

Heavy Quarks in Hot and Dense QCD Medium

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Content:

- Why Heavy Quark?
- Heavy Quark Production
- Heavy Quark Propagation
 - Collisional Energy Loss
 - Radiative Energy Loss
 - Heavy-flavoured hadron spectra (Jet Quenching)
- Chromoelectric Field Fluctuations
 - Collisional Energy gain
 - Heavy-flavoured Nuclear Suppression factor
- Conclusion

Why Heavy Quarks ?

Heavy Quarks ➤ Charm (c) & Bottom (b)

Charm: $M_c = 1.27_{-0.09}^{+0.07} \text{ GeV}$

Bottom: $M_b = 4.19_{-0.06}^{+0.18} \text{ GeV}$

Production time: $\tau_Q = 1/2M_Q \leq 0.1 \text{ fm}/c$

Thermalisation time (τ_{th}):

Successive equilibrium: $\tau_{th}^g < \tau_{th}^{u,d} < \tau_{th}^c < \tau_{th}^b < \tau_{th}^{life}$

[J. Alam, S. Raha & B. Sinha, PRL 73 (1994) 1895]

Therma. time for HQs $\tau_{th}^Q \sim \frac{M_Q}{T} \times \tau_{th}^{u,d}$ [G. Moore & D. Teaney, PRC 2005]

$\tau_{th}^{u,d} < \tau_{th}^Q$: no production at QGP and hadronic phase

Produced at very early time interactions in Hard Scattering of partons in Nucleons. Initial distribution of HQs ➤ frozen

g, u, d thermalize early and provide an expanding thermal background

Why Heavy Quarks ?

Heavy Quarks propagation:

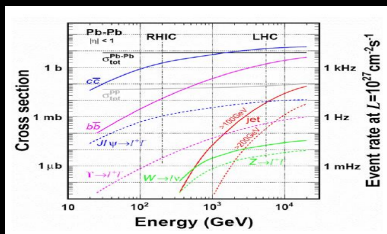
- ☞ in foreground with gluons & light quarks as an expanding thermal background for the non-equilibrated HQs ➤ expansion dynamics
- ☞ interacts with equilibrated degrees of freedom ➤ energy-loss of HQ
- ☞ requires dynamics ➤ final distrⁿ of HQs (Transport Eq.)

Uniqueness of Heavy Quarks:

- ☞ distinguishable down to lowest momenta ➤ medium/quark coupling (Hydro)
- ☞ cleaner energy-loss probe ➤ reflected in leading particle p_{\perp} spectra, flow
- ☞ tests understanding of mass dependence
- ☞ Quarkonia: QGP thermometer

True probe: Not glue or light quarks

Heavy Quark Production in Heavy-Ion Collisions:



Heavy-Ion Coll.: **ALICE: 0809.1062[nucl-ex]**

👉 Charm: **LHC ~ 10 × RHIC**

👉 Bottom: **LHC ~ 100 × RHIC**

Produced in pairs ($Q\bar{Q}$) at early times

QCD describes the structure and dynamics of hadrons in terms of their constituents: quarks and gluons

QCD provides a framework to compute production cross-section and distribution





Heavy Quark Propagation and Final distribution

Produced at early time ($\tau_Q < \tau_{th}$); No production at later time

- ➡ Total no. of HQ gets frozen very early in the history of collisions
- ➡ Immediately upon their production they will propagate through QGP
- ➡ One is left with the task of determining the HQ distribution
- ➡ Details of the distribution may reflect the characteristics and development of QGP

Fokker-Planck Equation

Fokker-Planck Eq. \blacktriangleright Simplified Boltzmann Eq.

-  Boltzmann Eq \blacktriangleright No external force and isotropic in space
-  Soft Scatt. (Landau approx.) \blacktriangleright Taylor expan. \blacktriangleright Landau Eq.
-  Landau Eq. \blacktriangleright an integro-diff^l Eq. involving transport coeffs. \blacktriangleright describes collision processes of two particles
-  Landau Eq. \blacktriangleright FP Eq. when distribution of the background particles (QGP) is thermal whereas foreground particle (HQs) is non-thermal

Standard Dynamics of Heavy Quarks in the QGP

c,b quarks



Brownian Motion?

