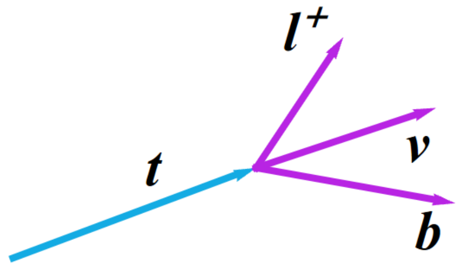
$$\delta C_{ij} \propto \frac{1}{\kappa^A \kappa^B} \frac{1}{\sqrt{N}}$$

Which decay channel?



Leptonic decay:

$$\boxed{\kappa_{\ell+} = 1.0}$$

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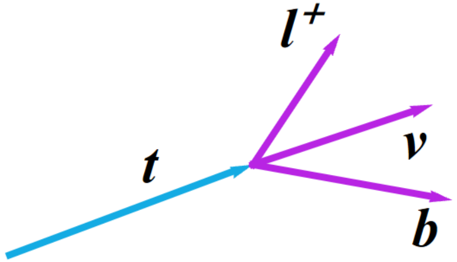

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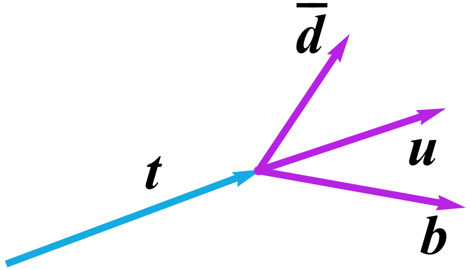

Hadronic decay:

$$\kappa_{\bar{d}} = 1.0$$
$$\kappa_u = 0.34$$

but cannot distinguish u, d

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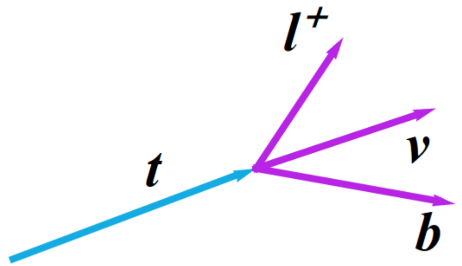

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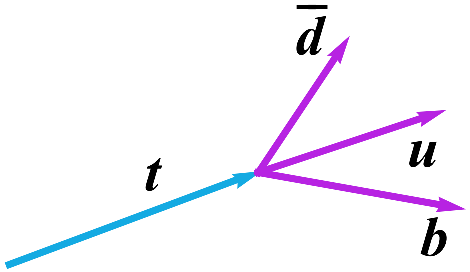

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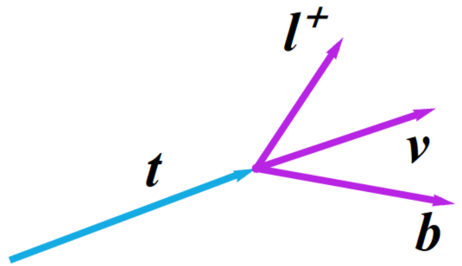
optimized direction:

$$\vec{\Omega}_{\rm opt}(\cos\theta_W) = P_{d\to p_{\rm soft}}(\cos\theta_W)\,\hat{p}_{\rm soft} + P_{d\to p_{\rm hard}}(\cos\theta_W)\,\hat{p}_{\rm hard}$$

(arXiv:1401.3021)

Which decay channel?



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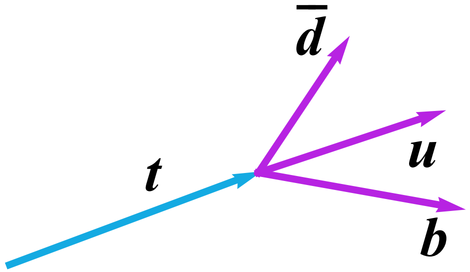

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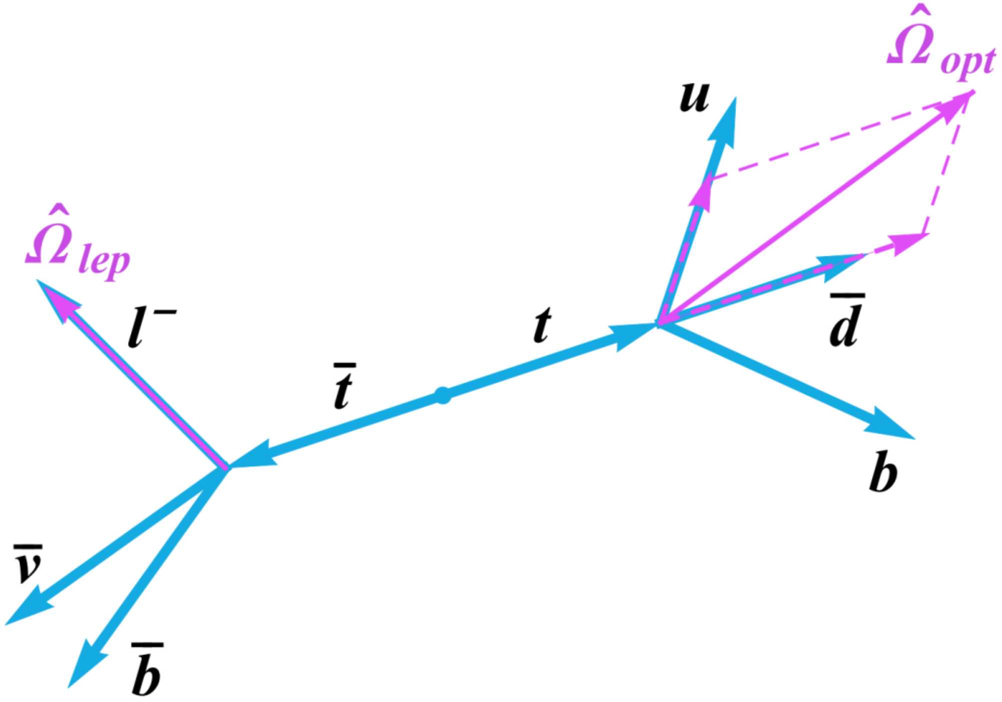
$$\Rightarrow \boxed{\kappa_{\rm opt} = 0.64}$$

(arXiv:1401.3021)

Semi-Leptonic Decay

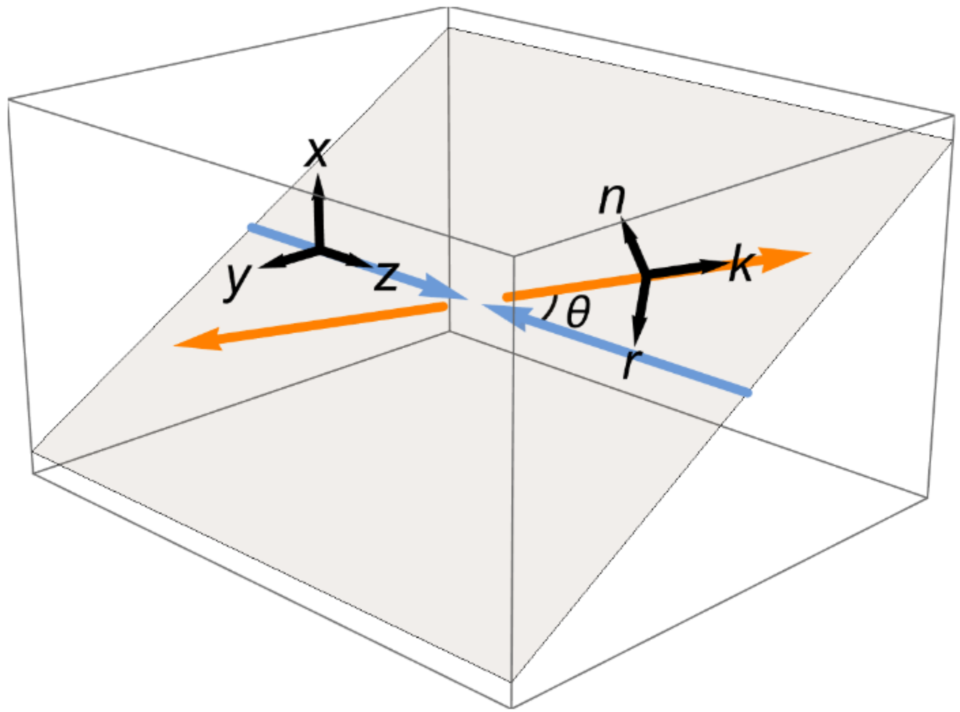


Larger branching ratio, higher significance

$$\frac{\text{significance}\ (t\bar{t} \to \ell j)}{\text{significance}\ (t\bar{t} \to \ell\ell)} = \frac{\kappa_\ell\, \kappa_{\text{opt}}}{\kappa_\ell\, \kappa_\ell} \sqrt{\frac{\text{BR}(t\bar{t} \to \ell j)}{\text{BR}(t\bar{t} \to \ell\ell)}}$$

$$\approx 0.64 \times \sqrt{6} = 1.6$$

Which basis?



Beam basis {x,y,z}:

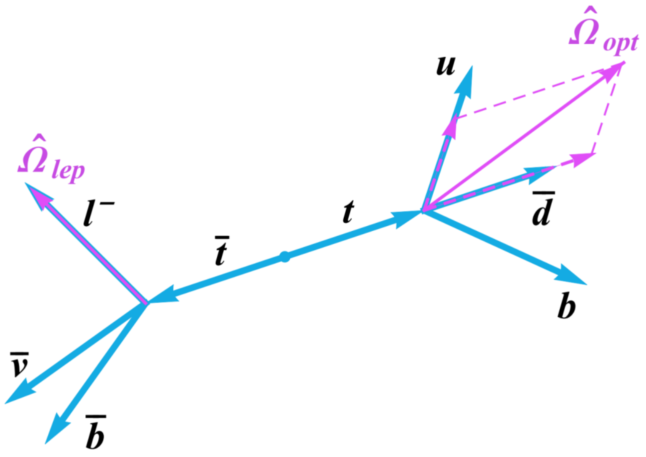
Cancellation leads to null result

Helicity basis {r,k,n} is more sensitive！

Maintain the entanglement with no cancellations after averaging over phase space

Outline

Measure $\hat{\Omega}_{lep}, \hat{\Omega}_{opt}$ → Extract C_{ij} → Calculate $\mathcal{C}(\rho)$, $|C_{11} \pm C_{33}|$