Fokker-Planck approach

$$\frac{\partial f_{c,b}}{\partial t} = \gamma \frac{\partial (pf_{c,b})}{\partial p} + D \frac{\partial^2 f_{c,b}}{\partial p^2} \quad T \ll m_Q$$

From scattering matrix $|M|^2$

$$\gamma p = \int d^3k |M(k,p)|^2 p$$

$$D = \frac{1}{2} \int d^3k |M(k,p)|^2 p^2 \quad \bullet \text{ from some theory...}$$

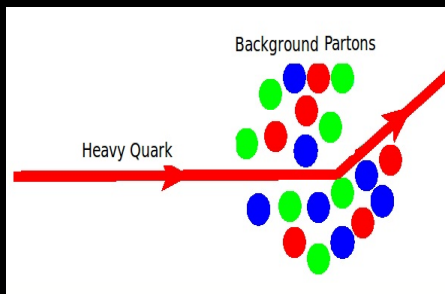
Extensive Body of Literatures

- \blacktriangleright Chakraborty & Syam, Lett. Nuov. Cim. '84
- \blacktriangleright Svetitsky, PRD '88
- \blacktriangleright Alam, Raha & Sinha, PRL '94
- \blacktriangleright Mustafa, Pal & Srivastava, PRC '97
- \blacktriangleright Mustafa PRC '05
- \blacktriangleright Moore & Teany PRC '05; Rapp & Hees '05

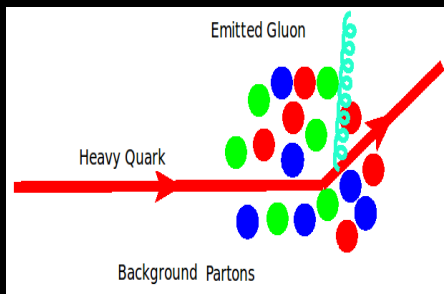
Propagation of Heavy Quark in QGP

Energy-Loss

Collisional Loss

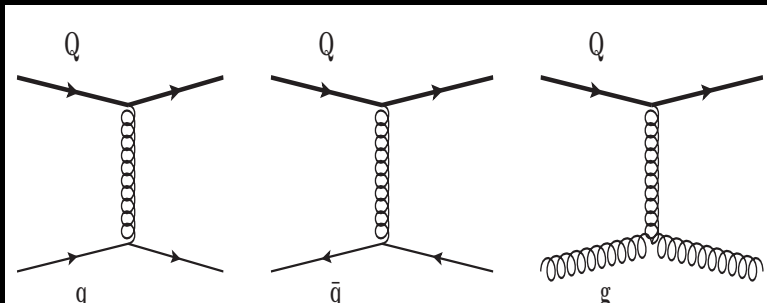


Radiative Loss




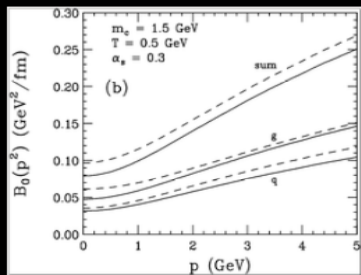
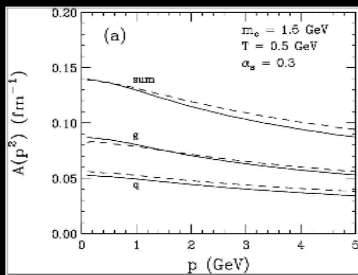
Collisional Processes

Feynman Diagram



Heavy Quark Transport Coefficients from FP Equation


 Drag $\gamma(p)$ and Diffusions $B_0(p)$



 Mustafa, Pal & Srivastava, PRC'97

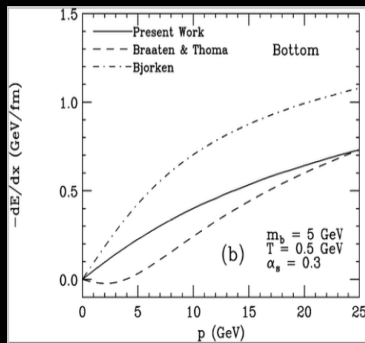
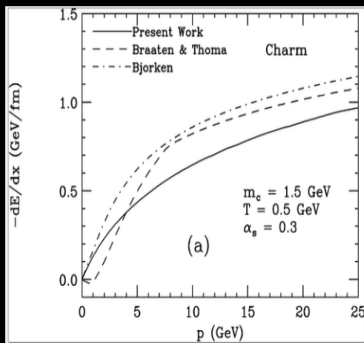
 Mustafa PRC'05