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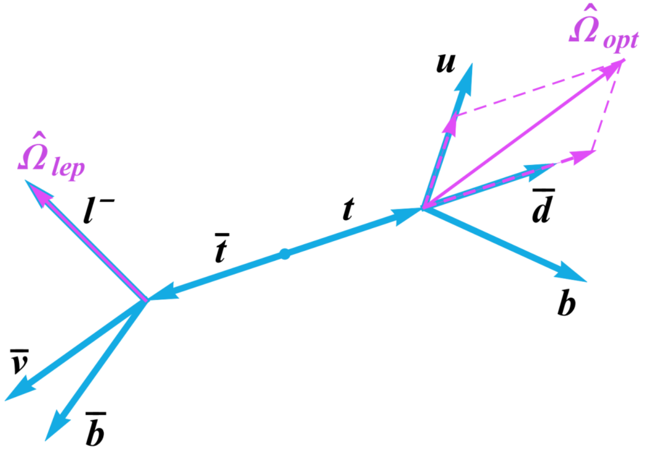

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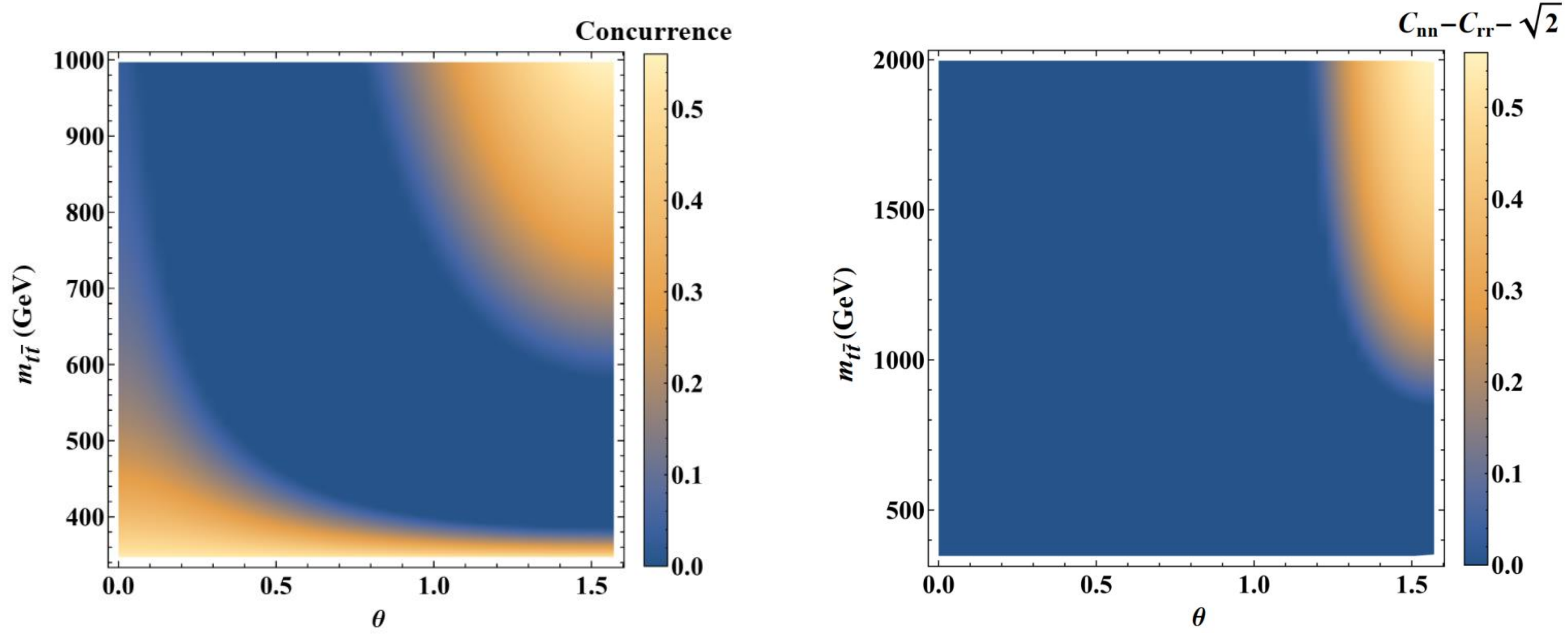
2302.00683
2307.09675
2307.14895
2106.01377
2209.14033
2209.13441
2307.13496
2209.13990
2304.02403
2306.17247