Differential Collisional Energy Loss

 Drag Coeff.: $\gamma(p) = -\frac{1}{p} \frac{dE}{dx}$

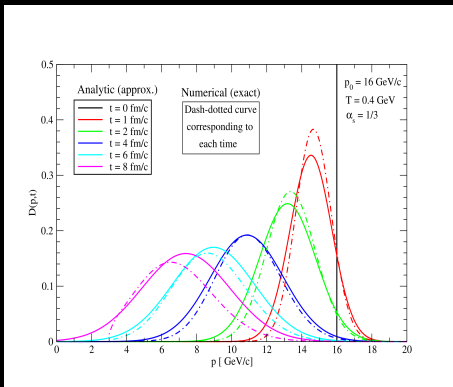
 Differential Collisional Energy-Loss: $\frac{dE}{dx}$

[Mustafa, Pal, Srivastava, PRC57 (1998) 889]



HQ Energy-Loss Distribution [Mustafa, PRC72 (2005) 014905]

Collisional Energy-Loss ➤ Transport-coeffs. ➤ Fokker-Planck Eq. ➤ Energy-Loss Distribution



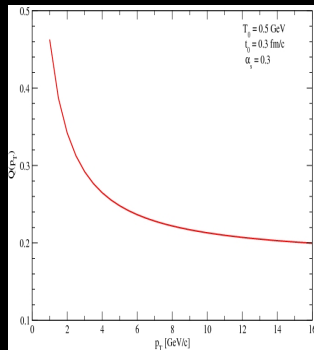
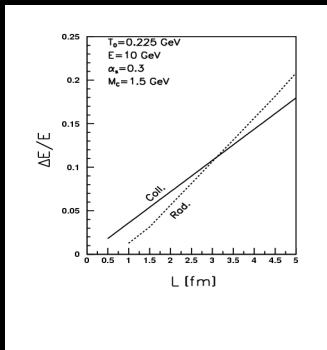
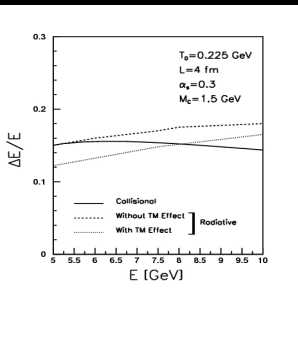
Mom. diffusion begins

Peak Shifts to lower p ➤ Energy Loss

Peak broadens ➤ Momentum Dispersion


Substantial Collisional E-Loss

 **Mustafa, PRC72 (2005) 014905**



 Coll. E-Loss is important

 Predicted R_{AA} but no data then

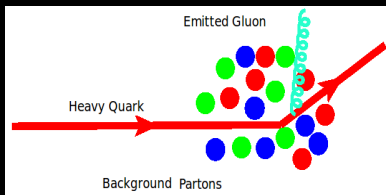
 For light quarks R_{AA} :

 **Mustafa et al., PRL100 (2008) 072301; Acta Phys.Hung. A22 (2005)**

Radiative Energy Loss

HQ encounters inelastic scattering as it traverses a medium

☞ Emits a gluon



☞ Differential Loss: dE/dx

☞ Very simple but first estimation:

☞ **Mustafa, Pal, Srivastava, Thoma, PLB 428 (1998) 234**

☞ Improvements in formalism

Radiative Energy Loss

Significant Points:

- ☞ The nature of the medium through which the energetic parton propagates ➤ thermally equilibrated perturbative medium with a collection of static scatt. centres with specified density
- ☞ Kinematic approximations for interaction between medium partons (thermal background) and the projectile parton that is propagating in the medium
- ☞ The virtuality and branching/splitting of hard parton to reduce its off-shellness ➤ multiple splitting may occur in the medium ➤ multigluon final state should include interference of emitted gluons

Suppression due to Mass and Dead Cone

General notion: heavy quark radiates less than light quark

$$qq' \rightarrow qq'$$

$$|\mathcal{M}_{qq' \rightarrow qq'}|^2 = \frac{8}{9} g^4 \frac{s^2}{t^2}$$

$$Qq \rightarrow Qq$$

$$|\mathcal{M}_{Qq \rightarrow Qq}|^2 = \frac{8}{9} g^4 \frac{s^2}{t^2} \left(1 - \frac{M^2}{s}\right)^2$$

$$qq' \rightarrow qq'g$$

$$|\mathcal{M}_{qq' \rightarrow qq'g}|^2 = 12g^2 \frac{8}{9} g^4 \frac{s^2}{t^2} \frac{1}{k_{\perp}^2}$$

$$Qq \rightarrow Qqg$$

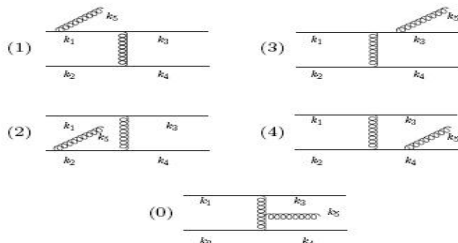
$$|\mathcal{M}_{Qq \rightarrow Qqg}|^2 = 12g^2 \frac{8}{9} g^4 \frac{s^2}{t^2} \frac{1}{k_{\perp}^2} \quad ?$$

2001 Dokshitzer and Kharzeev proposed (Phys. Lett. B 519, 199 (2001)) “dead cone” effect \Rightarrow heavy quark small energy loss.

$$\left(1 + \frac{\theta_0^2}{\theta^2}\right)^{-2} \quad \theta_0 = M/E$$

Generalised Dead Cone

 RHIC data (PHENIX) $\rightarrow R_{AA}^{HQ} \sim R_{AA}^{LQ} \rightarrow$ (Heavy quark puzzle)



Hierarchy employed in this study

$$\sqrt{s}, E \gg \sqrt{|t|} \sim q_{\perp} \gg \omega > k_{\perp} \gg m_D$$

Mass Range $0 < M/E < 1$

Emission angle Range $-\pi < \theta < +\pi$

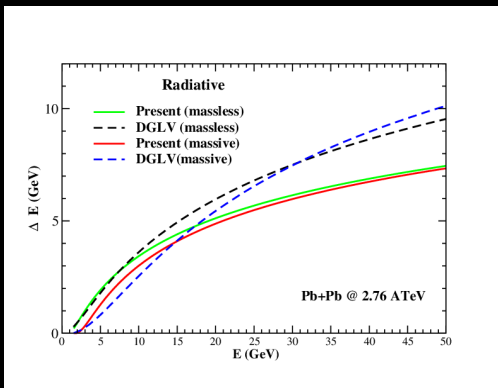
$$\begin{aligned}
 |\mathcal{M}_{Qq \rightarrow Qqg}|^2 &= 12g^2 |\mathcal{M}_{Qq \rightarrow Qq}|^2 \frac{1}{k_{\perp}^2} \left(1 + \frac{M^2}{s \tan^2(\frac{\theta}{2})} \right)^{-2} \\
 &= 12g^2 |\mathcal{M}_{Qq \rightarrow Qq}|^2 \frac{1}{k_{\perp}^2} \left(1 + \frac{M^2}{s} e^{2\eta} \right)^{-2}
 \end{aligned}$$


$$D = \left(1 + \frac{M^2}{s \tan^2(\frac{\theta}{2})} \right)^{-2}$$

..... *Dead Cone Factor*

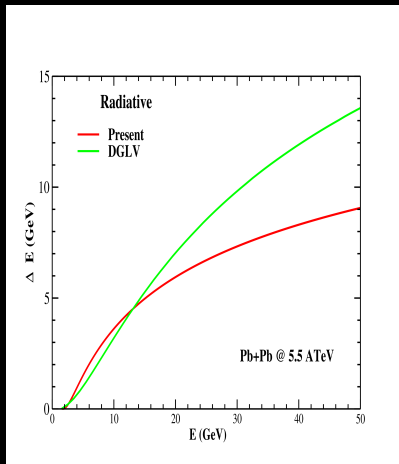
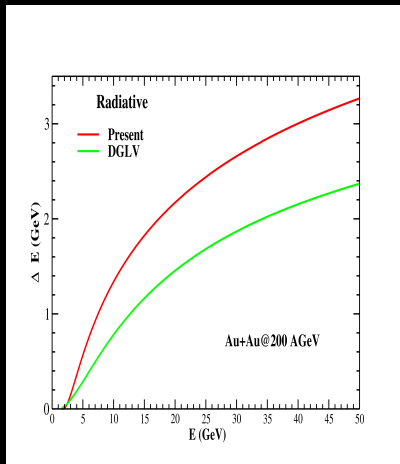
 **Abir, Greiner,
Martinez,
Mustafa,
Uphoff, PRD85
(2012) 054012**