Theoretic & parton level result



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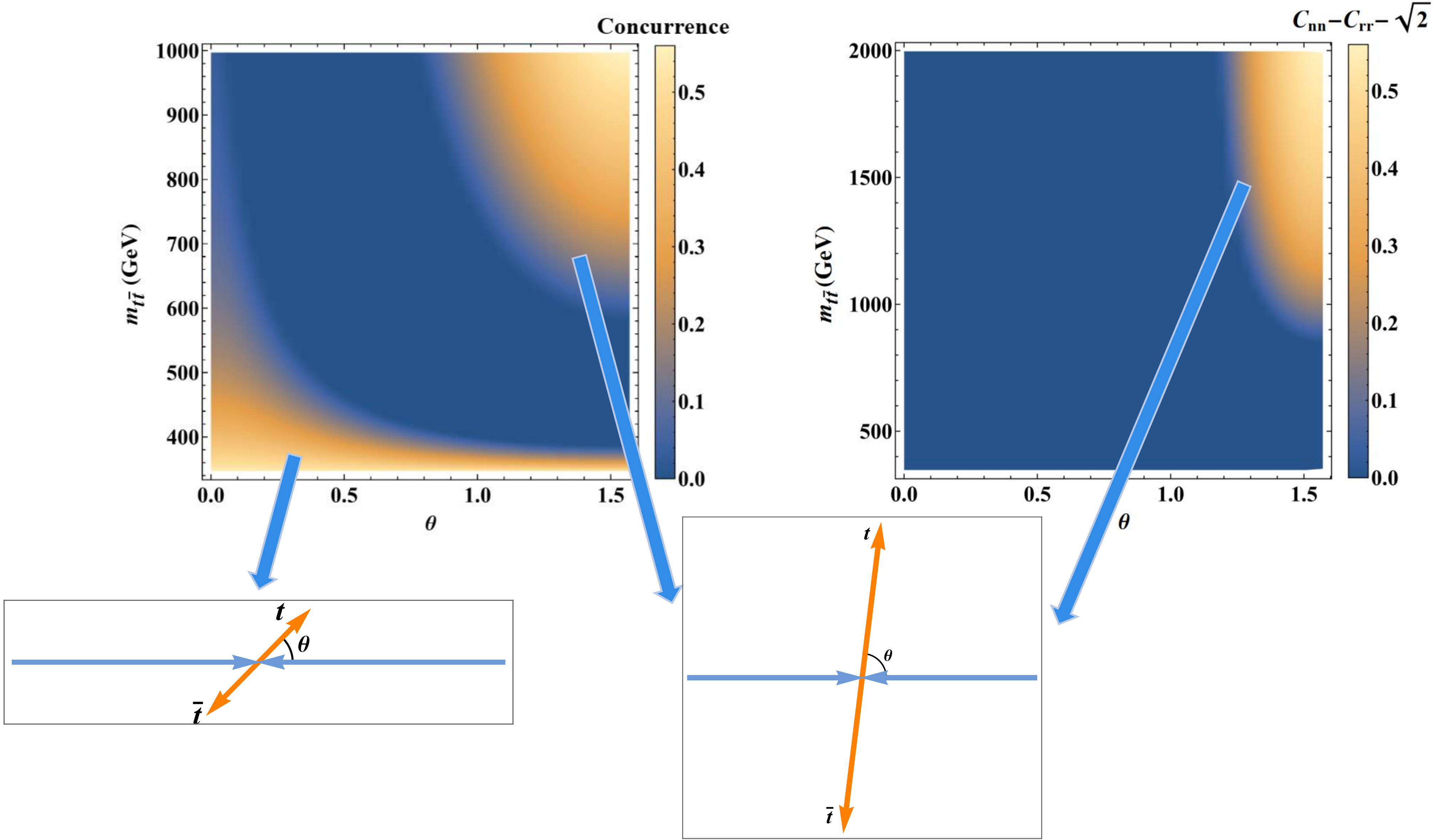

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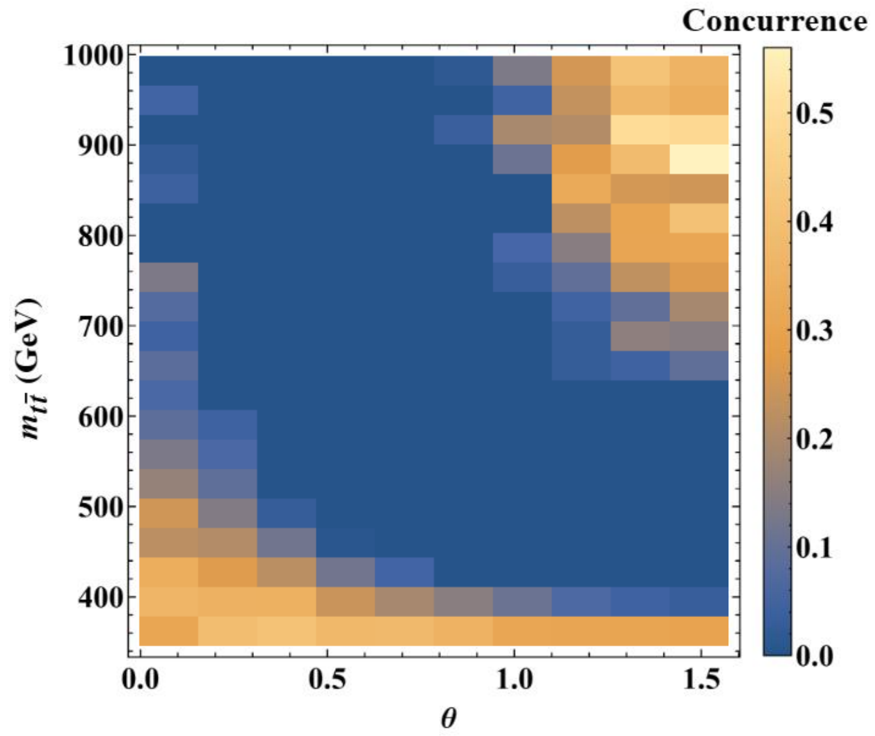