Radiative E-Loss [Abir,Jamil,Mustafa,Sivastava, PLB715 (2012) 183]

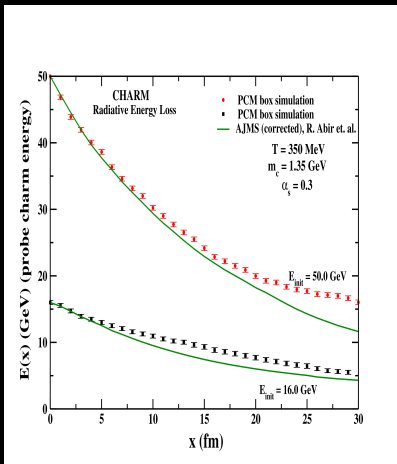


 DGLV: PRL85 (2000); NPA784 (2007); NPA783 (2000); NPA733 (2004)

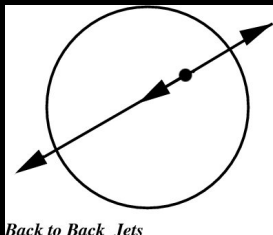
Radiative E-Loss [Abir,Jamil,Mustafa,Sivastava, PLB715 (2012) 183]



Comparison with Parton Cascade Model [Younus et al, Phys.Rev. C91 (2015) 024912]



Jet Quenching



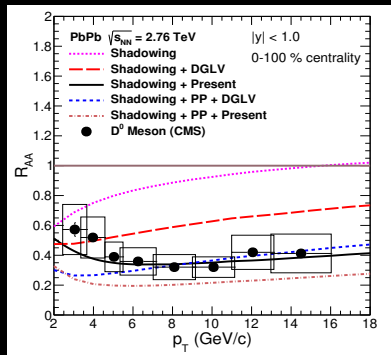
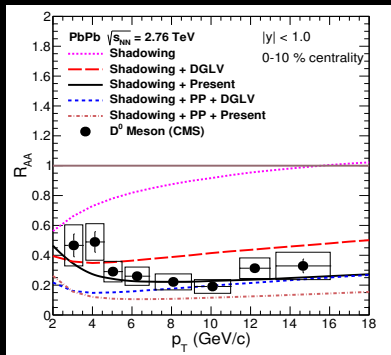
Back to Back Jets

- Jets are produced back to back
 - ☞ Awayside will pass through medium
 - ☞ Interact with the medium
 - ☞ Lose energy and quenched
 - ☞ Results in suppression of hadronic yields in AA than NN

■ Nuclear Suppression Factor:
$$R_{AA} = \frac{(\text{Yield})^{AA}}{N_{\text{coll}} (\text{Yield})^{NN}}$$

- Quenching depends on amount of energy-loss suffered in the medium

D-Meson @ LHC 2.76A TeV

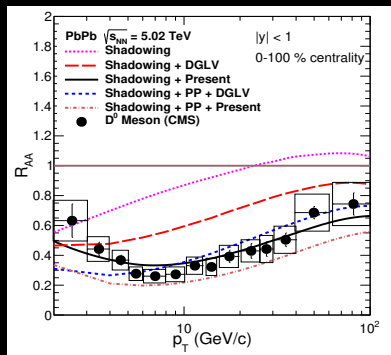
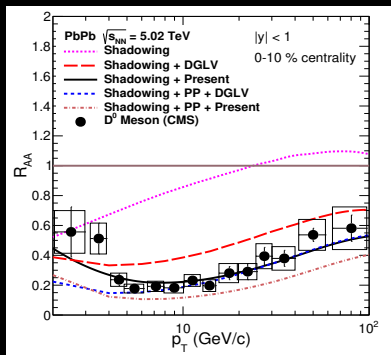


DATA: CMS Collaboration , CMS-PAS-HIN-16-011.