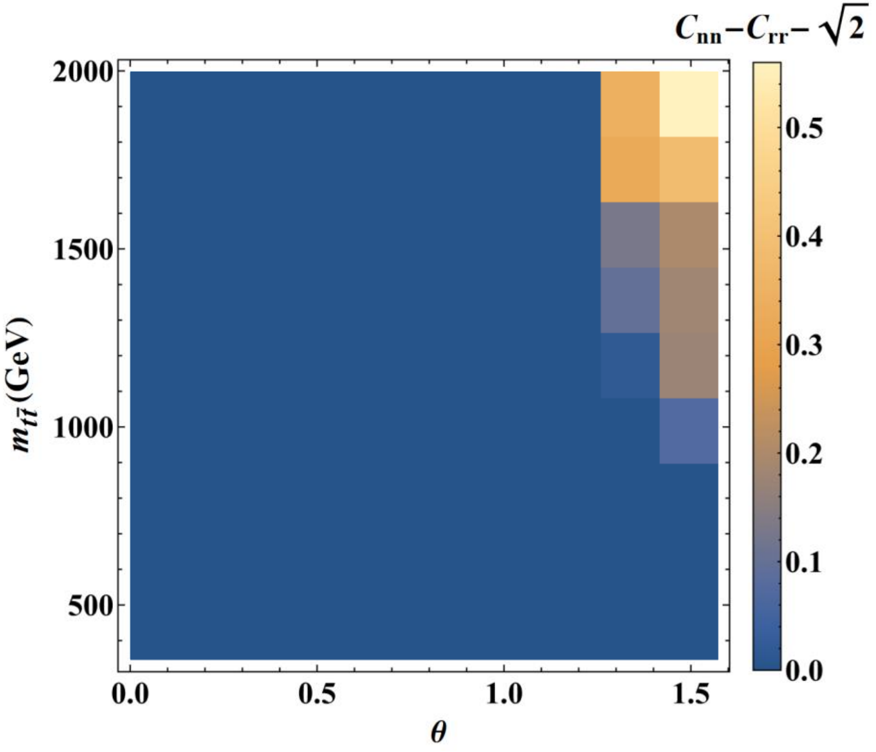

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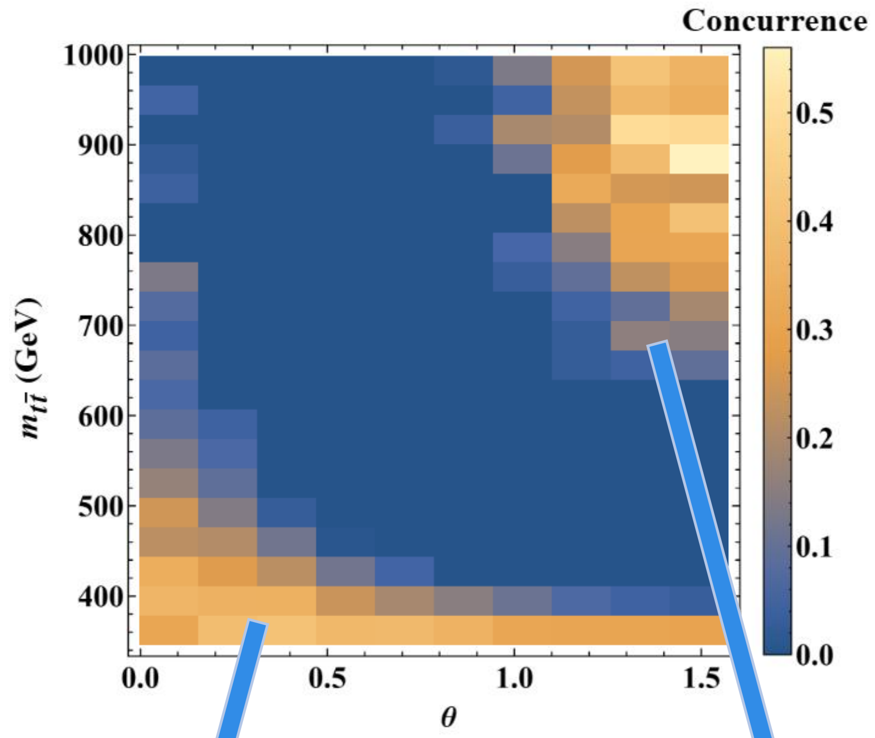

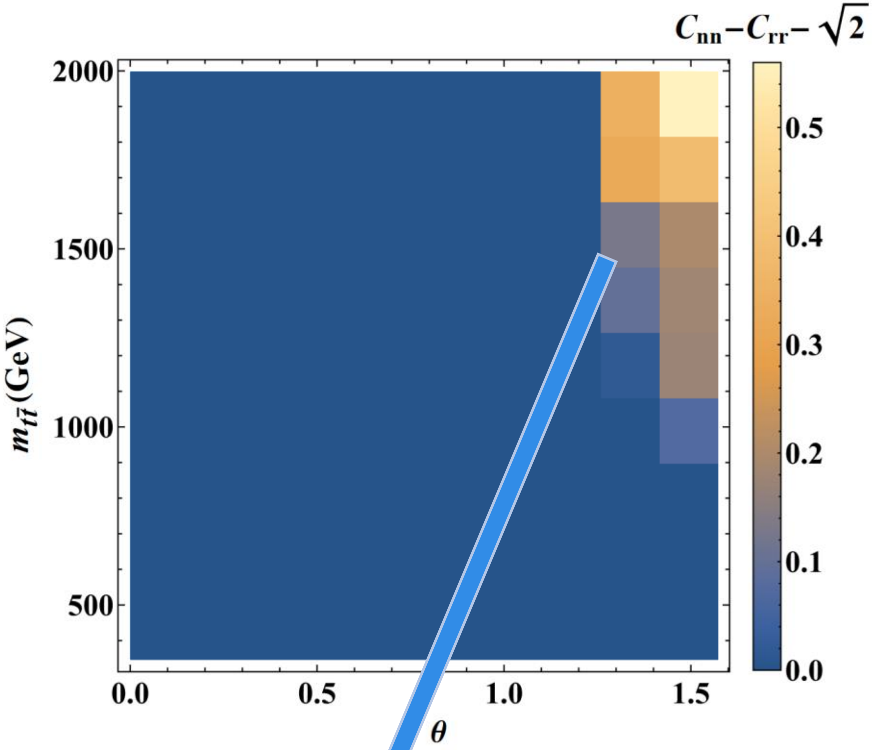

Precision: 1.6%

Precision: 4.8%

($139\mathrm{fb}^{-1}$)

Significance: 7.0σ

($3\mathrm{ab}^{-1}$)

(Uncertainties recalled by reconstruction efficiency from simulation)

Reconstruction

Full detector simulation with $\texttt{Madgraph 5} + \texttt{Pythia 8} + \texttt{Delphes 3}$
Reconstructed based on the pseudo-top algorithm.

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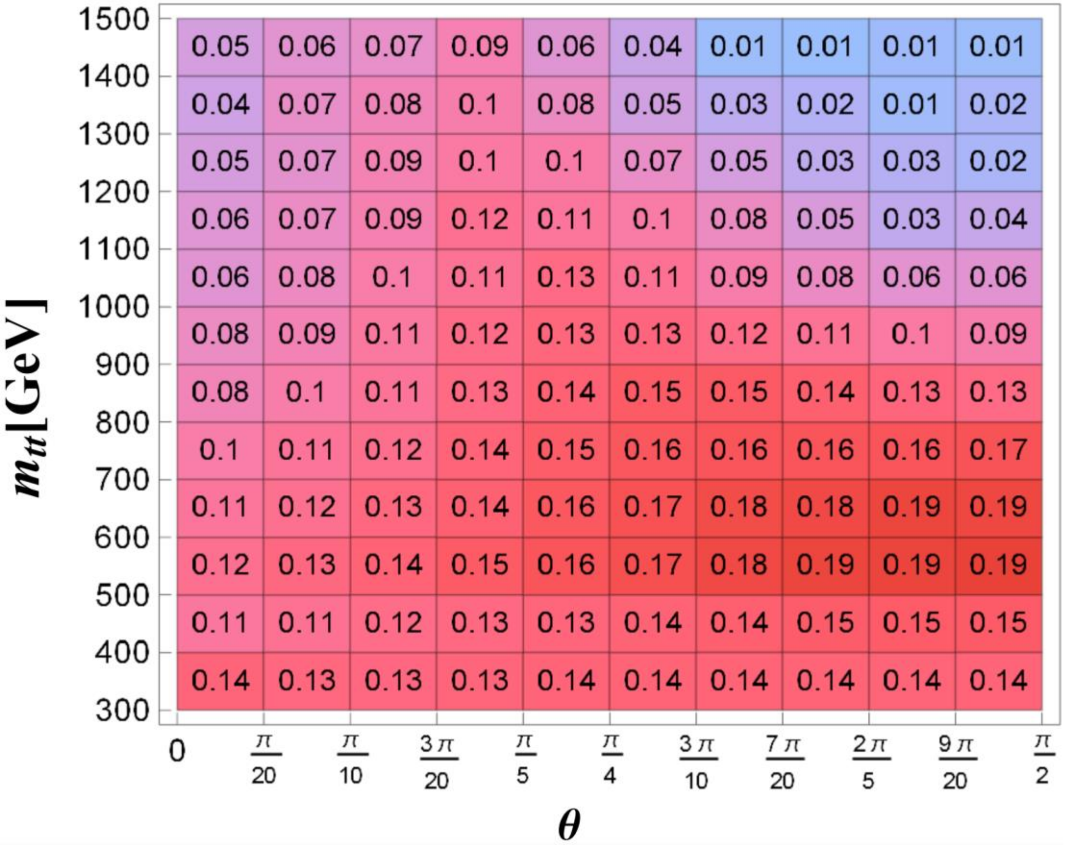
Difficulties: ① Low efficiency