Theory: Abir, Jamil, Mustafa, Srivastava, PLB715 (2012) 183

Saraswat, Shukla, Singh, NPA961(2017)169

D-Meson @ LHC 5.02A TeV



☛ Data: CMS Collaboration, CMS-PAS-HIN-16-001.

☛ Theory: Abir, Jamil, Mustafa, Srivastava, PLB715 (2012) 183

☛ Saraswat, Shukla, Singh, NPA961(2017)169

Collisional Energy Gain [Chakraborty, Mustafa, Thoma, PRC75 (2007) 064908]

❌ The collisional energy loss does not take into account :


- 👉 the field fluctuation in the plasma
- 👉 the particle recoil in collisions

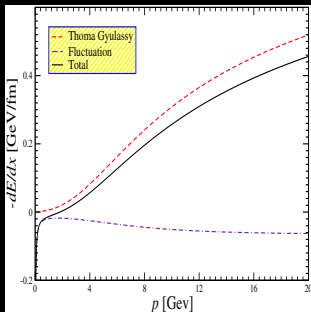
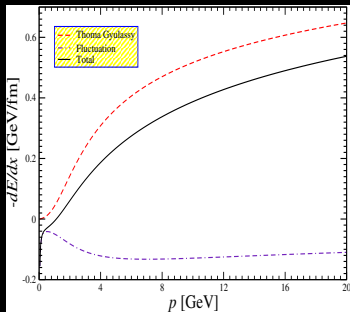
❌ One needs to take into account both :

👉 This leads to energy gain as

$$\begin{aligned} \left. \frac{dE}{dx} \right|_{fluc} &= \frac{C_F \alpha_s}{16\pi^3 E} \int d^3k \left[\frac{\partial}{\partial \omega} \langle w \vec{\mathcal{E}}_L^2 \rangle + \langle \vec{\mathcal{E}}_T^2 \rangle \right]_{\omega = \vec{k} \cdot \vec{v}} \\ &= 2\pi C_F \alpha_s^2 \left(1 + \frac{N_f}{6} \right) \frac{T^3}{E v^2} \ln \frac{1+v}{1-v} \ln \frac{k_{\max}}{k_{\min}} \end{aligned}$$

Collisional Energy Gain

 The field fluctuations lead to energy gain:



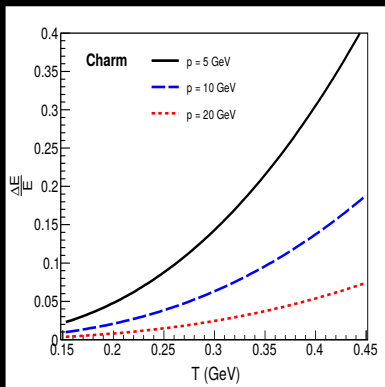
 Chakraborty, Mustafa, Thoma, PRC75 (2007) 064908

Fractional Energy Gain of Charm Quark


Coll. Gain/Energy = $\frac{\Delta E}{E}$

A Ikbal, Z Ahamed, M G Mustafa, : arXiv:1902.06989

Ikbal, Ahamed, Shukla, Mustafa, Phys .Rev. C98 (2018) 034915

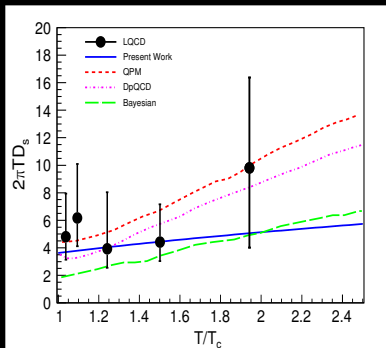


Diffusion Coefficient of Charm Quark

 Langevin Dynamics

A Ikbal, Z Ahamed, M G Mustafa, : arXiv:1902.06989


Ikbal, Ahamed, Shukla, Mustafa, PRC98 (2018) 034915



 Drag : $\gamma = -\frac{1}{p} \frac{dE}{dx}$

 FD Theorem:


Mom. Diff.: $D = 2\gamma ET$

 Einstein Relation:

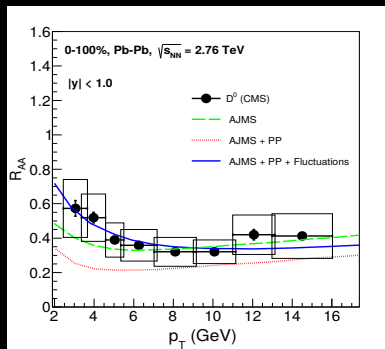
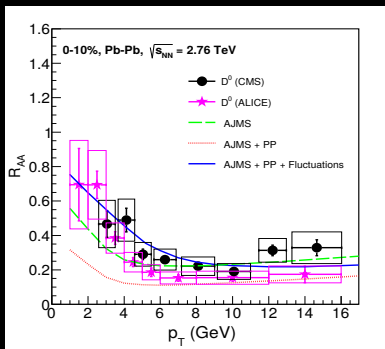
$$D_s \Big|_{p \rightarrow 0} = \frac{T}{\gamma M} = \frac{2T^2}{D}$$

 Banerjee, Datta, Gavai and Majumdar, PRD 85 014510 (2012)

Open Charm and Field Fluctuation

 Nuclear Modification factor (R_{AA}) for D Meson at 2.76 TeV

Ikbal, Ahamed, Shukla, Mustafa, *Phys. Rev. C* 98 (2018) 034915



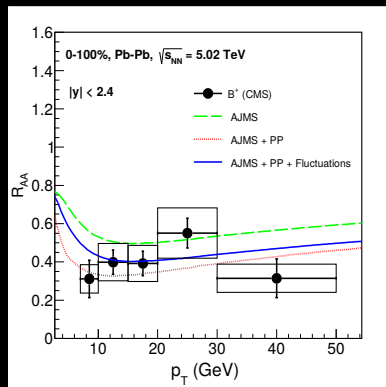
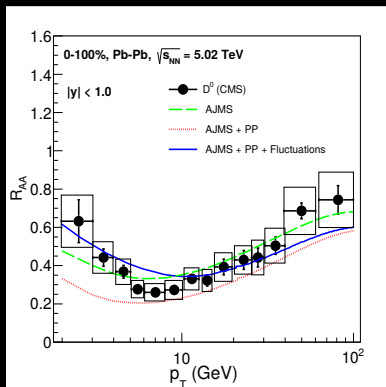
 DATA:CMS Collaboration , CMS-PAS-HIN-16-011

 ALICE Collaboration, JHEP 1603 (2016) 081, 2016

Open Charm/Beauty and Field Fluctuation

 Nuclear Modification factor (R_{AA}) for D & B-Meson at 5.02 TeV

Ikbal, Ahamed, Shukla, Mustafa, Phys .Rev. C98 (2018) 034915



 **DATA:CMS Collaboration , CMS-PAS-HIN-16-011**

Conclusion:

Discussed why are heavy quarks important

Heavy quark production in high energy HIC: X-section in pQCD

Collisional E-Loss & E-Gain

Relation to Transport Coeffs: Drag and Diffusion

Coll. E-Loss Distr. & importance of Coll. E-Loss

Radiative E-Loss

Generalised Dead Cone and radiative E-Loss

Light and heavy quark lose energy in a similar fashion

Importance Chromoelectric Field Fluctuations & Energy Gain

Nuclear suppression R_{AA} for various heavy quarked meson

Both D and B Meson R_{AA} @ LHC for 2.76ATeV and 5.02ATeV in Pb+Pb

Collaborators:

- Raktim Abir (AMU, Aligarh)
- Purnendu Chakraborty (Basirhat College, W. Bengal)
- Umme Jamil (D.R. College, Golaghata, Assam)
- Dipali Pal (USA)
- Asik Ikbal (VECC, Kolkata)
- D. K. Srivastava (VECC, Kolkata)
- Zubayer Ahmed (VECC, Kolkata)
- Prashant Shukla (BARC, Mumbai)

- Markus H. Thoma (Giessen Univ., Germany)
- Carsten Greiner (Goethe Univ. Frankfurt, Germany)
- Jan Uphoff (Goethe Univ., Frankfurt, Germany)
- M. Martinez (FIAS, Frankfurt, Germany)

THANK YOU