We need:

angular distribution in all directions

p_T, η cut:

throw away most of events :(

⇒ Efficiency: 0.01-0.2

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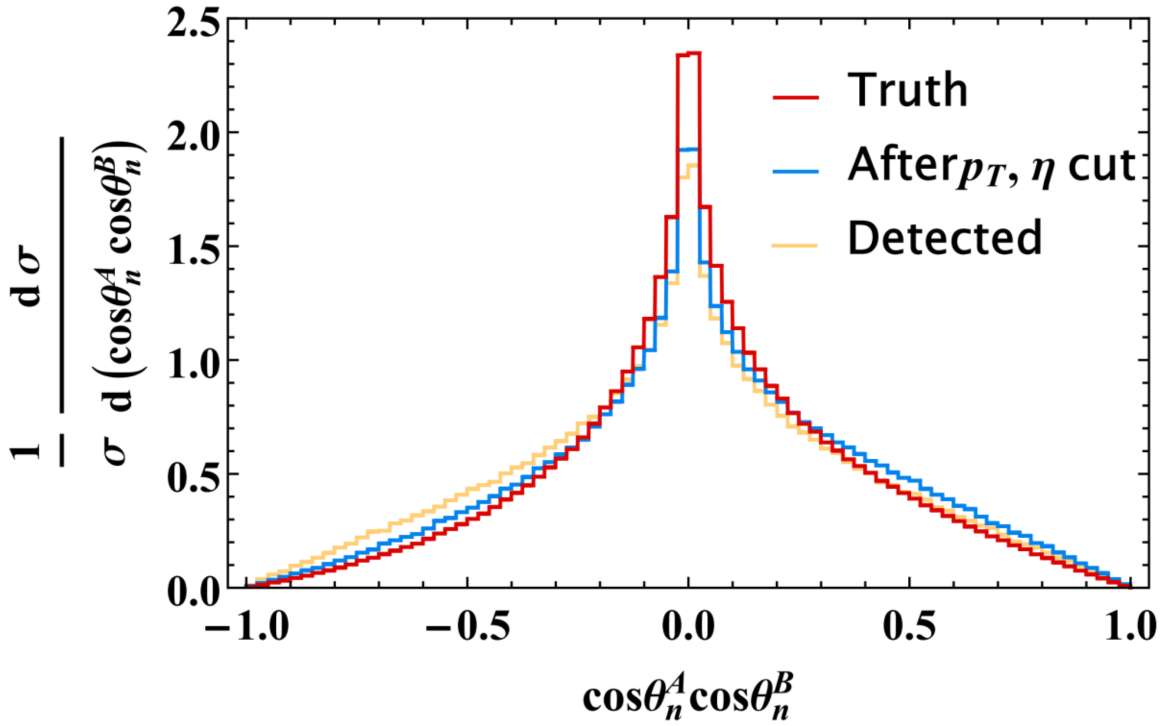
Reconstructed based on the pseudo-top algorithm.

Difficulties: ② Smeared angular distribution



An asymmetry of 0.5 was smeared to 0.05 :(

Reconstruction

Solution: Parametric Fit

Detector effects can be quantified by the Response Matrix R:

$$T_{\text{ruth}} \xrightarrow{\textbf{folded}} D_{\text{etected}} = R \cdot T_{\text{ruth}}$$

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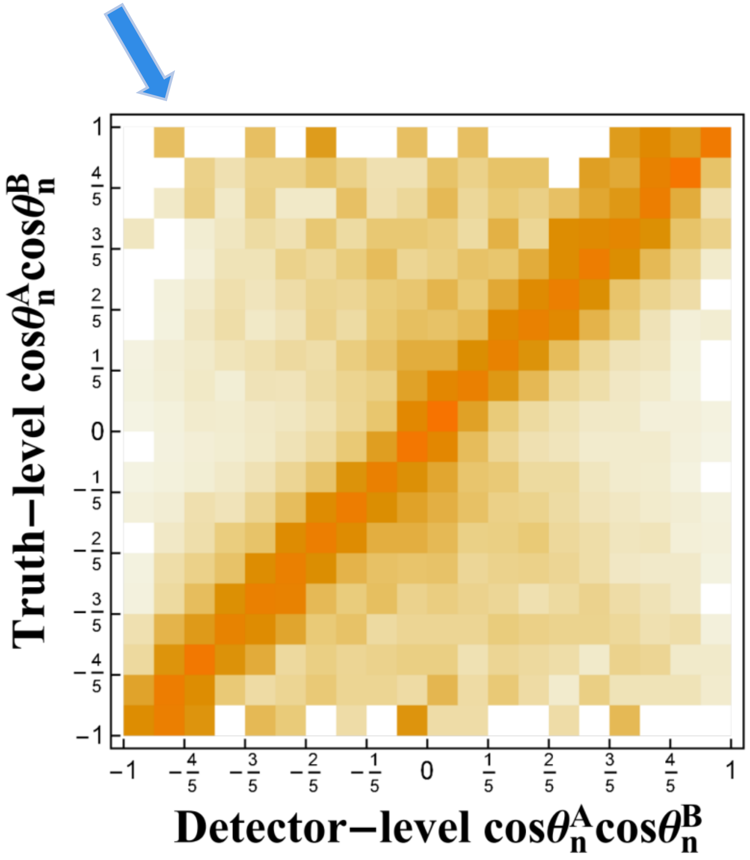

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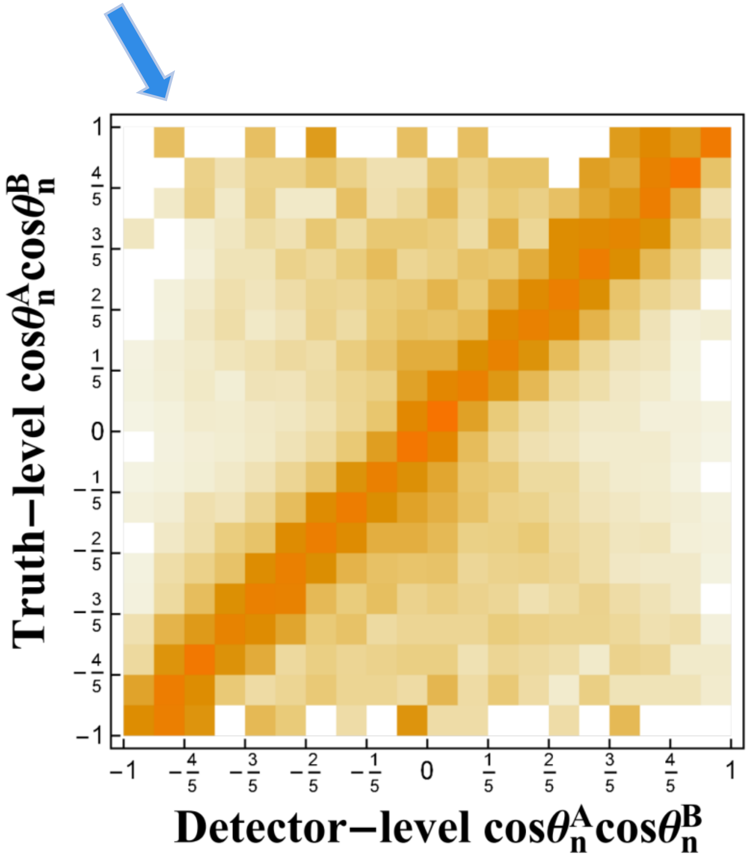

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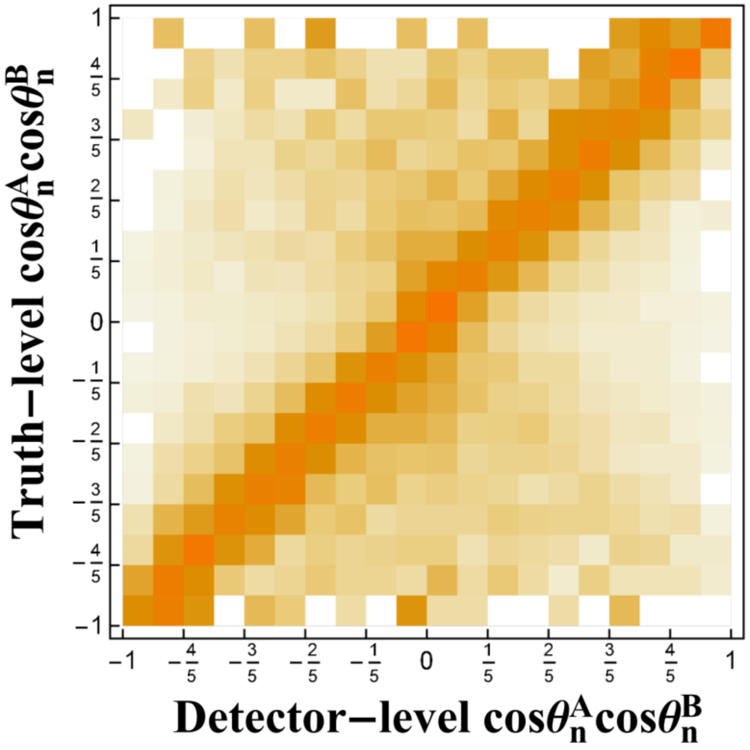
Fitting P to D solves the problem :)



Result

Entanglement

	Result$(139\mathrm{fb}^{-1})$	Precision
Boosted	0.276 ± 0.026	9.5%
Threshold	0.261 ± 0.008	3.0%

Bell's inequality violation

Result$(3\mathrm{ab}^{-1})$	Significance
0.23 ± 0.06	4.1σ

Conclusion

1. Semi-leptonic decay is much more sensitive than full leptonic decay.
2. There is strong detector effect in reconstructing spin correlation matrix.
3. 3% precision for entanglement and 4σ violation for Bell's inequality.

Thank you!